The pole structures of the X(1840)/X(1835) and the X(1880)

Niu Pengyu, Zhang Zhenyu, Li Yiyao, Wang Qian, Zhao Qiang

South China Normal University, Institute of Quantum Matter

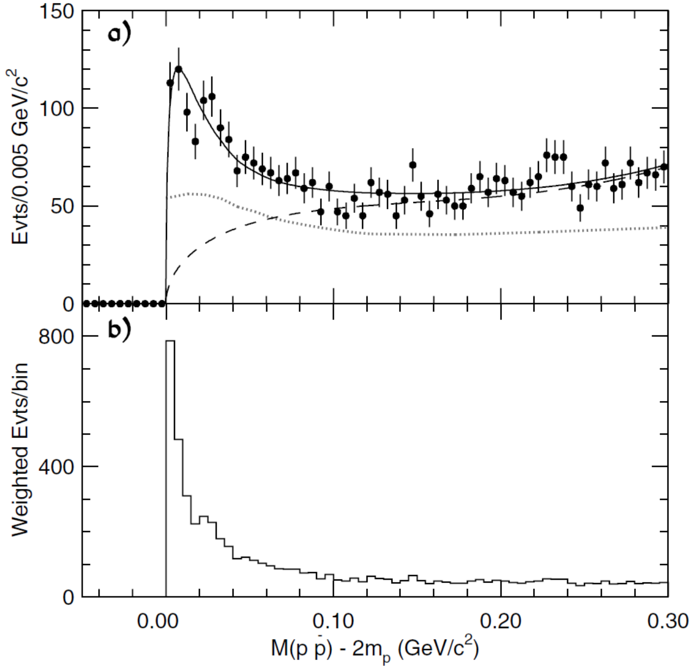
arXiv: 2408.14876

The 23rd International Conference on Few-Body Problems in Physics

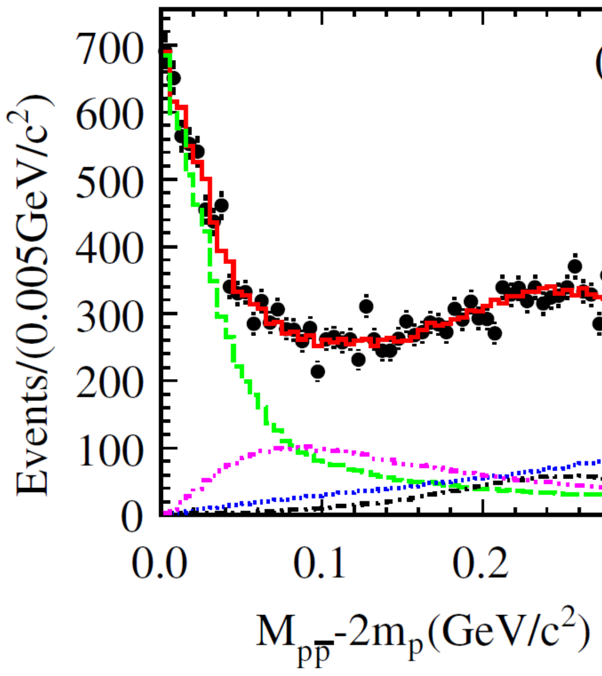
IHEP, Bei Jing, 2024.09.22-2024.09.27

Outline

The $p\bar{p}$ mass-threshold enhancement in the $J/\psi \to \gamma p\bar{p}$



BES, Phys. Rev. Lett. 91, 022001 (2003)



BESIII, Phys. Rev. Lett. 108, 112003 (2012)

- A narrow enhancement was observed near $2m_p$ for the first time.
- Using the S-wave fit:
  $M = 1859^{+3}_{-10}(\text{stat})^{+5}_{-25}(\text{syst})$ MeV
  $\Gamma < 30$ MeV

- The quantum number: 0^{-+}
- With the inclusion of Julich-FSI effects:
  $M = 1832^{+19}_{-5}(\text{stat})^{+18}_{-17}(\text{syst}) \pm 19$ MeV
  $\Gamma < 76$ MeV

The $p\bar{p}$ bound state?

Searching $p\bar{p}$ mass-threshold enhancement in various processes

processes	(Mass width) (MeV)		J^{PC}
$J/\psi \to \gamma p\bar{p}$	(1859,<30)[1]; (1861,<38)[2]; (1837,≈ 0)[3]; (1832,<76)[5]; (1831,<153)[6]	X(1835) superposition of two states?	0^{-+} [4]
$J/\psi \to \pi^0 p\bar{p}$	No similar structure [1,5]		-
$J/\psi \to \omega p\bar{p}$	No similar structure [7,8]		-
$J/\psi \to \eta p\bar{p}$	No similar structure [9]		-
$\psi(2S) \to X p\bar{p}$ $(X = \gamma, \pi^0, \eta)$	No similar structure [3]		-
$e^+e^- \to p\bar{p}$	near-threshold enhancement is observed [10]		-
$B \to X p\bar{p}$ $(X = \pi^+, K, K_s, D^{(*)})$	near-threshold enhancement is observed [11-17]		-

Refs.

[1]: BES, Phys. Rev. Lett. 91, 022001

[2]: BESIII, Chin. Phys. C 34, 421

[3]: CLEO, Phys. Rev. D 82, 092002

[4]: BES, Phys. Rev. D 80, 052004

[5]: BESIII, Phys. Rev. Lett. 108, 112003

[6]: BES, Phys.Rev.Lett. 95, 262001

[7]: BES, Eur.Phys.J.C 53, 15

[8]: BESIII, Phys. Rev. D 87,, 112004

[9]: BES, Phys.Lett.B 510, 75

[10]: BaBar, Phys. Rev. D 73, 012005

[11]: Belle, Phys. Rev. Lett. 88, 181803

[12]: Belle, Phys. Lett. B 659, 80

[13]: BaBar, Phys.Rev.D 72, 051101

[14]: Belle, Phys.Lett.B 617, 141-149

[15]: Belle, Phys.Rev.Lett. 89, 151802

[16]: BaBar, Phys.Rev.D 85, 092017

[17]: CLEO, Phys.Rev.D 82, 092002

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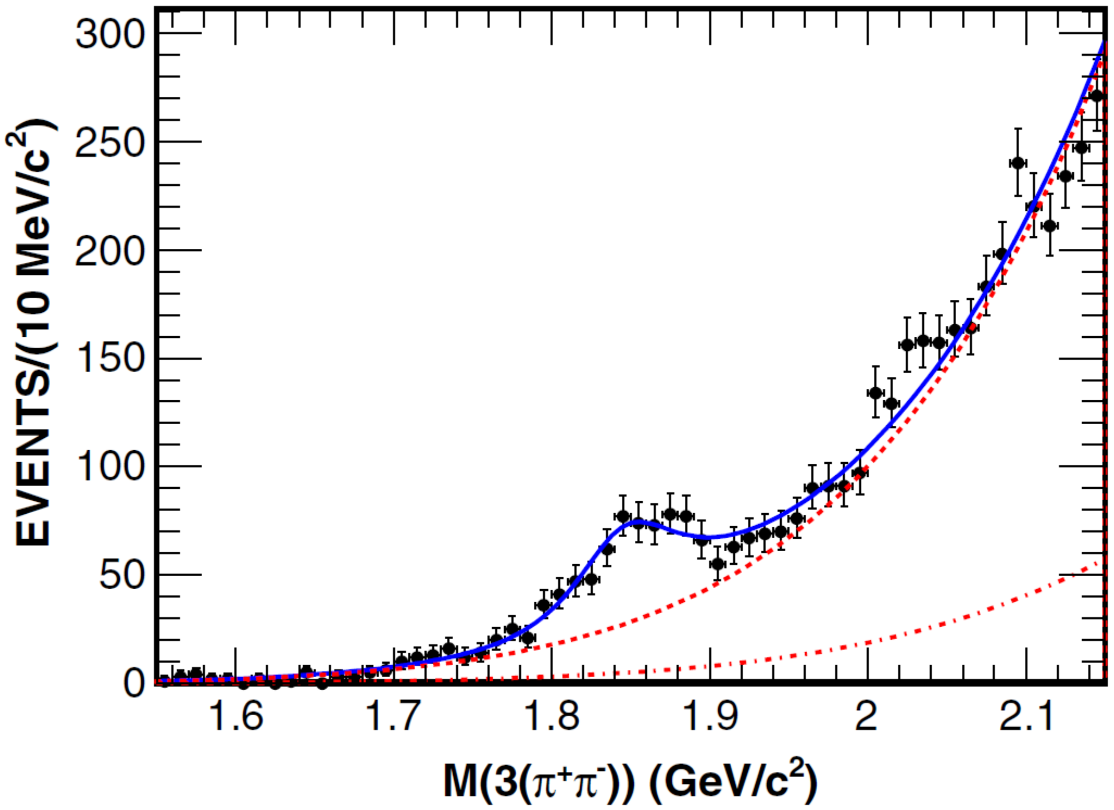
Observation of a structure at 1.84 GeV in $J/\psi \to \gamma 3(\pi^+\pi^-)$



BESIII, Phys. Rev. D 88, 091502(R) (2013)

- A structure at 1.84 GeV is observed in the $3(\pi^+\pi^-)$ mass spectrum in $J/\psi \to \gamma 3(\pi^+\pi^-)$ with a statistical significance of 7.6 σ.
- Modified Breit-Wigner function:

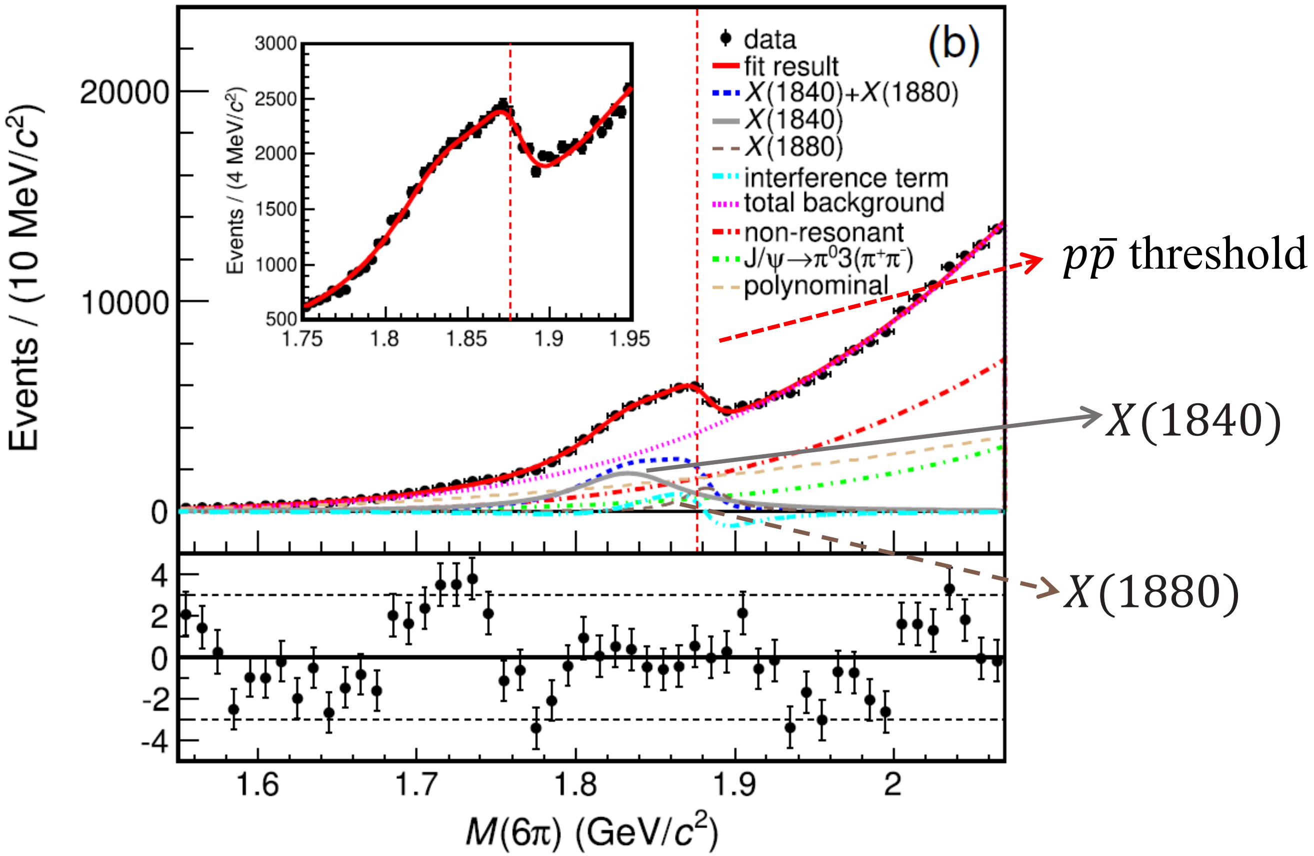
$M = 1842.2 \pm 4.2^{+7.6}_{-2.6}$ MeV

$\Gamma = 83 \pm 14 \pm 11$ MeV

$\boldsymbol{X(1840)}$

1. Background

The observation of $X(1880)$ in the process of $J/\psi \to \gamma 3(\pi^+\pi^-)$



BESIII, Phys. Rev. Lett. 132, 151901 (2024)

A simple resonant structure (Breit-Wigner) fails to describe the $M(6\pi)$ spectrum ($\chi^2/N_{\rm dof}$=399.0/45).

Model-I: one resonant structure ($\chi^2/N_{\rm dof}$=317.9/44):

$$A = \left| \frac{1}{M^2 - s - i\sum_j g_j^2 \rho_j} \right|^2$$

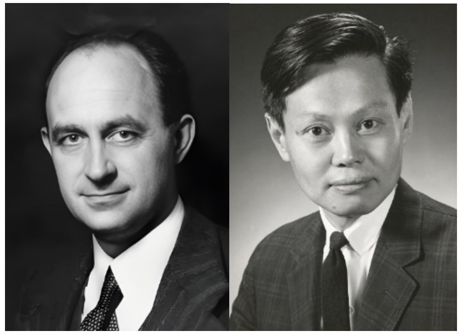
Model-II: two resonant structures ($\chi^2/N_{\rm dof}$=155.6/41):

$$A = \left| \frac{1}{M_1^2 - s - iM_1\Gamma_1} + \beta \frac{1}{M_2^2 - s - iM_2\Gamma_2} \right|^2$$

Particle	$X(1840)$	$X(1880)$
J^{PC}	0^{-+}	0^{-+}
Mass (MeV)	$1832.5 \pm 3.1 \pm 2.5$	$1882.1 \pm 1.7 \pm 0.7$
Width (MeV)	$80.7 \pm 5.2 \pm 7.7$	$30.7 \pm 5.5 \pm 2.4$

This result further supports the existence of a $p\bar{p}$ bound state.

Theoretical works



Are Mesons Elementary Particles?

E. Fermi and C. N. Yang*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

The hypothesis that π-mesons may be composite particles formed by the association of a nucleon with an anti-nucleon is discussed. From an extremely crude discussion of the model it appears that such a meson would have in most respects properties similar to those of the meson of the Yukawa theory.

Although a great effort has been put forward, the properties of the resonances in the mass region of [1.8,1.9] GeV is still remain controversial.

- **Final states interactions effects**

  G. Y. Chen and J. P. Ma, Phys. Rev. D 83, 094029
  G. Y. Chen, H. R. Dong and J. P. Ma, Phys. Lett. B 692, 136-142
  G. Y. Chen, H. R. Dong and J. P. Ma, Phys. Rev. D 78, 054022
  X. W. Kang, J. Haidenbauer and U. G. Meißner, Phys. Rev. D 91, 074003
  A. I. Milstein and S. G. Salnikov, Nucl. Phys. A 966, 54-63
  S. G. Salnikov and A. I. Milstein, Nucl. Phys. B 1002, 116539
  …

- **Pseudoscalar glueball**

  B. A. Li, Phys. Rev. D 74, 034019
  N. Kochelev and D. P. Min, Phys. Rev. D 72, 097502
  N. Kochelev and D. P. Min, Phys. Lett. B 633, 283-288
  X. G. He, X. Q. Li, X. Liu and J. P. Ma, Eur. Phys. J. C 49, 731
  G. Hao, C. F. Qiao and A. L. Zhang, Phys. Lett. B 642, 53-61
  L. C. Gui, J. M. Dong, Y. Chen and Y. B. Yang, Phys. Rev. D 100, 054511
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- **Radial excitations of η'**

  T. Huang and S. L. Zhu, Phys. Rev. D 73, 014023
  J. S. Yu, Z. F. Sun, X. Liu and Q. Zhao, Phys. Rev. D 83, 114007
  L. M.Wang, Q. S. Zhou, C. Q. Pang and X. Liu, Phys. Rev. D 102, 114034
  …

- **$3\,^1S_0\ q\bar{q}$ state**

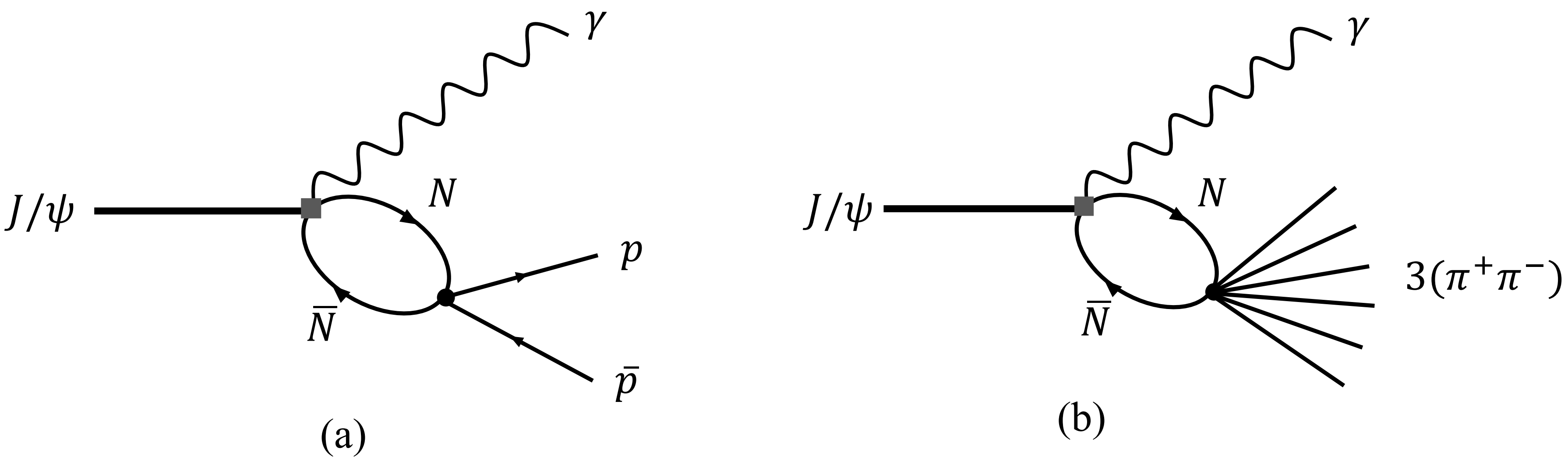
  D. M. Li and B. Ma, Phys. Rev. D 77, 074004

- **Reviews**

  Y. F. Liu and X. W. Kang, Symmetry 8, 14
  B. Q. Ma, arXiv:2406.19180
  …

- …

Can we describe $J/\psi \to \gamma p\bar{p}$ and $J/\psi \to \gamma 3(\pi^+\pi^-)$ simultaneously ?



- The $N\bar{N}$ and $3(\pi^+\pi^-)$ channels are considered dynamically and non-dynamically, respectively.
- We would not include the contribution of resonances but focus on the rescattering effect of the dynamic $N\bar{N}$ channel.
- Whether the dynamic generated states in the $N\bar{N}$ channel can describe the experimental data.

The $N\bar{N}$ interaction and the transition matrix $T_{\mathrm{N\bar{N}\to N\bar{N}}}$

The leading and next-leading order $N\bar{N}$ contact interactions in the chiral effective field theory:

$$L_{N\bar{N}}^{(0)} = C_s + C_T\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \,,$$

$$L_{N\bar{N}}^{(2)} = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{k}^2 + \left(C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{k}^2\right)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{i}{2} C_5 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot(\boldsymbol{q}\times\boldsymbol{k}) + C_6(\boldsymbol{q}\cdot\boldsymbol{\sigma}_1)(\boldsymbol{q}\cdot\boldsymbol{\sigma}_2) + C_7(\boldsymbol{k}\cdot\boldsymbol{\sigma}_1)(\boldsymbol{k}\cdot\boldsymbol{\sigma}_2).$$

The partial wave interaction:

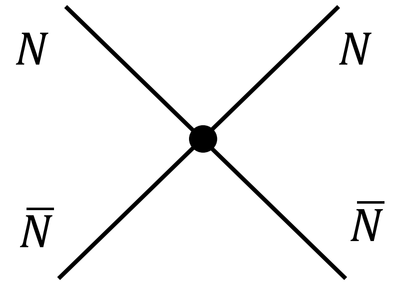
$$L(^1S_0) = C'_{01} + C'_{02}(\boldsymbol{p}+\boldsymbol{p}')$$

$$L(^3S_0) = C'_{11} + C'_{12}(\boldsymbol{p}+\boldsymbol{p}')$$

The $N\bar{N}$ interaction in the isospin basis

$$V^{I}_{^1S_0} = C_{I1} + C_{I2}\left(p^2 + P'^2\right), \mathrm{I} = 0,1$$

C_{I1}: complex number to take into account the annihilation contribution



Contact term

The isospin basis and the particle basis:

$$|I=1, I_3=0\rangle = \frac{1}{\sqrt{2}}(|p\bar{p}\rangle + |n\bar{n}\rangle)$$

$$|I=0, I_3=0\rangle = \frac{1}{\sqrt{2}}(|p\bar{p}\rangle - |n\bar{n}\rangle)$$

$$V_{p\bar{p}\to p\bar{p}} = V_{n\bar{n}\to n\bar{n}} = \frac{1}{2}(V^{I=1}_{^1S_0} + V^{I=0}_{^1S_0})$$

$$V_{p\bar{p}\leftrightarrow n\bar{n}} = \frac{1}{2}(V^{I=1}_{^1S_0} + V^{I=0}_{^1S_0})$$

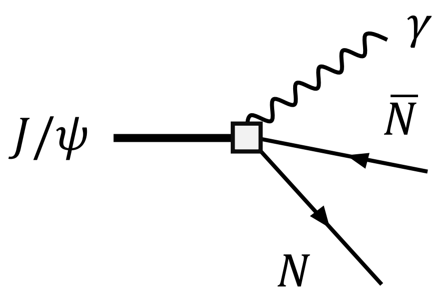
The transition matrix of $N\bar{N} \to N\bar{N}$ is obtained with the LS equation:

$$T_{\mathrm{N\bar{N}\to N\bar{N}}}(p,p') = V_{\mathrm{N\bar{N}\to N\bar{N}}}(p,p') + \int \frac{d^3\boldsymbol{p}''}{(2\pi)^3} V_{\mathrm{N\bar{N}\to N\bar{N}}}(p,\boldsymbol{p}'')G^+(E,\boldsymbol{p}'')T_{\mathrm{N\bar{N}\to N\bar{N}}}(\boldsymbol{p}'',p')$$

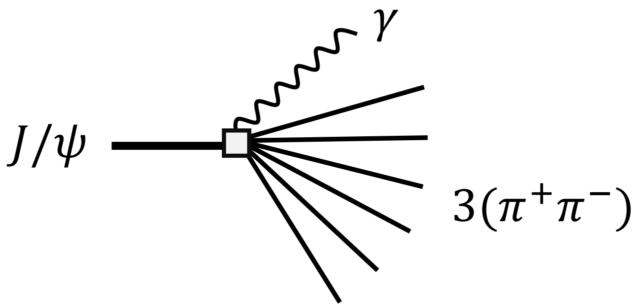
[1]. X. W. Kang, J. Haidenbauer and U. G. Meisner, JHEP 02, 113

[2]. L. Y. Dai, J. Haidenbauer and U. G. Meißner, JHEP 07, 078

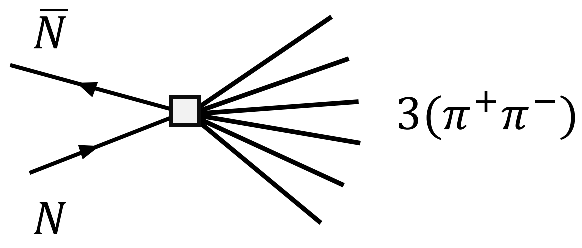
The production amplitudes



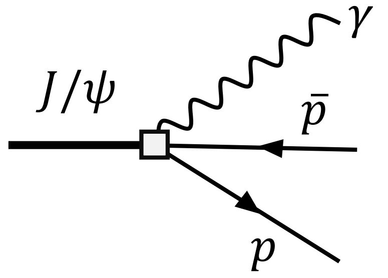
$$A^0_{J/\psi\to\gamma N\bar{N}} = C_{J/\psi\to\gamma N\bar{N}}$$



$$A^0_{J/\psi\to\gamma 3(\pi^+\pi^-)} = C_{J/\psi\to\gamma 3(\pi^+\pi^-)}$$



$$A_{N\bar{N}\to 3(\pi^+\pi^-)} = C_{N\bar{N}\to\gamma 6\pi}$$



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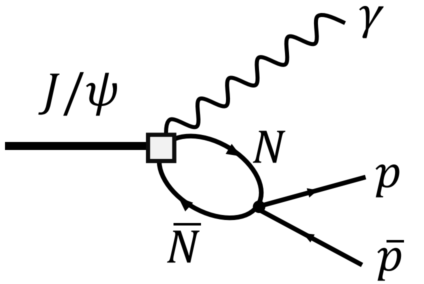

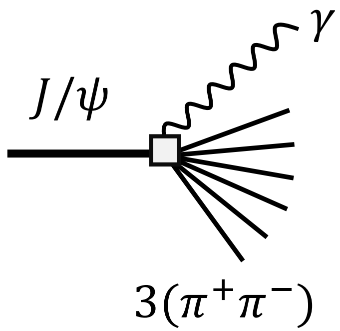

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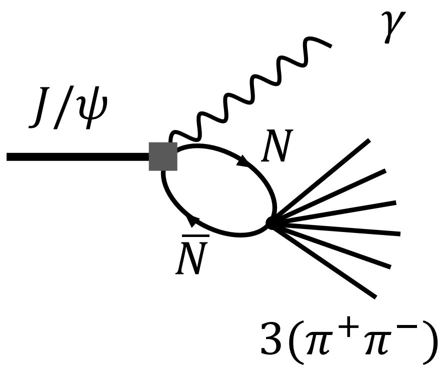


The physical decay amplitudes:

$$\widetilde{M}_{J/\psi\to\gamma p\bar{p}} = 8\pi^2\sqrt{E_{J/\psi}E_\gamma E_p E_{\bar{p}}}\, M_{J/\psi\to\gamma p\bar{p}},$$

$$\widetilde{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)} = 32\pi^{7/2}\sqrt{E_{J/\psi}E_\gamma E_2 E_3 E_4}\, M_{J/\psi\to 3(\pi^+\pi^-)},$$

where

$$M_{J/\psi\to\gamma N\bar{N}} = A^0_{J/\psi\to\gamma N\bar{N}} + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} A^0_{J/\psi\to\gamma N\bar{N}} \cdot G^+ \cdot T_{N\bar{N}\to N\bar{N}},$$

$$M_{J/\psi\to\gamma 3(\pi^+\pi^-)} = A^0_{J/\psi\to\gamma 3(\pi^+\pi^-)} + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} M_{J/\psi\to\gamma N\bar{N}} \cdot G^+ \cdot A_{N\bar{N}\to 3(\pi^+\pi^-)}.$$

[1]. Q. H. Yang, D. Guo and L. Y. Dai, Phys. Rev. D 107, no.3, 034030
[2]. L. Y. Dai, J. Haidenbauer and U. G. Meißner, Phys. Rev. D 98, 014005

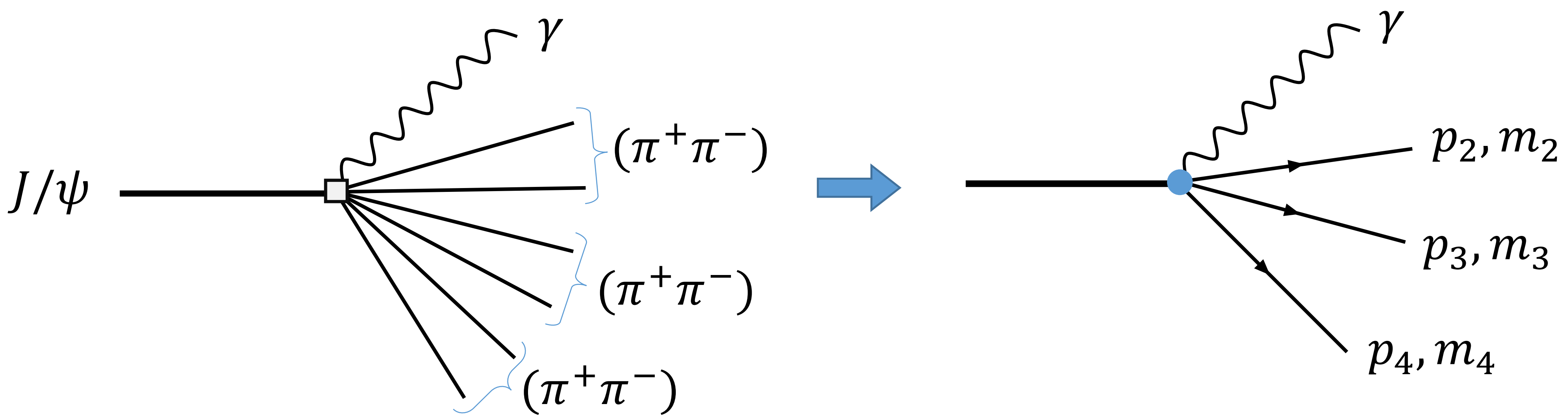
The fitting functions

1. Data 2003: $\text{Events}(m_{p\bar{p}}) = \text{fac1} \times \frac{d\Gamma_{J/\psi\to\gamma p\bar{p}}}{dm_{p\bar{p}}}$,
2. Data 2012: $\text{Events}(m_{p\bar{p}}) = \text{fac2} \times \frac{d\Gamma_{J/\psi\to\gamma p\bar{p}}}{dm_{p\bar{p}}}$,
3. Data 2024: $\text{Events}(m_{6\pi}) = \text{fac3} \times \frac{d\Gamma_{J/\psi\to\gamma 3(\pi^+\pi^-)}}{dm_{6\pi}} + \frac{d\Gamma^{\text{bg}}_{J/\psi\to\gamma 3(\pi^+\pi^-)}}{dm_{6\pi}}$.

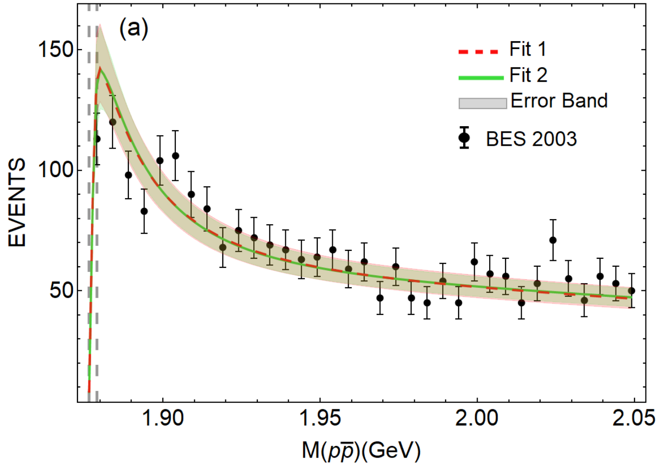
Background for the $J/\psi \to \gamma 3(\pi^+\pi^-)$:

$$bg_{J/\psi\to\gamma 3(\pi^+\pi^-)} = a + bQ + cQ^2.$$

Phase space: $d\Phi_7 \to d\Phi_4$:



The fitting results



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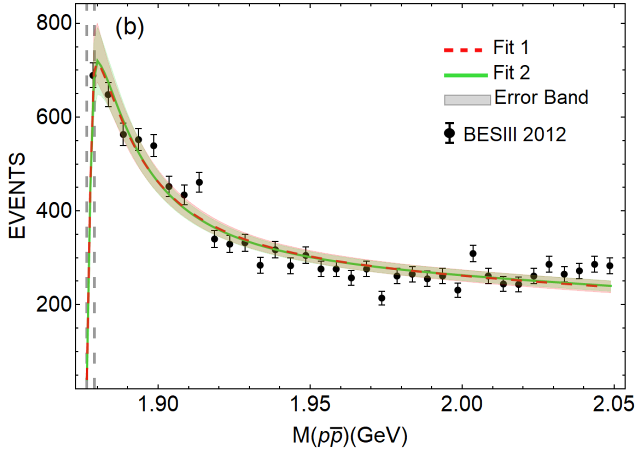

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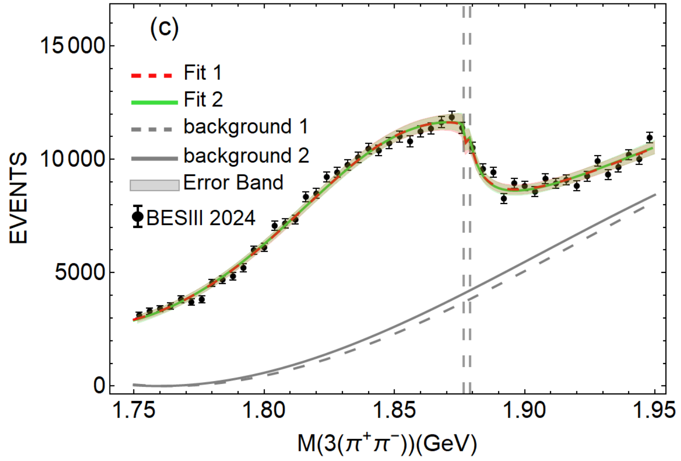


BESIII, Phys. Rev. Lett. 132, no.15, 151901 (2024)

Parameters	Solution-I	Solution-II
C_{01} (GeV^{-2})	$(87.41 \pm 0.32) - (6.08 \pm 0.10)i$	$(97.24 \pm 0.66) - (-6.72 \pm 0.16)i$
C_{02} (GeV^{-4})	-102.36 ± 0.21	-109.26 ± 0.47
C_{11} (GeV^{-2})	$(-33.47 \pm 0.31) + (0.56 \pm 0.76)i$	$(153.79 \pm 3.98) + (12.56 \pm 8.26)i$
C_{12} (GeV^{-4})	57.56 ± 16.44	247.88 ± 2.01
$C_{J/\psi\to\gamma p\bar{p}}$ (GeV^{-2})	168.06 ± 9.08	-88.22 ± 23.26
$C_{J/\psi\to\gamma n\bar{n}}$ (GeV^{-2})	-372.26 ± 8.23	428.47 ± 8.39
$C_{p\bar{p}\to 6\pi}$ (GeV$^{-7/2}$)	-330.66 ± 10.63	-111.08 ± 2.12
$C_{n\bar{n}\to 6\pi}$ (GeV$^{-7/2}$)	263.04 ± 10.75	81.65 ± 5.83
fac1 (10^{-3})	1.42 ± 0.10	0.85 ± 0.048
fac2 (10^{-3})	7.19 ± 0.50	4.32 ± 0.23
fac3 (10^{-3})	0.71 ± 0.04	3.14 ± 0.11
a (GeV^{-1})	$-2.58 \times 10^7 \pm 1.61 \times 10^4$	$-2.84 \times 10^7 \pm 9.47 \times 10^5$
b (GeV^{-2})	$2.54 \times 10^7 \pm 8.50 \times 10^3$	$2.84 \times 10^7 \pm 1.12 \times 10^3$
c (GeV^{-3})	$-6.17 \times 10^7 \pm 4.38 \times 10^3$	$-6.96 \times 10^6 \pm 2.62 \times 10^4$
χ^2/d.o.f	2.32(2.24)	2.33(2.31)

- Fitting program: Minuit2
- The two solutions can describe the experimental data almost equally well.

Riemann sheets of $T_{N\bar{N}\to N\bar{N}}$

Two channels

RS-I: $\mathrm{Im}\,k_1 > 0, \mathrm{Im}\,k_2 > 0,$

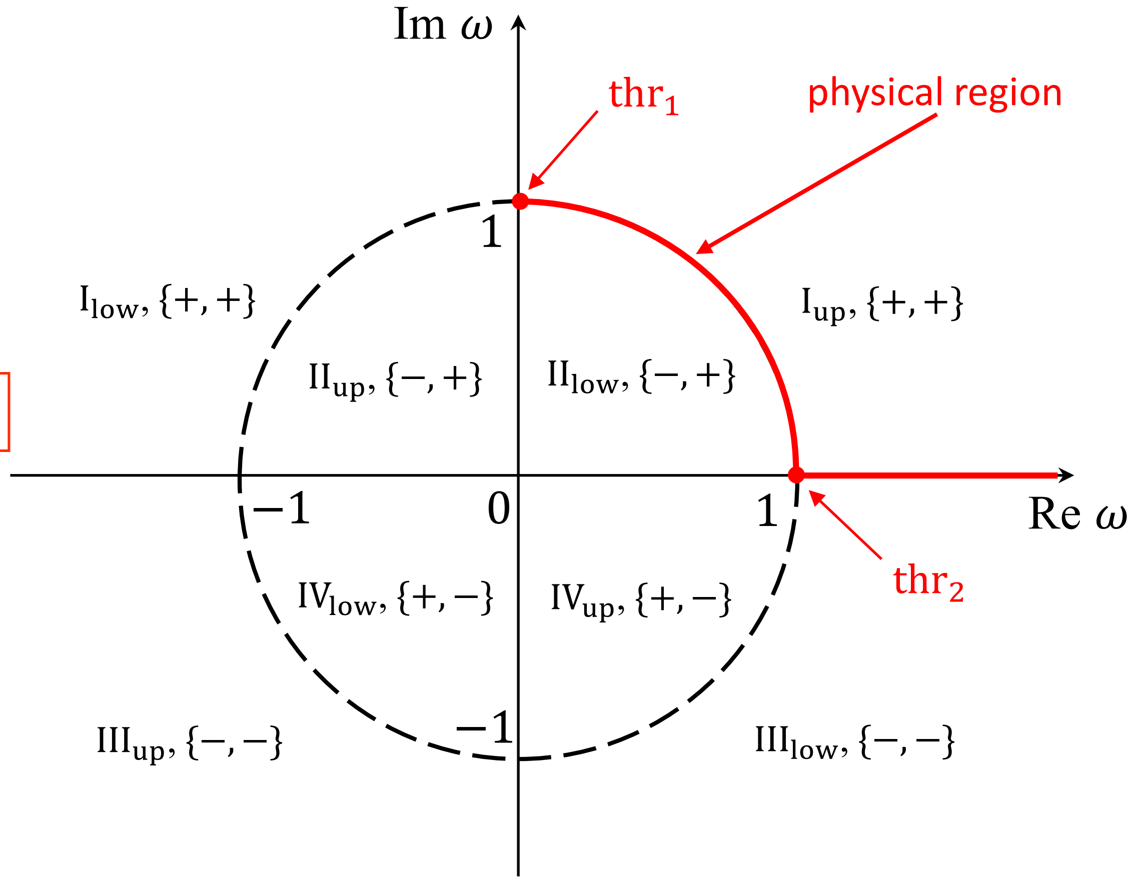
RS-II: $\mathrm{Im}\,k_1 < 0, \mathrm{Im}\,k_2 > 0,$

RS-III: $\mathrm{Im}\,k_1 < 0, \mathrm{Im}\,k_2 < 0,$

RS-IV: $\mathrm{Im}\,k_1 > 0, \mathrm{Im}\,k_2 < 0.$

$$k_1 = \sqrt{\frac{\mu_1\delta}{2}\left(\omega + \frac{1}{\omega}\right)},$$
$$k_2 = \sqrt{\frac{\mu_2\delta}{2}\left(\omega - \frac{1}{\omega}\right)}.$$

$\delta = \mathrm{thr}_1 - \mathrm{thr}_2$ =2.89 MeV



ω plane

- Bound state: pole below threshold on real axis of the first Riemann sheet of complex energy plane
- Virtual state: pole below threshold on real axis of the second Riemann sheet
- Resonance: pole in the complex plane on the second Riemann sheet

[1]. M. Kato, Annals Phys. 31, 130
[2]. V. Baru, E. Epelbaum, A. A. Filin, C. Hanhart, A. V. Nefediev and Q. Wang, Phys. Rev. D 99, 094013

3. Results

The pole positions and effective couplings

R. S.	I	IV	II	III
Solution-I (MeV)	$1851.90^{+3.02}_{-2.71} - 80.49^{+1.68}_{-1.63}i$	$1866.07^{+22.41}_{-7.20} + 86.34^{+8.65}_{-12.76}i$	$1875.46^{+20.61}_{-6.60} + 87.20^{+9.45}_{-12.92}i$	$1868.34^{+1.66}_{-0.55} - 0.82^{+1.17}_{-1.41}i$
$(g_{p\bar{p}}, g_{n\bar{n}})(\mathrm{GeV}^{-1/2})$	$(1.61, 1.64)$	$(3.42, 2.22)$	$(2.16, 3.42)$	$(0.98, 0.94)$
(g_1, g_0) $(\mathrm{GeV}^{-1/2})$	$(0.017, 2.37)$	$(3.32, 2.36)$	$(3.31, 2.32)$	$(1.35, 0.032)$
Solution-II (MeV)	$1852.90^{+3.57}_{-3.31} - 82.35^{+2.45}_{-2.80}i$	$1860.31^{+8.77}_{-8.34} + 61.23^{+6.18}_{-5.38}i$	$1855.23^{+8.49}_{-8.07} + 62.01^{+6.02}_{-5.29}i$	$1868.92^{+1.13}_{-1.48} - 2.58^{+1.70}_{-1.86}i$
$(g_{p\bar{p}}, g_{n\bar{n}})$ $(\mathrm{GeV}^{-1/2})$	$(1.85, 1.88)$	$(2.82, 1.80)$	$(1.76, 2.82)$	$(0.94, 0.90)$
(g_1, g_0) $(\mathrm{GeV}^{-1/2})$	$(0.013, 2.89)$	$(2.68, 1.99)$	$(2.67, 1.98)$	$(1.30, 0.033)$

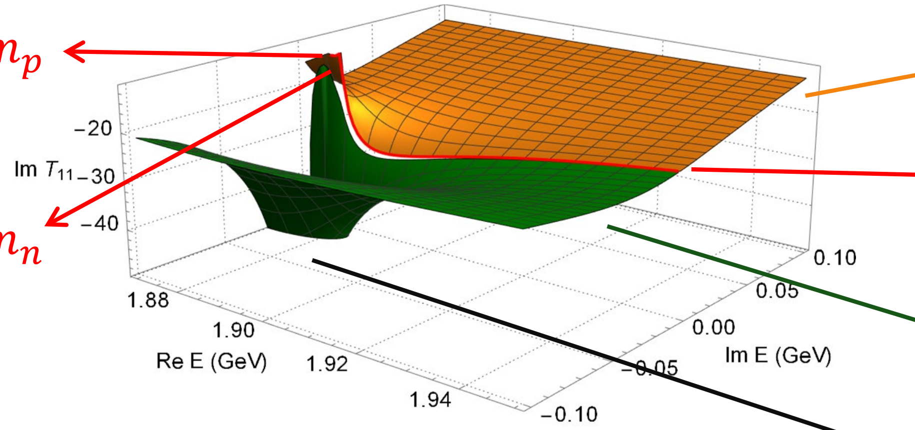
$2m_p$=1876.54 MeV

- Similar results
- All the poles positions are below the lowest threshold. Since the LECs C_{01} and C_{11} are set to be complex numbers.
- Only the pole on the physical sheet strongly couples to isospin singlet.

$T_{p\bar{p}\to p\bar{p}}$ on the complex energy plane

$\mathrm{thr}_1 = 2m_p$

$\mathrm{thr}_2 = 2m_n$



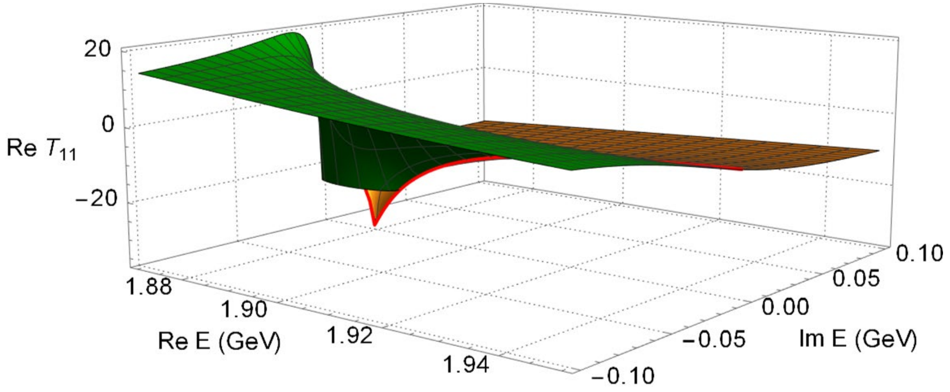
The upper half plane of the physical sheet

The physical range of the energy

The lower half plane of the third sheet

The pole position (1868.34 − 0.82i MeV)

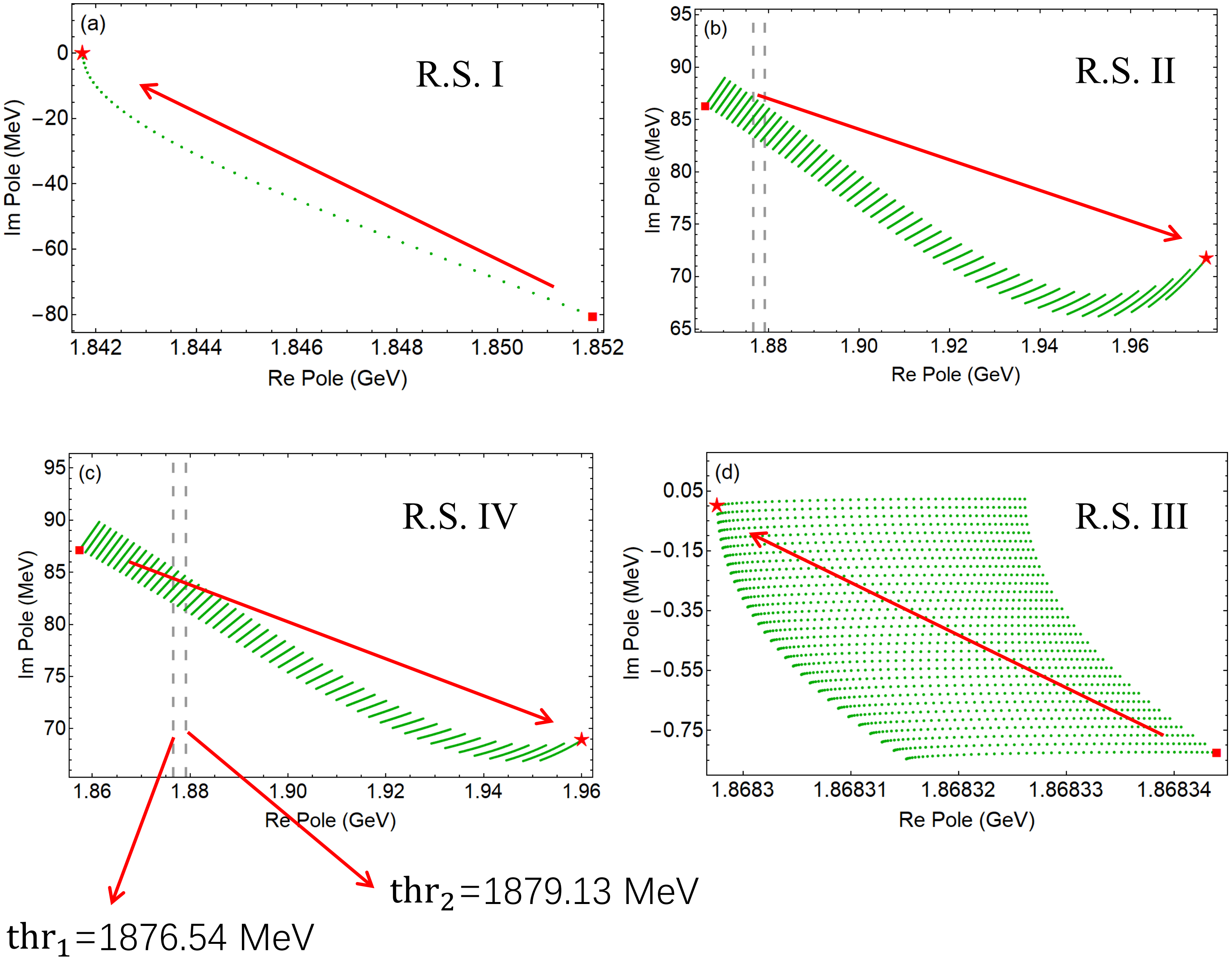
The imaginary parts $T_{p\bar{p}\to p\bar{p}}$



The real parts of $T_{p\bar{p}\to p\bar{p}}$

- We can clearly see how the pole on the third sheet results in the enhancement near $N\bar{N}$ threshold.
- Blow and far from the thr_1, only the pole on the first R.S. has significant affects on the line shape.

Poles trajectories on the complex energy plane

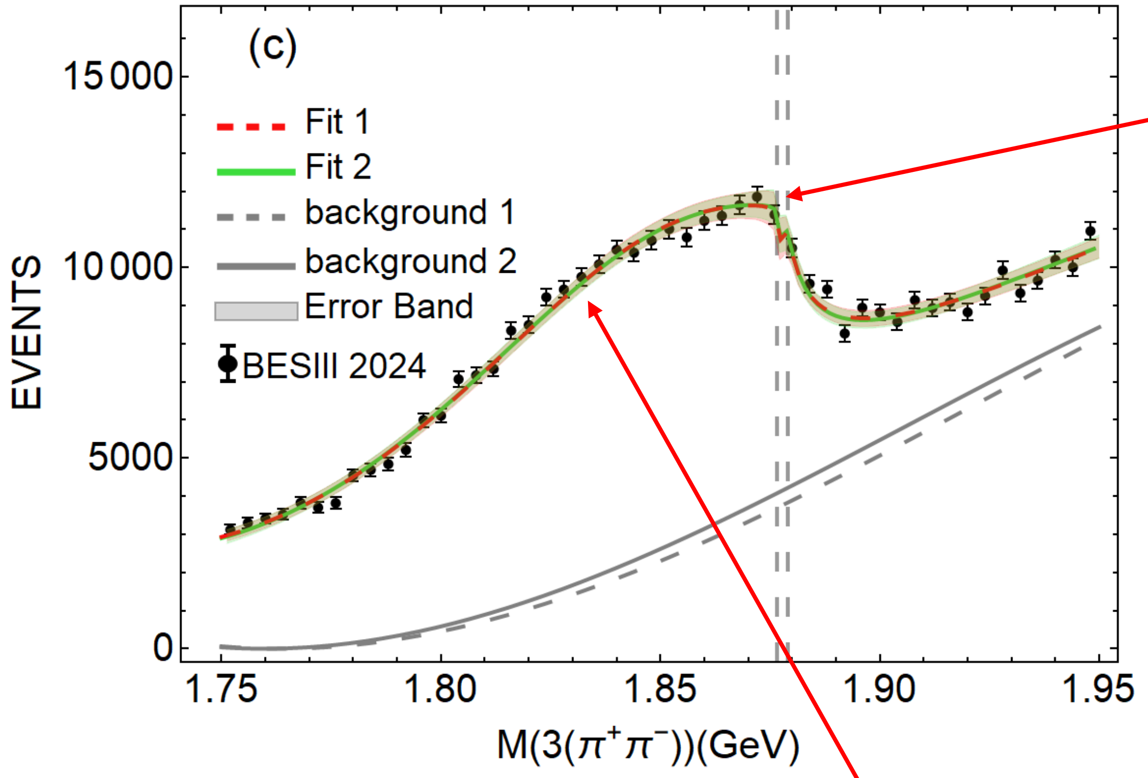


- The poles trajectories on the complex energy plane with the variation of Im C_{01} and Im C_{11} from the fitted values to zero in Solution-I.
- For Solution-II, the behaviors are similar to those of Solution-I.

> - The pole on the first R.S. can be treated as a bound state.
> - The pole one the third R.S. can be treated as a virtual state.
> - The other two poles are resonance.

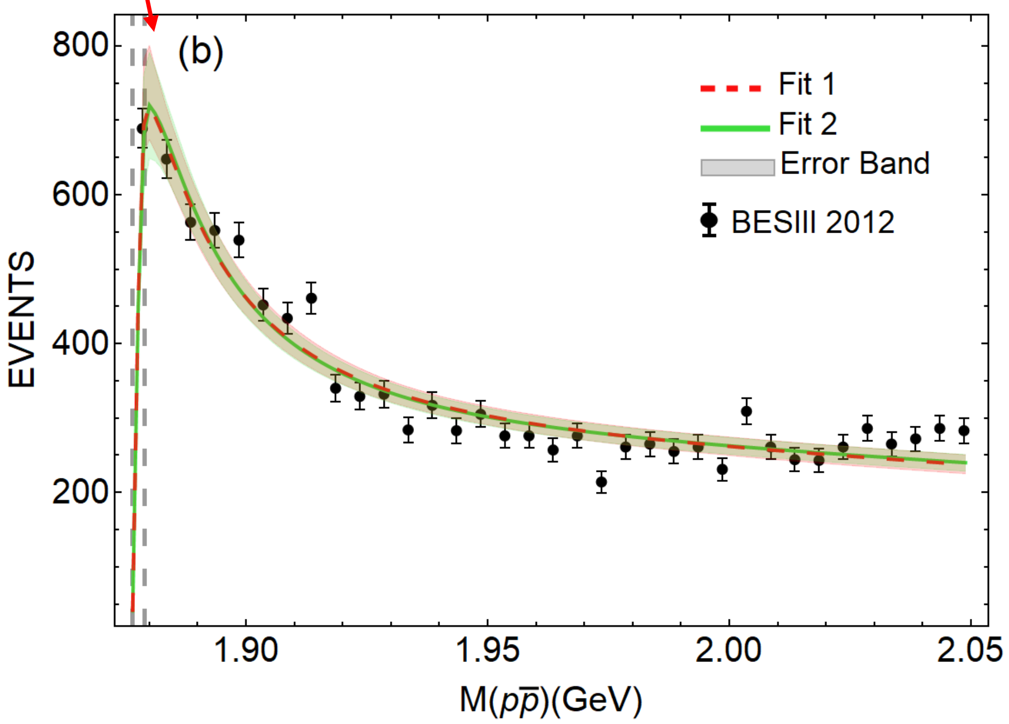
3. Results

The pole structures of the X(1840) and the X(1880)



$1868.34^{+1.66}_{-0.55} - 0.82^{+1.17}_{-1.41} i$

On the third R. S. of $T_{N\bar{N}}$(virtual state)



$1851.90^{+3.02}_{-2.71} - 80.49^{+1.68}_{-1.63} i$

On the first R. S. of $T_{N\bar{N}}$ (bound state)

The scattering length, effective range and compositeness

Effective-Range-Expansion:

$$T^{-1}(k) = -\frac{\mu}{2\pi}\left[-\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik + \mathcal{O}(k^4)\right]$$

$\Rightarrow$

scattering length: $a_0 = \frac{2\pi}{\mu} T(k)|_{k\to 0}$,

effective range: $r_0 = -\frac{2\pi}{\mu^2} \mathrm{Re}\left[\frac{dT^{-1}(E)}{dE}\right]$,

correction from isospin breaking effect:

$$r_0' = r_0 + \sqrt{\frac{1}{2\,\mu\,\Delta}}$$

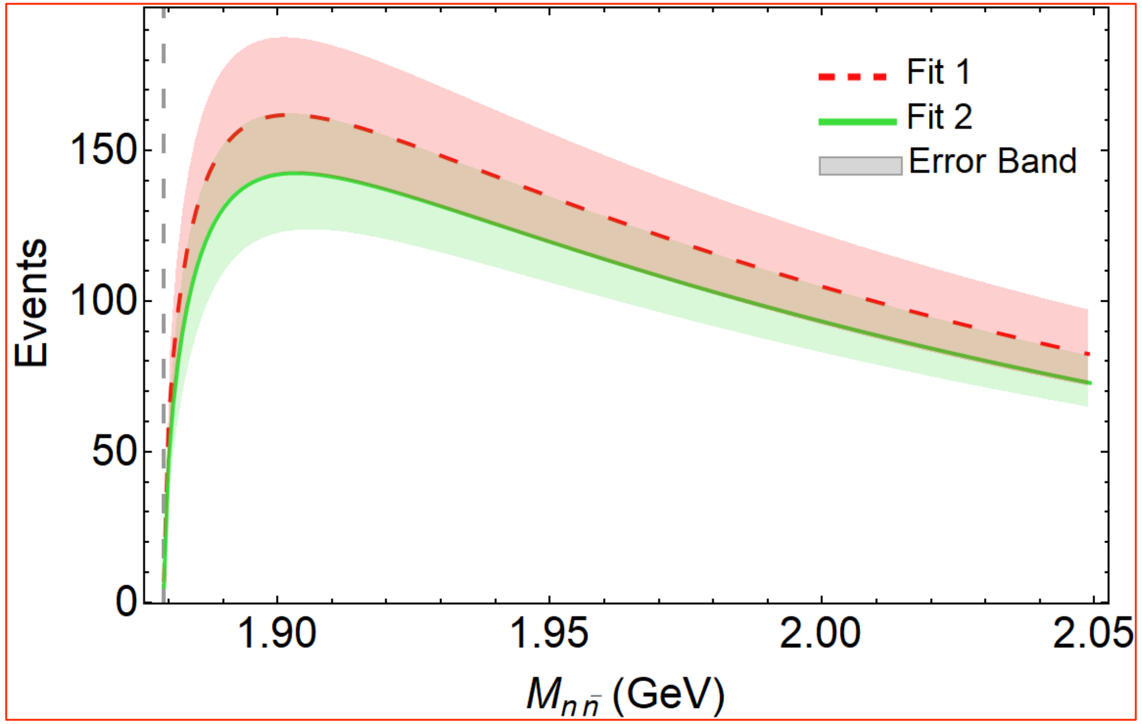
Compositeness: $\bar{X} = \frac{1}{\sqrt{1+2\left|\frac{r_0'}{\mathrm{Re}[a_0]}\right|}}$

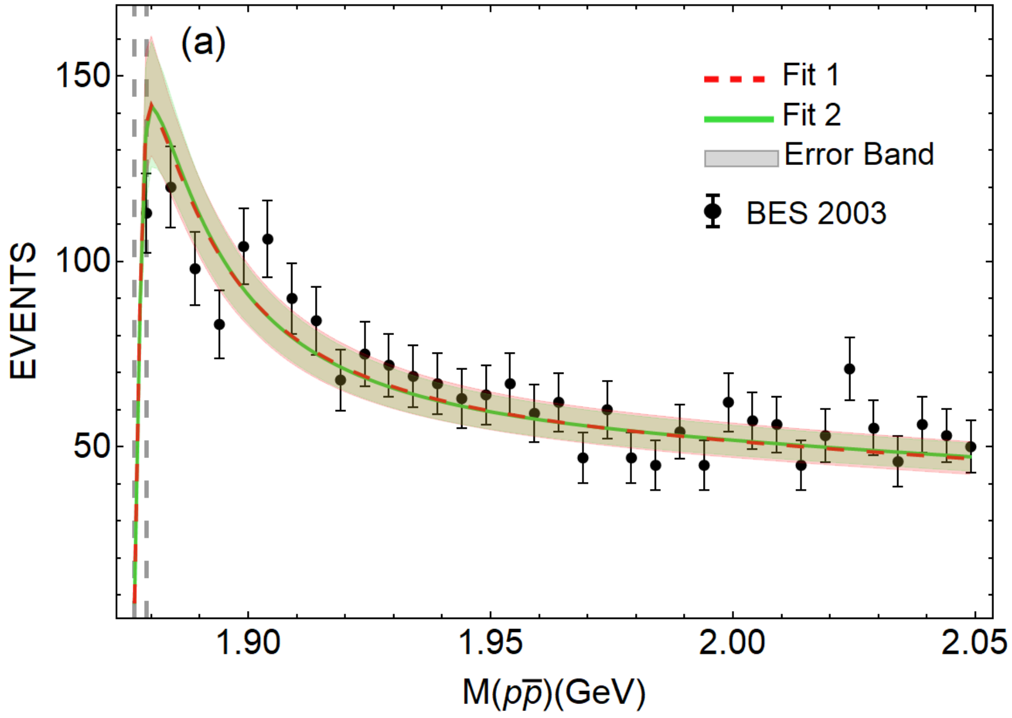
	r_0 (fm)	r_0' (fm)	a_0(fm)	$\bar{X}$
Solution-I	-2.30	1.70	-65.46-31.94i	0.98
Solution-II	1.50	5.50	-56.52-27.16i	0.91

There exists $p\bar{p}$ dynamical generated states.

[1]. V. Baru, X. K. Dong, M. L. Du, A. Filin, F. K. Guo,C. Hanhart, A. Nefediev, J. Nieves and Q. Wang, Phys. Lett. B 833,137290
[2]. M. L. Du, V. Baru, X. K. Dong, A. Filin, F. K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang, Phys. Rev. D 105, 014024

The predicted $n\bar{n}$ line shapes for the two solutions





- We can see a clear threshold enhancement in the $n\bar{n}$ channel, but not as significant as that in the $p\bar{p}$ channel.
- This result can be used to compare with the measurement of the future experiment.

4. Summary

- The $p\bar{p}$ threshold enhancement are both observed in $J/\psi \to \gamma p\bar{p}$ and $J/\psi \to \gamma 3(\pi^+\pi^-)$.
- By constructing the $N\bar{N}$ interaction respecting chiral symmetry, we extract the pole positions by fitting the experimental data.
- There exists $p\bar{p}$ dynamical generated states:
  - RS-I: $m = 1852$ MeV, $\Gamma = 160$ MeV (bound state)
  - RS-III: $m = 1868$ MeV, Γ =2 MeV (virtual state)
- We can see a clear threshold enhancement in the $n\bar{n}$ channel.

Thank you !