Entanglement suppression and low-energy scattering of heavy mesons

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Based on Phys. Rev. D 110, 014001 (2024) by Tao-Ran Hu, Su Chen and Feng-Kun Guo.

BACKGROUND

- Where does symmetry come from?
- Can symmetry be derived from even more fundamental principles?

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Accidental, emergent symmetries
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[Low:2021ufv] Ian Low and Thomas Mehen. **Symmetry from entanglement suppression.** Phys. Rev. D 104, 074014 (2021).

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- Can symmetry be derived from even more fundamental principles?

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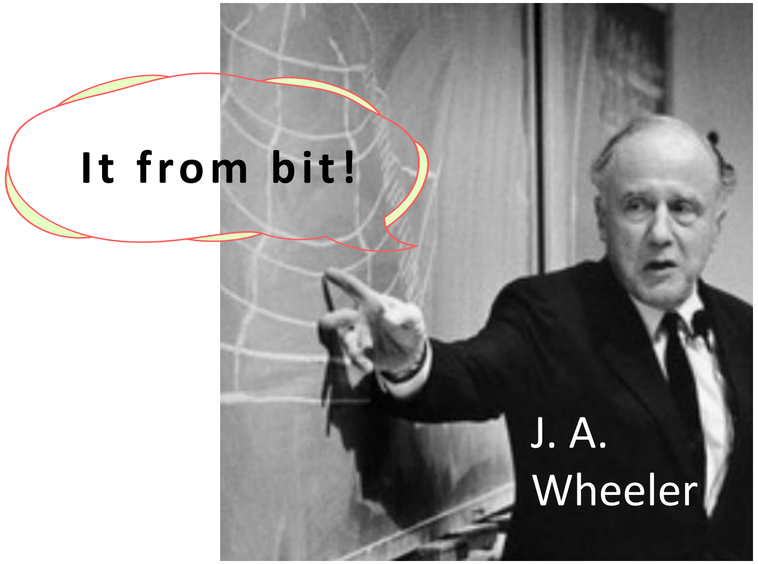
⬆

Applying tools in quantum information,
in particular the concept of entanglement



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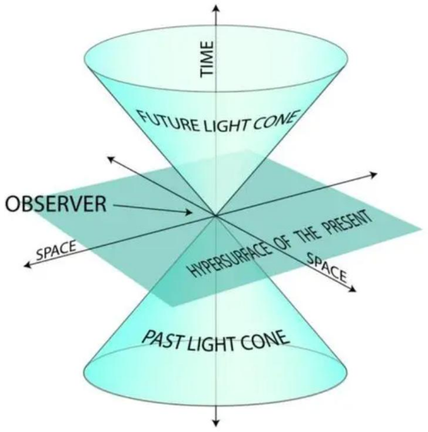
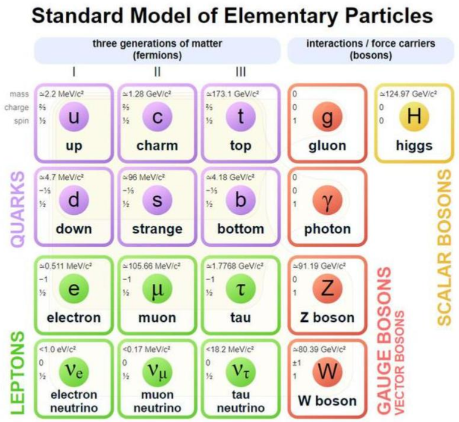
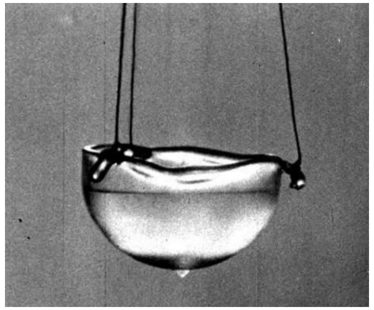
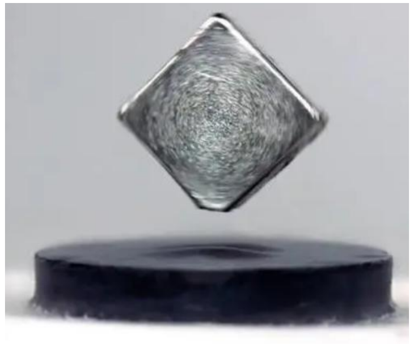
EXAMPLES

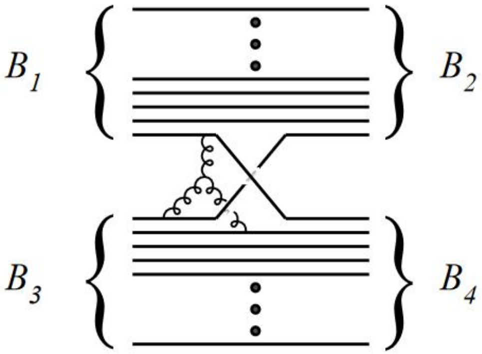




• O(1,3)	• NO
• U(1)×SU(2)×SU(3)	• NO
• Scale invariance during a second-order phase transition	• YES
• Superfluidity / superconductivity U(1)	• YES
• Wignar’s SU(4) in NN scattering	• YES
• All symmetries that large-N limit predicts	• YES







[Kaplan:1995yg]

MAIN IDEA

- In quantum information science, one can quantify the entanglement of an operator.
- S-Matrix is also an operator, so it is conceivable to assign an entanglement measure $E(S)$ to it.
- It is natural to consider extreme cases, such as when the entanglement of the S-matrix reaches its maximum or minimum value.

$$E(S) = 0\,?$$

FIRST APLICATION IN LOW-ENERGY QCD

- [Beane:2018oxh] reveals some intriguing clues:
- After requiring $E(S)=0$, the inherent $\mathrm{SU}(2)_{\mathrm{isospin}} \times \mathrm{SU}(2)_{\mathrm{spin}}$ symmetry of nucleon-nucleon scattering enlarges to Wigner's $\mathrm{SU}(4)$ symmetry.

$$(p\uparrow, p\downarrow, n\uparrow, n\downarrow) \text{ as fundamental representation}$$

- Leading to the conjecture:
- Entanglement suppression could be the origin of emergent symmetries.

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$$H \to G \supset H$$

DEFINITIONS (FOR BIPARTITE SYSTEM)

- Density matrix: $\rho = |\psi\rangle \langle\psi|$
- Reduced density matrix: $\rho_1 = \mathrm{Tr}_2(\rho)$ (tracing over subsystem 2)
- Entanglement measure (entropy): $E(|\psi\rangle) = 1 - \mathrm{Tr}_1[\rho_1^2]$
- Entanglement power: $E(U) = \overline{E\,(U\,|\psi\rangle)}, \quad |\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$
- The entanglement measure quantifies the entanglement in a quantum state $|\psi\rangle$, while the entanglement power measures the ability of a quantum-mechanical operator U to generate entanglement by averaging over all tensor-product states it acts on.

S-MATRIX

- In general, a low-energy scattering event can entangle position, spin, and flavor quantum numbers, and it is therefore natural to assign an entanglement power to the S-matrix for such a scattering process.

$$S = \sum_{I,J} P_{IJ} e^{2i\delta_{IJ}}$$

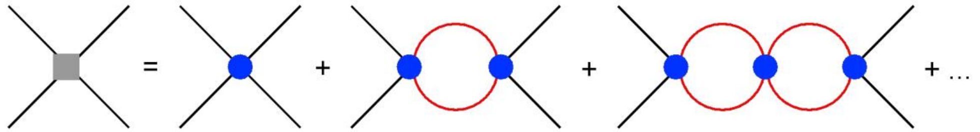
- We define P_{IJ} the projection operators onto subspaces of definite isospin and total angular momentum, and δ_{IJ} the corresponding phase shift.

$$S^\dagger S = 1$$

MATCHING TO AMPLITUDES

- Feynman rules for $\mathcal{L} = \sum_{i=1,2} \phi_i^\dagger \left(i\partial_0 - m_i + \frac{\nabla^2}{2m_i} \right) \phi_i - C_0\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2$

$$i\mathcal{M} = -iC_0 + (-iC_0)\,(iG)\,(-iC_0) + \cdots$$



$$= -\frac{i}{C_0^{-1} - G}$$

$$S = e^{2i\delta} = 1 + \frac{\mu p}{\pi} i\mathcal{M}$$

$$G = i \int \frac{dk^0 d^3\boldsymbol{k}}{(2\pi)^4} \left[\left(k^0 - \frac{\boldsymbol{k}^2}{2m_1} + i\epsilon \right) \left(E - k^0 - \frac{\boldsymbol{k}^2}{2m_2} + i\epsilon \right) \right]^{-1}$$

$$= -\frac{\mu}{2\pi} (\Lambda + ip), \ \text{PDS}$$

$$\mathcal{M} = \frac{2\pi}{\mu} \frac{1}{p \cot\delta - ip}$$

MATCHING TO AMPLITUDES

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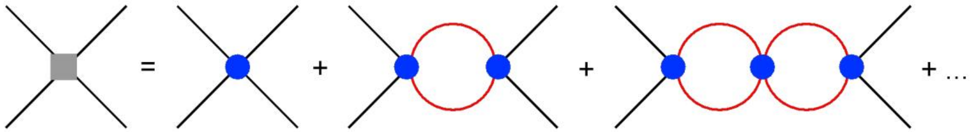

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$$= -\frac{\mu}{2\pi}(\Lambda + ip), \text{ PDS}$$

$$\mathcal{M} = \frac{2\pi}{\mu} \frac{1}{p\cot\delta - ip}$$

$$\boxed{p\cot\delta = -\frac{2\pi}{\mu C_0} - \Lambda}$$

ENTANGLEMENT SUPPRESSION

- Entanglement power of S-matrix: $E(S)$
- Require $E(S)$ to vanish: $E(S) = 0$

⬇

- Constraints for phase shifts δ

⬇

- Constraints for amplitudes $\mathscr{M}$

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- Constraints for Lagrangian $\mathcal{L}$

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Based on the assumption:
The physical region produces minimal entanglement.

A RECENT WORK

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

- [Liu:2022grf] Qiaofeng Liu, Ian Low, and Thomas Mehen. **Minimal entanglement and emergent symmetries in low-energy QCD.** Phys. Rev. C 107, 025204 (2023).

$$\begin{aligned} \mathcal{L}_{\mathrm{LO}}^{n_f=3} = & -c_1 \langle B_i^\dagger B_i B_j^\dagger B_j \rangle - c_2 \langle B_i^\dagger B_j B_j^\dagger B_i \rangle - c_3 \langle B_i^\dagger B_j^\dagger B_i B_j \rangle \\ & - c_4 \langle B_i^\dagger B_j^\dagger B_j B_i \rangle - c_5 \langle B_i^\dagger B_i \rangle \langle B_j^\dagger B_j \rangle \\ & - c_6 \langle B_i^\dagger B_j \rangle \langle B_j^\dagger B_i \rangle, \end{aligned}$$

- Entanglement suppression can be utilized to determine the relation between these six LECs.

RESULTS OF [Liu:2022grf]

Flavor subspaces	Minimal entanglement conditions
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$c_2 = -c_6 \quad \text{or} \quad c_1 + c_5 = -\frac{2\pi}{M\mu}, \quad c_2 + c_6 = \pm\frac{2\pi}{M\mu}$
$n\Sigma^-$ $p\Sigma^+$ $\Xi^-\Xi^0$	$c_1 = c_6 \quad \text{or} \quad -c_2 + c_5 = -\frac{2\pi}{M\mu}, \quad c_1 - c_6 = \pm\frac{2\pi}{M\mu}$
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$ $(\Sigma^-\Xi^0,\ \Xi^-\Sigma^0,\ \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+,\ \Xi^0\Lambda,\ \Xi^0\Sigma^0)$	$c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = c_6$ or $c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = -c_5 - \frac{2\pi}{M\mu} = c_6 \pm \frac{2\pi}{M\mu}$
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0n,\ \Lambda\Lambda)$	$c_1 = c_2 = c_3 = c_4 = c_6 = 0 \quad \text{or}$ $c_1 = c_2 = c_3 = c_4 = 0,\ c_5 = -2\pi/M\mu,\ c_6 = \pm 2\pi/M\mu$

The symmetry of the Lagrangian is determined by the relation between LECs.

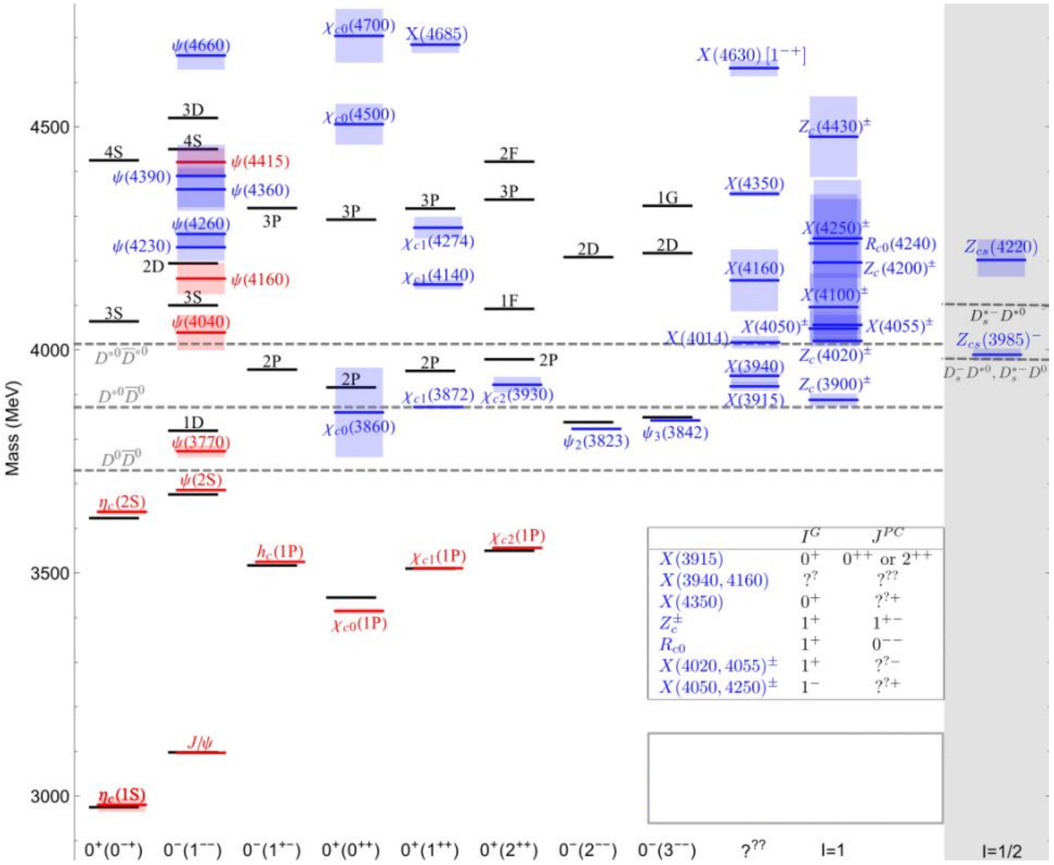
For instance, Lagrangian

$$\mathcal{L} = -(c_1 + c_5)\,\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle + c_1\langle B_i^\dagger B_j^\dagger\rangle\langle B_i B_j\rangle$$

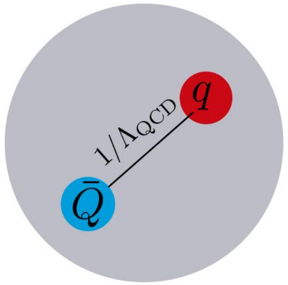
shows SO(8) symmetry.

OUR INTEREST

- Extend the study of entanglement suppression to heavy systems?



Heavy quark Q and light quark q



	(D,D^*)	(D_0^*,D_1)	(D_1,D_2^*)	Λ_c	(Σ_c,Σ_c^*)	$(\eta_c,J/\psi)$	$(h_c,\chi_{c0},\chi_{c1},\chi_{c2})$
$J^{P(C)}$	$(0^-,1^-)$	$(0^+,1^+)$	$(1^+,2^+)$	$\frac{1}{2}^+$	$\left(\frac{1}{2}^+,\frac{3}{2}^+\right)$	$(0^{-+},1^{--})$	$(1^{+-},0^{++},1^{++},2^{++})$
s_Q	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$(0,1)$	$(0,1,1,1)$
s_ℓ	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	0	1	0	1

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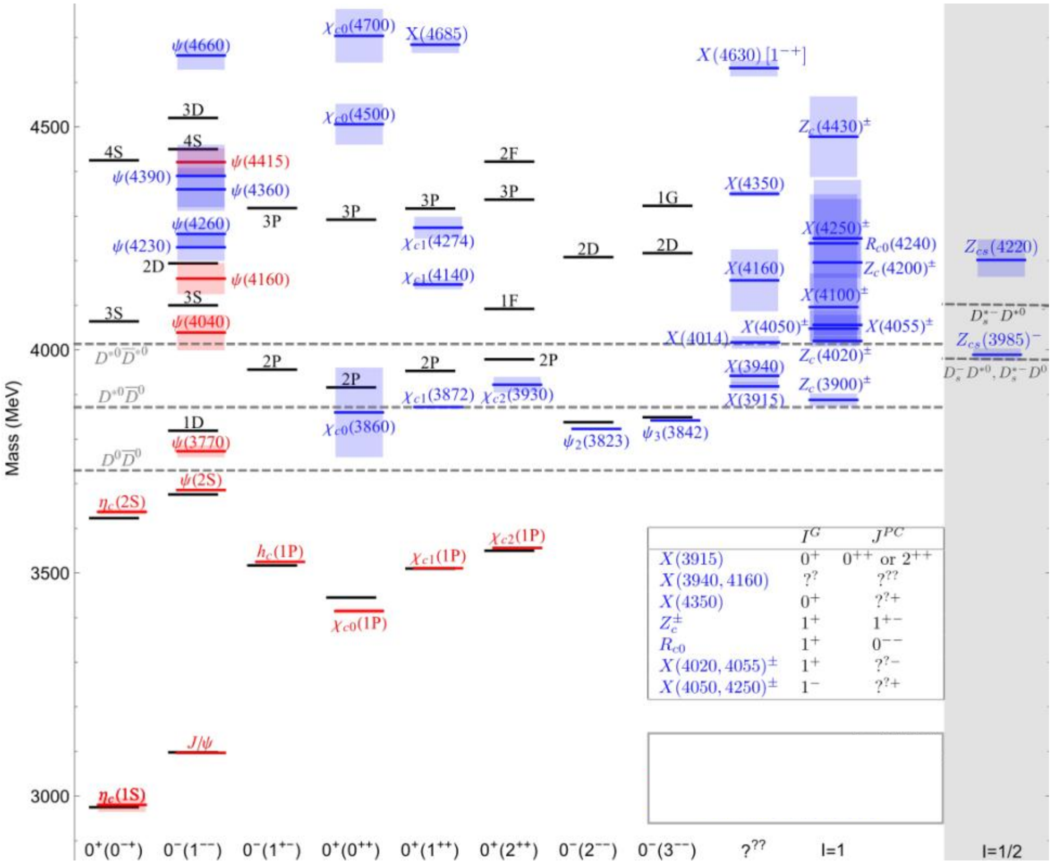

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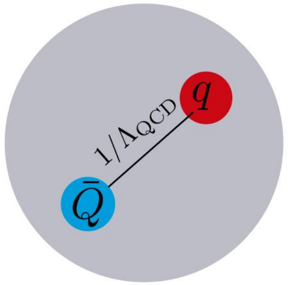

Numerous intriguing near-threshold structures!

	(D, D^*)	(D_0^*, D_1)	(D_1, D_2^*)	Λ_c	(Σ_c, Σ_c^*)	$(\eta_c, J/\psi)$	$(h_c, \chi_{c0}, \chi_{c1}, \chi_{c2})$
$J^{P(C)}$	$(0^-, 1^-)$	$(0^+, 1^+)$	$(1^+, 2^+)$	$\frac{1}{2}^+$	$\left(\frac{1}{2}^+, \frac{3}{2}^+\right)$	$(0^{-+}, 1^{--})$	$(1^{+-}, 0^{++}, 1^{++}, 2^{++})$
s_Q	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$(0, 1)$	$(0, 1, 1, 1)$
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Heavy quark Q and light quark q

Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays

S.-K. Choi *et al.* (Belle Collaboration)
Phys. Rev. Lett. **91**, 262001 – Published 23 December 2003

$$X(3872)$$

Numerous intriguing near-threshold structures!

Letter | Open access | Published: 16 June 2022

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration

Nature Physics **18**, 751–754 (2022) | Cite this article

$$T_{cc}(3875)^{+}$$

HADRONIC MOLECULES?

- The interaction between a pair of ground-state heavy mesons near threshold is closely related to formation of hadronic molecular states.

- Non-relativistic approximation
- Lowest partial wave (S-wave) dominance
- Constant contact potential at leading order

HEAVY-QUARK SPIN SYMMETRY

- Mass of heavy quark $m_Q \gg \Lambda_{\mathrm{QCD}}$
- Expanding $\Lambda_{\mathrm{QCD}}/m_Q$ yields heavy-quark effective theory.
- Leading order: heavy-quark limit $m_Q \to \infty$

- Spin-dependent interaction $\propto \dfrac{\vec{\sigma} \cdot \vec{B}^a}{m_Q} \xrightarrow{m_Q \to \infty} 0$

- Heavy-quark spin decouples, thus heavy-quark spin symmetry (HQSS).
- Light quark total angular momentum becomes good quantum number.

IMPACT OF HQSS

- Consider S-wave interaction of a pair of S-wave heavy mesons.
- Under heavy-quark limit:
- The interaction has only four independent LECs:

$$\langle \frac{1}{2}, \frac{1}{2}, s_l \left| \hat{\mathcal{H}}_I \right| \frac{1}{2}, \frac{1}{2}, s_l \rangle$$

- $s_l = 0, 1$ denotes light quark total angular momentum.
- $I = 0, 1$ denotes total isospin.

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Will entanglement suppression enlarge HQSS?

HEAVY MESON SCATTERING

$$H_a = P_a + \boldsymbol{P}_a^* \cdot \boldsymbol{\sigma},$$

$$\bar{H}^a = \bar{P}^a + \bar{\boldsymbol{P}}^{*a} \cdot \boldsymbol{\sigma}$$

- We consider $D^{(*)}D^{(*)}$ and $D^{(*)}\bar{D}^{(*)}$ systems.

$$\begin{aligned}\mathcal{L}_{HH} = &-\frac{D_{00}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b H^{b\dagger}H_a\right] - \frac{D_{01}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m H^{b\dagger}H_a\sigma^m\right] \\ &-\frac{D_{10}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b H^{c\dagger}H_d\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b - \frac{D_{11}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m H^{c\dagger}H_d\sigma^m\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b.\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{H\bar{H}} = &-\frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\right]\left(F_A\delta_a^b\delta_c^d + F_A^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right) \\ &+\frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\sigma^m\right]\left(F_B\delta_a^b\delta_c^d + F_B^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right)\end{aligned}$$

[Du:2021zzh] Meng-Lin Du, Vadim Baru, Xiang-Kun Dong, Arseniy Filin, Feng-Kun Guo, Christoph Hanhart, Alexey Nefediev, Juan Nieves, and Qian Wang. **Coupled-channel approach to Tcc+ including three-body effects.** Phys. Rev. D 105, 014024 (2022).

[Ji:2022uie] Teng Ji, Xiang-Kun Dong, Miguel Albaladejo, Meng-Lin Du, Feng-Kun Guo, and Juan Nieves. **Establishing the heavy quark spin and light flavor molecular multiplets of the X(3872), Zc(3900), and X(3960).** Phys. Rev. D 106, 094002 (2022).

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$$-\frac{D_{10}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b H^{c\dagger}H_d\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b - \frac{D_{11}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m H^{c\dagger}H_d\sigma^m\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b.$$

$$\mathcal{L}_{H\bar{H}} = -\frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\right]\left(F_A\delta_a^b\delta_c^d + F_A^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right)$$
$$+\frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\sigma^m\right]\left(F_B\delta_a^b\delta_c^d + F_B^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right)$$

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ISOSPIN COMBINATIONS

$$|DD, I=1\rangle = \begin{cases} D^+D^+ \\ -\frac{1}{\sqrt{2}}(D^+D^0 + D^0D^+) \\ D^0D^0 \end{cases};$$

$$|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),$$

$$|D^*D, I=1\rangle = \begin{cases} D^{*+}D^+ \\ -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+) \\ D^{*0}D^0 \end{cases};$$

$$|D^*D^*, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^{*0} - D^{*0}D^{*+}),$$

$$|D^*D^*, I=1\rangle = \begin{cases} D^{*+}D^{*+} \\ -\frac{1}{\sqrt{2}}(D^{*+}D^{*0} + D^{*0}D^{*+}) \\ D^{*0}D^{*0} \end{cases};$$

$$|D\bar{D}, I=0\rangle = -\frac{1}{\sqrt{2}}(D^+\bar{D}^+ + D^0\bar{D}^0),$$

$$|D\bar{D}, I=1\rangle = \begin{cases} -D^+\bar{D}^0 \\ -\frac{1}{\sqrt{2}}(D^+\bar{D}^+ - D^0\bar{D}^0) \\ D^0\bar{D}^+ \end{cases};$$

$$|D^*\bar{D}^*, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}\bar{D}^{*+} + D^{*0}\bar{D}^{*0}),$$

$$|D^*\bar{D}^*, I=1\rangle = \begin{cases} -D^{*+}\bar{D}^{*0} \\ -\frac{1}{\sqrt{2}}(D^{*+}\bar{D}^{*+} - D^{*0}\bar{D}^{*0}) \\ D^{*0}\bar{D}^{*+} \end{cases}.$$

Electrically neutral $D^{(*)}\bar{D}^{(*)}$ combinations should have definite C parities:

$$|\frac{1}{\sqrt{2}}\left(D\bar{D}^* \pm \bar{D}D^*\right), I=0\rangle = \begin{cases} -\frac{1}{2}(D^+\bar{D}^{*+} + D^0\bar{D}^{*0} + \bar{D}^+D^{*+} + \bar{D}^0D^{*0})[1^{+-}] \\ -\frac{1}{2}(D^+\bar{D}^{*+} + D^0\bar{D}^{*0} - \bar{D}^+D^{*+} - \bar{D}^0D^{*0})[1^{++}] \end{cases}$$

$$|\frac{1}{\sqrt{2}}\left(D\bar{D}^* \pm \bar{D}D^*\right), I=1\rangle = \begin{cases} -\frac{1}{2}(D^+\bar{D}^{*+} - D^0\bar{D}^{*0} + \bar{D}^+D^{*+} - \bar{D}^0D^{*0})[1^{+-}] \\ -\frac{1}{2}(D^+\bar{D}^{*+} - D^0\bar{D}^{*0} - \bar{D}^+D^{*+} + \bar{D}^0D^{*0})[1^{++}] \end{cases}$$

CONTACT POTENTIALS

$$T^{IJ=00}(DD) = \frac{1}{2}(D_{00} + 3D_{01} + D_{10} + 3D_{11}),$$

$$T^{IJ=01}(D^*D) = -2(D_{01} - 3D_{11}),$$

$$T^{IJ=11}(D^*D) = D_{00} + D_{01} + D_{10} + D_{11},$$

$$T^{IJ=01}(D^*D^*) = -2(D_{01} - 3D_{11}),$$

$$T^{IJ=10}(D^*D^*) = -\frac{1}{2}(D_{00} - 5D_{01} + D_{10} - 5D_{11}),$$

$$T^{IJ=12}(D^*D^*) = D_{00} + D_{01} + D_{10} + D_{11}.$$

$$T^{IJ=00}\left(D\bar{D}\right) = C_{0a}, \quad T^{IJ=10}\left(D\bar{D}\right) = C_{1a},$$

$$T_{-}^{IJ=01}\left(D\bar{D}^*\right) = C_{0a} - C_{0b}, \quad T_{-}^{IJ=11}\left(D\bar{D}^*\right) = C_{1a} - C_{1b},$$

$$T_{+}^{IJ=01}\left(D\bar{D}^*\right) = C_{0a} + C_{0b}, \quad T_{+}^{IJ=11}\left(D\bar{D}^*\right) = C_{1a} + C_{1b},$$

$$T^{IJ=00}\left(D^*\bar{D}^*\right) = C_{0a} - 2C_{0b}, \quad T^{IJ=10}\left(D^*\bar{D}^*\right) = C_{1a} - 2C_{1b},$$

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$$T^{IJ=02}\left(D^*\bar{D}^*\right) = C_{0a} + C_{0b}, \quad T^{IJ=12}\left(D^*\bar{D}^*\right) = C_{1a} + C_{1b},$$

CONSTRUCTION OF S-MATRIX

$$S = \sum_{I,J} P_{IJ} e^{2i\delta_{IJ}}$$

CONSTRUCTION OF S-MATRIX

$$S_{DD} = \mathcal{I}_0\, e^{2i\delta_{00}} + \mathcal{I}_1\, e^{2i\delta_{10}},$$

$$S_{D^*D} = \mathcal{I}_0\, e^{2i\delta_{01}} + \mathcal{I}_1\, e^{2i\delta_{11}},$$

$$S_{D^*D^*} = \sum_{I=0,1} \sum_{J=0,1,2} \mathcal{I}_I \otimes \mathcal{J}_J\, e^{2i\delta_{IJ*}}$$

$$S_{D\bar{D}} = \mathcal{I}_0\, e^{2i\bar{\delta}_{00}} + \mathcal{I}_1\, e^{2i\bar{\delta}_{10}},$$

$$S_{D\bar{D}^*\pm} = \mathcal{I}_0\, e^{2i\bar{\delta}_{01\pm}} + \mathcal{I}_1\, e^{2i\bar{\delta}_{11\pm}},$$

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Bose-Eienstein statistics:

$$\delta_{00} = 0 \text{ for } DD$$

$$\delta_{00*} = \delta_{02*} = \delta_{11*} = 0 \text{ for } D^*D^*$$

CONSTRUCTION OF S-MATRIX

$$S_{DD} = \mathcal{I}_0 \, e^{2i\delta_{00}} + \mathcal{I}_1 \, e^{2i\delta_{10}},$$

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Isospin space projectors

$$\mathcal{I}_0 \equiv \frac{1 - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{4}, \quad \mathcal{I}_1 \equiv \frac{3 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{4}$$

Spin space projectors

$$\mathcal{J}_0 \equiv -\frac{1}{3}\left[1 - (\boldsymbol{t}_1 \cdot \boldsymbol{t}_2)^2\right],$$

$$\mathcal{J}_1 \equiv 1 - \frac{1}{2}(\boldsymbol{t}_1 \cdot \boldsymbol{t}_2) - \frac{1}{2}(\boldsymbol{t}_1 \cdot \boldsymbol{t}_2)^2,$$

$$\mathcal{J}_2 \equiv \frac{1}{3}\left[1 + \frac{3}{2}(\boldsymbol{t}_1 \cdot \boldsymbol{t}_2) + \frac{1}{2}(\boldsymbol{t}_1 \cdot \boldsymbol{t}_2)^2\right]$$

TENSOR-PRODUCT ASSUMPTION

- Problem: Vector meson scatterings entangle both isospin and spin.
- Assumption: Entanglement being zero in a large space is equivalent to it being zero in all of its subspaces.

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For instance, applying $\mathcal{I}_{I'} \equiv \mathcal{I}_{I'} \otimes 1$ to $S = \sum_{I,J} \mathcal{I}_I \otimes \mathcal{J}_J \, e^{2i\delta_{IJ*}}$ gives

$$\mathcal{I}_{I'} S = \sum_{I,J} \mathcal{I}_{I'} \mathcal{I}_I \otimes \mathcal{J}_J e^{2i\delta_{IJ*}} = \mathcal{I}_{I'} \otimes \sum_J \mathcal{J}_J e^{2i\delta_{IJ'*}} \equiv \mathcal{I}_{I'} \otimes S_{I'}$$

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- Fix I, minimal entanglement in J space
- Fix J, minimal entanglement in I space

DENSITY MATRIX

$$\rho = |\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|, \; |\psi_{\text{out}}\rangle = S|\psi_{\text{in}}\rangle, \; |\psi_{\text{in}}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

- SU(2) case (isospin-1/2): $|\psi_i\rangle = \left(\cos\frac{\theta}{2}, e^{i\phi}\sin\frac{\theta}{2}\right)^T, \; \theta \in [0,\pi], \; \phi \in [0, 2\pi)$

- SO(3) case (spin-1): $|\psi_i\rangle = (\cos\beta\sin\alpha, e^{i\mu}\sin\beta\sin\alpha, e^{i\nu}\cos\alpha)^T,$

$$\alpha, \beta \in [0, \frac{\pi}{2}], \; \mu, \nu \in [0, 2\pi)$$

[Beane:2021zvo] Silas R. Beane, Roland C. Farrell, and Mira Varma. **Entanglement minimization in hadronic scattering with pions.** Int. J. Mod. Phys. A 36, 2150205 (2021).

ENTANGLEMENT POWER

- Entanglement power in the isospin-1/2 space:

$$E(S) = 1 - \int \frac{\mathrm{d}\Omega_1}{4\pi} \frac{\mathrm{d}\Omega_2}{4\pi} \mathrm{Tr}_1[\rho_1^2] \qquad S_J = \mathcal{I}_0 e^{2i\delta_{0J}} + \mathcal{I}_1 e^{2i\delta_{1J}}$$

- Entanglement power in the spin-1 space:

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$$\mathrm{d}\omega = \frac{2}{\pi^2} \cos\alpha \sin^3\alpha \mathrm{d}\alpha \cos\beta \sin\beta \mathrm{d}\beta \mathrm{d}\mu \mathrm{d}\nu$$

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ENTANGLEMENT POWER

- Entanglement power in the isospin-1/2 space:

$$E(S_J) = \frac{1}{6}\sin^2[2(\delta_{0J} - \delta_{1J})] \qquad S_J = \mathcal{I}_0 e^{2i\delta_{0J}} + \mathcal{I}_1 e^{2i\delta_{1J}}$$

- Entanglement power in the spin-1 space:

$$E(S_I) = \frac{1}{648}\Big\{156 - 6\cos[4(\delta_{I0*} - \delta_{I1*})] - 65\cos[2(\delta_{I0*} - \delta_{I2*})] \\ - 10\cos[4(\delta_{I0*} - \delta_{I2*})] - 60\cos[4(\delta_{I2*} - \delta_{I1*})] - 15\cos[2(\delta_{I0*} + \delta_{I2*} - 2\delta_{I1*})]\Big\}$$

$$S_I = \mathcal{J}_0 e^{2i\delta_{I0*}} + \mathcal{J}_1 e^{2i\delta_{I1*}} + \mathcal{J}_2 e^{2i\delta_{I2*}}$$

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$$|\delta_{I0*} - \delta_{I1*}| = |\delta_{I2*} - \delta_{I1*}| = 0 \quad \text{or} \quad \frac{\pi}{2}$$

HEAVY-MESON HADRONIC MOLECULES

- The near-threshold interaction between a pair of groundstate heavy mesons is closely related to formation of hadronic molecular states.
- The $X(3872)$ has been proposed as a candidate of an isoscalar $D\bar{D}^*$ hadronic molecule with quantum numbers $J^{PC}=1^{++}$.
- In 2021 the LHCb Collaboration announced the discovery of $T_{cc}(3875)^+$ with preferred quantum numbers $I(J^P)=0(1^+)$, a double-charm D^*D molecular candidate.

$$M_X - M_{D^0} - M_{\bar{D}^{*0}} = 0.00^{+0.09}_{-0.15}\,\mathrm{MeV},\;\; M_{T_{cc}^+} - M_{D^{*+}} - M_{D^0} = (-0.36 \pm 0.04)\,\mathrm{MeV}$$

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Implying that the near-threshold S-wave interactions in both channels approach the unitary limit!

INPUT: Tcc+ AS HADRONIC MOLECULE

- We assume Tcc+ to be a weakly bound isoscalar D^*D molecular state.
- Therefore we have $\delta_{01} = \frac{\pi}{2}$ for $|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$.
- This input reduces all possibilities into only two cases.

δ_{IJ}

1. All channels reach unitarity limit:

$$\delta_{01}(D^*D) = \delta_{01}(D^*D^*) = \delta_{10}(DD) = \delta_{11}(D^*D) = \delta_{10}(D^*D^*) = \delta_{12}(D^*D^*) = \frac{\pi}{2}$$

2. All isoscalar channels reach unitarity limit, while all amplitudes for isovector channels vanish:

$$\delta_{01}(D^*D) = \delta_{01}(D^*D^*) = \frac{\pi}{2},\ \delta_{10}(DD) = \delta_{11}(D^*D) = \delta_{10}(D^*D^*) = \delta_{12}(D^*D^*) = 0$$

PREDICTIONS

$\delta = 0:$
non-interacting
$\delta = \frac{\pi}{2}:$
at unitarity limit

- An additional D^*D^* zero-energy bound molecular state in the isoscalar $J^P = 1^+$ sector: T_{cc}^{*+}
- However, it is not a result of entanglement suppression but stems from HQSS: $T^{IJ=01}(D^*D) = T^{IJ=01}(D^*D^*)$
- The additional consequences of entanglement suppression is that the interaction strengths of the isovector channels are all the same, either non-interacting or at the unitarity limit.
- In the latter instance, we would also anticipate four extra weakly bound states near the $D^{(*)}D^{(*)}$ threshold.

[Albaladejo:2021vln] M. Albaladejo. **Tcc+ coupled channel analysis and predictions.** Phys. Lett. B 829, 137052 (2022).

INPUT: X(3872) AS HADRONIC MOLECULE

- Input: $\delta_{01+}\left(D\bar{D}^*\right)=\pi/2$
- Also two solutions:

1. All channels reach unitarity limit:

$$\delta_{I0}\left(D\bar{D}\right)=\delta_{I1\pm}\left(D\bar{D}^*\right)=\delta_{IJ}\left(D^*\bar{D}^*\right)=\frac{\pi}{2}$$

2. All isoscalar channels reach unitarity limit, while all amplitudes for isovector channels vanish:

$$I=0:\ \frac{\pi}{2},\ I=1:\ 0$$

PREDICTIONS

- In both scenarios, we conclude that X(3872) should have five spin partner states, all of them being isoscalar states, like the X(3872) itself.
- HQSS predicts only three isoscalar spin partners in the strict heavy-quark limit.
- Again, the interaction strengths of the isovector channels are all the same, either non-interacting or at the unitarity limit.
- If Nature chooses the latter case, there would be six isovector hadronic molecules in addition.
- $J^{PC} = 1^{+-}$: Zc(3900)? Zc(4020)?

ENLARGED SYMMETRY?

Number of independent LECs: $4 \to 2$

- Indeed, we see an emergent symmtry.
- The inherent heavy-quark spin symmetry leads to SU(2)×SU(2) symmetry for light-quark spins.
- Entanglement suppression predicts the same interaction strengths for 2×2=4 spin states in each isospin, therefore we conclude

$$\mathrm{SU}(2)\times\mathrm{SU}(2) \rightarrow \mathrm{SU}(4)$$

- This is referred to as the light-quark spin symmetry (LQSS).

LIGHT-QUARK SPIN SYMMETRY?

- The concept of LQSS was first introduced in heavy-antiheavy meson systems by M. Voloshin to explain the properties of the Zb(10610) and Zb(10650) states as potential hadronic molecular states.
- The existence and masses of hidden-charm pentaquark states Pc(4440), Pc(4457) and Pc(4312) further suggest that LQSS may also exist in the heavy baryon-antiheavy meson systems.
- Additionally, this approximate symmetry has been realized in the light vector meson exchange model (resonance saturation).

[Voloshin:2016cgm] M. B. Voloshin. **Light Quark Spin Symmetry in Zb Resonances?** Phys. Rev. D 93, 074011 (2016).
[Dong:2021juy] Xiang-Kun Dong, Feng-Kun Guo, and Bing-Song Zou. **A survey of heavy-antiheavy hadronic molecules.** Progr. Phys. 41, 65 (2021).

LQSS? NEED MORE STUDIES!!

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SUMMARY

- Conjecture: Minimal entanglement leads to enlarged symmetries.
- Framework: Non-relativistic effective Lagrangian manifesting HQSS, which includes only constant contact potentials at leading order.
- Input: X(3872) and Tcc(3875)+ as hadronic molecules.
- Results: HQSS → LQSS
- Predictions: More spin and isospin partners.
- Future: The LQSS and its predictions need to be confronted with experimental data and LQCD results to further test the conjecture.

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THANK YOU!