Doubly Charmed $\Lambda_c\Lambda_c$ Scattering from Lattice QCD

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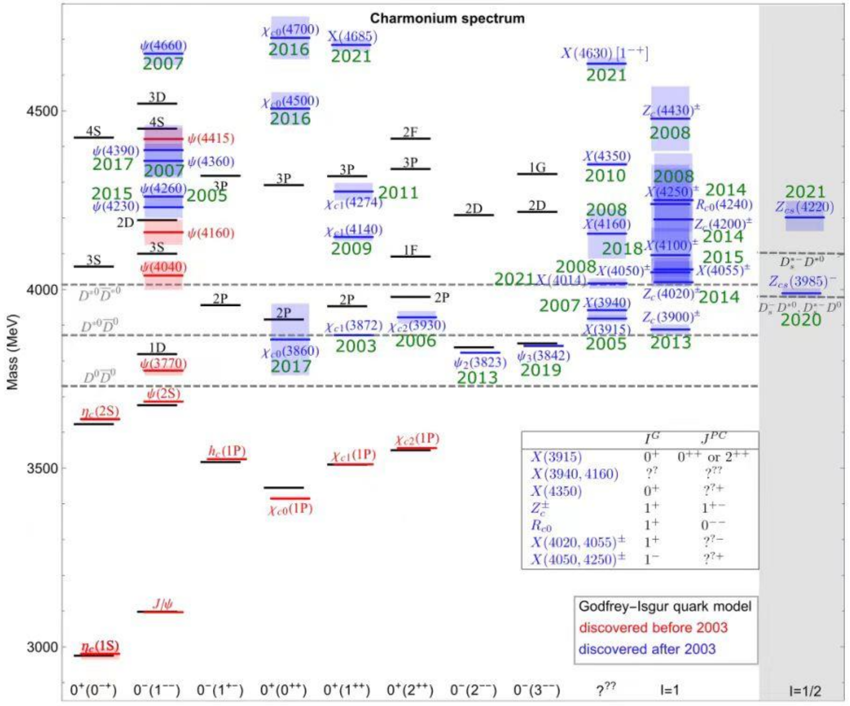
FB23, Sept 26th, 2024, Beijing

Outline

- Motivation
- Lattice Setup
- Formalism
- Analyses and Result
- Summary

- Since X(3872) was found in 2003, many exotic states beyond conventional quark model were found in experiments one after another.



[F.-K. Guo et al, PoS LATTICE2022(2023)232]

Also see talks:

[D. Wei, 24th 14:00-14:25 China Hall 2]

[L.-S. Geng, 24th 14:25-14:50 China Hall 2]

- Recently, many new exotic states were found, such as $T_{cc}^{+}(3875)$, $P_c(4440\&4457)$ and so on.

Also see talks: [F.-Z. Peng, 24th 14:50-15:10 China Hall 2]

[X. Zhang, 25th 11:25-11:45 China Hall 2]

[L.-R. Dai, 26th 9:45-10:05 China Hall 2]

- Since unquenched Lattice QCD was developed, many lattice study on hadronic spectrum have been pushed ahead.

  - Many lattice works on $T_{cc}^{+}(3875)$ are trying to explain it structure.

    [M. Padmanath and S. Prelovsek, Phys.Rev.Lett. 129 (2022) 3, 032002]

    [S.-Y. Chen, C.-J. Shi, Y. Chen at el, Phys.Lett.B 833 (2022) 137391]

    [Lyu Yan at el, Phys.Rev.Lett. 131 (2023) 16, 161901]

    [M.-L. Du, Phys.Rev.Lett. 131 (2023) 13, 13190]

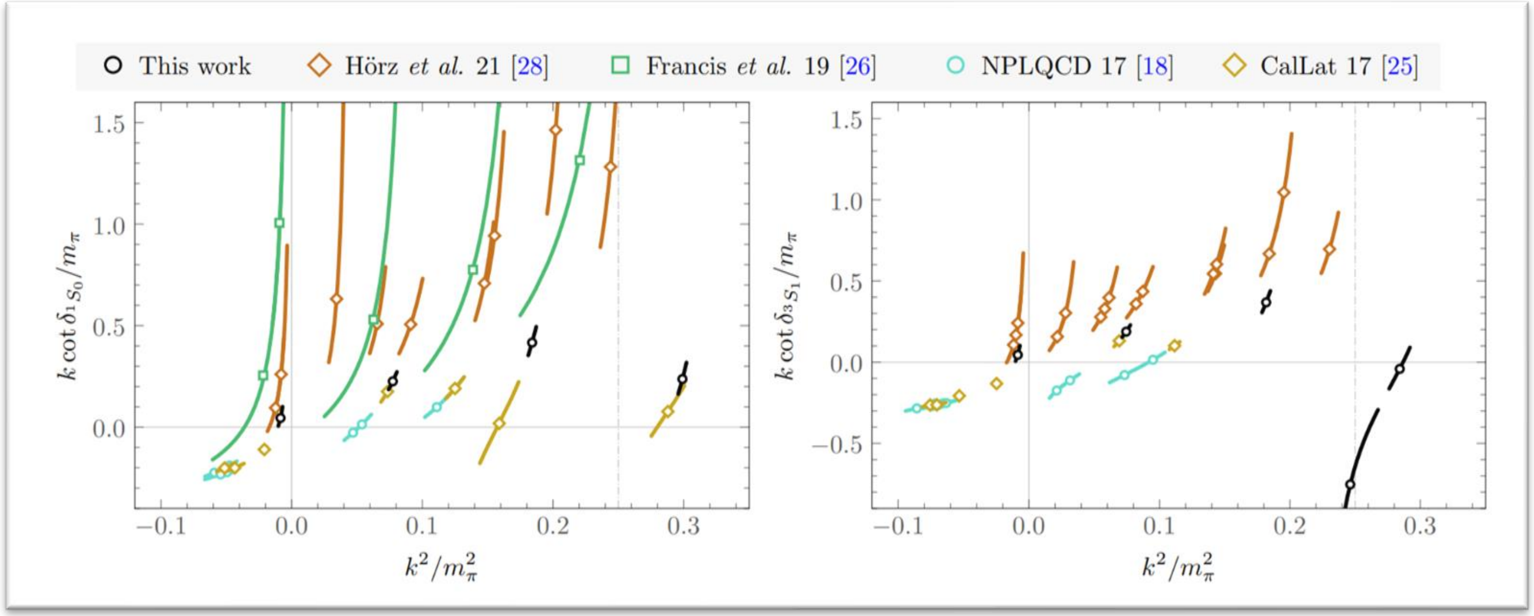
  - Lattice study on baryon-meson molecular states also have some results.

    [Lyu Yan at el, Phys.Rev.D 106 (2022) 7, 074507]

    [Hanyang Xing at el, arXiv 2210.08555]

Motivation

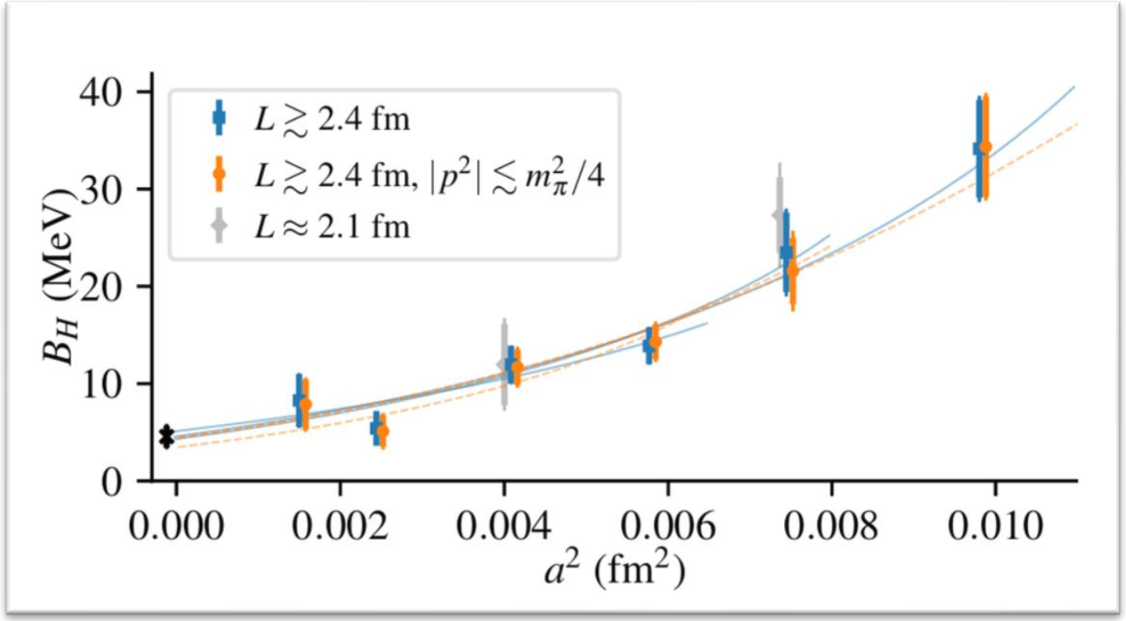
- Study of Baryon-Baryon interaction have many challenges on Lattice.
  - ➢ Deuteron bound state haven't been confirmed in the lattice calculation.

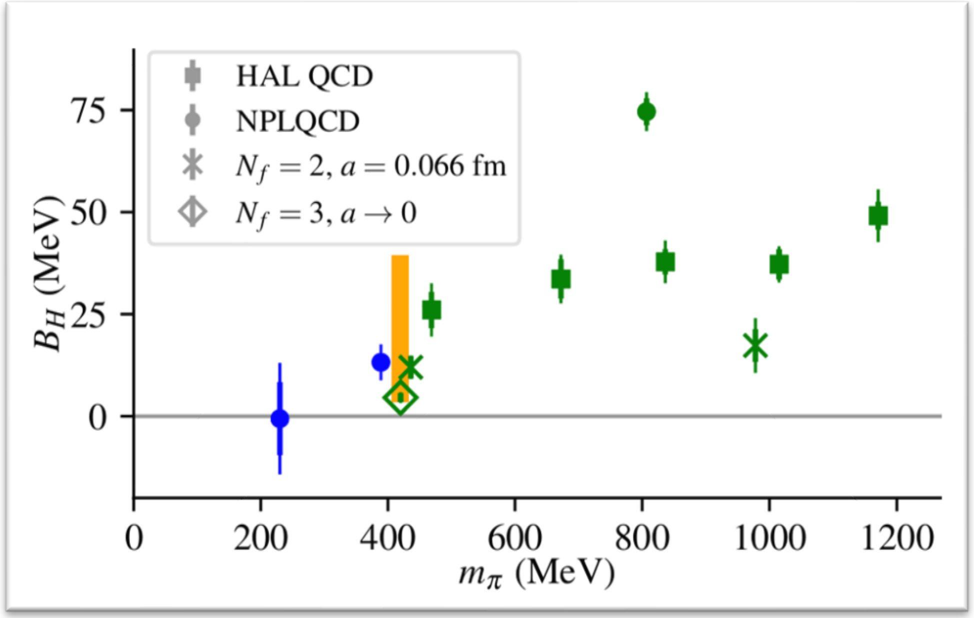


[Saman Amarasinghe et al. Phys.Rev.D. 107 (2023) 9, 094508]

Also see talk : Chen Chen, 26th 15:30-16:00 , China Hall 3

- $\Lambda\Lambda$ system, which is also known as H dibaryon.



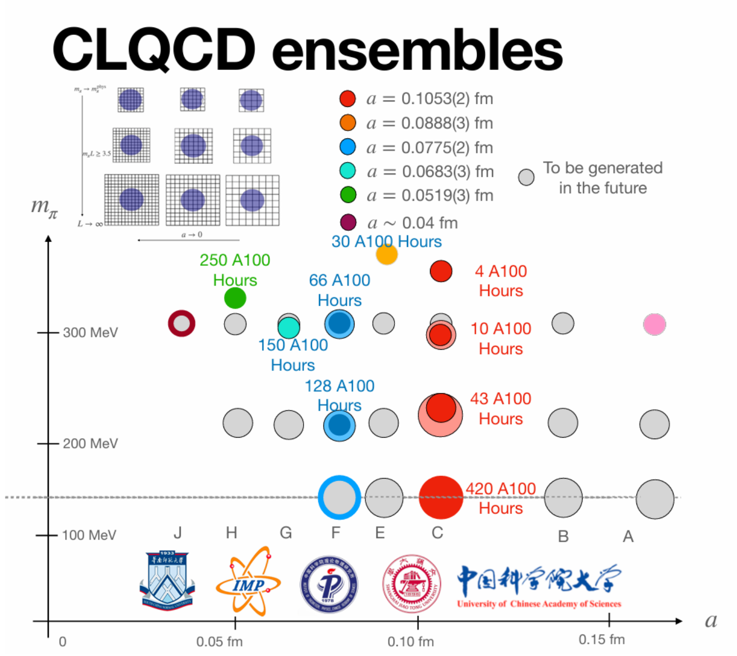


[Jeremy R. Green et al. Phys.Rev.Lett. 127 (2021) 24, 242003]

Also see talk : Hanyang Xing, 26th 12:00-12:30 , China Hall 3

What about $\Lambda_c\Lambda_c$?

Lattice Setup



[Z.C. Hu et al. PRD 109 (2024) 5, 054507]

In this work,

- Two Wilson-Clover lattice ensembles are used.
- Space-time symmetrical and 2+1 flavor.
- Same m_π and lattice spacing, but different volume.

ensemble	$(L/a)^3 \times T/a$	β	a(fm)	m_π(MeV)	m_K(MeV)	$m_\pi \times L$	N_{conf}
F32P30	$32^3 \times 96$	6.41	0.07746(18)	303.2(1.3)	524.6(1.8)	3.81	567
F48P30	$48^3 \times 96$	6.41	0.07746(18)	303.4(0.9)	523.6(1.4)	5.72	201

- **Lüscher's finite volume method** [M. Lüscher, Nucl. Phys. B. 354 (1991) 531-578]

$$\mathbf{det}[\mathbf{1} + \boldsymbol{i\rho T}(\mathbf{1} + \boldsymbol{i}\,\mathcal{M})] = \mathbf{0}$$

- ρ ~ phase space
- T ~ scattering amplitude
- $\mathcal{M}$ ~ Lüscher matrix

- For S wave rest-frame case: $$\mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi}L} Z_{00}(1;\tilde{q}^2)$$

Also see talk : Akski Rusetsky, 24th 12:00-12:30 , China Hall 2

➢ **A new modification on Lüscher equation is needed**

- Cause by discretized effect with $Z_{1(2)} \neq 1$:

$$E(k) = \sqrt{m_1^2 + Z_1\,p^2} + \sqrt{m_2^2 + Z_2\,p^2}$$

Formalism

[Yan Li, Jia-Jun Wu at el, JHEP 08 (2024) 178]

- General Lüscher matrix

$$\mathcal{M}_{lm,l'm'}(q,P) = \frac{1}{q}\left(\frac{1}{L^3}\sum_k - \mathcal{P}\int \frac{d^3k^r}{(2\pi)^3}\right)\frac{32\pi^2 E^*(q)\mathcal{J}^r}{4\omega_1(k^*)\omega_2(k^*)}\frac{Y_{lm}(\hat{k}^*)Y_{l'm'}(\hat{k}^*)\left(\frac{|k^*|}{q}\right)^{l+l'}}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)}$$

- Consider a translation of the limit

$$\lim_{k^*\to q}\frac{1}{4\omega_1(k^*)\omega_2(k^*)}\frac{q-k^*}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)} = \lim_{k^*\to q}\frac{1}{4q[Z_1\omega_2(q)+Z_2\omega_1(q)]}$$

$$\to \frac{1}{4\omega_1(k^*)\omega_2(k^*)}\frac{1}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)} = \frac{1}{2[Z_1\omega_2(q)+Z_2\omega_1(q)]}\frac{1}{q^2-k^{*2}}$$

$$\to \mathcal{M}_{lm,l'm'}(q,P) = \frac{16\pi^2}{q}\left(\frac{1}{L^3}\sum_k - \mathcal{P}\int \frac{d^3k^r}{(2\pi)^3}\right)\mathcal{J}^r\frac{Y_{lm}(\hat{k}^*)Y_{l'm'}(\hat{k}^*)\left(\frac{|k^*|}{q}\right)^{l+l'}}{q^2-k^{*2}}\frac{\boldsymbol{E^*(q)}}{\boldsymbol{Z_1\omega_2(q)+Z_2\omega_1(q)}}$$

- For S wave rest-frame case: $\mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi}L}Z_{00}(1;\tilde{q}^2)\times\frac{\boldsymbol{E^*(q)}}{\boldsymbol{Z_1\omega_2(q)+Z_2\omega_1(q)}}$

Distillation quark smearing method

- Advantages
  - Improve precision
  - Efficient for large numbers of operations

- Operate $\square(t)=V(t)V^{\dagger}(t)$ on quarks → $P = VGV^{\dagger}$

- G ~ propagator
- P ~ perambulator
- V ~ eigenvectors of ∇^2

- Dimension of V is $[N_t, N_c \times N_x^3, N_{ev}]$

[Hadron Spectrum Collaboration, Phys.Rev.D 80 (2009) 054506]

[Colin Morningstar, Phys.Rev.D 83 (2011) 114505]

- **Operator construction**
  - Single baryon $$B(k,x^0) = \sum_{\vec{x}} P_+ \, \varepsilon_{abc} \, r_{ax} \left[s_{bx}^T \, (C\gamma_5) t_{cx} \right] e^{-i\vec{k}\cdot\vec{x}}$$
  - Two baryons $$\mathcal{O}^{\Lambda}_{B_1 B_2}(|\vec{k}|) = \sum_j c_j^{\Lambda} B_1^T(|\vec{k}|) \, C\gamma_5 B_2(-|\vec{k}|)$$
- **Correlation function**

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^{\dagger}(0) \rangle \approx A e^{-E\,t}$$

- **Generalized eigenvalue problem(GEVP)**

$$C(t) v_{\alpha}(t,t_0) = \lambda_{\alpha}(t,t_0) C(t_0) v_{\alpha}(t,t_0)$$

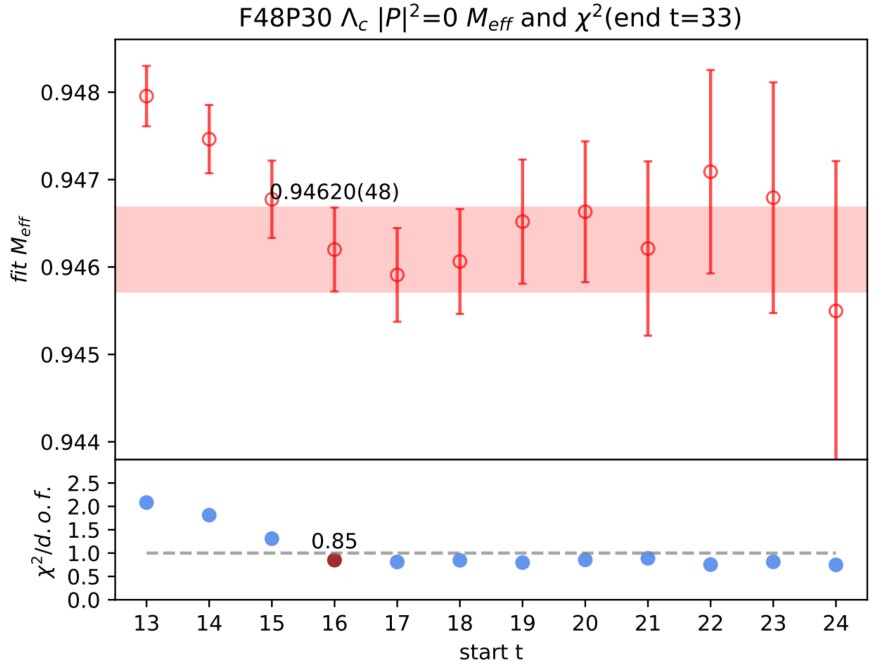
Energy Fitting Method

Fitting range selection

Correlation function of Single Λ_c then can be fitted as the parametrization

$$C(t) = A\exp[-M_{eff}t]$$

- Fitting window is $[\mathrm{t_{start}}, \mathrm{t_{end}}]$ while $\mathrm{t_{end}}$ is fixed.
- Starting time-slice should given a stable fitting result.
- Fitting window should be as wide as possible.

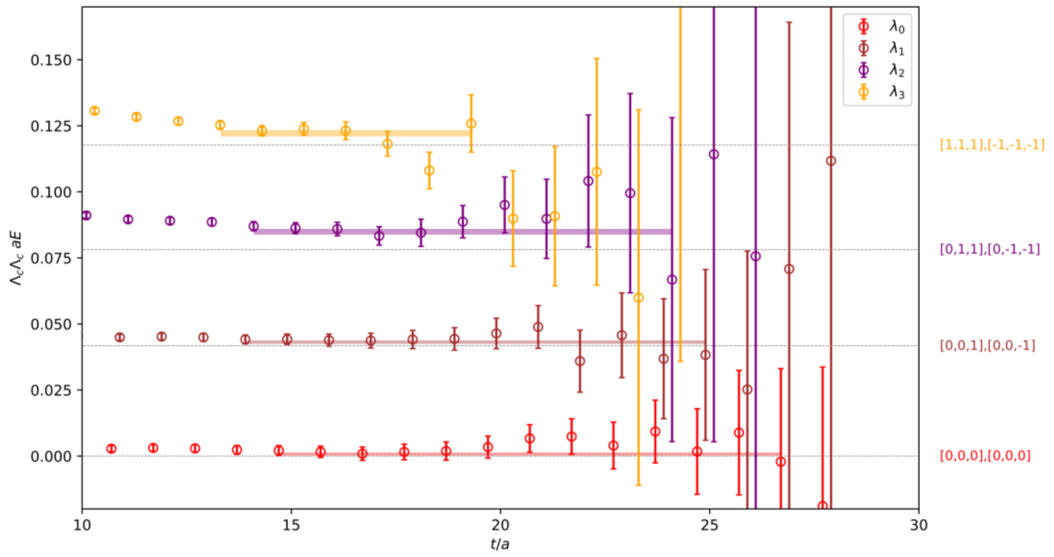


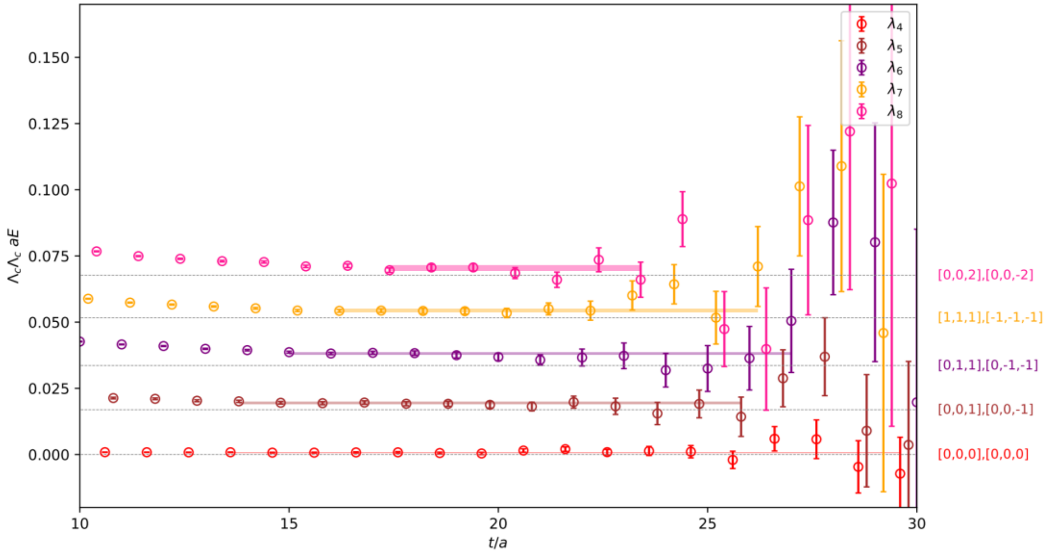
Energy Fitting Method

- Two baryons correlation function

Since these energy levels are quite close to their corresponding free energies, a ratio form fitting could perform better

$$C_R(t) = \frac{C_{BB}(t)}{C_{thre.}(t)} \simeq Ae^{-(E_{BB}-2m_{\Lambda_c})t} = Ae^{-\Delta E t}$$





Couple channel

To explore the states near $\Lambda_c\Lambda_c\ 0(0^+)$ threshold, the coupling with $\Xi_{cc}N$ is contained.

- single baryon $\boldsymbol{\Lambda_c\ 0\left(\frac{1}{2}^+\right), \Xi_{cc}, N\ \frac{1}{2}\left(\frac{1}{2}^+\right)}$ and $\boldsymbol{\Sigma_c\ 1\left(\frac{1}{2}^+\right)}$ → **Determine the $\Sigma_c\Sigma_c$ threshold cutoff**

- two baryons

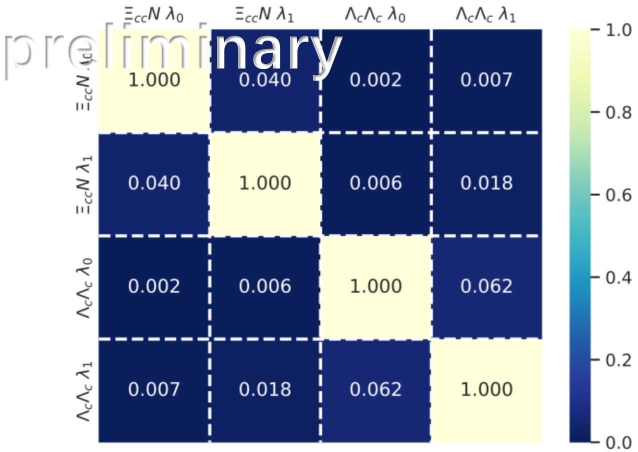
$$\Lambda_c\Lambda_c^{I=0} = [\Lambda_c\Lambda_c]$$

$$\Xi_{cc}N^{I=0} = \frac{1}{2}([p\Xi_{cc}^{+}] + [\Xi_{cc}^{+}p] - [n\Xi_{cc}^{++}] - [\Xi_{cc}^{++}n])$$

Base on identical particle exchange, the operator construction can be further simplified.

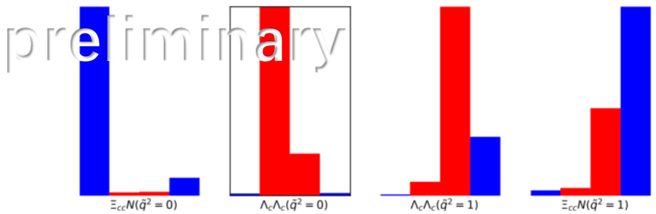
- Coupled with Ξ_{cc}N
  - ➢ Cross-correlation matrix

$$\widetilde{C}_{ij}(\vec{P},t) = C_{ij}(\vec{P},t)/\sqrt{|C_{ii}(\vec{P},t)C_{jj}(\vec{P},t)|}$$

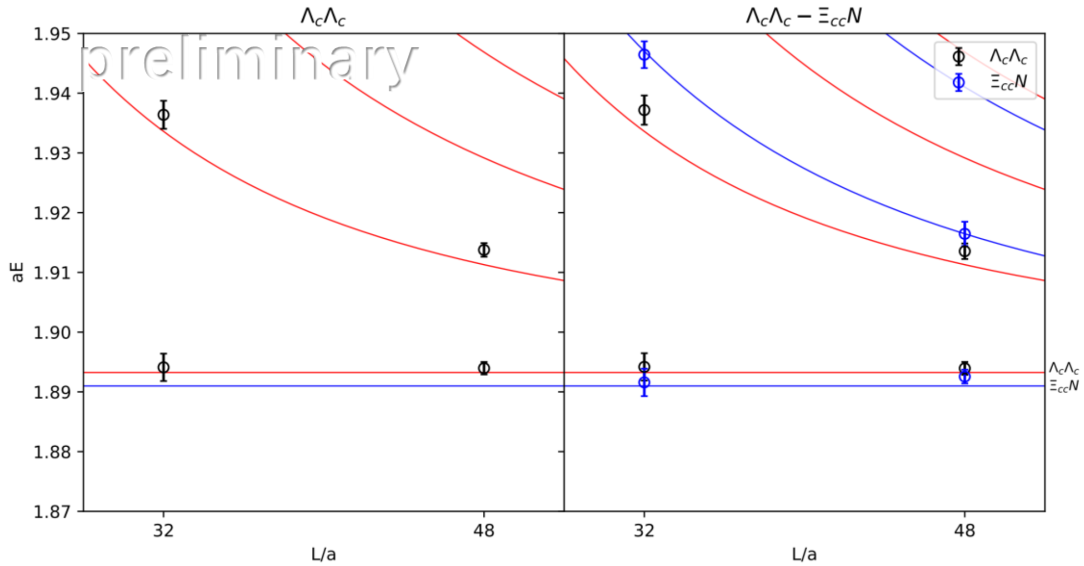


  - ➢ Overlap factor

$$Z_i^\alpha = \sqrt{2E_\alpha} e^{E_\alpha t_0/2} v_j^{\alpha *} C_{ji}(t_0)$$



- The coupling between Ξ_{cc}N and $\Lambda_c\Lambda_c$ is quite small.
- $\Lambda_c\Lambda_c$ energy levels haven't been shifted obviously.



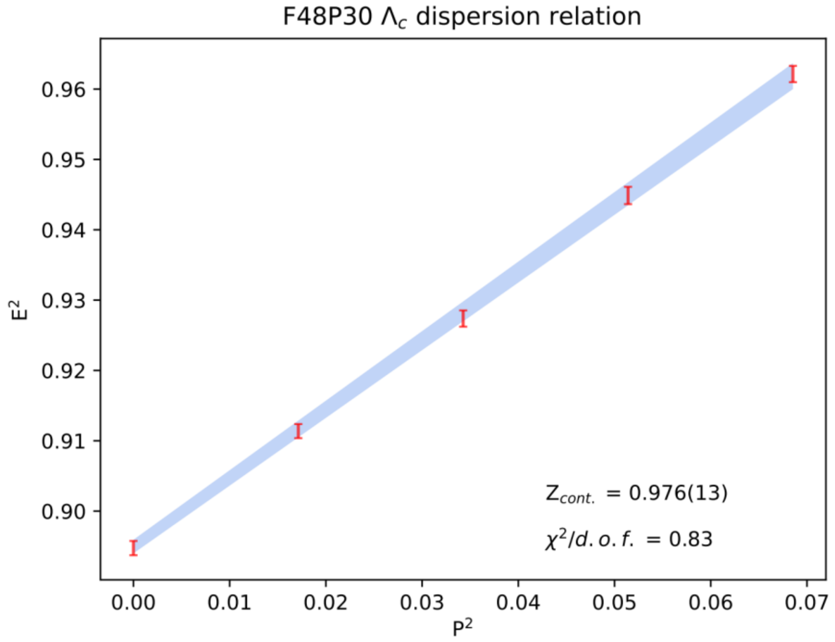
[C. Liu, L. Liu et al, Phys.Rev.D 101 (2020) 5, 054502]

[Jozef J. Dudek et al, Phys.Rev.D 82 (2010) 034508]

Discretization effect

- Dispersion relation $E^2(k) = m^2 + Z_{cont.}p^2$

where slope Z isn't always equal to 1 perfectly.



- Modification on Lüscher formula

Consider a translation of the limit

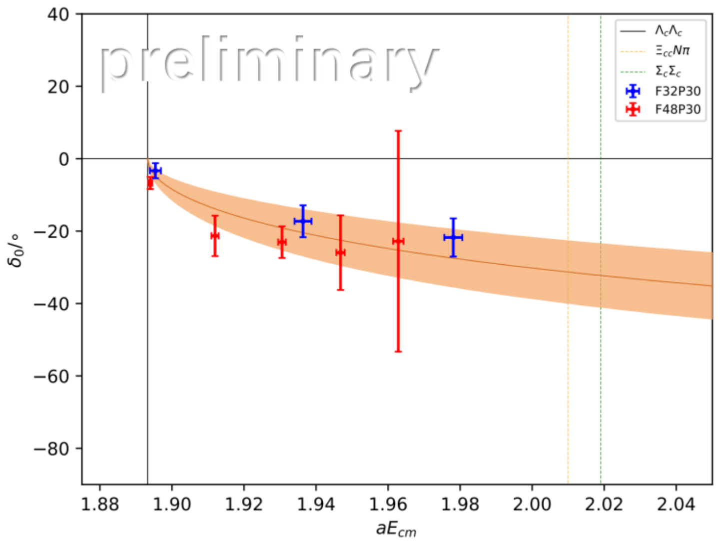
$$\lim_{k^*\to q} \frac{1}{4\omega_1(k^*)\omega_2(k^*)} \frac{q-k^*}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)} = \lim_{k^*\to q} \frac{1}{4q[Z_1\omega_2(q)+Z_2\omega_1(q)]}$$

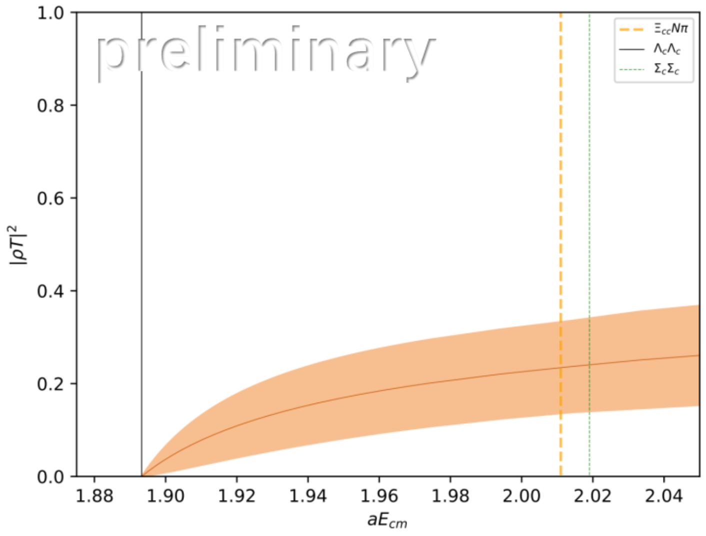
$$\to \frac{1}{4\omega_1(k^*)\omega_2(k^*)} \frac{1}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)} = \frac{1}{2[Z_1\omega_2(q)+Z_2\omega_1(q)]} \frac{1}{q^2-k^{*2}}$$

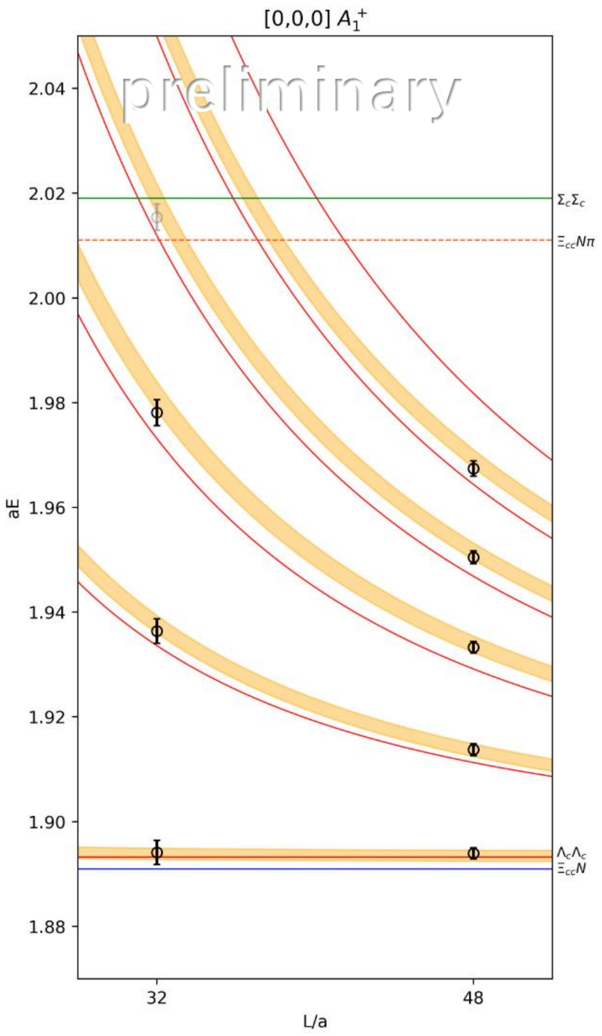
$$\to \mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi}L} \mathcal{Z}_{00}(1;\tilde{q}^2) \times \frac{E_1+E_2}{Z_1E_2+Z_2E_1}$$

[Yan Li, Jia-Jun Wu at el, JHEP 08 (2024) 178]

- Spectrum and Phase







Parameterization: $k\cot\delta_0 = \frac{1}{a_0}$.

Fitting result: $a_0 = -0.143(49)$ fm, with $\frac{\chi^2}{dof} = 0.86$.

Conclusion: **No bound state** in S wave $\Lambda_c\Lambda_c$ system.

- Repulsive interaction

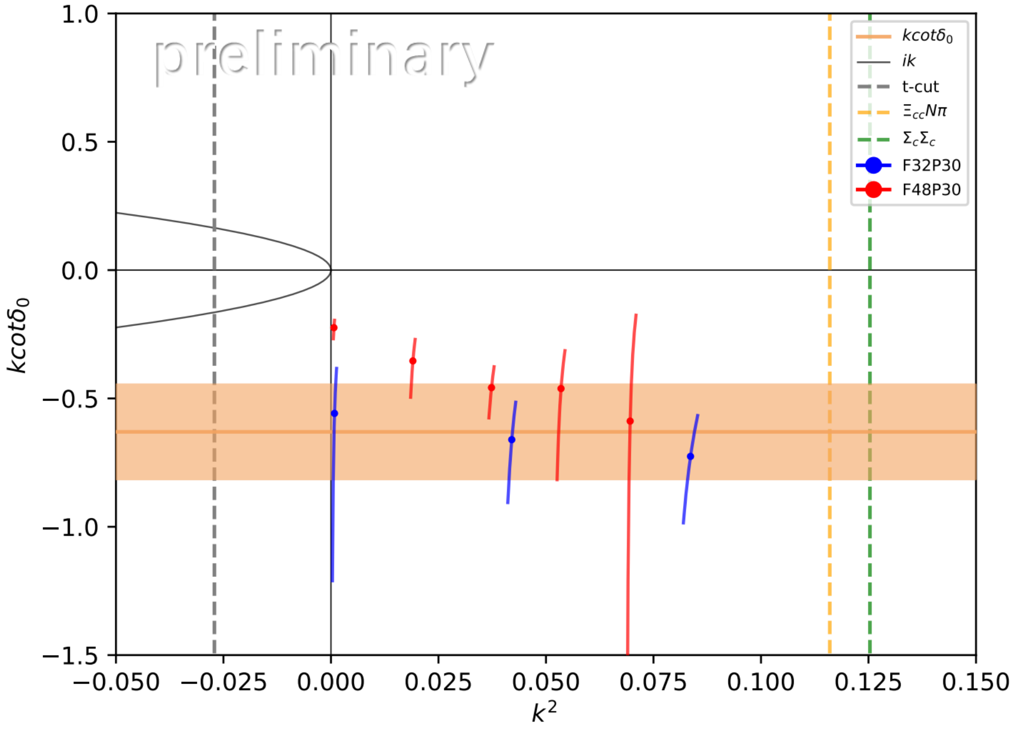
Summary

- Due to the **discretization** on lattice, a **modification on Lüscher equation** is proposed in this work.
- Coupled with $\Xi_{cc}N$, the energy levels of $\Lambda_c\Lambda_c$ haven't been shifted obviously. Therefore, **single channel** is contained.
- Showing a **repulsive interaction**, there's **no bound state** in this system.
- Scattering length $\boldsymbol{a_0 = -0.143(49)}$ **fm.**

Thanks!

Back-up

- Left-hand cut



The left-hand cut gives the limit of pole extraction with $k^2 = -\frac{m_\omega^2}{4}$ from the exchanging of omega boson.

[X.-K. Dong et al, Phys.Rev.D 105 (2022) 7, 074506]
[Also see talk in Aug 1st 11:30 and 11:50 in LT1]

- Finite volume effect
  - The scattering length are consistent in the two ensembles.
  - However, there's a little difference.