

Evgeny Epelbaum, Ruhr University Bochum

# Chiral EFT using Gradient Flow

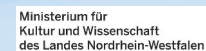
based on work done in collaboration with Hermann Krebs, e-Prints: 2311.10893; 2312.13932  
(both papers accepted for publication in PRC)

...opens an avenue for accurate  $\chi$ EFT calculations beyond the 2N system

- Introduction & state-of-the-art
- The need for a symmetry-preserving regulator
- Chiral EFT using gradient flow
- Summary & outlook

**FB23** THE 23<sup>rd</sup> INTERNATIONAL CONFERENCE ON  
FEW-BODY PROBLEMS IN PHYSICS (FB23)  
Sept. 22 -27, 2024 • Beijing, China

**Host** Institute of High Energy Physics, Chinese Academy of Sciences   Institute for Advanced Study, Tsinghua University   University of Chinese Academy of Sciences  
China Center of Advanced Science and Technology   Institute of Theoretical Physics, Chinese Academy of Sciences   South China Normal University  
**Co-host** Chinese Physical Society (CPS)   High Energy Physics Branch of CPS



# Deuteron as a bound state of quarks and gluons

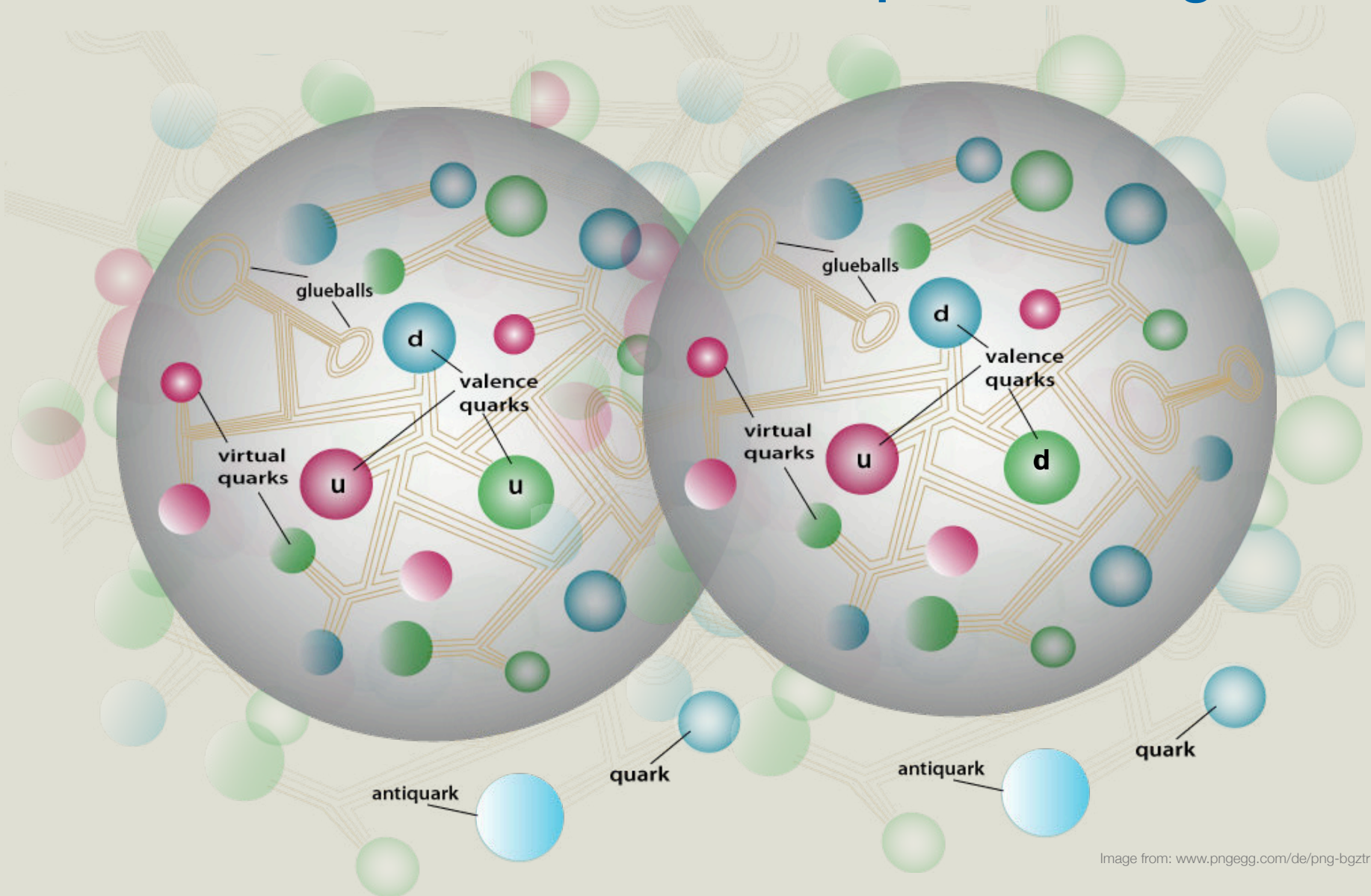


Image from: [www.pngegg.com/de/png-bgztr](http://www.pngegg.com/de/png-bgztr)

Is there a way to simplify the picture (without losing connection to QCD)?

# Chiral Effective Field Theory

The Standard Model (QCD, ...)

Schwinger-Dyson , large- $N_c$ , ...

Approximate chiral  $SU(2)_L \times SU(2)_R$  symmetry; introduce  $U(\pi) \rightarrow RU(\pi)L^\dagger$

effective chiral Lagrangian  $\mathcal{L}_{\text{eff}}(\pi, N)$

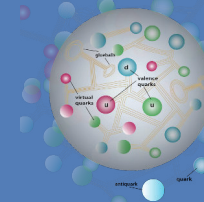
Chiral perturbation theory (usually using DimReg)

- S-matrix ( $\pi\pi$ ,  $\pi N$ ,  $\pi\pi N$ , ...)
- nuclear forces and currents

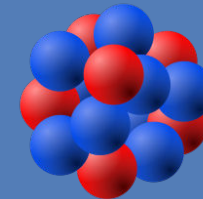
Few-body methods (Cutoff)

Lattice QCD

Hadron/nuclear structure and dynamics



proton



nuclei






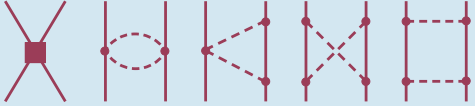





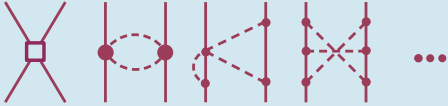
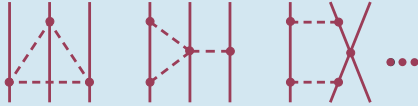

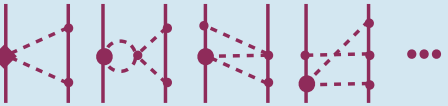


neutron stars

$E_{FV}$


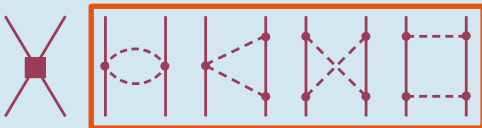
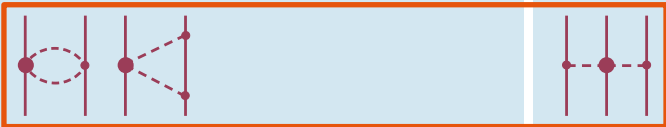

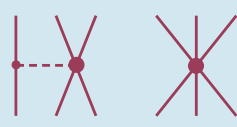
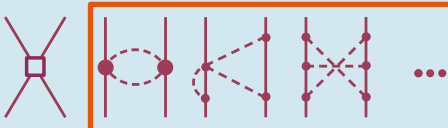
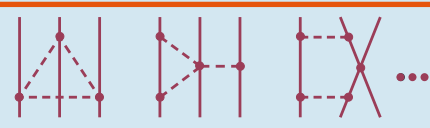

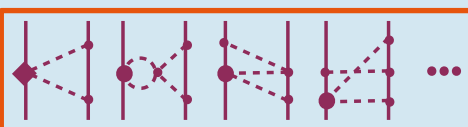
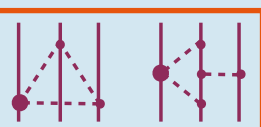
EFT

finite-volume methods

# Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:			
NLO:			
N <sup>2</sup> LO:			
N <sup>3</sup> LO:			
N <sup>4</sup> LO:			

# Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:		—	—
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Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes



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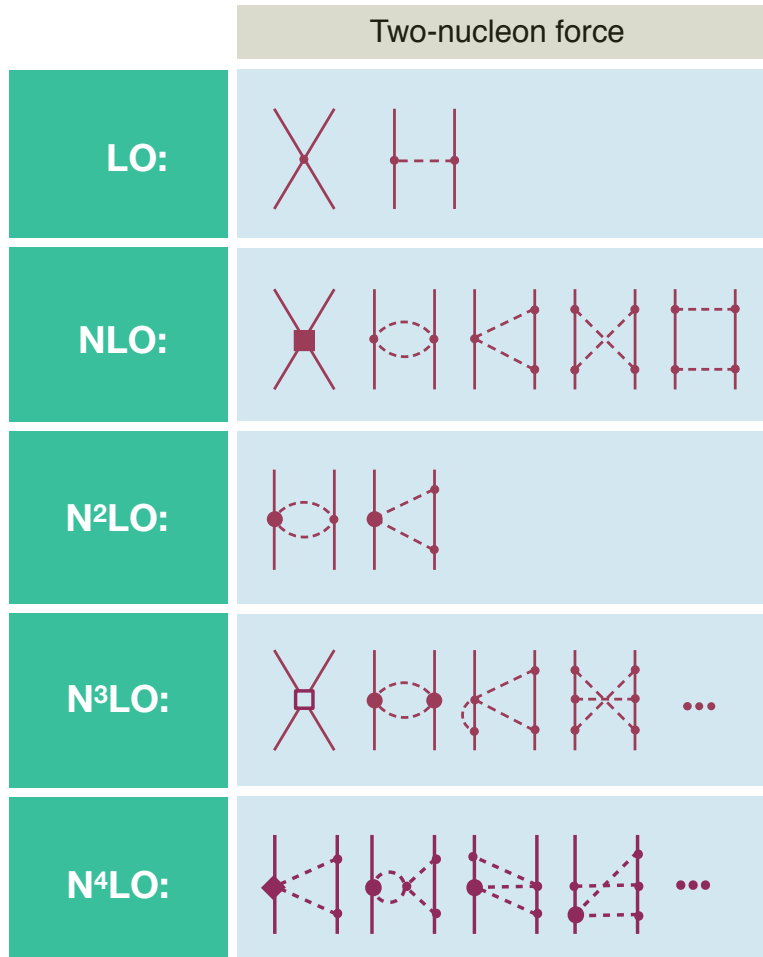
Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes



Short-range few-N interactions are tuned to experimental data

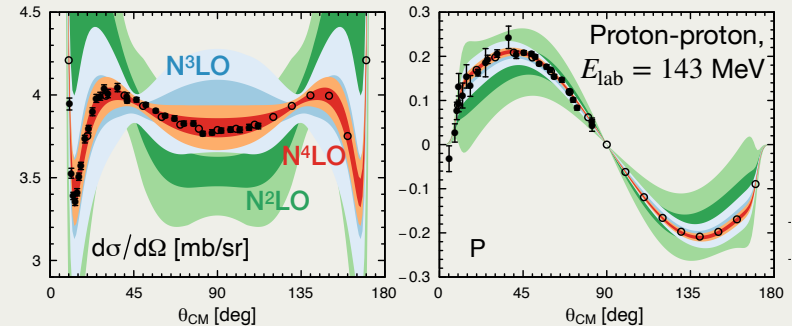


# Chiral expansion of nuclear forces



## $\chi$ EFT as a precision tool in the 2N sector

- N<sup>4</sup>LO+: currently most accurate and precise NN interactions on the market
- clear evidence of the TPEP from NN data
- Bayesian truncation-error estimation



- almost no residual cutoff dependence

⇒ precision 2N physics from  $\chi$ EFT

Semi-local regularization in momentum space Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2} e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction},$$

$$V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2} e^{-\frac{\vec{q}^2 + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

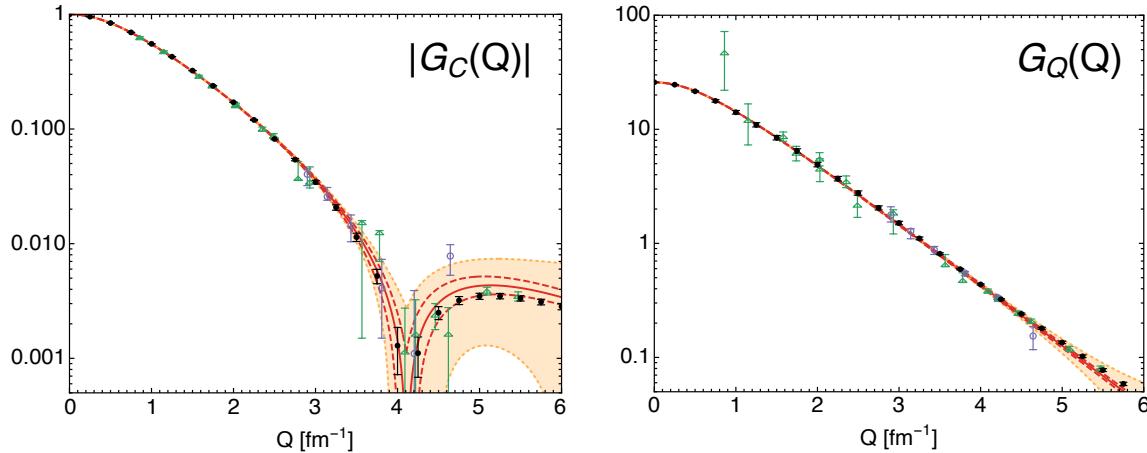




# Precision 2N physics: Deuteron FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

## Charge and quadrupole form factors of the deuteron at N<sup>4</sup>LO



Extracted quadrupole moment:

$$Q_d = 0.2854^{+0.0038}_{-0.0017} \text{ fm}^2$$

EFT truncation, choice of fitting range,  
NN,  $\pi$ N and  $\gamma$ NN LECs

to be compared with experiment

$$Q_d^{\text{exp}} = 0.285699(15)(18) \text{ fm}^2$$

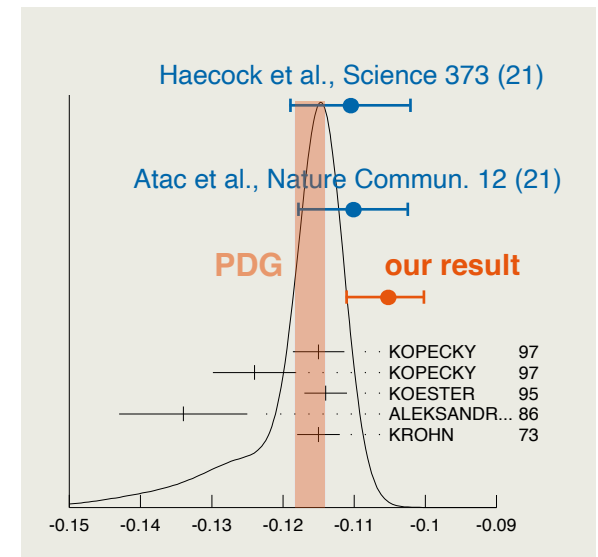
Puchalski et al., PRL 125 (2020)

The charge and structure radius:

$$r_d^2 = (-6) \frac{\partial G_C(Q^2)}{\partial Q^2} \Big|_{Q^2=0} = r_{\text{str}}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

Combining our result  $r_{\text{str}} = 1.9729^{+0.0015}_{-0.0012} \text{ fm}$  with very precise isotope-shift spectroscopy data for  $r_d^2 - r_p^2$ , we determine the neutron m.s. charge radius:

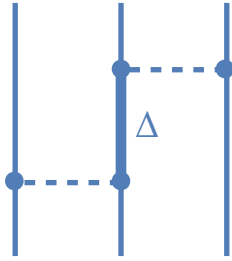
$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{ fm}^2$$



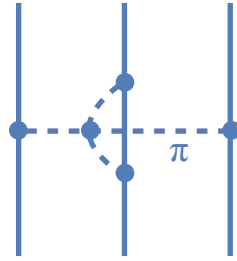
# 3-body force: A frontier in nuclear & atomic physics

Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, e-Print: 2405.09807 [nucl-th]

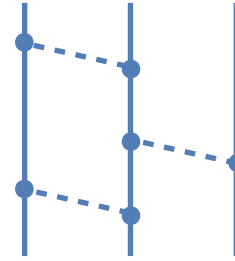
- Three-nucleon forces (3NF) are small **but important** corrections to the dominant NN forces
- 3NF mechanisms:



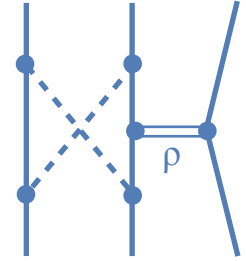
intermediate  $\Delta$ -excitation  
Fujita, Miyazawa '57



multi-pion interactions



off-shell behavior of the  $V_{NN}$   
 $V_{\text{ring}} = \mathcal{A}_{3\pi} - V_{\pi} G_0 V_{\pi} G_0 V_{\pi}$

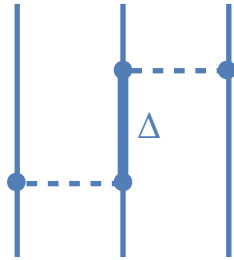


short-range

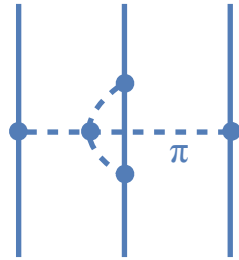
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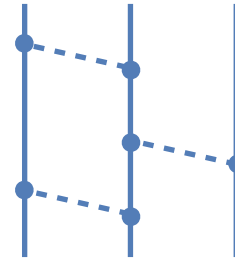
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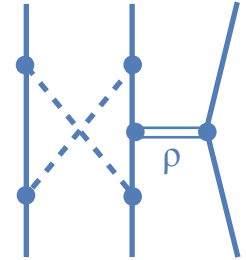
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off-shell behavior of the  $V_{NN}$   
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short-range

- **Difficult to model: None of the existing 3NFs allow to describe of 3N data...**
  - scarcer database compared to the NN sector ← talk by Kimiko Sekiguchi
  - high computational cost of solving the Faddeev equation
  - complicated structure:

$$V_{3N}^{\text{non-local}} = \sum_{i=1}^{320} O_i \times f_i = \sum_{i=1}^{68} O_i \times \tilde{f}_i + \text{perm.} \xrightarrow{\text{antisymm.}} \sum_{i=1}^{14} O_i \times \tilde{\tilde{f}}_i + \text{perm.}$$

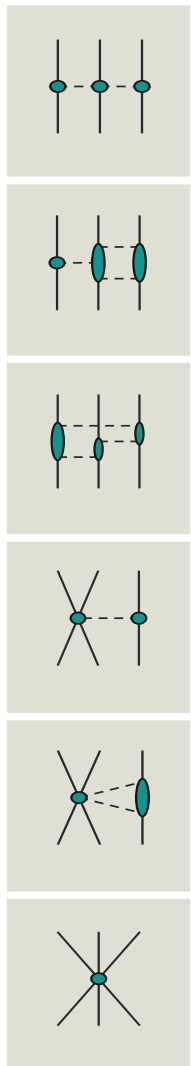
Topolnicki '17
Krebs, EE, in preparation

*spin-momentum-isospin functions of 5 momenta*

⇒ Guidance from theory indispensable — an opportunity for  $\chi$ EFT!

# 3-body force: A frontier in nuclear & atomic physics

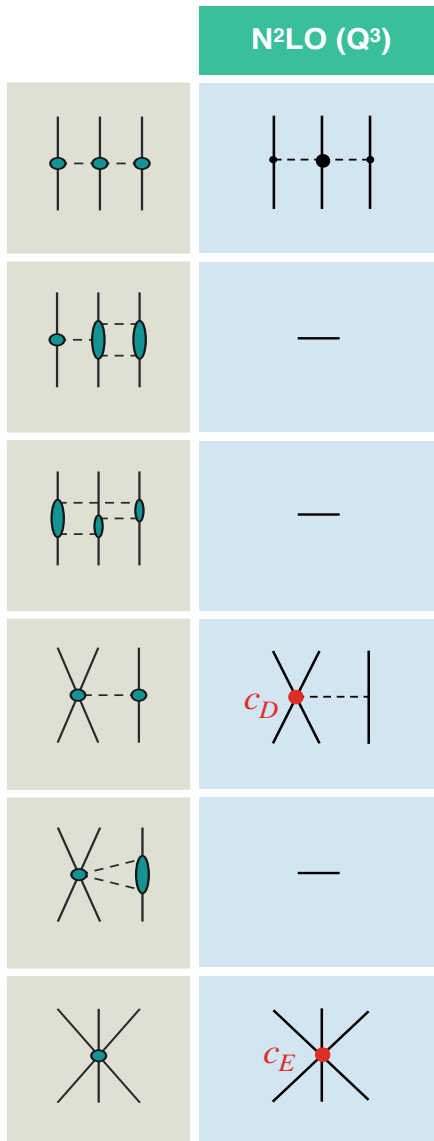
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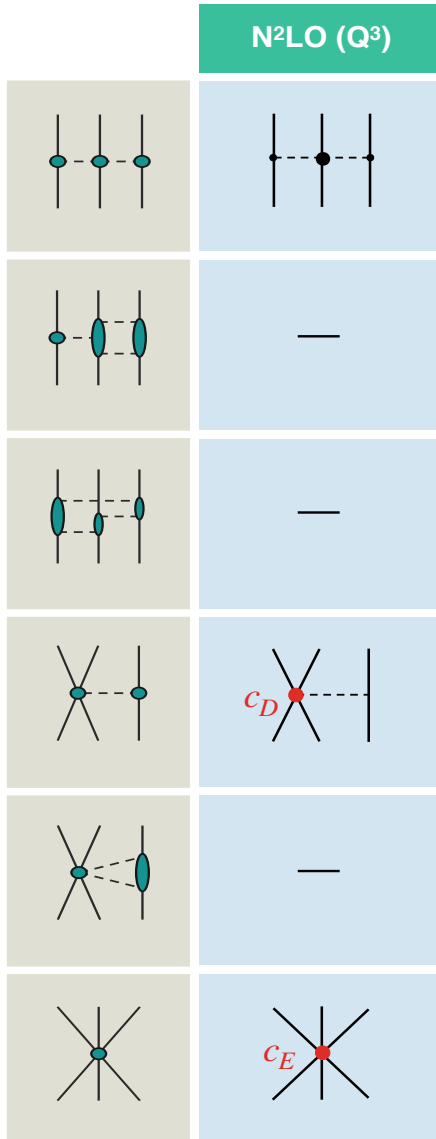
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, e-Print: 2405.09807 [nucl-th]



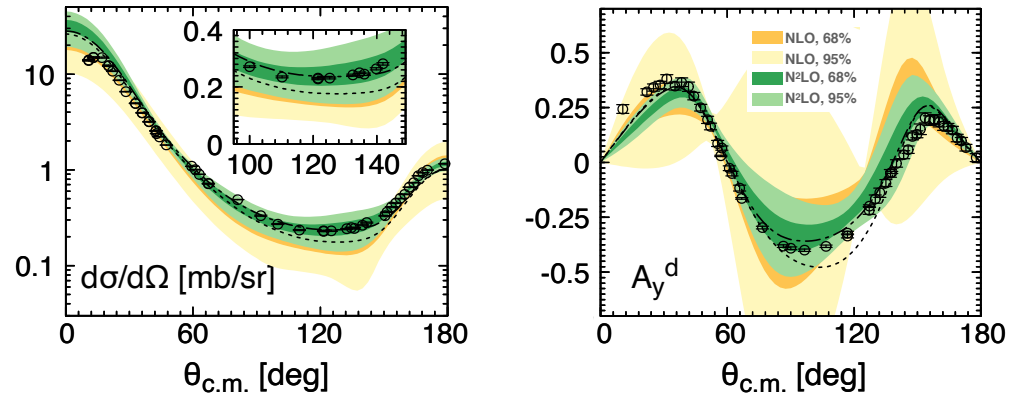
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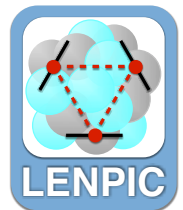
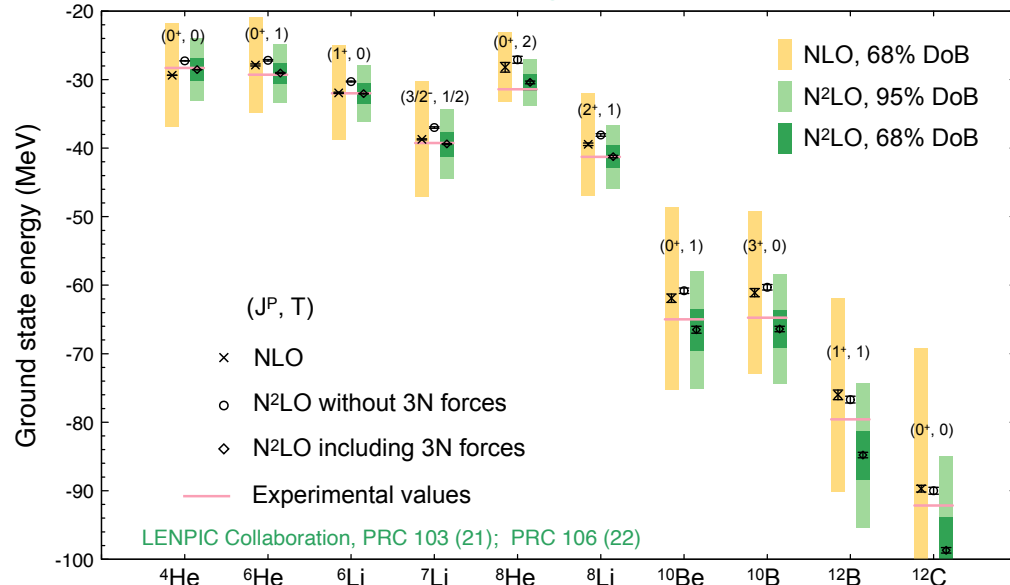
Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, e-Print: 2405.09807 [nucl-th]



Elastic Nd scattering at 135 MeV



Predictions for light p-shell nuclei



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Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, e-Print: 2405.09807 [nucl-th]

	N <sup>2</sup> LO (Q <sup>3</sup> )	N <sup>3</sup> LO (Q <sup>4</sup> )	N <sup>4</sup> LO (Q <sup>5</sup> )
		 Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08	 Krebs, Gasparyan, EE '12
	—	 Bernard, EE, Krebs, Meißner '08	 Krebs, Gasparyan, EE '13
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		 Bernard, EE, Krebs, Meißner '11	 Krebs, Gasparyan, EE '13
	—	 Bernard, EE, Krebs, Meißner '11	 Krebs, Gasparyan, EE '13
		—	 13 LECs Girlanda, Kievski, Viviani '11

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	$c_D$		
	Bernard, EE, Krebs, Meißner '11	Bernard, EE, Krebs, Meißner '11	
	—		
		Bernard, EE, Krebs, Meißner '11	

mixing DimReg with Cutoff in the Schrödinger equation breaks  $\chi$ -symmetry [EE, Krebs, Reinert '19]  
 $\Rightarrow$  need to be re-derived using symmetry-preserving Cutoff regularization  
 (also applies to nuclear currents at N<sup>3</sup>LO and beyond...)



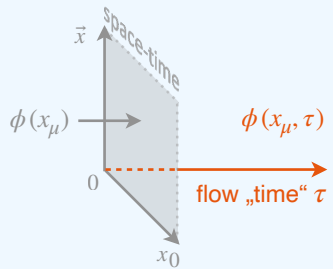
**DANGER:** momentum cutoff for pions breaks chiral symmetry!

# Gradient flow

Gradient flows: methods for smoothing manifolds

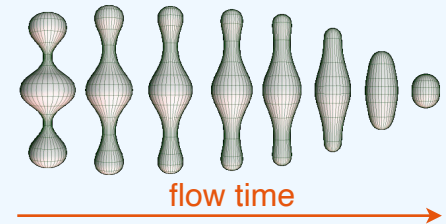
(e.g., Ricci flow used in the proof of the Poincaré conjecture)

Gradient flow as a regulator in field theory



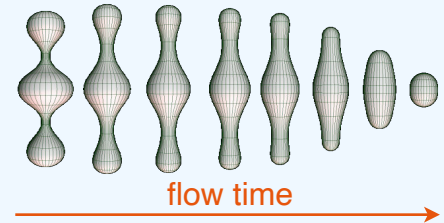
$$\text{Flow equation: } \frac{\partial}{\partial \tau} \phi(x, \tau) = - \left. \frac{\delta S[\phi]}{\delta \phi(x)} \right|_{\phi(x) \rightarrow \phi(x, \tau)}$$

subject to the boundary condition  $\phi(x, 0) = \phi(x)$

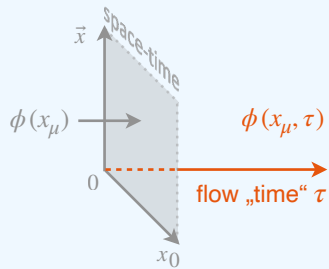


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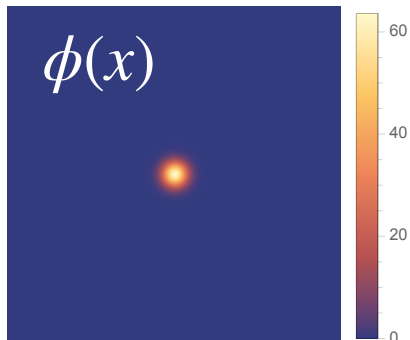
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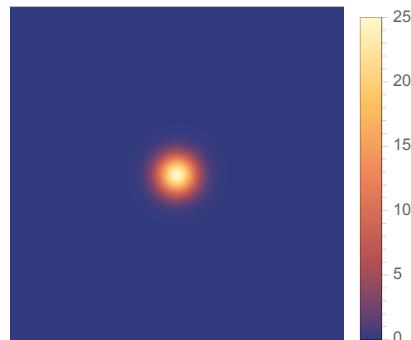
Free scalar field:

$$G(x, \tau) = \frac{\theta(\tau)}{16\pi^2 \tau^2} e^{-\frac{x^2 + 4M^2 \tau^2}{4\tau}}$$

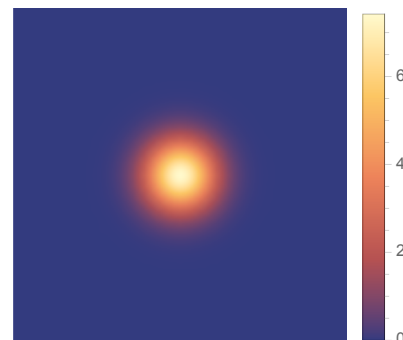
$$[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)] \phi(x, \tau) = 0 \quad \Rightarrow \quad \phi(x, \tau) = \int \underbrace{d^4 y G(x - y, \tau)}_{\text{heat kernel}} \phi(y) \quad \Rightarrow \quad \tilde{\phi}(q, \tau) = e^{-\tau(q^2 + M^2)} \tilde{\phi}(q)$$



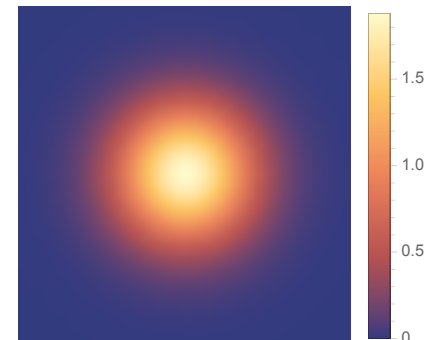
$\tau = 0$



$\tau = 1$



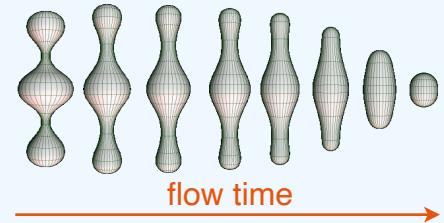
$\tau = 2$



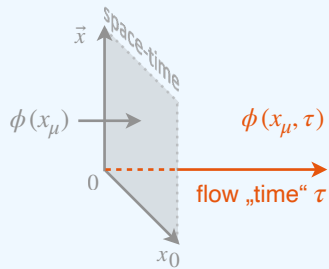
$\tau = 4$

# Gradient flow

Gradient flows: methods for smoothing manifolds  
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



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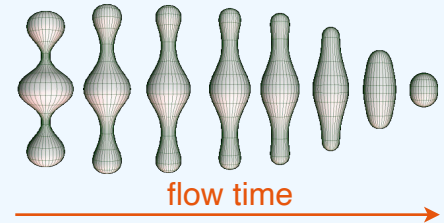
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YM gradient flow Narayanan, Neuberger '06, Lüscher, Weisz '11:  $\partial_\tau A_\mu(x, \tau) = D_\nu G_{\nu\mu}(x, \tau)$  ← extensively used in LQCD

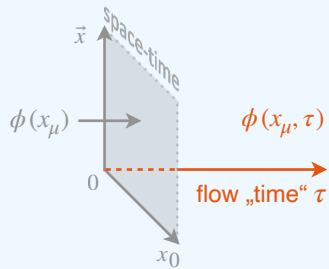
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YM gradient flow Narayanan, Neuberger '06, Lüscher, Weisz '11:  $\partial_\tau A_\mu(x, \tau) = D_\nu G_{\nu\mu}(x, \tau)$  ← extensively used in LQCD

Chiral gradient flow Krebs, EE, 2312.13932, to appear in PRC

$$\text{Generalize } U(x), U(x) \rightarrow RU(x)L^\dagger \text{ to } W(x, \tau): \quad \partial_\tau W = - \underbrace{i \overline{w} \text{EOM}(\tau)}_{\sqrt{W}} w, \quad W(x, 0) = U(x)$$

We have proven  $\forall \tau: W(x, \tau) \in \text{SU}(2), W(x, \tau) \rightarrow RW(x, \tau)L^\dagger$



# Chiral gradient flow

Solving the chiral gradient flow equation  $\partial_\tau W = -iw \text{EOM}(\tau) w$

– most general parametrization of  $U$ :  $U = 1 + \frac{i}{F} \boldsymbol{\tau} \cdot \boldsymbol{\pi} - \frac{\boldsymbol{\pi}^2}{2F^2} - \alpha \frac{i}{F^3} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \boldsymbol{\pi}^2 + \dots$

– similarly, write  $W = 1 + i\boldsymbol{\tau} \cdot \boldsymbol{\phi} - \boldsymbol{\phi}^2 - i\alpha \boldsymbol{\tau} \cdot \boldsymbol{\phi} \boldsymbol{\phi}^2 + \dots$  and make an ansatz  $\boldsymbol{\phi} = \sum_{n=0}^{\infty} \frac{\boldsymbol{\phi}^{(n)}}{F^n}$

$\Rightarrow$  recursive (perturbative) solution of the GF equation in  $1/F$

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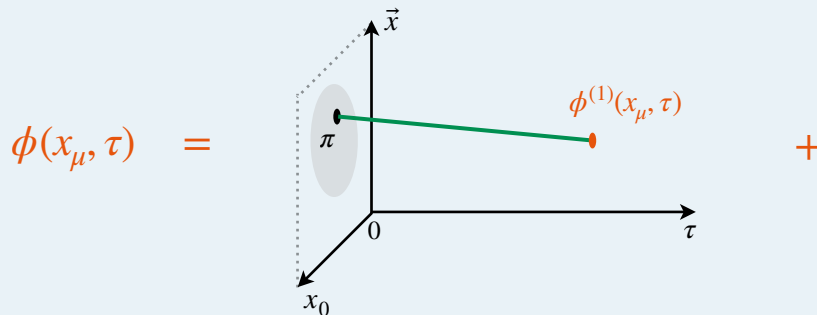
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Leading order  $\boldsymbol{\phi}^{(1)}$  (no external sources):

$$\left. \begin{aligned} [\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)] \boldsymbol{\phi}^{(1)}(x, \tau) &= 0 \\ \boldsymbol{\phi}^{(1)}(x, 0) &= \boldsymbol{\pi}(x) \end{aligned} \right\} \Rightarrow \boldsymbol{\phi}^{(1)}(x, \tau) = \int d^4y \underbrace{G(x-y, \tau)}_{G(x, \tau)} \boldsymbol{\pi}(y)$$

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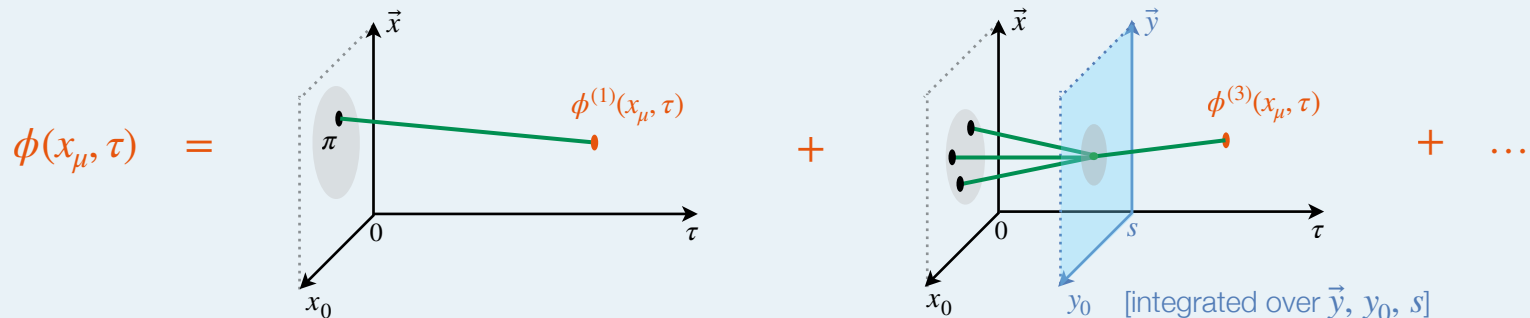
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# Nuclear forces using chiral gradient flow

Regularization is achieved by requiring N to „live“ at a fixed  $\tau$ :  $\mathcal{L}_{\pi N} \rightarrow \mathcal{L}_{\phi N}(\tau) = \mathcal{L}_{\pi N} \Big|_{U \rightarrow W(\tau)}$

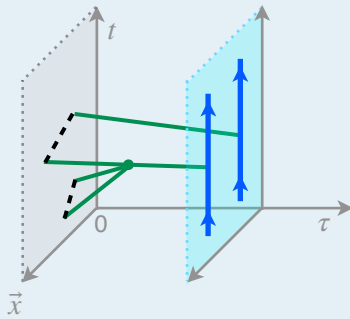
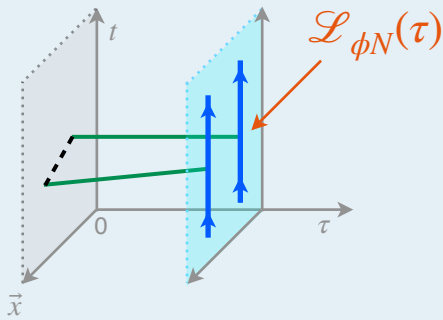
Notice: chiral symmetry manifest since  $W(\tau) \rightarrow RW(\tau)L^\dagger$  for all  $\tau$ .

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## Local field theory in 5d

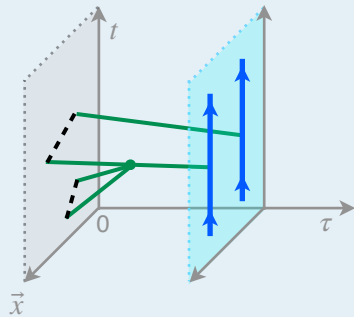
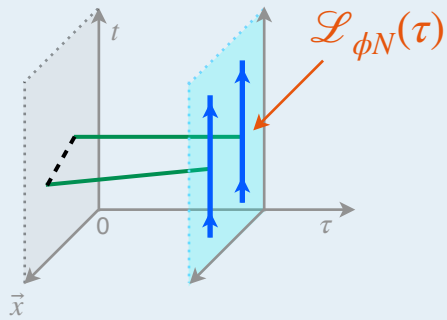


# Nuclear forces using chiral gradient flow

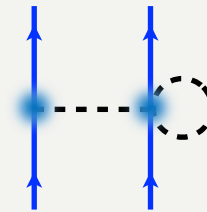
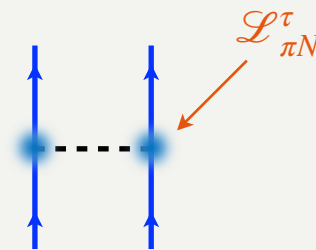
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## Local field theory in 5d



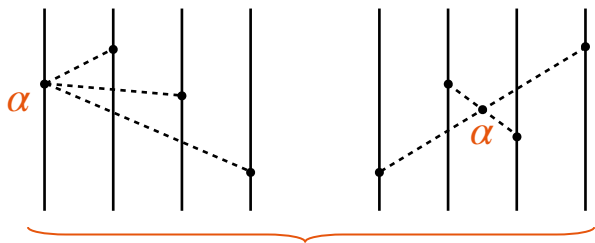
## Smearred (non-local) theory in 4d





# Chiral symmetry and the 4N force

unregularized



The sum of two diagrams **must** be  $\alpha$ -independent

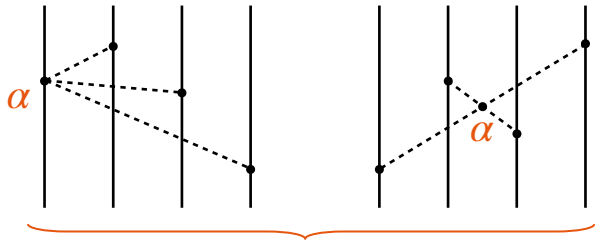
Unregularized expression for this 4NF [EE, EPJA 34 \(2007\)](#):

$$\begin{aligned}
 V^{4N} = & -\frac{g^4}{64F^6} \frac{\hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]}}{(\vec{q}_2^2 + M^2)(\vec{q}_3^2 + M^2)(\vec{q}_4^2 + M^2)} \vec{\sigma}_1 \cdot \vec{q}_{12} \\
 & + \frac{g^4}{128F^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]}}{(\vec{q}_1^2 + M^2)(\vec{q}_2^2 + M^2)(\vec{q}_3^2 + M^2)(\vec{q}_4^2 + M^2)} (M^2 + \vec{q}_{12}^2) + 23 \text{ perm.}
 \end{aligned}$$

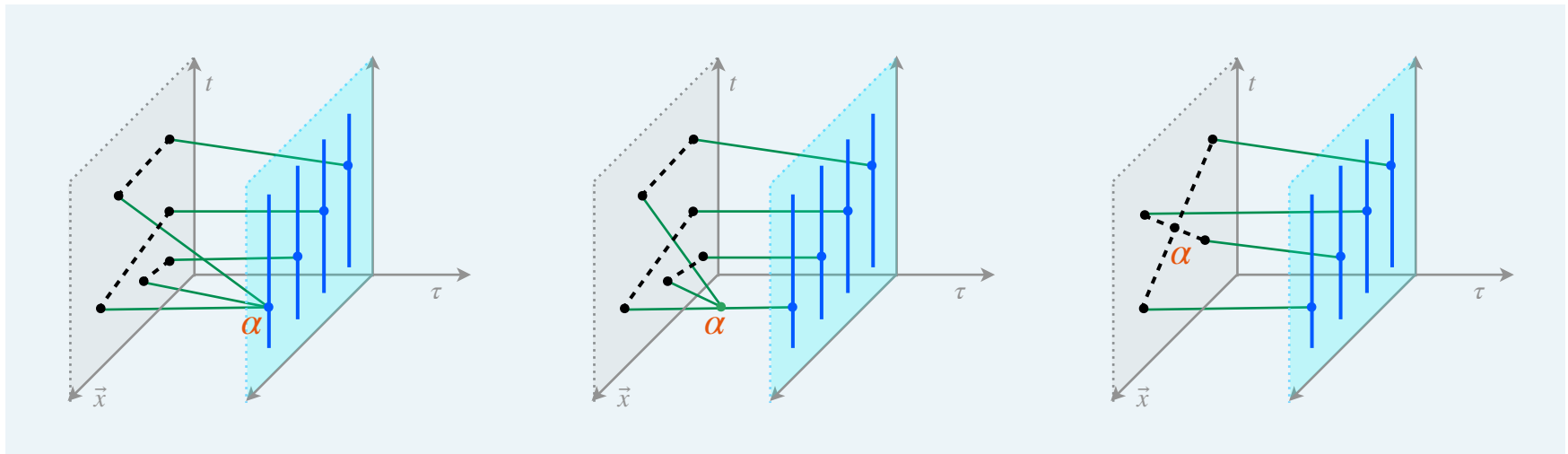
$\hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]} = \tau_1 \cdot \tau_2 \tau_3 \cdot \tau_4 \vec{\sigma}_2 \cdot \vec{q}_2 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_4 \cdot \vec{q}_4$

# Chiral symmetry and the 4N force

unregularized

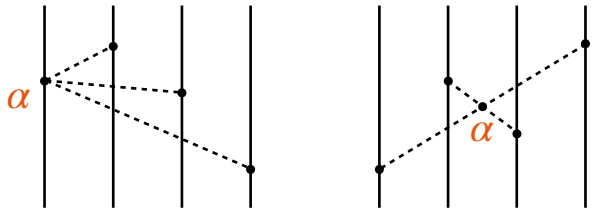


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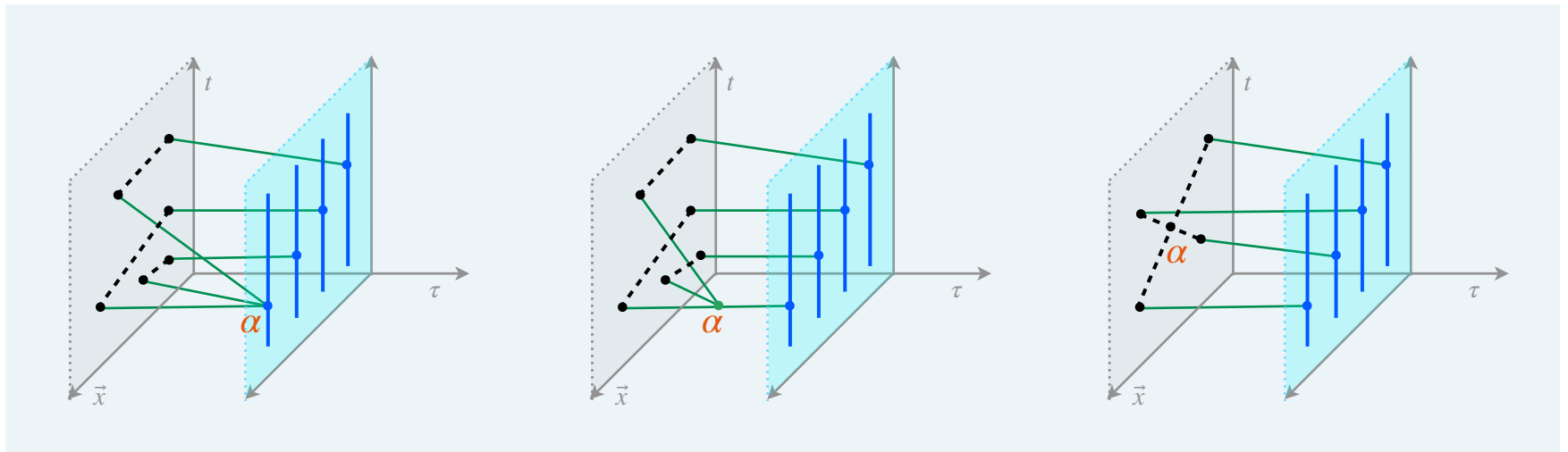
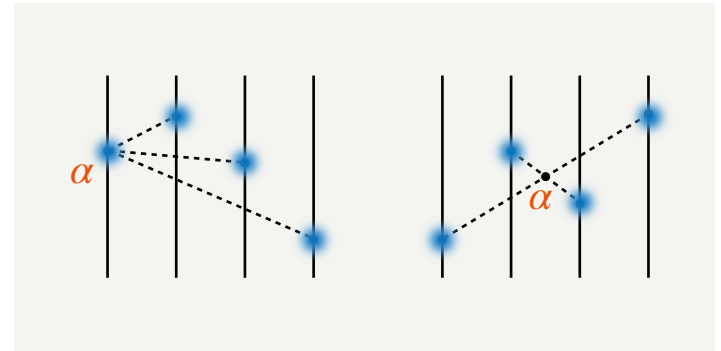


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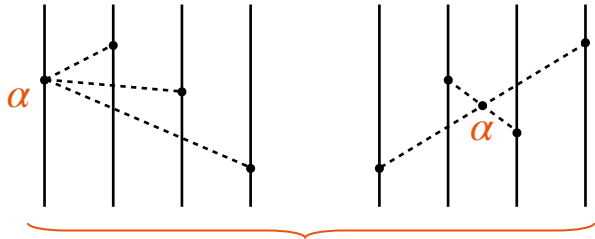


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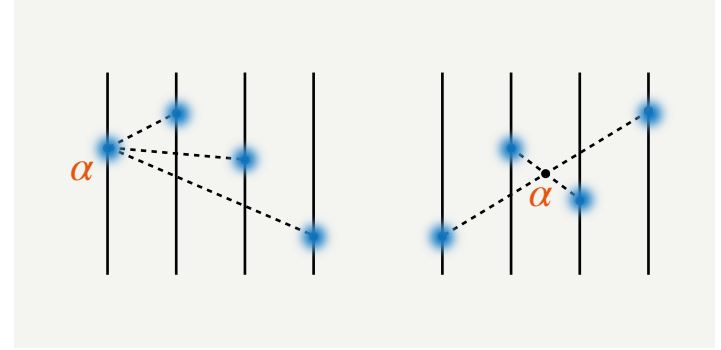


# Chiral symmetry and the 4N force

unregularized



The sum of two diagrams **must** be  $\alpha$ -independent



Regularized expression (ready to use in the A-body Schrödinger equation):

$$\begin{aligned}
 V_{\Lambda}^{4N} = & \frac{g^4}{64F^6} \frac{\hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]}}{(\vec{q}_2^2 + M^2)(\vec{q}_3^2 + M^2)(\vec{q}_4^2 + M^2)} \left[ \vec{\sigma}_1 \cdot \vec{q}_1 (2g_{\Lambda} - 4f_{\Lambda}^{123} + 2f_{\Lambda}^{134} - f_{\Lambda}^{234}) - \vec{\sigma}_1 \cdot \vec{q}_2 f_{\Lambda}^{234} \right. \\
 & + 2\vec{\sigma}_1 \cdot \vec{q}_1 (5M^2 + \vec{q}_1^2 + \vec{q}_2^2 + \vec{q}_3^2 + \vec{q}_4^2 + \vec{q}_{34}^2) \frac{g_{\Lambda} - f_{\Lambda}^{134}}{2M^2 + \vec{q}_1^2 + \vec{q}_3^2 + \vec{q}_4^2 - \vec{q}_2^2} \\
 & \left. - 4\vec{\sigma}_1 \cdot \vec{q}_1 (3M^2 + \vec{q}_1^2 + \vec{q}_2^2 + \vec{q}_3^2 + \vec{q}_4^2 - \vec{q}_{34}^2) \frac{g_{\Lambda} - f_{\Lambda}^{124}}{2M^2 + \vec{q}_1^2 + \vec{q}_2^2 + \vec{q}_4^2 - \vec{q}_3^2} \right] \\
 + & \frac{g^4}{128F^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]}}{(\vec{q}_1^2 + M^2)(\vec{q}_2^2 + M^2)(\vec{q}_3^2 + M^2)(\vec{q}_4^2 + M^2)} (M^2 + \vec{q}_{12}^2) (4f_{\Lambda}^{123} - 3g_{\Lambda}) + 23 \text{ perm.}, \\
 & f_{\Lambda}^{ijk} = e^{-\frac{\vec{q}_i^2 + M^2}{\Lambda^2}} e^{-\frac{\vec{q}_j^2 + M^2}{\Lambda^2}} e^{-\frac{\vec{q}_k^2 + M^2}{\Lambda^2}} \quad \uparrow \quad \uparrow \quad e^{-\frac{\vec{q}_1^2 + M^2}{2\Lambda^2}} e^{-\frac{\vec{q}_2^2 + M^2}{2\Lambda^2}} e^{-\frac{\vec{q}_3^2 + M^2}{2\Lambda^2}} e^{-\frac{\vec{q}_4^2 + M^2}{2\Lambda^2}}
 \end{aligned}$$

(reduces to the unregularized result in the  $\Lambda \rightarrow \infty$  limit)

# Summary and outlook

## New formulation of nuclear chiral EFT:

- gradient flow regularized formulation of chiral EFT Krebs, EE, 2312.13932, to appear in PRC
- path integral method to perform QM reduction of QFT Krebs, EE, 2311.10893, to appear in PRC
  - ⇒ regularized 3N, 4N forces and currents, which are consistent with the SMS NN potentials and respect chiral & gauge symmetries

## Already done:

- NN at N<sup>2</sup>LO, long-range 3NF (still needs to be implemented...) and 4NF at N<sup>3</sup>LO

## Work in progress:

- $\pi$ N scattering inside the Mandelstam triangle (LECs), 3N scattering at N<sup>3</sup>LO

The new method can also be useful for improving convergence of SU(3) BChPT

# Thank you for your attention