



2024/9/25

The 23rd International Conference on Few-Body Problems in Physics

Plan of spin correlation coefficient measurements of deuteron-proton scattering at intermediate energies

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- ❖ **Kyushu Univ.** : T. Wakasa, K. Nishibata, K. Aradono, K. Hirasawa, Y. Nagao, S. Sakaguchi
- ❖ **Miyazaki Univ.**: Y. Maeda ❖ **Saitama Univ.** : S. Otsuka



Contents

- 1. Three-Nucleon Force and deuteron-proton (d - p) elastic scattering**
- 2. Measurement of Spin Correlation Coefficients in \vec{d} - \vec{p} elastic scattering**
 - ❖ Polarized Proton Target ❖ KuJyaku Detector System
 - ❖ Polarized Deuteron Beam
- 3. Experiment with the new systems in January 2024
(d - \vec{p} elastic scattering @ 135 MeV/Nucleon)**
- 4. proton- ${}^3\text{He}$ (p - ${}^3\text{He}$) elastic scattering experiments**
- 5. Summary**





1. Three-Nucleon Force and deuteron-proton elastic scattering

Two-Nucleon Force (NN) and Three-Nucleon Force ($3NF$)

Realistic NN potentials : Argonne ν_{18} (AV18), CD Bonn (CDB), Nijmegen I, II

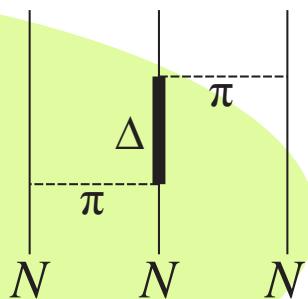
- reproduce 4000 high-precision NN scattering data with accuracy of $\chi^2/\text{data} \sim 1$
- excellent descriptions of deuterons ($A = 2$)



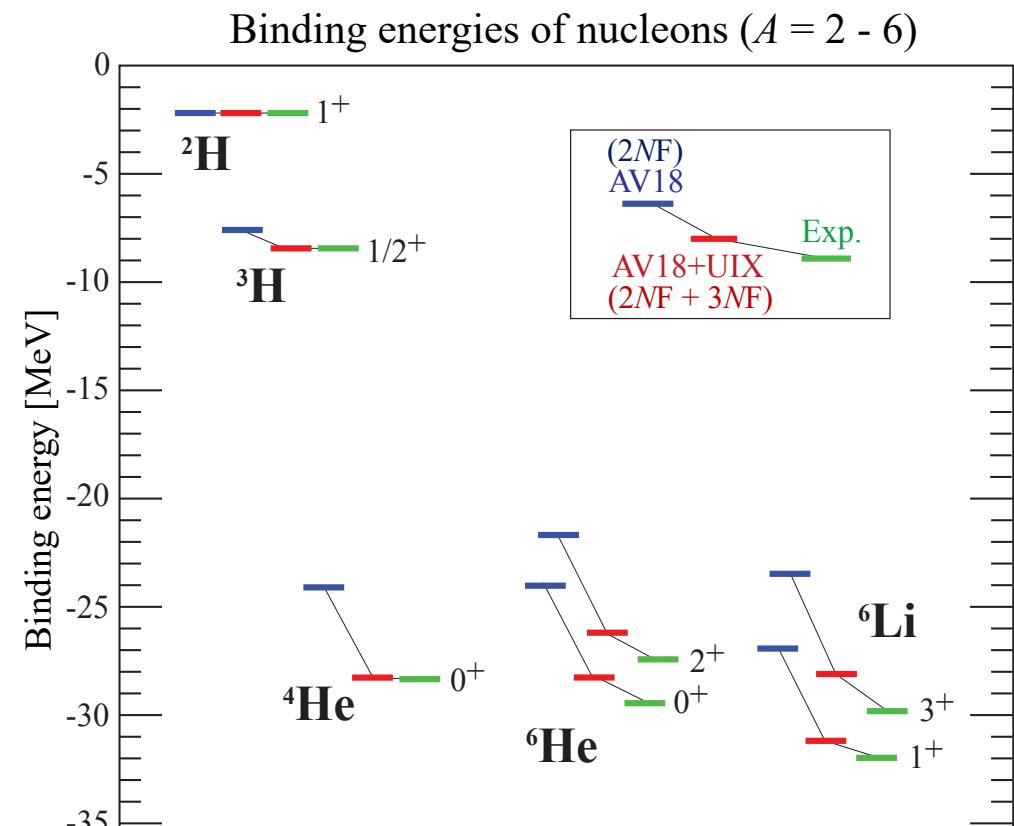
fail to describe properties of $A \geq 3$ nucleon systems
eg. binding energies, nuclear matters

Three-Nucleon Force ($3NF$)

- Typical $3NF$: Fujita-Miyazawa (2π -exchange) type
- Development of Urbana IX (UIX), Tucson-Melbourne (TM)
→ potentials based on 2π -exchange type $3NF$



good descriptions of $A \geq 3$ nuclear binding energies



S. C. Pieper et al., Phys. Rev. C **64**, 014001 (2001).



1. Three-Nucleon Force and deuteron-proton elastic scattering

Study of 3NFs via few-nucleon scattering experiments

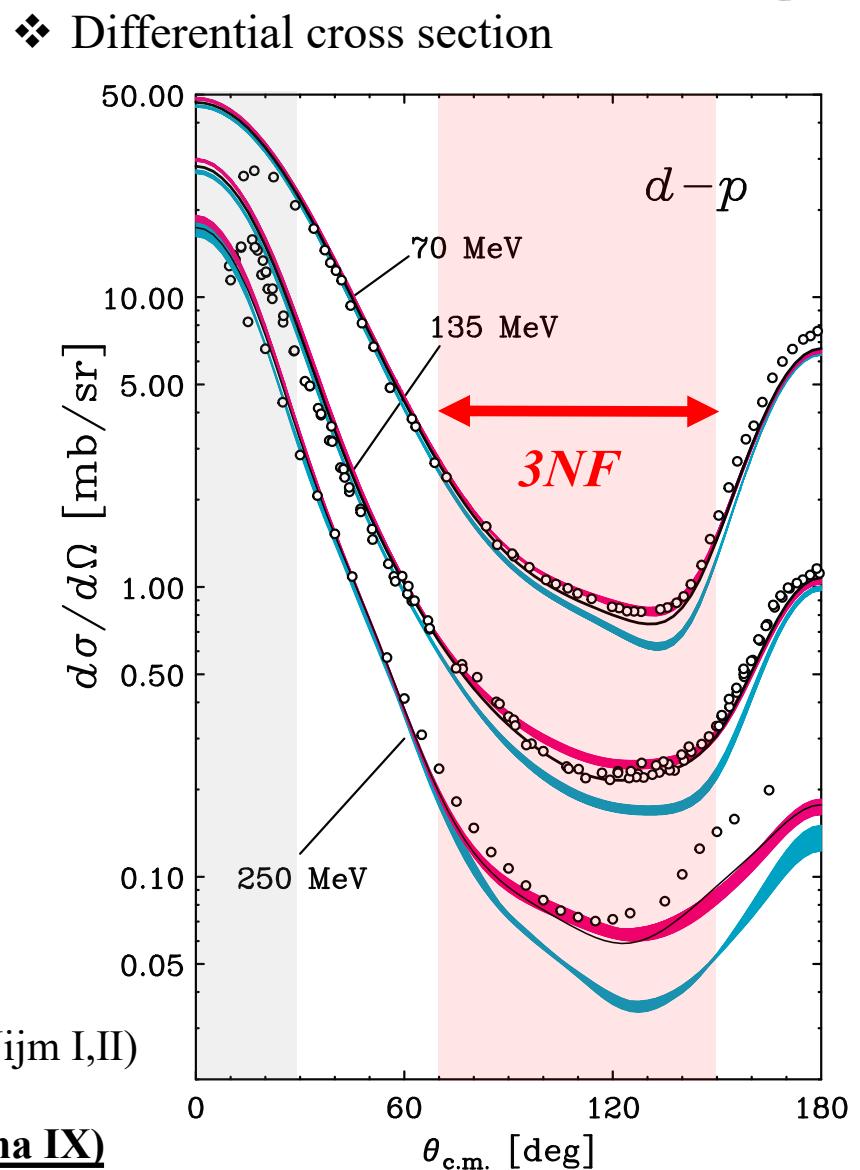
- momentum, spin, isospin dependence of the 3NFs
Direct comparison between...
 - high-precision data** in few-nucleon scattering (differential cross sections, spin observables)
 - theoretical predictions** based on rigorous numerical calculations
- d-p* elastic scattering (E/A 70–300 MeV)**

Differential cross section
→3NF effect observed at cross section minimum

***d-p* elastic scattering :**
effective probe for investigation of the 3NFs

- data
- 2NF (CDB, AV18, Nijm I,II)
- 2NF + 3NF (TM)
- AV18 + 3NF (Urbana IX)

K. Sekiguchi *et al.*, PRC **65**, 034003 (2002).





1. Three-Nucleon Force and deuteron-proton elastic scattering Chiral Effective Field Theory and Three-Nucleon Force

Chiral Effective Field Theory (χ EFT)

- Newly progressing theory based on low energy QCD

χ EFT's $2N$: achieved to level of high precision (N^4LO)

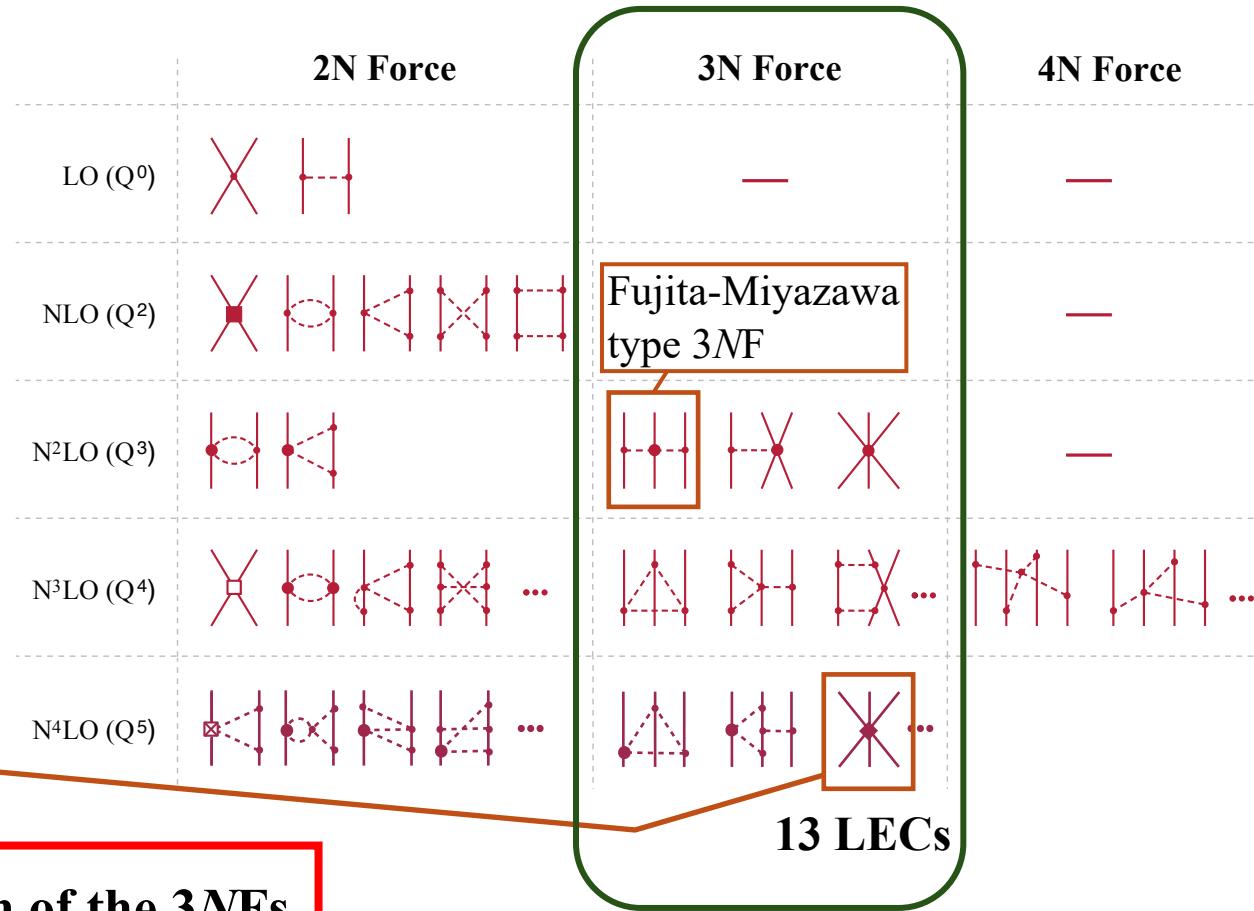
P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018).

χ EFT's $3N$: aim to develop an accurate potential at N^4LO
→ **low energy constants (LECs)** must be fixed
from experimental data

11/13 LECs can be probed in Nd scattering

***d-p* elastic scattering : good probe for investigation of the 3NFs**
→ determination of 11 LECs in N^4LO 's 3NF sector

→ New Experiment necessary!



E. Epelbaum, arXiv:1908.09349 (2019).

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(d - \vec{p} elastic scattering @ 135 MeV/Nucleon)

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5. Summary





2. Measurement of Spin Correlation Coefficients ($C_{i,j}$) in $\vec{d}\text{-}\vec{p}$ elastic scattering

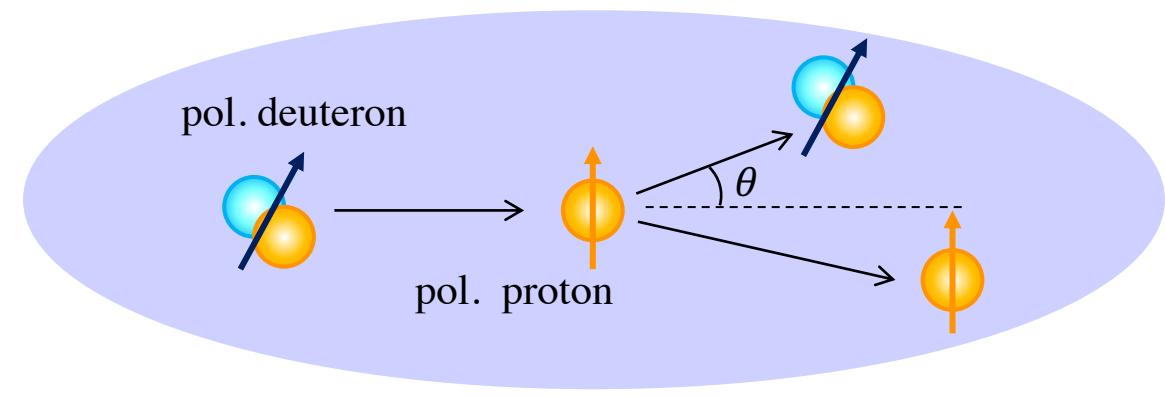
New $d\text{-}p$ experiment for the determination of the 3NF

pd and nd elastic scattering at 65-300 MeV/nucleon

Observable	50	100	200	300	[MeV/N]
$\frac{d\sigma}{d\Omega}$	● ● ● ● ●	● ● ● ● ●	● ● ● ● ●	● ●	
$\vec{p}_n \cdot A_y^p$	● ● ● ● ●	● ● ● ● ●	● ● ● ● ●	● ●	
$\vec{n} \cdot A_y^n$			●	●	
\vec{d}	● ● ●	● ● ●	● ● ●	● ● ●	
iT_{11}	● ● ●	● ● ●	● ● ●	● ● ●	
T_{20}	● ● ●	● ● ●	● ● ●	● ● ●	
T_{22}	● ● ●	● ● ●	● ● ●	● ● ●	
T_{21}	● ● ●	● ● ●	● ● ●	● ● ●	
$\vec{p} \rightarrow \vec{p}'$					
$K_x^{x'} K_y^{y'}$				●	
$K_z^{z'} K_x^{x'} K_z^{z'}$				●	
$\vec{d} \rightarrow \vec{p}$					
$K_y^{y'} K_{yy}^{y'}$	●	●			
$K_{xx}^{y'} K_{xz}^{y'}$		●	●		
$\vec{p}\vec{d}$					
$C_{x,x} C_{y,y} C_{z,z}$		●	●		
$C_{x,z} C_{z,z}$		●	●		
$C_{xx,y} C_{yy,y}$		●	●		
$C_{xz,y} C_{yz,x} C_{xy,x}$		●	●		

Measurement of spin correlation coefficients in $d\text{-}p$ elastic scattering @ RIKEN

➤ for the determination of LECs in N4LO 3NFs



Polarization of both beam and target necessary!

- ❖ Polarized deuteron beam (\vec{d}) : polarized ion source @ RIKEN
- ❖ Polarized proton target (\vec{p}) : newly developed based on triplet-DNP
- ❖ New detector system (KuJyaku) developed for measurement of L, R, U, D at wide angular (θ) range



2. Measurement of Spin Correlation Coefficients ($C_{i,j}$) in \vec{d} - \vec{p} elastic scattering **Polarized Proton Target (\vec{p})**

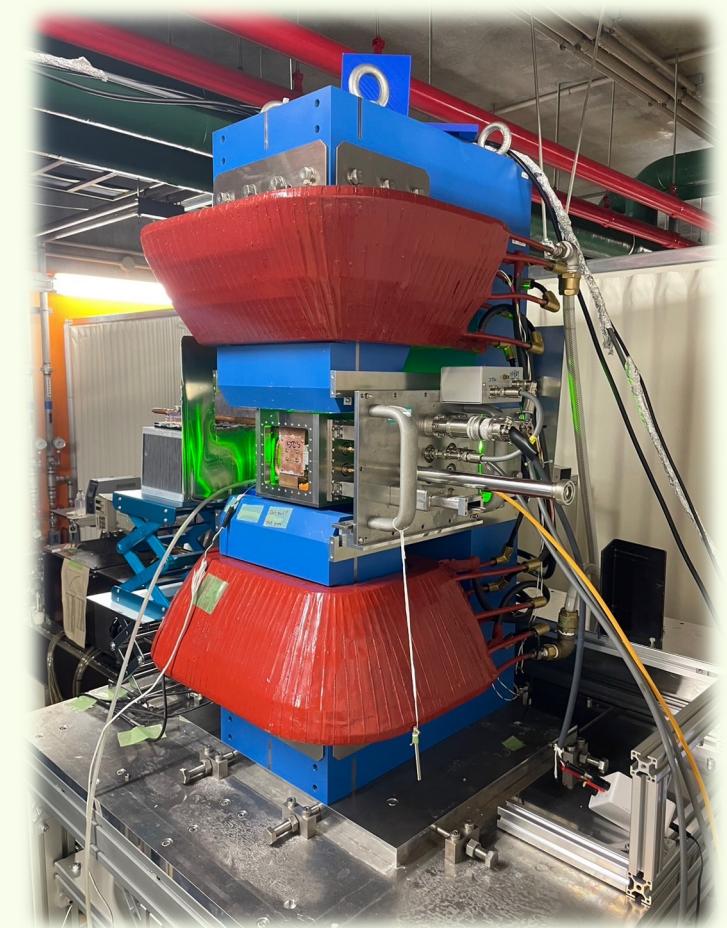
→based on the **Triplet-Dynamic Nuclear Polarization (triplet-DNP) Method**

Past achievements : Proton polarization of 34% @ 0.4 T and ~300K

K. Tateishi *et al.*, www.pnas.org/cgi/doi/10.1073/pnas.1315778111

necessary conditions for \vec{d} - \vec{p} elastic scattering experiment

- **measurement at wide angular range ($\theta_{lab} : \pm 60^\circ$)**
- **detection of scattered particles with relatively low energy (60-200 MeV)**
→ static field under ~1 T
- **polarization around 10 %**

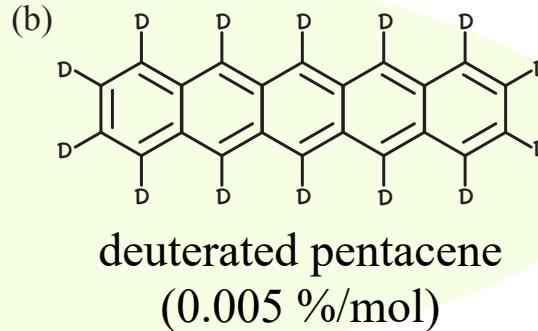
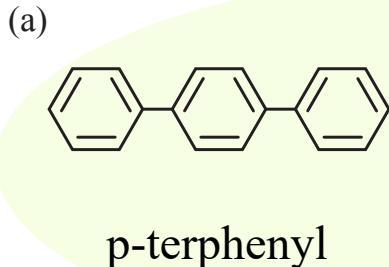


2. Measurement of Spin Correlation Coefficients ($C_{i,j}$) in $\vec{d}\text{-}\vec{p}$ elastic scattering

Polarized Proton Target (\vec{p})



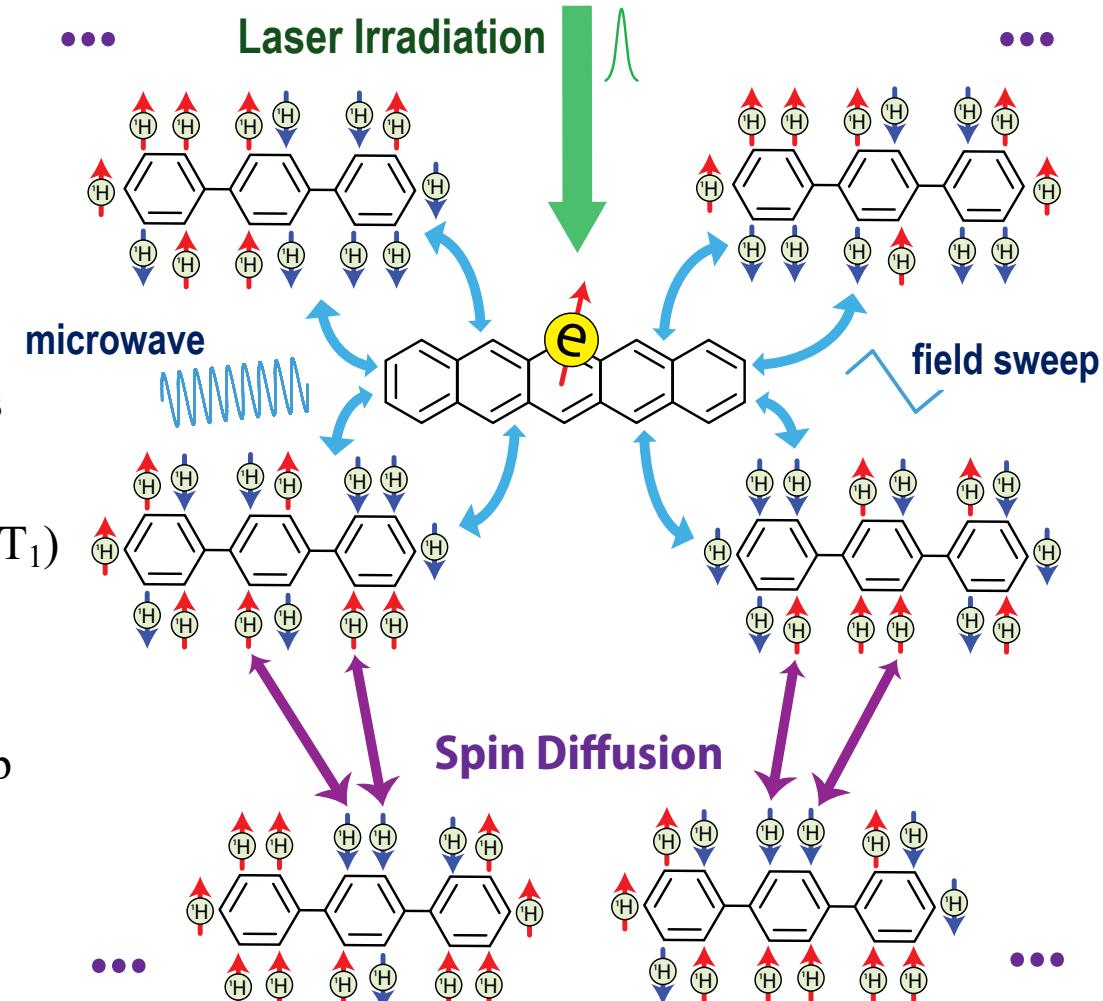
Target : pentacene doped *p*-terphenyl single crystal



Size : $\varphi 10 \times 2.5 \text{ mm}^3$

- ① Optical excitation of electrons in pentacene and decay to triplet state (T_1)
→ **Laser Irradiation** of ~ 85%
- ② **Polarization transfer** from electrons to protons
: integrated solid effect (ISE) = Microwave irradiation + field sweep
- ③ Proton polarization localized around pentacene after ISE
→ spontaneously diffused and averaged out = **Spin diffusion**

Aim : proton polarization of around 10%

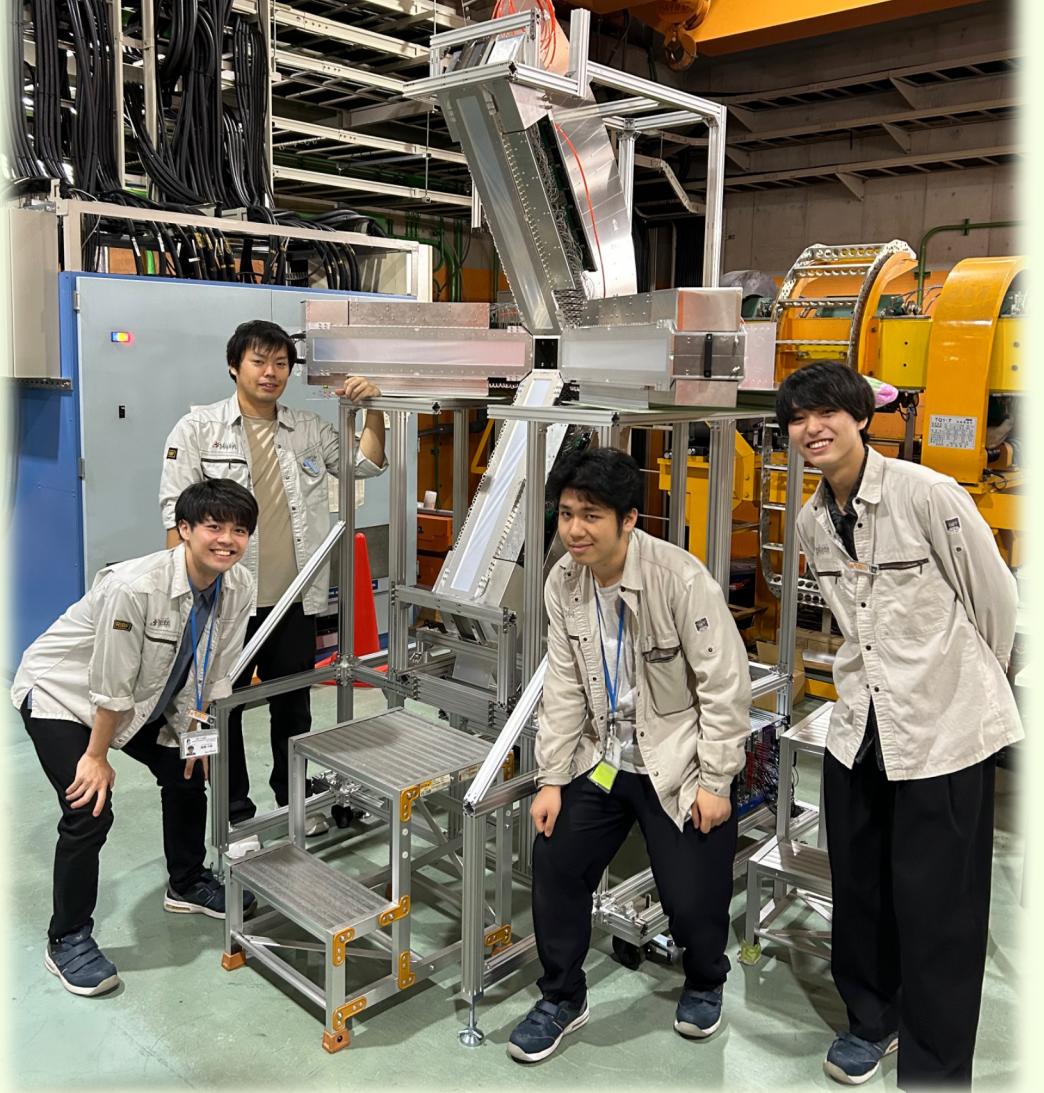


2. Measurement of Spin Correlation Coefficients ($C_{i,j}$) in \vec{d} - \vec{p} elastic scattering

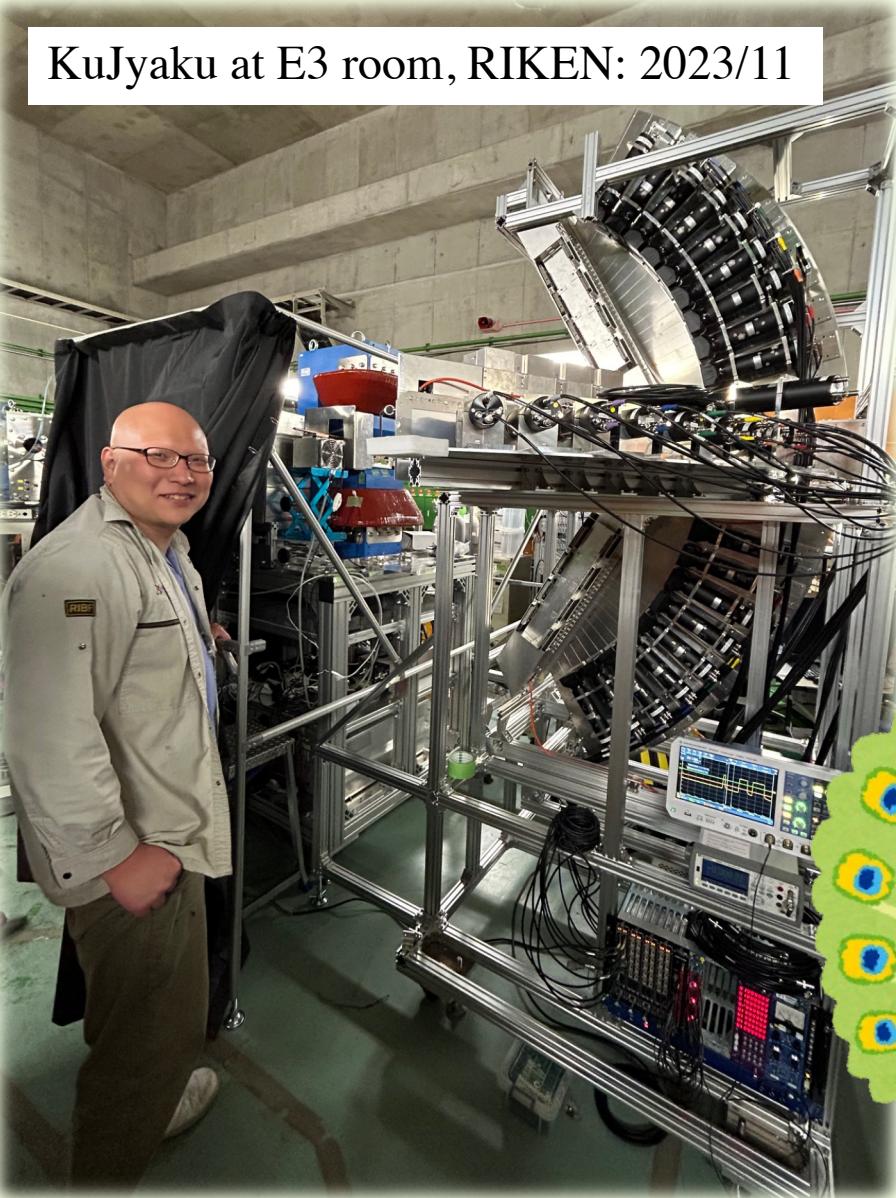
KuJyaku Detector System



Placement of MWDCs (L,R,U,D): 2023/9

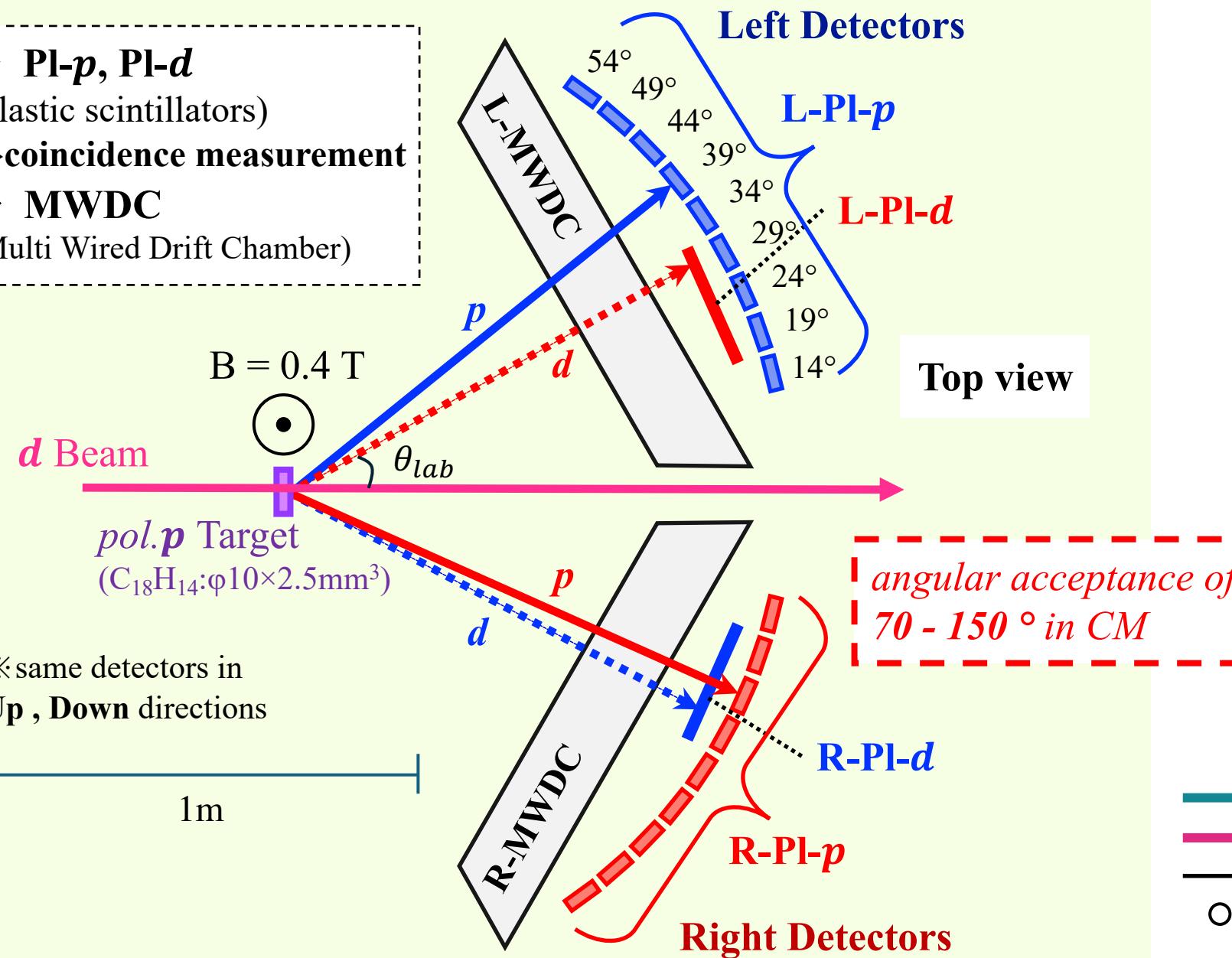


KuJyaku at E3 room, RIKEN: 2023/11

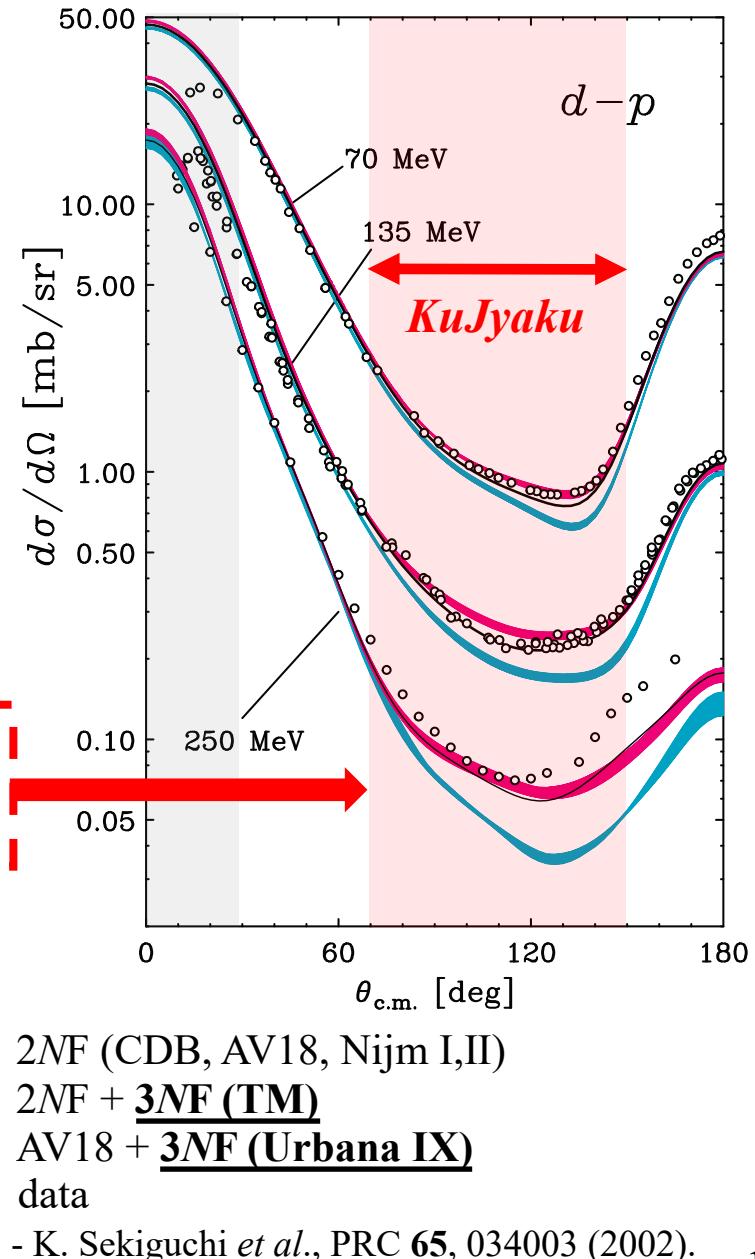


2. KuJyaku Detector System

- ❖ Pl- p , Pl- d
(plastic scintillators)
→coincidence measurement
- ❖ MWDC
(Multi Wired Drift Chamber)



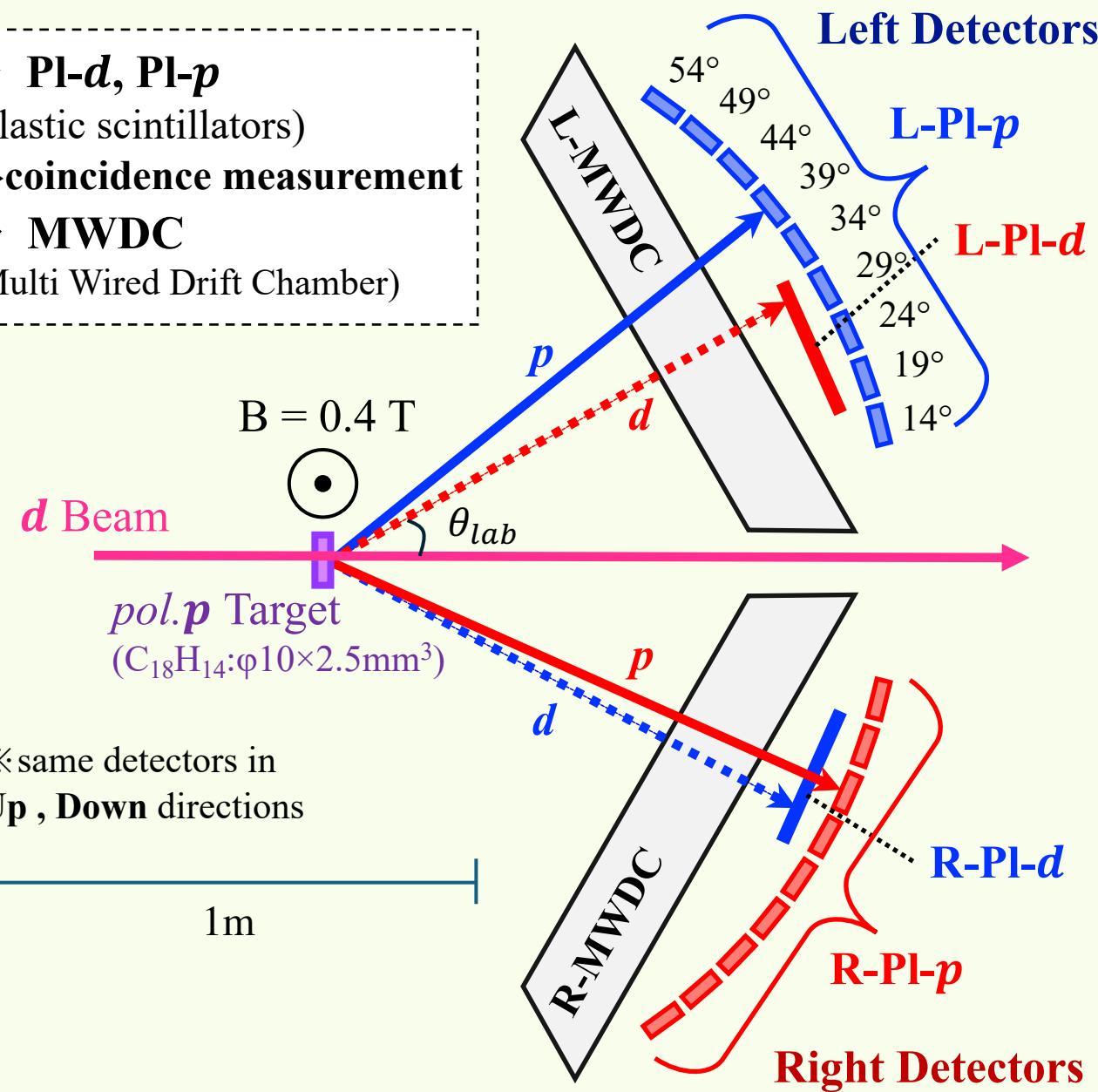
- ❖ Differential cross section





2. KuJyaku Detector System

- ❖ Pl- d , Pl- p
(plastic scintillators)
→ coincidence measurement
- ❖ MWDC
(Multi Wired Drift Chamber)



- ❖ trajectories of d and p
→ bent at target (0.4T)

- simulation via:
 - TOSCA (OPERA-3d)
 - GEANT4

- MWDC:
→ for tracking trajectories of d and p

→ consistency between simulation and measured trajectory

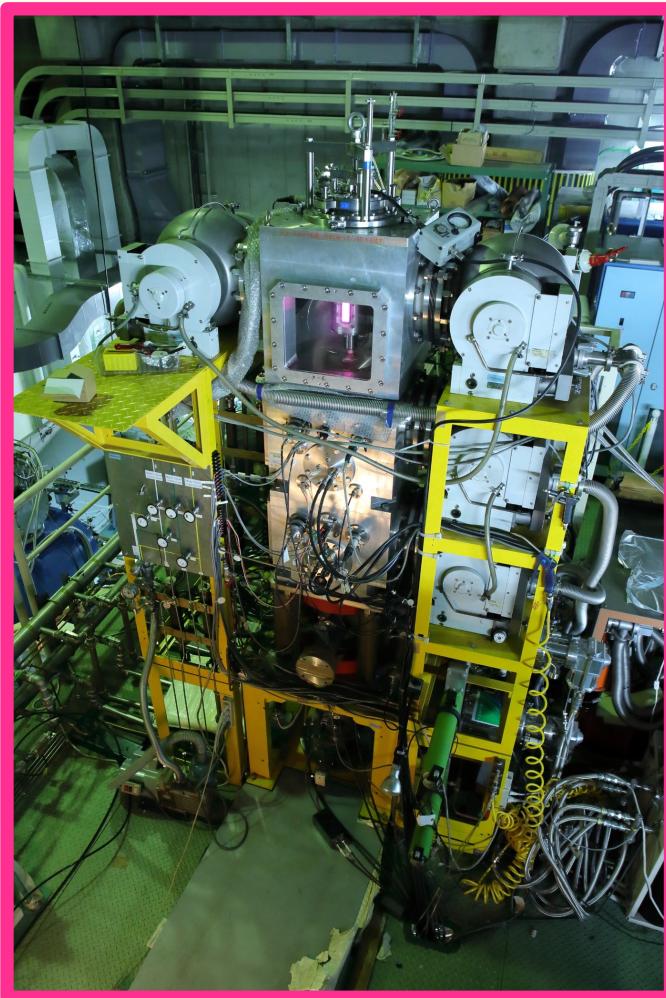
→ analysis ongoing

2. Measurement of Spin Correlation Coefficients ($C_{i,j}$) in $\vec{d}\text{-}\vec{p}$ elastic scattering

Polarized deuteron beam (\vec{d}) : Polarized ion source



Polarized ion source



3 m

~2012

- polarization : **60-80%** of theoretically maximum values

2023/7~

- water leakage from cooling pipe inside vacuum chamber
→ repairing / maintenance

2024/9/14 → beam test to check the deuteron polarization !

- polarization : **50-80%** (preliminary)



**Ready for the
Spin Correlation Coefficients Measurement**

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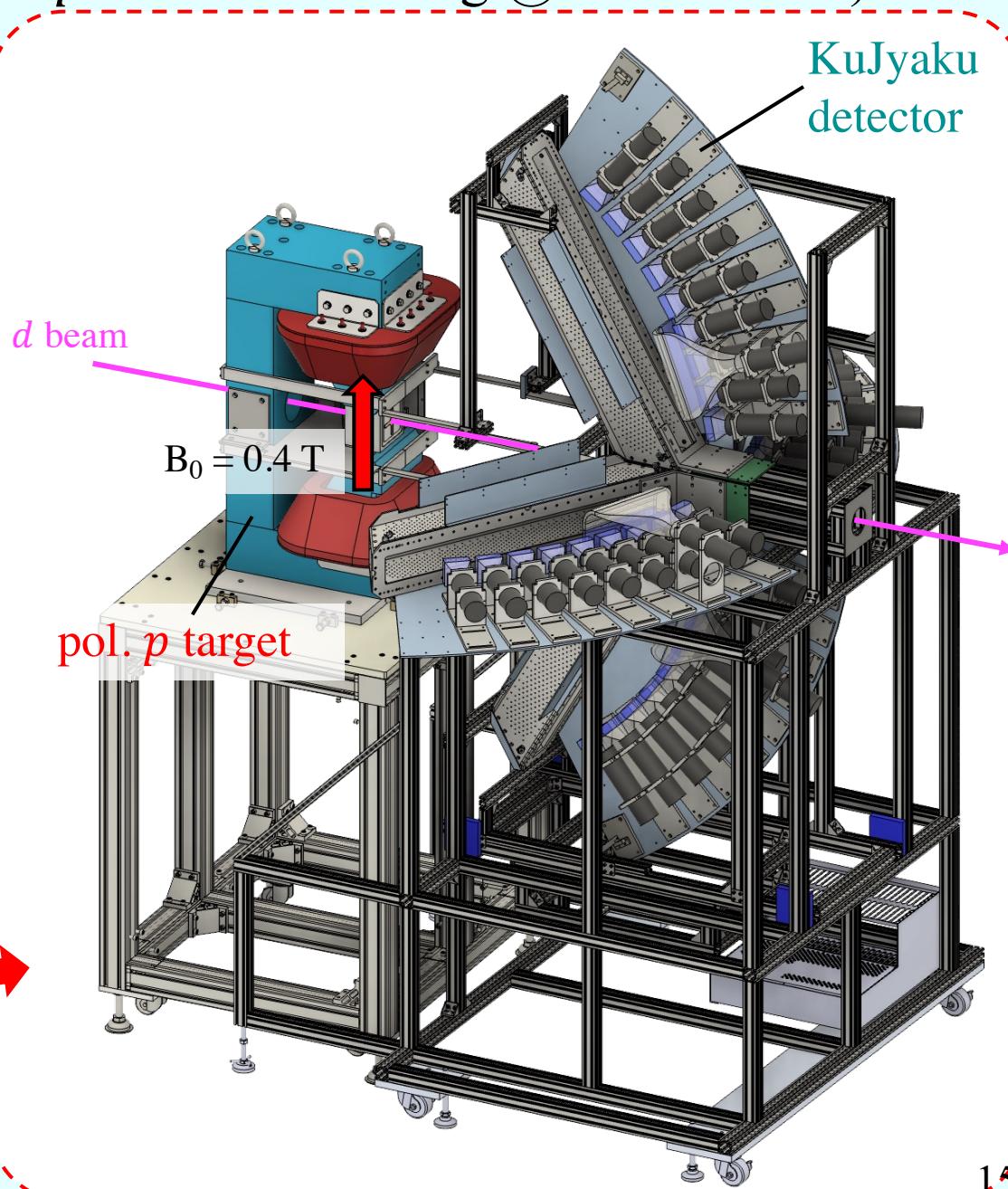
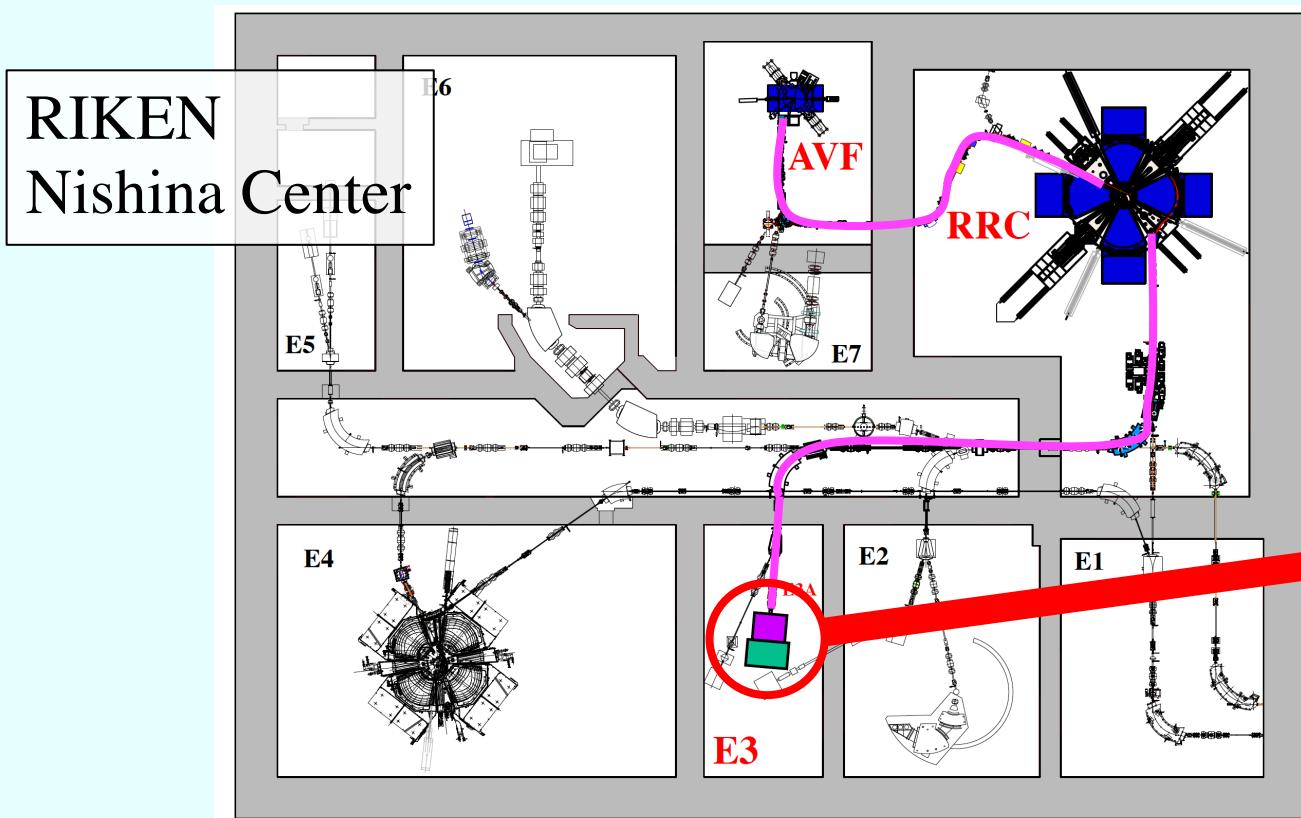


TOMOE₁₄

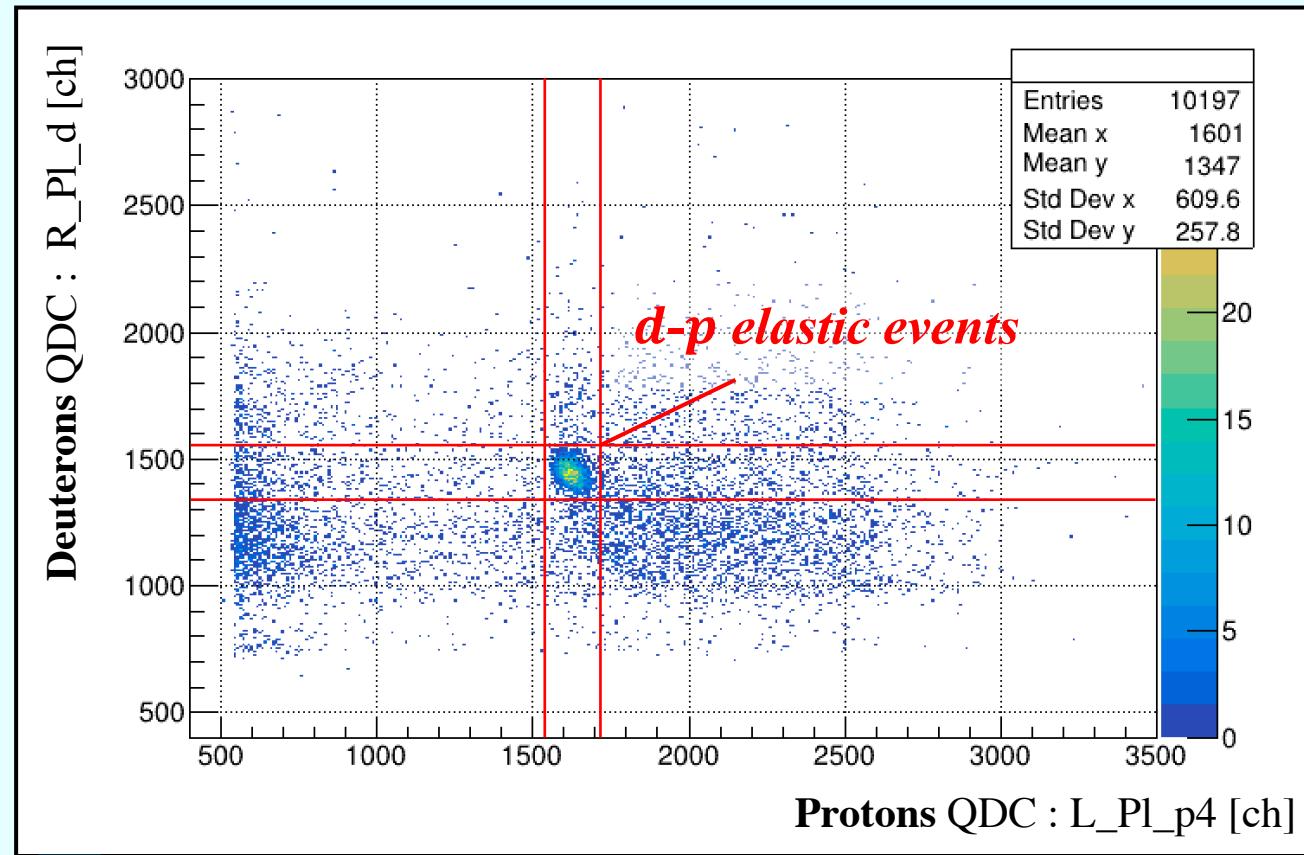
3. Experiment with the new systems in January 2024 (d - \vec{p} elastic scattering @ 135 MeV/N)

➤ First beam test on target/detector systems using d beam

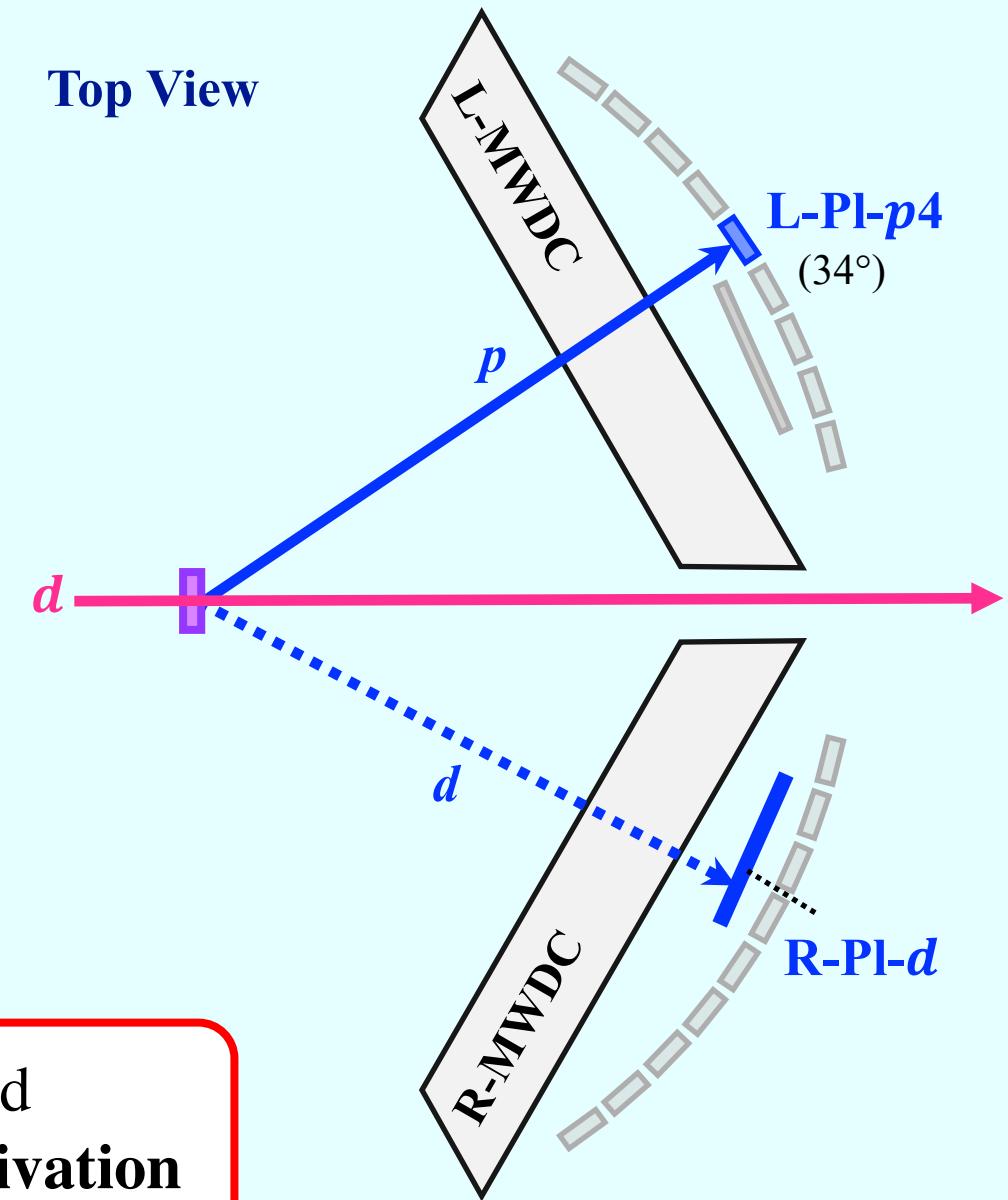
Observables	$d\sigma/d\Omega$, A_y , target polarization
Machine time	1/8 9:00 - 1/10 9:00
beam	135 MeV/Nucleon unpolarized d beam
target	pol. p target (C ₁₄ H ₁₈ crystal : $\phi 10$, 2.5 mm)
detector	KuJyaku system (plastic scintillators, MWDCs)



3. Experiment with the new systems in January 2024 (d - \vec{p} elastic scattering @ 135 MeV/N) Event Selection



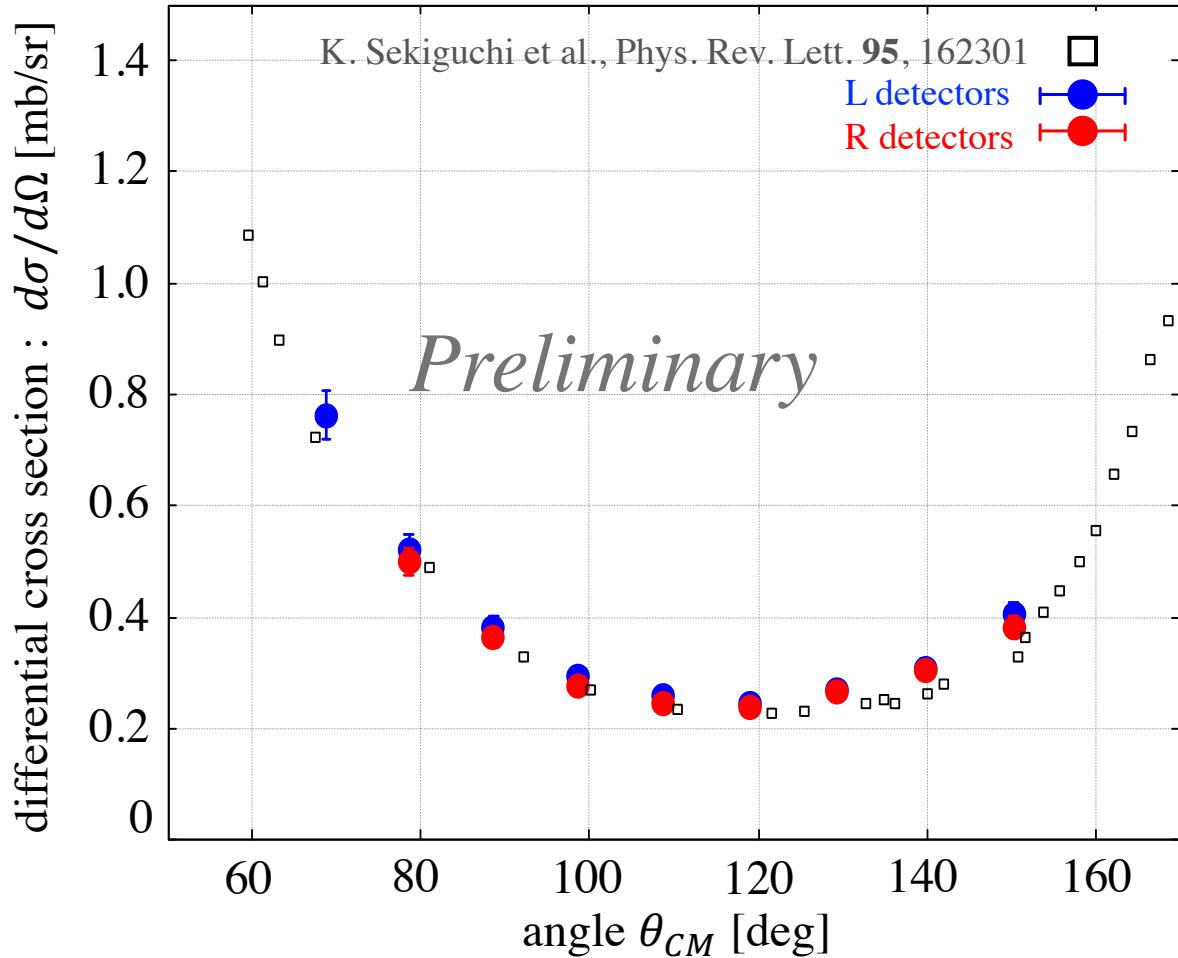
Top View



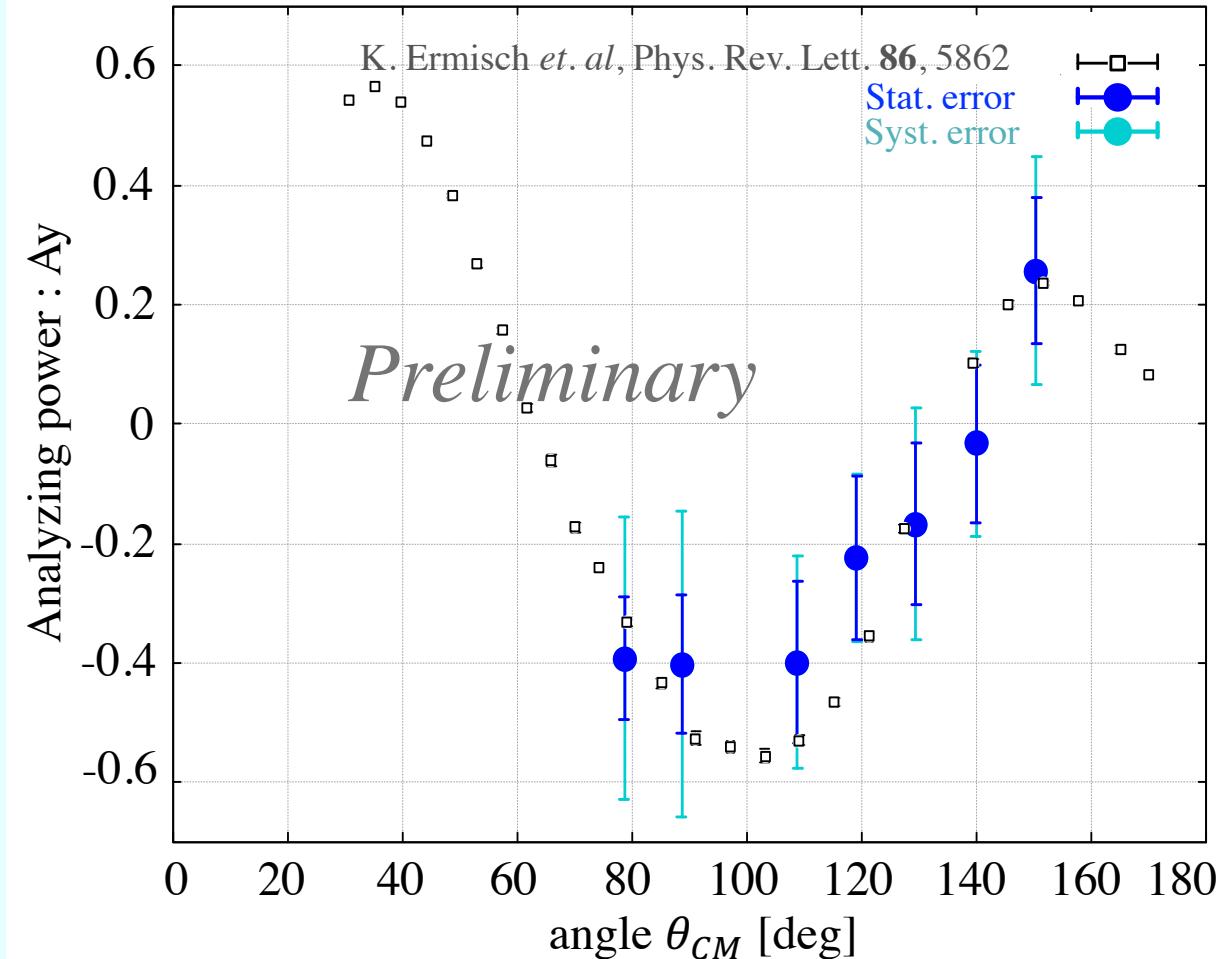
d - p elastic events successfully observed
→ yield used for $d\sigma/d\Omega$ and A_y derivation

3. Experiment with the new systems in January 2024 (d - \vec{p} elastic scattering @ 135 MeV/N)

Preliminary Results



※error bars: systematic error
(static error within circles)



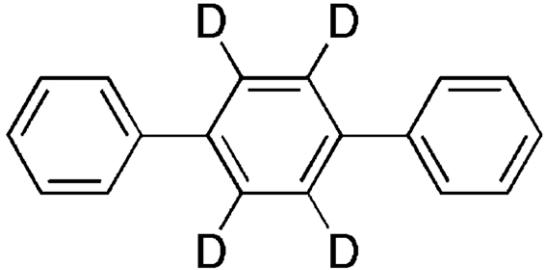
※large error due to *low polarization of proton target (~2-3%)*
→improvements necessary!



Future plans for the Measurement of Spin Correlation Coefficients in $\vec{d}\text{-}\vec{p}$ elastic scattering

❖ *Necessity of improvements in target polarization*

- ✓ new material for target crystal : ***p*-terphenyl-*d*₄**
 - under development
 - polarization expected to rise to ~10%

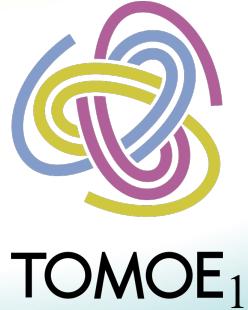


❖ **Measurement of Spin Correlation Coefficients in $\vec{d}\text{-}\vec{p}$ elastic scattering**

- ✓ Next : $\vec{d} + \vec{p}$ @ 100 & 135 MeV/Nucleon → $C_{y,y}, C_{x,x}, C_{yy,y}$
- ✓ Future : $\vec{d} + \vec{p}$ @ 100 & 135 MeV/Nucleon → $C_{z,x}, C_{xx,y}, C_{xy,x}, C_{yz,x}, C_{xz,y}$

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4. proton- ${}^3\text{He}$ (p - ${}^3\text{He}$) scattering experiments

Isospin Dependence of 3NFs

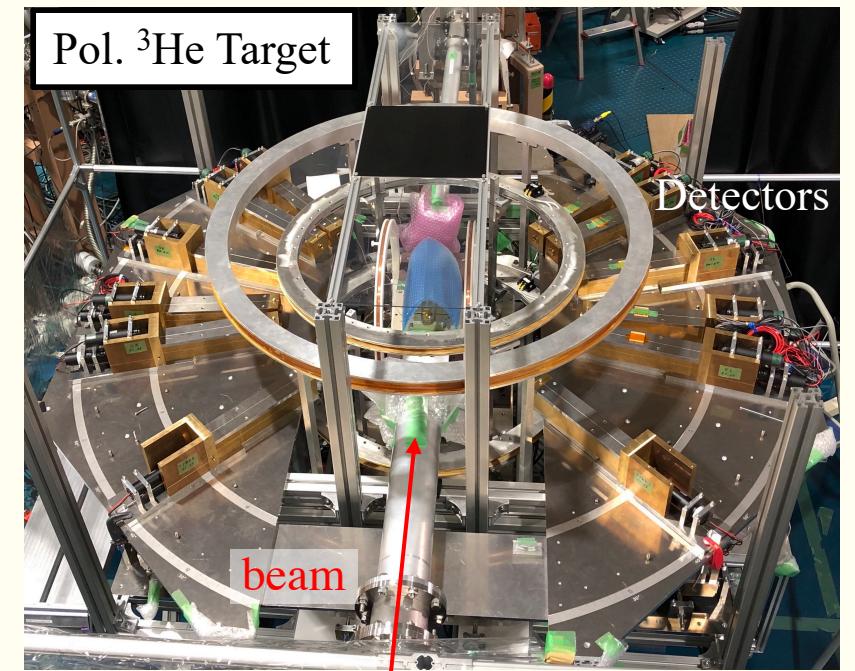
- $d + p$: isospin channel limited to $T = 1/2$
- **$T = 3/2$ channel** of 3NFs : Important roles for exploring **neutron-rich nuclei & pure neutron matter**

Proton- ${}^3\text{He}$ elastic Scattering @ 65-100 MeV for investigation of $T = 3/2$ channel in 3NFs

Our Experiments

Incident Energy	65 MeV	70 MeV	65 MeV	100 MeV
Beams	pol. p	p	pol. p	pol. p
Observables	$d\sigma/d\Omega, A_y^p$	$A_y^{{}^3\text{He}}$	$A_y^p, A_y^{{}^3\text{He}}, C_{y,y}$	$A_y^p, A_y^{{}^3\text{He}}, C_{y,y}$
Measured Angles ($\theta_{\text{c.m.}}$)	27° – 170°	46° – 141°	46° – 133°	47° – 149°
Facilities	RCNP, Osaka Univ.	CYRIC, Tohoku Univ.	RCNP, Osaka Univ.	RCNP, Osaka Univ.
Exp. Course	WS course	41 course	ENN course	ENN course

- **pol. ${}^3\text{He}$ gas target** : Alkali-Hybrid SEOP type
→ polarization : **30-40%** as of 2018



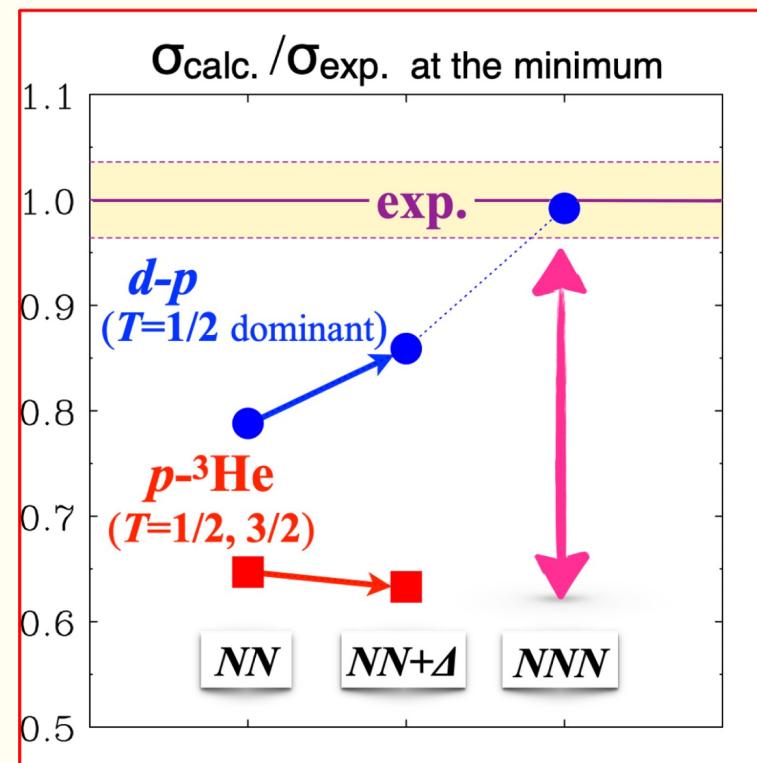
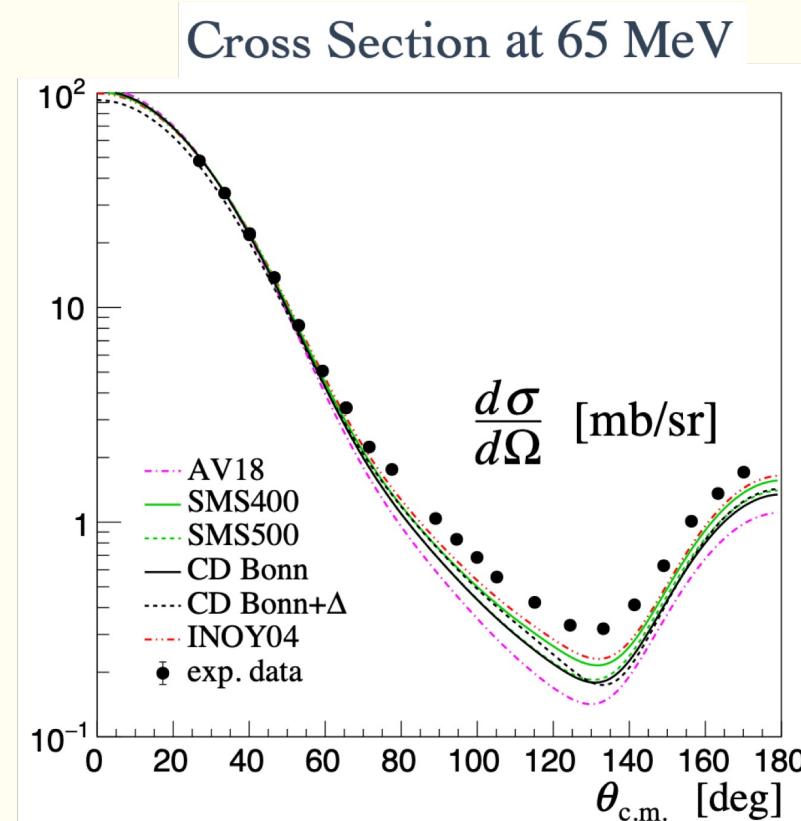
4. proton- ${}^3\text{He}$ (p - ${}^3\text{He}$) scattering experiments



Isospin Dependence of 3NFs

- $d + p$: isospin channel limited to $T = 1/2$
- **$T = 3/2$ channel** of 3NFs : Important roles for exploring **neutron-rich nuclei & pure neutron matter**

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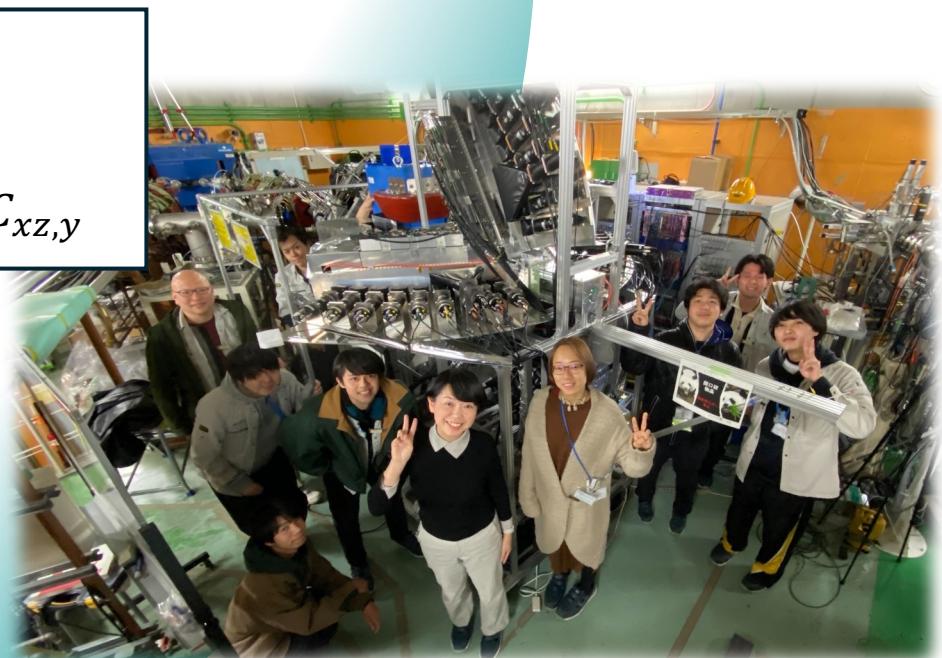


- Measurement of spin correlation coefficients in \vec{d} - \vec{p} elastic scattering to determine 11 LECs in N⁴LO of the 3NF sector in Chiral EFT
 - ❖ Polarized Deuteron Beam ❖ Polarized Proton Target ❖ KuJyaku Detector System
- Experiment with the new systems in January 2024 (d - \vec{p} elastic scattering @ 135 MeV/Nucleon, RIKEN)
 - ✓ First beam test on the target and detector systems via deuteron beam
 - ✓ Analysis on the differential cross sections / Analyzing Powers → ongoing
 - ✓ New material for target crystal : ***p*-terphenyl-*d*₄**

❖ Plan for Measurement of Spin Correlation Coefficients

- ✓ Next : $\vec{d} + \vec{p}$ @ 100 & 135 MeV/Nucleon → $C_{y,y}$, $C_{x,x}$, $C_{yy,y}$
- ✓ Future : $\vec{d} + \vec{p}$ @ 100 & 135 MeV/Nucleon → $C_{z,x}$, $C_{xx,y}$, $C_{xy,x}$, $C_{yz,x}$, $C_{xz,y}$

- ***p*-³He elastic scattering**
→ for investigation of $T = 3/2$ channel in 3NFs



Thank you for your attention.



Back ups



TOMOE₂₄



JST ERATO Three-Nucleon Force Project (PI : Kimiko Sekiguchi)



Nuclear Medicine



RI production

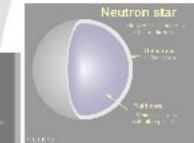
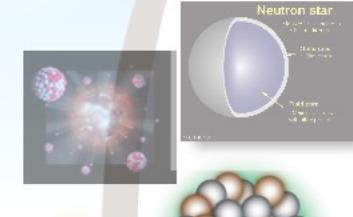


Engineering

Nuclear fusion & fission

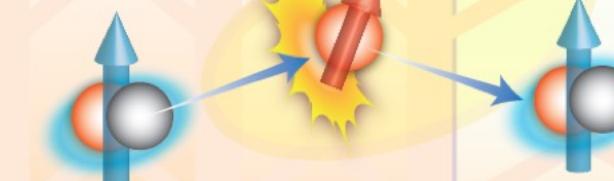
Nucleosynthesis

Neutron star



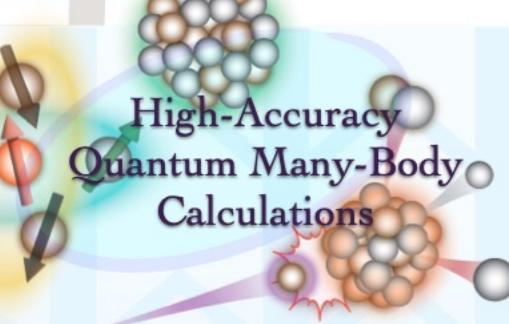
Applied Science Evolution of Nuclear Data

Polarization Experiment
- Few-Nucleon Systems -



High Precision
NN+NNN
Force

Nuclear Forces
from Chiral Effective Field
Theory



Ultra Cold Atom
Experiment



Fundamental Science Descriptions of Nuclei from First Principles

Establishment of Quantum Many-Body Simulation Tool of Nuclear Phenomena
with High-predictive Power

2. Measurement of Spin Correlation Coefficients ($C_{i,j}$) in $\vec{d} - \vec{p}$ elastic scattering → polarization of **beam** and **target** necessary

❖ Polarized deuteron beam (\vec{d}) : polarized ion source @ RIKEN RIBF

H. Okamura *et al.*, AIP Conf. Proc. **293**, 84 (1994).

❖ Polarized proton target (\vec{p}) : newly developed based on triplet-DNP method (2021~)

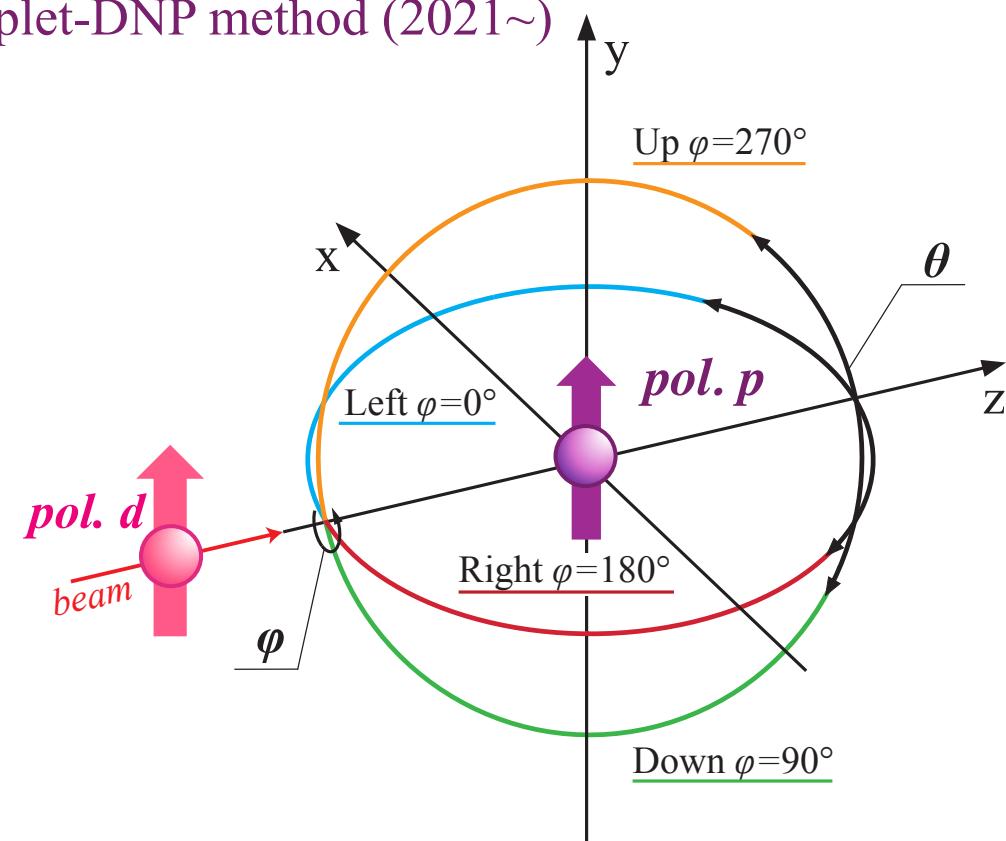
Polarized cross sections of Left, Right, Up, Down directions

$$L(\theta) = L_0(\theta) \left\{ 1 + \frac{3}{2} p_y (A_y^d(\theta) + p_y^T C_{y,y}(\theta)) + p_y^T A_y^p(\theta) + \frac{1}{2} p_{yy} (A_{yy}^d(\theta) + p_y^T C_{yy,y}(\theta)) \right\},$$

$$R(\theta) = R_0(\theta) \left\{ 1 + \frac{3}{2} p_y (-A_y^d(\theta) + p_y^T C_{y,y}(\theta)) - p_y^T A_y^p(\theta) + \frac{1}{2} p_{yy} (A_{yy}^d(\theta) - p_y^T C_{yy,y}(\theta)) \right\},$$

$$U(\theta) = U_0(\theta) \left\{ 1 + \frac{3}{2} p_y p_y^T C_{x,x}(\theta) + \frac{1}{2} p_{yy} A_{xx}^d(\theta) \right\},$$

$$D(\theta) = D_0(\theta) \left\{ 1 + \frac{3}{2} p_y p_y^T C_{x,x}(\theta) + \frac{1}{2} p_{yy} A_{xx}^d(\theta) \right\}.$$



❖ New detector system (KuJyaku) developed for Measurement of L, R, U, D at wide angular (θ) range

2. Detector System for $\vec{d} - \vec{p}$ scattering experiment

→ derivation of $C_{i,j}$
 → measurement of
 pol. cross sections

Coincidence measurement of \vec{d} and \vec{p}

※ for reduction of background events
 from target crystal ($C_{10}H_8$)

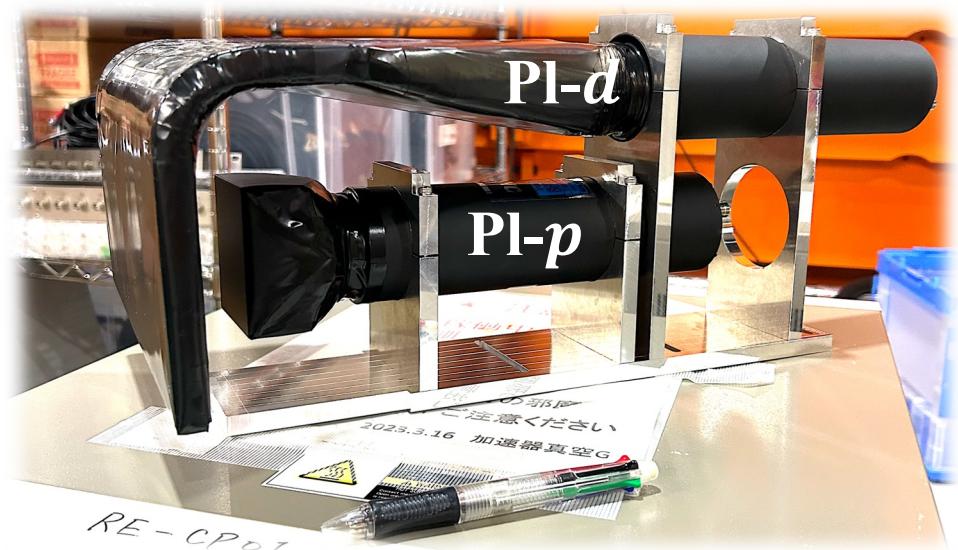
Pl-p (plastic scintillators): → recoil protons

Size	70mm ^L × 70mm ^H × 25mm ^t
PMT	H7195

$$\Delta\Omega : \pm 2[\text{deg}] \\ (3.8 \times 10^{-3} [\text{sr}])$$

Pl-d (plastic scintillators): → scattered deuterons

Size	250mm ^L × 70mm ^H × 10mm ^t
PMT	H7195



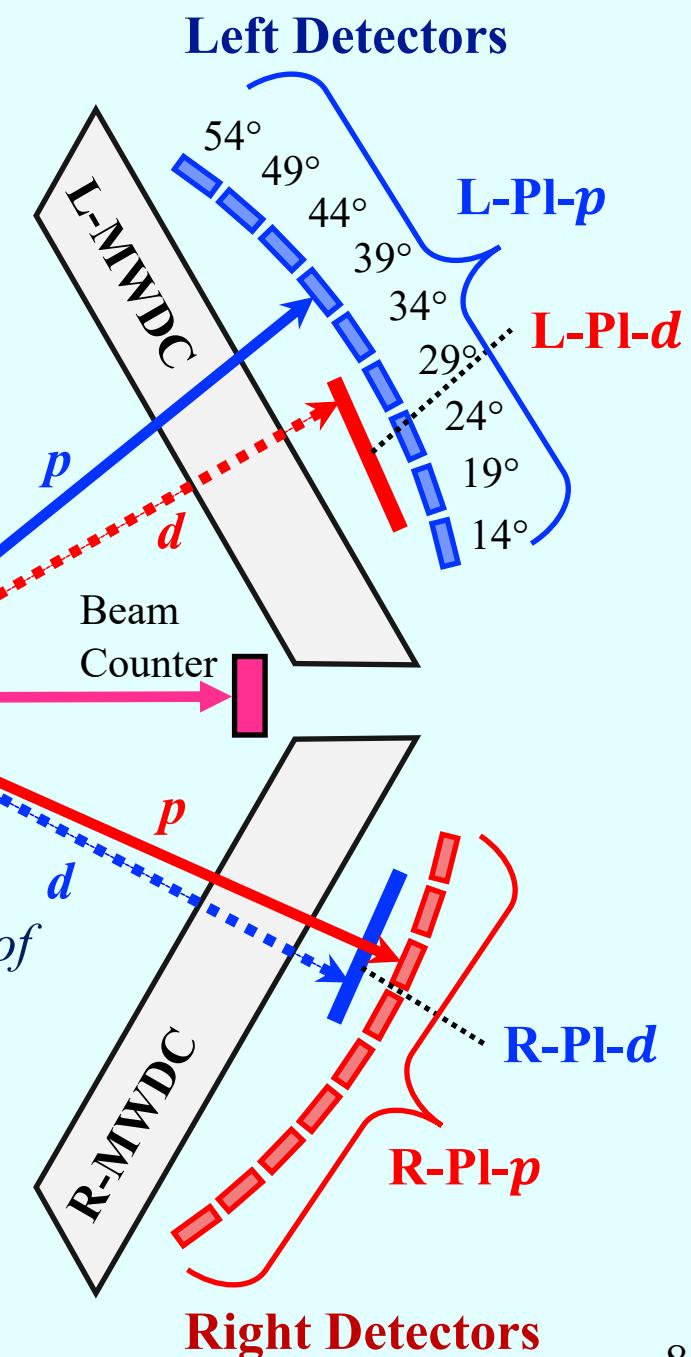
$$B = 0.4 \text{ T}$$

pol.d
Beam

pol.p Target
($C_{10}H_8: \phi 10 \times 3 \text{ mm}^3$)

angular acceptance of
70 - 150 ° in CM

※ same detectors in
Up , Down directions



2. Detector System for $\vec{d} - \vec{p}$ scattering experiment

→ derivation of $C_{i,j}$
 → measurement of
 pol. cross sections

Coincidence measurement of \vec{d} and \vec{p}

※ for reduction of background events
 from target crystal ($C_{10}H_8$)

Pl-p (plastic scintillators): → recoil protons

Size	70mm ^L × 70mm ^H × 25mm ^t
PMT	H7195

$$\Delta\Omega : \pm 2[\text{deg}] \\ (3.8 \times 10^{-3} [\text{sr}])$$

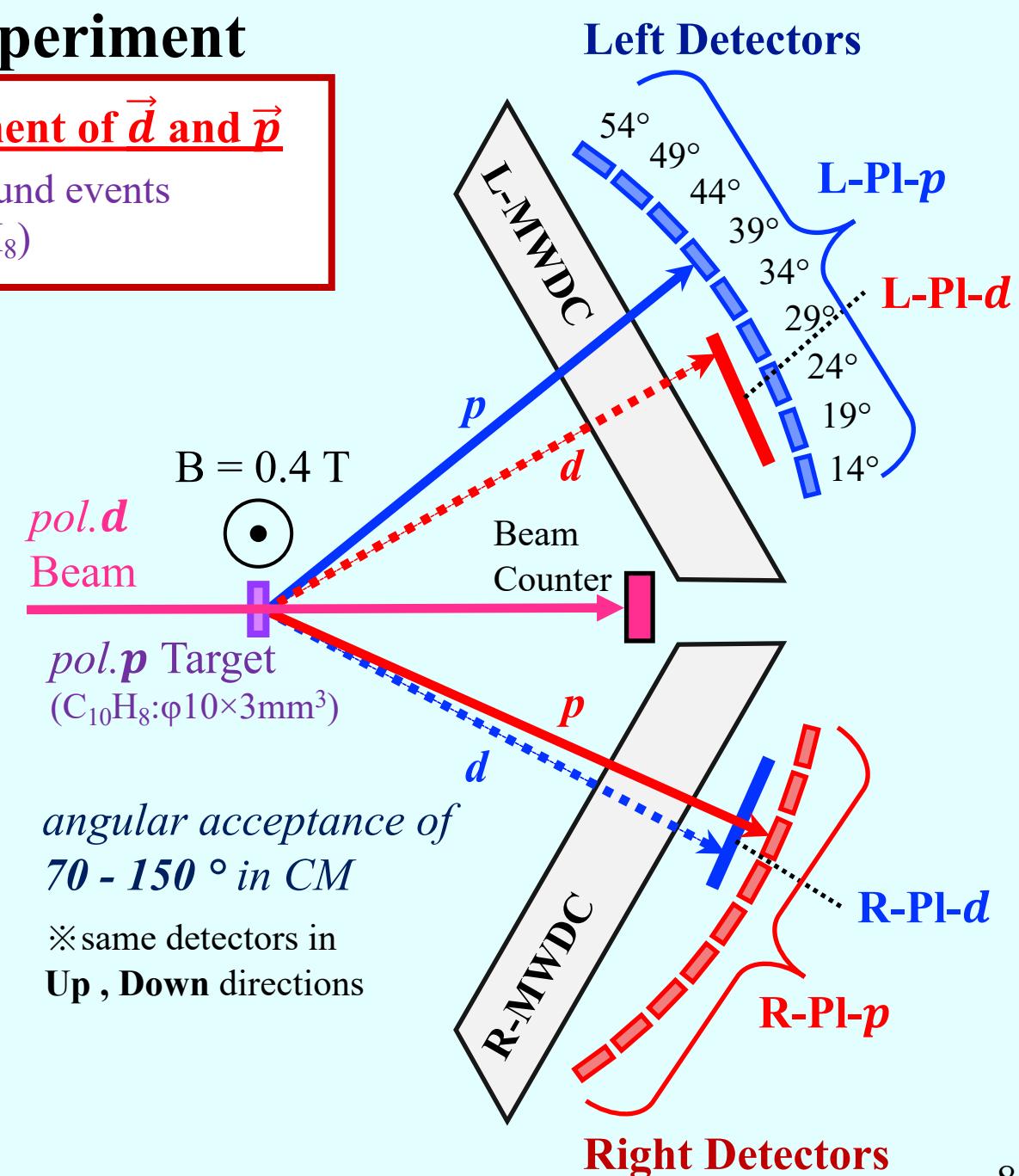
Pl-d (plastic scintillators): → scattered deuterons

Size	250mm ^L × 70mm ^H × 10mm ^t
PMT	H7195

MWDC (Multi Wired Drift Chamber):

→ for tracking trajectories of d and p bent at target (0.3T)

Wire configuration	X(31)-X'(32)-X(33)-X'(32)
Cell shape	Hexagonal
Active area	878mm ^L × 70mm ^H
Sense wire spacing	24mm (X-X' : 12mm)
Sense wires	30μm φ Au-plated W wire
Cathode / Shield wires	100μm φ Au-plated Be-Cu wire
Gas	Ar(50%) + C ₂ H ₆ (50%)



angular acceptance of
70 - 150 ° in CM

※ same detectors in
Up , Down directions

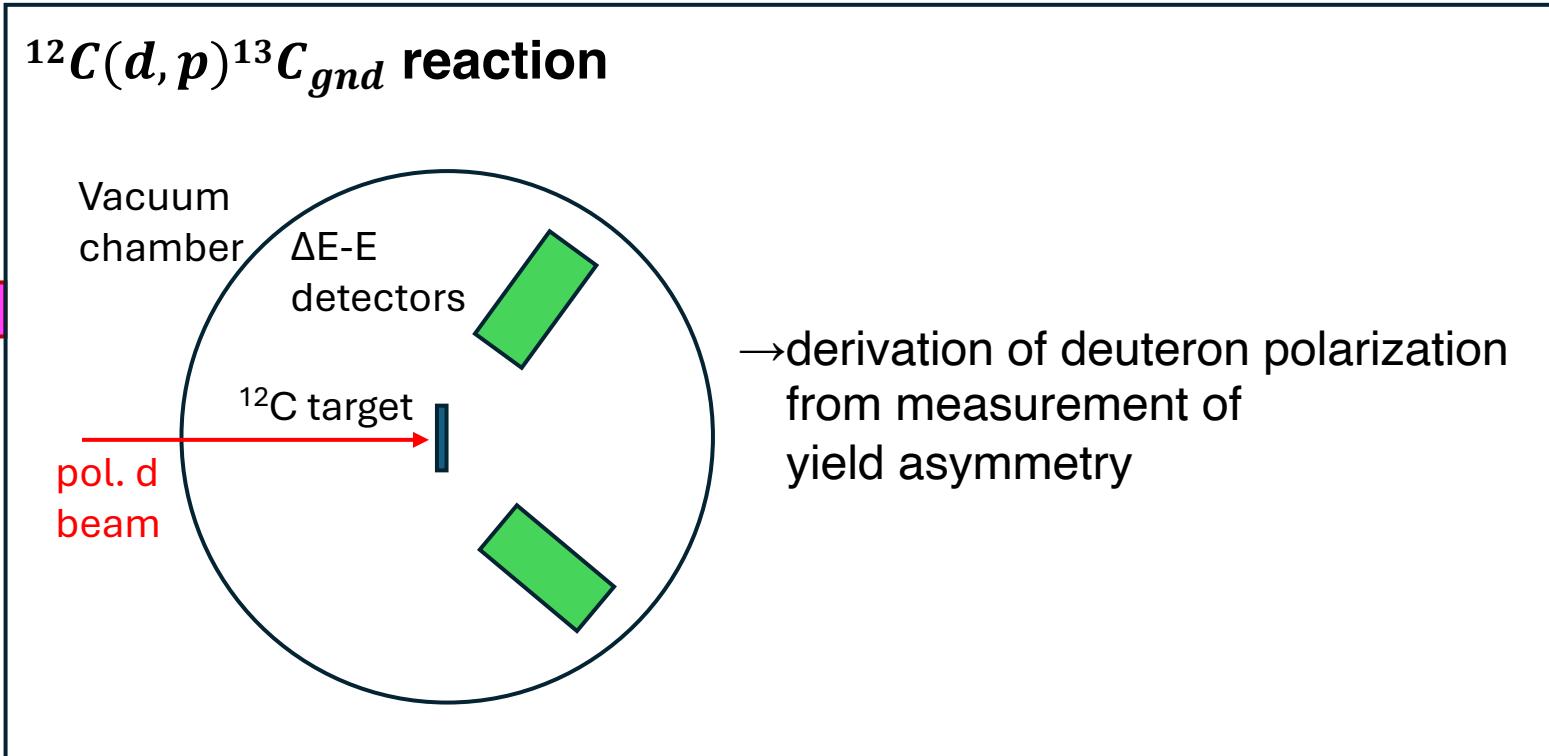
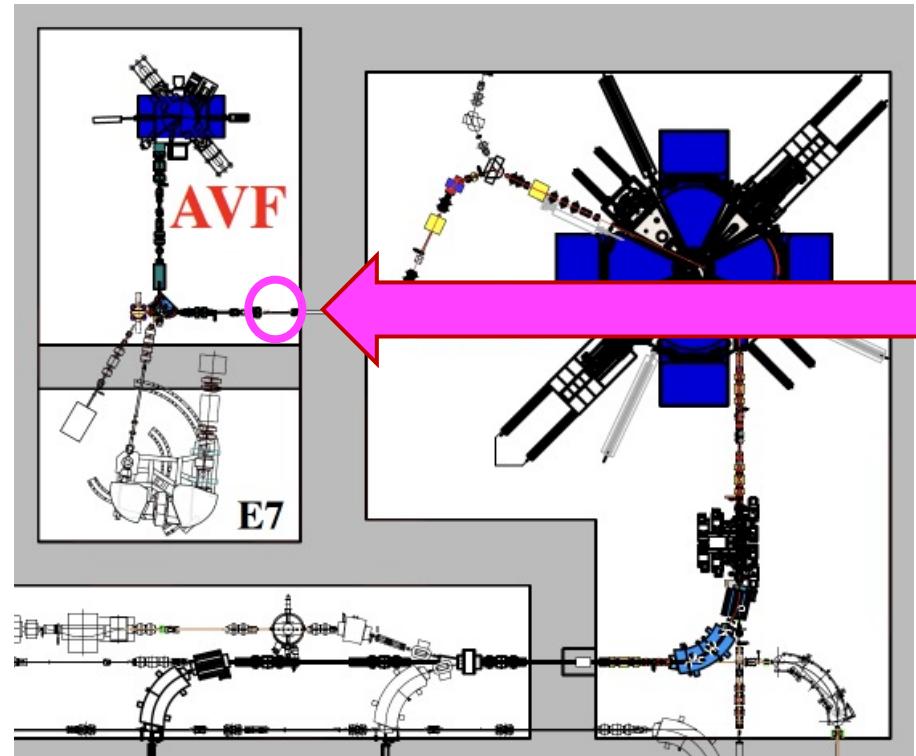
Deuteron polarization beam test (2024/9/14) @ RIKEN

Observables	d polarization
Machine time	9/14 0:00 - 9/15 0:00
beam	7 MeV/Nucleon polarized d beam
target	^{12}C target (0.5 mg/cm ²)
detector	ΔE -E detectors

polarization : 50-80% (preliminary)

Ready for the
Spin Correlation Coefficients Measurement

RIKEN Nishina Center



Laser spot size on target

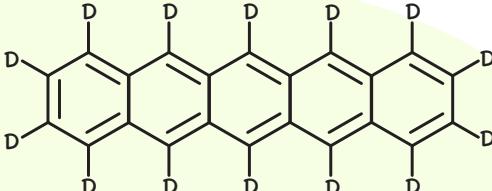
Target : pentacene doped *p*-terphenyl single crystal

(a)



p-terphenyl

(b)

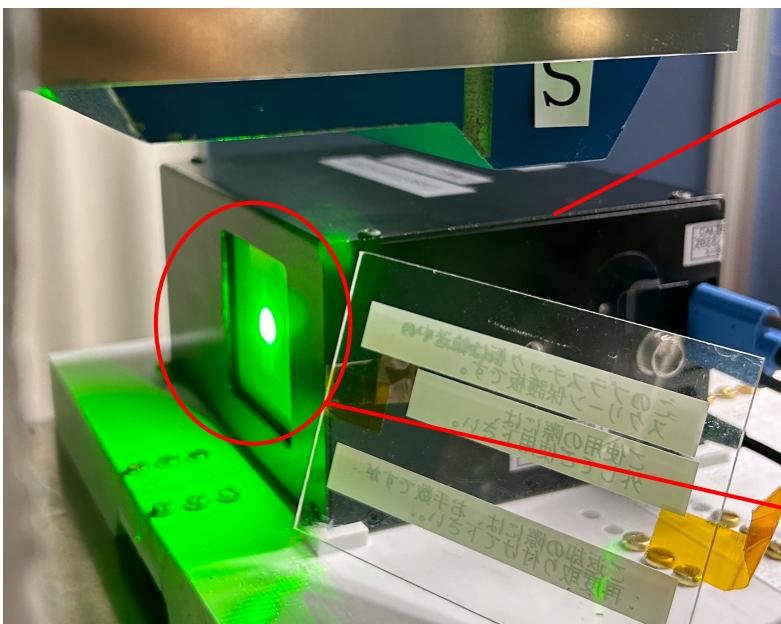


deuterated pentacene
(0.005 %/mol)



Size : $\varphi 10 \times 2.5 \text{ mm}^3$

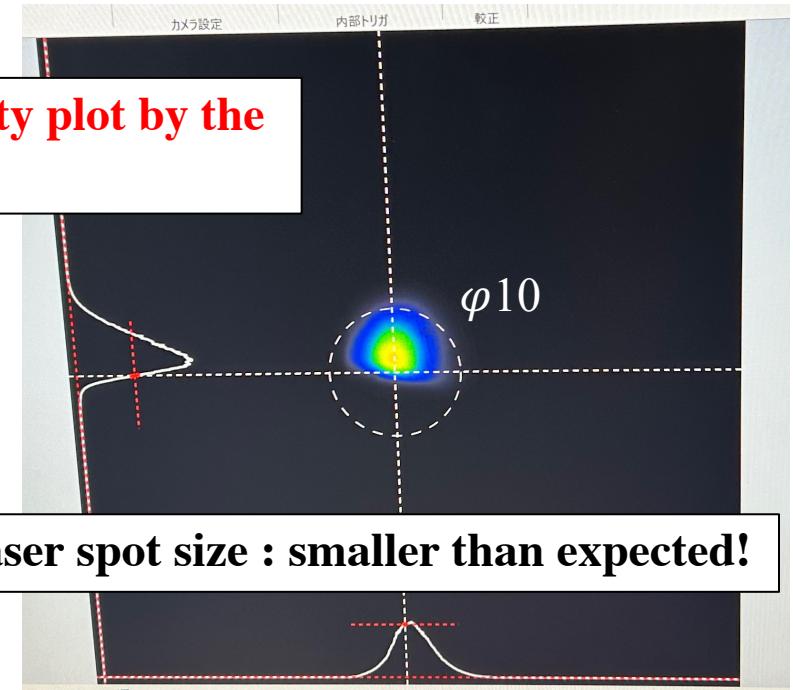
Want laser to cover
the full $\varphi 10 \times 2.5 \text{ mm}^3$ range



Beam profiler

To the eye,
Laser spot looks like $\varphi 10$

Laser intensity plot by the
laser profiler



Laser spot size : smaller than expected!

Motivation : The Spin Correlation Coefficients $C_{i,j}$

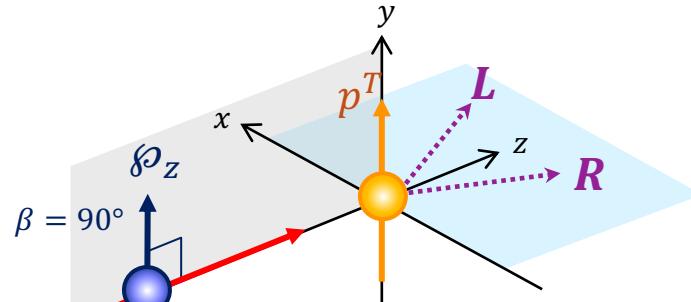
Form of spin 1 – spin $\frac{1}{2}$ correlation experiments

$$\begin{aligned} \frac{I(\theta, \phi)}{I_0(\theta)} = 1 + \frac{3}{2} & \beta_z [\sin\beta \cos\phi \mathbf{A}_y + p^T \{ -\sin\beta \sin\phi \sin(\phi + \phi') C_{x,x} + \sin\beta \cos\phi \cos(\phi + \phi') C_{y,y} + \cos\beta \sin(\phi + \phi') C_{z,x} \} + p_z^T \{ -\sin\beta \sin\phi C_{x,z} + \cos\beta C_{z,z} \}] \\ & + \beta_{zz} \left\{ \frac{1}{6} \{ (-\cos^2\phi + (3\sin^2\beta - 1)\sin^2\phi) \mathbf{A}_{xx} + (-\sin^2\phi + (3\sin^2\beta - 1)\cos^2\phi) \mathbf{A}_{yy} + (3\cos^2\beta - 1) \mathbf{A}_{zz} \} - \sin\beta \cos\beta \sin\phi \mathbf{A}_{xz} \right\} \\ & + p^T \left[\frac{1}{6} \sin(\phi + \phi') \{ (-\cos^2\phi + (3\sin^2\beta - 1)\sin^2\phi) C_{xx,y} + (-\sin^2\phi + (3\sin^2\beta - 1)\cos^2\phi) C_{yy,y} + (3\cos^2\beta - 1) C_{zz,y} \} \right. \\ & \quad \left. + \{ -\sin^2\beta \sin 2\phi \sin(\phi + \phi') C_{xy,x} + 2\sin\beta \cos\beta \cos\phi \sin(\phi + \phi') C_{yz,x} - 2\sin\beta \cos\beta \sin\phi \cos(\phi + \phi') C_{xz,y} \} \right] \\ & + p_z^T \{ -\sin^2\beta \sin 2\phi C_{xy,z} + 2\sin\beta \cos\beta \cos\phi C_{yz,z} \} + p^T \cos(\phi + \phi') \mathbf{A}_y^T \end{aligned}$$

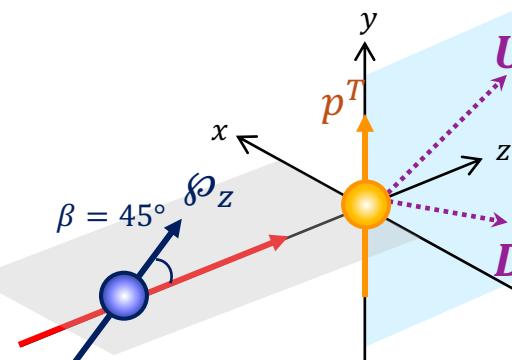
Total $C_{i,j}$'s : 12 (8 will be measured)

→ **polarized beam & target** necessary

$$C_{y,y} = \frac{1}{6\beta_z p^T} \left(\frac{\mathbf{L}^{u^T} + \mathbf{R}^{u^T}}{I_0^{u^T}} - \frac{\mathbf{L}^{d^T} + \mathbf{R}^{d^T}}{I_0^{d^T}} \right)$$



$$C_{yz,x} = \frac{1}{2\beta_{zz} p^T} \left(\frac{\mathbf{U}^{u^T} + \mathbf{D}^{u^T}}{I_0^{u^T}} - \frac{\mathbf{U}^{d^T} + \mathbf{D}^{d^T}}{I_0^{d^T}} \right)$$



→ **Yield unsymmetry (L&R, U&D)** must be measured