Antinucleon-nucleon interactions in covariant chiral effective field theory

arxiv: 2406.01292

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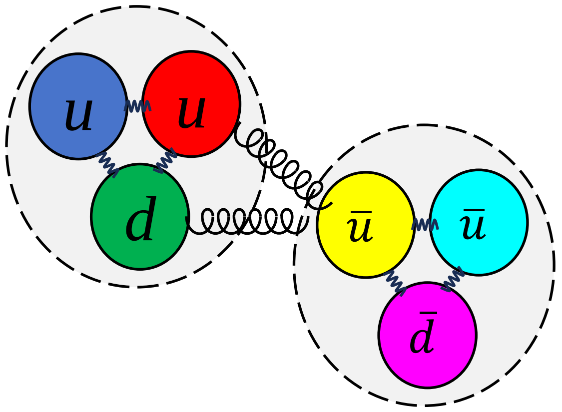
Outline

- **Introduction**
- **Theoretical framework**
- **Results and discussion**
  - Description of phase shifts
  - Possible $\bar{N}N$ structures
- **Summary and Outlook**

$\bar{N}N$ interactions is of fundamental interest

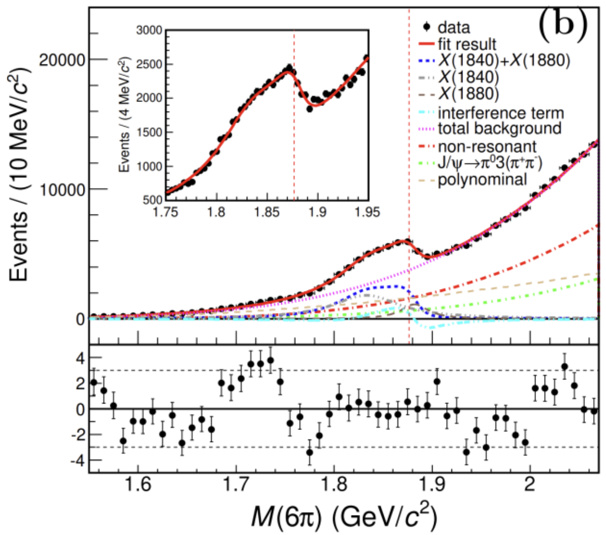
- **Antinucleon-nucleon** ($\bar{N}N$) interactions (residue quark-gluon strong force)
  - Acts between anti-nucleons and nucleons
  - Binds antinucleons-nucleons into antinucleonic atoms (possible)
  - Plays an important role in antinulceon related physics
    - **Hardon Spectroscopy**
    - **Nucleon electromagnetic formfactors**
    - **Neutron-antineutron oscillations**
    - …



Precise understanding of the $\bar{N}N$ interactions is essential!

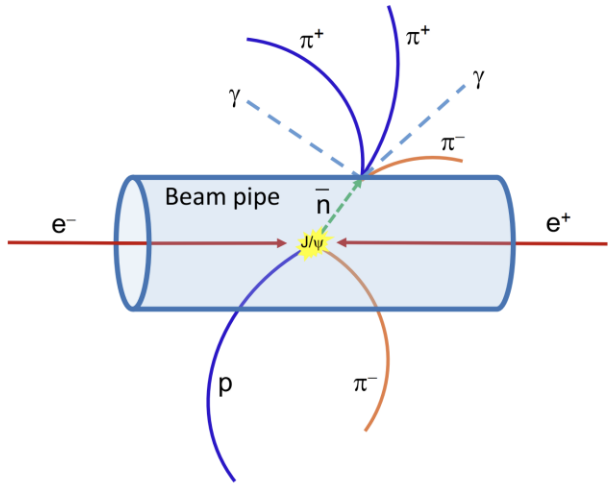
Experiment advances provide fresh opportunities

- **New** measurements
  - Near-threshold $\bar{N}N$ enhancement.



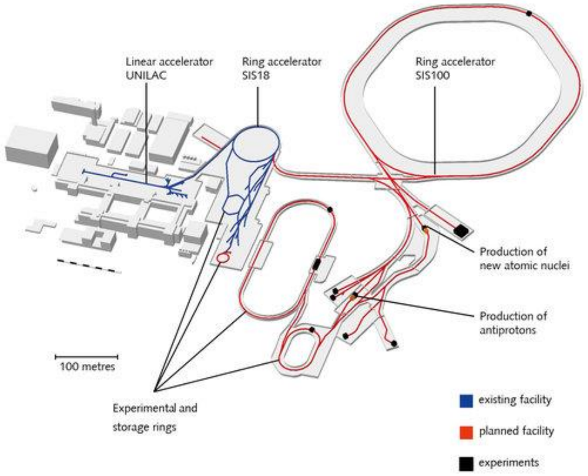
BESIII, PRL. 132, 151901 (2024)

- **Novel** proposal
  - Super *J/psi* factory (beam source production)



C.-Z. Yuan and M. Karliner, PRL. 127, 012003 (2021)

- **Next** generation facilities
  - FAIR (antiproton-nucleus collision)



C. Sturm et al., NPA 834, 682c(2010)

Ongoing interest in the studies of $\bar{N}N$ interactions.

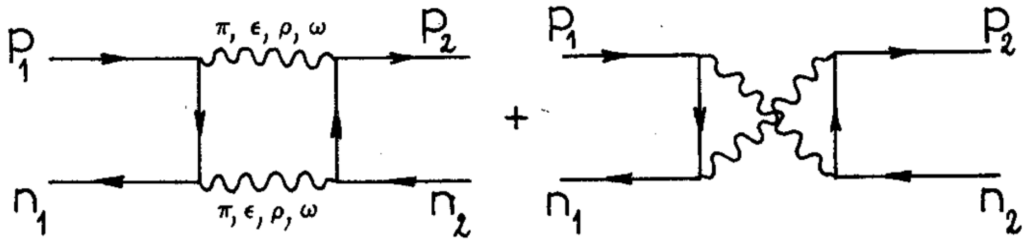
Theoretical studies I

- **Phenomenological** $\bar{N}N$ interactions
  - Optical model : Paris *J. Cote, et al., Phys. Rev. Lett. 48, 1319 (1982)*

$$V_{\bar{N}N} = U_{\bar{N}N} - iW_{\bar{N}N}$$

$U_{\bar{N}N}$: ***G* transformation of Paris *NN* potential**

$W_{\bar{N}N}$: **Meson theory**

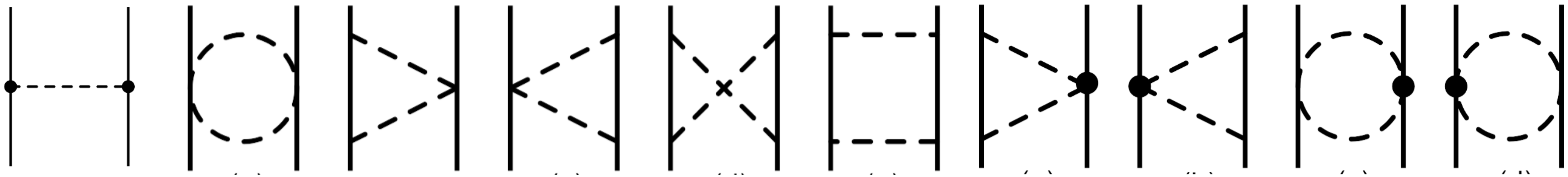


  - Partial wave analysis (PWA): Nijmegen *Daren Zhou, et al., Phys. Rev. C 86, 044003 (2012)*

Short-range effects : Parametrization

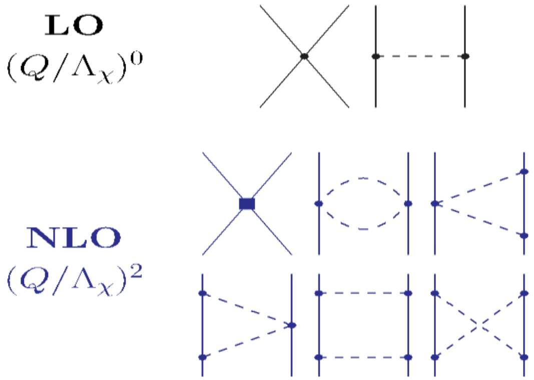
Long-range effects : N^{2}LO chiral EFT pion-exchange potentials

Most accurate to date

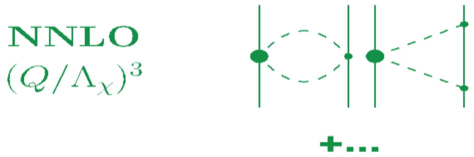


Theoretical studies II

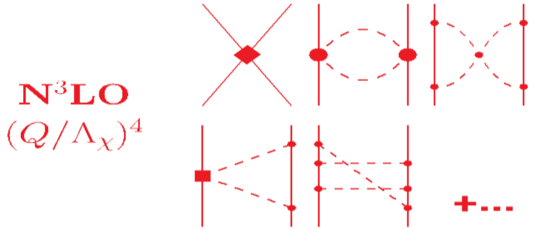
- **Chiral** $\bar{N}N$ interactions **(low energy EFT of QCD)**
  - **Heavy baryon** chiral $\bar{N}N$ interactions (**Weinberg power counting**)



GYC & JPM Phys. Rev. D 83, 094029 (2011)



WXK et al., JHEP 02, 113(2014)



LYD et al., JHEP 07, 078(2017)

- **Long-range : *G* transformation of *NN* pion-exchange potentials**
- **Short-range : Complex contact terms similar to *NN* case**

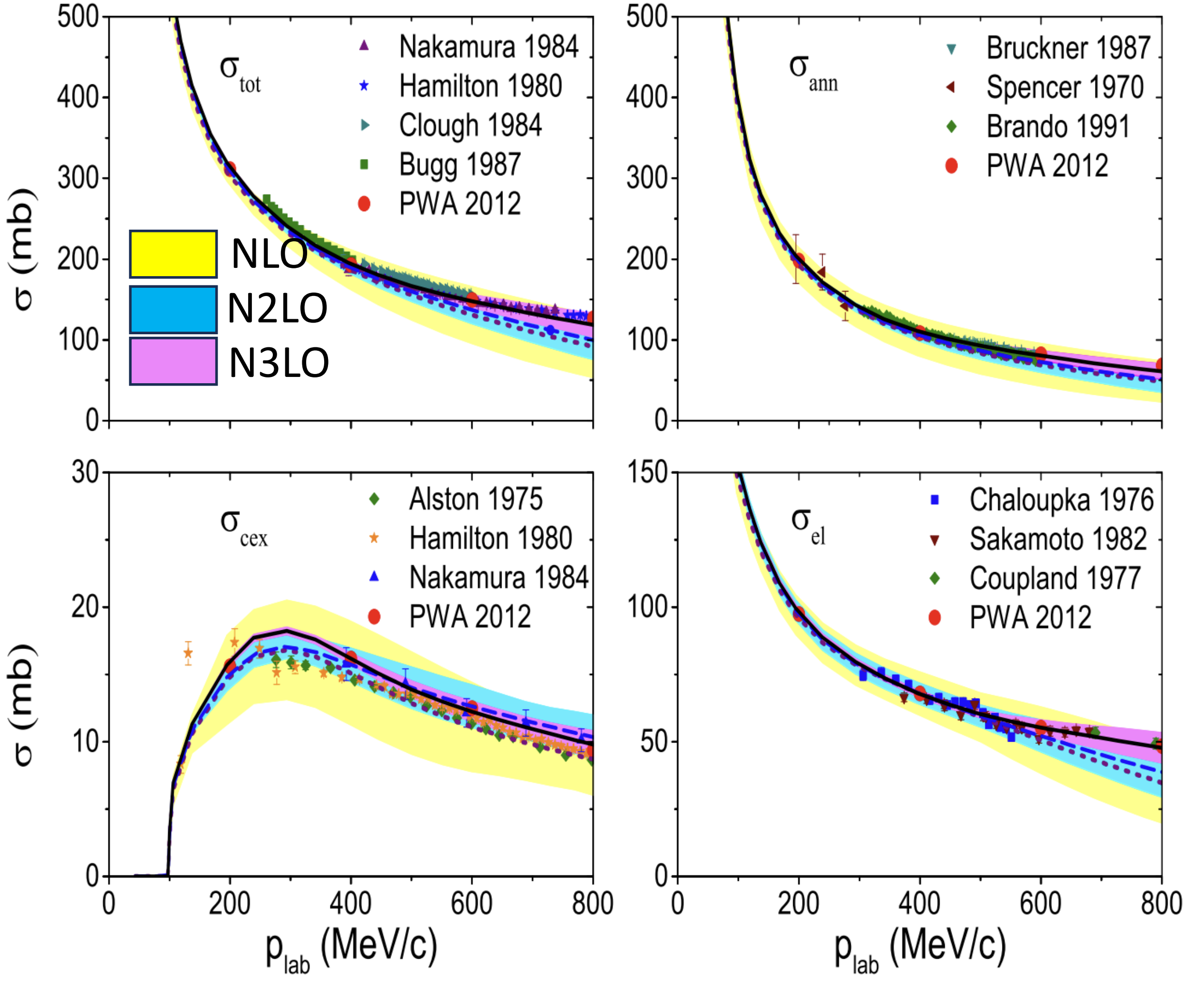
2011 — **2 body: NLO**

2014 — **2 body: N^2LO**

2017 — **2 body: N^3LO**

The chiral potential nowadays of high precision

$\bar{p}p$ cross section



LYD et al., JHEP 07, 078(2017)

- **An equally $\bar{N}N$ good description of the data as in the Nijmegen PWA.**
- **The chiral potential is increasingly being used in antinucleonic related studies.**

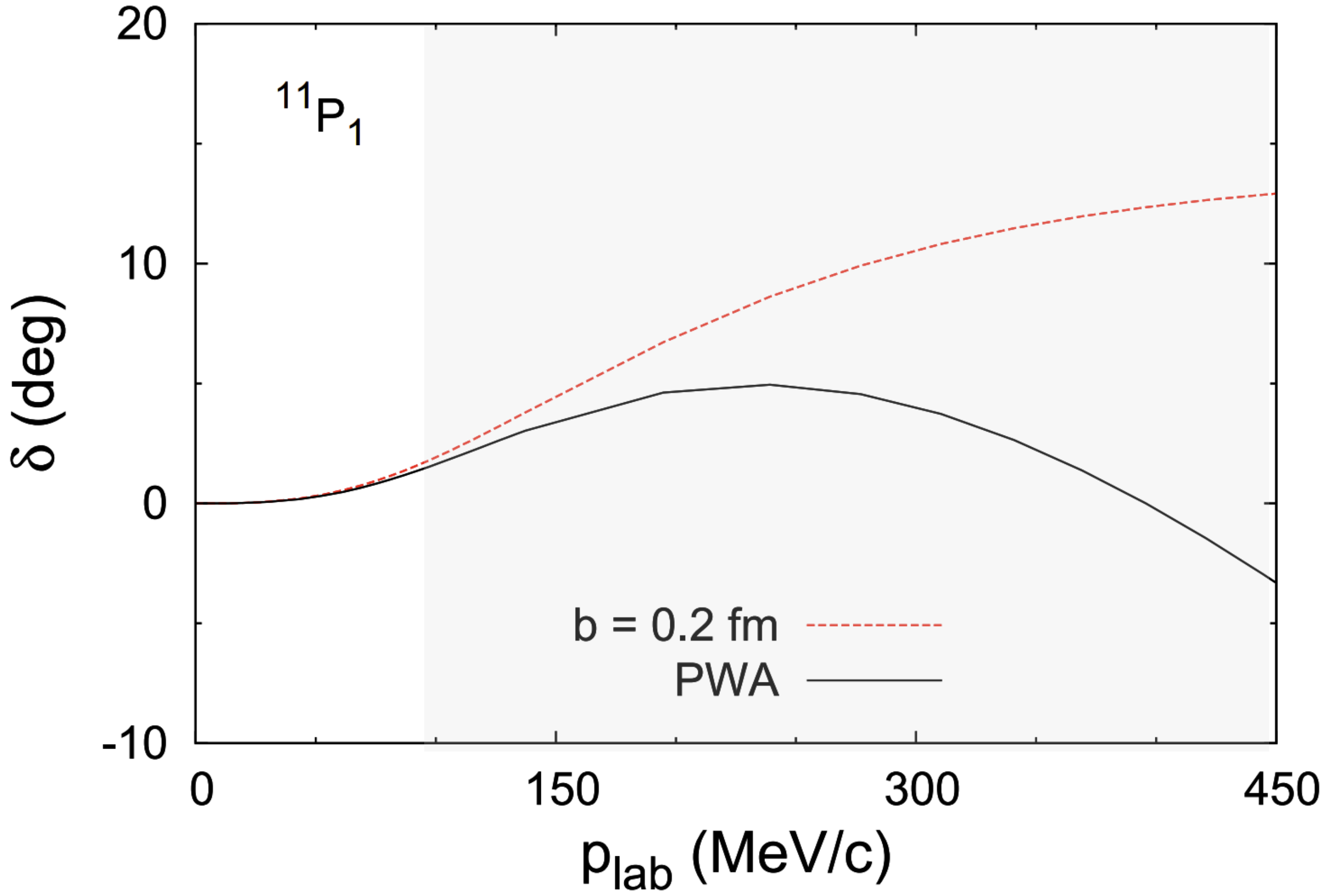
F. Oosterhof, et al., Phys. Rev. Lett. 122, 172501 (2019)

QH Yang et al., Sci. bull. 68 2728 (2023)

…

Why covariant

LO chiral results for I=0 1P1 waves (Weinberg p.c.)



D. Zhou et al., Phys. Rev. C 105, 054005 (2022)

- **In analogy to the *NN* case, renormalization challenging Weinberg power counting.**
- ***NN* experiences suggest covariance might important.**

We constructing LO covariant chiral interactions

- Expecting a faster convergence.

$\bar{N}N$ interactions at leading order

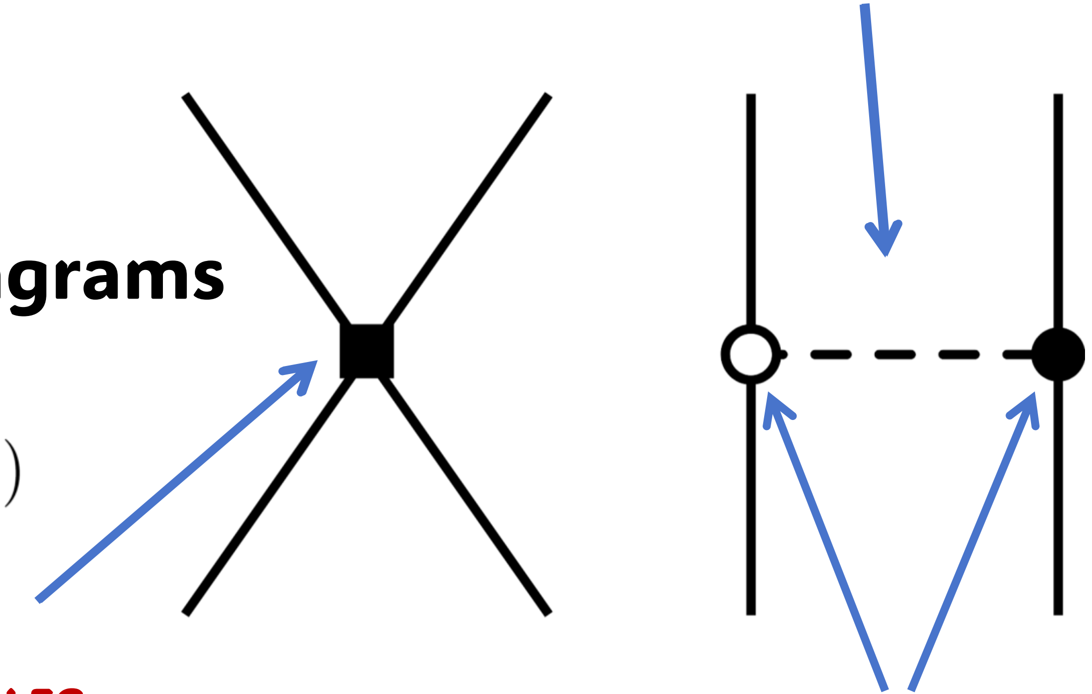
- **Covariant chiral Lagrangians**

$$\mathcal{L}_{\mathrm{eff.}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\bar{N}}^{(1)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\bar{N}N}^{(0)},$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + \left(U + U^\dagger\right) m_\pi^2\right]$$

- **LO Lagrangians & Feynman diagrams**

$$\begin{aligned}\mathcal{L}_{\bar{N}N}^{(0)} =& C_S \left(\bar{\Psi}\Psi\right)\left(\bar{\Psi}\Psi\right) + C_A \left(\bar{\Psi}\gamma_5\Psi\right)\left(\bar{\Psi}\gamma_5\Psi\right) \\ &+ C_V \left(\bar{\Psi}\gamma_\mu\Psi\right)\left(\bar{\Psi}\gamma^\mu\Psi\right) \\ &+ C_{AV} \left(\bar{\Psi}\gamma_\mu\gamma_5\Psi\right)\left(\bar{\Psi}\gamma^\mu\gamma_5\Psi\right) \\ &+ C_T \left(\bar{\Psi}\sigma_{\mu\nu}\Psi\right)\left(\bar{\Psi}\sigma^{\mu\nu}\Psi\right),\end{aligned}$$

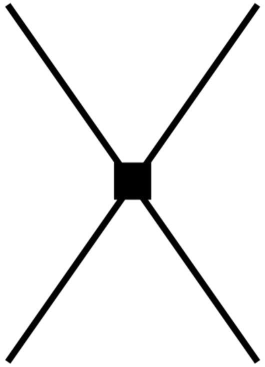


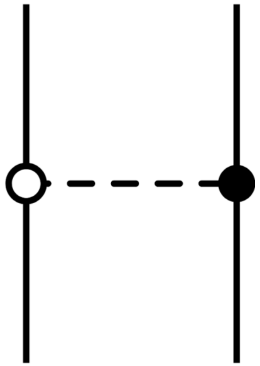
5 * 2 LECs

$$\mathcal{L}_{\pi\bar{N}}^{(1)} = \mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\not{D} - M + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi,$$

Chiral potentials

$$V_{\bar{N}N}^{(0)} = \bar{v}_1 \bar{u}_2 \{$$





$$\} v_1 u_2$$

$$\bar{v}_1 \bar{u}_2 v_1 u_2 \coloneqq \bar{v}_1 v_1 \bar{u}_2 u_2$$

$$u(\boldsymbol{p}, s) = N \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E + m_N} \end{pmatrix} \chi_s, N = \sqrt{\frac{E + m_N}{m_N}},$$

$$v(\boldsymbol{p}, s) = \gamma_0 C\, u^*(\boldsymbol{p}, s).$$

One-pion exchange potential is trivial

$$V_{\mathrm{OPE}}^{NN}(\boldsymbol{p},\boldsymbol{p}') = -\frac{g_A^2}{4f_\pi^2}\frac{[\bar{u}(\boldsymbol{p},s_1)\boldsymbol{\tau}_1\gamma_\mu\gamma_5 q^\mu u(\boldsymbol{p}',s_1')][\bar{u}(-\boldsymbol{p}',s_2')\boldsymbol{\tau}_2\gamma_\nu\gamma_5 q^\nu u(-\boldsymbol{p},s_2)]}{(E_{p'}-E_p)^2-(\boldsymbol{p}'-\boldsymbol{p})^2-m_\pi^2}$$

Ren X.L. et al., Chinese Physics C 42, 1, 014103 (2018)

G-parity *Fermi E, Yang CN, Phys Rev. 76:1739–43 (1949)*

$$V_{\mathrm{OPE}}^{\bar{N}N}(\boldsymbol{p},\boldsymbol{p}') = \frac{g_A^2}{4f_\pi^2}\frac{[\bar{v}(\boldsymbol{p},s_1)\boldsymbol{\tau}_1\gamma_\mu\gamma_5 q^\mu v(\boldsymbol{p}',s_1')][\bar{u}(-\boldsymbol{p}',s_2')\boldsymbol{\tau}_2\gamma_\nu\gamma_5 q^\nu u(-\boldsymbol{p},s_2)]}{(E_{p'}-E_p)^2-(\boldsymbol{p}'-\boldsymbol{p})^2-m_\pi^2}$$

One-pion exchange potential is of an opposite sign as the *NN* case

Contact terms needs careful consideration

- **Normal ordering**

$$
\begin{aligned}
V_{\mathrm{CT}}^{\bar{N}N}(\boldsymbol{p},\boldsymbol{p}') = & C_S\left[\bar{v}(\boldsymbol{p},s_1)\,v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,u(-\boldsymbol{p},s_2)\right]+C_A\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma_5 u(-\boldsymbol{p},s_2)\right] \\
& +C_V\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^\mu u(-\boldsymbol{p},s_2)\right]+C_{AV}\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^5\gamma_5 u(-\boldsymbol{p},s_2)\right] \\
& +C_T\left[\bar{v}(\boldsymbol{p},s_1)\,\sigma_{\mu\nu} v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\sigma^{\mu\nu} u(-\boldsymbol{p},s_2)\right],
\end{aligned}
$$

- **All arrangements**

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i v(\boldsymbol{p}',s_1')\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i v(-\boldsymbol{p}',s_2')\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i v(-\boldsymbol{p}',s_2')\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

Contact terms needs careful consideration

- **Normal ordering**

$$\begin{aligned}V_{\mathrm{CT}}^{\bar{N}N}(\boldsymbol{p},\boldsymbol{p}') =& C_S\left[\bar{v}(\boldsymbol{p},s_1)\,v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,u(-\boldsymbol{p},s_2)\right] + C_A\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma_5 u(-\boldsymbol{p},s_2)\right]\\ &+C_V\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^\mu u(-\boldsymbol{p},s_2)\right] + C_{AV}\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^5\gamma_5 u(-\boldsymbol{p},s_2)\right]\\ &+C_T\left[\bar{v}(\boldsymbol{p},s_1)\,\sigma_{\mu\nu} v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\sigma^{\mu\nu} u(-\boldsymbol{p},s_2)\right],\end{aligned}$$

- **All arrangements**

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i v(\boldsymbol{p}',s_1')\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i v(-\boldsymbol{p}',s_2')\right],$$

$$\sum_i C_i \ldots\,\Gamma_i v(-\boldsymbol{p}',s_2')\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

All necessary ?

- **Reminds of Fierz identities**

$$(\bar{\Psi}^{(1)}\Gamma^A\Psi^{(2)})(\bar{\Psi}^{(3)}\Gamma^B\Psi^{(4)}) = -\sum_{C,D} C_{CD}^{AB}(\bar{\Psi}^{(1)}\Gamma^C\Psi^{(4)})(\bar{\Psi}^{(3)}\Gamma^D\Psi^{(2)}),$$

- **But Exchanged spinors of different type**

Contact terms needs careful consideration

- **Normal ordering**

$$
\begin{aligned}
V_{\text{CT}}^{\bar{N}N}(\boldsymbol{p},\boldsymbol{p}') =& C_S\left[\bar{v}(\boldsymbol{p},s_1)\,v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,u(-\boldsymbol{p},s_2)\right] + C_A\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma_5 u(-\boldsymbol{p},s_2)\right] \\
&+C_V\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^\mu u(-\boldsymbol{p},s_2)\right] + C_{AV}\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^5\gamma_5 u(-\boldsymbol{p},s_2)\right] \\
&+C_T\left[\bar{v}(\boldsymbol{p},s_1)\,\sigma_{\mu\nu} v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\sigma^{\mu\nu} u(-\boldsymbol{p},s_2)\right],
\end{aligned}
$$

- **All arrangements**

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i v(\boldsymbol{p}',s_1')\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i v(-\boldsymbol{p}',s_2')\right],$$

$$\sum_i C_i \cdots)\,\Gamma_i v(-\boldsymbol{p}',s_2')\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

All necessary ?

✓ **Interchange a pair of u-spinors to v-spinors in quadrilinear is possible**

- **Generalized Fierz identities**

$$\boldsymbol{e}(1234) = \boldsymbol{K}^{(abcd)}\boldsymbol{e}(abcd)$$

J. F. Nieves et al, Am. J. Phys. 72, 1100 (2004).

Contact terms needs careful consideration

- **Normal ordering**

$$\begin{aligned} V_{\mathrm{CT}}^{\bar{N}N}(\boldsymbol{p},\boldsymbol{p}') = & C_S\left[\bar{v}(\boldsymbol{p},s_1)\,v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,u(-\boldsymbol{p},s_2)\right] + C_A\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma_5 u(-\boldsymbol{p},s_2)\right] \\ & + C_V\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^\mu u(-\boldsymbol{p},s_2)\right] + C_{AV}\left[\bar{v}(\boldsymbol{p},s_1)\,\gamma_\mu\gamma_5 v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\gamma^5\gamma_5 u(-\boldsymbol{p},s_2)\right] \\ & + C_T\left[\bar{v}(\boldsymbol{p},s_1)\,\sigma_{\mu\nu} v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\sigma^{\mu\nu} u(-\boldsymbol{p},s_2)\right], \end{aligned}$$

Only need this one !

- **All arrangements**

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i v(\boldsymbol{p}',s_1')\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(-\boldsymbol{p}',s_2')\,\Gamma^i v(\boldsymbol{p}',s_1')\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i u(-\boldsymbol{p},s_2)\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i v(-\boldsymbol{p}',s_2')\right],$$

$$\sum_i C_i\left[\bar{v}(\boldsymbol{p},s_1)\,\Gamma_i v(-\boldsymbol{p}',s_2')\right]\left[\bar{u}(\boldsymbol{p}',s_1')\,\Gamma^i u(-\boldsymbol{p},s_2)\right],$$

Chiral potentials in *LSJ* basis

- **Elastic potential (10 LECs)**

$$V^{\mathrm{R}}_{1S0} = \xi\left[C_{1S0}\left(R_p^2+R_{p'}^2\right)+\hat{C}_{1S0}\left(1+R_p^2R_{p'}^2\right)\right],$$
$$V^{\mathrm{R}}_{3P0} = \xi C_{3P0}R_pR_{p'},$$
$$V^{\mathrm{R}}_{1P1} = 2\xi\left(C_{3S1}-3\hat{C}_{3S1}\right)R_pR_{p'},$$
$$V^{\mathrm{R}}_{3P1} = \frac{4}{3}\xi\left(C_{1S0}-\hat{C}_{1S0}\right)R_pR_{p'},$$
$$V^{\mathrm{R}}_{3S1} = \xi\left[C_{3S1}\left(R_p^2+R_{p'}^2\right)+\hat{C}_{3S1}\left(9+R_p^2R_{p'}^2\right)\right],$$
$$V^{\mathrm{R}}_{3D1} = 8\xi\hat{C}_{3S1}R_p^2R_{p'}^2,$$
$$V^{\mathrm{R}}_{3S1-3D1} = 2\sqrt{2}\xi\left(2C_{3S1}R_p^2+\hat{C}_{3S1}R_p^2R_{p'}^2\right),$$
$$V^{\mathrm{R}}_{3D1-3S1} = 2\sqrt{2}\xi\left(2C_{3S1}R_{p'}^2+\hat{C}_{3S1}R_p^2R_{p'}^2\right),$$

$$\xi = -4\pi N_p^2N_{p'}^2 \quad R_p = |\mathbf{p}|/\epsilon_p \quad \epsilon_p = E_p + m_N$$

- **Annihilation potential (16 LECs)**

(from unitary constrains)

$$V^{\mathrm{I}}_{1S0} = -i\left(C^a_{1S0}+\hat{C}^a_{1S0}\frac{p^2}{4m_N^2}\right)\left(C^a_{1S0}+\hat{C}^a_{1S0}\frac{p'^2}{4m_N^2}\right),$$
$$V^{\mathrm{I}}_{3P0} = -i\left(C^a_{3P0}\right)^2\frac{pp'}{4m_N^2},$$
$$V^{\mathrm{I}}_{1P1} = -i\left(C^a_{1P1}\right)^2\frac{pp'}{4m_N^2},$$
$$V^{\mathrm{I}}_{3P1} = -i\left(C^a_{3P1}\right)^2\frac{pp'}{4m_N^2},$$
$$V^{\mathrm{I}}_{3S1} = -i\left(C^a_{3S1}+\hat{C}^a_{3S1}\frac{p^2}{4m_N^2}\right)\left(C^a_{3S1}+\hat{C}^a_{3S1}\frac{p'^2}{4m_N^2}\right),$$
$$V^{\mathrm{I}}_{3S1-3D1} = -i\left(C^a_{3S1}+\hat{C}^a_{3S1}\frac{p^2}{4m_N^2}\right)C^a_{\epsilon_1}\frac{p'^2}{4m_N^2},$$
$$V^{\mathrm{I}}_{3D1-3S1} = -iC^a_{\epsilon_1}\frac{p^2}{4m_N^2}\left(C^a_{3S1}+\hat{C}^a_{3S1}\frac{p'^2}{4m_N^2}\right),$$
$$V^{\mathrm{I}}_{3D1} = -i\left(C^a_{\epsilon_1}\right)^2\frac{p^2p'^2}{16m_N^4}.$$

Kang XW, JHEP02113 (2014)

- **One-pion exchange potential is of a opposite sign as the *NN* case**

Ren XL et al., Chin.Phys.C 42 1, 014103 (2018)

T-matrix & phase shifts

- **T-matrix is obtained by solving the Kadyshevsky equation**

$$T^{SJ}_{L',L}(p',p) = V^{SJ}_{L',L}(p',p) + \sum_{L''}\int_0^{+\infty}\frac{k^2\mathrm{d}k}{(2\pi)^3}V^{SJ}_{L',L}(p',k)$$
$$\times\frac{m_N^2}{2(k^2+m_N^2)}\frac{1}{\sqrt{p^2+m_N^2}-\sqrt{k^2+m_N^2}+i\epsilon}\times T^{SJ}_{L'',L}(k,p)$$

- **Potential regularized using a non-local gaussian type cutoff**
  - **Covariant pion-exchange potential is non-local due to retardation effects and the presence of Dirac spinors**

$$f^{\Lambda}(p,p') = \exp\left[-\left(p^6+p'^6\right)/\Lambda^6\right]$$

Kang XW, JHEP02113 (2014)

- **On-shell S-matrix and phase-shifts**

$$S^{SJ}_{L',L}(p) = \delta_{L',L} - i\frac{p\, m_N^2}{8\pi^2 E_p}T^{SJ}_{L',L}(p)$$

$$\mathrm{Re}(\delta_L) = \frac{1}{2}\arctan\frac{\mathrm{Im}(S_L)}{\mathrm{Re}(S_L)},$$
$$\mathrm{Im}(\delta_L) = -\frac{1}{2}\log|S_L|.$$

Couple channel: stapp parameterization

Numerical details

- **26 LECs are determined by fitting**
  - **NPWA**: $\bar{p}p$ scattering phase shifts
  - **14** partial waves : $J = 0, 1$ (twice as *NN* case because the absence of Pauli exclusion principle)
    - 144 data points : 10 for each partial waves ($T_{lab} \leq 125$ MeV) & 4 additional for $^{31}S_0$, $^{33}P_0$ waves ($^{(2I+1)(2S+1)}L$)
- **Fitted results**
  - Some constants

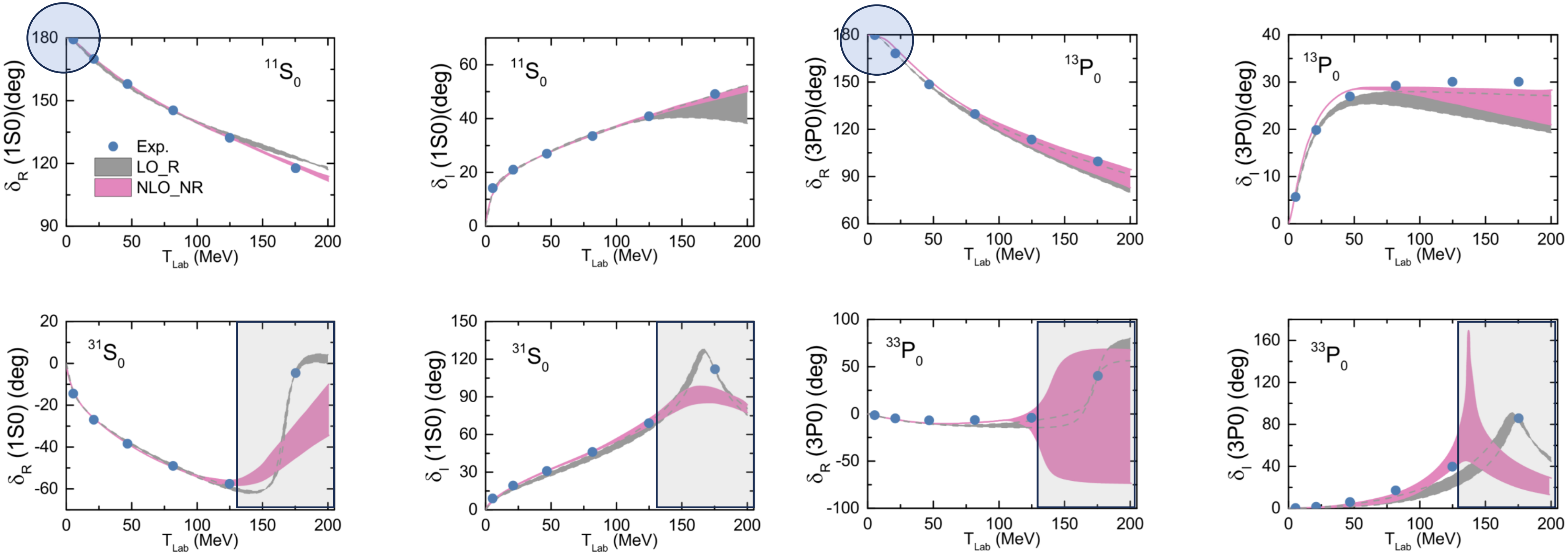
Λ = 450 ~ 600 MeV
g_A=1.29 f_π=92.4 MeV
m_N = 939 MeV m_π=138 MeV

	LEC	$\Lambda = 450$ MeV	$\Lambda = 600$ MeV
$I=0$	C_{1S0}	0.213	0.154
	$\hat{C}_{1S0}$	0.031	0.013
	C^a_{1S0}	−1.080	0.668
	$\hat{C}^a_{1S0}$	20.987	−9.199
	C_{3P0}	−0.019	−0.117
	C^a_{3P0}	1.472	0.971
	C^a_{1P1}	1.281	1.478
	C^a_{3P1}	0.737	0.447
	C_{3S1}	−0.043	−0.025
	$\hat{C}_{3S1}$	0.001	0.0002
	C^a_{3S1}	0.207	−0.388
	$\hat{C}^a_{3S1}$	4.533	3.326
	$C^a_{\epsilon 1}$	−1.982	0.743

$I=1$	C_{1S0}	−0.051	0.016
	$\hat{C}_{1S0}$	−0.004	0.025
	C^a_{1S0}	−0.398	1.270
	$\hat{C}^a_{1S0}$	4.730	−14.110
	C_{3P0}	0.242	0.179
	C^a_{3P0}	1.177	0.570
	C^a_{1P1}	1.270	1.066
	C^a_{3P1}	1.336	1.214
	C_{3S1}	0.014	0.032
	$\hat{C}_{3S1}$	0.001	0.001
	C^a_{3S1}	0.211	−0.081
	$\hat{C}^a_{3S1}$	9.208	−4.512
	$C^a_{\epsilon 1}$	1.720	−2.078

C (10^{-4} GeV^{-2}) & C^a (10^{-2} GeV^{-1})

Description of $J = 0$ phase shifts



A Better description than NLO Julich results

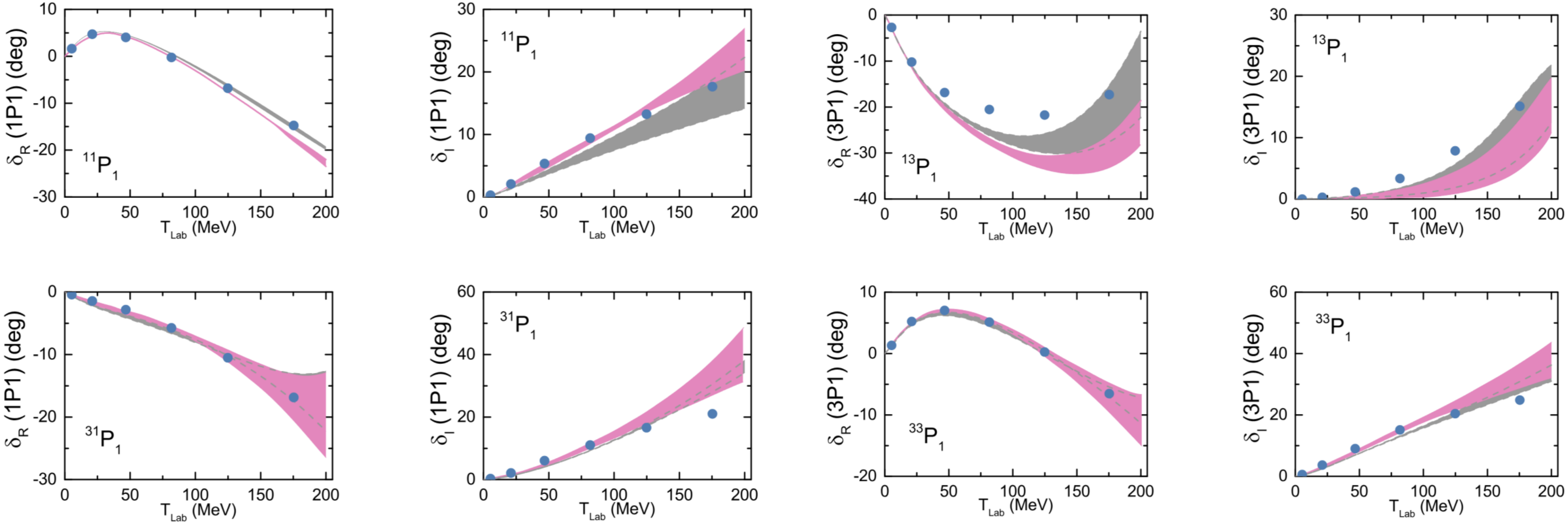
e.g. $^{31}S_0$ & $^{33}P_0$ partial wave

Phase shfits suggest some structures

e.g. Bound states in $^{11}S_0$, $^{13}P_0$ partial waves

P. W.	E_B(MeV)
$^{11}S_0$	(-102.2,-152.5)-I(79.1,199.3)
$^{13}P_0$	(-1.5,-2.1)-I(20.2,21.0)

Description of $J = 1$ phase shifts (single channel)



No free parameters for the elastic process

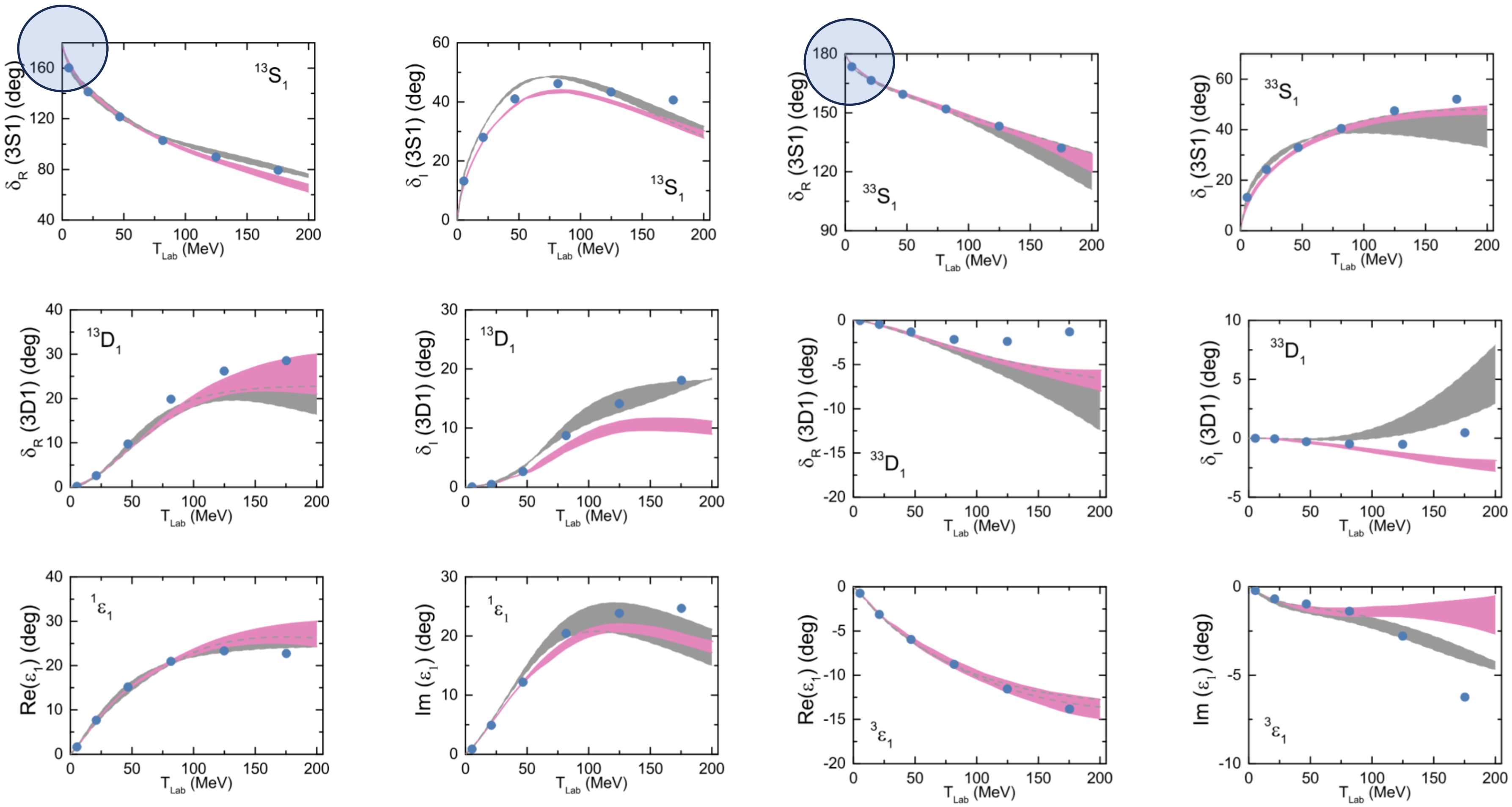
$^1P_1$ & $^3P_1$ elastic potential is controlled by the **S wave LECs**

LO covariant & NLO Julich comparable

$$V^{R}_{1P1} = 2\xi\left(C_{3S1} - 3\hat{C}_{3S1}\right) R_p R_{p'},$$

$$V^{R}_{3P1} = \frac{4}{3}\xi\left(C_{1S0} - \hat{C}_{1S0}\right) R_p R_{p'},$$

Description of $J = 1$ phase shifts (coupled channel)



Possible S wave bound states

P. W.	E_B(MeV)
$^{13}S_1$	(-7.1,28.8)-I(45.5,49.2)
$^{33}S_1$	(-17.6,7.0)-I(128.9,134.4)

S wave generally well reproduced

D wave elastic potential is controlled by the **S wave LECs**

LO covariant & NLO Julich comparable

$$V^{R}_{3S1} = \xi\left[C_{3S1}\left(R_p^2 + R_{p'}^2\right) + \hat{C}_{3S1}\left(9 + R_p^2 R_{p'}^2\right)\right],$$
$$V^{R}_{3D1} = 8\xi\hat{C}_{3S1}R_p^2 R_{p'}^2,$$
$$V^{R}_{3S1-3D1} = 2\sqrt{2}\xi\left(2C_{3S1}R_p^2 + \hat{C}_{3S1}R_p^2 R_{p'}^2\right),$$
$$V^{R}_{3D1-3S1} = 2\sqrt{2}\xi\left(2C_{3S1}R_{p'}^2 + \hat{C}_{3S1}R_p^2 R_{p'}^2\right),$$

Summary & Outlook

- **We have constructed the covariant chiral antinucleon-nucleon interactions at leading order**
  - Comparable (even better) description of P.S. than NLO Julich potentials.
  - **A deeply bound state in $^{11}S_0$ channel**, quantum number consistent with X(1835), X(1840), X(1880).
- **We are working on**
  - **Exploring more suitable regular functions** on the long-range interactions

    P. Reinert, et al., EPJA54(2018)86
  - Constructing a more consistent annihilation potential
- **Our ultimate goal is**
  - **Build a high precision covariant chiral potential**
  - Study antinucleonic related physics

Thank you !

Backups

Relations between covariant LECs

$$C_{1S0} = C_A + C_{AV} - 6C_T + 3C_V\,,$$
$$\hat{C}_{1S0} = 3C_{AV} + C_S - 6C_T + C_V\,,$$
$$C_{3P0} = -2\left(C_A - 4C_{AV} + C_S - 12C_T - 4C_V\right),$$
$$C_{3S1} = \frac{1}{3}\left(-C_A - C_{AV} - 2C_T + C_V\right),$$
$$\hat{C}_{3S1} = \frac{1}{9}\left(-C_{AV} + C_S + 2C_T + C_V\right).$$

Annihilation potentials

$$V = \sum_{X=2\pi,3\pi,\ldots} V_{\bar{N}N\to X} G_X V_{X\to\bar{N}N},$$

$$\frac{1}{x \pm i\epsilon} = \mathcal{P}\frac{1}{x} \mp i\pi\delta\left(x\right),$$

$$\mathrm{Im}V = -\pi \sum_X V_{\bar{N}N\to X} V_{X\to\bar{N}N}.$$

Possible bound states

Partial Wave	E_B (MeV)	
	LO relativistic	NLO non-relativistic [33]
$^{11}S_0$	$(-102.2, -152.5) - \mathrm{i}\,(79.1, 199.3)$	/ [a]
$^{13}P_0$	$(-1.5, -2.1) - \mathrm{i}\,(20.2, 21.0)$	$(-1.1, 1.9) - \mathrm{i}\,(17.8, 22.4)$
$^{13}S_1$	$(-7.1, 28.8) - \mathrm{i}\,(45.5, 49.2)$	$(5.6, 7.7) - \mathrm{i}\,(49.2, 60.5)$
$^{33}S_1$	$(-17.6, 7.0) - \mathrm{i}\,(128.9, 134.4)$	/ [a]

[a] It is unclear whether a $\bar{N}N$ bound states can be observed in this channel because the possible structures are only searched for near $\bar{N}N$ threshold.

Generalized Fierz identities *J. F. Nieves and P. B. Pal, Am. J. Phys. 72, 1100 (2004).*

- **Some notations & standard Fierz identities**

$$e_I\left(1234\right)=\left(\bar{\Psi}_1\Gamma_I\Psi_2\right)\left(\bar{\Psi}_3\Gamma^I\Psi_4\right) \qquad e_I\left(1234\right)=\sum_J F_{IJ}e_J\left(1432\right)$$

$$\begin{aligned}\Gamma_S &= \mathbb{1},\\ \Gamma_V &= \gamma_\mu,\\ \Gamma_T &= \sigma^{\mu\nu},\\ \Gamma_{AV} &= i\gamma^\mu\gamma_5,\\ \Gamma_A &= \gamma_5.\end{aligned}$$

- **Generalized case (a simple example)**

$$e_I\left(2^c1^c34\right)=\left(\bar{\Psi}^c\Gamma_I\Psi^c\right)\left(\bar{\Psi}\Gamma^I\Psi\right) \qquad \Psi^c=\gamma_0 C\Psi^*$$

$$C^{-1}\Gamma_I C=\eta_I\Gamma_I^{\mathrm{T}} \qquad \bar{\Psi}\Gamma_I\Psi=-\eta_I\bar{\Psi}^c\Gamma_I\Psi^c$$

$$\eta_I=\begin{cases}+1 & I=S,AV,A\\ -1 & I=V,T\end{cases}$$

$$e_I\left(1234\right)=\sum_J S_{IJ}e_J\left(2^c1^c34\right)$$

Interchange a pair of u-spinors to v-spinors in quadrilinear is possible

Generalized Fierz identities

$$\boldsymbol{e}\,(1234) = \boldsymbol{K}^{(abcd)}\boldsymbol{e}(abcd)$$

$$\boldsymbol{F} = \frac{1}{4}\begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 & 1 \\ 4 & -2 & 0 & -2 & -4 \\ 12 & 0 & -2 & 0 & 12 \\ -4 & -2 & 0 & -2 & 4 \\ 1 & -1 & \frac{1}{2} & 1 & 1 \end{pmatrix}.$$

$$\boldsymbol{S} = \mathrm{diag}\,(-1, +1, +1, -1, -1)\,.$$

Final order	$\boldsymbol{K}$
(1234)	$\mathbb{1}$
(1432)	$\boldsymbol{F}$
(2^c1^c34)	$\boldsymbol{S}$
(124^c3^c)	$\boldsymbol{S}$
(13^c2^c4)	$\boldsymbol{SFS}$
(13^c4^c2)	$\boldsymbol{SF}$
(142^c3^c)	$\boldsymbol{FS}$
$(2^c1^c4^c3^c)$	$\boldsymbol{SS} = \mathbb{1}$
(31^c2^c4)	$\boldsymbol{SF}$
(31^c4^c2)	$\boldsymbol{SFS}$
$(4^c1^c2^c3^c)$	$\boldsymbol{F}$
(4^c1^c32)	$\boldsymbol{FS}$

Phase shifts

single channel

$$\mathrm{Re}\left(\delta_L\right) = \frac{1}{2}\arctan\frac{\mathrm{Im}\left(S_L\right)}{\mathrm{Re}\left(S_L\right)},$$

$$\mathrm{Im}\left(\delta_L\right) = -\frac{1}{2}\log|S_L|.$$

Coupled channel

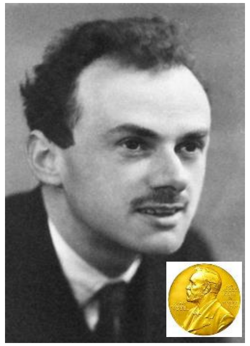
$$\mathrm{Re}\left(\delta_{L\pm1}\right) = \frac{1}{2}\arctan\frac{\mathrm{Im}\left(\eta_{L\pm1}\right)}{\mathrm{Re}\left(\eta_{L\pm1}\right)},$$

$$\eta_L = \frac{S_{L,L}}{\cos 2\epsilon_J}$$

$$\mathrm{Im}\left(\delta_{L\pm1}\right) = -\frac{1}{2}\log|\eta_{L\pm1}|,$$

$$\epsilon_J = \frac{1}{2}\arctan\left(\frac{i\left(S_{L-1,L-1} + S_{L+1,L+1}\right)}{2\sqrt{S_{L-1,L-1}S_{L+1,L+1}}}\right)$$

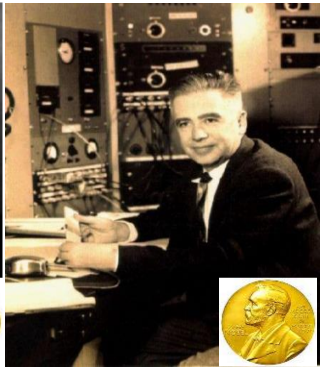
First observation of antinucleon



P.A.M. Dirac

$$(i\gamma^{\mu}\partial_{\mu} - m)\varphi = 0$$

O. Chamberlain



E. G. Segrè