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An accurate relativistic chiral nuclear force

Jun-Xu Lu @ Beihang U.

Co-operators: Dr. Chun-Xuan Wang, Dr. Yang Xiao, Prof. Li-Sheng Geng

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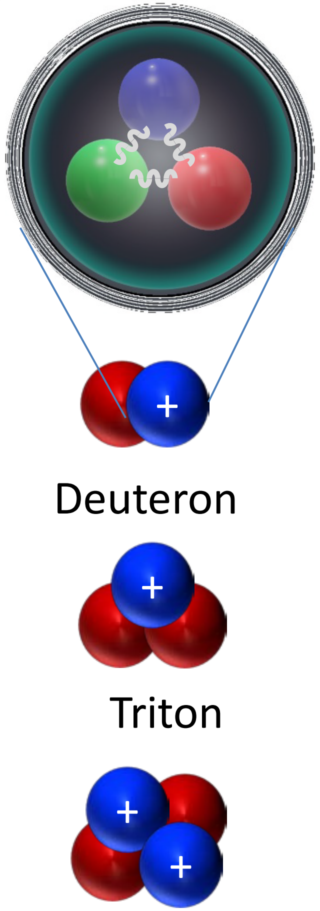
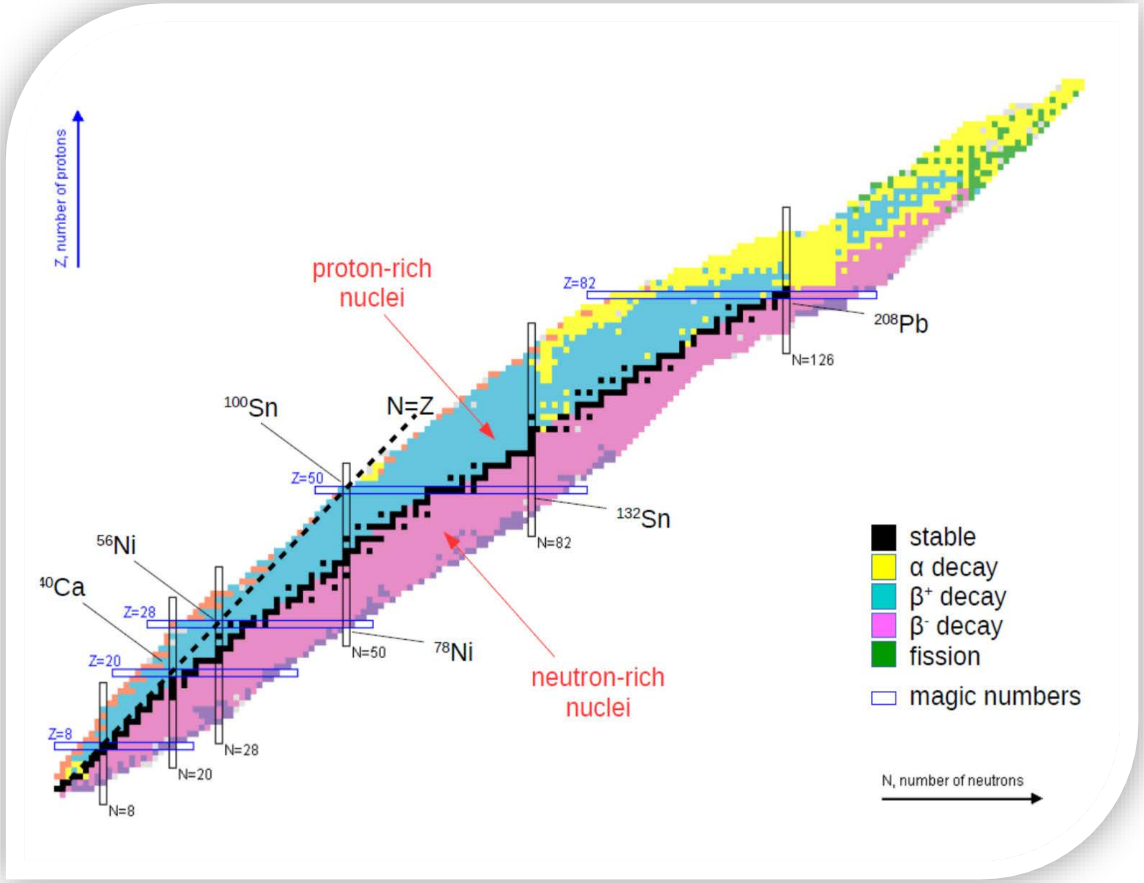
Outline

- Introduction
- Framework: chiral nuclear force
- Results and discussion
- Summary and prospect

☐ The key for nuclear physics

- Nuclear matter, structure, reaction, astrophysics…





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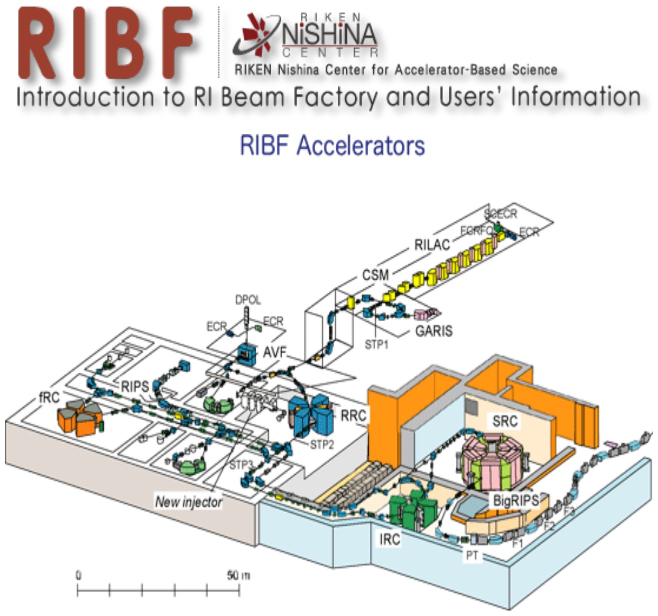
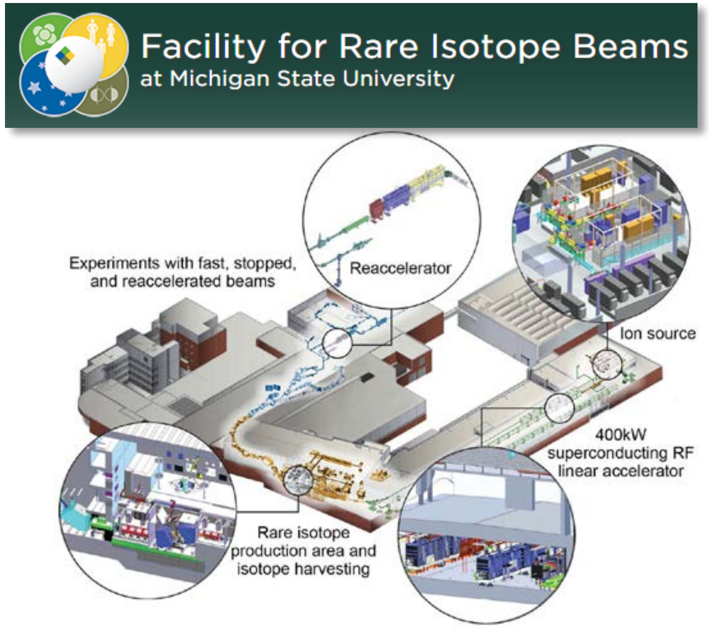
The major concern of various facilities

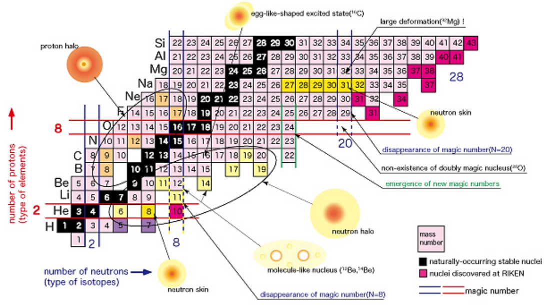




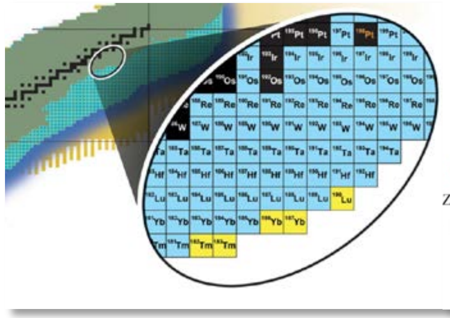
A new era of discovery----Exotic nuclei

four-neutron system

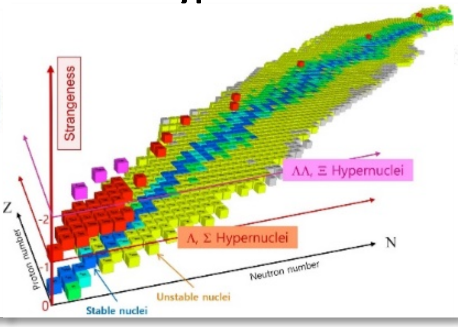
halo nucleus



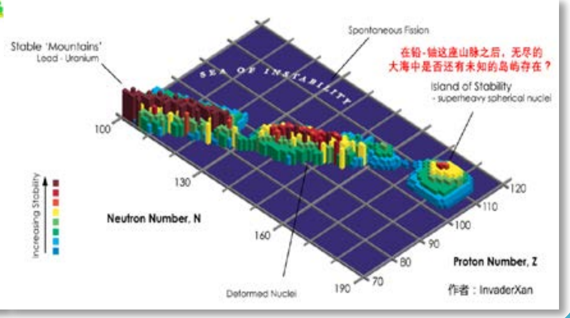
nuclei near drip line



Hyper-nuclei

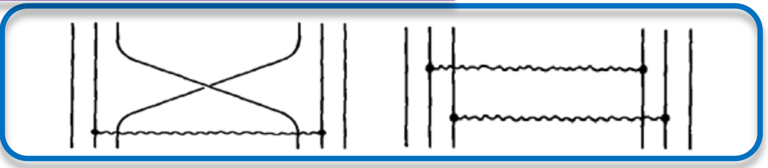


Super heavy elements



One of the most difficult problem

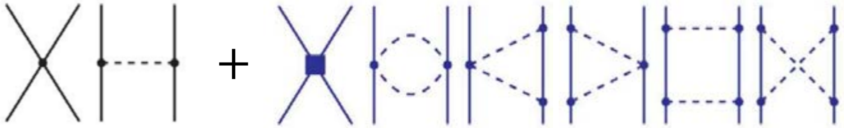
- 1935 Yukawa, Meson theory
- 1950-1960' One pion exchange, One boson exchange
- 1980' Quark model



- 1990' Weinberg, **ChEFT**

S. Weinberg, PLB1990

Modern NF

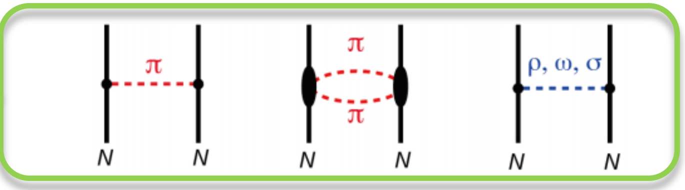


- 1994 **High precision pheno. Models**: **AV18/Reid93 (operator parameterization)**

V. Stoks, PRC1994 R. Wiringa,PRC1994

$$V_{NN} = V_c(r)\hat{1} + V_\sigma(r)\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2} + V_{LS}(r)\,\boldsymbol{L}\cdot\boldsymbol{S} + V_T(r)\,\boldsymbol{\sigma_1}\cdot\boldsymbol{q}\boldsymbol{\sigma_2}\cdot\boldsymbol{q} + \cdots\cdots$$

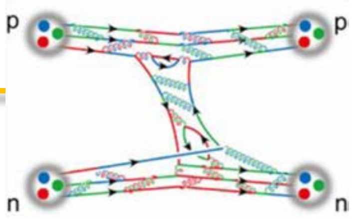
- 2001 **High precision pheno. Models** : **CD-Bonn (Meson Exchange)**



R. Machleidt, PRC2001

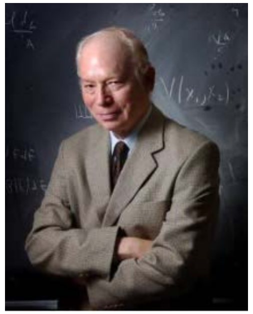
- 2006 **Lattice QCD with full dynamics**

S.R.Beane PRL2006



Hans Bethe

Yukawa



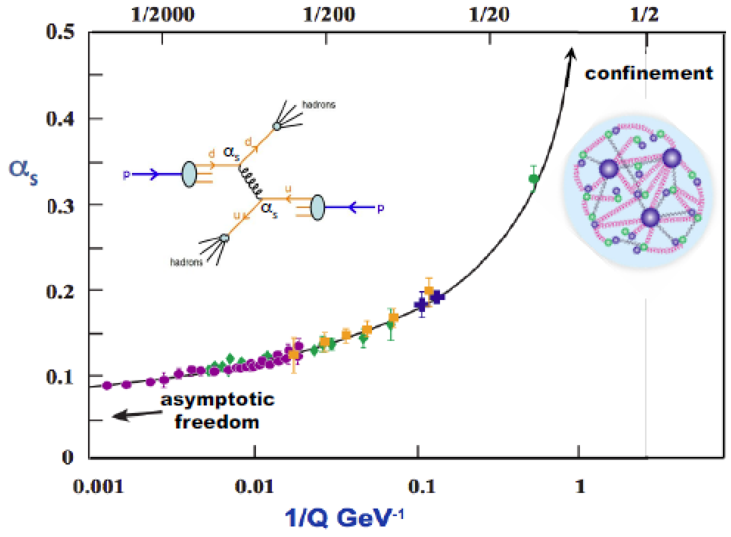
Weinberg

Nobel Prize 1967/1949/1979

QCD and chiral EFT

❑QCD: Asymptotic freedom and confinement

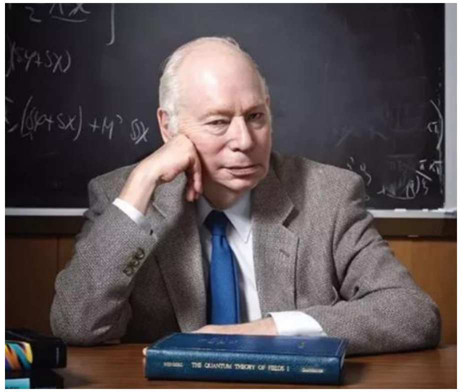
- In higher energy regions
  - "free" quarks
  - perturbative QCD
- In lower energy regions
  - **non-perturbative nature**
  - **hadrons** as degrees of freedom for interactions



❑Chiral effective field theory

One has to write **the most general Lagrangian** consistent with **the assumed symmetry principles**, particularly the **(broken) chiral symmetry of QCD**.

- Chiral symmetry and its spontaneously breaking——Goldstone bosons
- EFT——Interactions in low-energy regime do not depend on the details of those in high-energy regimes



Steven Weinberg

Nobel Prize 1979

closely related to QCD	Degree of freedom	Expansion parameters
$\mathcal{L}_{QCD} \rightarrow$ $\mathcal{L}_{\chi EFT} = \sum c_i O_i$	Quarks & Gluons ⬇ **Hadrons**	$Q/\Lambda \ll 1$ • **Hard scale Λ** • **Soft scale Q**

Perturbative treatment of low-energy strong interaction

- In powers of external momenta and light quark masses——**power counting rule**

$$p_i \to \alpha\, p_i \qquad m_q \to \alpha^2\, m_q$$

$$\mathcal{A}(\alpha p_i, \alpha^2 m_q) = \alpha^D \mathcal{A}(p_i, m_q)$$

$$D = 4L + \sum_n n\, V_n - 2N_M - N_B$$

- D: chiral order of certain diagram
- L: number of loops
- N_M: number of meson propagators
- N_B: number of baryon propagators
- V_n: number of n-th order vertices

Advantages

- Closer connected to QCD
- Systematically improvement
- Uncertainty estimation
- Self-consistent 3-body force

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❑ Advantages

➢ Closer connected to QCD ➢ Systematically improvement ➢ Uncertainty estimation ➢ Self-consistent 3-body force

❑ For baryon systems —— power counting breaking

- Non-vanishing baryon mass at chiral limit

Heavy baryon (HB)
- Heavy static nucleon
- Non-relativistic $p^\mu = m_B v^\mu + k^\mu$

Jenkins and Manohar, **PLB** 255 (1991) 558.

Infrared regularization (IR)
- Covariance
- Analytical problem

Becher and Leutwyler, EPJC 9 (1999) 643

Extended-on-mass-shell (EOMS)
- Covariance
- Faster convergence

Gegelia, Fuchs et al. in **PRD60**(1999), **PRD68** (2003)

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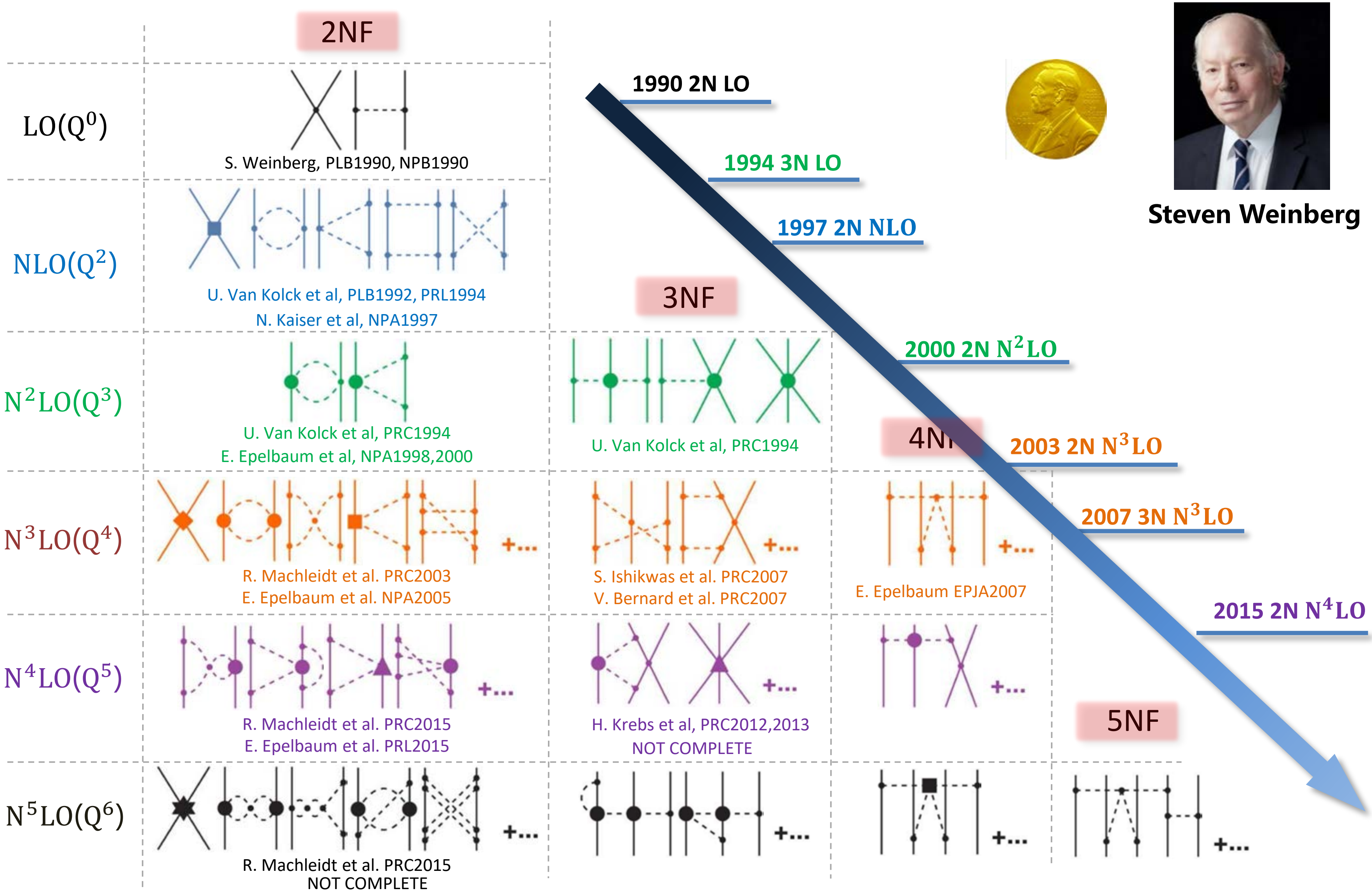
◻ Non-relativistic chiral nuclear force



Steven Weinberg

van Kolck

Kaiser

Epelbaum

Machleidt

Ulf-G Meissner

Adopted from D.R. Entem et al Front.in Phys. 8 (2020) 57

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Non-relativistic chiral nuclear force

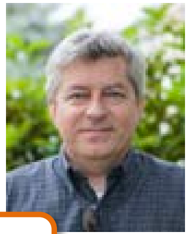
2NF

$LO(Q^0)$

S. Weinberg, PLB1990, NPB1990

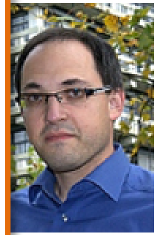
1990 2N LO

1994 3N LO



...n Kolck

Kaiser



...elbaum

Machleidt

Ulf-G Meissner

TABLE II. χ^2/datum for the reproduction of the 1999 *np* database [38] below 290 MeV by various *np* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

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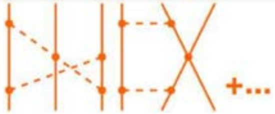
0–290	2057	1.50	35.4	80.1	1.38

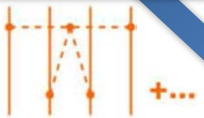
D.R. Entem, R. Machleidt et al. PRC2003

$N^3LO(Q^4)$

R. Machleidt et al. PRC2003
E. Epelbaum et al. NPA2005



S. Ishikwas et al. PRC2007
V. Bernard et al. PRC2007



E. Epelbaum EPJA2007

2007 3N N^3LO

2015 2N N^4LO

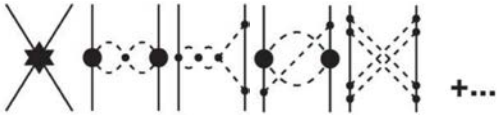
N^3LO: Non-relativistic Chiral NF reached the level of most refined phenomenological forces

R. Machleidt et al. PRC2015
E. Epelbaum et al. PRL2015

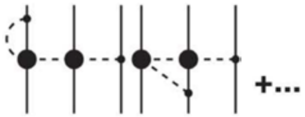
H. Krebs et al, PRC2012,2013
NOT COMPLETE

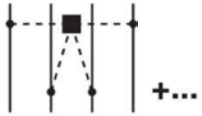
5NF

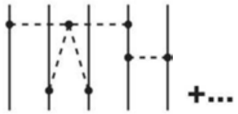
$N^5LO(Q^6)$



R. Machleidt et al. PRC2015
NOT COMPLETE





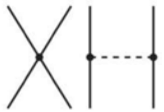


Adopted from D.R. Entem et al Front.in Phys. 8 (2020) 57

Non-relativistic chiral nuclear force

2NF

LO(Q^0)



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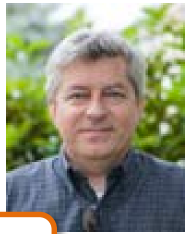

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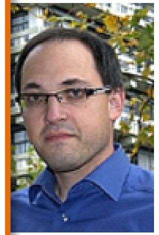
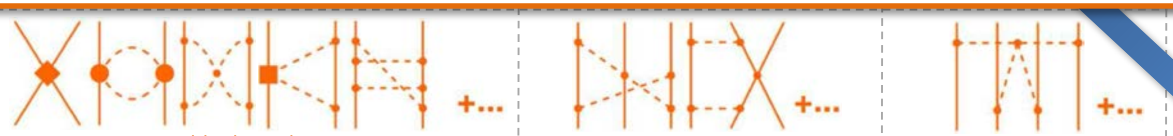


...elbaum

Machleidt

N^{3}LO(Q^4)



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N^{3}LO: Non-relativistic Chiral NF reached the level of most refined phenomenological forces

R. Machleidt et al. PRC2015

H. Krebs et al. PRC2012,2013

5NF

Convenience for ab-initio calculation

- **Local/non-local/semi-local**
- **Momentum/Coordinate**
- **Optimization**
- **Softening(SRG)**

R. Machleidt et al. PRC2015

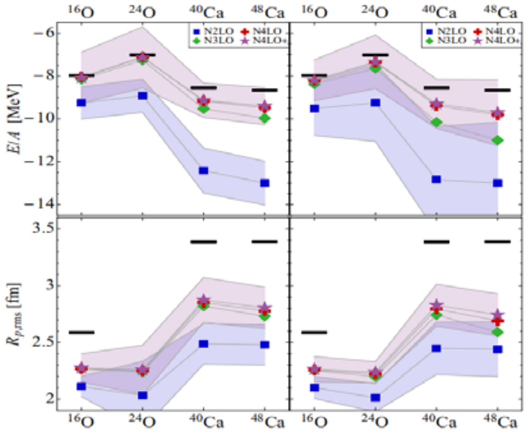
NOT COMPLETE

Adopted from D.R. Entem et al Front.in Phys. 8 (2020) 57

Huge success, but still less satisfying

Slow convergence

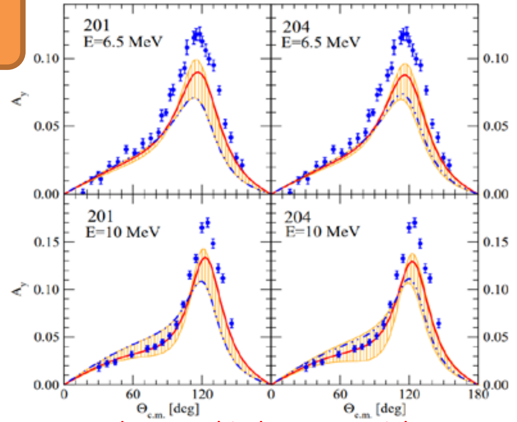
At least N^4LO:
To achieve desired accuracy



P. Maris et al. PRC103(2021)054001

Ay puzzle in n-d scattering

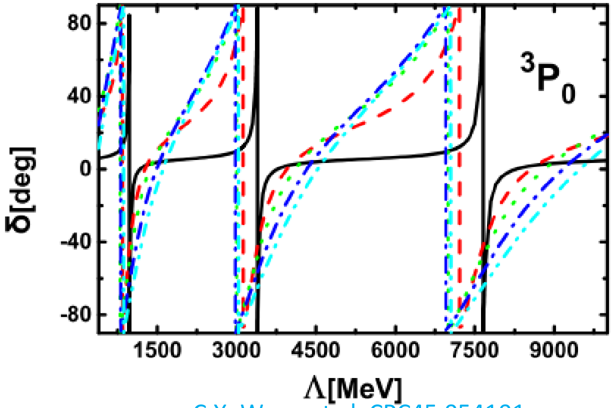
Consistent N^3LO:
2N+3N
does not solve the puzzle


Red: N3LO chiral NN potential
Blue: N3LO chiral NN + full 3N potential

L. Girlanda et al. PRC2019 J. Golak et al. EPJA50,177

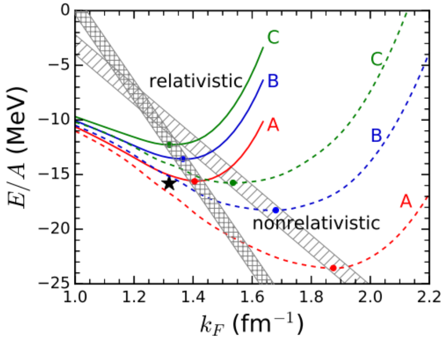
Renormalization group invariance

LS equation is **NOT** renormalizable when potentials are truncated up to certain order

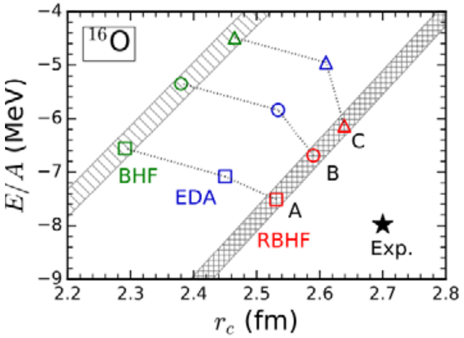


C.X. Wang et al. CPC45,054101

Advantages of relativistic ab-initio calculations



Equation of state of symmetric nuclear matter

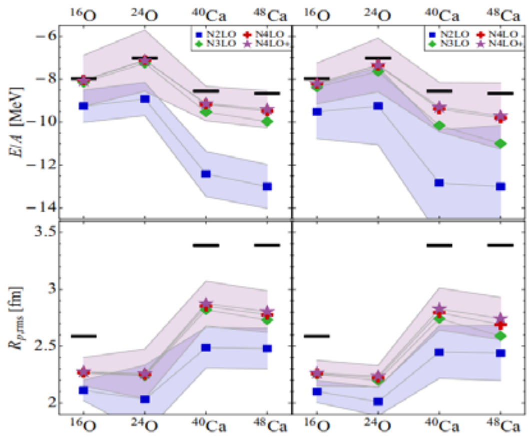


Energy per nucleon

Huge success, but still less satisfying

Slow convergence

At least N^4LO:
To achieve desired accuracy



P. Maris et al. PRC103(2021)054001

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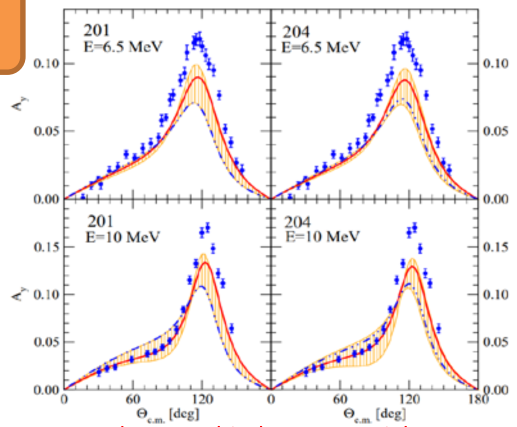

L. Girlanda et al. PRC2019 J. Golak et al. EPJA50,177

Red: N3LO chiral NN potential
Blue: N3LO chiral NN + full 3N potential

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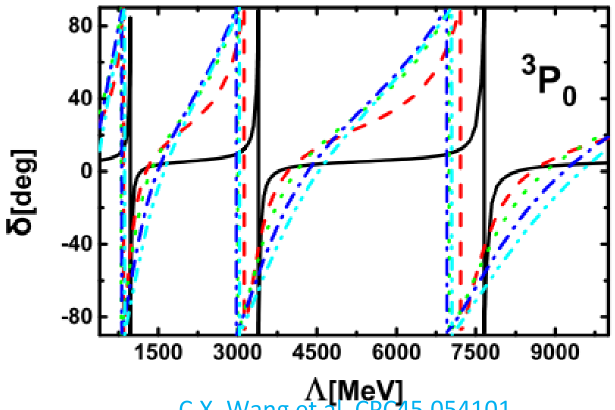

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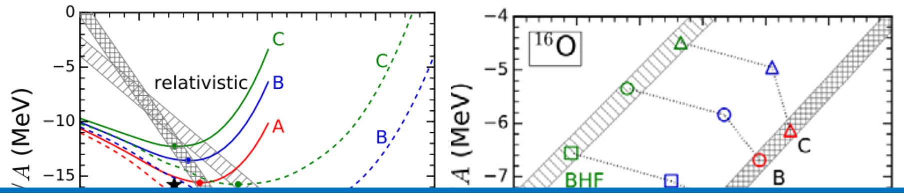


But now, the only available input for relativistic ab-initio calculations —— Bonn potential

Call for accurate relativistic nuclear force!

Relativistic framework

Dynamically and kinematically

- ✓ Lorentz invariance
- ✓ Faster convergence
- ✓ Improved renormalization group invariance
- ✓ Available for relativistic ab-initio calculations

Chiral effective lagrangian: Dirac Spinor/Clifford algebra

$$u(p,s) = N_p \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m} \end{pmatrix} \chi_s$$

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{O}$	0	1	0	0	0	–	0	1
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+

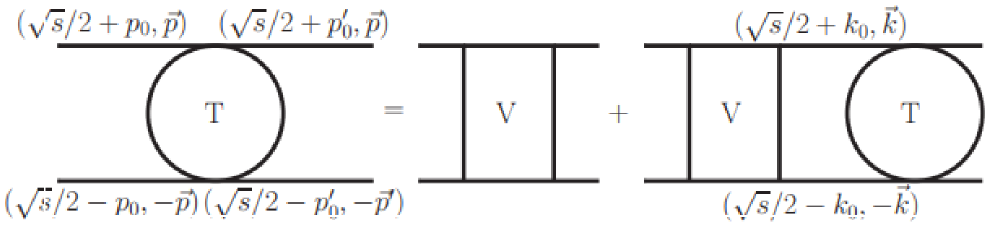
Covariant scheme to restore the broken power counting

Extended-on-mass-shell(EOMS) scheme

$$C_i = C_i^b + C_i^d R + C_i^{\mathrm{PCB}}$$

C_i: LEC C_i^b: the bare value C_i^d: for UV divergence C_i^{PCB}: for PCB terms

Non-perturbative treatment: relativistic scattering equation (Blankenbecler-Sugar equation)



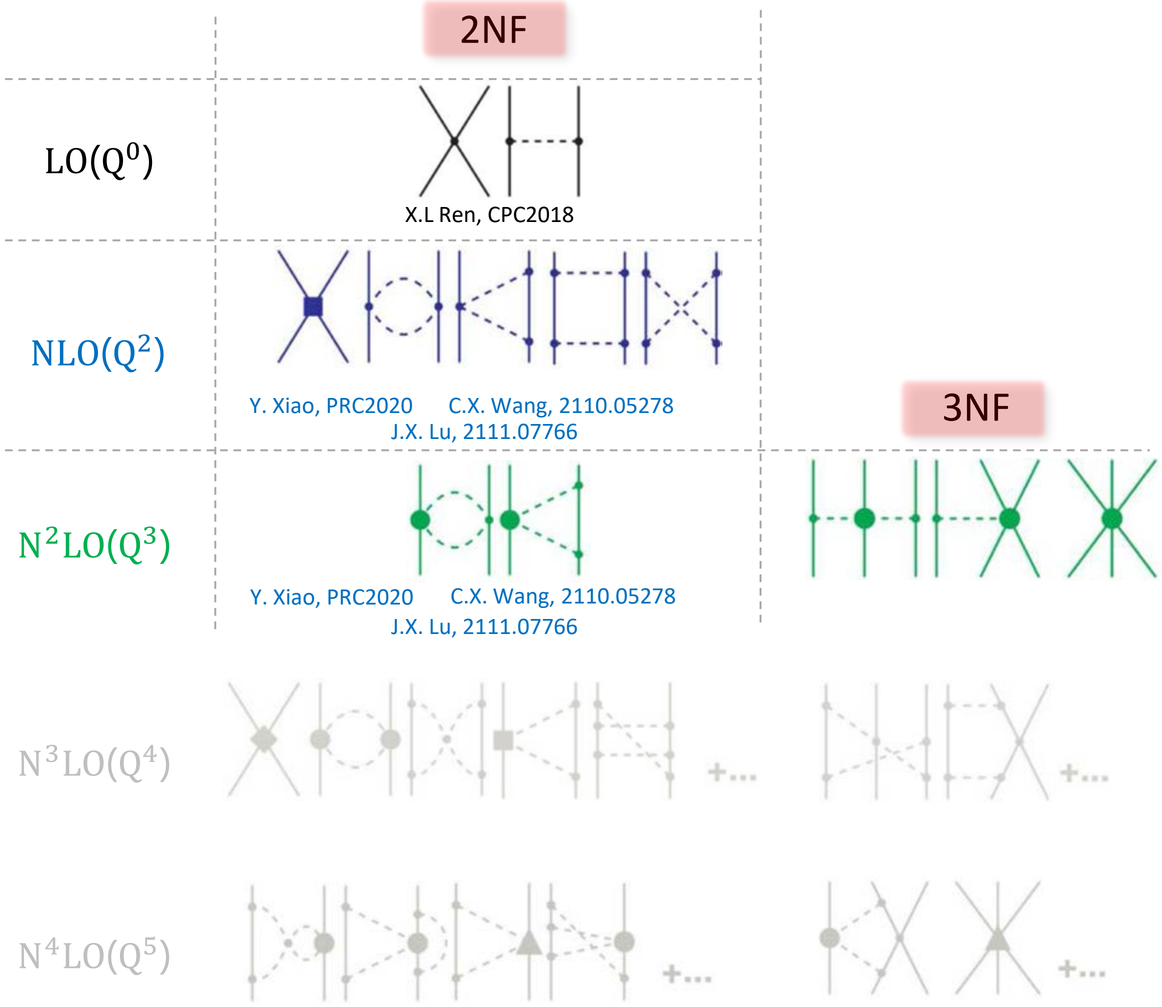
$$\mathcal{T}(p',p|W) = \mathcal{A}(p',p|W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p',k|W) G(k|W) \mathcal{T}(k,p|W),$$

$$\mathcal{T} = \mathcal{V} + \mathcal{V} g \mathcal{T}, \qquad \mathcal{V} = \mathcal{A} + \mathcal{A}(G-g)\mathcal{V}$$

$$g = \frac{\pi i \delta(k^0) \Lambda_+^1(\boldsymbol{k}) \Lambda_+^2(-\boldsymbol{k})}{2E_k(E_k^2 - s/4 - i\epsilon)}$$

Covariant framework

☐ **Relativistic** chiral nuclear force



Relativistic Nuclear force

2018: ***Leading order*** *relativistic chiral nucleon-nucleon interaction*
X.L Ren et al CPC42,014103

2019: *Covariant chiral nucleon-nucleon contact* ***Lagrangian*** *up to order* $\mathcal{O}(q^4)$
Y. Xiao et al Phys.Rev.C 99,024004

2020: ***Two-pion exchange*** *contributions to the nucleon-nucleon interaction in covariant baryon ChPT*
Y. Xiao et al Phys.Rev.C 102,054001

2021: ***Non-perturbative two-pion exchange*** *contributions to the nucleon-nucleon interaction in covariant baryon ChPT*
C.X. Wang et al Phys.rev.C 105,014003

An accurate *relativistic chiral nucleon-nucleon interaction up to NNLO*
J.X Lu et al Phys.rev.lett. 128,142002

2023: ***Saturation of nuclear matter*** *in the relativistic Brueckner-Hatree-Fock approach with* ***a leading order covariant chiral nuclear force***
W.J. Zou et al. Phys.Lett.B 854 (2024) 138732

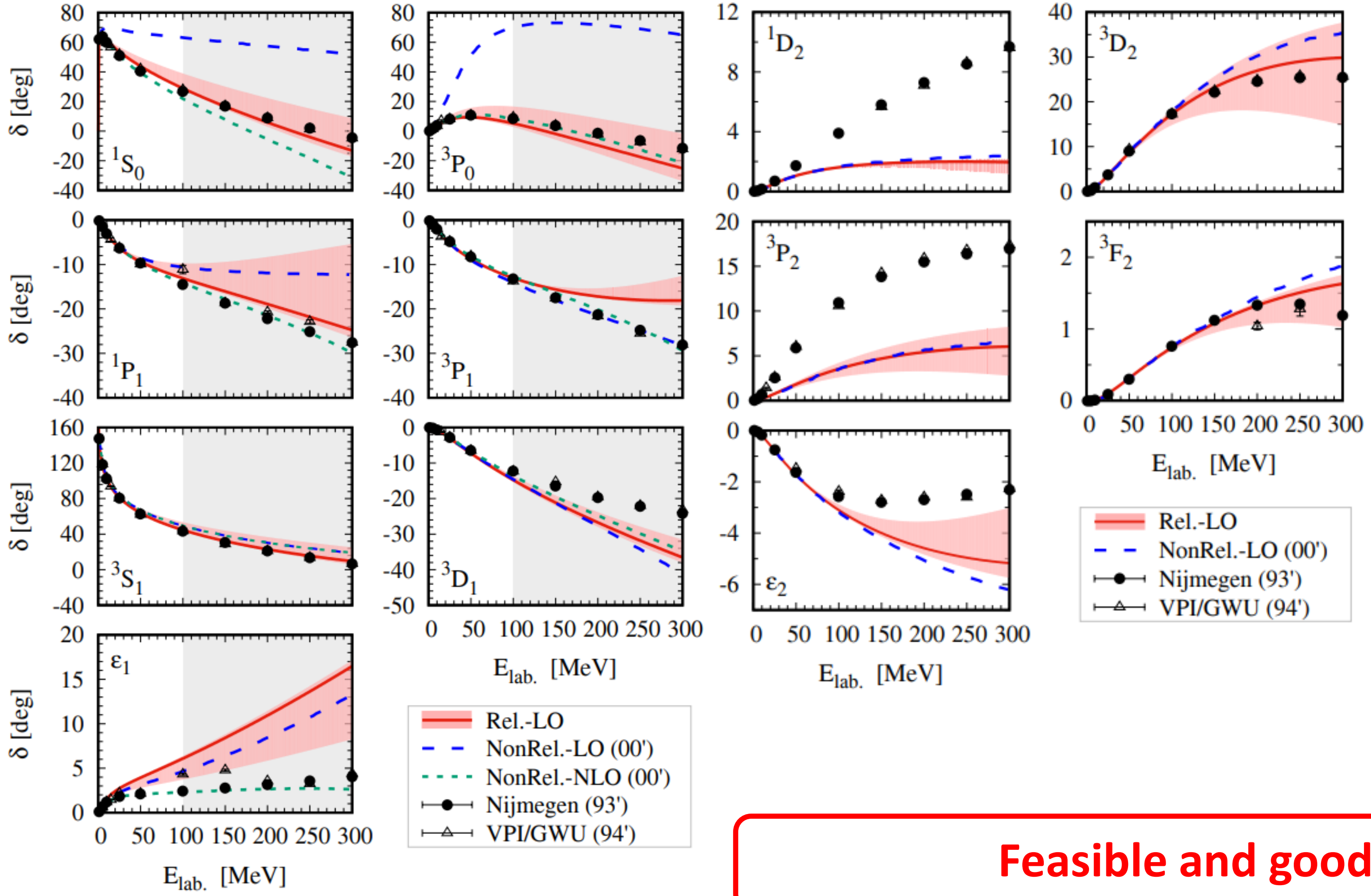
2024: ***Antinucleon-nucleon interactions*** *in covariant chiral effective field theory*
Y. Xiao et al. arXiv: 2406.01292

LO Relativistic chiral nuclear force

- CON(4)+OPE
- Phase shifts for n-p scattering

Ren et al. CPC42,014103

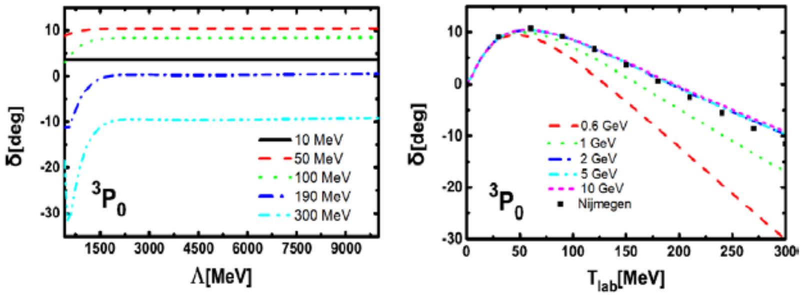
- **Compatible to NR NLO for J=0,1**
- **Not good enough for J=2**



- RGI analysis

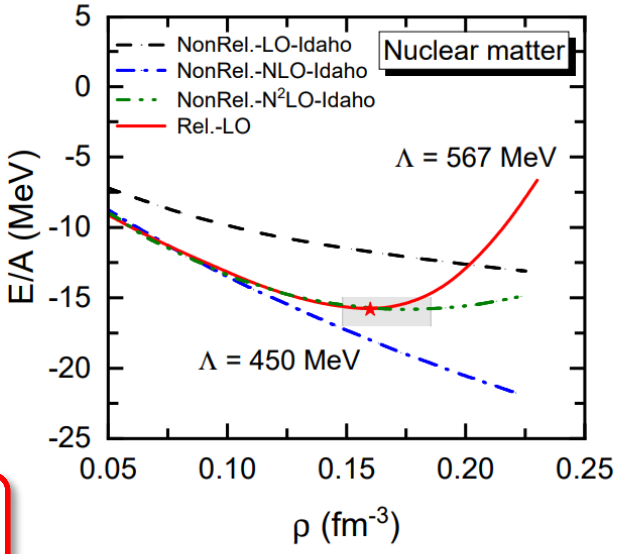
Wang et al. CPC45,054101

- $^3S_1$, $^3D_1$, $^1P_1$, $^3P_0$



- Nuclear matter saturation

Zou et al. PLB 854 (2024) 138732



Feasible and good

☐ NNLO Relativistic chiral nuclear force: contact terms

$$V = V_{\rm CT}^{\rm LO} + V_{\rm CT}^{\rm NLO} + V_{\rm OPE} + V_{\rm TPE}^{\rm NLO} + V_{\rm TPE}^{\rm NNLO} - V_{\rm IOPE}$$

Xiao et al. Phys.Rev.C 99,024004

- Relativistic NLO NF **(4+13)**

LO	$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$
LO	$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$
LO	$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$
LO	$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$
NLO	$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$
NLO	$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_\mu\psi\right)$
NLO	$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_\nu\psi\right)$
NLO	$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^\mu\psi\right)\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
NLO	$\widetilde{O}_9$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\alpha}\psi\right)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
NLO	$\widetilde{O}_{10}$	$\frac{1}{4m^2}\left(\bar\psi\psi\right)\partial^2\left(\bar\psi\psi\right)$
NLO	$\widetilde{O}_{11}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu\psi\right)\partial^2\left(\bar\psi\gamma_\mu\psi\right)$
NLO	$\widetilde{O}_{12}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$
NLO	$\widetilde{O}_{13}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
NLO	$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_1$
NLO	$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_2$
NLO	$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_3$
NLO	$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_4$

- Non-relativistic $\mathrm{N^3LO}$ NF **(2+7+15)**

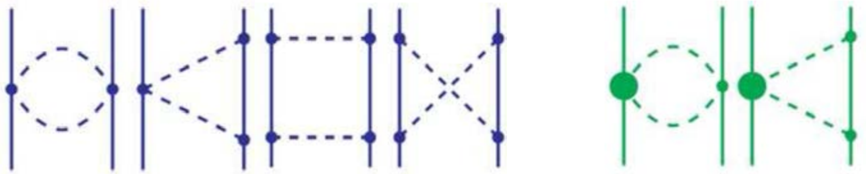
LO	O_S	$(N^\dagger N)(N^\dagger N)$
LO	O_T	$(N^\dagger\boldsymbol{\sigma}N)\cdot(N^\dagger\boldsymbol{\sigma}N)$
NLO	O_1	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}^2N)+\text{h.c.}$
NLO	O_2	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
NLO	O_3	$i(N^\dagger\boldsymbol{\sigma}N)\cdot(N^\dagger\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)$
NLO	O_4	$(N^\dagger\sigma^jN)(N^\dagger\sigma^j\overrightarrow{\nabla}^2N)+\text{h.c.}$
NLO	O_5	$(N^\dagger\sigma^jN)(N^\dagger\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
NLO	O_6	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)+\text{h.c.}$
NLO	O_7	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}N)$
N³LO	O_8	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}^4N)+\text{h.c.}$
N³LO	O_9	$(N^\dagger\overrightarrow{\nabla}^2N)(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)+\text{h.c.}$
N³LO	O_{10}	$(N^\dagger\overrightarrow{\nabla}^2N)(N^\dagger\overleftarrow{\nabla}^2N)$
N³LO	O_{11}	$(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
N³LO	O_{12}	$i(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)(N^\dagger\overrightarrow{\nabla}^2N)+\text{h.c.}$
N³LO	O_{13}	$i(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
N³LO	O_{14}	$(N^\dagger\sigma^jN)(N^\dagger\sigma^j\overrightarrow{\nabla}^4N)+\text{h.c.}$
N³LO	O_{15}	$(N^\dagger\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)(N^\dagger\sigma^j\overrightarrow{\nabla}^2N)+\text{h.c.}$
N³LO	O_{16}	$(N^\dagger\sigma^j\overrightarrow{\nabla}^2N)(N^\dagger\sigma^j\overleftarrow{\nabla}^2N)$
N³LO	O_{17}	$(N^\dagger\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)(N^\dagger\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
N³LO	O_{18}	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\overrightarrow{\nabla}^2N)+\text{h.c.}$
N³LO	O_{19}	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}\overrightarrow{\nabla}^2N)+\text{h.c.}$
N³LO	O_{20}	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)+\text{h.c.}$
N³LO	O_{21}	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\overleftarrow{\nabla}^2N)+\text{h.c.}$
N³LO	O_{22}	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$

Num. of LECs:

NR-NNLO < R-NNLO < NR-$\mathrm{N^3LO}$

NLO/NNLO Relativistic chiral nuclear force: TPE

$$V = V_{\rm CT}^{\rm LO} + V_{\rm CT}^{\rm NLO} + V_{\rm OPE} + V_{\rm TPE}^{\rm NLO} + V_{\rm TPE}^{\rm NNLO} - V_{\rm IOPE}$$



- Non-relativistic NLO NF

$$V_{\rm NLO}^{\rm TPEP} = -\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{384\pi^2 f_\pi^4} L(q)\left\{4M_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\right\} - \frac{3g_A^4}{64\pi^2 f_\pi^4}L(q)\{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q} - q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\} + P(\boldsymbol{k},\boldsymbol{q}),$$

$$L(q) = \frac{1}{q}\sqrt{4M_\pi^2+q^2}\ln\frac{\sqrt{4M_\pi^2+q^2}+q}{2M_\pi}$$

- Relativistic NLO NF

$$\mathcal{A} = \text{Bilinear} \times \text{Loop integral}$$

- Bilinear(**114 terms for planar box diagram**)

$$\bar u_3 u_1\cdot\bar u_4 u_2 \quad \bar u_3 \not p_i u_1\cdot\bar u_4 u_2 \quad \bar u_3\not p_i\not p_j u_1\cdot\bar u_4 u_2 \quad \bar u_3\gamma^\mu u_1\cdot\bar u_4\gamma_\mu u_2 \quad \bar u_3\not p_i\gamma^\mu u_1\cdot\bar u_4\not p_j\gamma_\mu u_2$$

$$\bar u_3 u_1\cdot\bar u_4\not p_i u_2 \quad \bar u_3 u_1\cdot\bar u_4\not p_i\not p_j u_2 \quad \bar u_3\not p_i u_1\cdot\bar u_4\not p_j u_2 \quad \bar u_3\gamma^\mu u_1\cdot\bar u_4\not p_i\gamma_\mu u_2 \quad \bar u_3\gamma^\mu u_1\cdot\bar u_4\not p_i\not p_j\gamma_\mu u_2$$

more...

- Loop integral—— scalar and tensor integrals (**29 terms for planar box diagram**)

$$A_0, B_0, B_{00}, C_0, C_1, C_2, C_{00}, C_{11}, C_{12}, C_{22}, D_0, D_1, D_2, D_{00}, D_{11}, D_{12}, D_{22}, D_{23}$$

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Results of NLO/NNLO relativistic NF

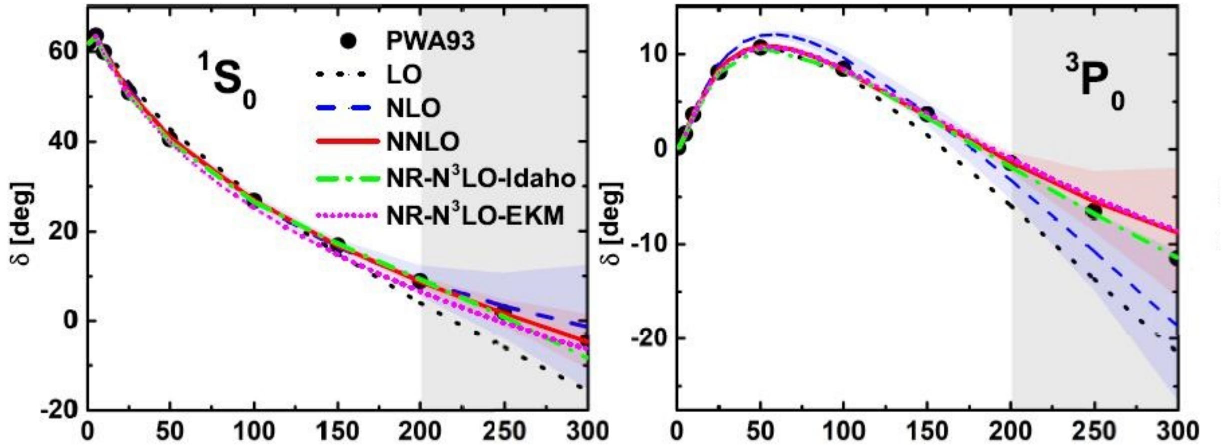
TABLE III. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\rm PWA93})^2$ of different chiral forces for partial waves up to $J \leq 2$.

	Total	$^1S_0$	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	$^3D_1$	ϵ_1	$^1D_2$	$^3D_2$	$^3P_2$	$^3F_2$	ϵ_2
NLO	17.02	1.02	7.04	0.46	0.33	1.80	1.69	0.15	2.18	1.35	0.95	0.01	0.04
NNLO	16.61	0.18	0.30	1.07	1.55	3.36	0.26	0.03	0.01	9.56	0.01	0.27	0.01
NR-N^{3}LO-Idaho	8.84	1.53	0.30	2.41	0.04	2.33	1.00	0.02	0.57	0.42	0.17	0.03	0.02
NR-N^{3}LO-EKM	16.08	13.45	0.29	0.34	0.06	0.01	0.13	0.01	0.02	0.43	0.12	1.22	0.00

Relativistic NNLO compatible to non-relativistic N^{3}LO

- NR-N^{3}LO-Idaho: R. Machleidt and D. R. Entem, Phys.Rev.C(2003), Phys.Rept.(2011)
- NR-N^{3}LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur.Phys.J.A(2015), Phys.Rev.Lett. (2015).

Phase shifts for J=0



- **Uncertainties are estimated via Bayesian model**
- **NLO** is compatible with **NNLO** below 200MeV
- Below 200MeV: **NNLO, NR-N^{3}LO-Idaho, NR-N^{3}LO-EKM** almost overlap

Fitting results

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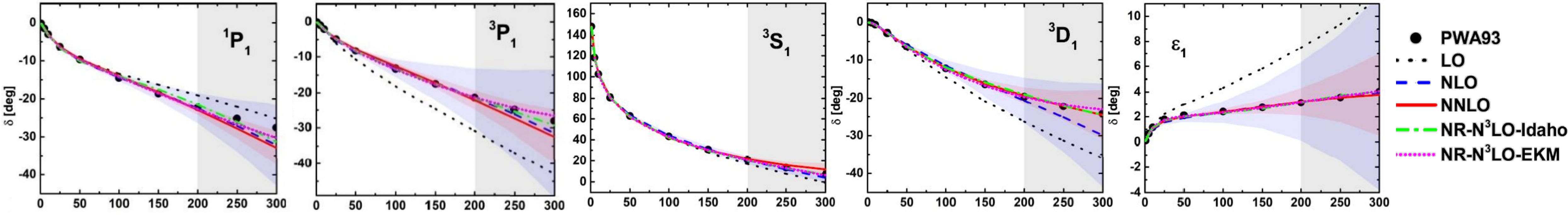
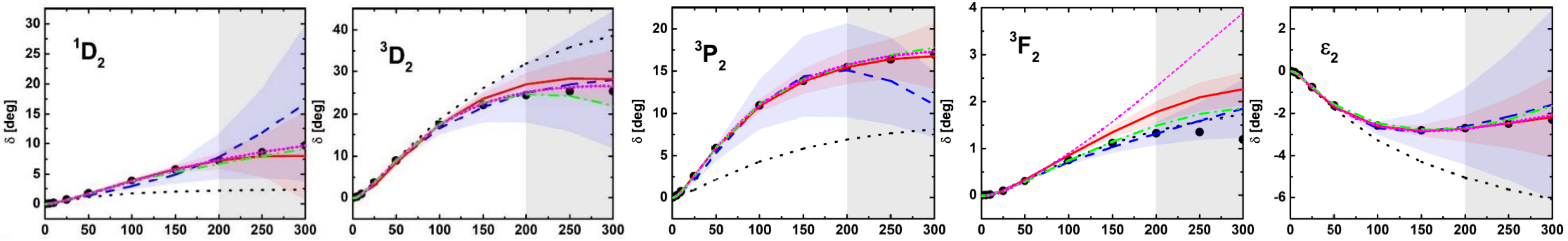
Phase shifts for J=1



- **NLO** is compatible with **NNLO** below 200MeV
- Below 200MeV: **NNLO, NR-N^{3}LO-Idaho, NR-N^{3}LO-EKM** almost overlap

Phase shifts for J=2



For higher kinetic energies

- $^1D_2$, $^3P_2$: **NLO** slightly worse
- $^3D_2$: **NNLO** slightly worse

- Below 200MeV: **NNLO, NR-N^{3}LO-Idaho, NR-N^{3}LO-EKM** almost overlap

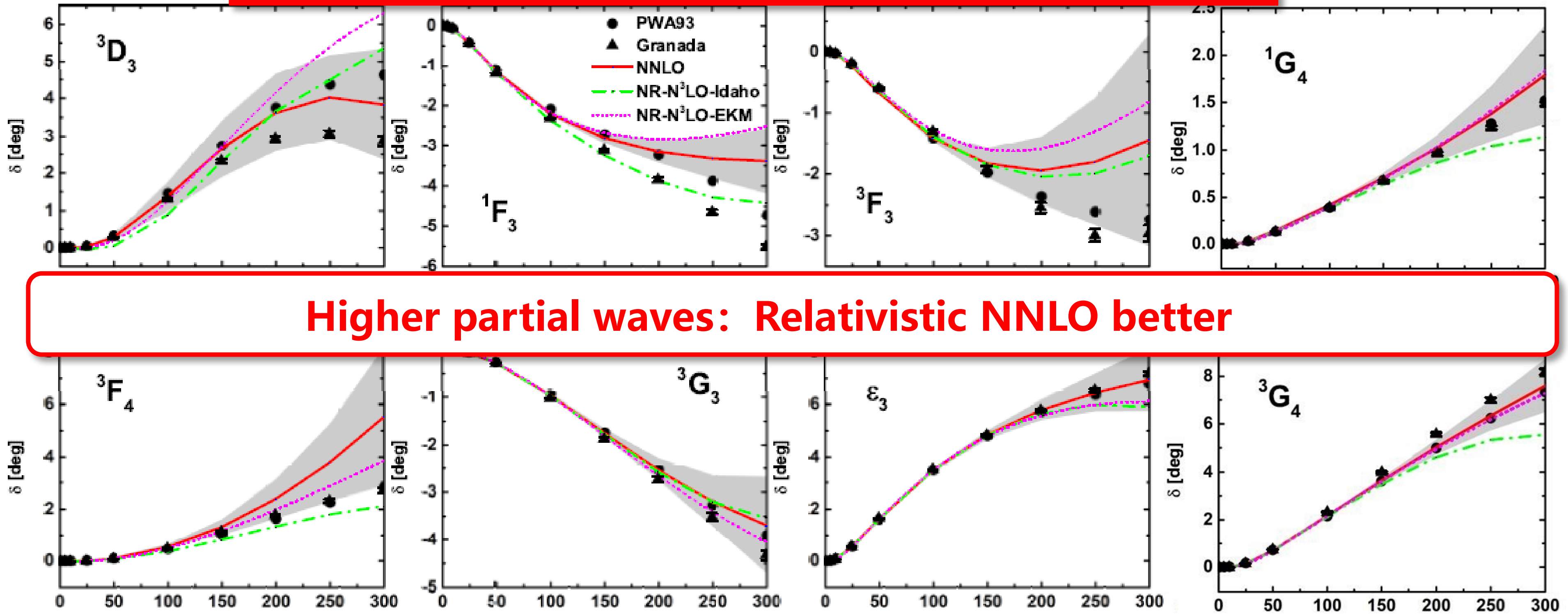
- $^3F_2$: **NR-N^{3}LO-Idaho** best but with fine-tuned $\mathbf{c}_{2,3,4}$

- Predictions for phase shifts for $J \leq 4$ and $L \leq 4$

No contact terms up to NNLO

TABLE II. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\mathrm{PWA93}})^2$ of different chiral forces for partial waves shown in Fig. 6.

	Total	$^3D_3$	$^1F_3$	$^3F_3$	$^3F_4$	$^3G_3$	ϵ_3	$^1G_4$	$^3G_4$
NNLO	0.98	0.03	0.03	0.21	0.70	0.00	0.01	0.00	0.00
NR-N^{3}LO-Idaho	1.73	0.58	0.73	0.13	0.12	0.00	0.01	0.01	0.15
NR-N^{3}LO-EKM	1.39	0.23	0.18	0.79	0.15	0.01	0.02	0.00	0.00



Higher partial waves: Relativistic NNLO better

- **NNLO** is better in $^3D_3$, $^1F_3$ but well above the Nijmegen data in $^3F_4$.
- $^1G_4$, $^3G_4$, ε_3: **NR-N^{3}LO-Idaho** tend to yield smaller values at higher energies

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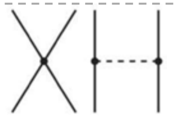
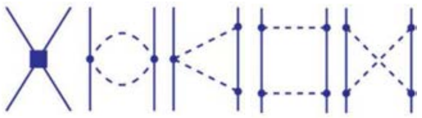
Relativistic NNLO p-p scattering

- Isospin breaking effect： Epelbaum et al. NPA747362
  - Strong part —— $\mathbf{m_u - m_d}$
  - Electromagnetic part —— charges of u,d quarks

	n-p: 2NF	p-p extra term
$\mathrm{LO}(Q^0)$	 X.L Ren, CPC2018	
$\mathrm{NLO}(Q^2)$	 Y. Xiao, PRC2020 C.X. Wang, 2110.05278 J.X. Lu, 2111.07766	$M_\pi^\pm \neq M_\pi^0$ in OPE p-p coulomb force
$\mathrm{N^2LO}(Q^3)$	 Y. Xiao, PRC2020 C.X. Wang, 2110.05278 J.X. Lu, 2111.07766	Contact terms strong part $\propto \epsilon M_\pi^2$ Independent on momenta

$$\epsilon = \frac{m_u - m_d}{m_u + m_d}$$

Correspond to LO contact terms

- Non-relativistic frame: only $^1S_0$

Equivalently redefine the LECs

$$\langle {}^1S_0, \mathrm{np}|V^{\mathrm{np}}_{\mathrm{cont}}|{}^1S_0, \mathrm{np}\rangle = \tilde{C}^{\mathrm{np}}_{1S0} + C_{1S0}(p^2 + p'^2)$$

$$\langle {}^1S_0, \mathrm{pp}|V^{\mathrm{pp}}_{\mathrm{cont}}|{}^1S_0, \mathrm{pp}\rangle = \tilde{C}^{\mathrm{pp}}_{1S0} + C_{1S0}(p^2 + p'^2)$$

- Relativistic frame: $^1S_0$, $^3P_0$, $^3P_1$

$$V^{LO}_{1S0} = 2\pi\left(2C_1^{1S0} + C_2^{1S0}\frac{\Gamma}{m^2}\right)$$

$$V^{LO}_{3P0} = -\frac{2\pi C^{3P0} pp'}{m^2}$$

$$V^{LO}_{3P1} = -\frac{4\pi C^{3P1} pp'}{3m^2}$$

$\Rightarrow$ $C^{1S0}_{1,pp}$ $C^{1S0}_{2,pp}$ C^{3P0}_{pp} C^{3P1}_{pp}

3 independent

□ Relativistic NNLO p-p scattering

Epelbaum et al. NPA747362

np, pp simultaneously

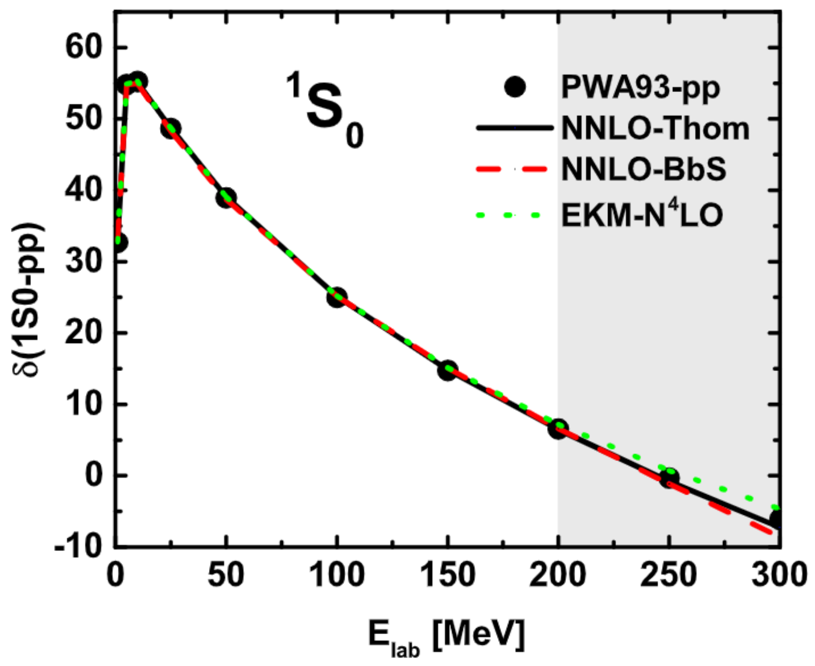
Thompson Equation; Gaussian, Λ=0.7GeV

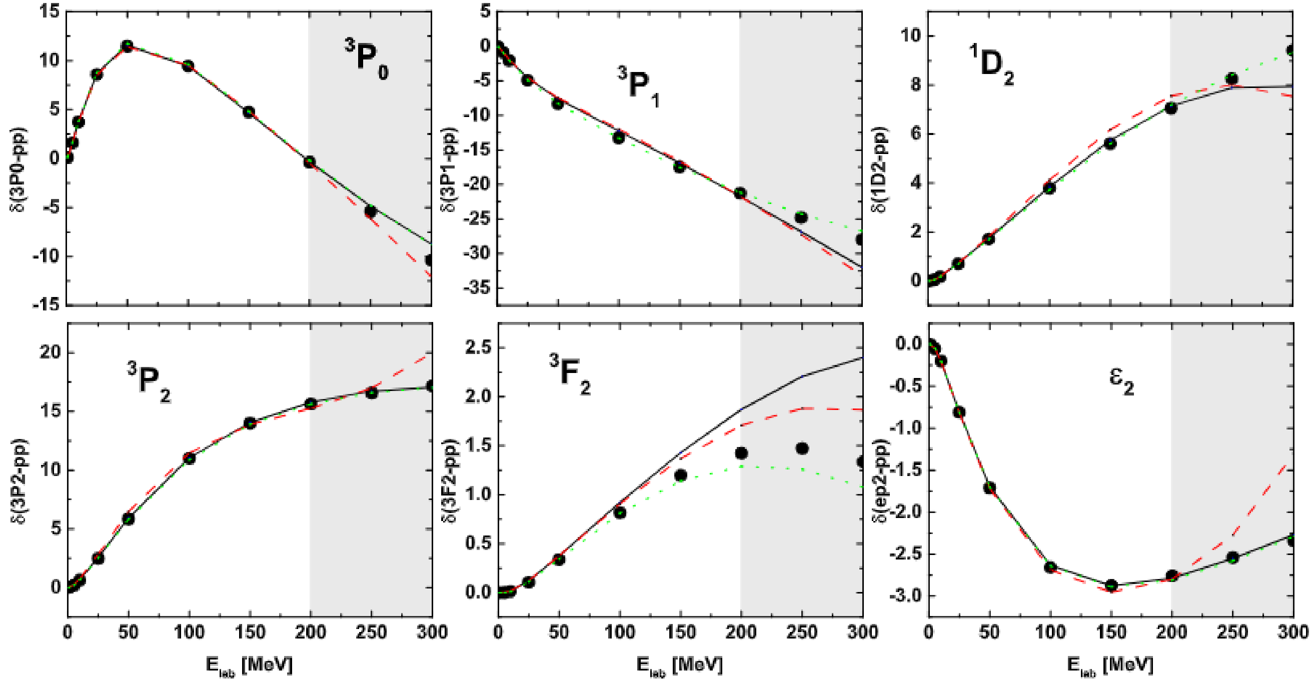
Blankenbecler-Sugar Equation; sharp, Λ=0.9GeV

TABLE IV. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\mathrm{PWA93}})^2$ of different chiral forces for partial waves up to $J \leq 2$ for pp scattering.

	Total	$^1S_0$	$^3P_0$	$^3P_1$	$^1D_2$	$^3P_2$	$^3F_2$	ϵ_2
NNLO-Thom	5.67	0.50	0.04	3.43	0.70	0.87	0.12	0.01
NNLO-BbS	2.60	0.07	0.04	2.15	0.03	0.04	0.27	0.00
NR-N^{4}LO-EKM	0.87	0.60	0.17	0.03	0.02	0.03	0.02	0.00

- For $^1S_0$, all are in good agreement with experimental data. **NNLO** is slightly flawed for $^3P_1$
- **NNLO** overlap with **NR-N4LO-EKM** in lower energy region, but slightly worse **beyond 200MeV**



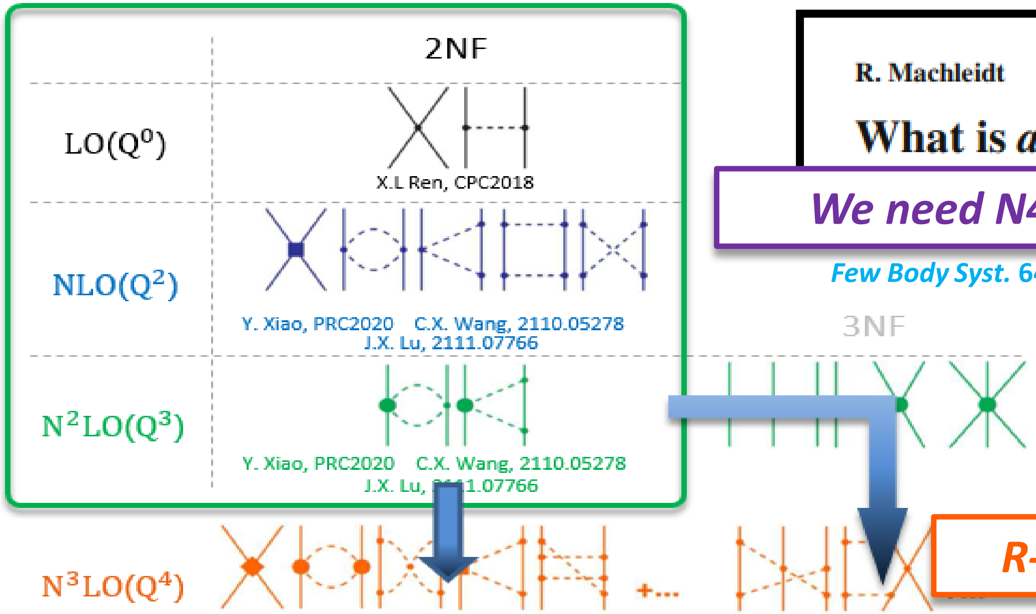


Summary and prospect

- We construct a relativistic chiral nuclear force up to NNLO
  - We maintain the relativity **both dynamically and kinematically**
  - For **np scattering**, it is compatible with **non-relativistic N^3LO results** for lower energies.
  - For **pp scattering**, the phase shifts for **$^1S_0$** agrees quite well with Nijmegen93

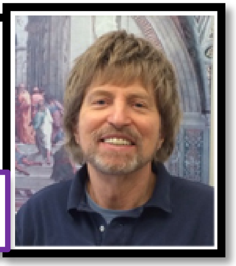
• Lorentz invariance • Fewer parameters • Lower chiral order

- What's next?



R. Machleidt

What is *ab initio*?



We need N4LO !

Few Body Syst. 64 (2023) 4, 77

R-ChNF-N^{3}LO ongoing !

Thanks for your attention

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Uncertainties

Bayesian truncation uncertainties

- The expansion for an observable X of EFT

$$X = X_{\rm ref}\sum_{n=0}^{\infty} c_n Q^n = X^{(0)} + \Delta X^{(2)} + \ldots, \qquad Q = {\rm Max}\{\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\}$$

R.J.Furnstahl et al. PRC2015, PRC2019
E. Epelbaum et al. PRL2015,EPJA2019
P. Maris et al. PRC2021

$$X_{\rm ref} = {\rm Max}\{|X^{\rm LO}|, \frac{|X^{\rm LO} - X^{\rm NLO}|}{Q^2}, \frac{|X^{\rm NLO} - X^{\rm NNLO}|}{Q^3}\},$$

- **$k^{\rm th}$ order truncation uncertainty**

$$\Delta_k = \sum_{n=k+1}^{\infty} c_n Q^n$$

- **Bayesian model**: encode the expectation of the expansion coefficients c_i in a "prior pdf" $\mathbf{pr(c_i|\bar{c})}$

$$\mathrm{pr}_h(\Delta|c_{i\le k}) = \frac{\int_0^\infty d\bar{c}\,\mathrm{pr}_h(\Delta|\bar{c})\mathrm{pr}(\bar{c})\prod_{i\in A}\mathrm{pr}(c_i|\bar{c})}{\int_0^\infty d\bar{c}\,\mathrm{pr}(\bar{c})\prod_{i\in A}\mathrm{pr}(c_i|\bar{c})},$$

DoB

$$p\% = \int_{-d_k^{(p)}}^{d_k^{(p)}} p(\Delta_k|c_0, c_1, \ldots, c_k)\mathrm{d}\Delta_k \qquad \Delta X^{(k)} = X_{\rm ref} d_k^{(p)}$$

Advantages

- ✓ Statistically well established
- ✓ Up to **arbitrary order** vs only **known order**

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Relativistic NNLO p-p scattering

Epelbaum et al. NPA747362

np, pp simultaneously

TABLE IV. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\rm PWA93})^2$ of different chiral forces for partial waves up to $J \le 2$ for pp scattering.

	Total	$^1S_0$	$^3P_0$	$^3P_1$	$^1D_2$	$^3P_2$	$^3F_2$	ϵ_2
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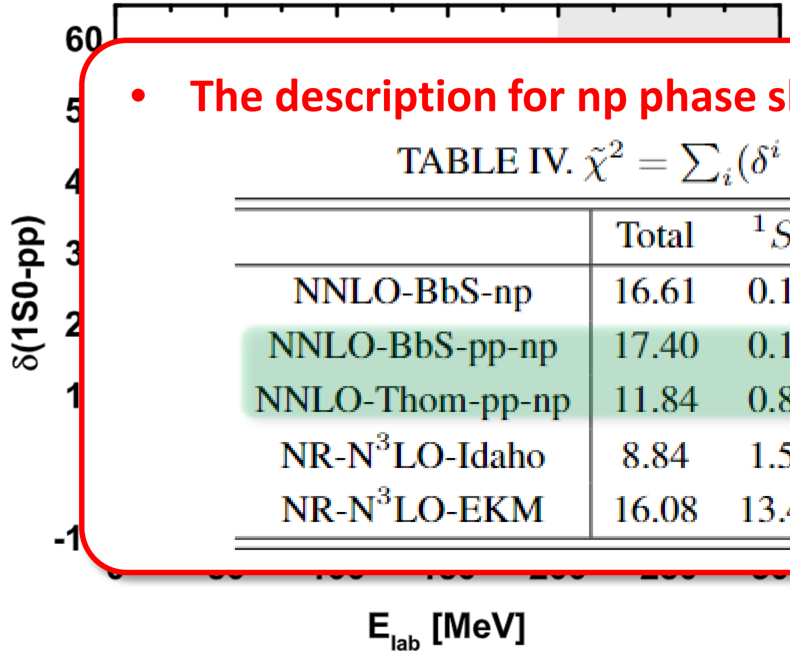
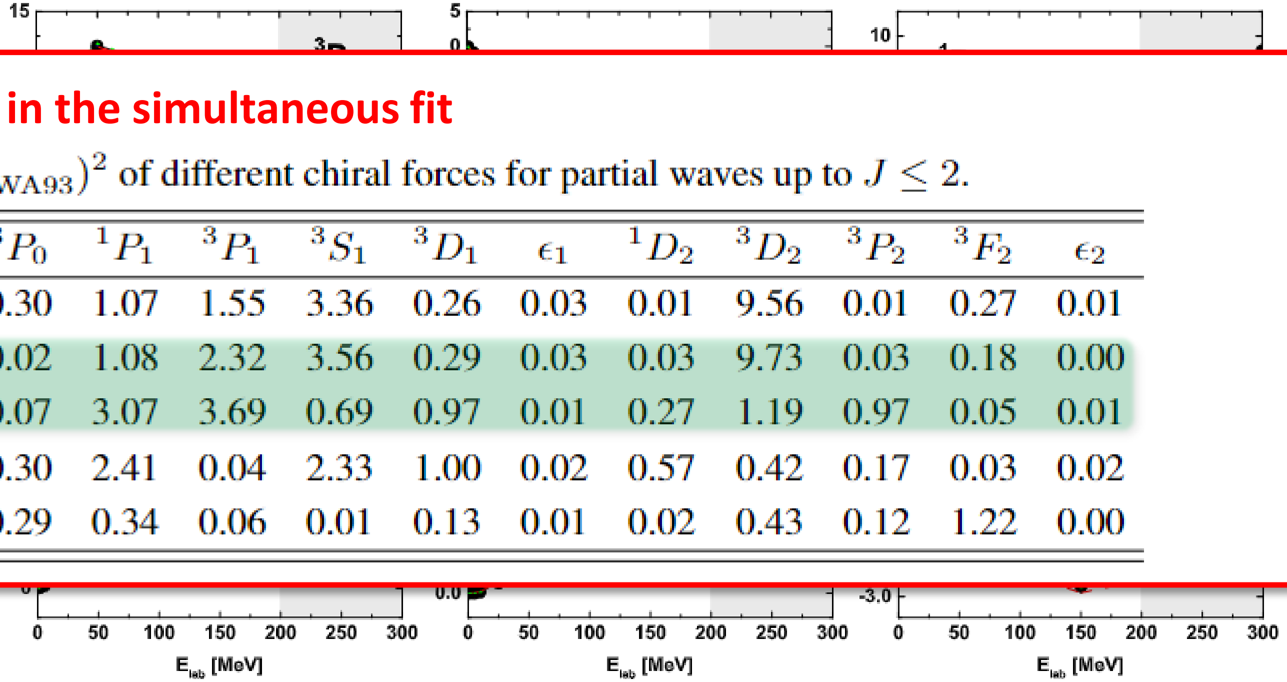
NR-N^{4}LO-EKM	0.87	0.60	0.17	0.03	0.02	0.03	0.02	0.00

Gaussian, Λ=0.7GeV

sharp, Λ=0.9GeV

- For $^1S_0$, all are in good agreement with experimental data. **NNLO** is slightly flawed for $^3P_1$
- **NNLO** overlap with **NR-N4LO-EKM** in lower energy region, but slightly worse **beyond 200MeV**





- **The description for np phase shifts in the simultaneous fit**

TABLE IV. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\rm PWA93})^2$ of different chiral forces for partial waves up to $J \le 2$.

	Total	$^1S_0$	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	$^3D_1$	ϵ_1	$^1D_2$	$^3D_2$	$^3P_2$	$^3F_2$	ϵ_2
NNLO-BbS-np	16.61	0.18	0.30	1.07	1.55	3.36	0.26	0.03	0.01	9.56	0.01	0.27	0.01
NNLO-BbS-pp-np	17.40	0.14	0.02	1.08	2.32	3.56	0.29	0.03	0.03	9.73	0.03	0.18	0.00
NNLO-Thom-pp-np	11.84	0.86	0.07	3.07	3.69	0.69	0.97	0.01	0.27	1.19	0.97	0.05	0.01
NR-N^{3}LO-Idaho	8.84	1.53	0.30	2.41	0.04	2.33	1.00	0.02	0.57	0.42	0.17	0.03	0.02
NR-N^{3}LO-EKM	16.08	13.45	0.29	0.34	0.06	0.01	0.13	0.01	0.02	0.43	0.12	1.22	0.00

Fitting results

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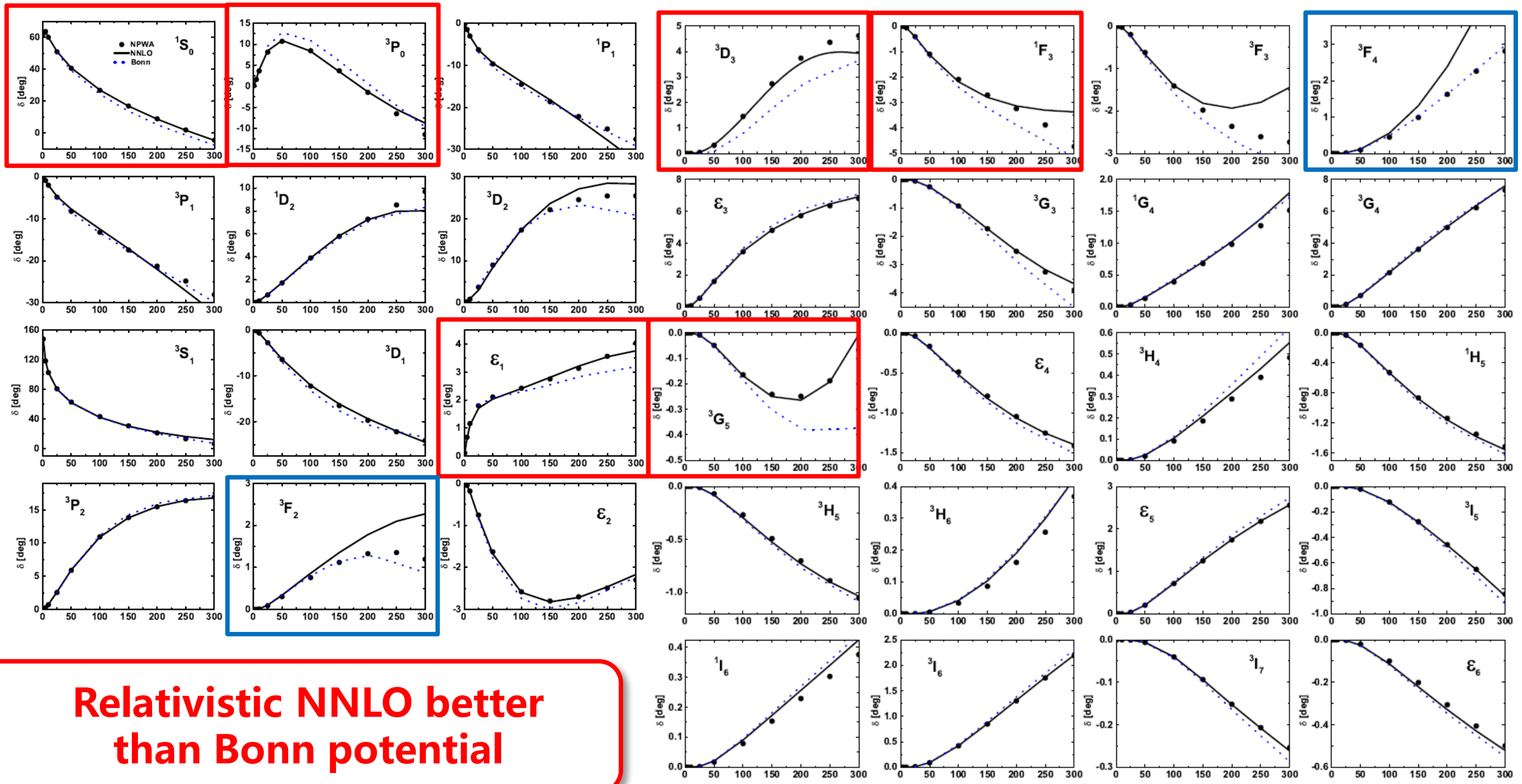
Comparison with **Bonn potential**

R. Machleidt et al, Phys.Rept.1987
S-H. Shen et al, Prog.Part.Nucl.Phys.2019

- So far, Bonn potential developed in 1980’ is the most common choice for relativistic many-body studies



Relativistic NNLO better than Bonn potential