Baryon properties from direct solutions of the Poincaré-covariant three-body Faddeev equation

Si-Xue Qin

（秦思学）

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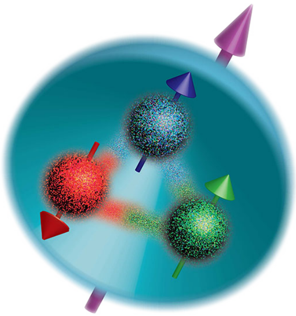
Introduction: Baryons and their properties



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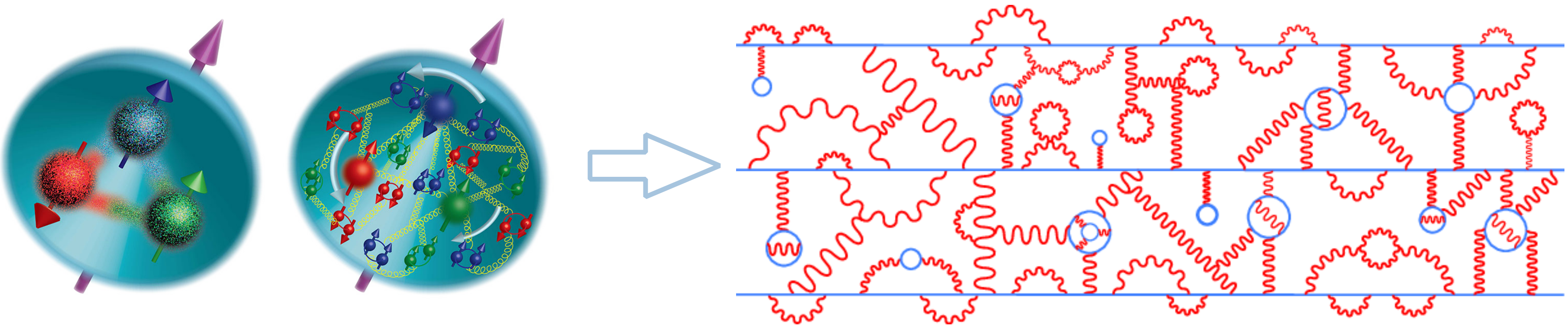

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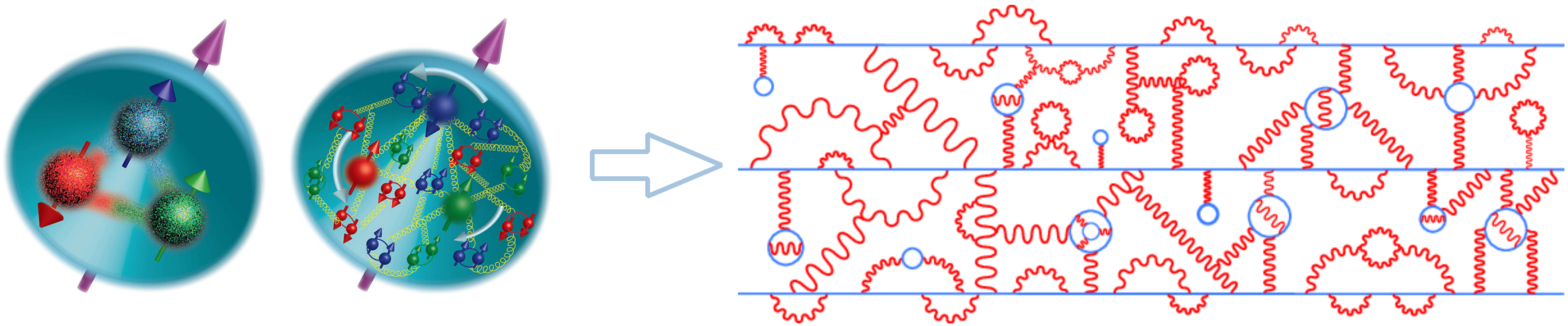

✓ Quarks are **complex** objects which have many intrinsic degrees of freedom, e.g, spins, colors, flavors, and etc, and are **strongly bound** by gluons.

✓ Baryons are (infinitely) **many-body** systems of quarks, whose dynamics is a well-known **difficult** problem, even in the classical level.

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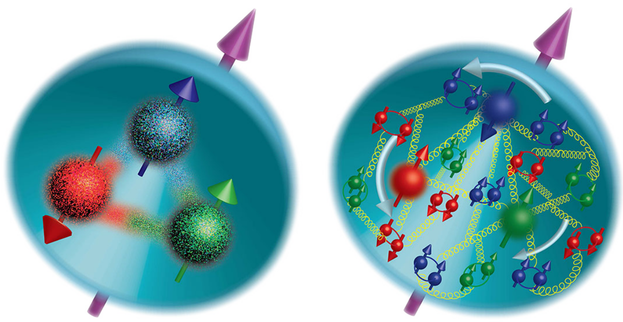

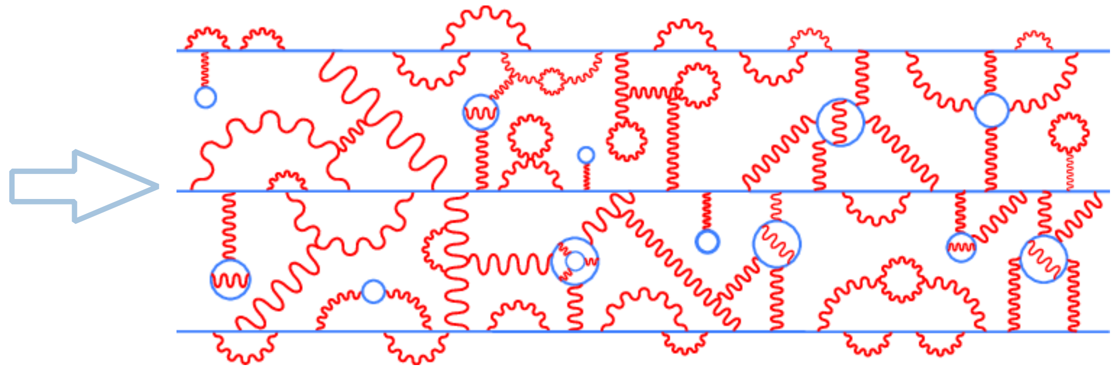

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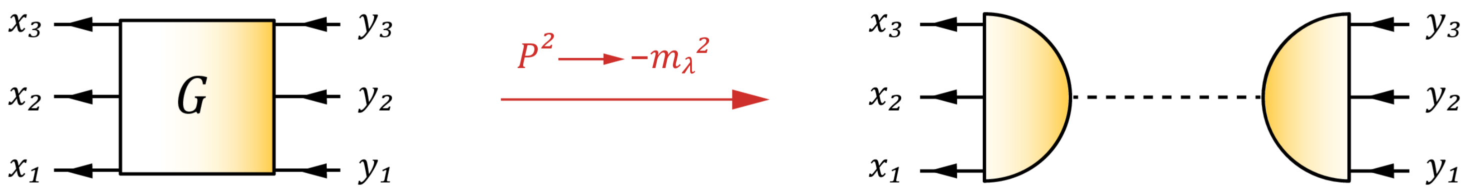
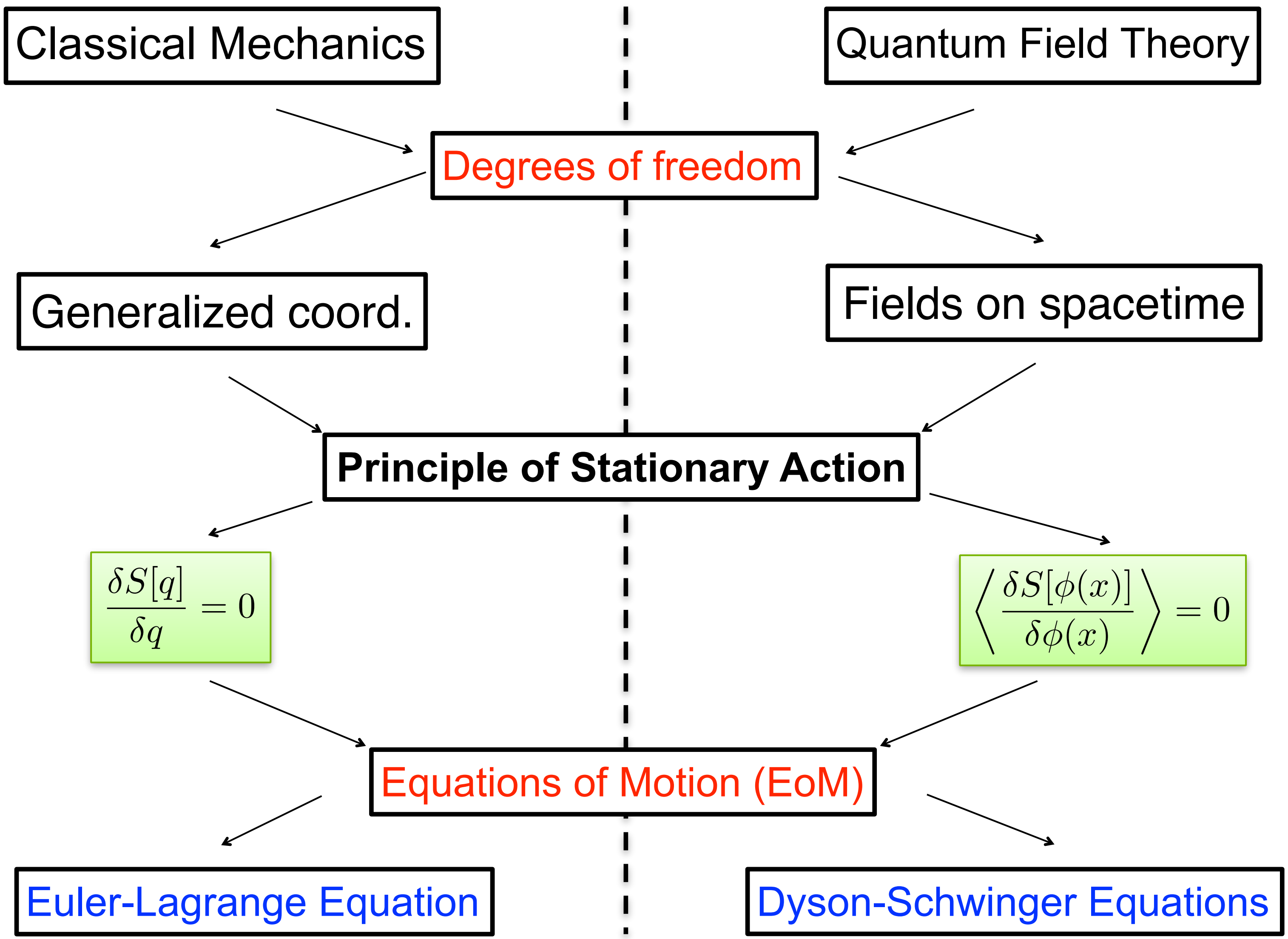
Relativistic strongly-coupled many-body systems

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

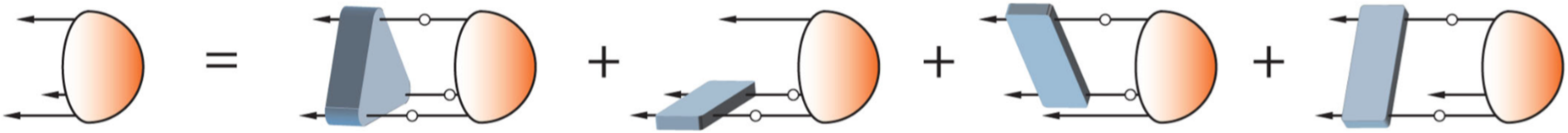


$$G^{(6)}(x_1, x_2, x_3, y_1, y_2, y_3) = \langle \Omega | q(x_1) q(x_2) q(x_3) q(y_1) q(y_2) q(y_3) | \Omega \rangle$$

Introduction: Nonperturbative QCD Framework



Introduction: Three-Body Faddeev Equation



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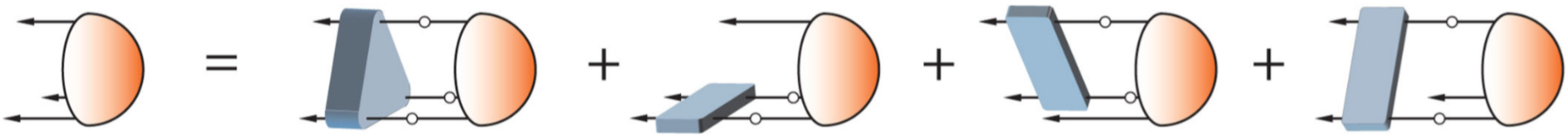

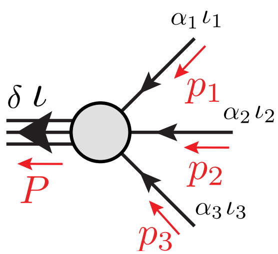

$$ {}_{c_1 c_2 c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P) = \frac{1}{\sqrt{6}}\varepsilon_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P)\,, $$

Spinors: $4\times4\times4\times4=256$

Flavors: $2\times2\times2\times2=16$

128-terms

Jacobi Coordinates: P_μ, p_μ, q_μ

$P^2=-M_N^2; p^2, q^2, Pq, Pp, pq$

5-dim

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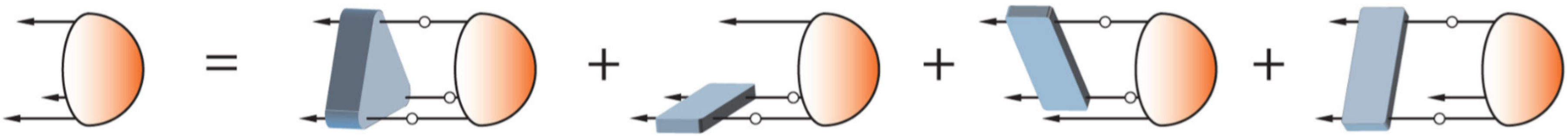

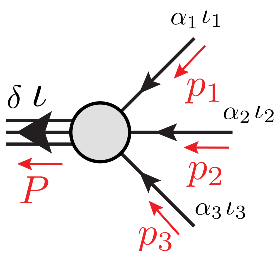

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Highly complex algebra + Cutting edge numerical technology **1TB**

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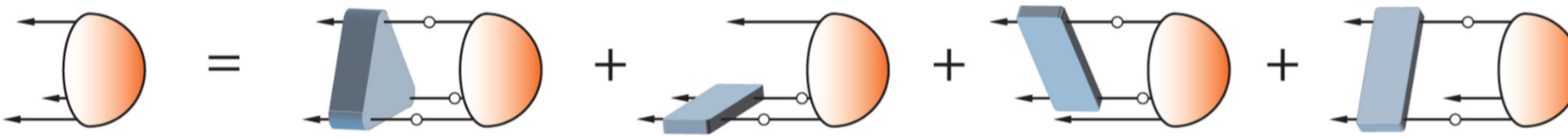

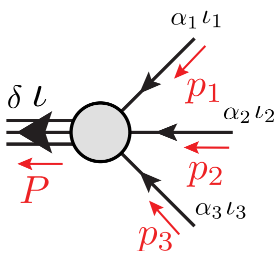

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Diquark approximation: Reduce three-body problem to two two-body ones.

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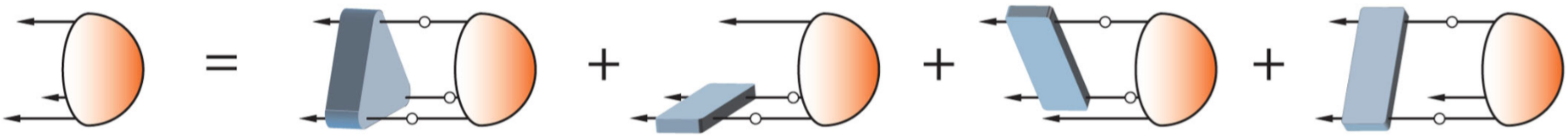

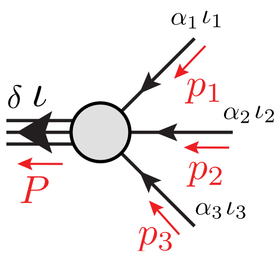

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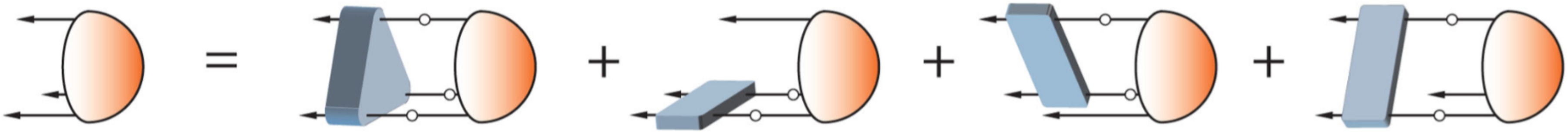

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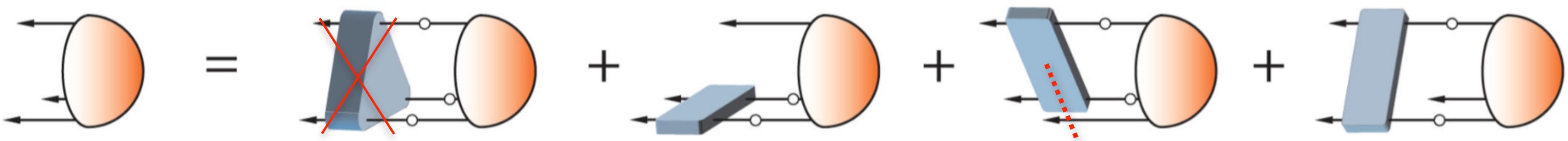
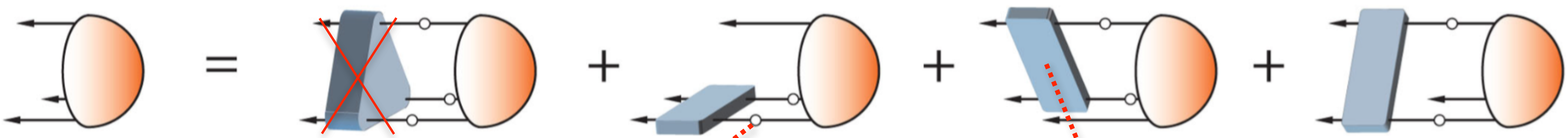


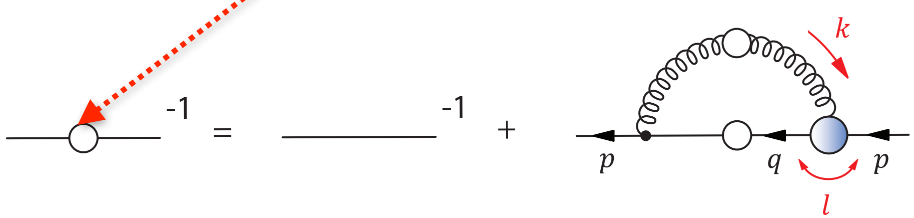
Scattering kernel

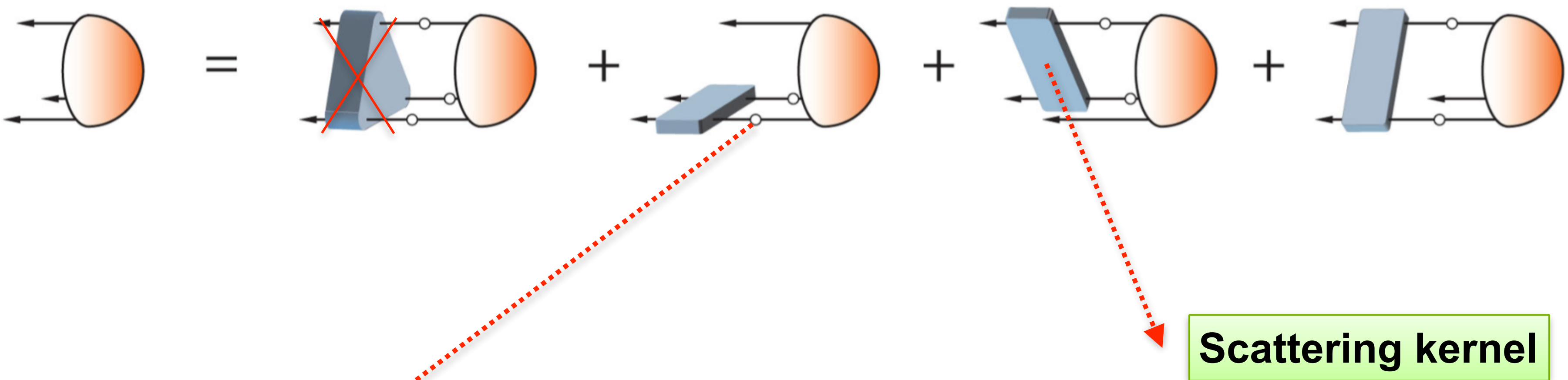
Introduction: Three-Body Faddeev Equation



Scattering kernel

One-body gap equation for inputs





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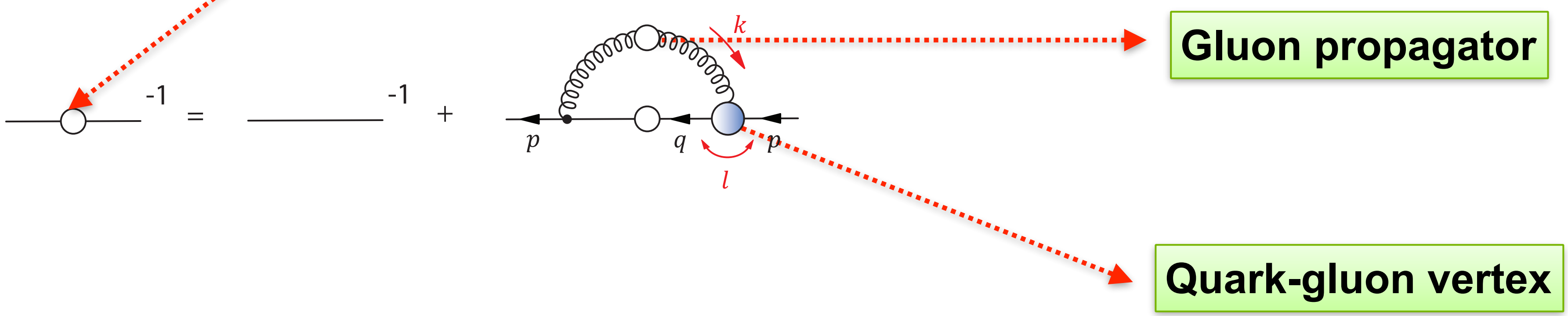
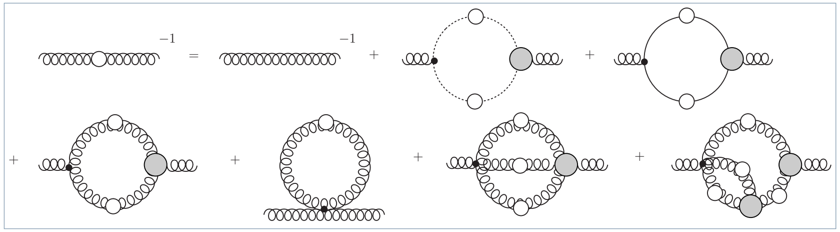
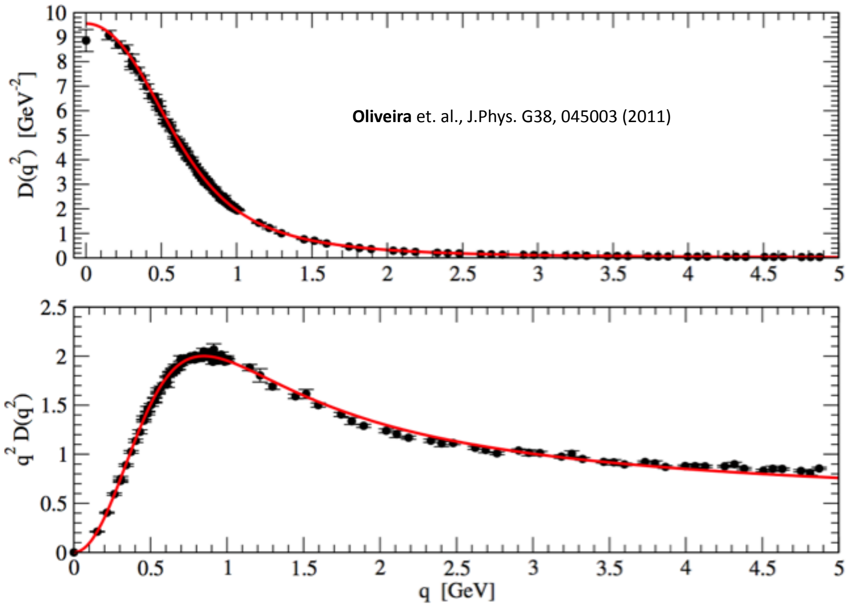


Basics: Gluons are massive quasi-particles

Gluon gap equation: **Aguilar**, Binosi, Papavassiliou and Rodriguez-Quintero



Lattice QCD simulations:



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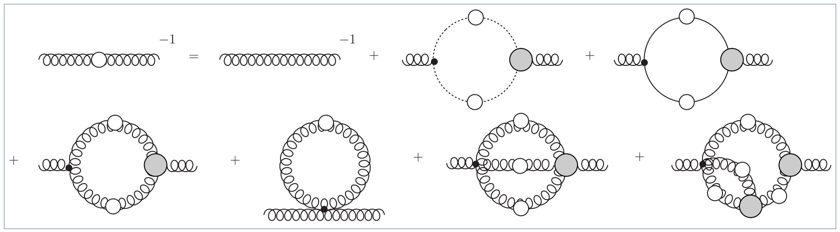

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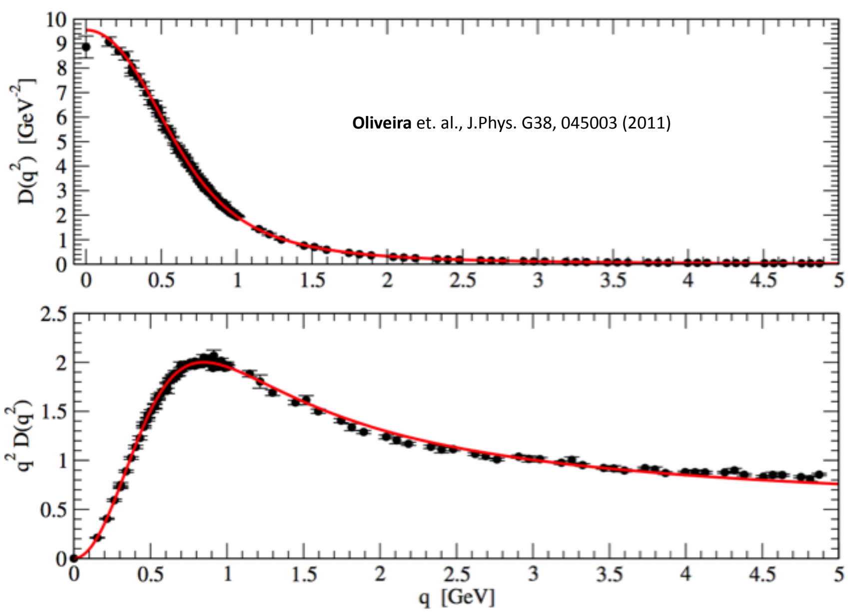


- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

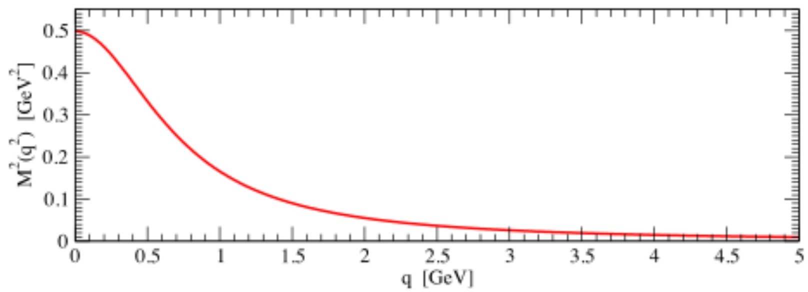
$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

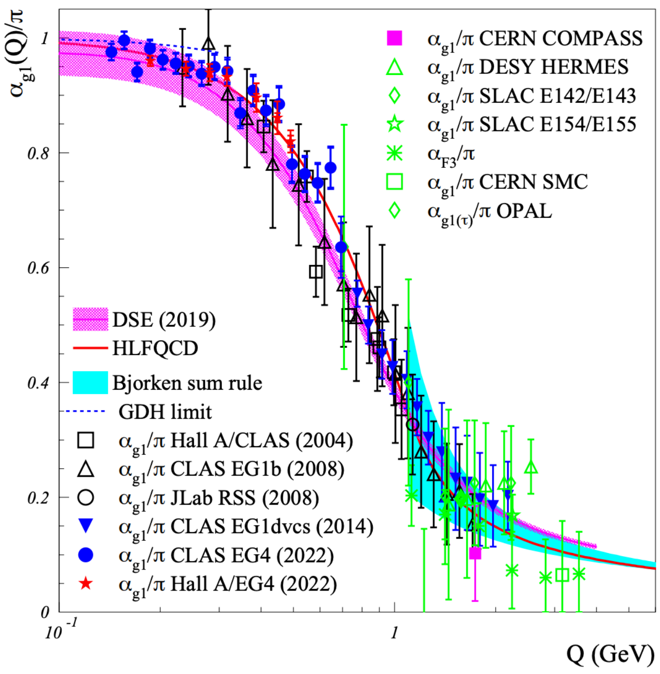
- In QCD: Gluons are ***cannibals*** — a particle species whose members become **massive** by eating each other — **quasi-particles**!

Basics: Gluons are massive quasi-particles

Gluon mass function: **Oliveira** et. al., J.Phys. G38, 045003 (2011)



Running coupling: **Deur, Brodsky, Roberts,** PPNP, 104081 (2024)



See, e.g., PRC 84, 042202(R) (2011)

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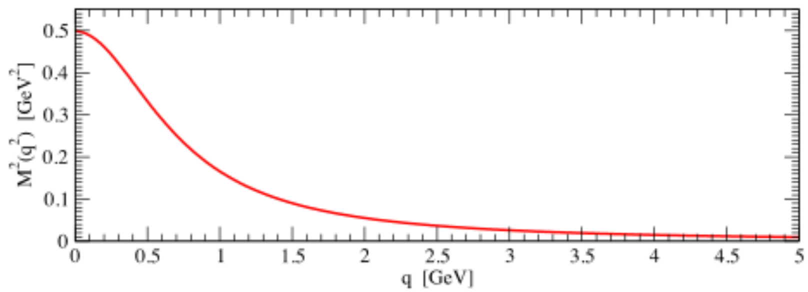

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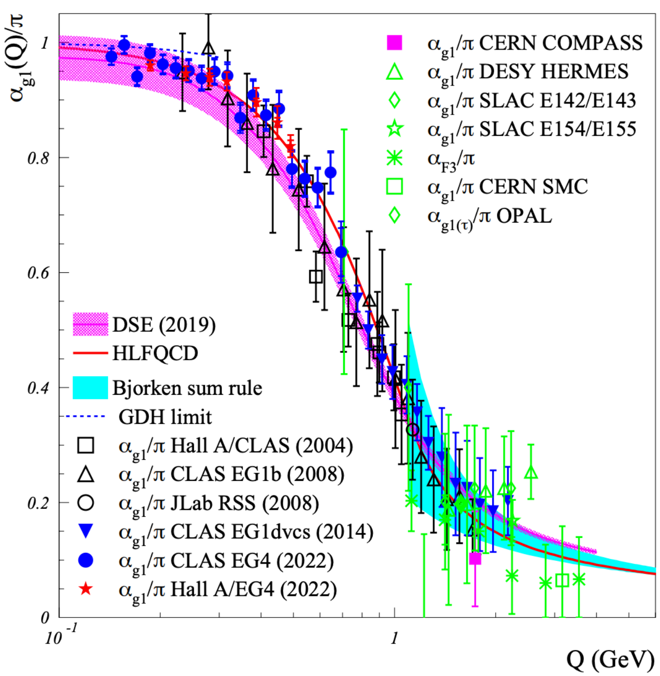


1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

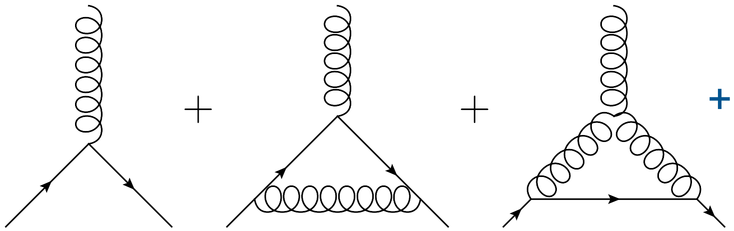
$$M_g \sim 700 \text{ MeV}$$

2. The effective running coupling **saturates** in the infrared limit.

- converge to: $\alpha_s(0) \sim \pi$
- transition at: $Q \sim 1$ GeV

See, e.g., PRC 84, 042202(R) (2011)

Basics: Vertex has DCSB-rendered appearance

Quark-gluon vertex:  + … = ?

See, e.g., PLB722, 384 (2013)

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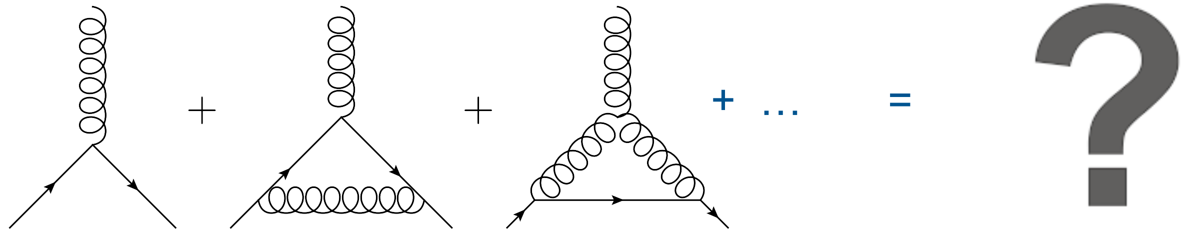
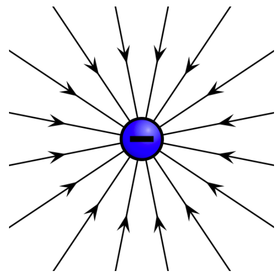
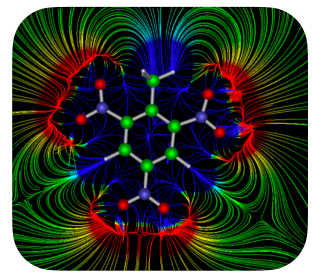


point charge





distributed charges

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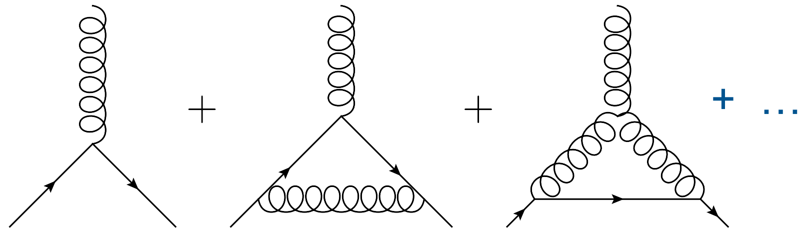

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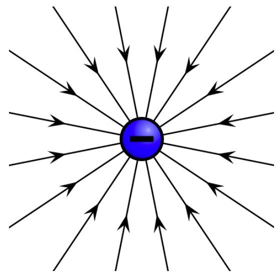

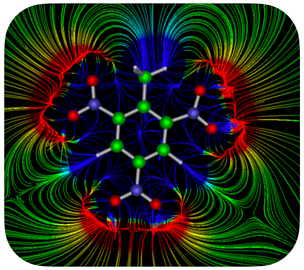

distributed charges

- The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^\mu(P',P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

12 terms

- The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous moment** is proportional to the **Pauli** term.

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance

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Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$\gamma^{\mu}\Sigma_A + i\sigma_{\mu\nu}Q^{\nu}\Delta_B + \text{etc.} \sim \Gamma^{\mu}$$

Ward identities (Symmetries)

See, e.g., PLB722, 384 (2013)

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Ward identities (Symmetries)

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**. **DCSB** is closely related to the **Pauli term**:

$$F_2 \sim \text{DCSB}$$

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Ward identities (Symmetries)

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**. **DCSB** is closely related to the **Pauli term**:

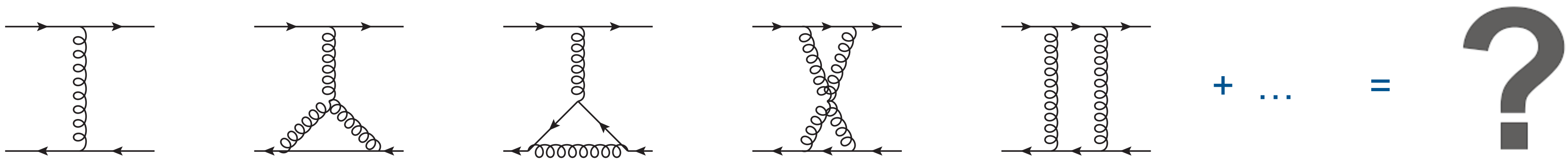
$$F_2 \sim \text{DCSB}$$

2. The **appearance** of the vertex is dramatically modified by the **dynamics**. The vertex can be phenomenologically expressed as:

$$\Gamma^\mu \sim \gamma^\mu + i\eta\sigma_{\mu\nu} Q^\nu \Delta_B$$

See, e.g., PLB722, 384 (2013)

Basics: Kernel has the Dirac and Pauli terms



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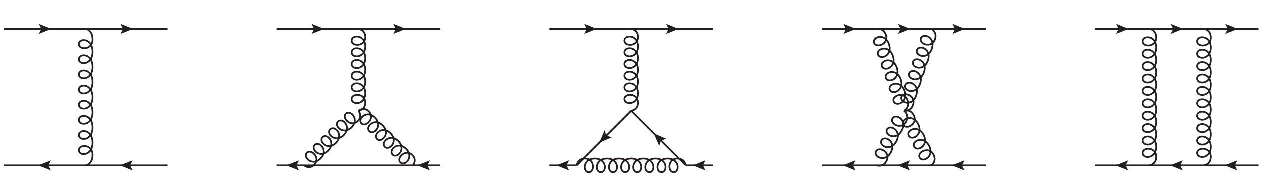

+ … = ?

✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry	Gauge symmetry
C-, P-, T-symmetry	Chiral symmetry

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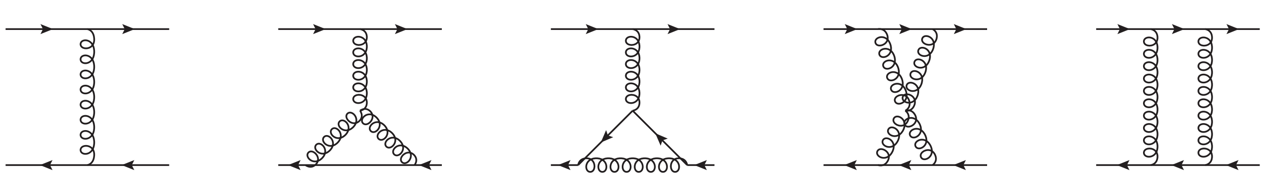

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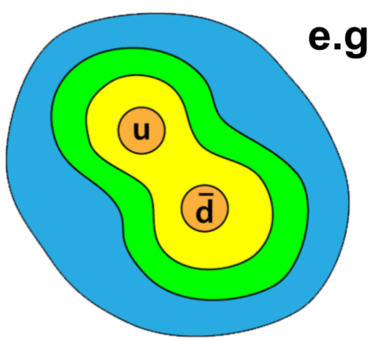

e.g., pion

1. **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

2. **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

Basics: Kernel has the Dirac and Pauli terms

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✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_+ K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_+ K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_- K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_- K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_\pm := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

discrete

✦ Deformed WTIs:

$$\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{K_{L0}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)}\right\} \\
0 &= \int_{dq} \left\{K_{L0}^{(+)}[\sigma_B(q_-)]K_{R0}^{(-)} - K_{L0}^{(-)}[\sigma_B(q_+)]K_{R0}^{(+)} + K_{L2}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(+)}\right\} \\
[\Sigma_A(k_+) - \Sigma_A(k_-)] &= \int_{dq} \left\{K_{L0}^{(+)}[-\sigma_B(q_+)]K_{R0}^{(-)} + K_{L0}^{(-)}[\sigma_B(q_-)]K_{R0}^{(+)} + K_{L2}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(-)}\right\} \\
-\Sigma_B(k_-) &= \int_{dq} \left\{K_{L0}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(+)} + K_{L1}^{(-)}[\sigma_B(q_-)]K_{R1}^{(-)} + K_{L1}^{(+)}[-\sigma_B(q_+)]K_{R1}^{(+)}\right\}
\end{aligned}$$

continuous

See, e.g., CPL 38 (2021) 7, 071201

Basics: Kernel has the Dirac and Pauli terms

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$$
\begin{aligned}
K^{(2)} = & \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_{+} K_{R1}^{(-)}\right] \\
& + \left[K_{L1}^{(+)} \otimes_{+} K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_{-} K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_{-} K_{R2}^{(+)}\right]
\end{aligned}
$$

$$
\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_{\pm} := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)
$$

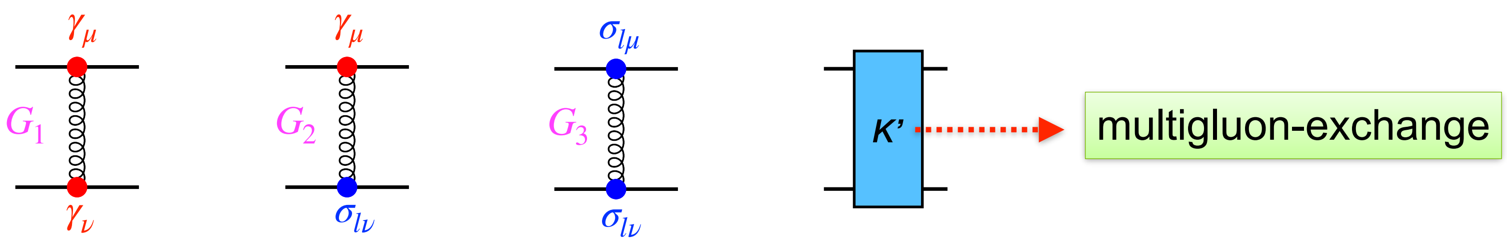
discrete

✦ Deformed WTIs:

$$
\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{ K_{L0}^{(+)}[\Delta^{\pm}_{\sigma_A}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)} \right\} \\
0 &= \int_{dq} \left\{ K_{L0}^{(+)}[\sigma_B(q_-)]K_{R0}^{(-)} - K_{L0}^{(-)}[\sigma_B(q_+)]K_{R0}^{(+)} + K_{L2}^{(+)}[\Delta^{\pm}_{\sigma_A}]K_{R2}^{(+)} \right\} \\
[\Sigma_A(k_+) - \Sigma_A(k_-)] &= \int_{dq} \left\{ K_{L0}^{(+)}[-\sigma_B(q_+)]K_{R0}^{(-)} + K_{L0}^{(-)}[\sigma_B(q_-)]K_{R0}^{(+)} + K_{L2}^{(-)}[\Delta^{\pm}_{\sigma_A}]K_{R2}^{(-)} \right\} \\
-\Sigma_B(k_-) &= \int_{dq} \left\{ K_{L0}^{(-)}[\Delta^{\pm}_{\sigma_A}]K_{R0}^{(+)} + K_{L1}^{(-)}[\sigma_B(q_-)]K_{R1}^{(-)} + K_{L1}^{(+)}[-\sigma_B(q_+)]K_{R1}^{(+)} \right\}
\end{aligned}
$$

continuous

1. A realistic kernel must involves the Dirac and Pauli structures:



See, e.g., CPL 38 (2021) 7, 071201

Basics: Kernel has the Dirac and Pauli terms

✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_{+} K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_{+} K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_{-} K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_{-} K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_{\pm} := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

discrete

✦ Deformed WTIs:

$$\begin{aligned}
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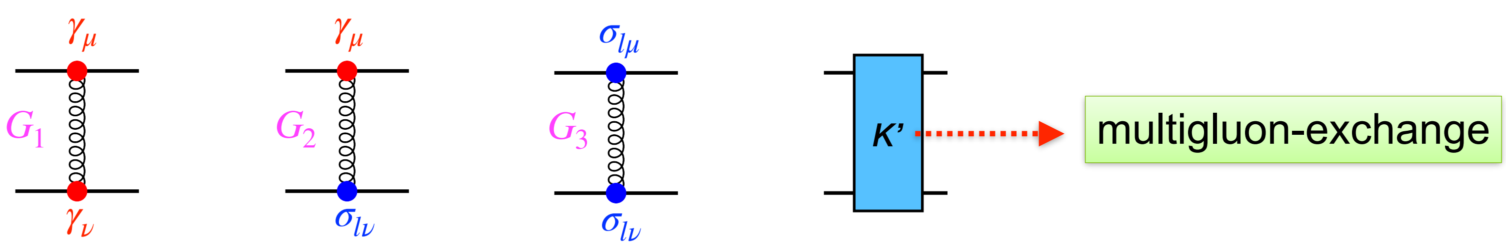
\end{aligned}$$

continuous

1. A realistic kernel must involves the Dirac and Pauli structures:



2. G_2 and G_3 are proportional to the Pauli term in the vertex, and thus to **DCSB**:

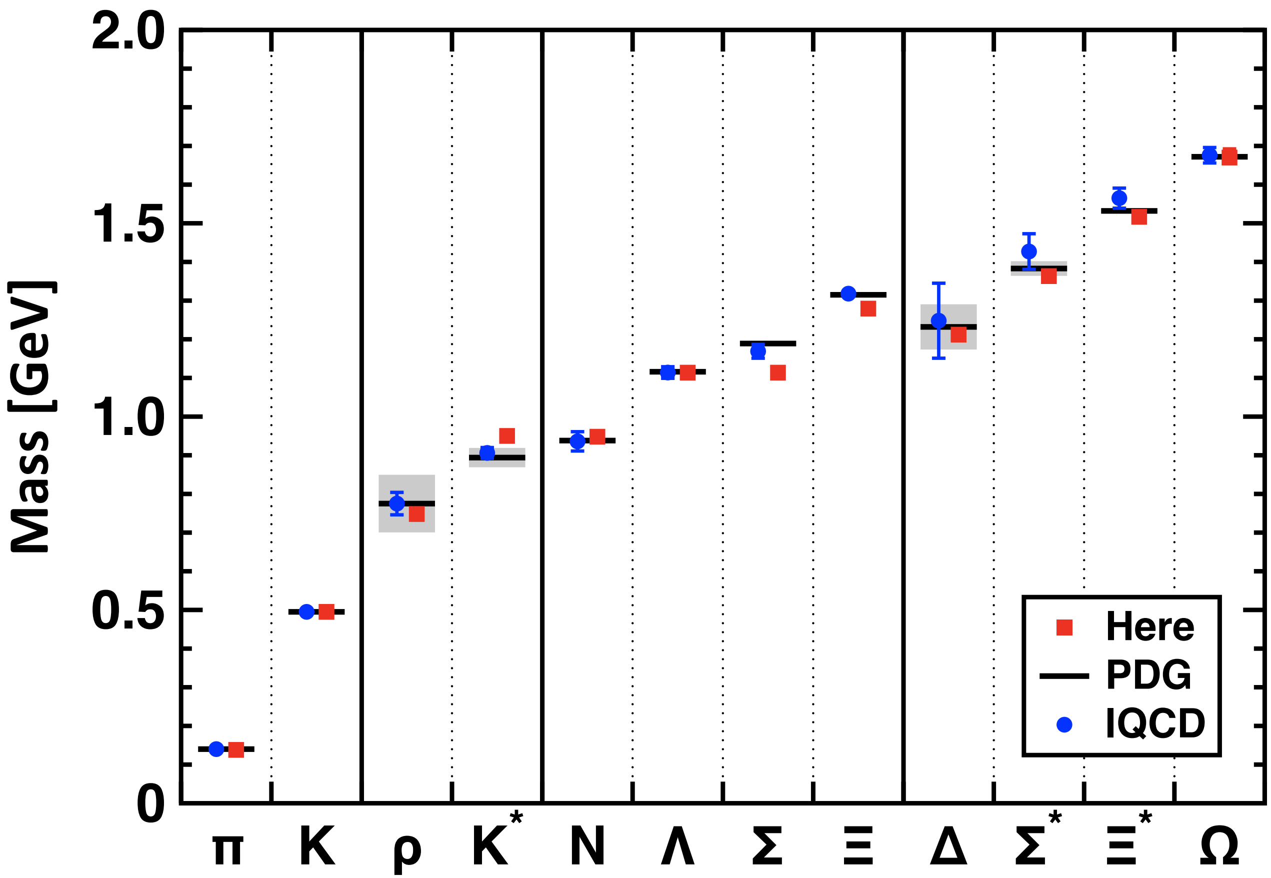
$$G_2, G_3 \sim \text{DCSB}$$

See, e.g., CPL 38 (2021) 7, 071201

Ground states

Ground states: Light & Strange flavor spectra

CHONGQING UNIVERSITY

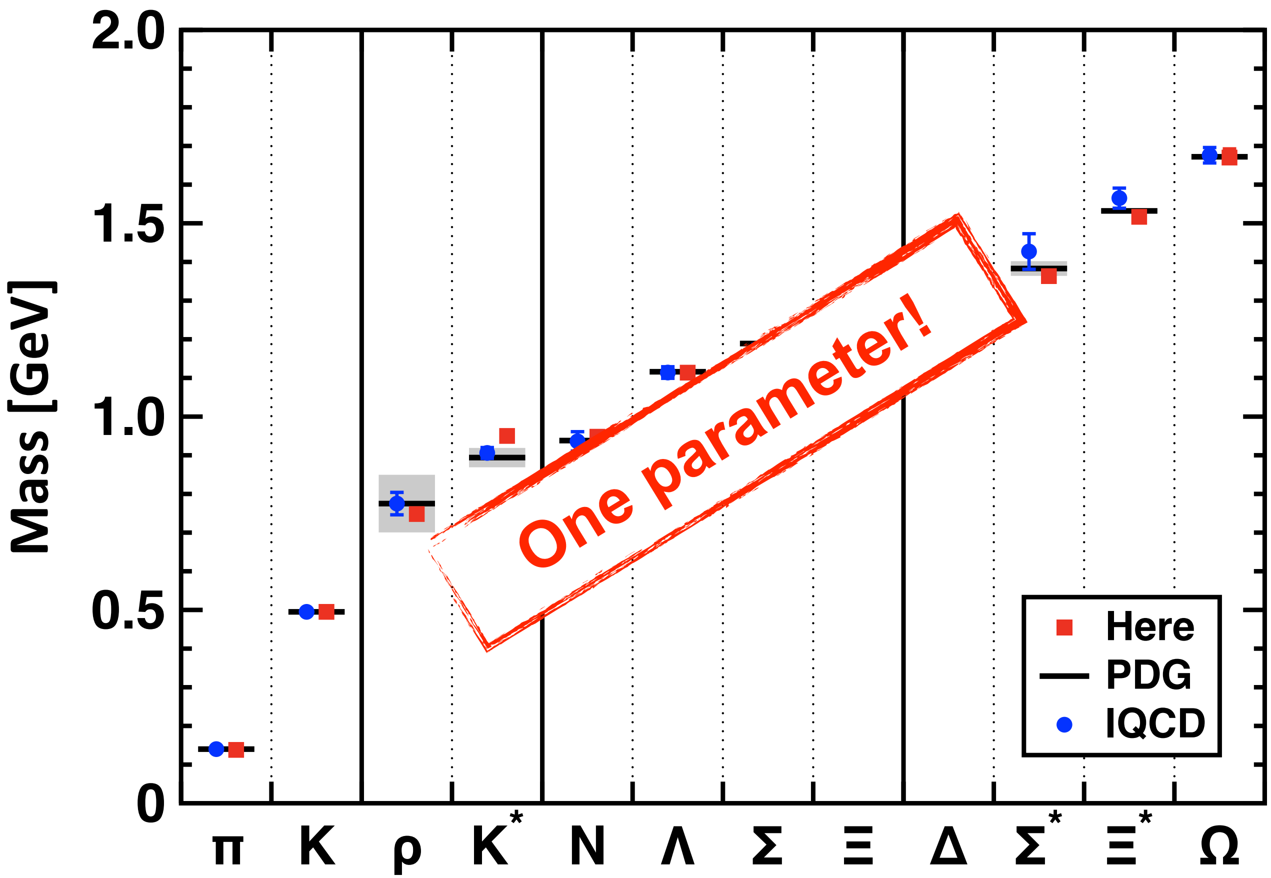


The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Light & Strange flavor spectra

CHONGQING UNIVERSITY



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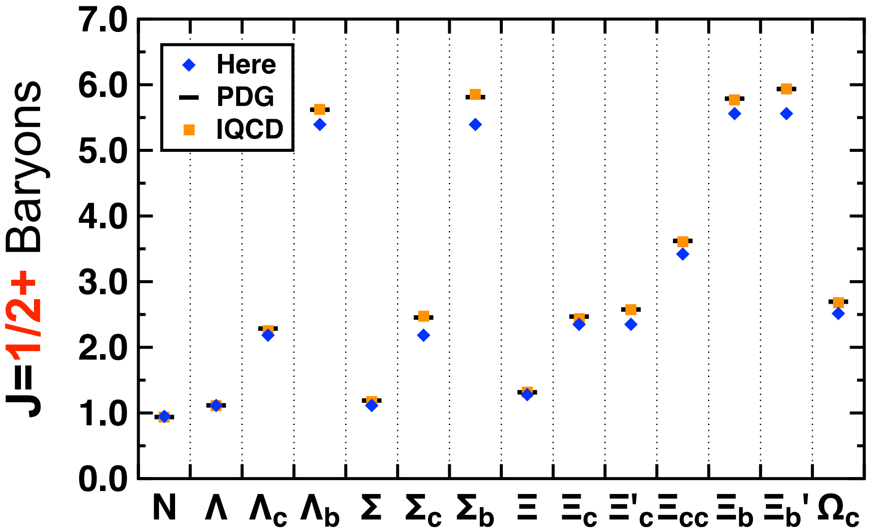
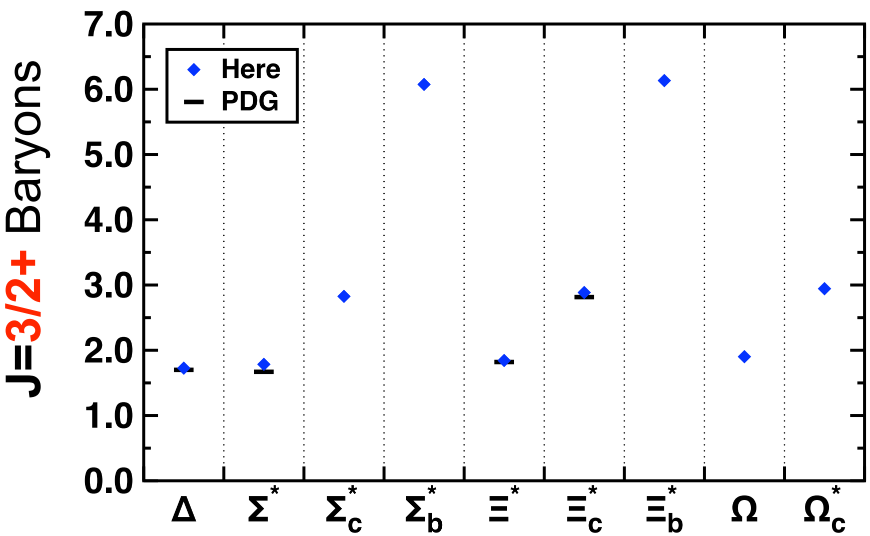
Ground states: Charm & Bottom flavor spectra





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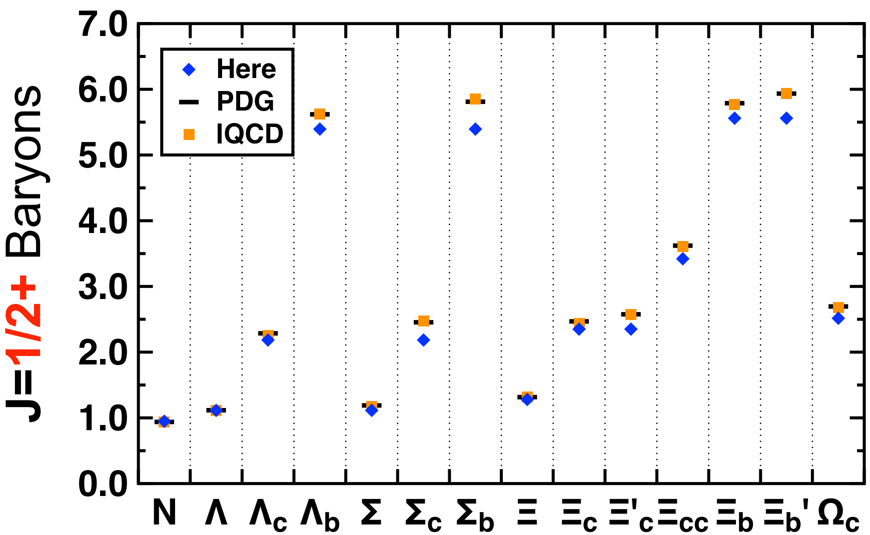
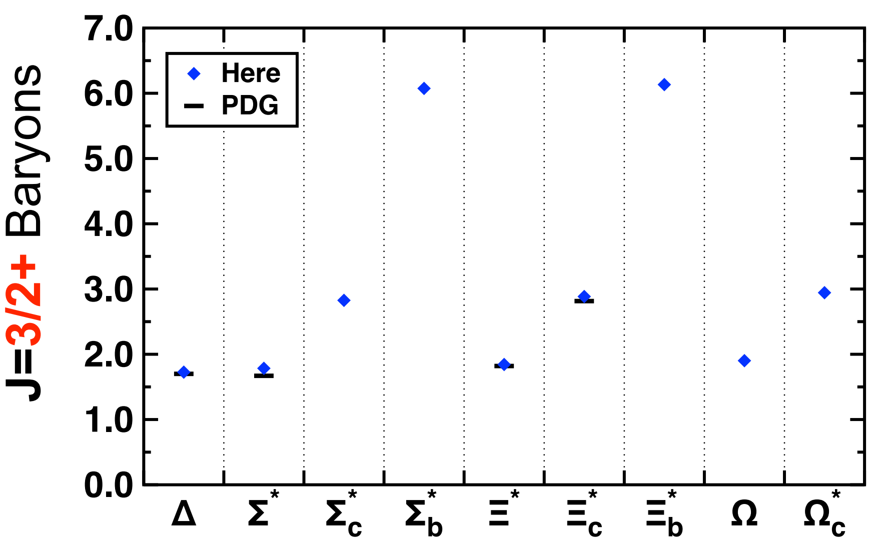
Ground states: Charm & Bottom flavor spectra

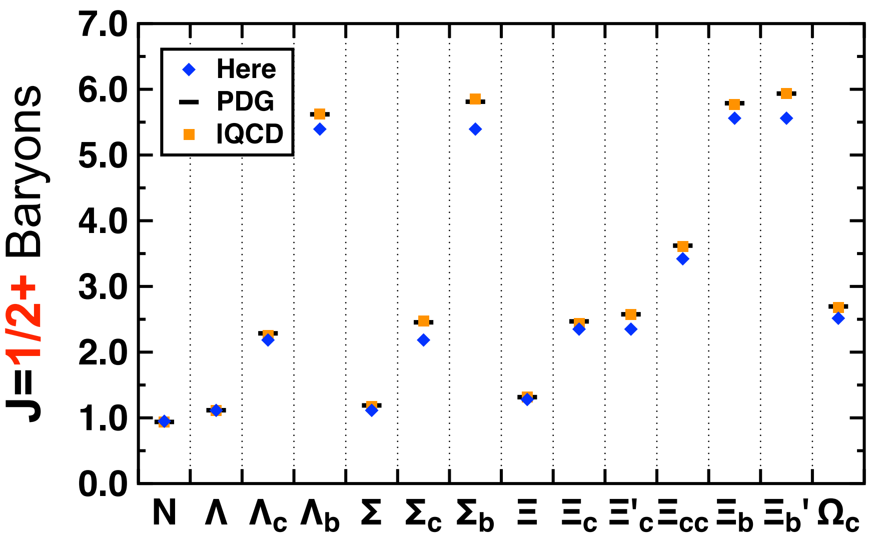


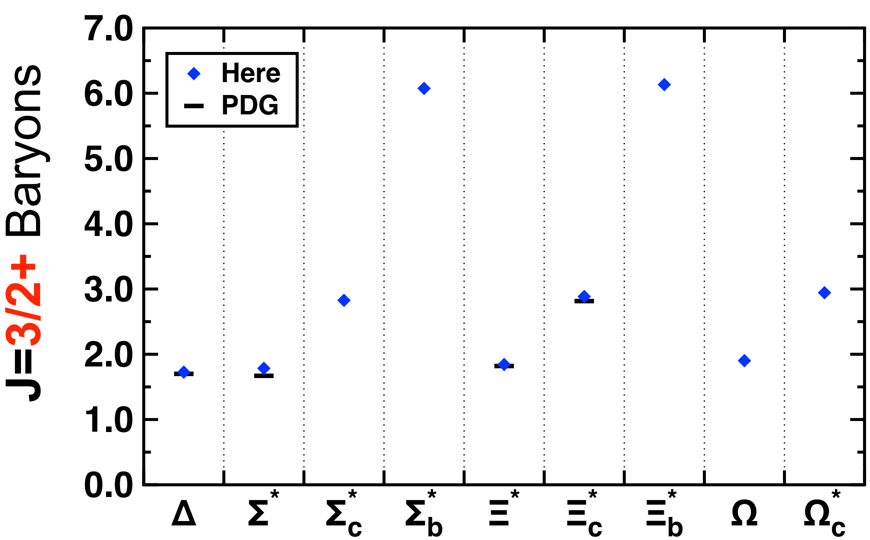


- The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Charm & Bottom flavor spectra

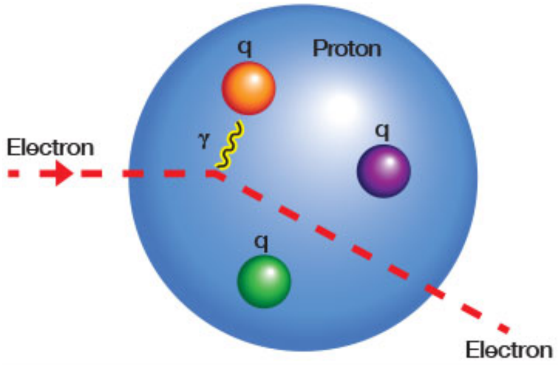


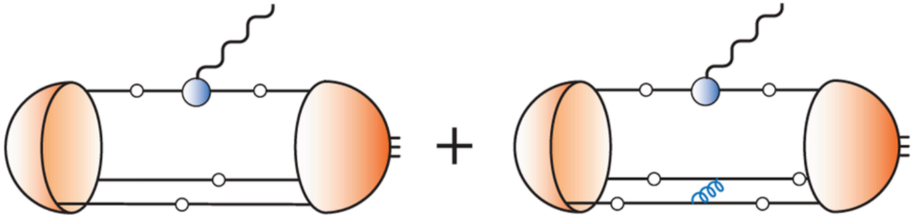


- ✦ The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.
- ✦ The ground spectra is NOT sensitive to the structures beyond the leading terms in the vertex and the kernel.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: EM form factors

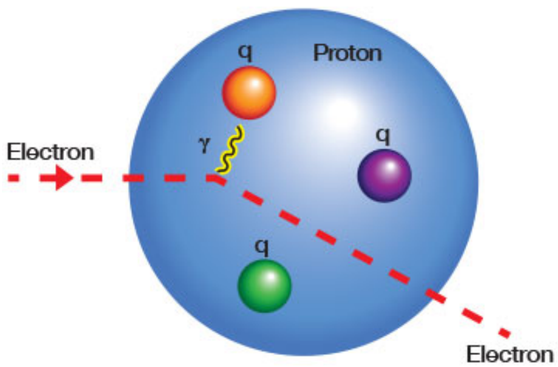


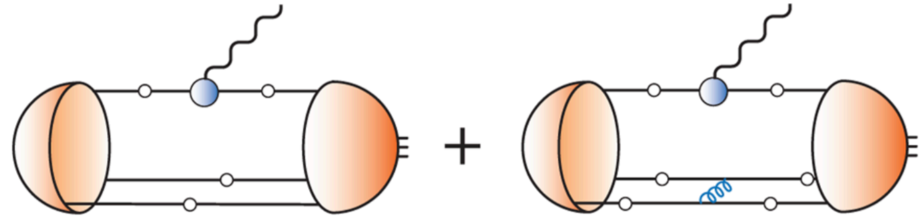


$$\langle N(P')|J^\mu(x)|N(P)\rangle \sim \Gamma^\mu(P',P)$$

See, e.g., Eichmann, PRD 84, 014014 (2011)

Ground states: EM form factors





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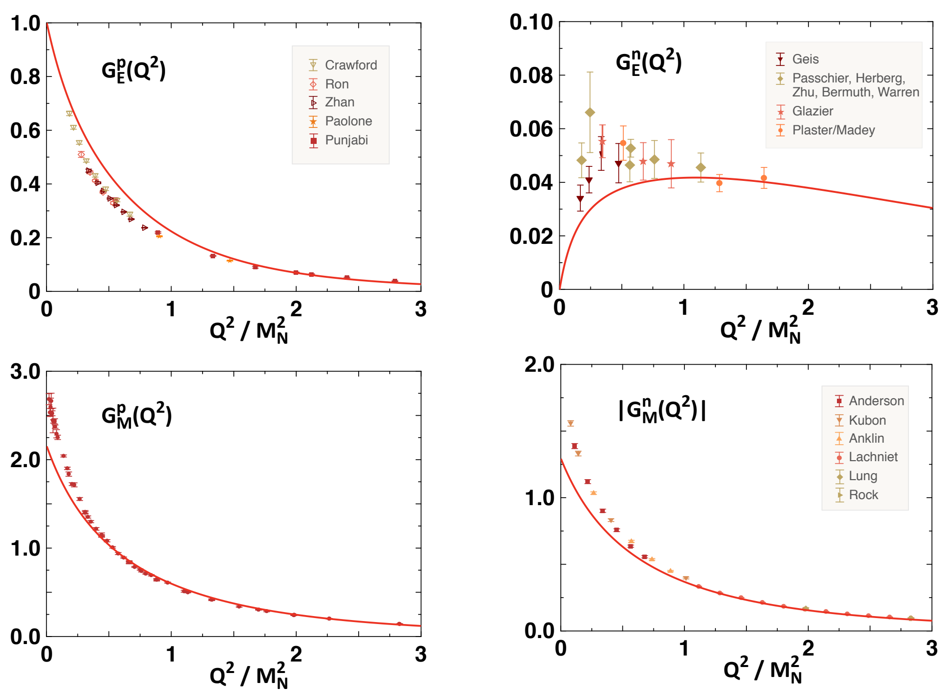


The two Sachs form factors read:

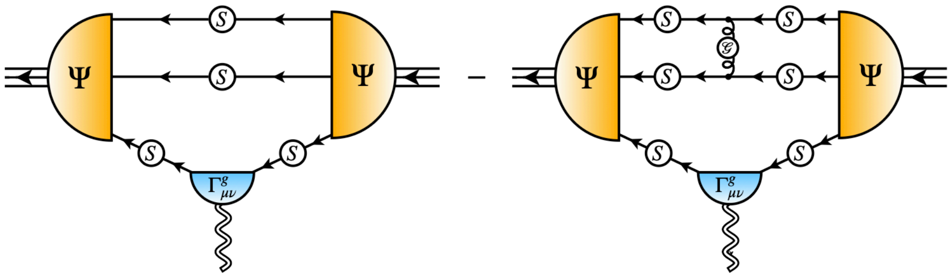
$$G_E(Q^2) := F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) := F_1(Q^2) + F_2(Q^2)$$

For charge & magnetization densities

See, e.g., Eichmann, PRD 84, 014014 (2011)

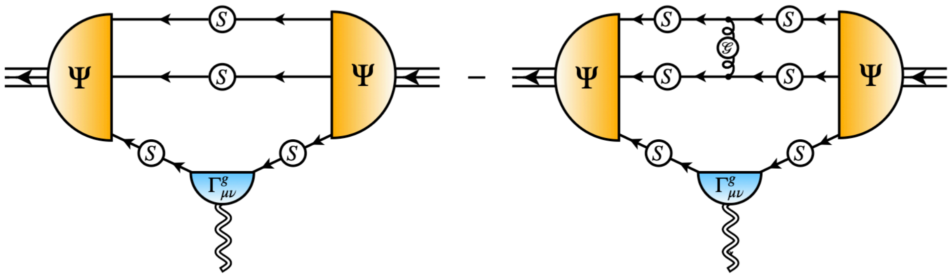
Ground states: Gravitational form factors



For energy, angular momentum, pressure, and shear force densities

See, e.g., Yao (姚照千), et al (NJU and ECT*), arXiv:2409.15547 (2024)

Ground states: Gravitational form factors



For energy, angular momentum, pressure, and shear force densities

$$\langle N(\mathbf{p}', s')|\hat{T}^{\mu\nu}|N(\mathbf{p}, s)\rangle = \frac{1}{m}\bar{u}(\mathbf{p}', s')\left[P^\mu P^\nu \boxed{A(t)} + iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho \boxed{J(t)} + \frac{1}{4}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)\boxed{D(t)}\right]u(\mathbf{p}, s)$$

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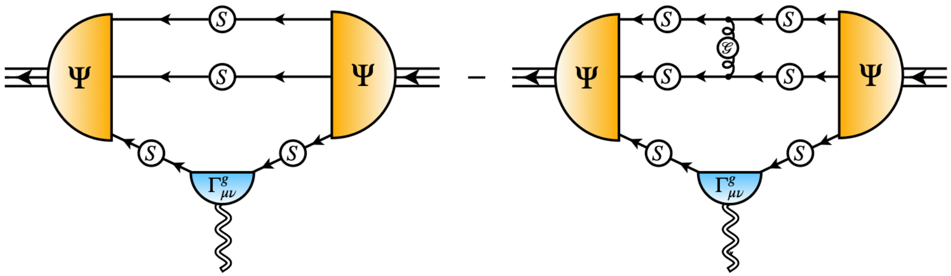

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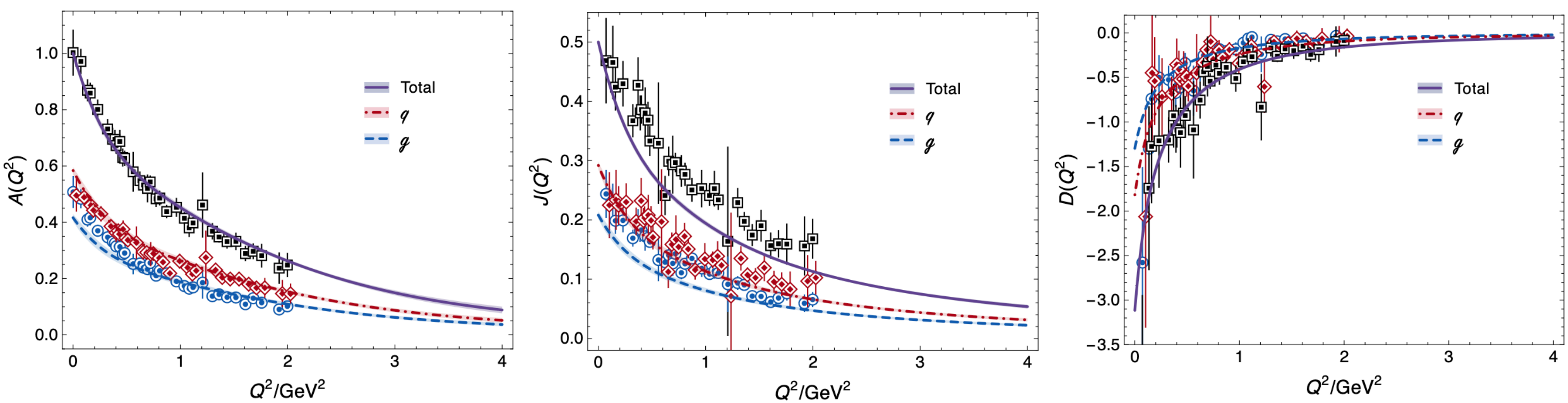

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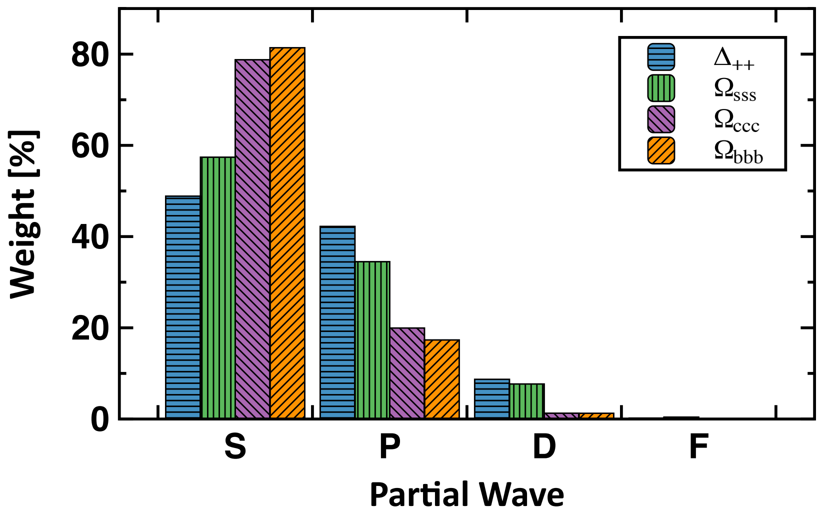
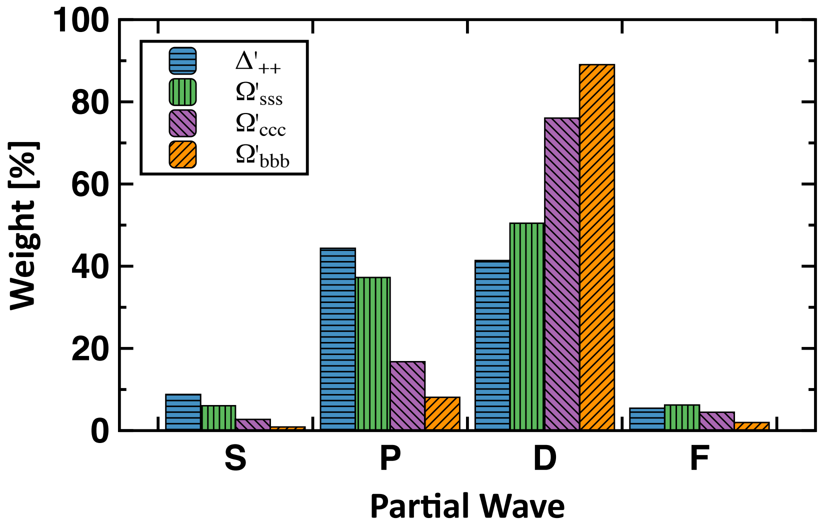
Excited states

Excited states: Multiple partial waves



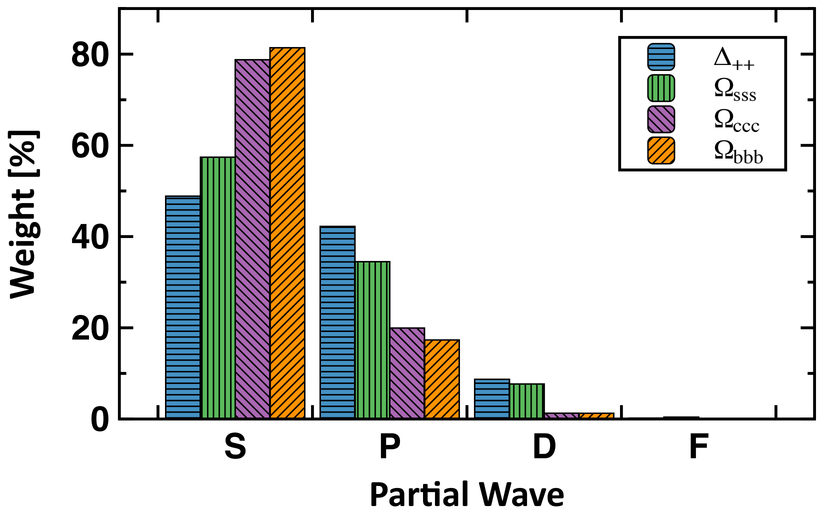
✓ S-waves dominate for ground states, but P-waves grow for light baryons.

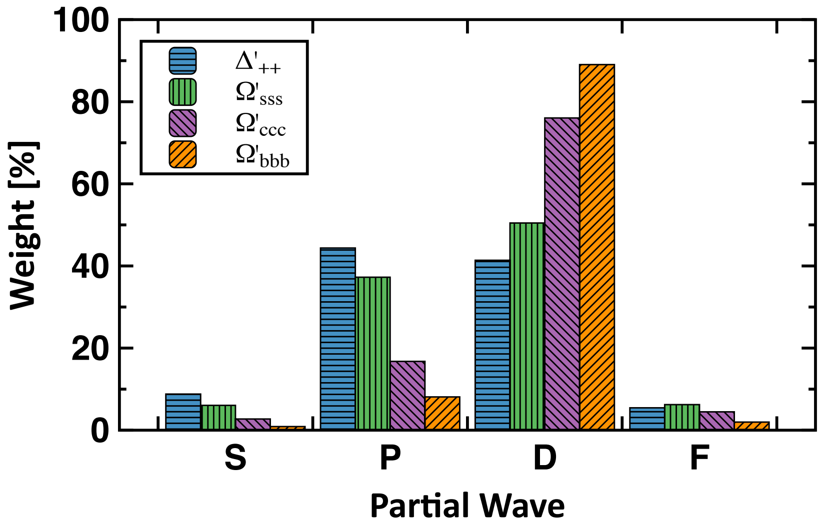


✓ D-waves dominate for excited states, but P-waves grow for light baryons.

See, e.g., PRD 97, 114017 (2018)

Excited states: Multiple partial waves





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Why NR potential models work

✓ D-waves dominate for excited states, but P-waves grow for light baryons.

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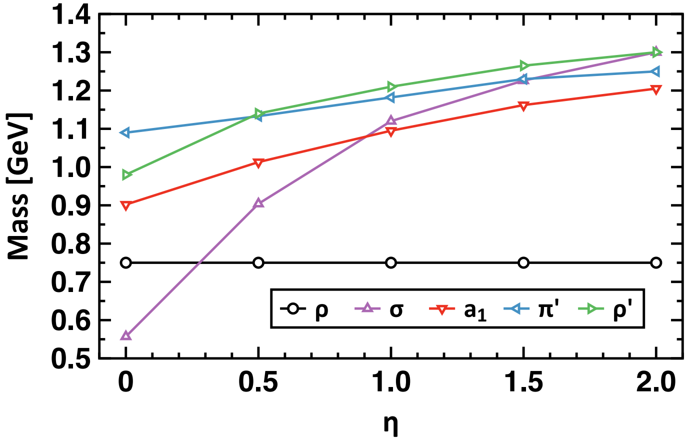
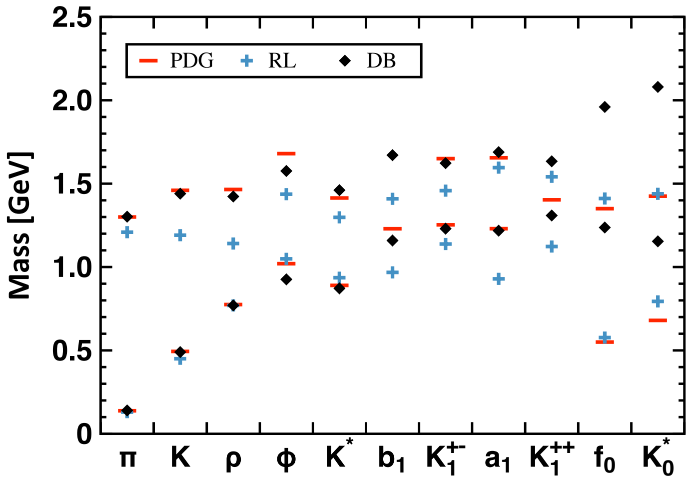
Excited states: Spin-orbit interaction

➡ Impact of the Pauli term (**a**nomalous **m**oment):



➡ Light & strange meson spectrum:



See, e.g., CPL 38, 071201 (2021) & EPJA 59, 39 (2023)

Excited states: Spin-orbit interaction

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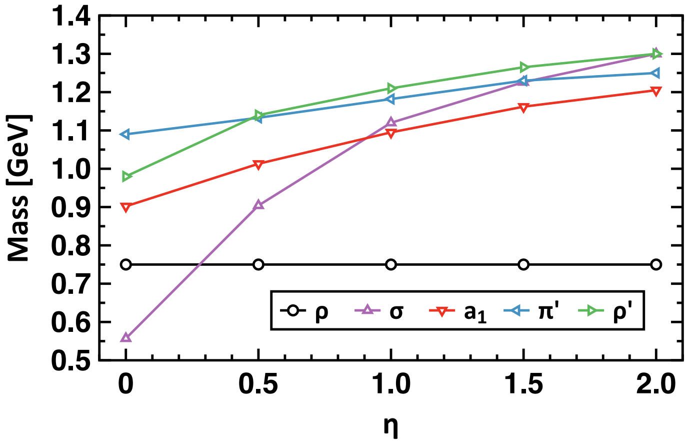

✦ With increasing the AM strength, the $a_1-\rho$ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

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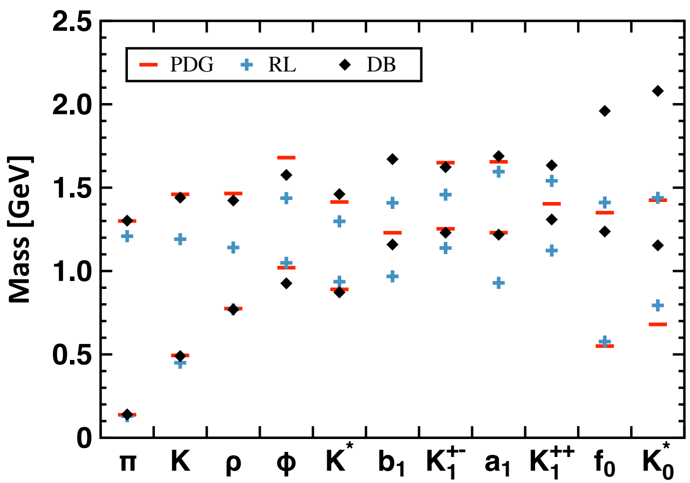

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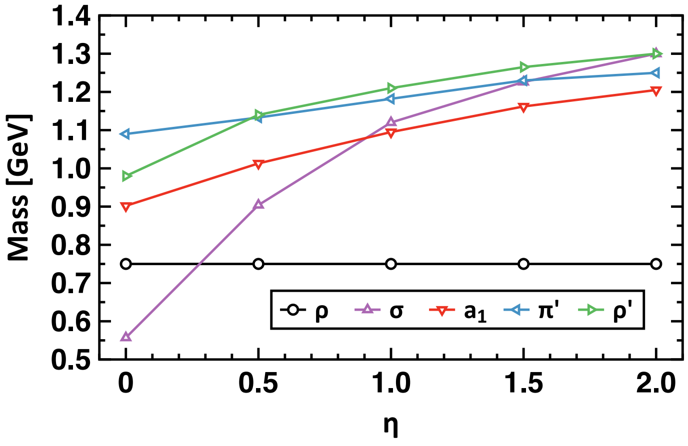

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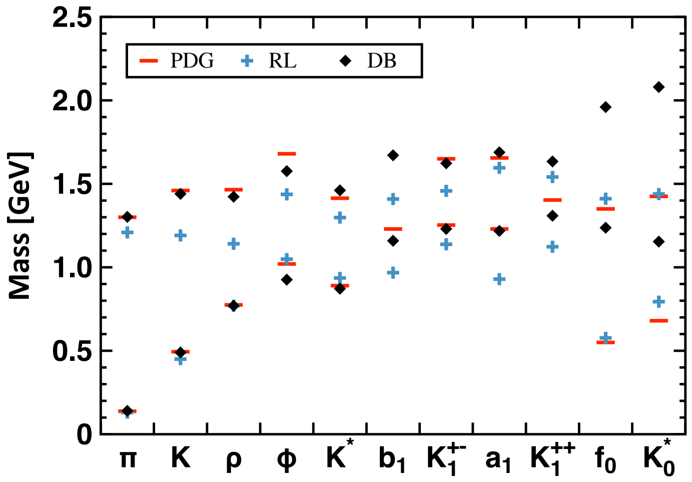
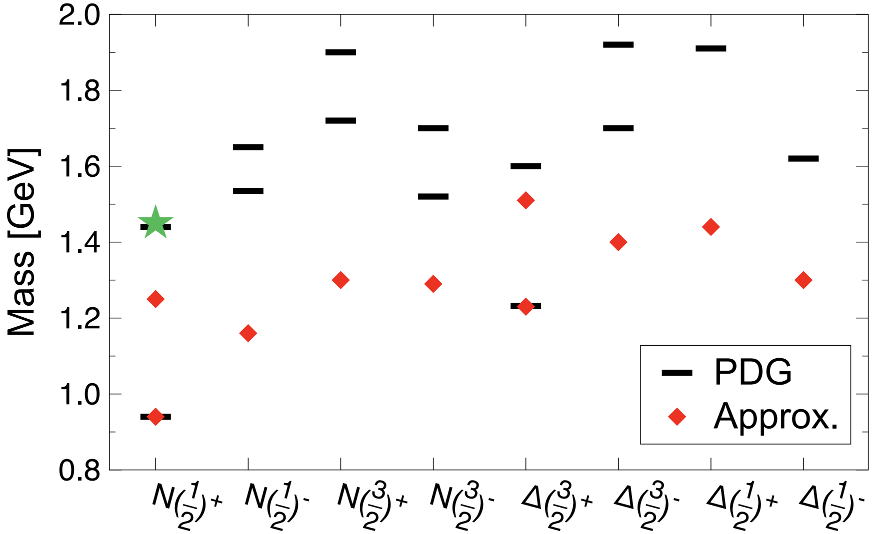
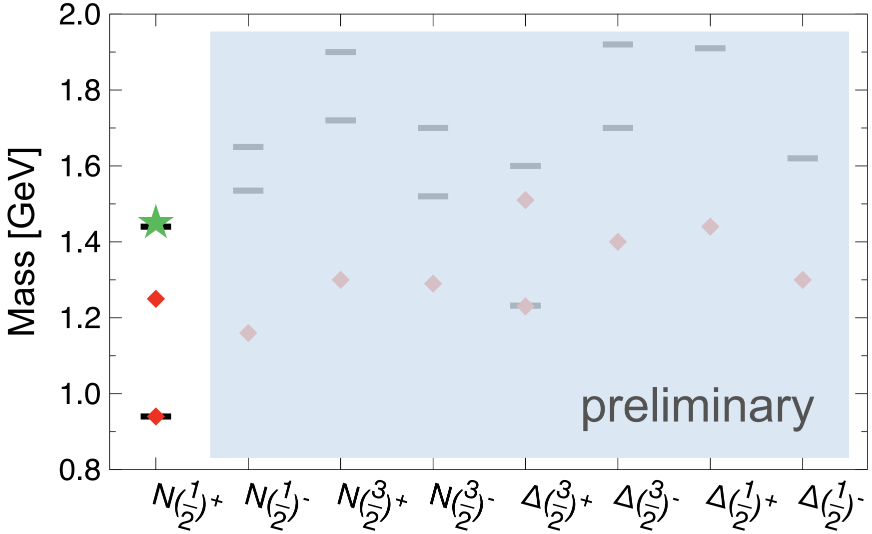


✦ The magnitude and ordering of all excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38, 071201 (2021) & EPJA 59, 39 (2023)

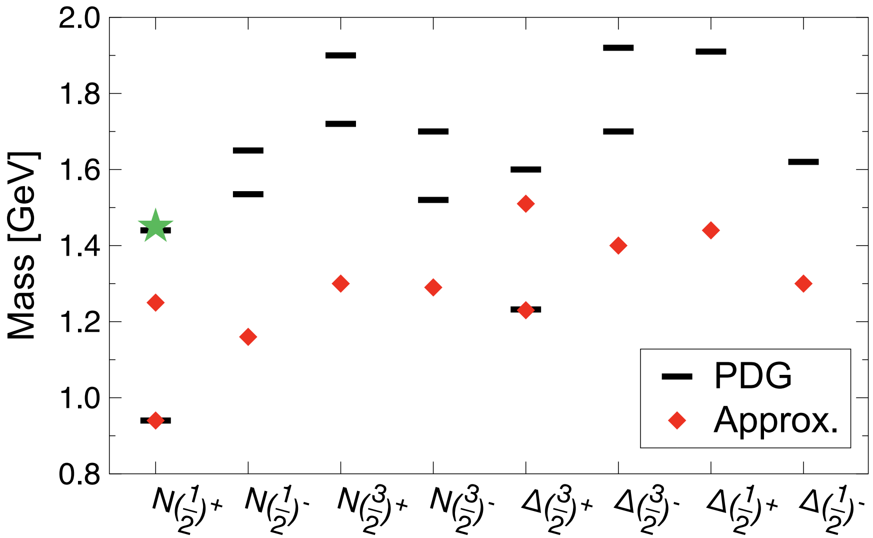
Excited states: DCSB-rendered spectra

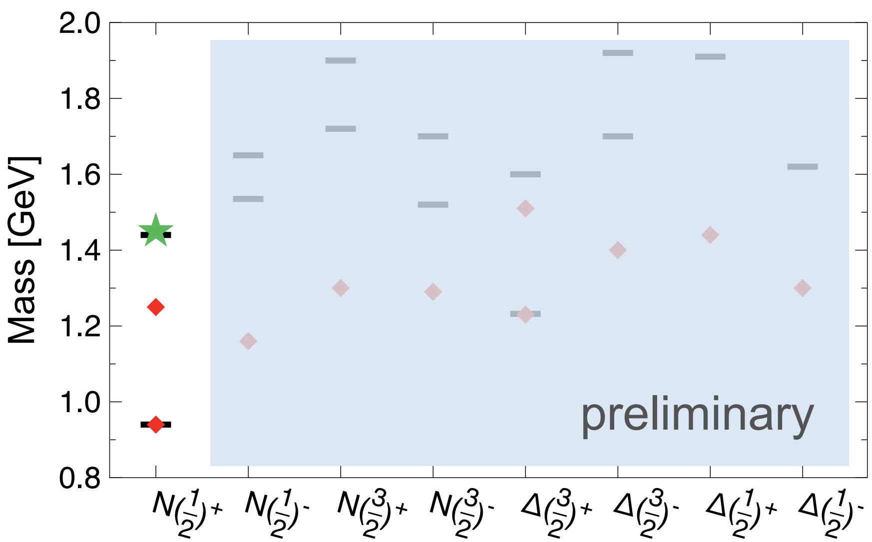




In progress

Excited states: DCSB-rendered spectra

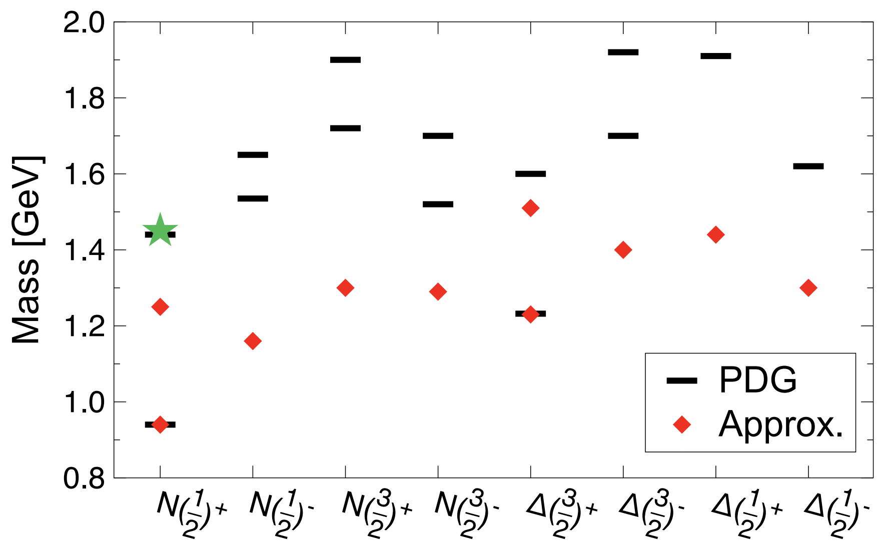




- The magnitude and ordering of radial or angular excitation states are WRONG in the approximation lacking of DCSB effect.

In progress

Excited states: DCSB-rendered spectra



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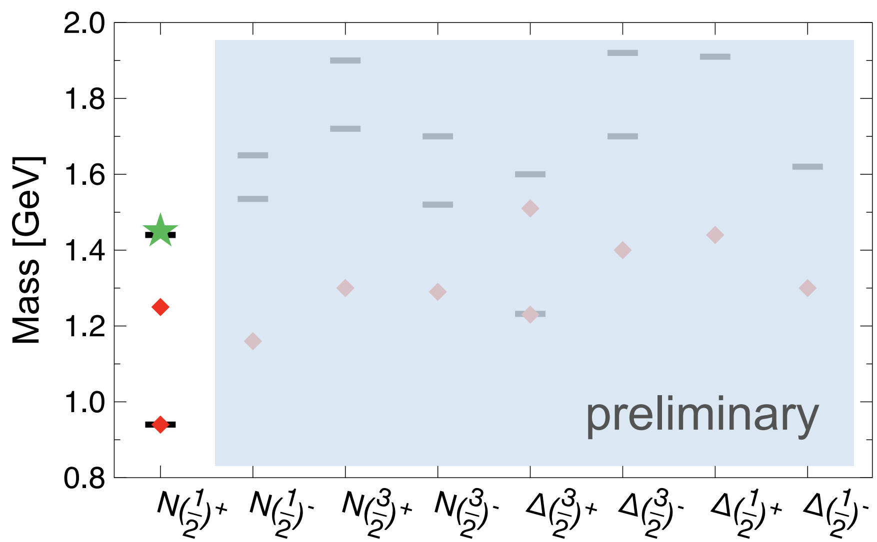

- The DCSB-enhanced kernel boost up 1st excitation nucleon, and can potentially fix the full spectra.

In progress

Summary

- The framework of three-body Faddeev equation, which describes baryons in continuum QCD, and its basics (e.g., gluon, vertex, kernel) are introduced.

- Baryon properties are studied: **a**) ground baryons — full mass spectrum of J=1/2 and J=3/2, nucleon EM and gravitational form factors; **b**) excited baryons — partial waves, spin-orbit interaction, DCSB-rendered spectra.

Summary

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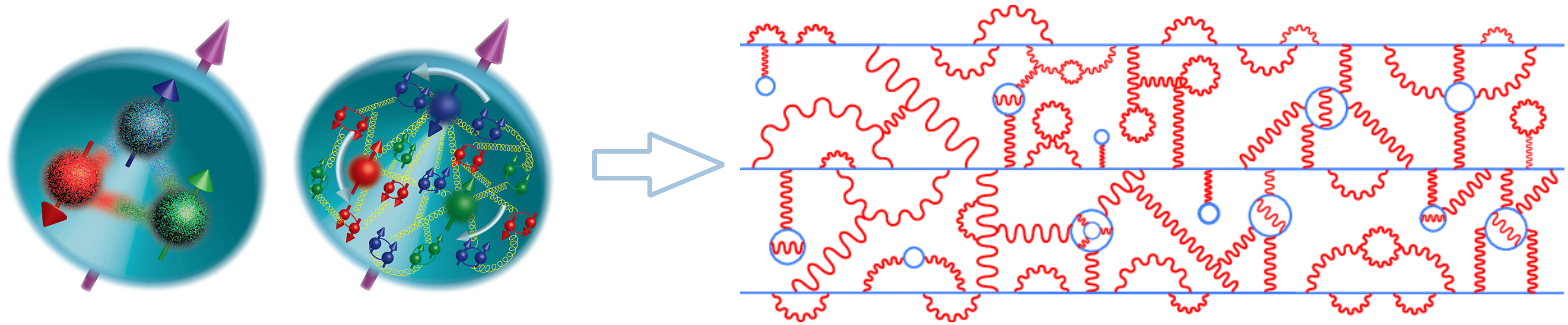
Outlook

- Use the three-body Faddeev equation to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.

- Hopefully, iterating with future high precision experiments on **light** and **heavy** hadrons, from spectroscopy to structures, we may provide a **faithful path** to understand **QCD**.

Introduction: Baryons and their properties

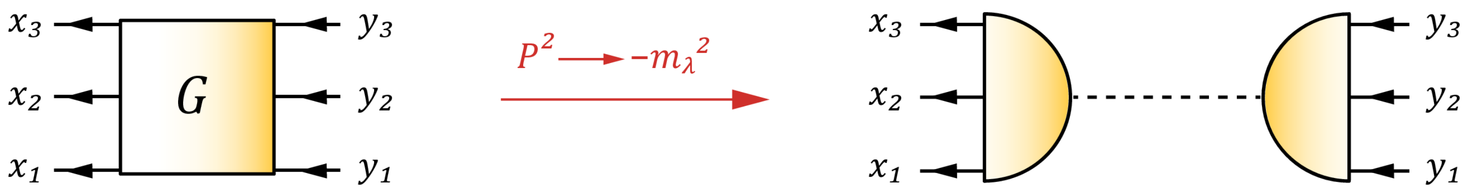


✓ Quarks are **complex** objects which have many intrinsic degrees of freedom, e.g, spins, colors, flavors, and etc, and are **strongly bound** by gluons.

✓ Baryons are (infinitely) **many-body** systems of quarks, whose dynamics is a well-known **difficult** problem, even in the classical level.

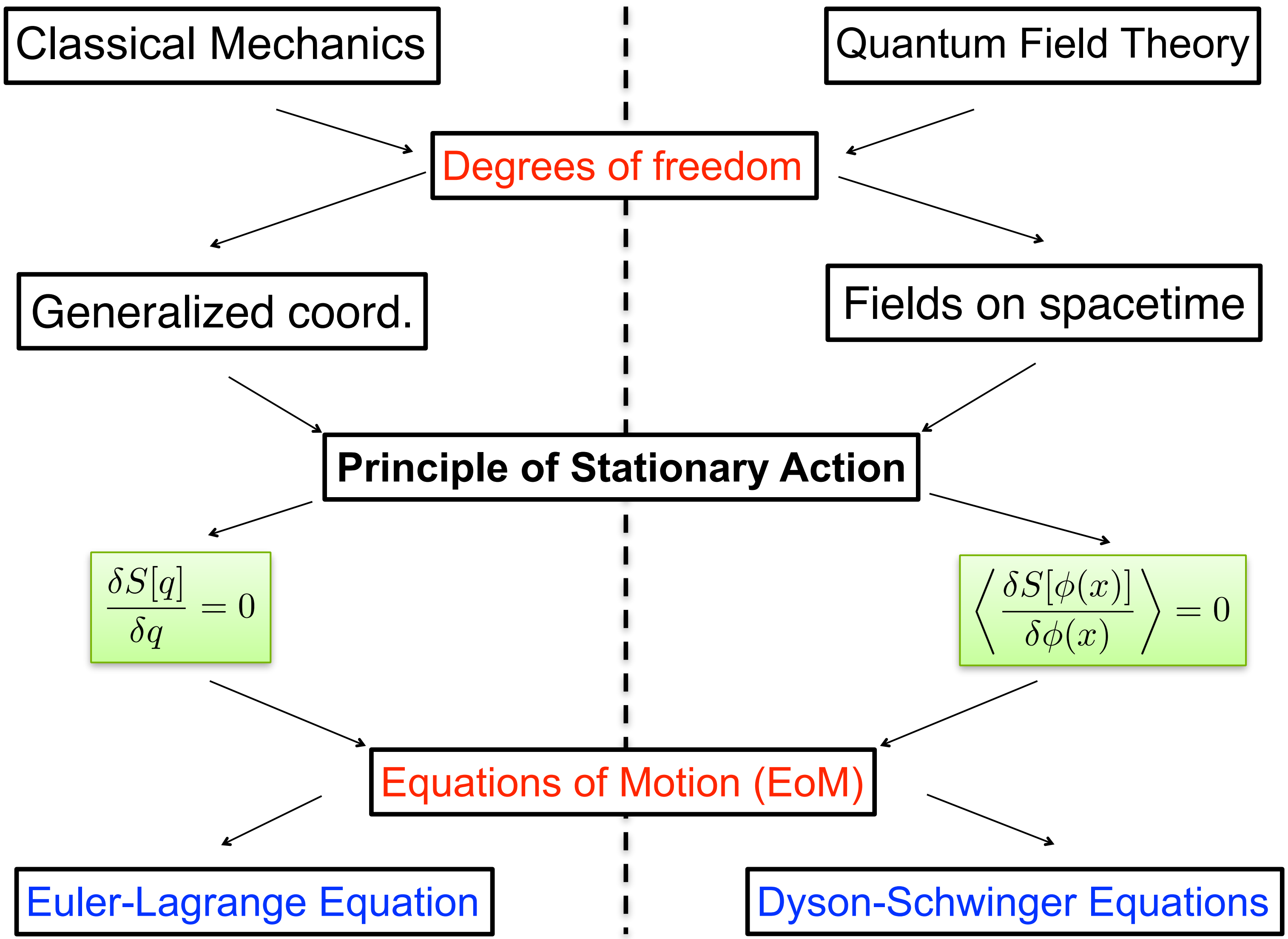
Relativistic strongly-coupled many-body systems

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

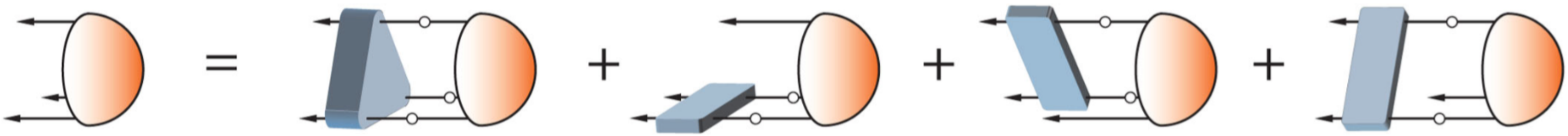


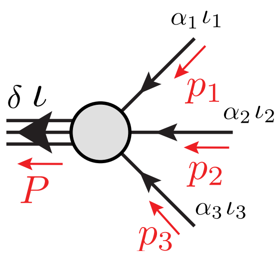
$$G^{(6)}(x_1, x_2, x_3, y_1, y_2, y_3) = \langle \Omega | q(x_1) q(x_2) q(x_3) q(y_1) q(y_2) q(y_3) | \Omega \rangle$$

Introduction: Nonperturbative QCD Framework



Introduction: Three-Body Faddeev Equation





$$ {}_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P) = \frac{1}{\sqrt{6}}\varepsilon_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P)\,, $$

Spinors: $4\times4\times4\times4=256$
Flavors: $2\times2\times2\times2=16$

128-terms

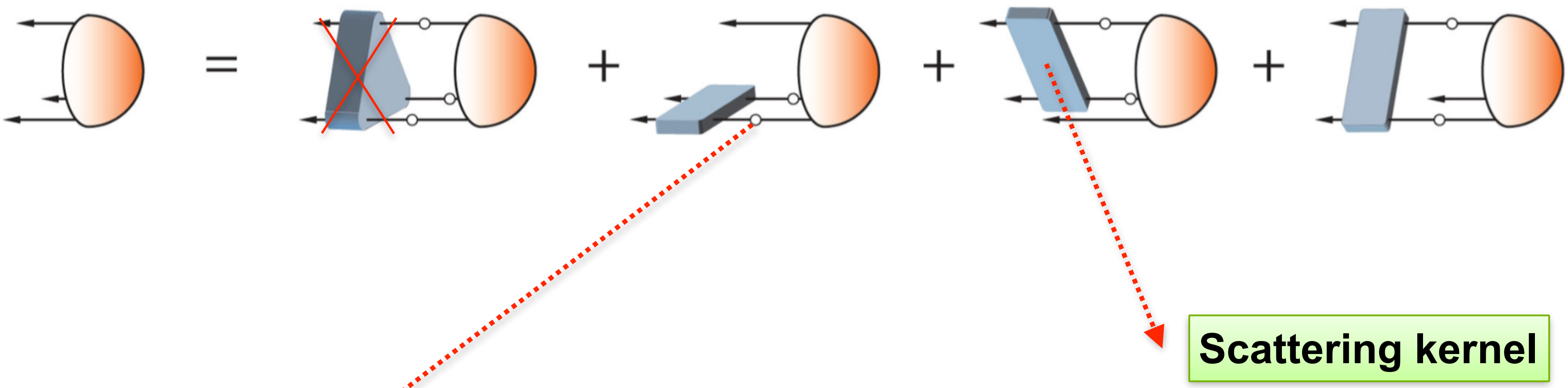
Jacobi Coordinates: P_μ, p_μ, q_μ
$P^2=-M_N^2; p^2, q^2, Pq, Pp, pq$

5-dim

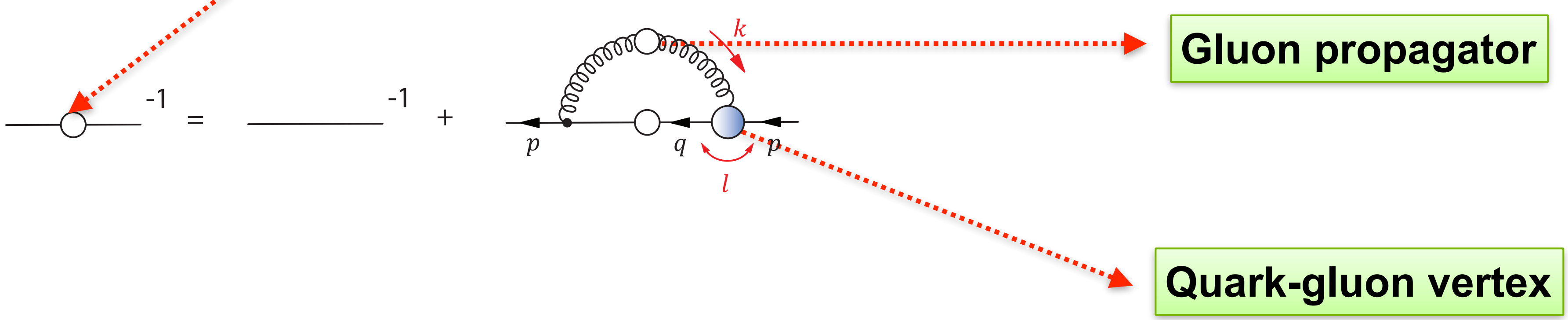
Highly complex algebra + Cutting edge numerical technology **1TB**

Diquark approximation: Reduce three-body problem to two two-body ones.

Introduction: Three-Body Faddeev Equation

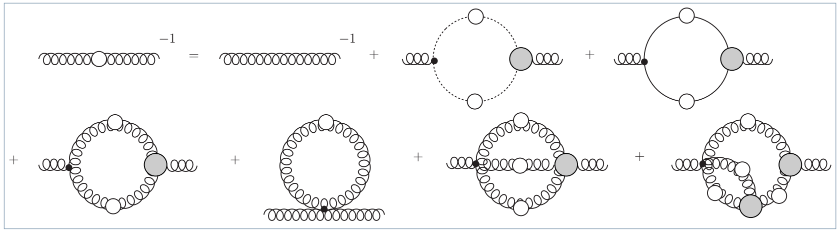


One-body gap equation for inputs

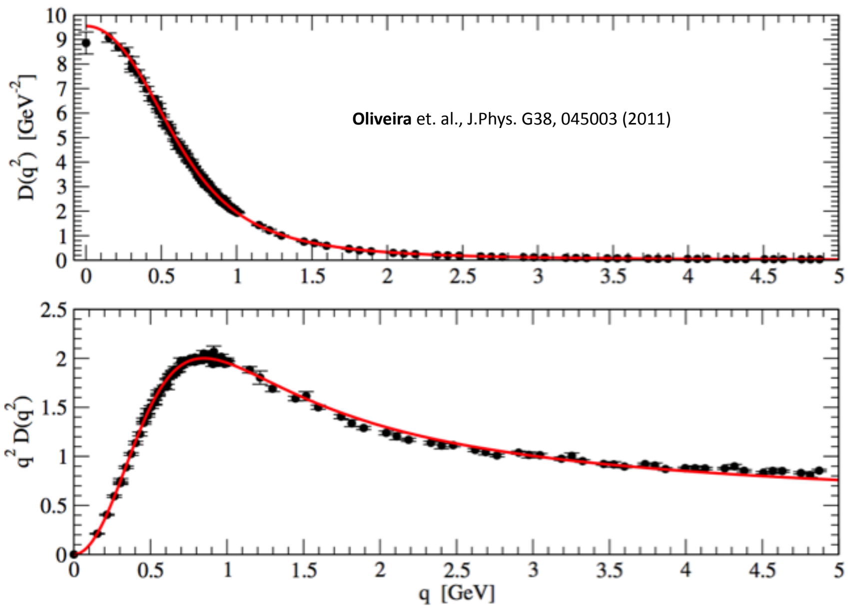


Basics: Gluons are massive quasi-particles

Gluon gap equation: **Aguilar**, Binosi, Papavassiliou and Rodriguez-Quintero



Lattice QCD simulations:



- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

- In QCD: Gluons are ***cannibals*** — a particle species whose members become **massive** by eating each other — **quasi-particles**!

Basics: Gluons are massive quasi-particles

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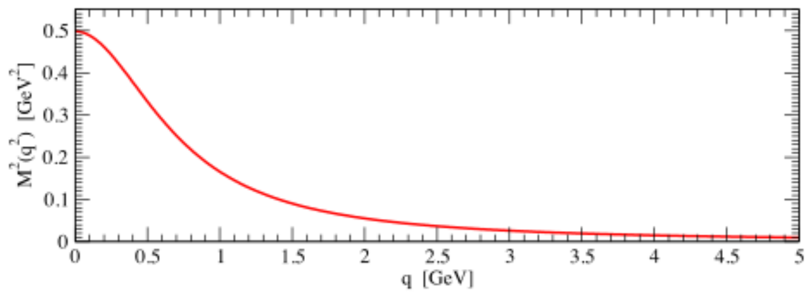
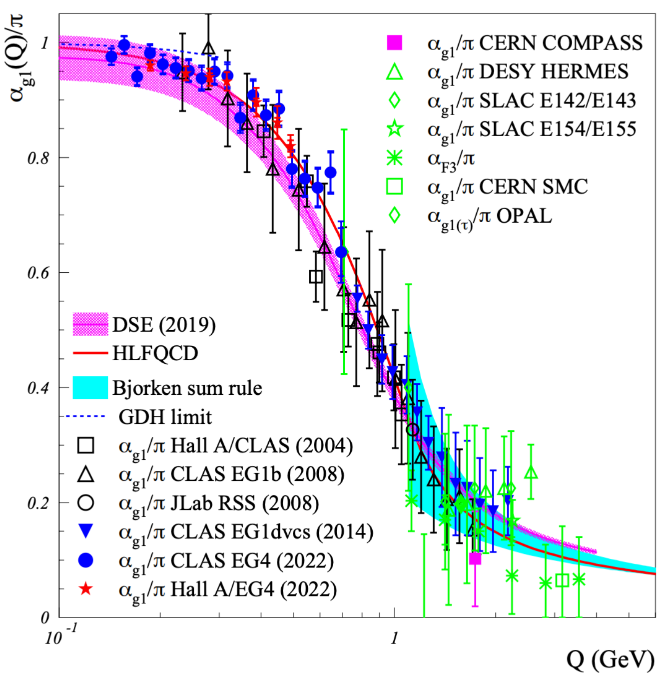
Gluon mass function: **Oliveira** et. al., J.Phys. G38, 045003 (2011)



Running coupling: **Deur, Brodsky, Roberts,** PPNP, 104081 (2024)



1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

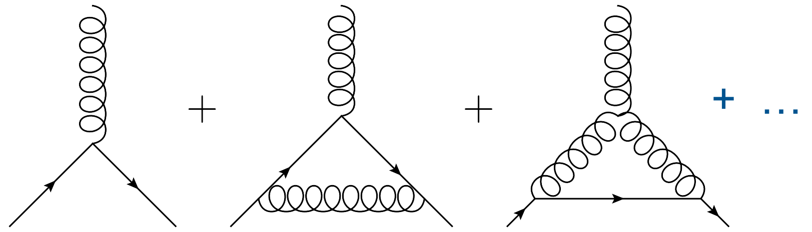
$$M_g \sim 700 \text{ MeV}$$

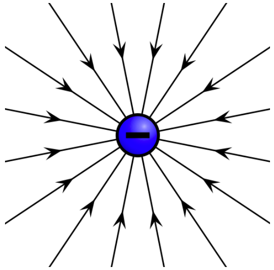
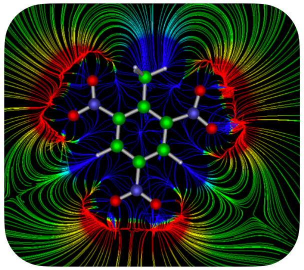
2. The effective running coupling **saturates** in the infrared limit.

- converge to: $\alpha_s(0) \sim \pi$
- transition at: $Q \sim 1$ GeV

See, e.g., PRC 84, 042202(R) (2011)

Basics: Vertex has DCSB-rendered appearance

Quark-gluon vertex:  = ?

point charge  →  distributed charges

✦ The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^{\mu}(P', P) = \gamma^{\mu} F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^{\nu} F_2(Q^2)$$

12 terms

✦ The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous moment** is proportional to the **Pauli** term.

See, e.g., PLB722, 384 (2013)

Basics: Vertex has DCSB-rendered appearance

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Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma \cdot p A(p^2) \ + \ B(p^2)}$$

$$\gamma^\mu \Sigma_A \ + \ i\sigma_{\mu\nu} Q^\nu \Delta_B \ + \ \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

1. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**. **DCSB** is closely related to the **Pauli term**:

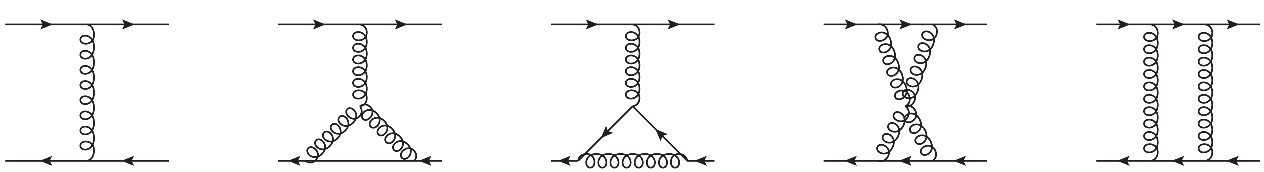
$$F_2 \sim \text{DCSB}$$

2. The **appearance** of the vertex is dramatically modified by the **dynamics**. The vertex can be phenomenologically expressed as:

$$\Gamma^\mu \sim \gamma^\mu + i\eta\sigma_{\mu\nu} Q^\nu \Delta_B$$

See, e.g., PLB722, 384 (2013)

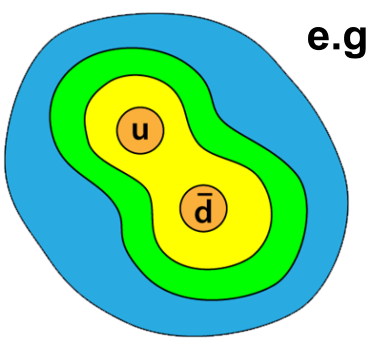
Basics: Kernel has the Dirac and Pauli terms



+ … = ?

✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry	Gauge symmetry
C-, P-, T-symmetry	Chiral symmetry



e.g., pion

1. **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

2. **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

Basics: Kernel has the Dirac and Pauli terms

✦ Proper decomposition:

$$K^{(2)} = \left[K_{L0}^{(+)} \otimes K_{R0}^{(-)}\right] + \left[K_{L0}^{(-)} \otimes K_{R0}^{(+)}\right] + \left[K_{L1}^{(-)} \otimes_+ K_{R1}^{(-)}\right] + \left[K_{L1}^{(+)} \otimes_+ K_{R1}^{(+)}\right] + \left[K_{L2}^{(-)} \otimes_- K_{R2}^{(-)}\right] + \left[K_{L2}^{(+)} \otimes_- K_{R2}^{(+)}\right]$$

$$\text{with} \quad \gamma_5 K^{(\pm)} \gamma_5 = \pm K^{(\pm)}, \quad \otimes_\pm := \tfrac{1}{2}(\otimes \pm \gamma_5 \otimes \gamma_5)$$

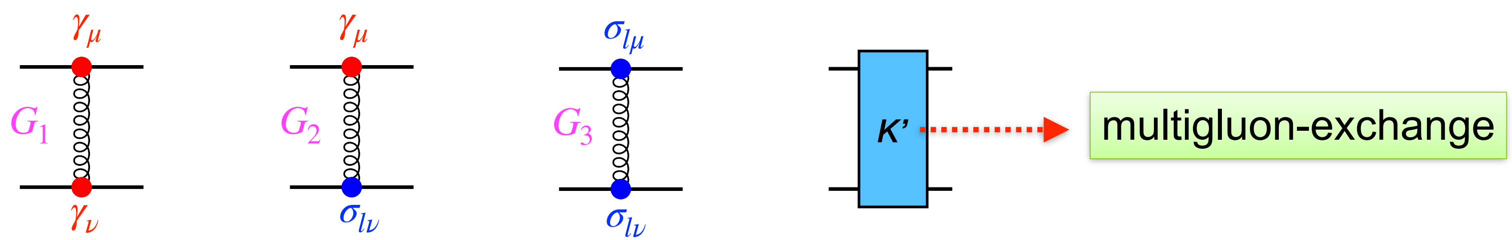
discrete

✦ Deformed WTIs:

$$\begin{aligned}
\Sigma_B(k_+) &= \int_{dq} \left\{K_{L0}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(-)} - K_{L1}^{(-)}[\sigma_B(q_+)]K_{R1}^{(-)} + K_{L1}^{(+)}[\sigma_B(q_-)]K_{R1}^{(+)}\right\} \\
0 &= \int_{dq} \left\{K_{L0}^{(+)}[\sigma_B(q_-)]K_{R0}^{(-)} - K_{L0}^{(-)}[\sigma_B(q_+)]K_{R0}^{(+)} + K_{L2}^{(+)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(+)}\right\} \\
[\Sigma_A(k_+) - \Sigma_A(k_-)] &= \int_{dq} \left\{K_{L0}^{(+)}[-\sigma_B(q_+)]K_{R0}^{(-)} + K_{L0}^{(-)}[\sigma_B(q_-)]K_{R0}^{(+)} + K_{L2}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R2}^{(-)}\right\} \\
-\Sigma_B(k_-) &= \int_{dq} \left\{K_{L0}^{(-)}[\Delta_{\sigma_A}^{\pm}]K_{R0}^{(+)} + K_{L1}^{(-)}[\sigma_B(q_-)]K_{R1}^{(-)} + K_{L1}^{(+)}[-\sigma_B(q_+)]K_{R1}^{(+)}\right\}
\end{aligned}$$

continuous

1. A realistic kernel must involves the Dirac and Pauli structures:



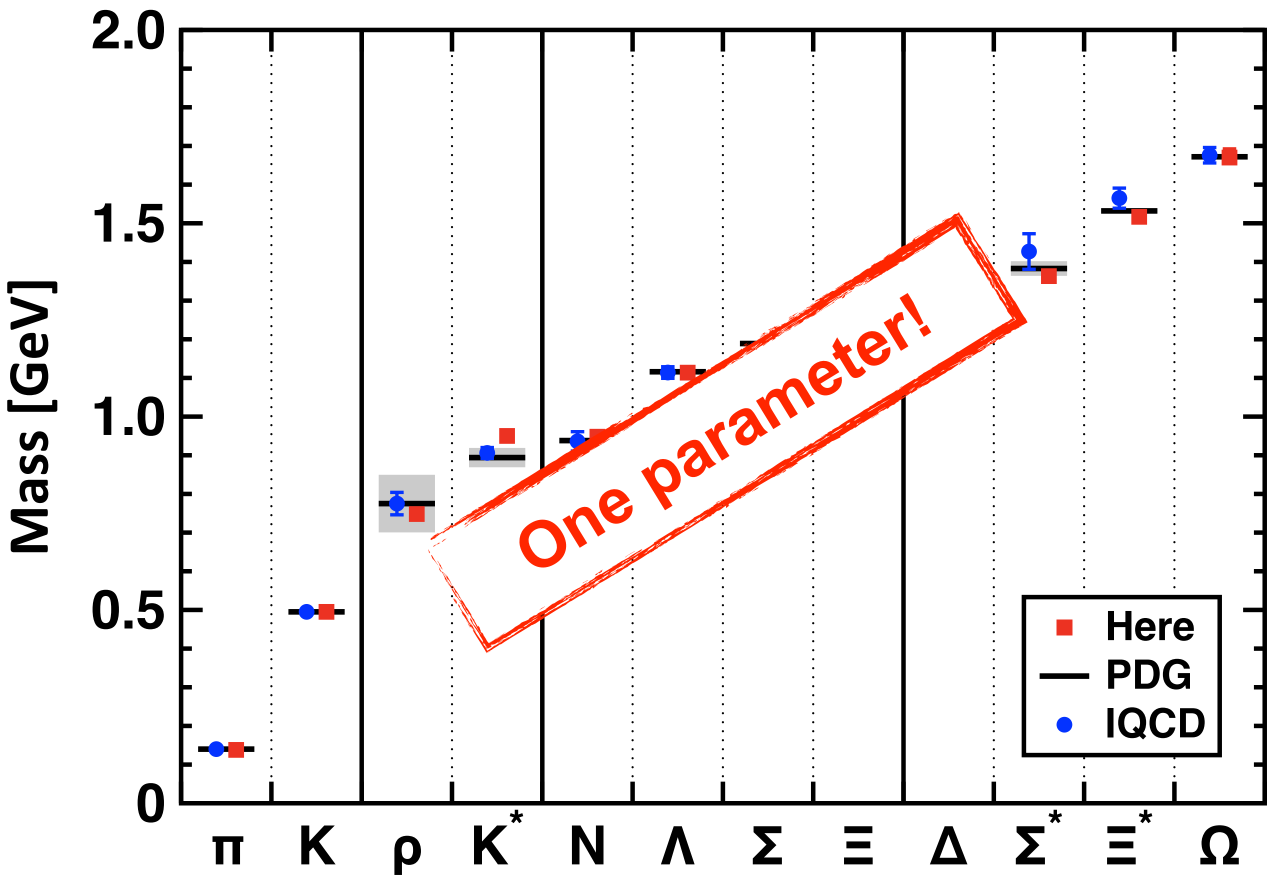
2. G_2 and G_3 are proportional to the Pauli term in the vertex, and thus to **DCSB**:

$$G_2, G_3 \sim \text{DCSB}$$

See, e.g., CPL 38 (2021) 7, 071201

Ground states

Ground states: Light & Strange flavor spectra



The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

See, e.g., Few-Body Syst 60, 26 (2019)

Ground states: Charm & Bottom flavor spectra

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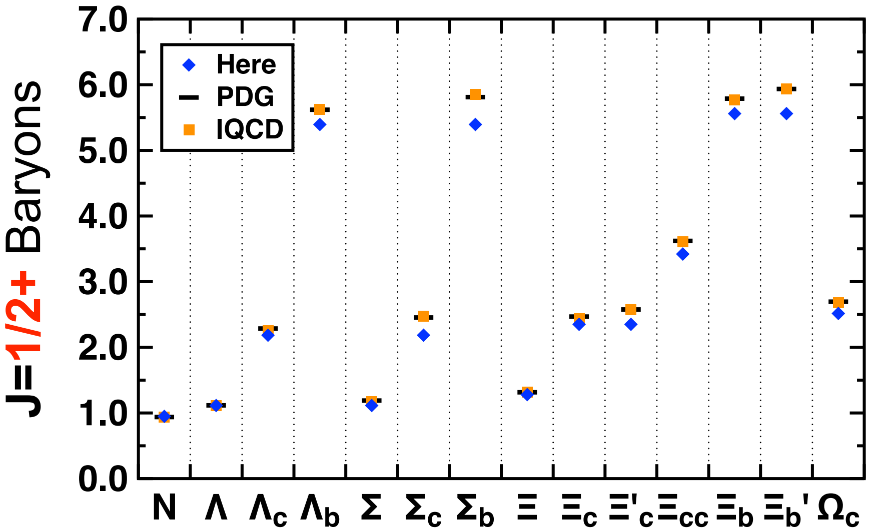
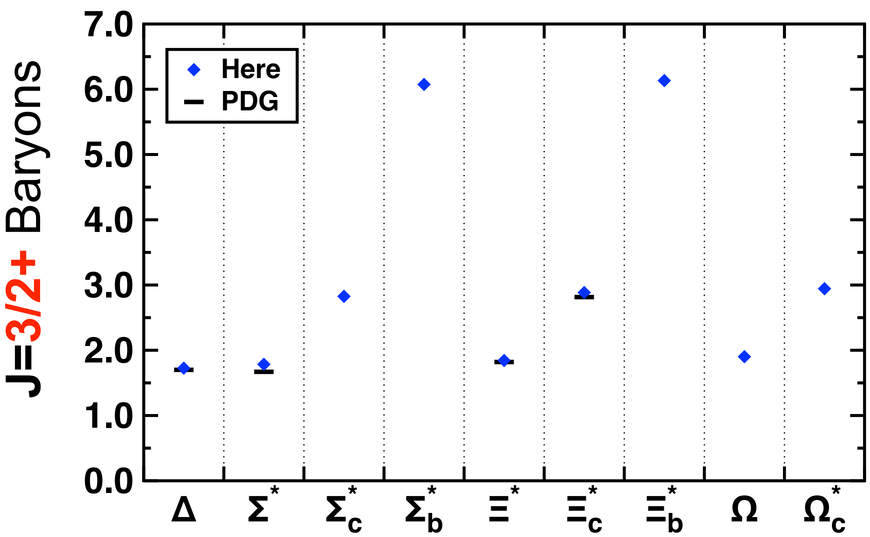
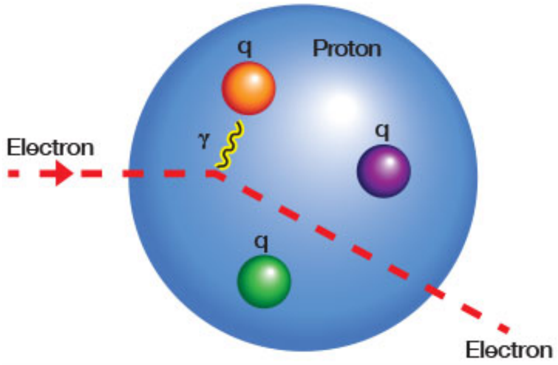
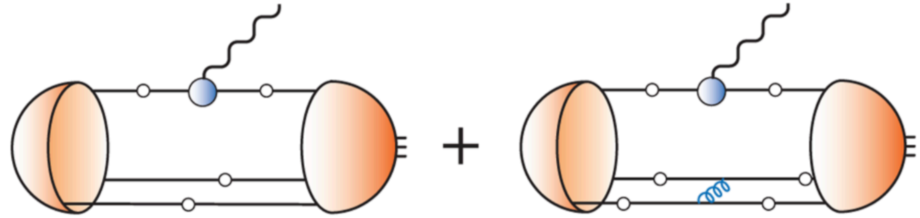




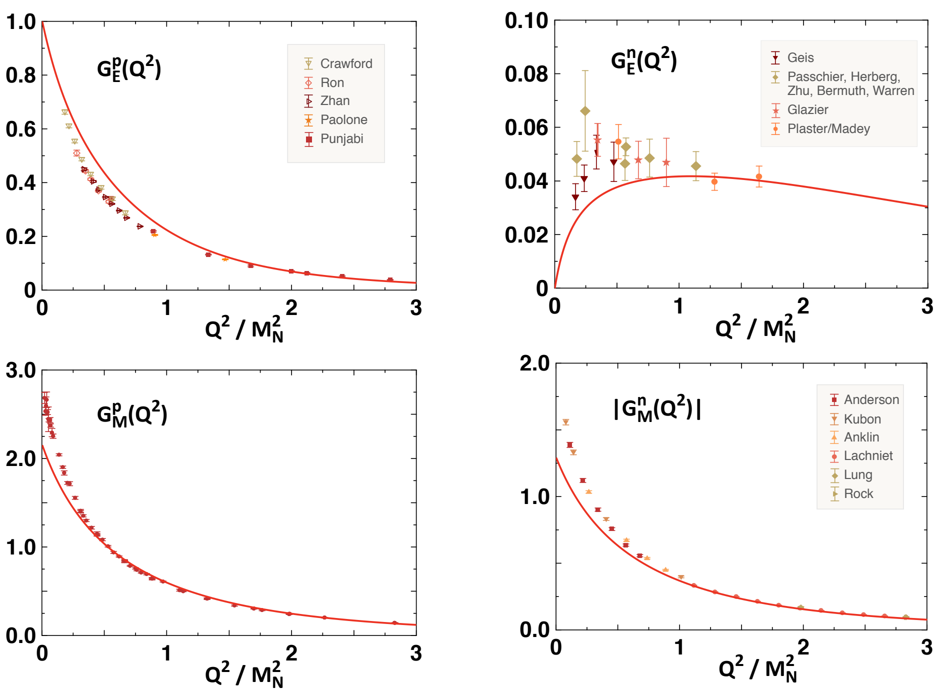
- ✦ The mean-absolute-relative-difference between the calculated values for the ground-states and the known empirical masses is about 5%.
- ✦ The ground spectra is NOT sensitive to the structures beyond the leading terms in the vertex and the kernel.

See, e.g., Few-Body Syst 60, 26 (2019)





$$\langle N(P')|J^\mu(x)|N(P)\rangle \sim \Gamma^\mu(P',P)$$



The two Sachs form factors read:

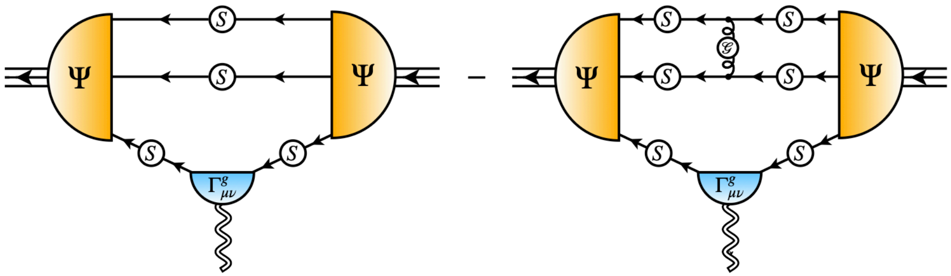
$$G_E(Q^2) := F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) := F_1(Q^2) + F_2(Q^2)$$

For charge & magnetization densities

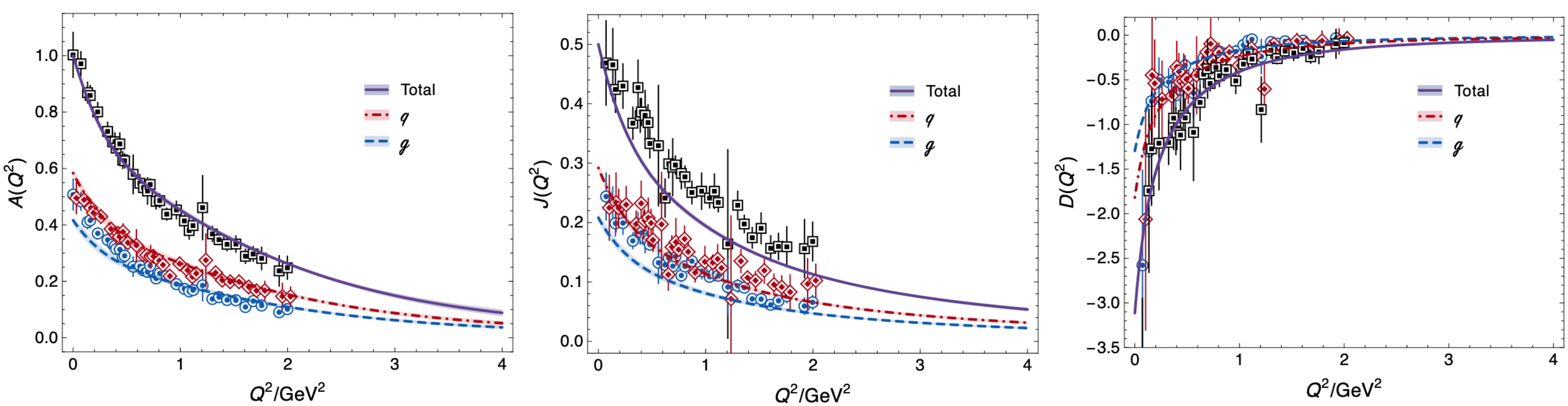
See, e.g., Eichmann, PRD 84, 014014 (2011)

Ground states: Gravitational form factors



For energy, angular momentum, pressure, and shear force densities

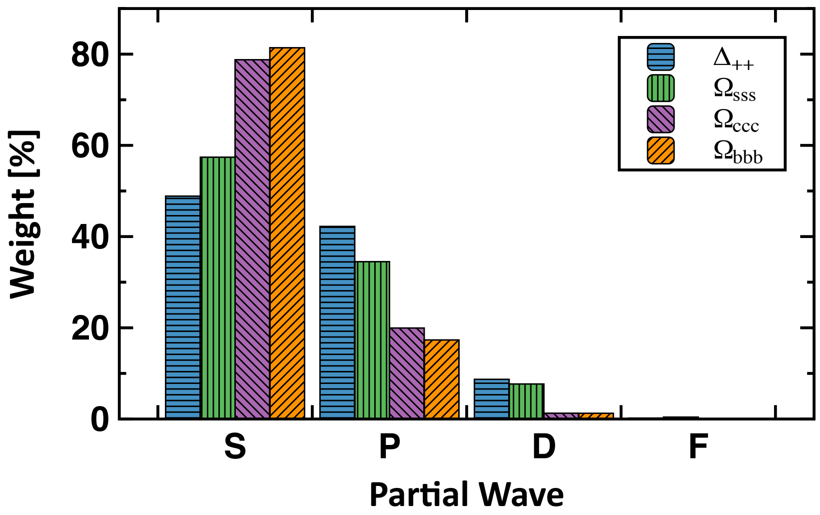
$$\langle N(\mathbf{p}',s')|\hat{T}^{\mu\nu}|N(\mathbf{p},s)\rangle = \frac{1}{m}\bar{u}(\mathbf{p}',s')\left[P^{\mu}P^{\nu}\boxed{A(t)} + iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_{\rho}\boxed{J(t)} + \frac{1}{4}(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2})\boxed{D(t)}\right]u(\mathbf{p},s)$$



See, e.g., Yao (姚照千), et al (NJU and ECT*), arXiv:2409.15547 (2024)

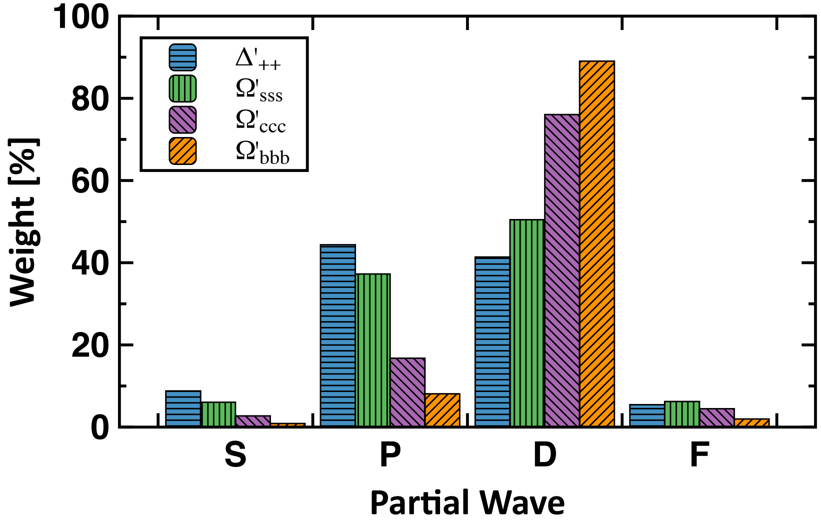
Excited states

Excited states: Multiple partial waves



✓ S-waves dominate for ground states, but P-waves grow for light baryons.

Why NR potential models work

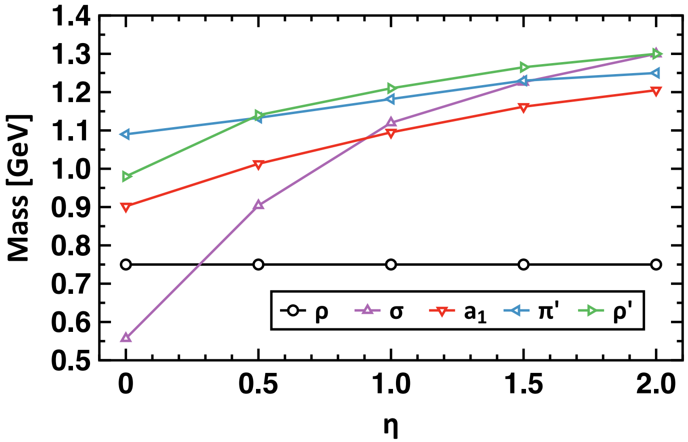


✓ D-waves dominate for excited states, but P-waves grow for light baryons.

See, e.g., PRD 97, 114017 (2018)

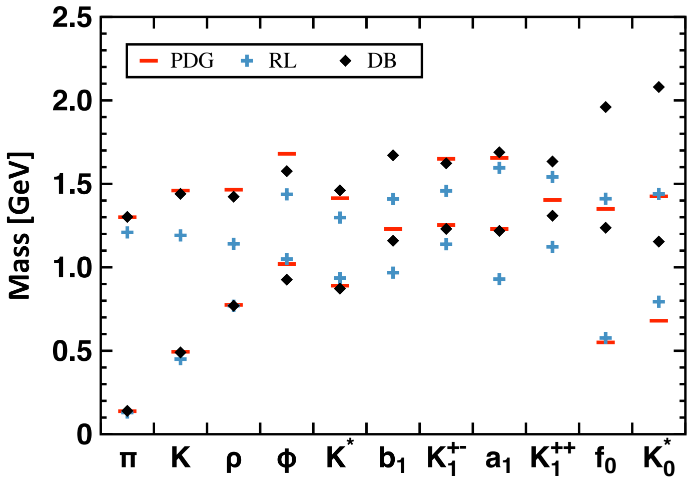
Excited states: Spin-orbit interaction

➡ Impact of the Pauli term (**a**nomalous **m**oment):



✦ With increasing the AM strength, the $a_1-\rho$ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

➡ Light & strange meson spectrum:



✦ The magnitude and ordering of all excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38, 071201 (2021) & EPJA 59, 39 (2023)

Excited states: DCSB-rendered spectra

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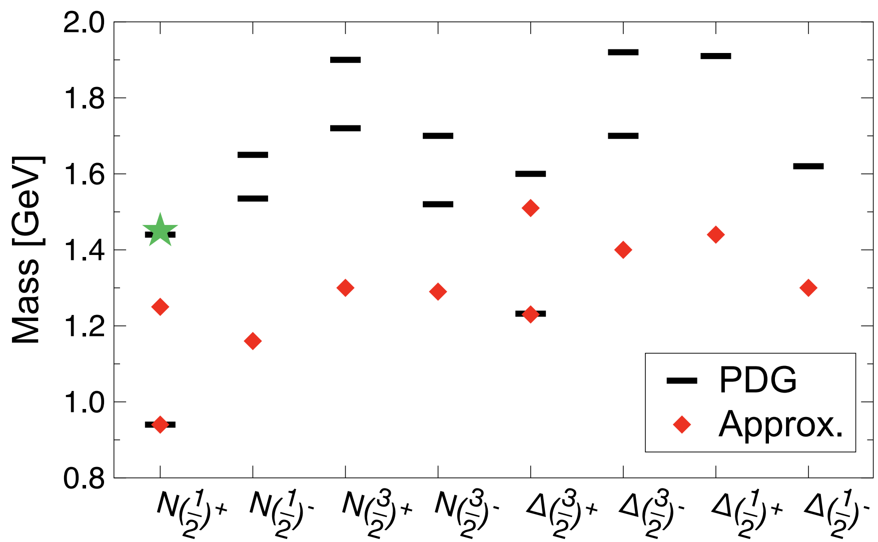
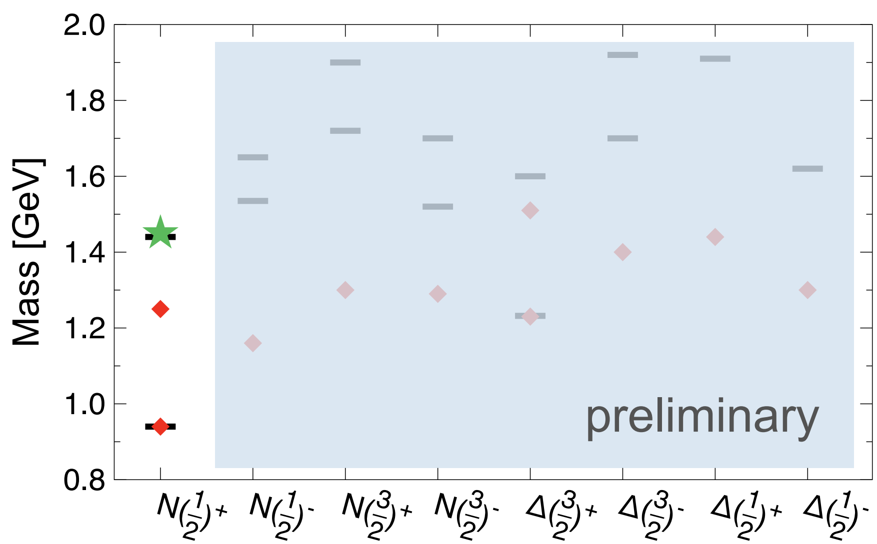


✦ The magnitude and ordering of radial or angular excitation states are WRONG in the approximation lacking of DCSB effect.



✦ The DCSB-enhanced kernel boost up 1st excitation nucleon, and can potentially fix the full spectra.

In progress

Summary

- ✦ The framework of three-body Faddeev equation, which describes baryons in continuum QCD, and its basics (e.g., gluon, vertex, kernel) are introduced.

- ✦ Baryon properties are studied: **a**) ground baryons — full mass spectrum of J=1/2 and J=3/2, nucleon EM and gravitational form factors; **b**) excited baryons — partial waves, spin-orbit interaction, DCSB-rendered spectra.

Outlook

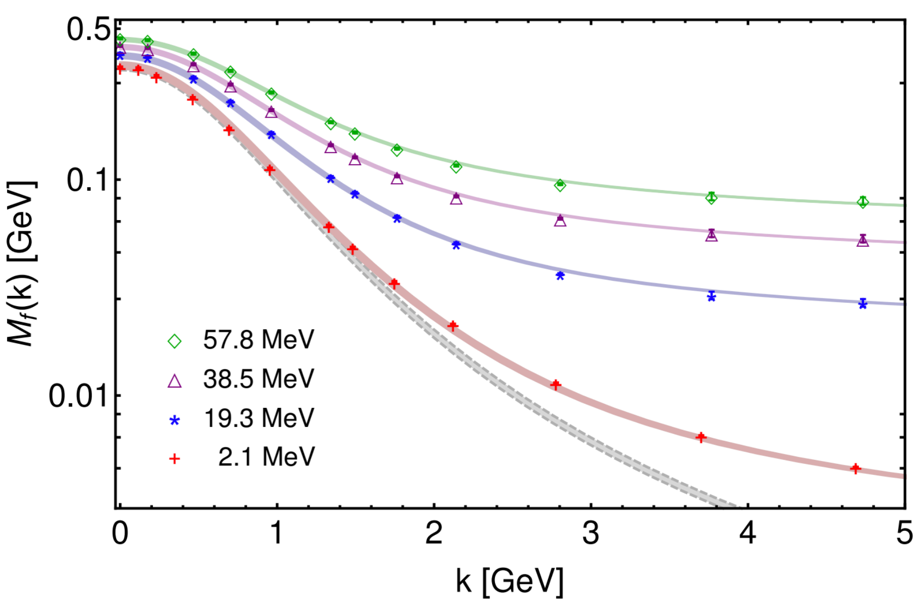
- ◆ Use the three-body Faddeev equation to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.

- ◆ Hopefully, iterating with future high precision experiments on **light** and **heavy** hadrons, from spectroscopy to structures, we may provide a **faithful path** to understand **QCD**.

Backup: Quark running mass function

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Chang, Yang, et. al., PRD 104, 094509 (2021)



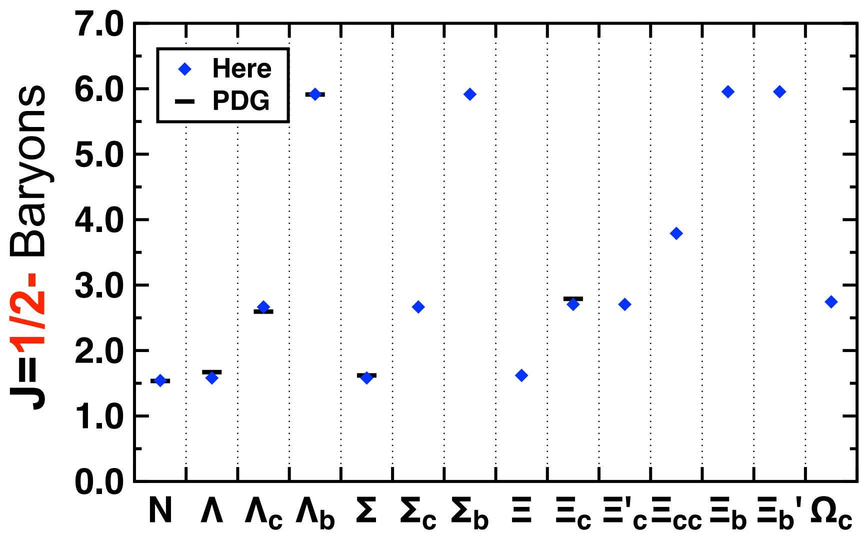
✦ **Now:**

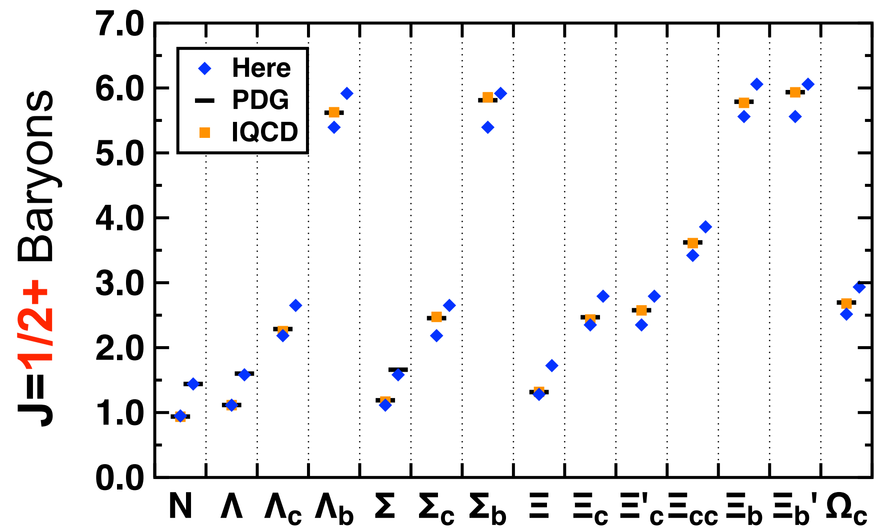
1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

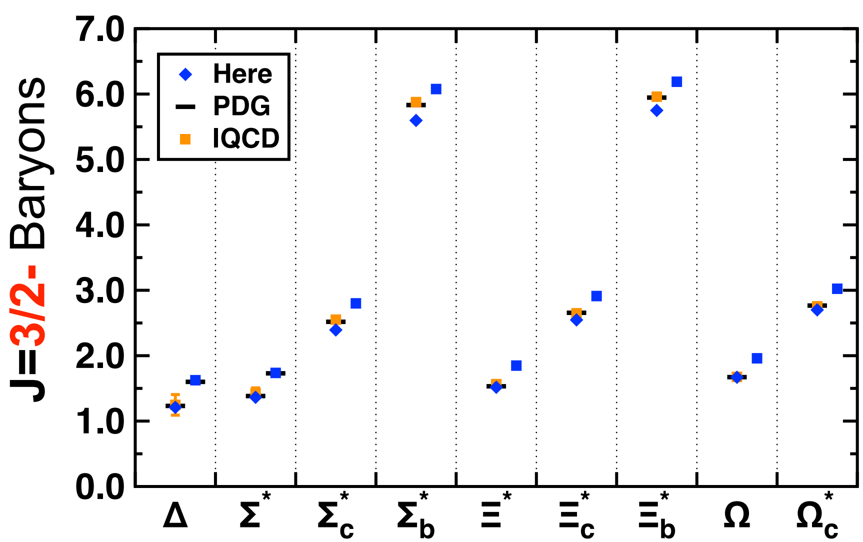
✦ **Next:**

1. What is the **infrared scale** of quark mass function?
2. How does the **transition** connect the non-perturbative and perturbative regions?

Backup: mimic-DCSB spectra







✦ To mimic the **spin-orbit** repulsion, we can decrease the effective interaction strength to calculate the spectra.

See, e.g., Few-Body Syst 60, 26 (2019)