Mass spectra of strange double charm pentaquark

FB23 , Beijing, China

Speaker: Zi-Yan Yang
Institute of Quantum Matter, South China Normal University

Based on **PRD 110, 056022(2024)**

Co-authors: Prof. Qian Wang, Prof. Wei Chen

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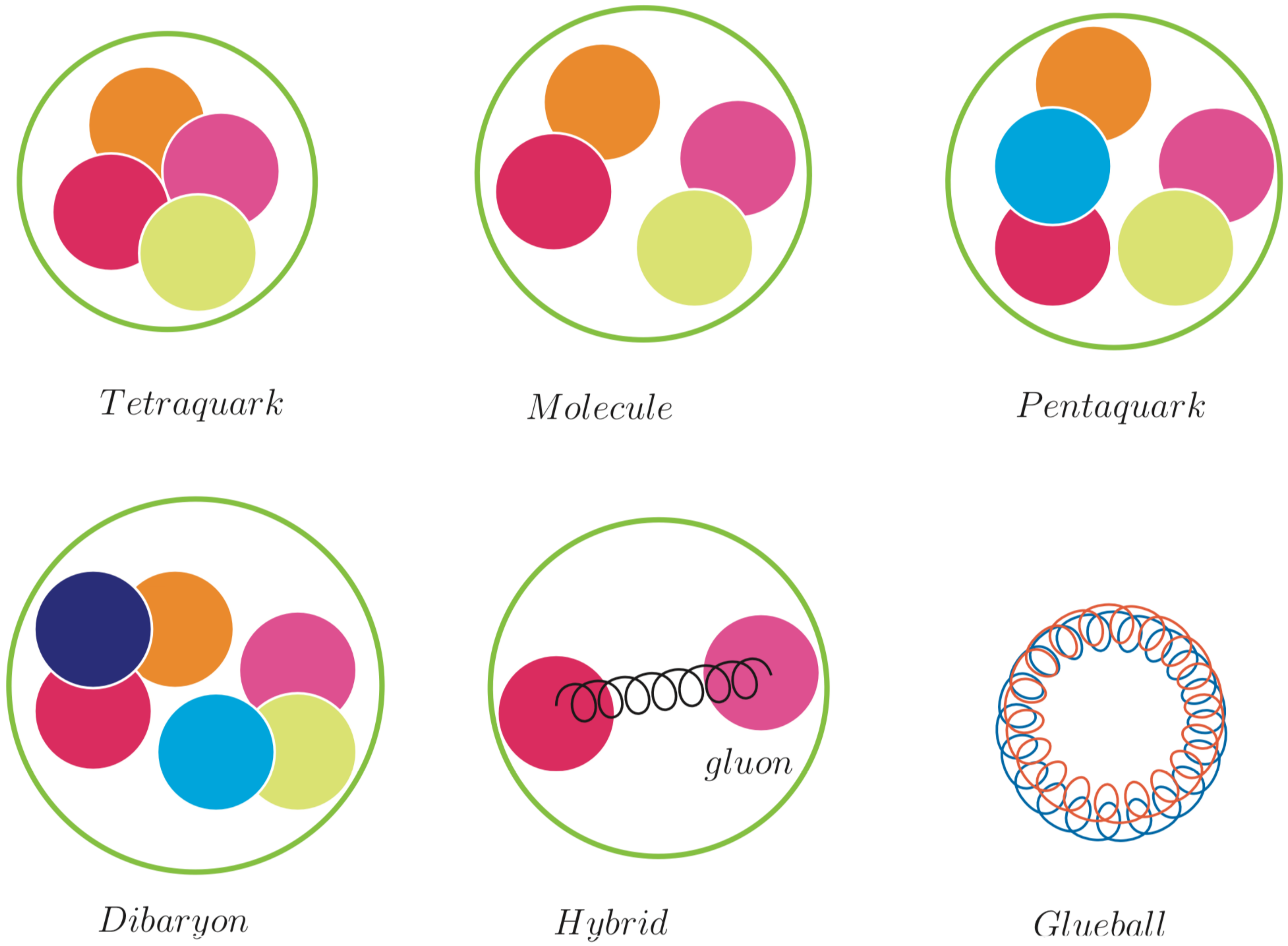
Content

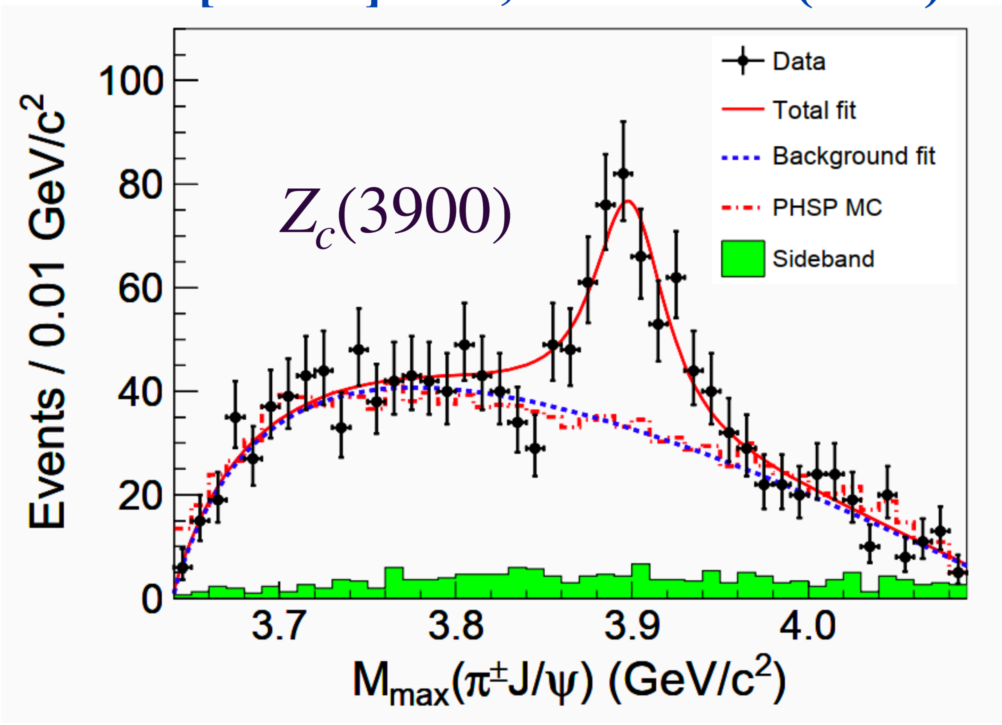
Background: Exotic Hadron



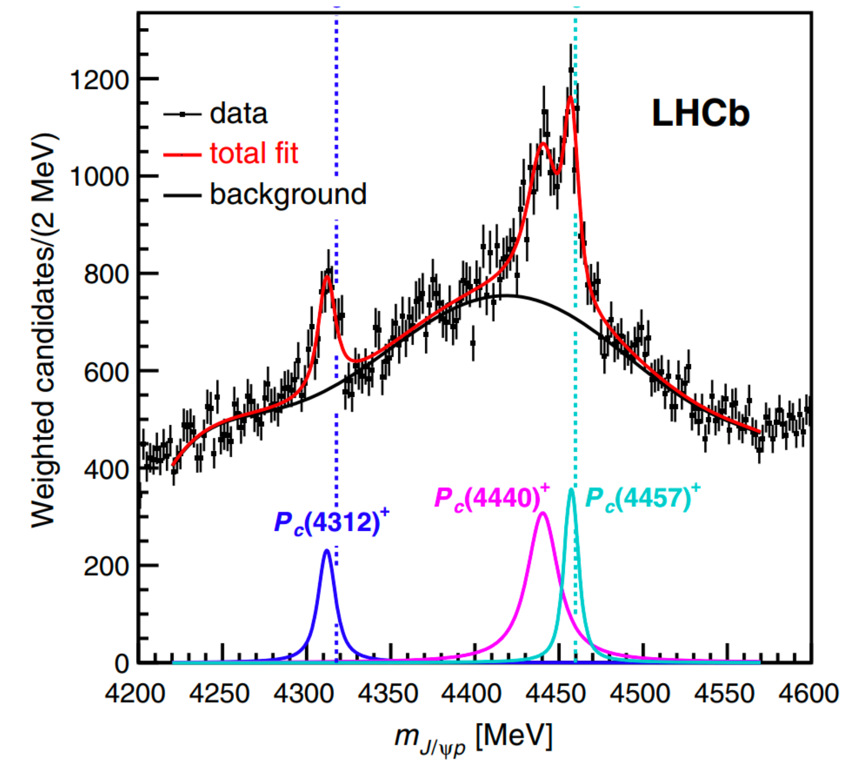
- Exotic Hadron: Tetraquark, Pentaquark, Hybrid, Glueball$\cdots$

Background: Exotic Hadron

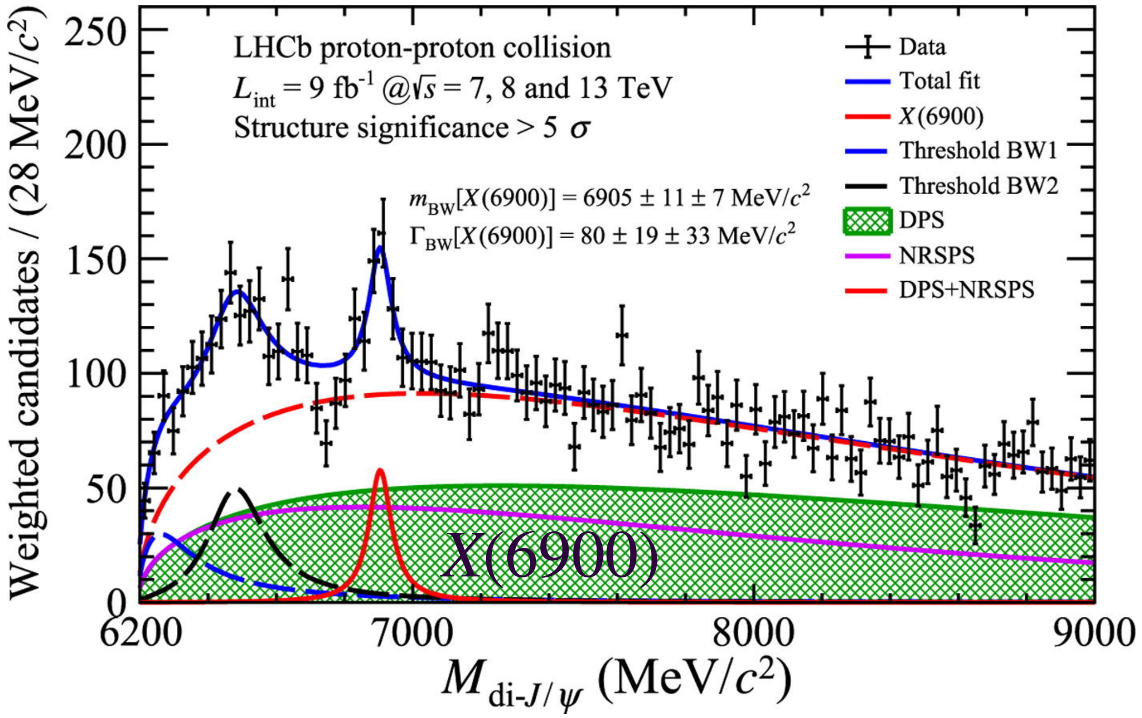
[BESIII] *PRL*, 110.252001(2013)



[LHCb] *PRL*, 122.222001(2019)



[LHCb] *Sci. Bull.*,65.1983(2020)



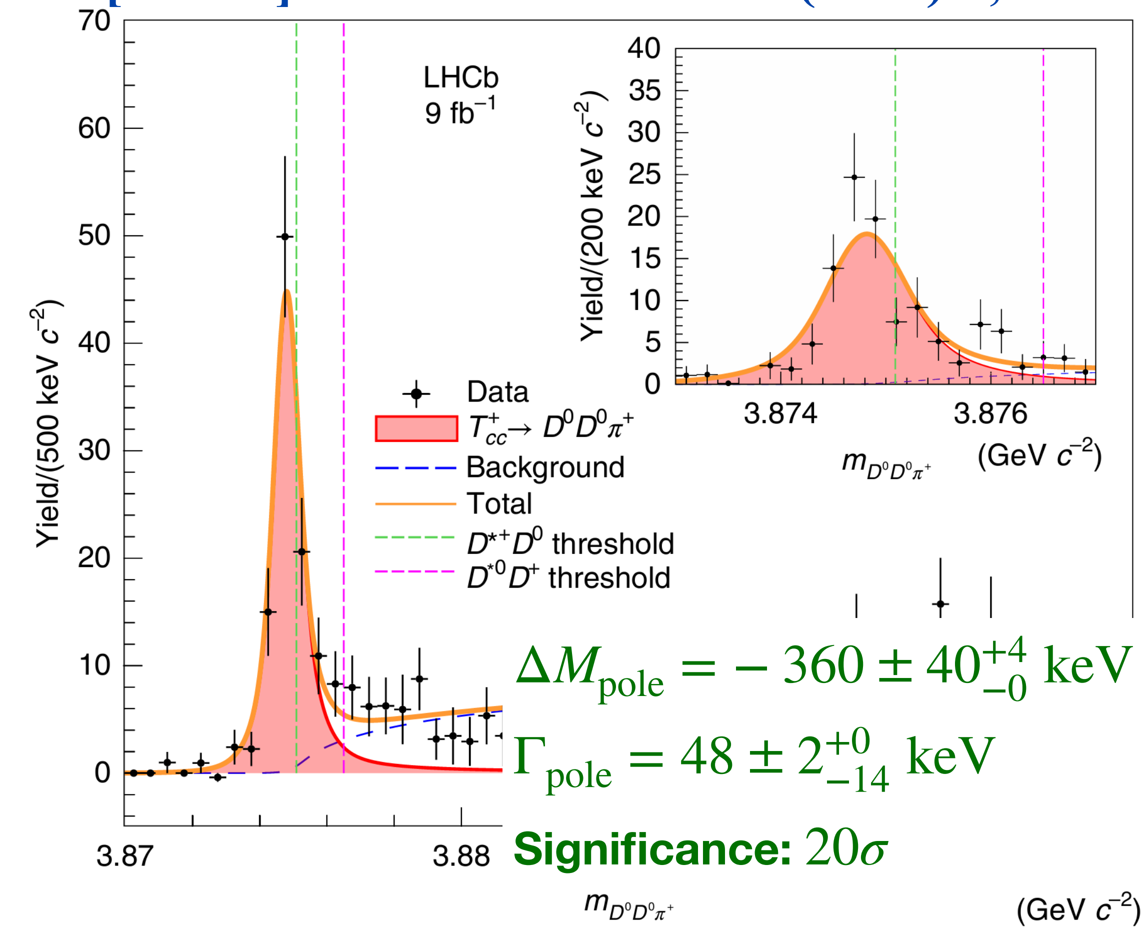
- Exotic Hadron: Tetraquark, Pentaquark, Hybrid, Glueball···

Motivation: T_{cc}, $T_{c\bar{s}}$ and P_{ccs}

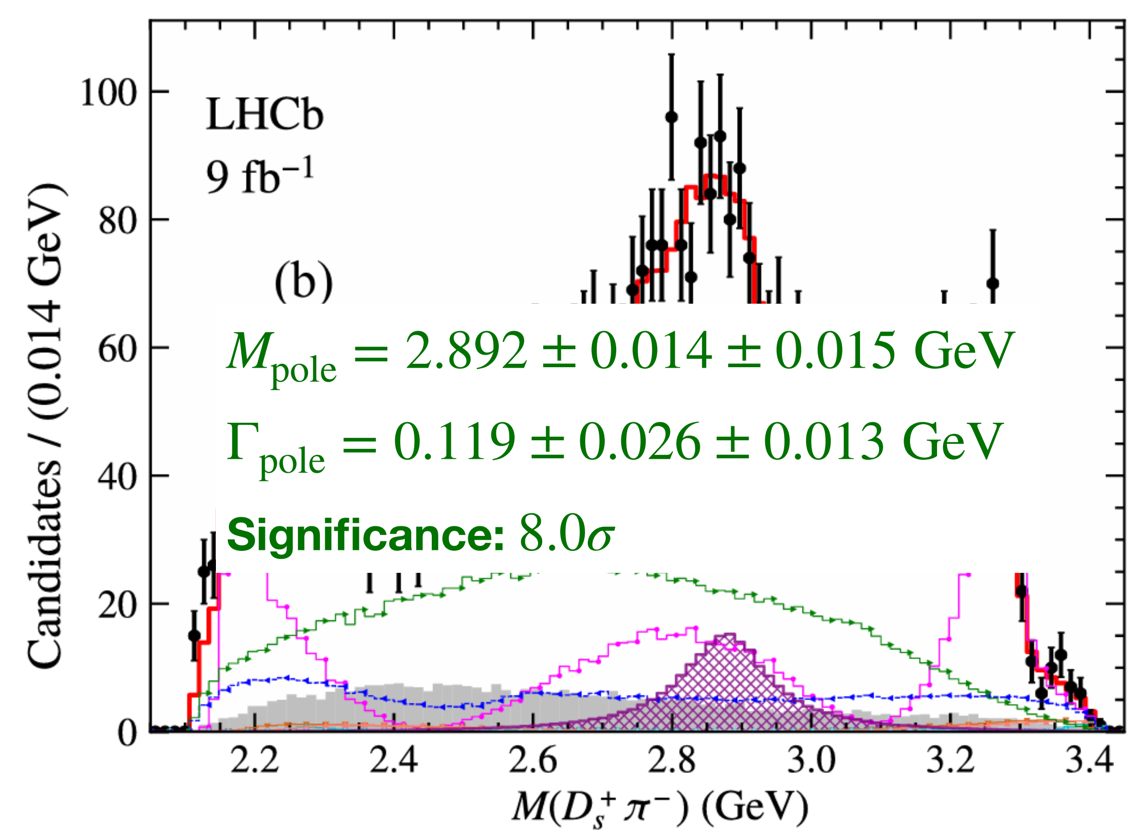
- Recent discovery of the double charm tetraquark $T_{cc}^{+}(3875)$ raises the question whether **double charm pentaquark** exists or not.
- Heavy antiquark-diquark symmetry(HADS) states that a color triplet **double heavy diquark** behaves like a **heavy antiquark** in color space.

  - The observed $T_{c\bar{s}}(2900)^0$ with quark content $cd\bar{u}\bar{s}$ indicates the potential existence of strange double charm pentaquark $ccus\bar{d}$

[LHCb] *Nature Commun.* 13 (2022) 1, 3351



[LHCb] PRD 108 (2023) 1, 012017



Previous theoretical studies...

- Many theoretical attempts on the mass spectrum from molecular picture[1] and compact pentaquark picture[2].
- **[XK Dong etc, CTP 73.125201(2021)]**, Bethe-Salpeter equation with interaction respecting heavy quark spin symmetry, predicts several **bound states**.

[1] Yan et al. Phys. Rev. D 98, 091502(2018)
Dong et al. Commun. Theor. Phys. 73, 125201 (2021)
Chen et al. Phys. Rev. D 96, 116012 (2017)
Guo, Phys. Rev. D 96, 074004 (2017)
Zhu et al. Phys. Lett. B 797,134869 (2019)
Shen et al. Eur.Phys.J.C 83,70 (2023)
Wang et al. Phys. Rev. D 109, 074035 (2024)
Duan et al. Phys. Rev. D 109, 094018 (2024)
[2] Chen et al. Phys. Lett. B 822, 136693 (2021)
Xing et al. Eur. Phys. J. C 81, 978 (2021)
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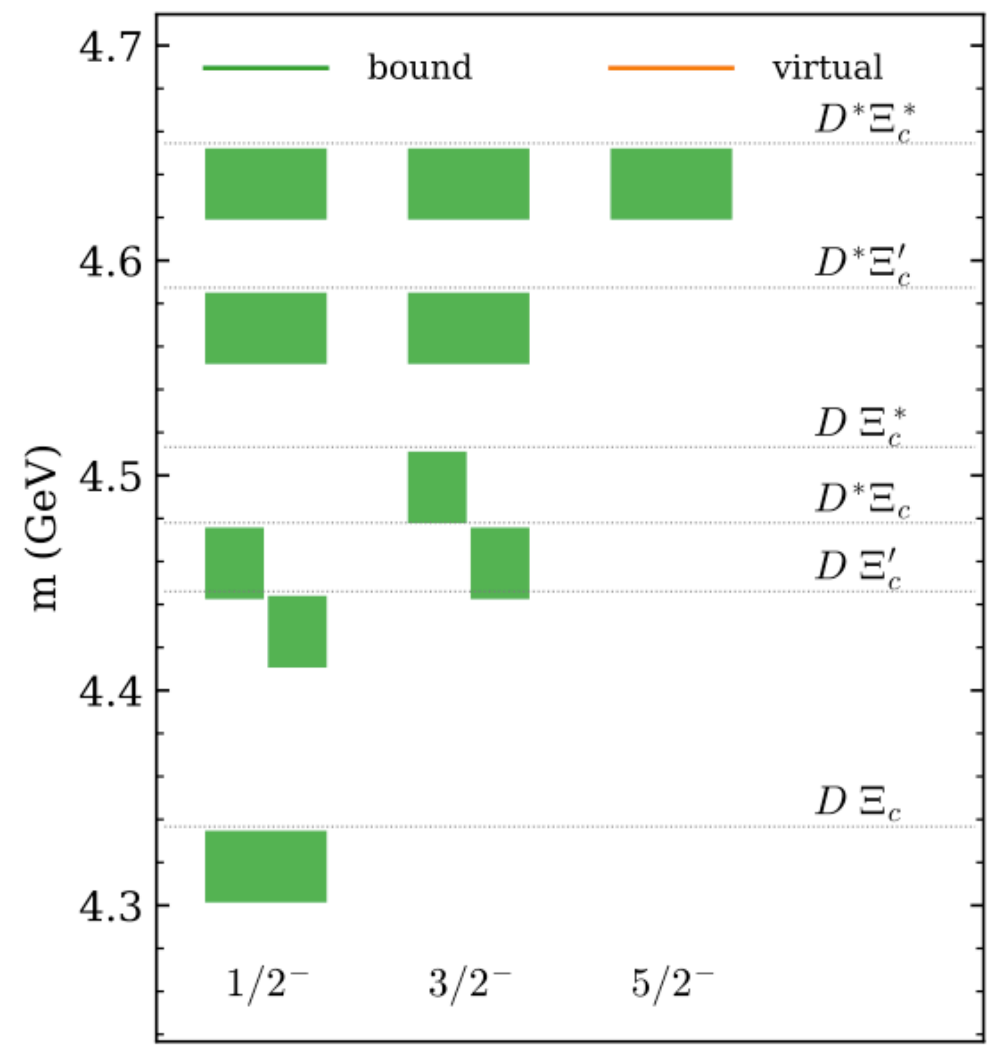

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- **[XK Dong etc, CTP 73.125201(2021)]**, Bethe-Salpeter equation with interaction respecting heavy quark spin symmetry, predicts several **bound states.**
- **[QS Zhou etc, PRC. 98.045204(2018)]**, color-magnetic interaction, predicts **several compact $ccsq\bar{q}$ states**

[1] Yan et al. Phys. Rev. D 98, 091502(2018)
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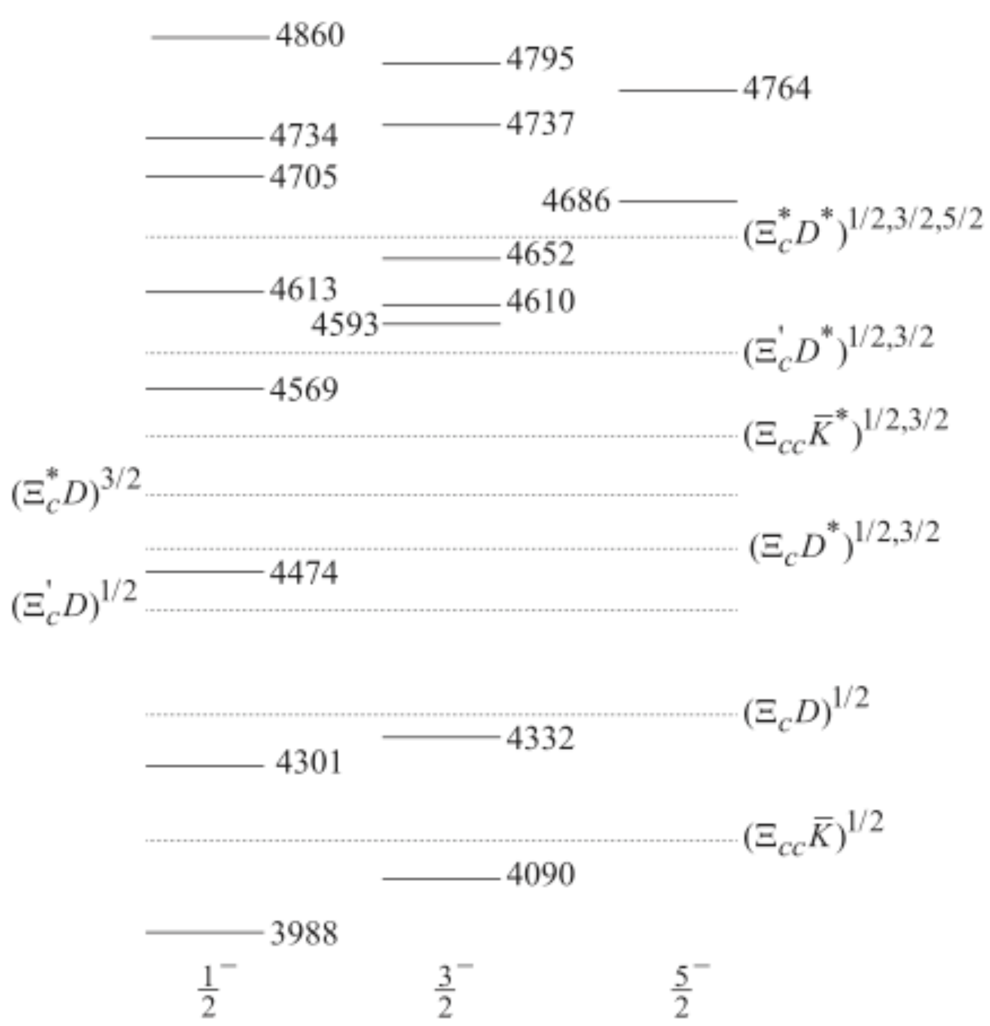


Previous theoretical studies...

Pentaquark (J^P, I)	Mass				Threshold
	This work	ChEFT [19]	CMI [15]	QCDSR [20]	
$cc\bar{s}nn(\frac{1}{2}^-, 0)$	4.092 ± 0.298	3.957	4.702	–	$\Xi_{ccq}\bar{K}$
$cc\bar{n}nn(\frac{1}{2}^-, \frac{1}{2})$	3.841 ± 0.290	3.816	4.578	4.21	$\Xi_{ccq}\pi$
$cc\bar{n}sn(\frac{1}{2}^-, 1/0)$	4.125 ± 0.301	4.112	4.854	–	$\Xi_{ccq}K$
$cc\bar{s}sn(\frac{1}{2}^-, \frac{1}{2})$	4.409 ± 0.307	3.816	4.968	–	$\Omega_{ccq}K$
$cc\bar{s}nn(\frac{3}{2}^-, 0)$	4.496 ± 0.338	–	4.355	–	$\Xi^*_{ccq}\bar{K}$
$cc\bar{n}nn(\frac{3}{2}^-, \frac{1}{2})$	4.393 ± 0.340	–	3.970	4.27	$\Xi^*_{ccq}\pi$
$cc\bar{n}sn(\frac{3}{2}^-, 1/0)$	4.600 ± 0.348	–	4.802	–	$\Xi^*_{ccq}K$
$cc\bar{s}sn(\frac{1}{2}^-, \frac{1}{2})$	4.762 ± 0.342	–	4.955	–	$\Omega^*_{ccq}K$

- **[Y Xing etc, EPJC 81.978(2021)]**, double heavy triquark-diquark framework with SU(3) flavor symmetry, predicts several stable double charm pentaquark (for instance a $J^P = 1/2^-$ $ccsn\bar{n}$ pentaquark)

Previous theoretical studies…

- Many theoretical attempts on the mass spectrum from molecular picture**[1]** and compact pentaquark picture**[2]**.

Current	J^P	$s_0[\mathrm{GeV}^2]$	$M_B^2[\mathrm{GeV}^2]$	Pole (%)	CVG (%)	Mass [GeV]	Two-hadron threshold [GeV]	$f_X[\mathrm{GeV}^6]$
$J^{\Lambda_c D}$	$\frac{1}{2}^-$	19.5(±5%)	2.83–3.43	>16.9	<5	$4.13^{+0.10}_{-0.09}$	4.15	$0.77^{+0.16}_{-0.16}\times10^{-3}$
$J^{\Sigma_c D}$	$\frac{1}{2}^-$	18.3(±5%)	3.40–3.70	>5.9	<5	$4.08^{+0.18}_{-0.13}$	4.32	$0.28^{+0.08}_{-0.08}\times10^{-3}$
$J^{\Sigma_c D^*}$	$\frac{3}{2}^-$	20.3(±5%)	3.17–3.47	>11.9	<10	$4.14^{+0.18}_{-0.15}$	4.46	$0.27^{+0.08}_{-0.08}\times10^{-3}$
$J^{\Sigma_c^* D}$	$\frac{3}{2}^-$	22.8(±5%)	3.82–4.22	>13.4	<2	$4.47^{+0.11}_{-0.10}$	4.39	$1.43^{+0.31}_{-0.30}\times10^{-3}$
$J^{\Lambda_c D^*}$	$\frac{3}{2}^-$	21.0(±5%)	3.55–3.95	>12.6	<5	$4.31^{+0.11}_{-0.10}$	4.29	$0.95^{+0.21}_{-0.21}\times10^{-3}$
$J^{\Lambda_c^* D}$	$\frac{3}{2}^-$	22.8(±5%)	2.91–3.51	>25.0	<10	$4.42^{+0.13}_{-0.12}$	4.73	$0.79^{+0.16}_{-0.15}\times10^{-3}$
$J^{\Lambda_c^* D^*}$	$\frac{5}{2}^-$	22.1(±5%)	3.09–3.69	>15.5	<10	$4.41^{+0.17}_{-0.14}$	4.86	$0.86^{+0.21}_{-0.19}\times10^{-3}$
$J^{\Sigma_c^* D^*}$	$\frac{5}{2}^-$	25.0(±5%)	4.0–4.6	>12.5	<2	$4.69^{+0.12}_{-0.11}$	4.53	$2.48^{+0.56}_{-0.54}\times10^{-3}$

- **[FB Duan etc, PRD 109.094018(2024)]**, QCD sum rule, consider the potential $ccud\bar{d}$ pentaquarks.

- ……

Interpolating currents: Spin of HADS partner for $T_{c\bar{s}}$

- In HADS, insuring the charm diquark has the same color structure $\bar{3}$, the spin structure of diquark should be symmetric due to Pauli principle, which is $S_{cc} = 1$.
- The strange charm tetrequark $T_{c\bar{s}}(2900)^0$ with quark content $[u\bar{c}][s\bar{d}]$ is a spin singlet state, the spin structure should be $[u\bar{c}]_0[s\bar{d}]_0$ or $[u\bar{c}]_1[s\bar{d}]_1$
- The spin structure of corresponding HADS pentaquark partner:

$$\mathbf{1}_{[cc]} \otimes \frac{\mathbf{1}}{\mathbf{2}}_{[u]} \otimes \mathbf{0}_{[s\bar{d}]} = \frac{\mathbf{1}}{\mathbf{2}}_{[ccus\bar{d}]} \oplus \frac{\mathbf{3}}{\mathbf{2}}_{[ccus\bar{d}]}$$

Or

$$\mathbf{1}_{[cc]} \otimes \frac{\mathbf{1}}{\mathbf{2}}_{[u]} \otimes \mathbf{1}_{[s\bar{d}]} = \frac{\mathbf{1}}{\mathbf{2}}_{[ccus\bar{d}]} \oplus \frac{\mathbf{3}}{\mathbf{2}}_{[ccus\bar{d}]} \oplus \frac{\mathbf{5}}{\mathbf{2}}_{[ccus\bar{d}]}$$

Analogous discussion can be made for the molecular structure and the compact structure. We consider these spins of pentaquark states in this work.

Interpolating currents: Flavor configuration

- Five flavor configurations

molecule $[\bar{d}_d s_d][\epsilon^{abc} Q_a Q_b u_c], [\bar{d}_d Q_d][\epsilon^{abc} Q_a u_b s_c], [\bar{d}_d u_d][\epsilon^{abc} Q_a Q_b s_c]$

compact $\epsilon^{aij}\epsilon^{bkl}\epsilon^{abc}[Q_i u_j][Q_k s_l]\bar{d}_c, \epsilon^{aij}\epsilon^{bkl}\epsilon^{abc}[Q_i Q_j][u_k s_l]\bar{d}_c$

Which can be related by the Fierz transformation.

$$\delta^{de}\epsilon^{abc} = \delta^{da}\epsilon^{ebc} + \delta^{db}\epsilon^{aec} + \delta^{dc}\epsilon^{abe}$$

Interpolating currents

Molecule $\Xi_c^{(\prime,*)+} D^{+(*)}$

$$\eta_1 = \frac{1}{\sqrt{2}}\epsilon_{abc}\left[\left(u_a^T C\gamma_5 s_b - s_a^T C\gamma_5 u_b\right) Q_c\right]\left[\bar{d}_d\gamma_5 Q_d\right],$$
$$\eta_2 = \frac{1}{\sqrt{2}}\epsilon_{abc}\left[\left(u_a^T C\gamma_\mu\gamma_5 s_b - s_a^T C\gamma_\mu\gamma_5 u_b\right)\gamma_\mu Q_c\right]\left[\bar{d}_d\gamma_5 Q_d\right],$$
$$\eta_3 = \frac{1}{\sqrt{2}}\epsilon_{abc}\left[\left(u_a^T C\gamma_5 s_b - s_a^T C\gamma_5 u_b\right)\gamma_\mu Q_c\right]\left[\bar{d}_d\gamma_\mu Q_d\right],$$
$$\eta_{4\mu} = \frac{1}{\sqrt{2}}\epsilon_{abc}\left[\left(u_a^T C\gamma_\nu\gamma_5 s_b - s_a^T C\gamma_\nu\gamma_5 u_b\right)\gamma_\nu Q_c\right]\left[\bar{d}_d\gamma_\mu Q_d\right],$$
$$\eta_{5\mu} = \sqrt{\frac{2}{3}}\epsilon_{abc}\left[(s_a^T C\gamma_\mu u_b)\gamma_5 Q_c + (u_a^T C\gamma_\mu Q_b)\gamma_5 s_c + (Q_a^T C\gamma_\mu s_b)\gamma_5 u_c\right]\left[\bar{d}_d\gamma_5 Q_d\right],$$
$$\eta_6 = \sqrt{\frac{2}{3}}\epsilon_{abc}\left[(s_a^T C\gamma_\mu u_b)\gamma_5 Q_c + (u_a^T C\gamma_\mu Q_b)\gamma_5 s_c + (Q_a^T C\gamma_\mu s_b)\gamma_5 u_c\right]\left[\bar{d}_d\gamma_\mu Q_d\right],$$
$$\eta_{7,\mu\nu} = \sqrt{\frac{2}{3}}\epsilon_{abc}\left[(s_a^T C\gamma_\mu u_b)\gamma_5 Q_c + (u_a^T C\gamma_\mu Q_b)\gamma_5 s_c + (Q_a^T C\gamma_\mu s_b)\gamma_5 u_c\right]\left[\bar{d}_d\gamma_\nu Q_d\right] + (\mu \leftrightarrow \nu),$$

Molecule $\Xi_{cc}^{(*)++} \bar{K}^{0(*)}$

$$\xi_1 = \left[\epsilon_{abc}(Q_a^T C\gamma_\mu Q_b)\gamma_\mu\gamma_5 u_c\right]\left[\bar{d}_d\gamma_5 s_d\right],$$
$$\xi_{2\mu} = \left[\epsilon_{abc}(Q_a^T C\gamma_\nu Q_b)\gamma_\nu\gamma_5 u_c\right]\left[\bar{d}_d\gamma_\mu s_d\right],$$
$$\xi_{3\mu} = \frac{1}{\sqrt{3}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_\mu Q_b\right)\gamma_5 Q_c + \left(Q_a^T C\gamma_\mu Q_b\right)\gamma_5 u_c\right]\left[\bar{d}_d\gamma_5 s_d\right],$$
$$\xi_4 = \frac{1}{\sqrt{3}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_\mu Q_b\right)\gamma_5 Q_c + \left(Q_a^T C\gamma_\mu Q_b\right)\gamma_5 u_c\right]\left[\bar{d}_d\gamma_\mu s_d\right],$$
$$\xi_{5,\mu\nu} = \frac{1}{\sqrt{3}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_\mu Q_b\right)\gamma_5 Q_c + \left(Q_a^T C\gamma_\mu Q_b\right)\gamma_5 u_c\right]\left[\bar{d}_d\gamma_\nu s_d\right] + (\mu \leftrightarrow \nu),$$

Molecule $\Omega_{cc}^{(*)+} \pi^+(\rho^+)$

$$\psi_i = \xi_i\ (u \leftrightarrow s),$$

Compact

$$J_{1,2} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu Q_j\right)\left(u_k^T C\gamma_\mu s_l\right)\gamma_5 C\bar{d}_c^T,$$
$$J_{1,3} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu Q_j\right)\left(u_k^T C\gamma_5 s_l\right)\gamma_\mu C\bar{d}_c^T,$$
$$J_{1,5\mu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu Q_j\right)\left(u_k^T C\gamma_5 s_l\right)\gamma_5 C\bar{d}_c^T,$$
$$J_{1,8\mu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\nu Q_j\right)\left(u_k^T C\gamma_\nu s_l\right)\gamma_\mu C\bar{d}_c^T,$$
$$J_{1,9\mu\nu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu Q_j\right)\left(u_k^T C\gamma_\nu s_l\right)\gamma_5 C\bar{d}_c^T + (\mu \leftrightarrow \nu),$$

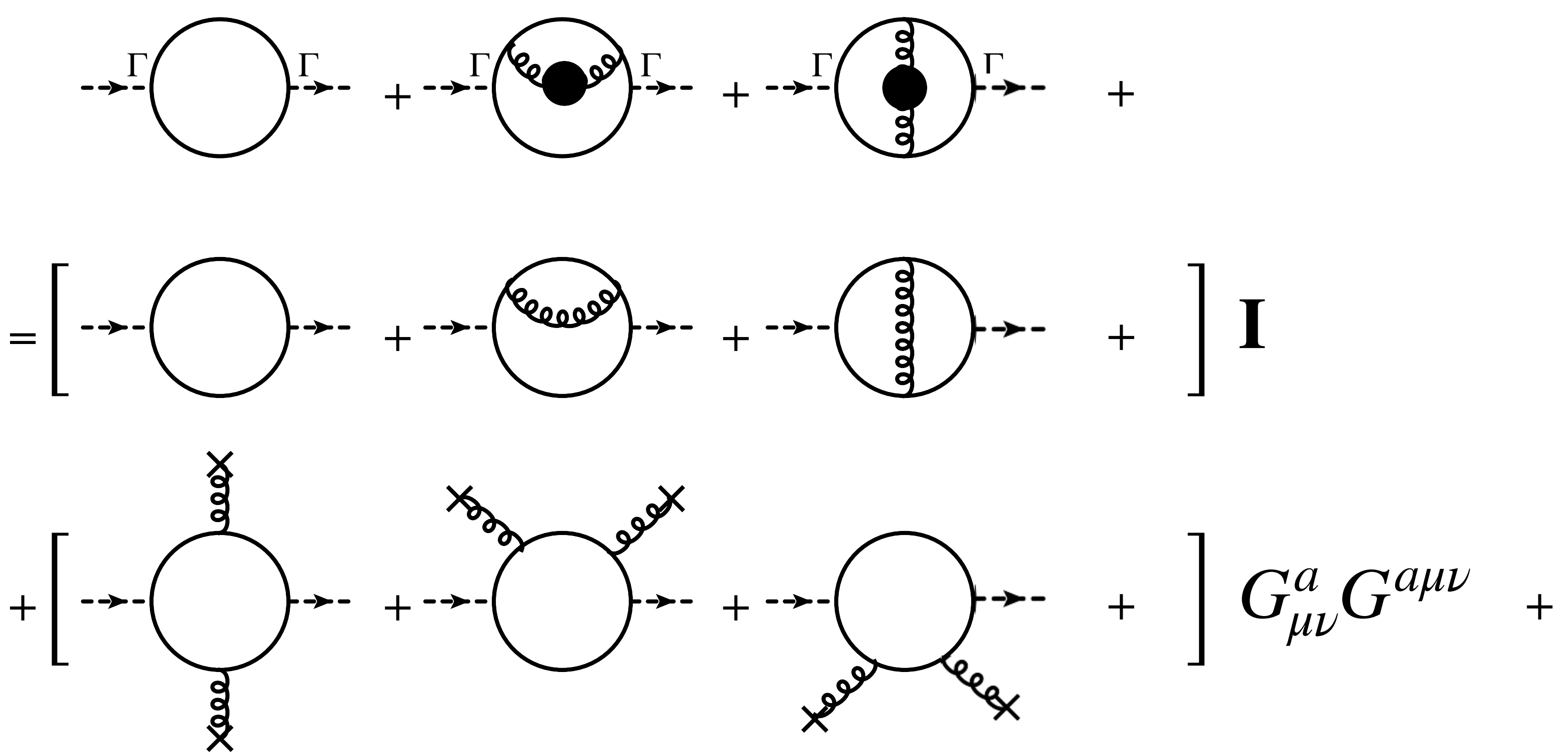
$$J_{2,1} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_5 u_j\right)\left(Q_k^T C\gamma_5 s_l\right)\gamma_5 C\bar{d}_c^T,$$
$$J_{2,2} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu u_j\right)\left(Q_k^T C\gamma_\mu s_l\right)\gamma_5 C\bar{d}_c^T,$$
$$J_{2,3} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu u_j\right)\left(Q_k^T C\gamma_5 s_l\right)\gamma_\mu C\bar{d}_c^T,$$
$$J_{2,4} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_5 u_j\right)\left(Q_k^T C\gamma_\mu s_l\right)\gamma_\mu C\bar{d}_c^T,$$
$$J_{2,5\mu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu u_j\right)\left(Q_k^T C\gamma_5 s_l\right)\gamma_5 C\bar{d}_c^T,$$
$$J_{2,6\mu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_5 u_j\right)\left(Q_k^T C\gamma_\mu s_l\right)\gamma_5 C\bar{d}_c^T,$$
$$J_{2,7\mu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_5 u_j\right)\left(Q_k^T C\gamma_5 s_l\right)\gamma_\mu C\bar{d}_c^T,$$
$$J_{2,8\mu\nu} = \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}\left(Q_i^T C\gamma_\mu u_j\right)\left(Q_k^T C\gamma_\nu s_l\right)\gamma_5 C\bar{d}_c^T + (\mu \leftrightarrow \nu),$$

QCD sum rule

- One of the most widely used methods to obtain the information about hadrons properties (Shifman,Vainshtein, Zakharov 1979)
- Based on the operator product expansion(OPE) of the correlator of interpolating currents

$$\Pi_{\mu\nu\ldots}(q^2) = \mathrm{i}\int \mathrm{d}^4x e^{iq\cdot x}\langle\Omega|T[J_\Gamma(x)J^\dagger_{\Gamma'}(0)]|\Omega\rangle = T_{\mu\nu\ldots}\Pi(q^2)$$

- Main point of the QCDSR philosophy is the implementation of the interaction of the high virtual valence quark-gluon system with the **soft vacuum quark and gluon fields**, whose strength is determined by the values of the vacuum condenstates.



Operator product expansion

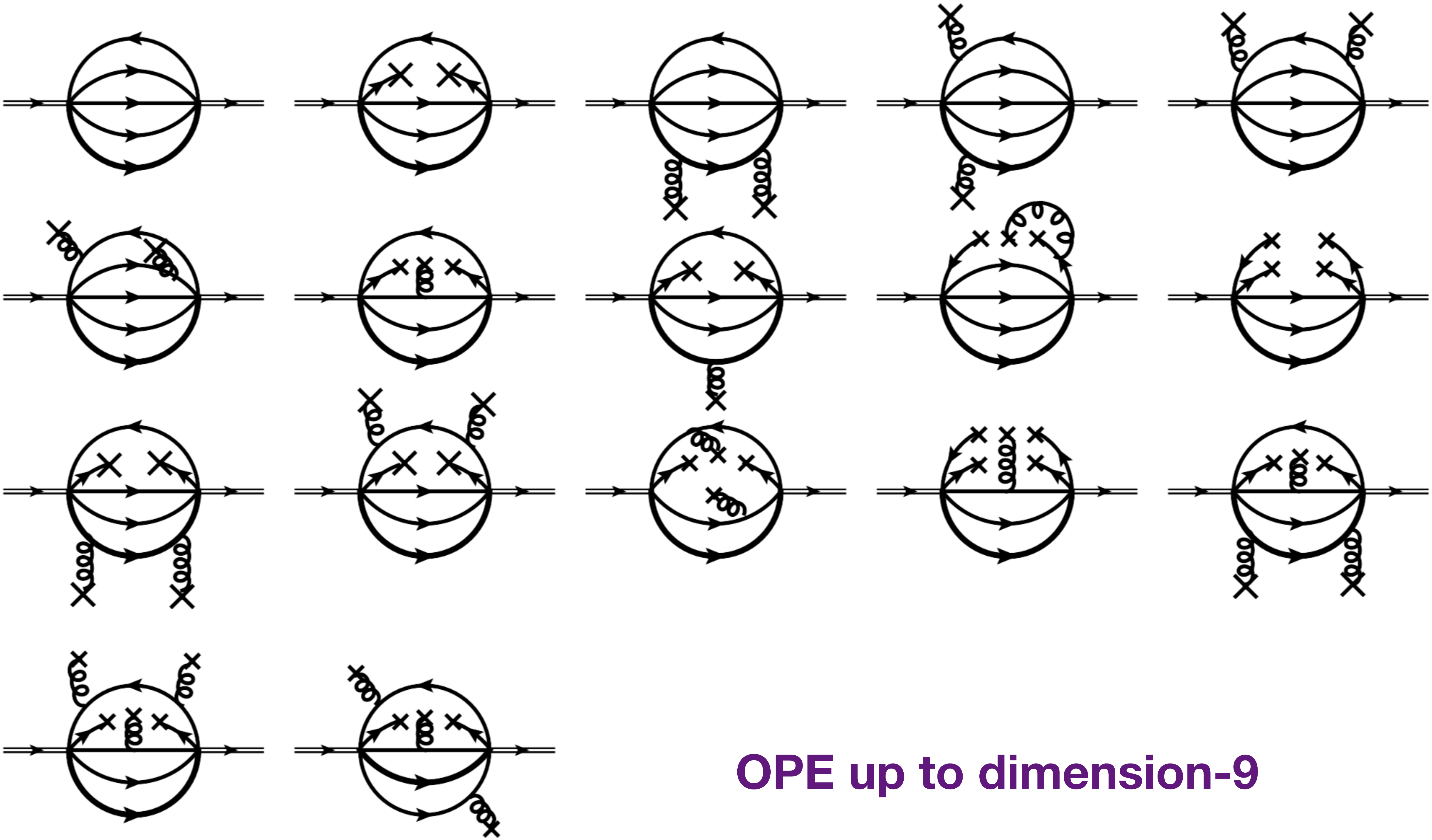
The correlation function can be perturbative evaluated via operator product expansion method (OPE) in timelike space $Q^2 = -q^2 \to \infty$. (Wilson 1969)

$$i\int d^4x e^{iq\cdot x}\langle\Omega|T[J_\Gamma(x)J_{\Gamma'}^\dagger(0)]|\Omega\rangle = \sum_d C_d(q^2)\langle\Omega|\mathcal{O}_d(0)|\Omega\rangle$$

$C_d(q^2)$ are Wilson coefficients, containing high q^2 QCD perturbative effects, local gauge invariant operators $\mathcal{O}_d(0)$ contain QCD non-perturbative effects.

$$\begin{aligned}
\mathcal{O}_3 &= :\bar{q}(0)q(0):\equiv \bar{q}q \\
\mathcal{O}_4 &= :g_s^2 G^n_{\alpha\beta}(0)G^n_{\alpha\beta}(0):\equiv g_s^2G^2 \\
\mathcal{O}_5 &= :\bar{q}(0)g_s\sigma^{\alpha\beta}\frac{\lambda^n}{2}G^n_{\alpha\beta}(0)q(0):\equiv \bar{q}\sigma Gq \\
\mathcal{O}_6^q &= :\bar{q}(0)q(0)\bar{q}(0)q(0):\equiv \bar{q}q\bar{q}q \\
\mathcal{O}_6^G &= :f_{ijk}g_s^3G^i_{\alpha\beta}(0)G^j_{\beta\gamma}(0)G^k_{\gamma\alpha}(0):\equiv g_s^3G^3 \\
&\vdots
\end{aligned}$$

Operator product expansion



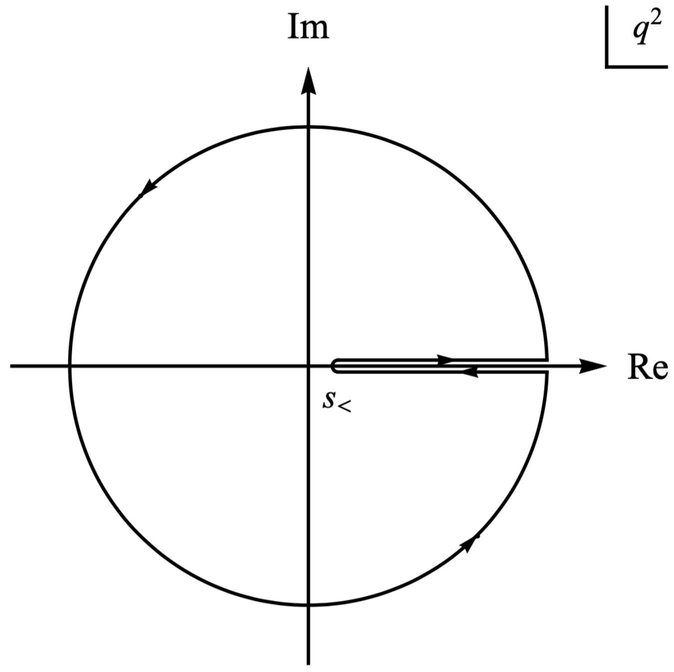
OPE up to dimension-9

Dispersion relation

Quark-Gluon level

We evaluate the correlation function in timelike space $Q^2 = -q^2 \to \infty$,

$$\Pi^{\text{OPE}}(q^2) = \sum_d C_d(q^2)\langle\Omega|\mathcal{O}_d(0)|\Omega\rangle$$



but we are more interested in physical region $q^2 > 0$

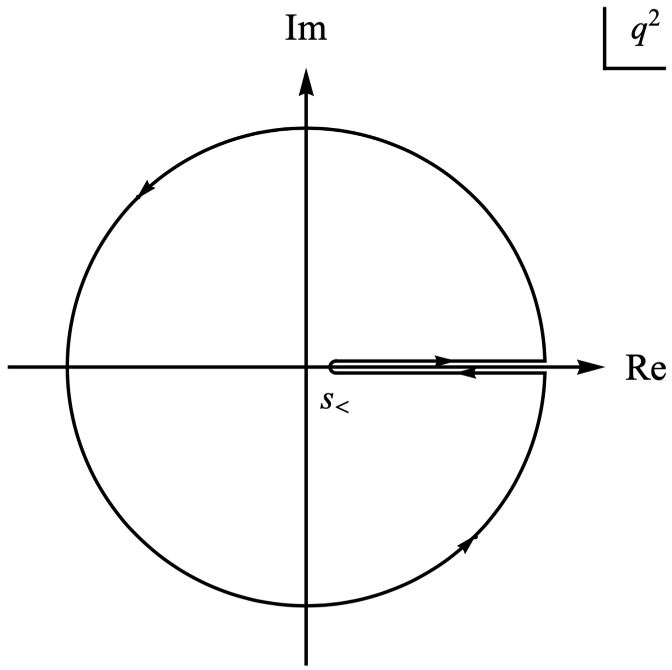
Hadronic level

the unitarity of $\mathcal{S}$ matrix $\to$ branch cut only on positive real axis

$$2\text{Im}\Pi(q^2) = \sum_n \langle\Omega|J_\Gamma(x)|n\rangle\langle n|J_\Gamma(0)|\Omega\rangle(2\pi)^4 \text{d}\Phi_n \delta^{(4)}(q - p_n)$$

Dispersion relation

To connect the CF at **two levels**, we consider the analyticity of CF in the q^2 complex plane



Cauchy's integral formula

$$\Pi(q^2) = \frac{1}{2\pi \mathrm{i}} \oint_C \mathrm{d}s \frac{\Pi(s)}{s - q^2}$$

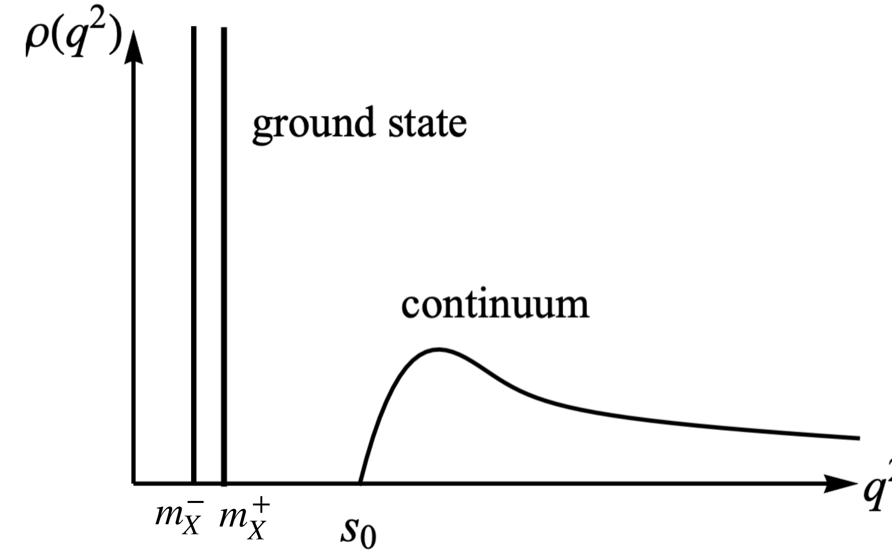
For $\lim_{R\to\infty} |\Pi(q^2)| \to 0$, the dispersion relation is

$$\Pi^{\mathrm{OPE}}(q^2) = \frac{1}{\pi} \int_{s_<}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

Ultraviolet divergence in CF, the dispersion relation is

$$\Pi(q^2) = \frac{(q^2)^N}{\pi} \int_{s_<}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s^N(s - q^2 - i\epsilon)} + \sum_{k=0}^{N-1} \frac{\Pi^{(k)}(0)}{k!}(q^2)^k$$

Dispersion relation



Optical theorem

$$\mathrm{Im}\Pi(q^2) \sim s\sigma(p_{\mathrm{in}} \to \text{anything})$$

We use narrow resonance approximation for

baryon system:(double pole+continuum)

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s) = f_X^{-2}(\not{p} + m_X^-)\delta(s - m_X^{-2}) + f_X^{+2}(\not{p} + m_X^+)\delta(s - m_X^{+2}) + \text{continuum}\,.$$

The sum rules for baryon system:

parity projected sum rules(Jido, Kodama, Oka 1996)

$$\mathscr{L}_k(s_0^+, M_B^2, +) \equiv \frac{1}{2}\int_{s_<}^{s_0^+} e^{-s/M_B^2}\left[\sqrt{s}\rho_A^{\mathrm{OPE}}(s) + \rho_B^{\mathrm{OPE}}(s)\right] s^k ds = \lambda_+^2 m_+^{2k+1}\exp\left[-\frac{m_+^2}{M_B^2}\right]$$

$$\mathscr{L}_k(s_0^-, M_B^2, -) \equiv \frac{1}{2}\int_{s_<}^{s_0^-} e^{-s/M_B^2}\left[\sqrt{s}\rho_A^{\mathrm{OPE}}(s) - \rho_B^{\mathrm{OPE}}(s)\right] s^k ds = \lambda_-^2 m_-^{2k+1}\exp\left[-\frac{m_-^2}{M_B^2}\right]$$

The extracted mass: $$m_\pm(s_0^\pm, M_B) = \sqrt{\frac{\mathscr{L}_1(s_0^\pm, M_B^2, \pm)}{\mathscr{L}_0(s_0^\pm, M_B^2, \pm)}}$$

Numerical Analysis

Values for various Condensates

Standard value at 1 GeV

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3$$

$$\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$$

$$\langle g_s^2 GG \rangle = (0.48 \pm 0.14)^4 \text{GeV}^4$$

$$\langle g_s \bar{q}\sigma \cdot Gq \rangle = -M_0^2 \langle \bar{q}q \rangle$$

$$M_0^2 = (0.8 \pm 0.2) \text{GeV}^2$$

Energy-scale from RGE

$$m_s(\mu) = m_s(2\ \text{GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2\ \text{GeV})}\right]^{\frac{12}{33-2n_f}},$$

$$m_c(\mu) = m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}},$$

$$m_b(m_b) = m_b(m_b) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{33-2n_f}},$$

$$\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1\ \text{GeV}) \left[\frac{\alpha_s(1\ \text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},$$

$$\langle \bar{s}s \rangle(\mu) = \langle \bar{s}s \rangle(1\ \text{GeV}) \left[\frac{\alpha_s(1\ \text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},$$

$$\langle \bar{q}g_s\sigma \cdot Gq \rangle(\mu) = \langle \bar{q}g_s\sigma \cdot Gq \rangle(1\ \text{GeV}) \left[\frac{\alpha_s(1\ \text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},$$

$$\langle \bar{s}g_s\sigma \cdot Gs \rangle(\mu) = \langle \bar{s}g_s\sigma \cdot Gs \rangle(1\ \text{GeV}) \left[\frac{\alpha_s(1\ \text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},$$

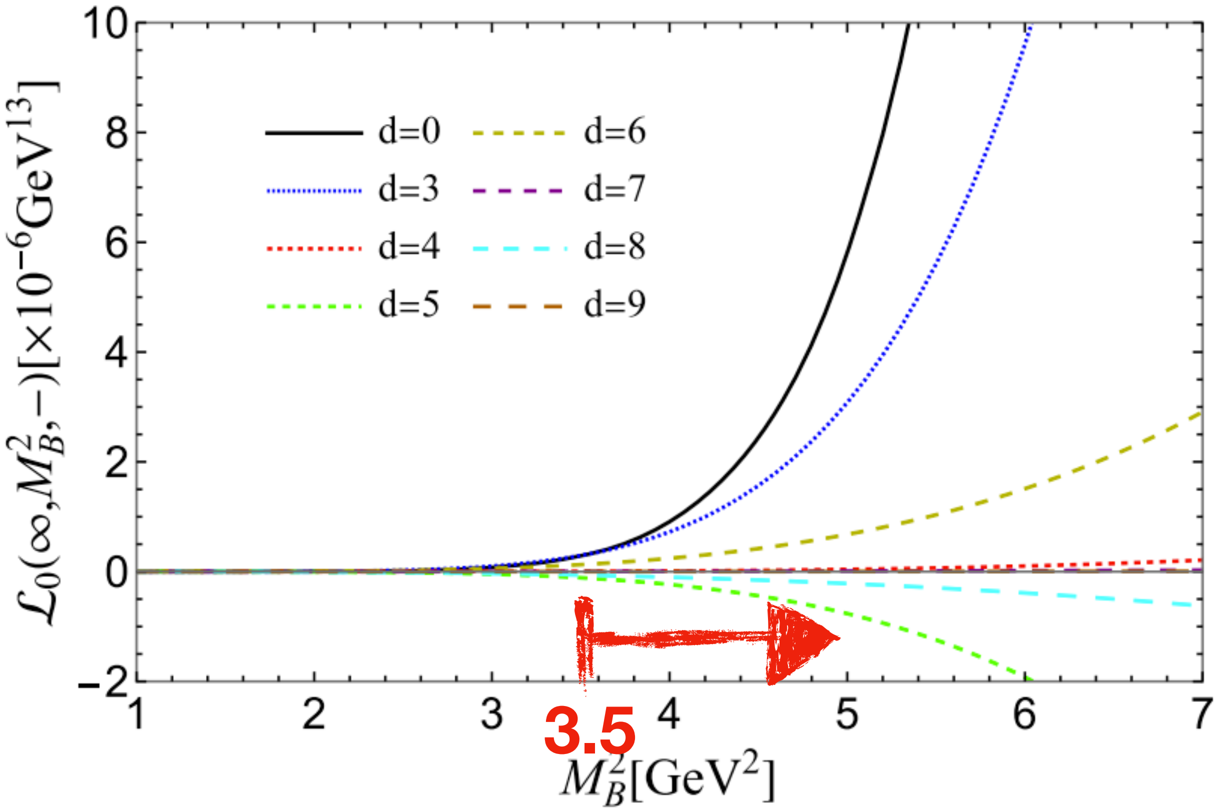
$$\alpha_s(\mu) = \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right],$$

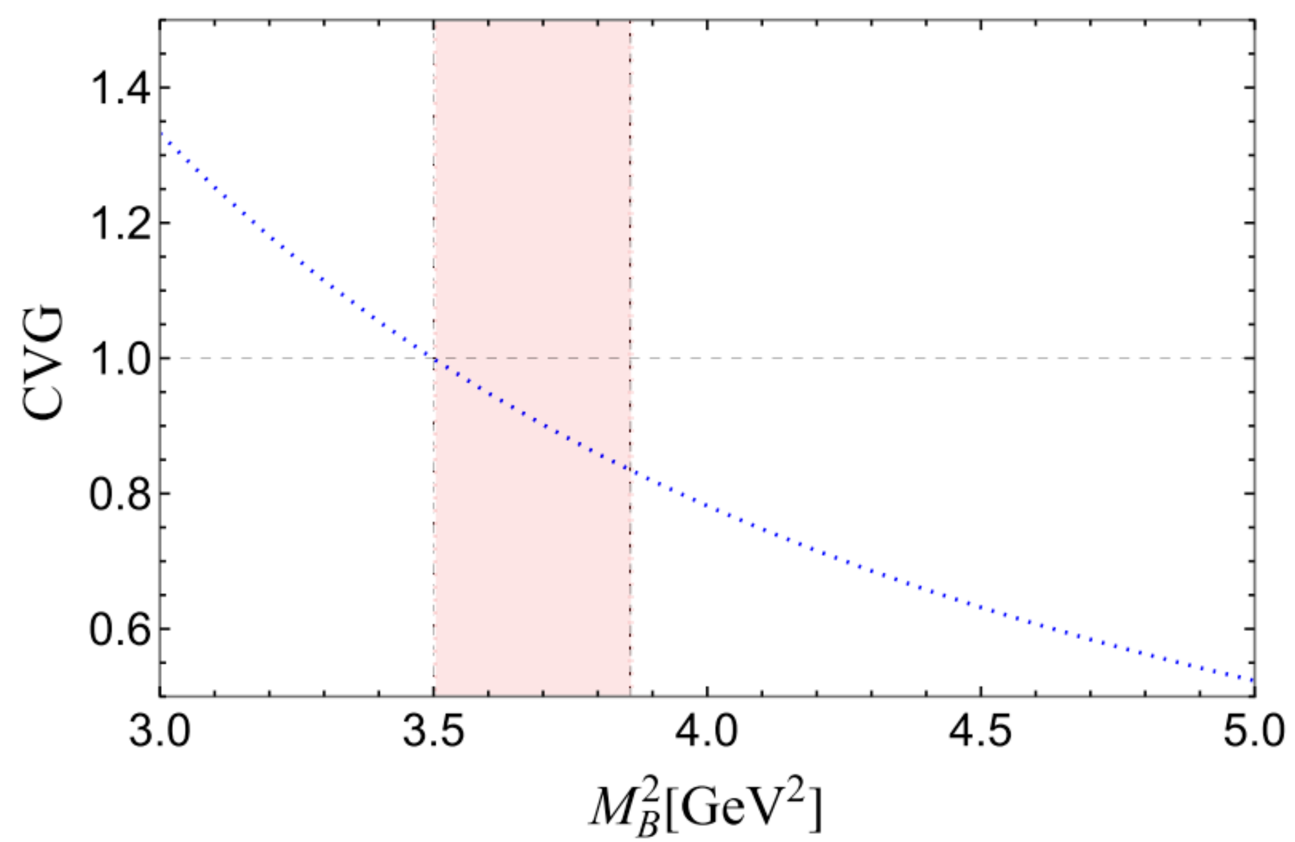
We evolve all input at $\mu = 2m_Q$

Numerical Analysis

As an example: $J_{1,2}$ with $J^P = 1/2^-$

The contributions of the perturbative term and various condensate terms to the correlation function with respect to M_B^2 when $s_0 \to \infty$

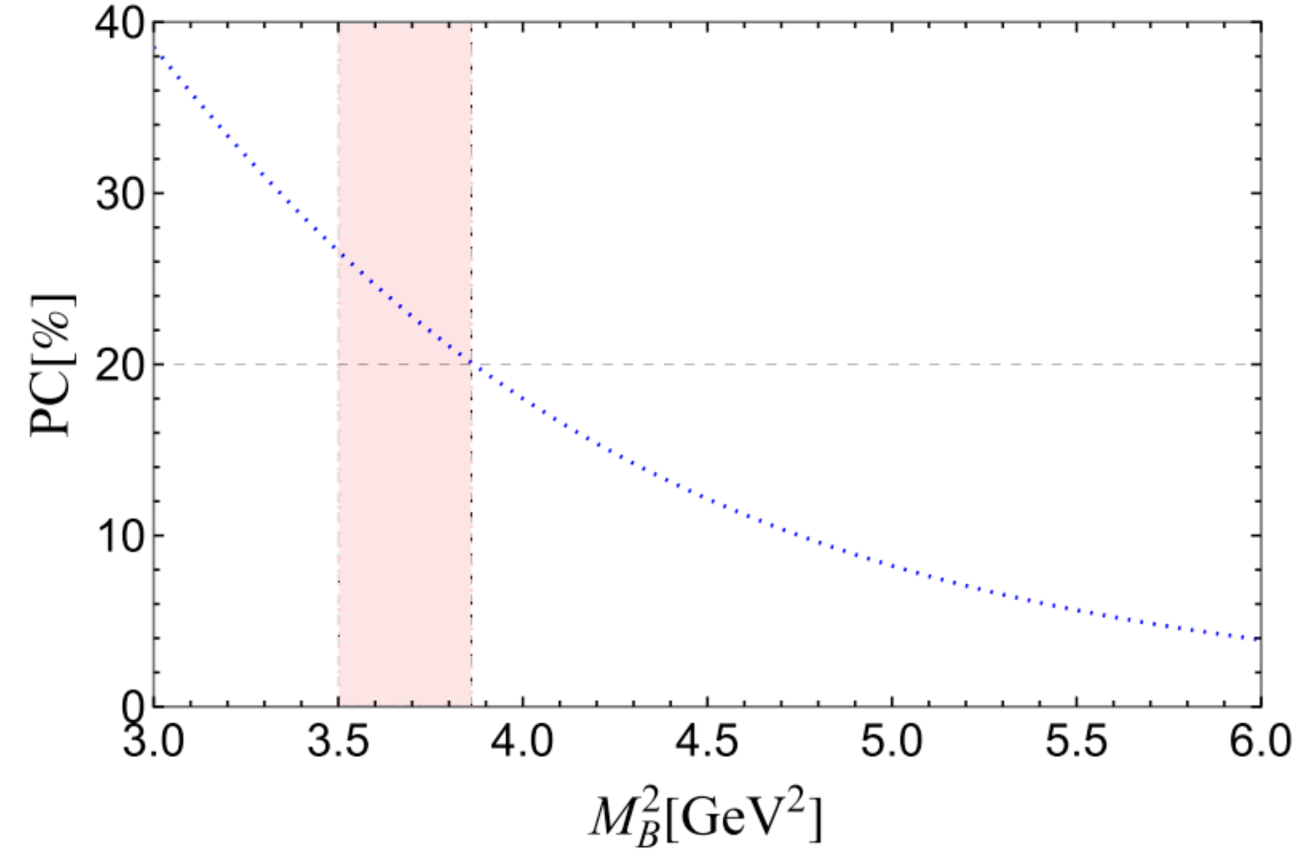


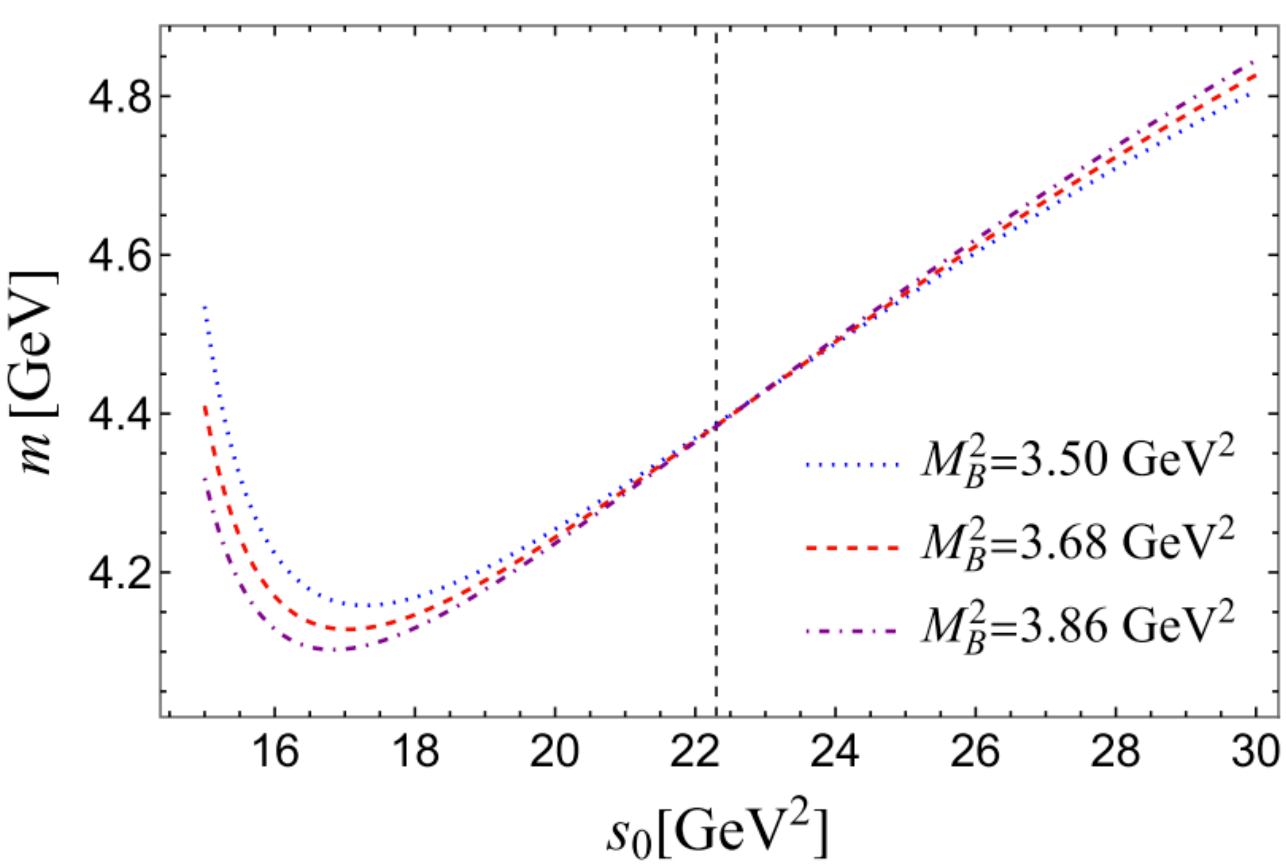


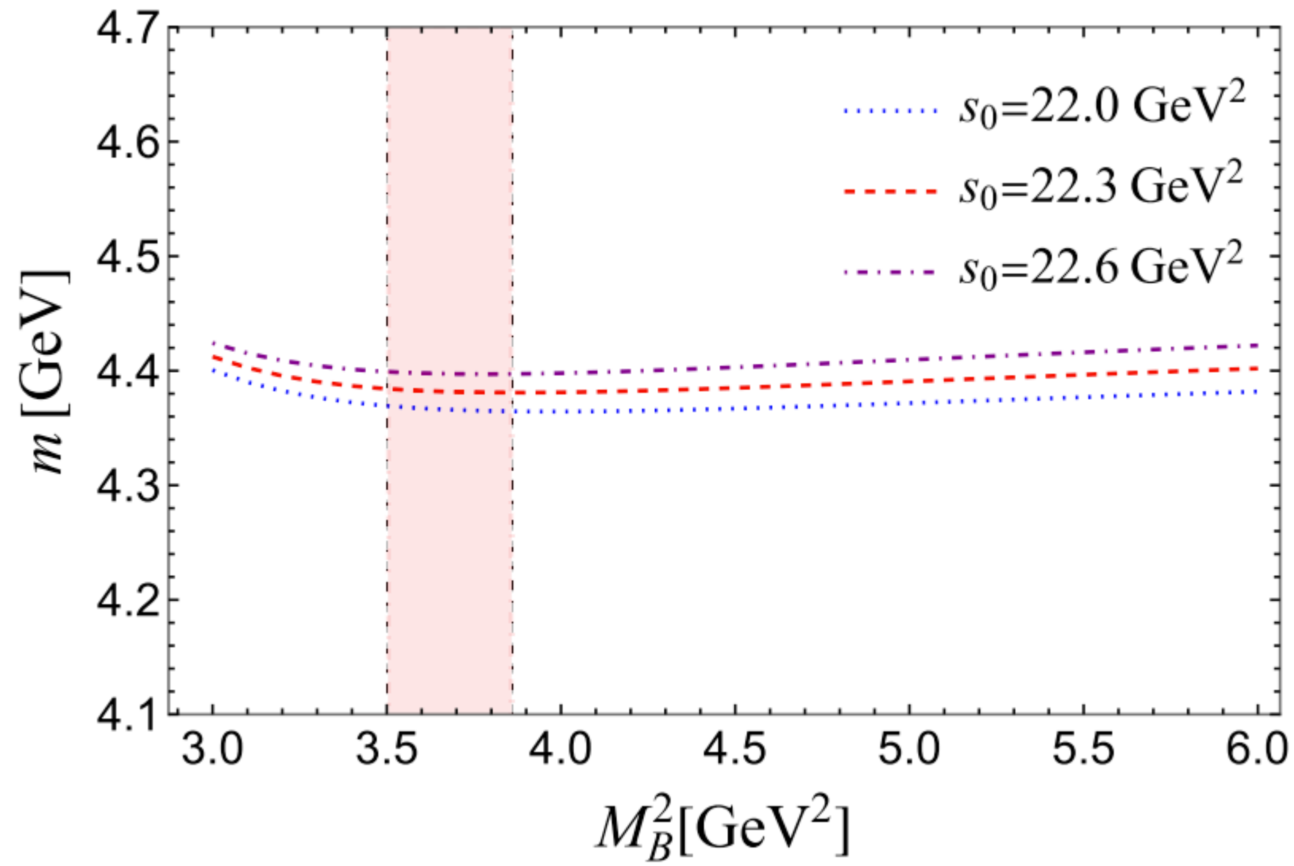
To ensure the convergence of OPE, we require the contribution of the perturbative term is larger than the quark condensate term

Numerical Analysis

We also require that the pole contribution(PC) to be larger than 20%, thus our working Borel Window is 3.50~3.86 GeV^2

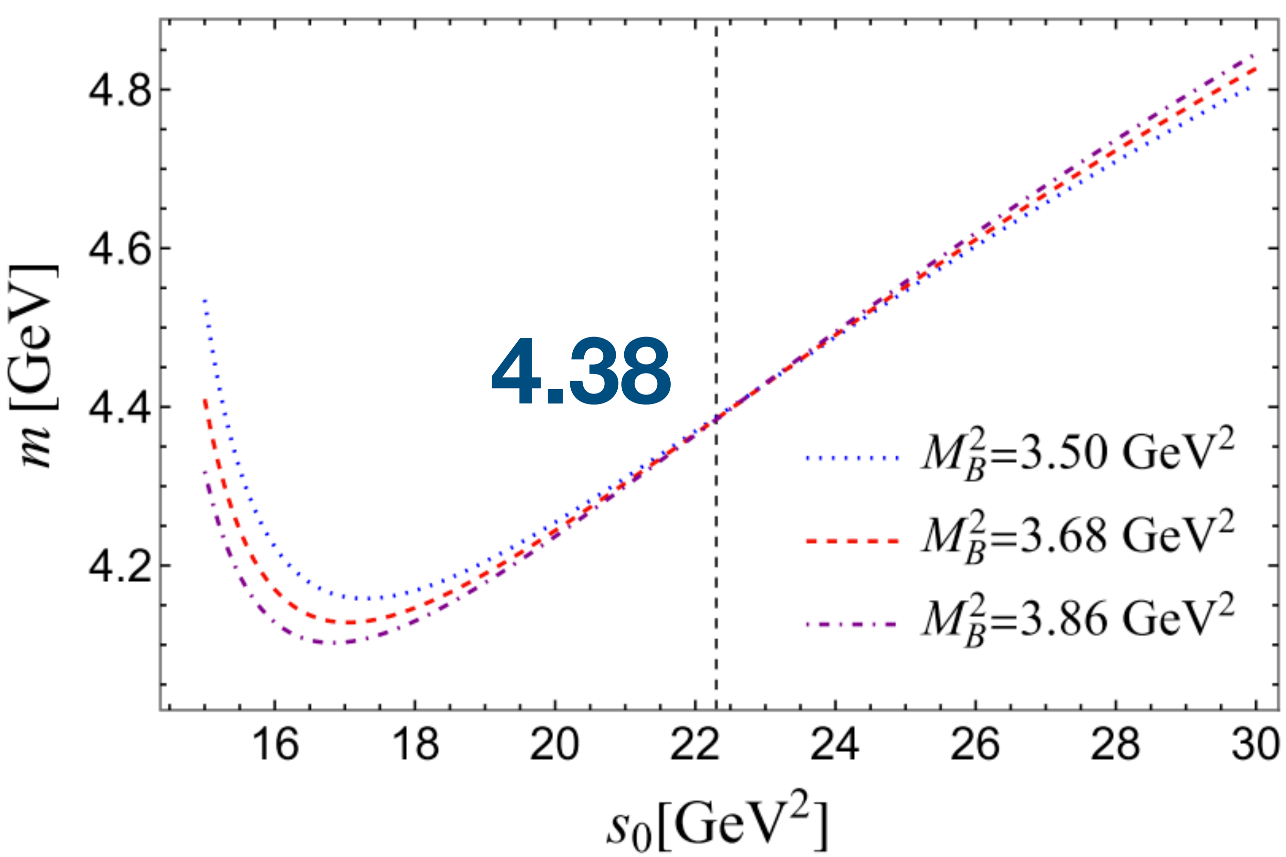


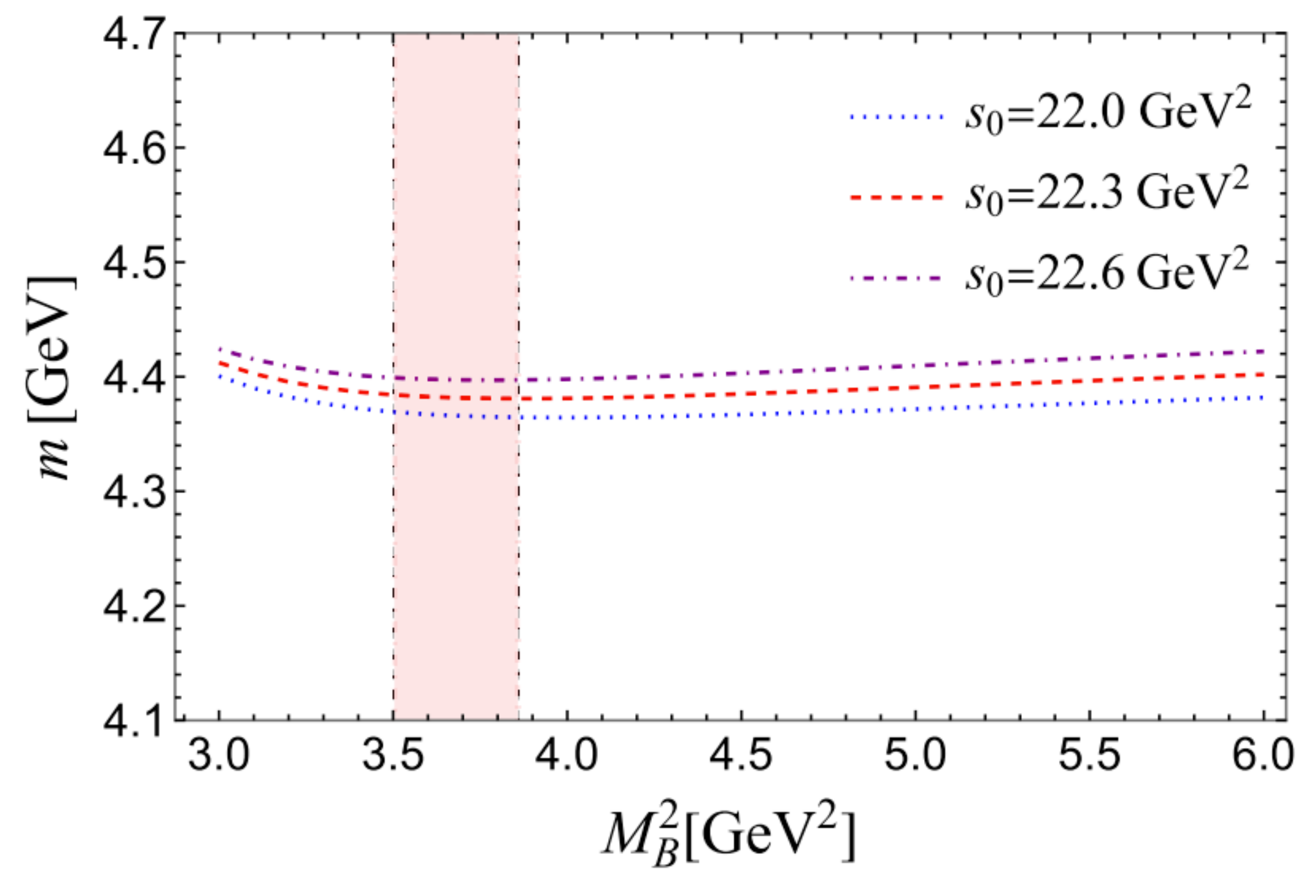




Numerical Analysis

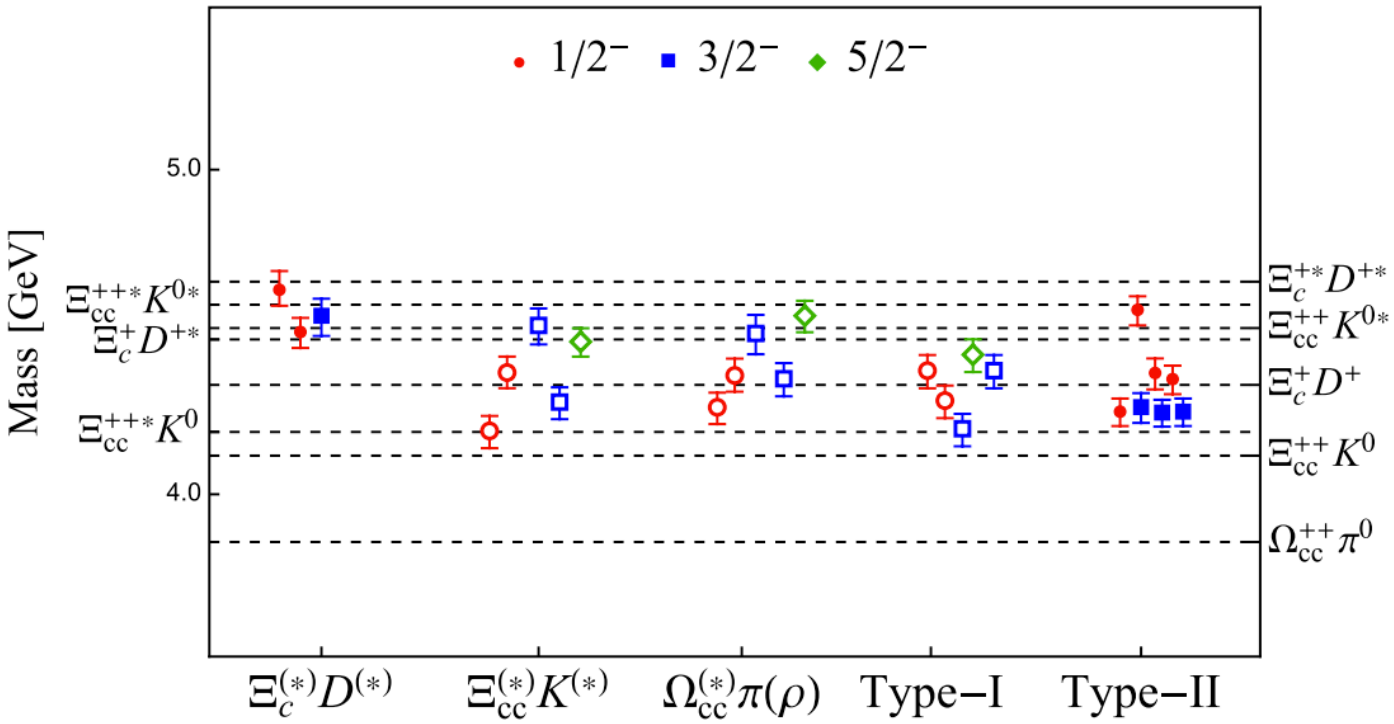
- The variation of the mass with respect to M_B^2 should be minimized to obtain the optimal value of the continuum threshold s_0
- We can find that the optimized value can be chosen as $s_0 \approx 22.3\mathrm{GeV}^2$
- Our mass sum rules are established to be very stable in the above parameter regions





Discussion and Conclusion

Mass spectra for strange double charm pentaquark

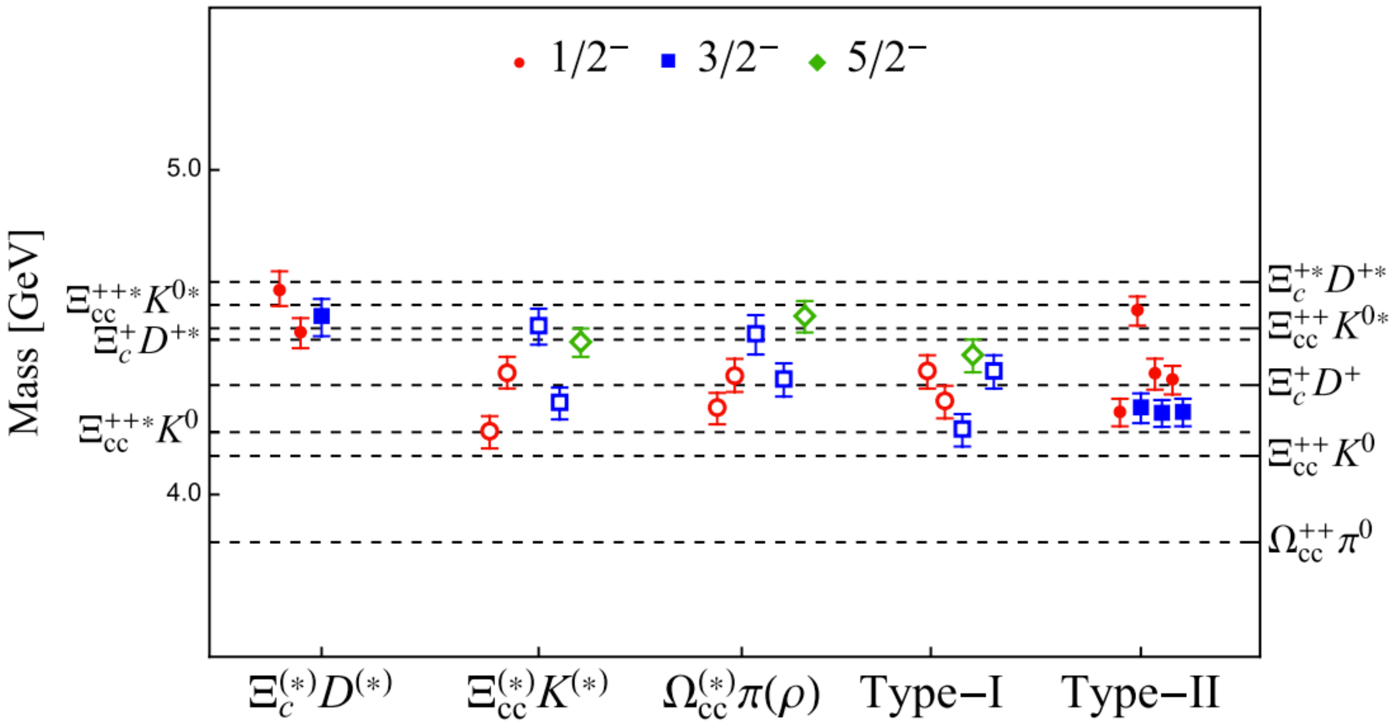


Current	Structure	J^P	$M_B^2(\mathrm{GeV}^2)$	$s_0(\mathrm{GeV}^2)$	Mass (GeV)
$J_{1,2}$	$[cc]_1[us]_1\bar{d}$	$\frac{1}{2}^-$	3.50–3.86	22.3	$4.38^{+0.04}_{-0.05}$
$J_{1,3}$	$[cc]_1[us]_0\bar{d}$	$\frac{1}{2}^-$	3.14–4.22	21.3	$4.29^{+0.05}_{-0.05}$
$J_{1,5\mu}$	$[cc]_1[us]_0\bar{d}$	$\frac{3}{2}^-$	3.50–3.82	21.3	$4.27^{+0.05}_{-0.05}$
$J_{1,8\mu}$	$[cc]_1[us]_1\bar{d}$	$\frac{3}{2}^-$	3.50–4.10	22.3	$4.38^{+0.05}_{-0.05}$
$J_{1,9\mu\nu}$	$[cc]_1[us]_1\bar{d}$	$\frac{5}{2}^-$	3.46–4.25	23.3	$4.43^{+0.05}_{-0.05}$
$J_{2,1}$	$[cu]_0[cs]_0\bar{d}$	$\frac{1}{2}^-$	3.00–4.05	21.3	$4.25^{+0.04}_{-0.04}$
$J_{2,2}$	$[cu]_1[cs]_1\bar{d}$	$\frac{1}{2}^-$	3.00–5.08	24.3	$4.57^{+0.05}_{-0.04}$
$J_{2,3}$	$[cu]_1[cs]_0\bar{d}$	$\frac{1}{2}^-$	3.78–4.28	23.3	$4.44^{+0.05}_{-0.04}$
$J_{2,4}$	$[cu]_0[cs]_1\bar{d}$	$\frac{1}{2}^-$	4.00–4.11	22.3	$4.35^{+0.05}_{-0.04}$
$J_{2,5\mu}$	$[cu]_1[cs]_0\bar{d}$	$\frac{3}{2}^-$	3.00–4.03	21.3	$4.27^{+0.05}_{-0.04}$
$J_{2,6\mu}$	$[cu]_0[cs]_1\bar{d}$	$\frac{3}{2}^-$	3.00–4.31	22.3	$4.31^{+0.04}_{-0.04}$
$J_{2,7\mu}$	$[cu]_0[cs]_0\bar{d}$	$\frac{3}{2}^-$	3.00–4.05	21.3	$4.25^{+0.04}_{-0.04}$

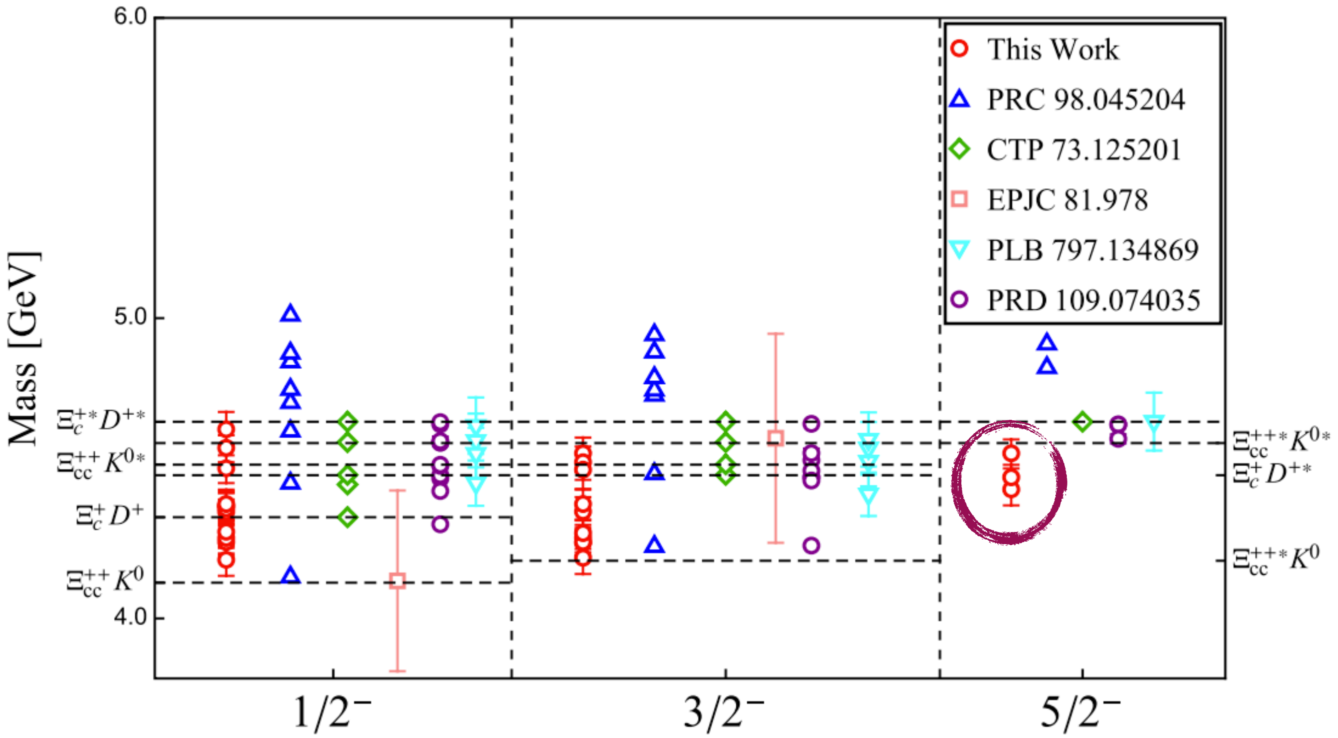
Current	Structure	J^P	$M_B^2(\mathrm{GeV}^2)$	$s_0(\mathrm{GeV}^2)$	Mass (GeV)	Threshold (MeV)
η_3	$\Xi_c^+D^{*+}$	$\frac{1}{2}^-$	3.20–4.38	24.3	$4.50^{+0.05}_{-0.04}$	4477
$\eta_{4\mu}$	$\Xi_c^{\prime+}D^{*+}$	$\frac{3}{2}^-$	3.30–4.25	24.3	$4.55^{+0.05}_{-0.04}$	4588
η_6	$\Xi_c^{*+}D^{*+}$	$\frac{1}{2}^-$	3.00–4.75	24.3	$4.63^{+0.06}_{-0.05}$	4655
ξ_1	$\Xi_{cc}^{++}\bar{K}^0$	$\frac{1}{2}^-$	3.16–4.05	20.3	$4.20^{+0.05}_{-0.05}$	4120
$\xi_{2\mu}$	$\Xi_{cc}^{++}\bar{K}^{*0}$	$\frac{3}{2}^-$	3.12–4.23	24.3	$4.52^{+0.06}_{-0.05}$	4512
$\xi_{3\mu}$	$\Xi_{cc}^{*++}\bar{K}^0$	$\frac{3}{2}^-$	3.38–4.23	21.3	$4.28^{+0.05}_{-0.05}$	4192
ξ_4	$\Xi_{cc}^{*++}\bar{K}^{*0}$	$\frac{1}{2}^-$	3.00–3.76	22.3	$4.37^{+0.05}_{-0.05}$	4584
$\xi_{5\mu\nu}$	$\Xi_{cc}^{*++}\bar{K}^{*0}$	$\frac{5}{2}^-$	3.00–4.45	24.3	$4.47^{+0.05}_{-0.04}$	4584
ψ_1	$\Omega_{cc}^+\pi^+$	$\frac{1}{2}^-$	3.18–4.25	21.3	$4.27^{+0.05}_{-0.05}$	3853
$\psi_{2\mu}$	$\Omega_{cc}^+\rho^+$	$\frac{3}{2}^-$	3.16–3.63	24.3	$4.50^{+0.06}_{-0.06}$	4488
$\psi_{3\mu}$	$\Omega_{cc}^{*+}\pi^+$	$\frac{3}{2}^-$	3.04–4.41	22.3	$4.36^{+0.05}_{-0.05}$	3925
ψ_4	$\Omega_{cc}^{*+}\rho^+$	$\frac{1}{2}^-$	3.00–3.77	22.3	$4.37^{+0.05}_{-0.05}$	4560
$\psi_{5\mu\nu}$	$\Omega_{cc}^{*+}\rho^+$	$\frac{5}{2}^-$	3.20–4.63	25.3	$4.55^{+0.05}_{-0.05}$	4560

Discussion and Conclusion

Mass spectra for strange double charm pentaquark



Compare with other works



- $\Xi_c^{(\prime)}D^*, \Xi_c^*D^*, \Xi_{cc}^*\bar{K}^*, \Omega_{cc}^*\rho$ may form as bound molecular states
- $J^P = 5/2^-$ state lies below the thresholds of strong decay channels, could be a narrow state and easy to be identified in experiment.
- The best observed channel is the semileptonic decay to double charm baryon.

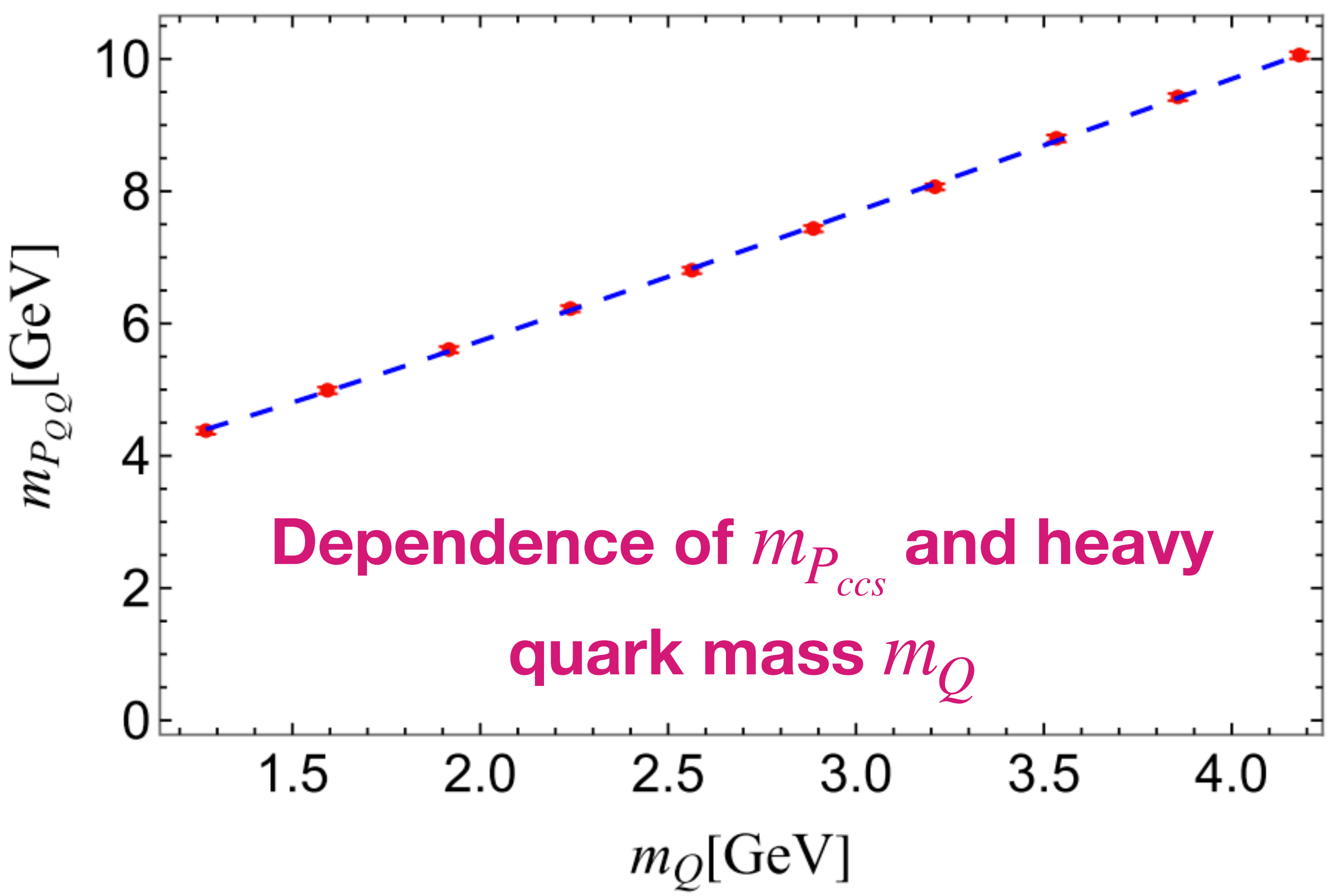
Discussion and Conclusion

- Furthermore, we consider the dependence of the mass on the heavy quark mass by varying the heavy quark mass to perform QCDSR analysis.

Heavy Quark Spin Symmetry

$$m_{P_{cc}} = 2m_Q + \bar{\Lambda} + \frac{\Delta m^2}{4m_Q} + O(1/m_Q^2) \xrightarrow{\text{fit with}} m_{P_{cc}} = 2m_Q + b + \frac{c}{m_Q}$$

- We choose ten testing point from m_c to m_b.
- The results from QCDSR fit well with expectation from HQSS.



Discussion and Conclusion

Heavy Quark Spin Symmetry

$$m_{P_{cc}} = 2m_Q + \bar{\Lambda} + \frac{\Delta m^2}{4m_Q} + O(1/m_Q^2) \xrightarrow[2m_Q \to m_Q]{\text{HADS}} m_{T_c} = m_Q + \bar{\Lambda} + \frac{\Delta m^2}{2m_Q} + O(1/m_Q^2)$$

Current	J^P	Structure	b(GeV)	c (GeV^2)	$m_{P_{bbs}}$(GeV)	$m_{T_{c\bar{s}}}$(GeV)	$J^P(T_{c\bar{s}})$
η_3	$\frac{1}{2}^-$	$\Xi_c^+ D^{*+}$	1.65	0.40	$10.16^{+0.06}_{-0.05}$	–	–
$\eta_{4\mu}$	$\frac{3}{2}^-$	$\Xi_c'^+ D^{*+}$	1.79	0.25	$10.31^{+0.06}_{-0.05}$	–	–
η_6	$\frac{1}{2}^-$	$\Xi_c^{*+} D^{*+}$	2.33	−0.35	$10.68^{+0.06}_{-0.05}$	–	–
ξ_1	$\frac{1}{2}^-$	$\Xi_{cc}^{++} \bar{K}^0$	1.48	0.24	$9.93^{0.06}_{0.05}$	2.94	0^+
$\xi_{2\mu}$	$\frac{3}{2}^-$	$\Xi_{cc}^{++} \bar{K}^{*0}$	1.64	0.44	$10.17^{0.06}_{0.06}$	3.26	1^+
$\xi_{3\mu}$	$\frac{3}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^0$	1.46	0.38	$9.96^{+0.06}_{-0.05}$	3.03	1^+
ξ_4	$\frac{1}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	1.04	1.07	$9.59^{+0.07}_{-0.07}$	3.16	0^+
$\xi_{5\mu\nu}$	$\frac{5}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	1.63	0.38	$9.99^{+0.06}_{-0.05}$	3.20	$0^+, 1^+, 2^+$
ψ_1	$\frac{1}{2}^-$	$\Omega_{cc}^{*+} \pi^+$	1.48	0.30	$9.96^{+0.06}_{-0.05}$	2.99	0^+
$\psi_{2\mu}$	$\frac{3}{2}^-$	$\Omega_{cc}^{+} \rho^+$	1.57	0.51	$10.04^{+0.06}_{-0.06}$	3.25	1^+
$\psi_{3\mu}$	$\frac{3}{2}^-$	$\Omega_{cc}^{*+} \pi^+$	1.45	0.47	$9.96^{+0.06}_{-0.05}$	3.09	1^+
ψ_4	$\frac{1}{2}^-$	$\Omega_{cc}^{*+} \rho^+$	1.16	0.88	$9.71^{+0.06}_{-0.06}$	3.12	0^+
$\psi_{5\mu\nu}$	$\frac{5}{2}^-$	$\Omega_{cc}^{*+} \rho^+$	1.80	0.25	$10.14^{+0.07}_{-0.06}$	3.27	$0^+, 1^+, 2^+$

Current	J^P	Structure	b (GeV)	c (GeV^2)	$m_{P_{bbs}}$ (GeV)	$m_{T_{c\bar{s}}}$ (GeV)	$J^P(T_{c\bar{s}})$
$J_{1,2}$	$\frac{1}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.61	0.30	$10.06^{+0.06}_{-0.05}$	3.11	$0^+, 1^+$
$J_{1,3}$	$\frac{1}{2}^-$	$[cc]_1[us]_0\bar{d}$	1.52	0.29	$10.00^{+0.06}_{-0.05}$	3.02	0^+
$J_{1,5\mu}$	$\frac{3}{2}^-$	$[cc]_1[us]_0\bar{d}$	1.53	0.17	$9.97^{+0.06}_{-0.05}$	2.94	1^+
$J_{1,8\mu}$	$\frac{3}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.61	0.30	$10.06^{+0.06}_{-0.05}$	3.11	1^+
$J_{1,9\mu\nu}$	$\frac{5}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.66	0.29	$10.10^{+0.06}_{-0.06}$	3.15	2^+
$J_{2,1}$	$\frac{1}{2}^-$	$[cu]_0[cs]_0\bar{d}$	1.79	−0.16	$10.21^{+0.07}_{-0.06}$	...	...
$J_{2,2}$	$\frac{1}{2}^-$	$[cu]_1[cs]_1\bar{d}$	1.87	0.14	$10.33^{+0.07}_{-0.06}$	...	...
$J_{2,3}$	$\frac{1}{2}^-$	$[cu]_1[cs]_0\bar{d}$	1.74	0.07	$10.18^{+0.07}_{-0.06}$	...	...
$J_{2,4}$	$\frac{1}{2}^-$	$[cu]_0[cs]_1\bar{d}$	1.72	0.08	$10.17^{+0.06}_{-0.06}$	...	...
$J_{2,5\mu}$	$\frac{3}{2}^-$	$[cu]_1[cs]_0\bar{d}$	1.65	0.07	$10.08^{+0.06}_{-0.06}$	...	...
$J_{2,6\mu}$	$\frac{3}{2}^-$	$[cu]_0[cs]_1\bar{d}$	1.61	0.08	$10.08^{+0.06}_{-0.06}$	...	...
$J_{2,7\mu}$	$\frac{3}{2}^-$	$[cu]_0[cs]_0\bar{d}$	1.79	−0.16	$10.21^{+0.07}_{-0.06}$	...	...

Discussion and Conclusion

Heavy Quark Spin Symmetry

$$m_{P_{cc}} = 2m_Q + \bar{\Lambda} + \frac{\Delta m^2}{4m_Q} + O(1/m_Q^2) \quad \xrightarrow{\text{HADS}} \quad m_{T_c} = m_Q + \bar{\Lambda} + \frac{\Delta m^2}{2m_Q} + O(1/m_Q^2)$$

HADS partner doublet for Tcs(2900)

Current	J^P	Structure	b(GeV)	c (GeV^2)	$m_{P_{bbs}}$(GeV)	$m_{T_{c\bar{s}}}$(GeV)	$J^P(T_{c\bar{s}})$
$\eta_{[illegible]}$	$\frac{1}{2}^-$	$\Xi^+ D^{*+}$	1.65	0.40	$10.16^{+0.06}$	–	–
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	–
η_6	$\frac{1}{2}^-$	$\Xi_c^{*+} D^{*+}$	2.33	−0.35	$10.68^{+0.06}_{-0.05}$	–	–
ξ_1	$\frac{1}{2}^-$	$\Xi_{cc}^{++} \bar{K}^0$	1.48	0.24	$9.93^{0.06}_{0.05}$	2.94	0^+
$\xi_{2\mu}$	$\frac{3}{2}^-$	$\Xi_{cc}^{++} \bar{K}^{*0}$	1.64	0.44	$10.17^{0.06}_{0.06}$	3.26	1^+
$\xi_{3\mu}$	$\frac{3}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^0$	1.46	0.38	$9.96^{+0.06}_{-0.05}$	3.03	1^+
ξ_4	$\frac{1}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	1.04	1.07	$9.59^{+0.07}_{-0.07}$	3.16	0^+
$\xi_{5\mu\nu}$	$\frac{5}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	1.63	0.38	$9.99^{+0.06}_{-0.05}$	3.20	$0^+, 1^+, 2^+$
ψ_1	$\frac{1}{2}^-$	$\Omega_{cc}^{*+} \pi^+$	1.48	0.30	$9.96^{+0.06}_{-0.05}$	2.99	0^+
$\psi_{2\mu}$	$\frac{3}{2}^-$	$\Omega_{cc}^{+} \rho^+$	1.57	0.51	$10.04^{+0.06}_{-0.06}$	3.25	1^+
$\psi_{3\mu}$	$\frac{3}{2}^-$	$\Omega_{cc}^{*+} \pi^+$	1.45	0.47	$9.96^{+0.06}_{-0.05}$	3.09	1^+
ψ_4	$\frac{1}{2}^-$	$\Omega_{cc}^{*+} \rho^+$	1.16	0.88	$9.71^{+0.06}_{-0.06}$	3.12	0^+
$\psi_{5\mu\nu}$	$\frac{5}{2}^-$	$\Omega_{cc}^{*+} \rho^+$	1.80	0.25	$10.14^{+0.07}_{-0.06}$	3.27	$0^+, 1^+, 2^+$

Current	J^P	Structure	b (GeV)	c (GeV^2)	$m_{P_{bbs}}$ (GeV)	$m_{T_{c\bar{s}}}$ (GeV)	$J^P(T_{c\bar{s}})$
$J_{1,2}$	$\frac{1}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.61	0.30	$10.06^{+0.06}_{-0.05}$	3.11	$0^+, 1^+$
$J_{1,3}$	$\frac{1}{2}^-$	$[cc]_1[us]_0\bar{d}$	1.52	0.29	$10.00^{+0.06}_{-0.05}$	3.02	0^+
$J_{1,5\mu}$	$\frac{3}{2}^-$	$[cc]_1[us]_0\bar{d}$	1.53	0.17	$9.97^{+0.06}_{-0.05}$	2.94	1^+
$J_{1,8\mu}$	$\frac{3}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.61	0.30	$10.06^{+0.06}_{-0.05}$	3.11	1^+
$J_{1,9\mu\nu}$	$\frac{5}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.66	0.29	$10.10^{+0.06}_{-0.06}$	3.15	2^+
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	…
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	…
$J_{2,3}$	$\frac{1}{2}^-$	$[cu]_1[cs]_0\bar{d}$	1.74	0.07	$10.18^{+0.07}_{-0.06}$	…	…
$J_{2,4}$	$\frac{1}{2}^-$	$[cu]_0[cs]_1\bar{d}$	1.72	0.08	$10.17^{+0.06}_{-0.06}$	…	…
$J_{2,5\mu}$	$\frac{3}{2}^-$	$[cu]_1[cs]_0\bar{d}$	1.65	0.07	$10.08^{+0.06}_{-0.06}$	…	…
$J_{2,6\mu}$	$\frac{3}{2}^-$	$[cu]_0[cs]_1\bar{d}$	1.61	0.08	$10.08^{+0.06}_{-0.06}$	…	…
$J_{2,7\mu}$	$\frac{3}{2}^-$	$[cu]_0[cs]_0\bar{d}$	1.79	−0.16	$10.21^{+0.07}_{-0.06}$	…	…

HADS partner triplet for tetraquarks with mass about 3.1 GeV

Summary

- Motivated by the observation of $T_{c\bar{s}}(2900)^0$, we study the mass spectrum of its HADS counter parts, strange double charm pentaquarks.
- The masses for strange double charm pentaquarks with $J^P = 1/2^-, 3/2^-, 5/2^-$ are within the energy region 4.2-4.6 GeV, 4.2-4.5 GeV and 4.4-4.5 GeV, respectively.
- The $5/2^-$ states are below the threshold of its two-hadron strong decay channel and can be viewed as a narrow state. The best observed channel is the semileptonic decay to double charm baryon.

Thanks for your attention!

Backup

Operator product expansion

The correlation function can be perturbative evaluated via operator product expansion method **(OPE) in timelike space $Q^2 = -q^2 \to \infty$. (Wilson 1969)**

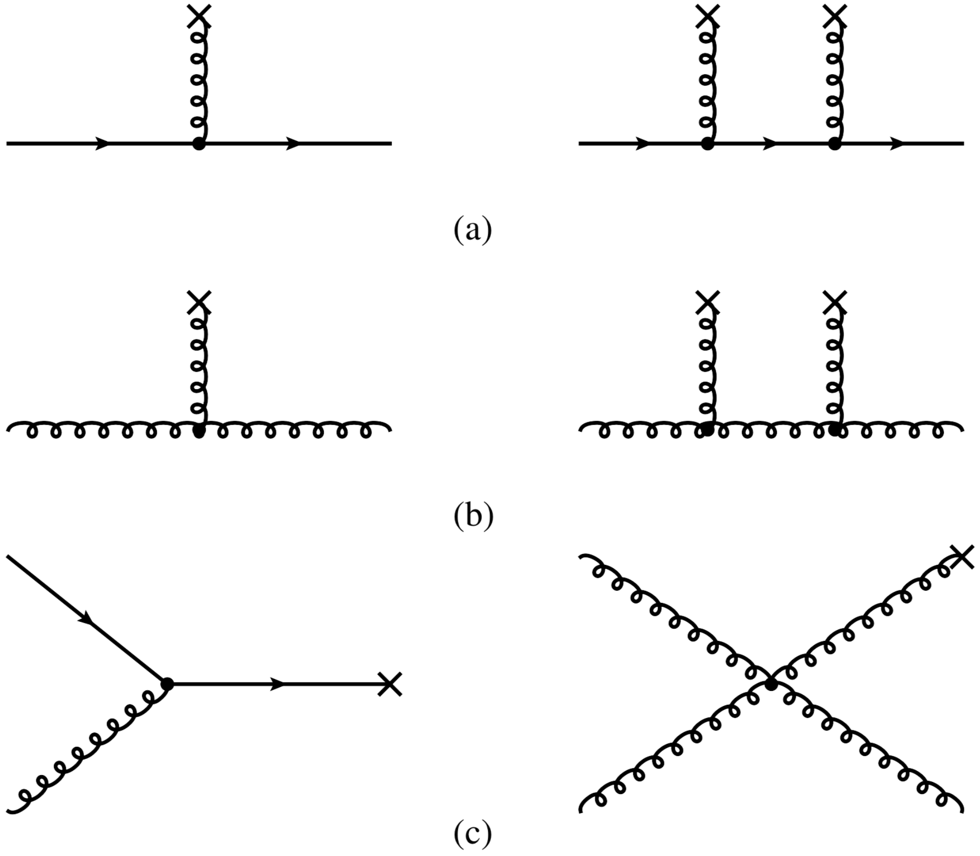
$$i\int d^4x e^{iq\cdot x}\langle\Omega|T[J_\Gamma(x)J^\dagger_{\Gamma'}(0)]|\Omega\rangle = \sum_d C_d(q^2)\langle\Omega|\mathcal{O}_d(0)|\Omega\rangle$$

$\mathcal{O}_d(0)$ are ordered by increasing dimension d, and $C_d(q^2)$ fall off by corresponding power of q^2.

$$C_d(q^2) \sim \frac{1}{q^{2d}} \sim x^{2d}$$

To suppress the contributions of high dimension operators, we can work in deep Euclidean region $Q^2 = -q^2 \to \infty$. **The** $C_d(q^2)$ can be calculated in perturbation theory.

Wilson coefficient in Fock-Schwinger gauge



Background field method

$$A_\mu^a \to A_\mu^a + \phi_\mu^a$$

$$q \to q + \eta$$

Fock-Schwinger gauge (fixed-point gauge)

$$(x - x_0)^\mu A_\mu(x) = 0$$

External gauge field A_μ can be expressed as

$$A_\mu(x) = \int_0^1 t\mathrm{d}t G_{\nu\mu}(tx)x^\nu$$

Taylor expansion at small x

$$A_\mu(x) = \frac{1}{2}x^\nu G_{\nu\mu}(0) + \frac{1}{3}x^\alpha x^\nu D_\alpha G_{\nu\mu}(0) + \frac{1}{8}x^\alpha x^\beta x^\nu D_\alpha D_\beta G_{\nu\mu}(0) + \cdots$$

Wilson coefficient in Fock-Schwinger gauge

As for soft quark field, which interact with the QCD vacuum with small momentum transfer to form condensates, they can be expanded at small x

$$q(x) = q(0) + x^{\alpha_1}\overrightarrow{D}_{\alpha_1}q(0) + \frac{1}{2}x^{\alpha_1}x^{\alpha_2}\overrightarrow{D}_{\alpha_1}\overrightarrow{D}_{\alpha_2}q(0) + \cdots$$

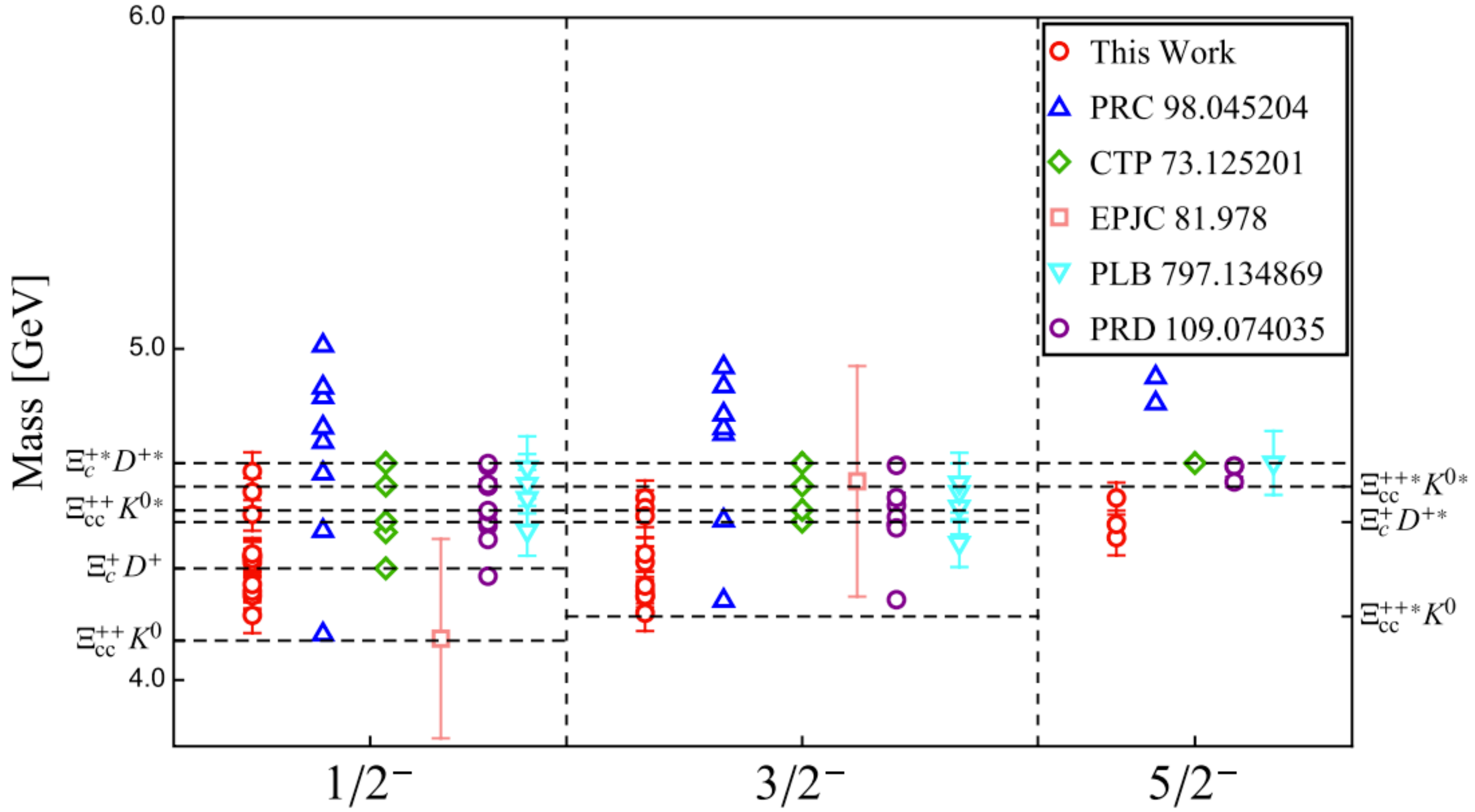
$$\bar{q}(x) = \bar{q}(0) + x^{\alpha_1}\bar{q}(0)\overleftarrow{D}_{\alpha_1} + \frac{1}{2}x^{\alpha_1}x^{\alpha_2}\bar{q}(0)\overleftarrow{D}_{\alpha_1}\overleftarrow{D}_{\alpha_2} + \cdots$$

With the expansion, the CF becomes like

$$\Pi_{\mu\nu\ldots}(x) = \langle \Omega \,|\, T[J_\Gamma(x)J^\dagger_{\Gamma'}(0)] \,|\, \Omega \rangle$$

$$= \text{(some product of propagator)} + \text{(some product of propagator)}\langle \Omega \,|\, \bar{q}(x)q(0) \,|\, \Omega \rangle + \cdots$$

Discussion and Conclusion



- PRC： color-magnetic interaction in Schroedinger equation by variational method, which is well-known to only give the upper limit of a given state.
- PLB: Non-relativistic constituent quark model by solving multi body Schroedinger equation. Overlooked linear confinement potential and induce two free parameters. And variational method.