Charmed meson mass and decay constants from CLQCD ensembles

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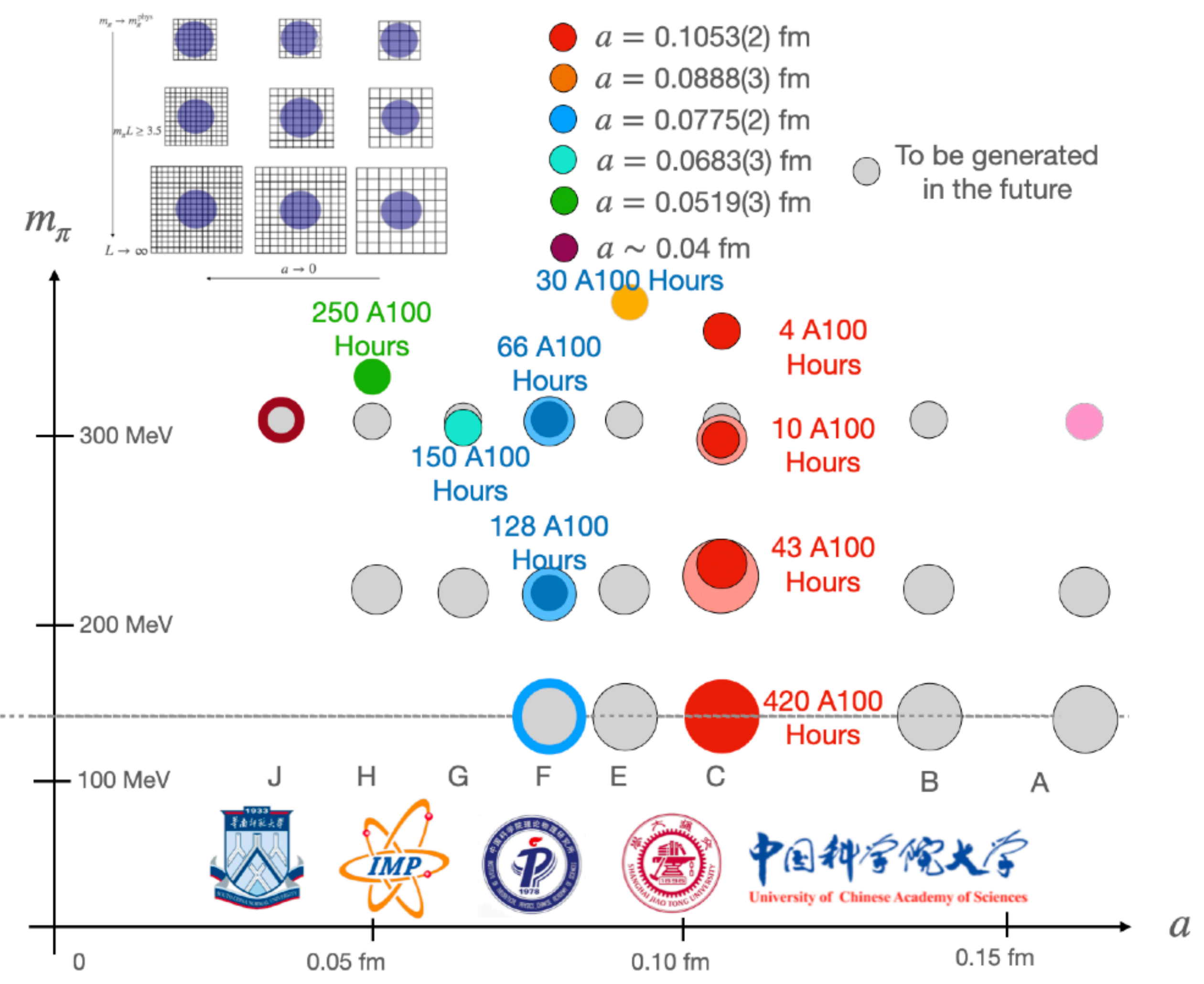
With Hai-Yang Du, Bo-Lun Hu, Peng Sun, et.al.,

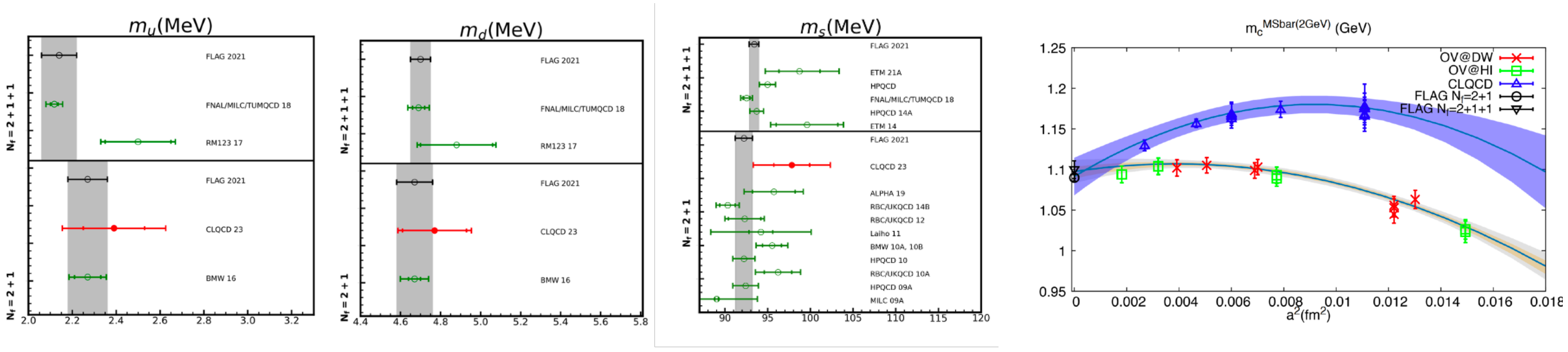
For CLQCD collaboration

CLQCD

Outline

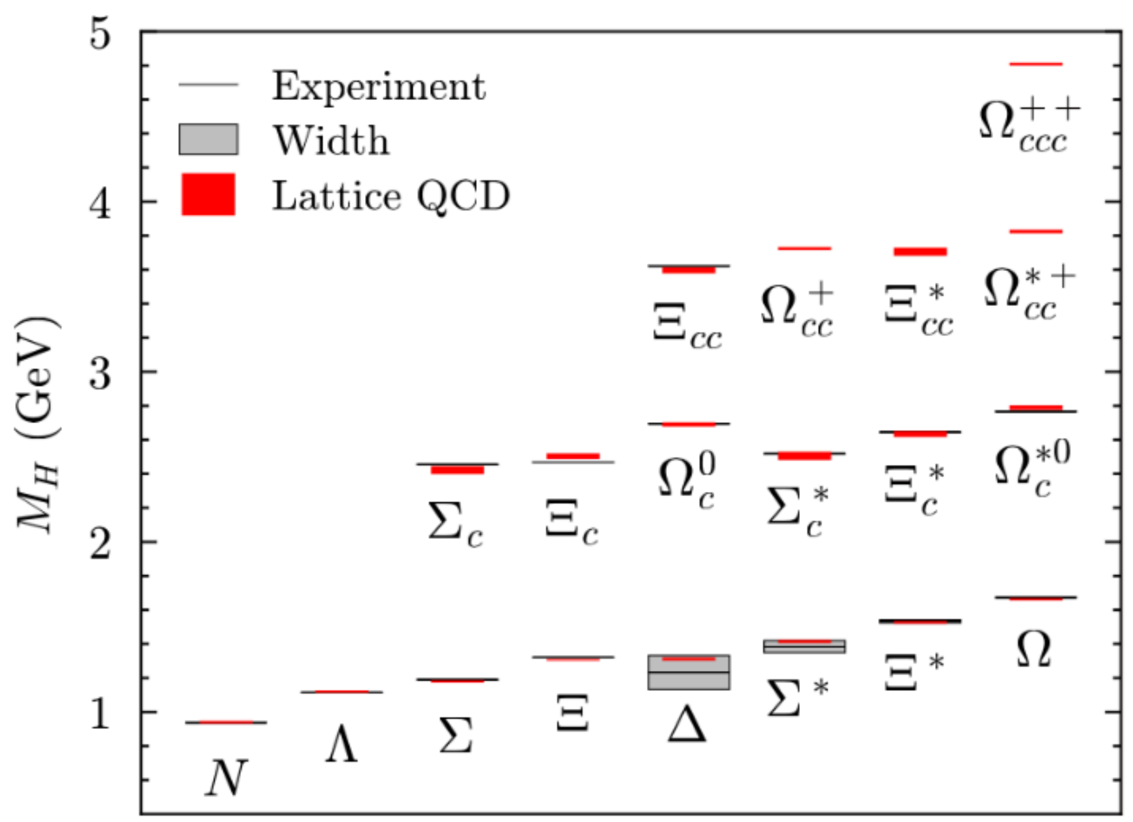
- **LQCD background and CLQCD ensembles**

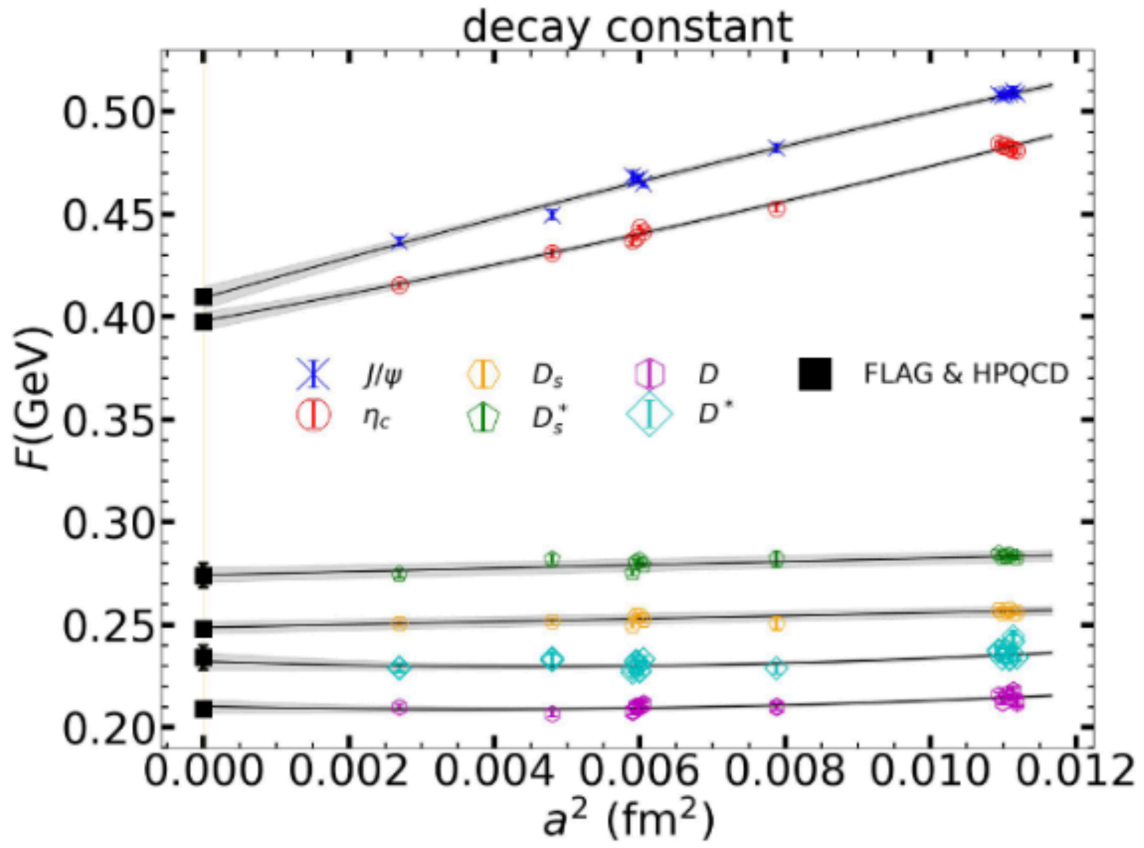




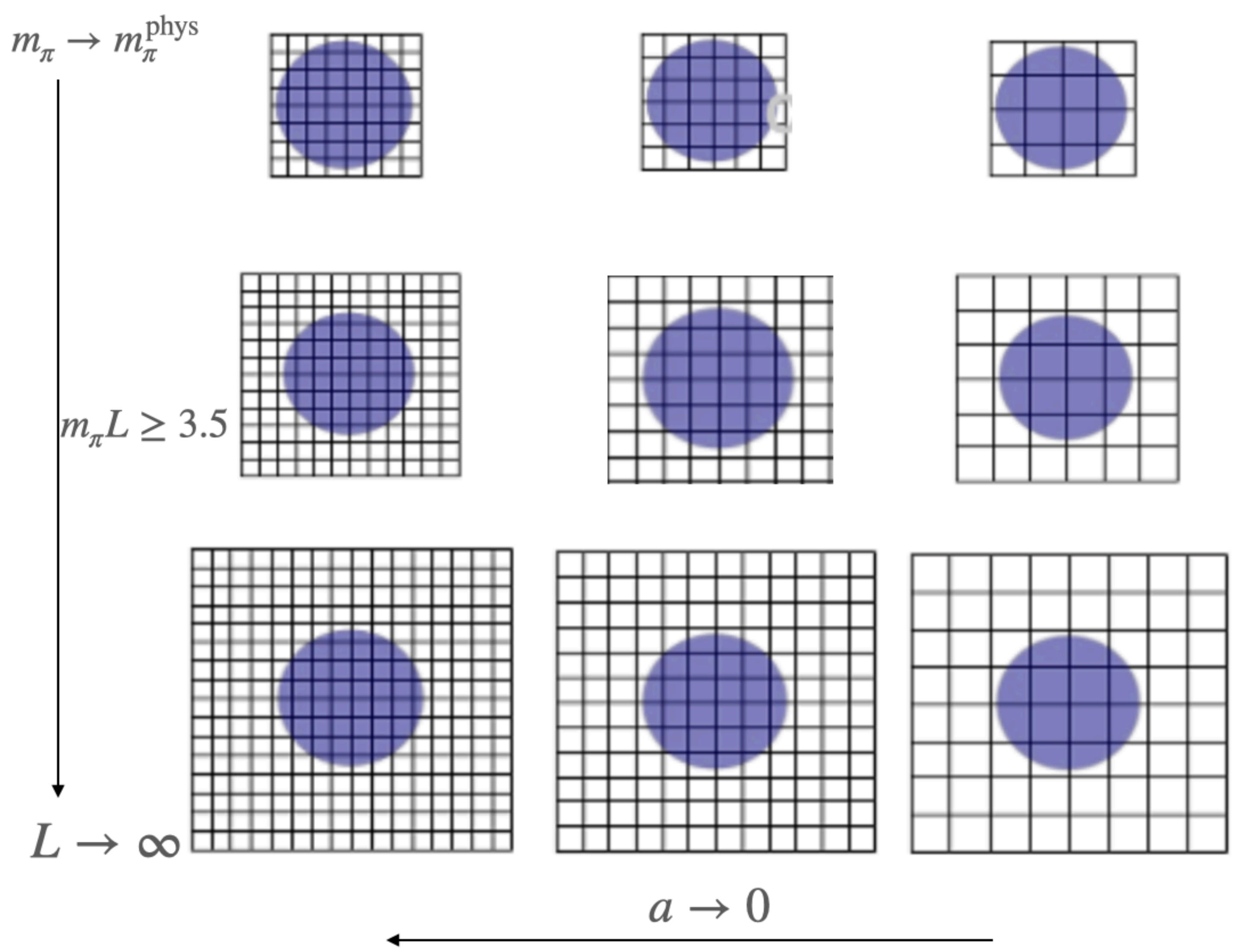
- Quark mass determinations

- Hadron masses and decay constants



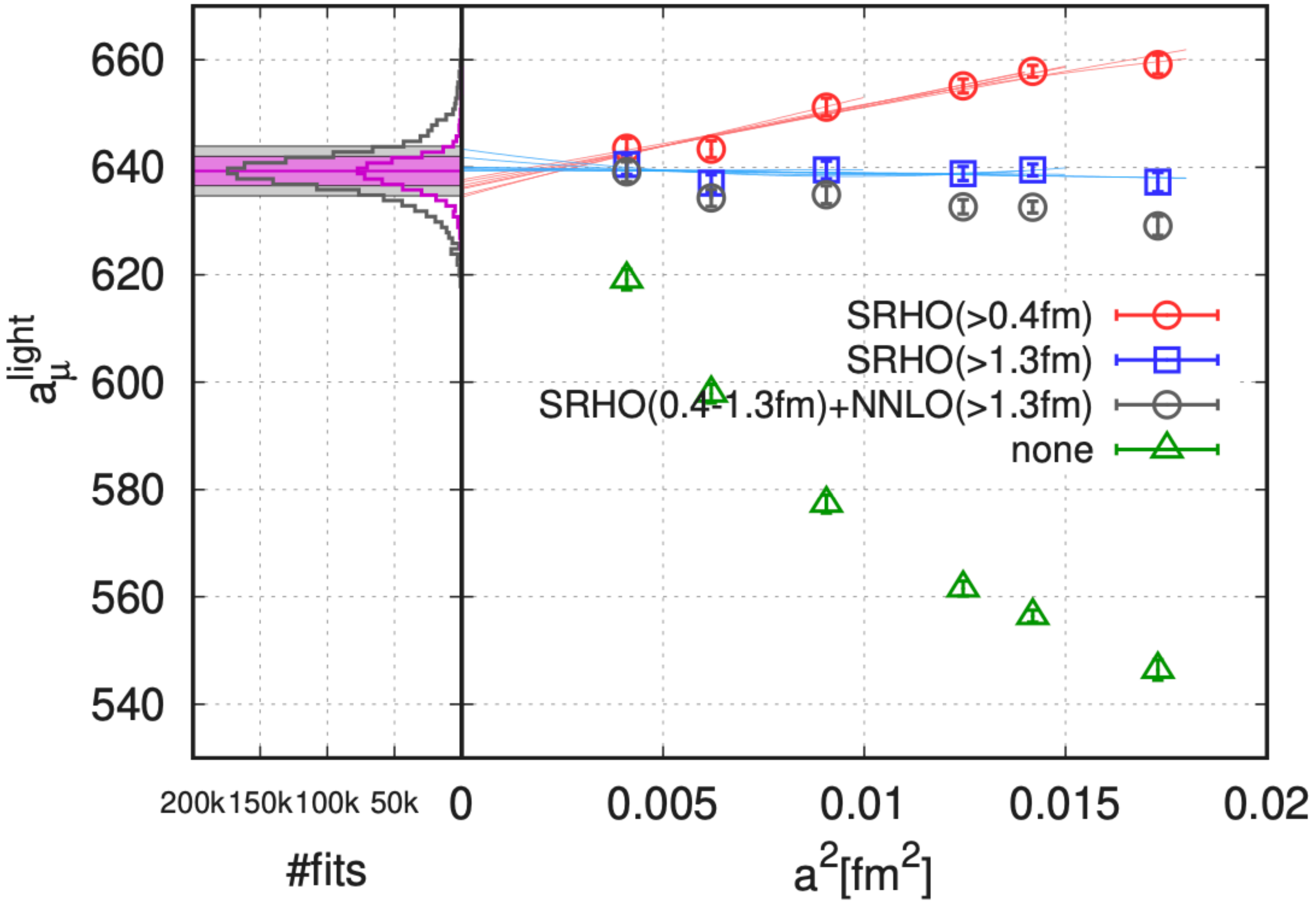


Lattice QCD



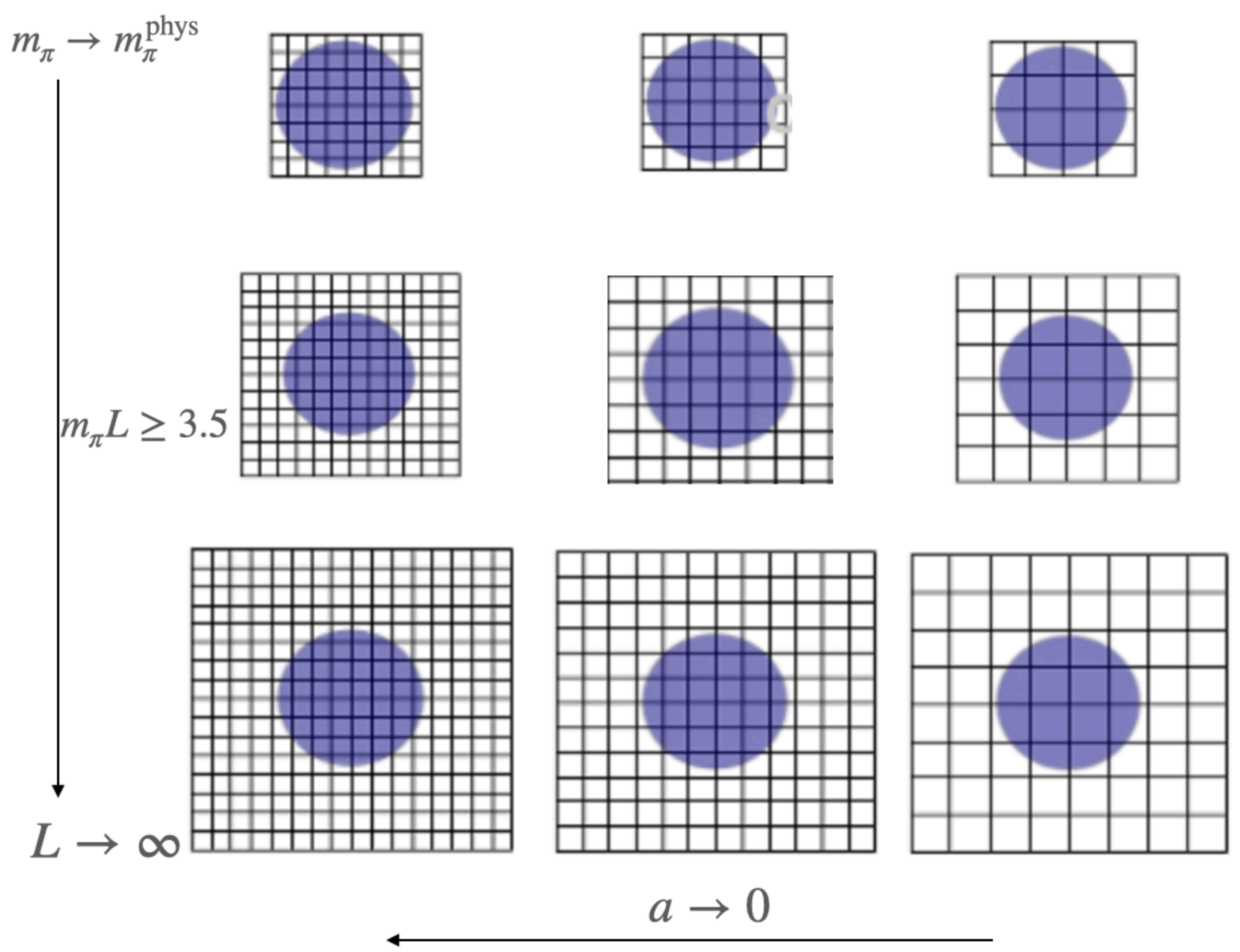
Discretization error

- Lattice calculation will suffer from the discretization error, which is usually $\mathcal{O}(a^2\Lambda_{QCD}^2)$.
- If we reduce the lattice spacing a by a factor of 2, the cost of calculation will increase by a factor of at least 16.
- The current FLAG “green star” requires at least three lattice spacings and at least two points below 0.1 fm and a range of lattice spacings satisfying $a_{\max}^2/a_{\min}^2 \geq 2$.

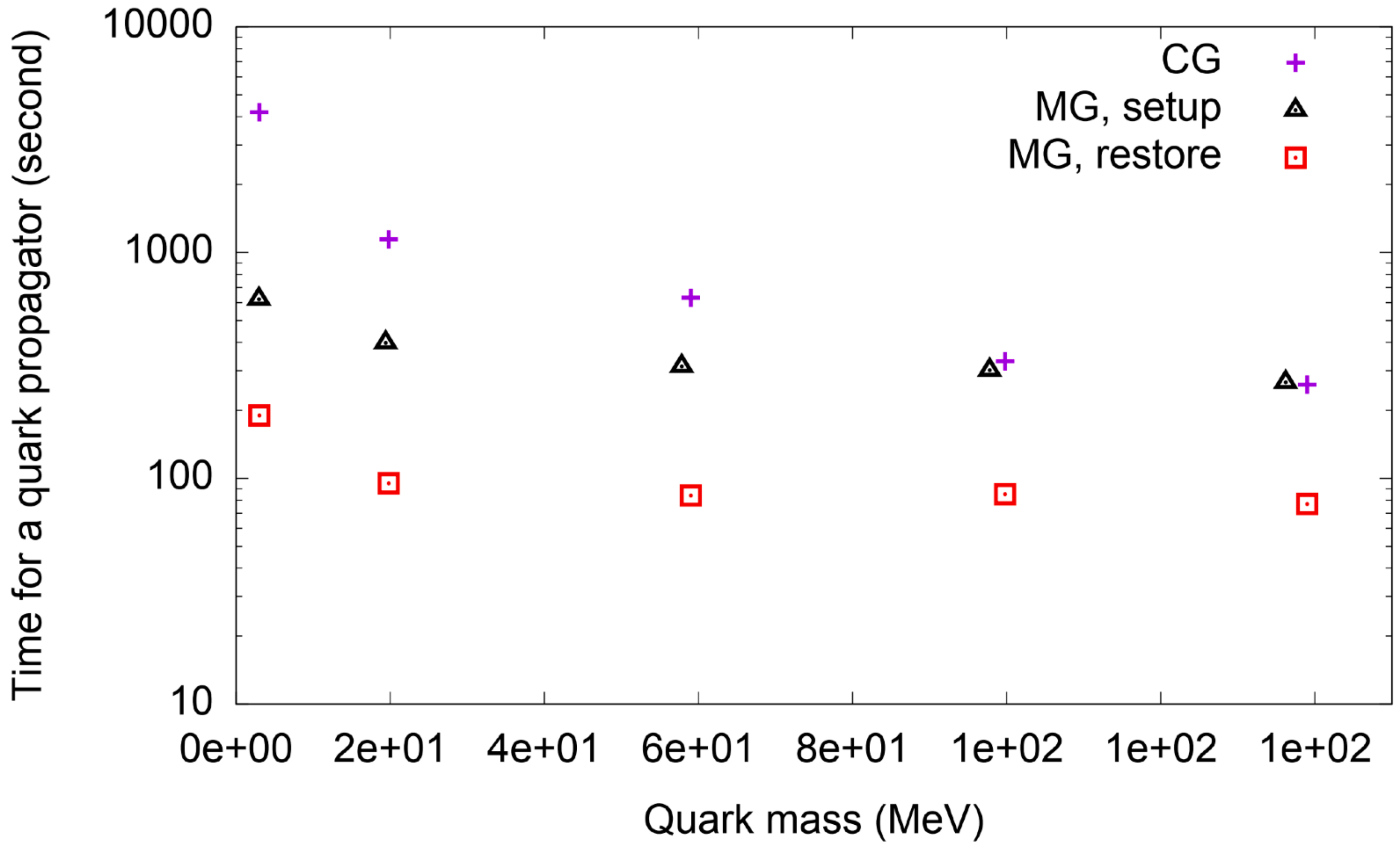


BMWc, Nature 593(2021)51

Lattice QCD

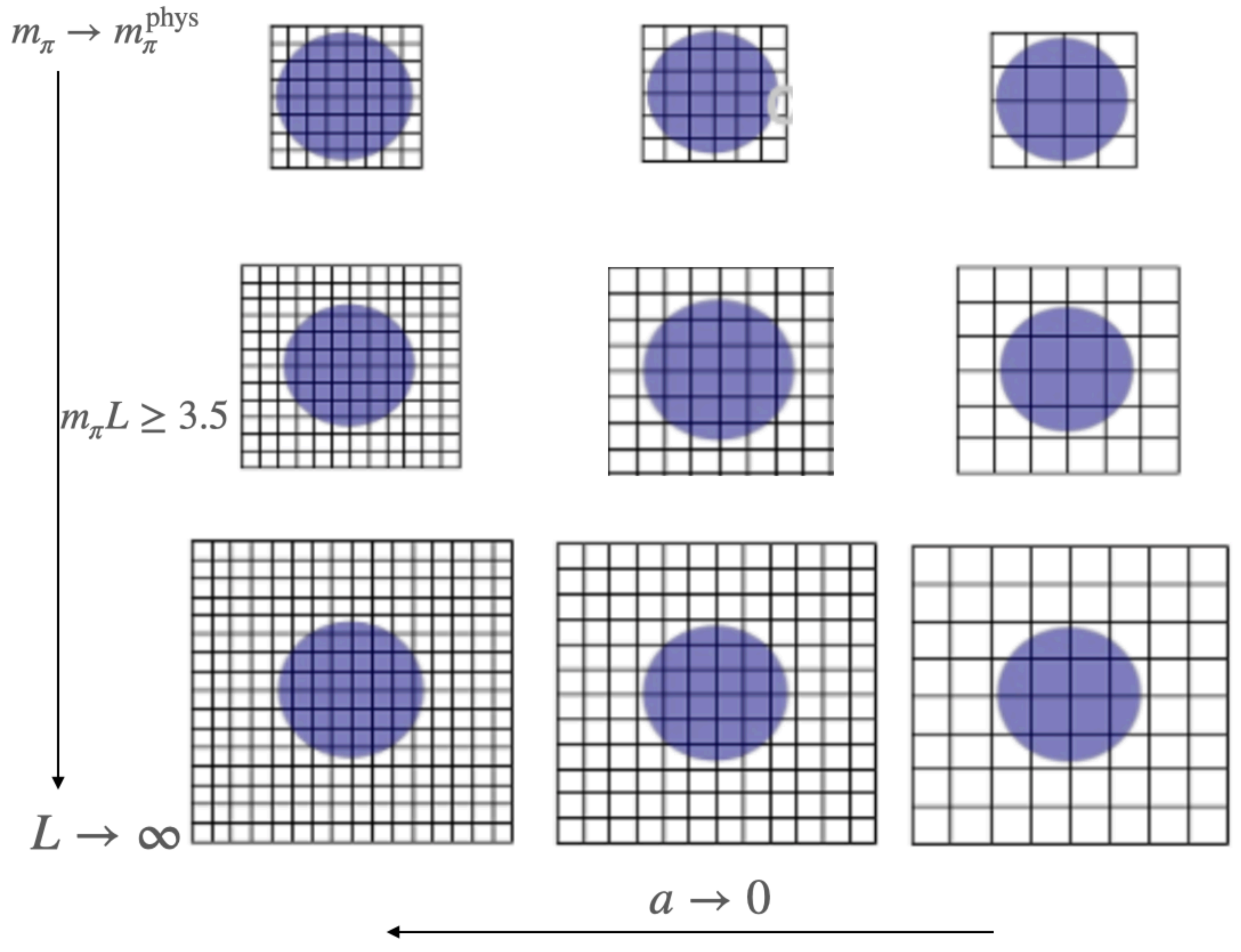


Chiral extrapolation

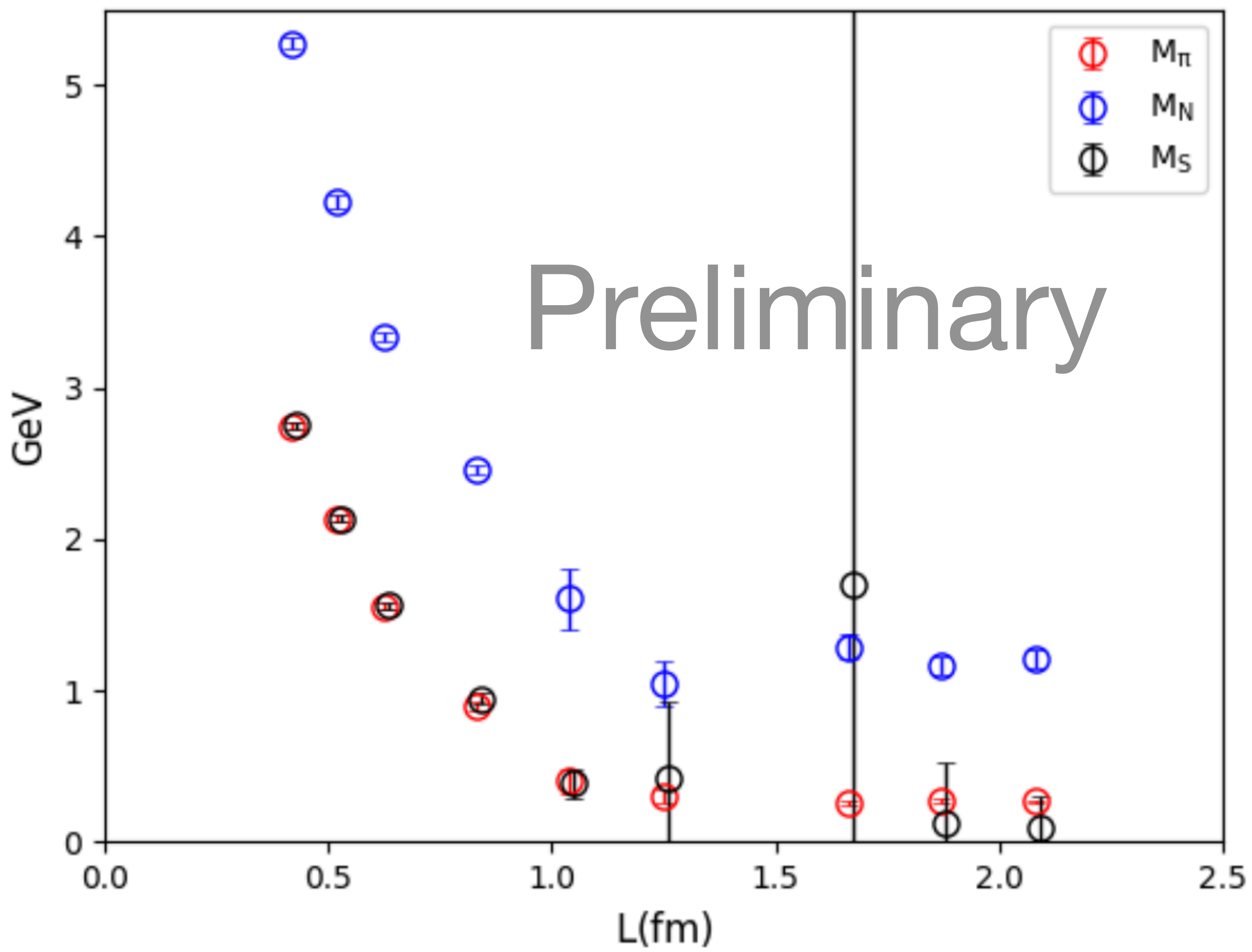


- The cost to simulate light quark can be an order of magnitude larger than that of the strange quark.
- Non-trivial algorithm likes multigrid can speed up the calculation of the light quark for certain fermion actions.
- The current FLAG “green star” requires $m_{\pi,\min} < 200$ MeV with at least three m_π in the chiral extrapolation, **or** $m_{\pi,\text{ case1}} = 135 \pm 10$ MeV and $m_{\pi,\text{ case2}} < 200$ MeV.

Lattice QCD

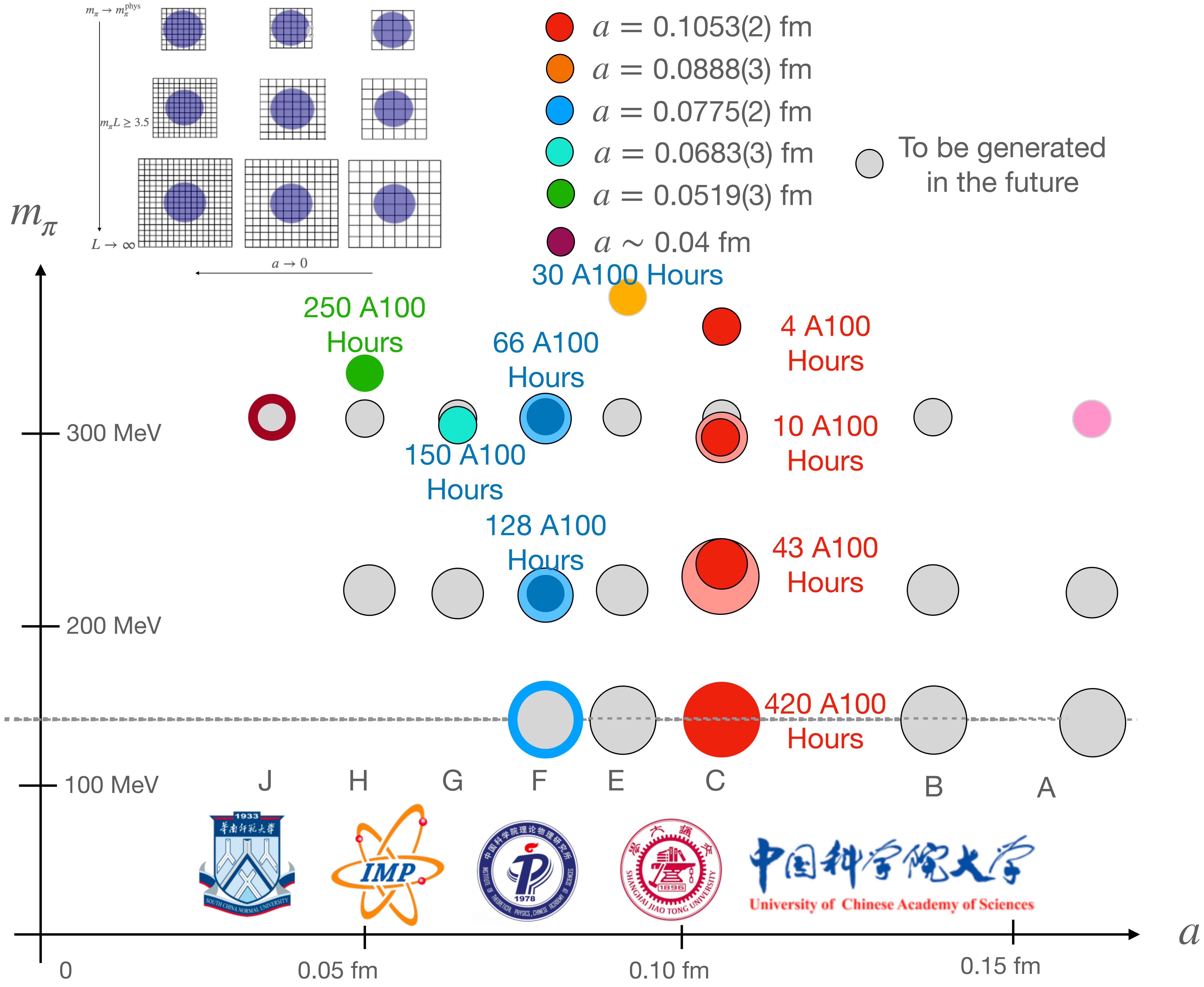


Finite volume effect



- Hadron mass can have very strong dependence on spatial size L, especially when $L \leq \Lambda_{\mathrm{QCD}}^{-1}$;
- The finite volume chiral perturbative theory suggest an $e^{-m_\pi L}$ correction when $m_\pi L \geq 3$, it means that the volume required by $m_\pi \sim$ 135 MeV is ~ 11 times of that required by $m_\pi \sim$ 300 MeV.
- The current FLAG “green star” requires $m_\pi L \sim$ 3.2 for $m_\pi \sim$ 135 MeV, **or** at least three volumes.

CLQCD ensembles



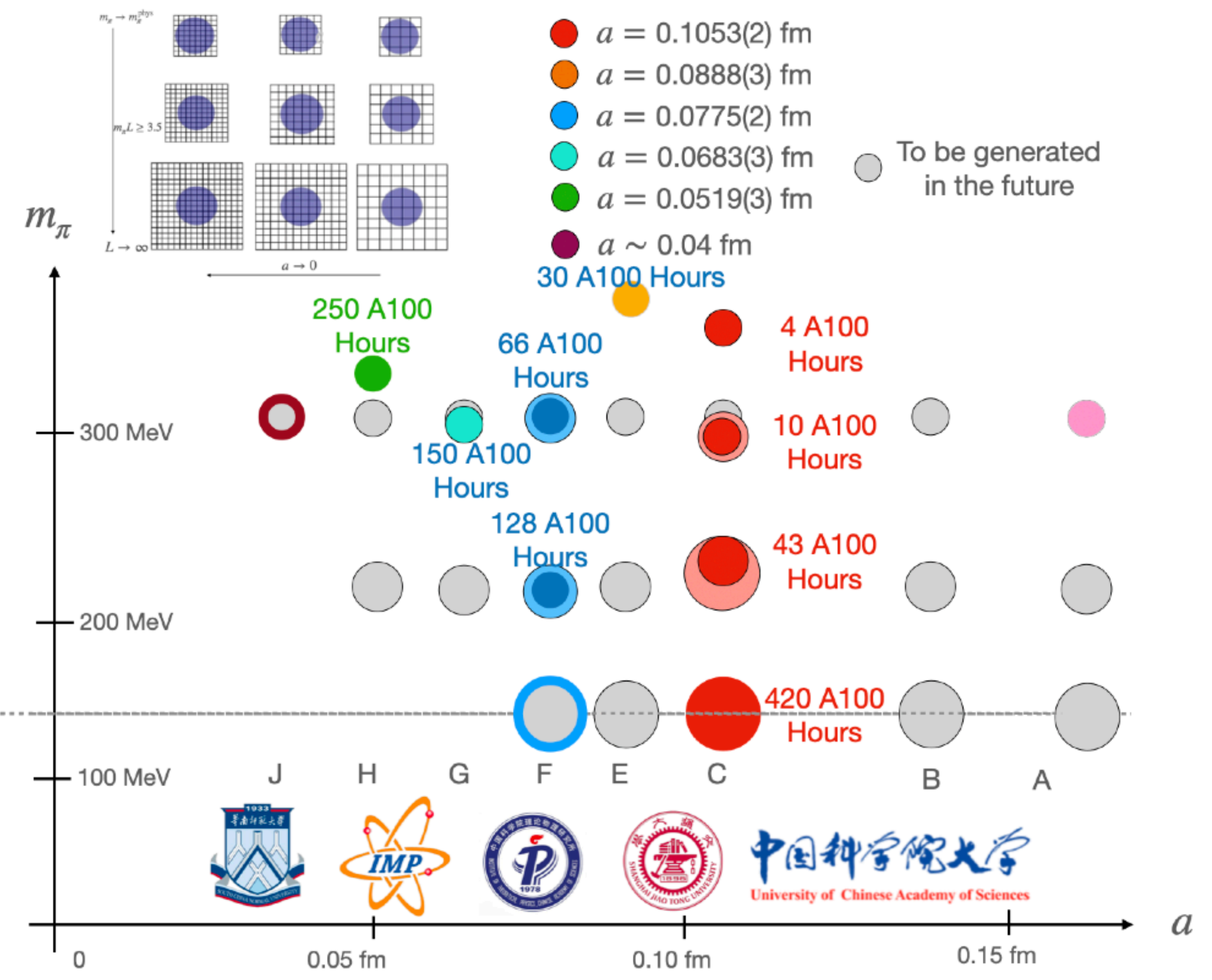
Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

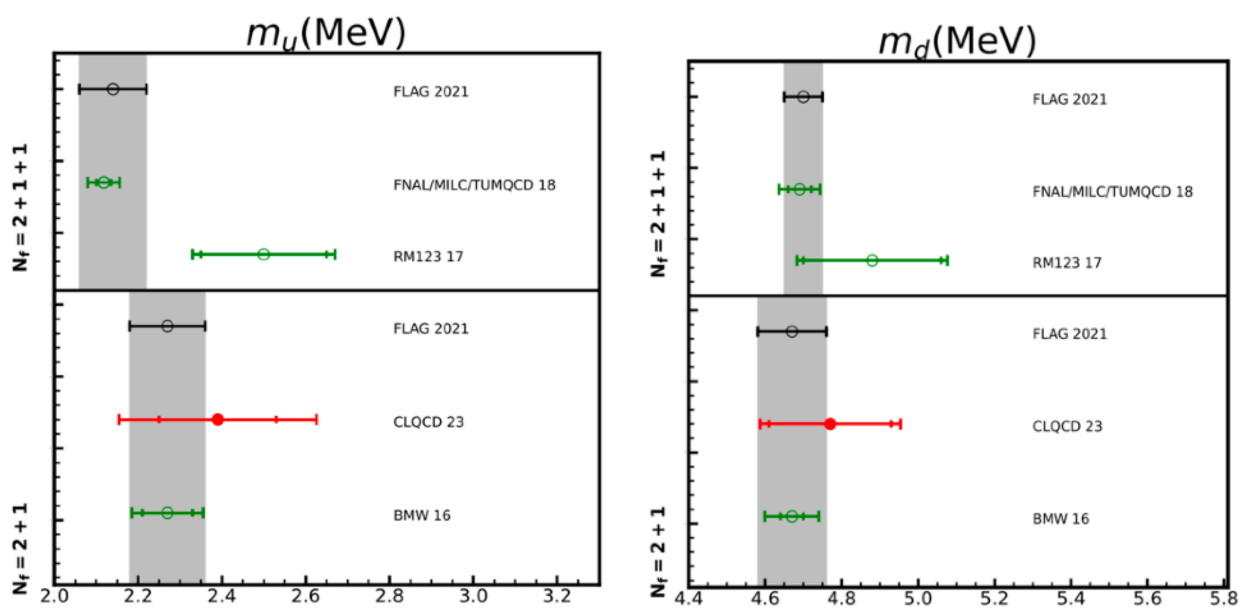
Informations

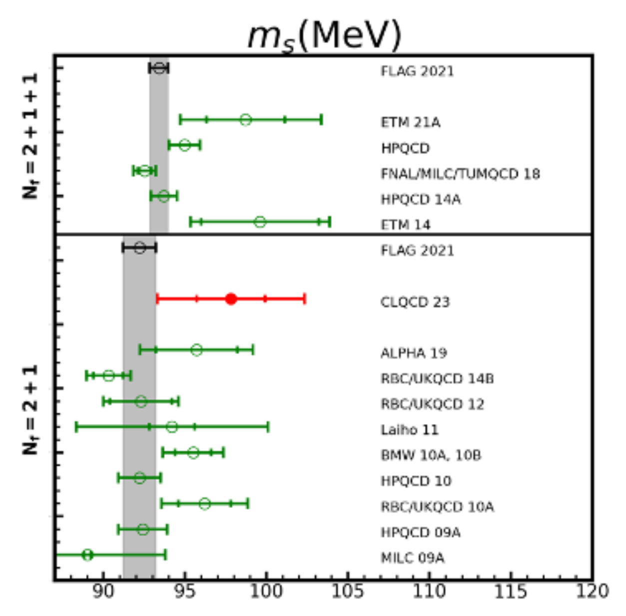
- Cost:
- That of an independent configuration (per 10 traj.'s with $\tau = 1.0$, converted to A100 GPU hours) is shown on the figure;
- Used more than equivalently 5,000,000 A100 GPU hours;
- Working on the Sugon machines to avoid the embargo of A100 GPU.

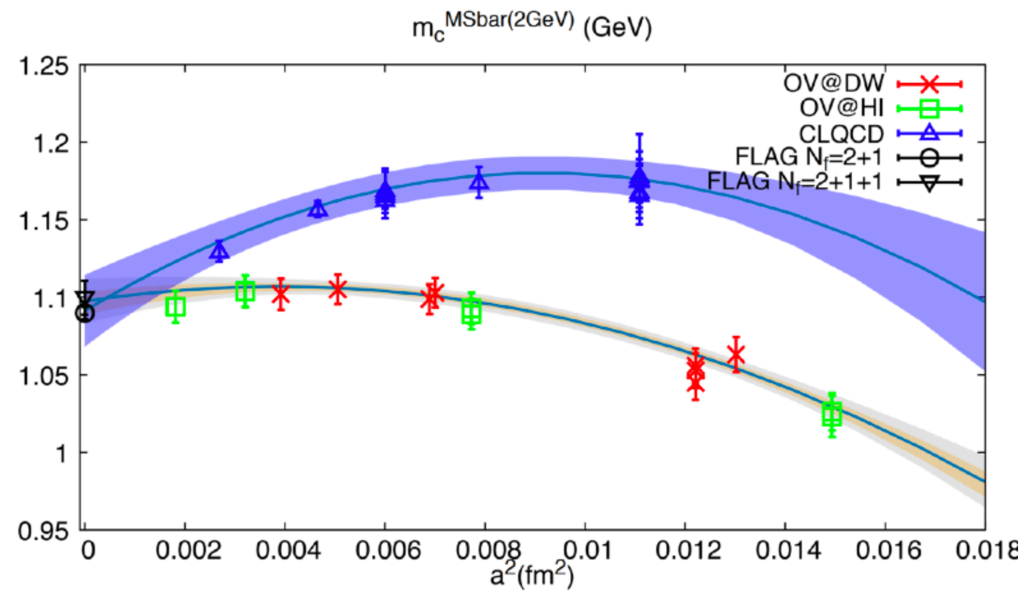
Outline

- LQCD background and CLQCD ensembles



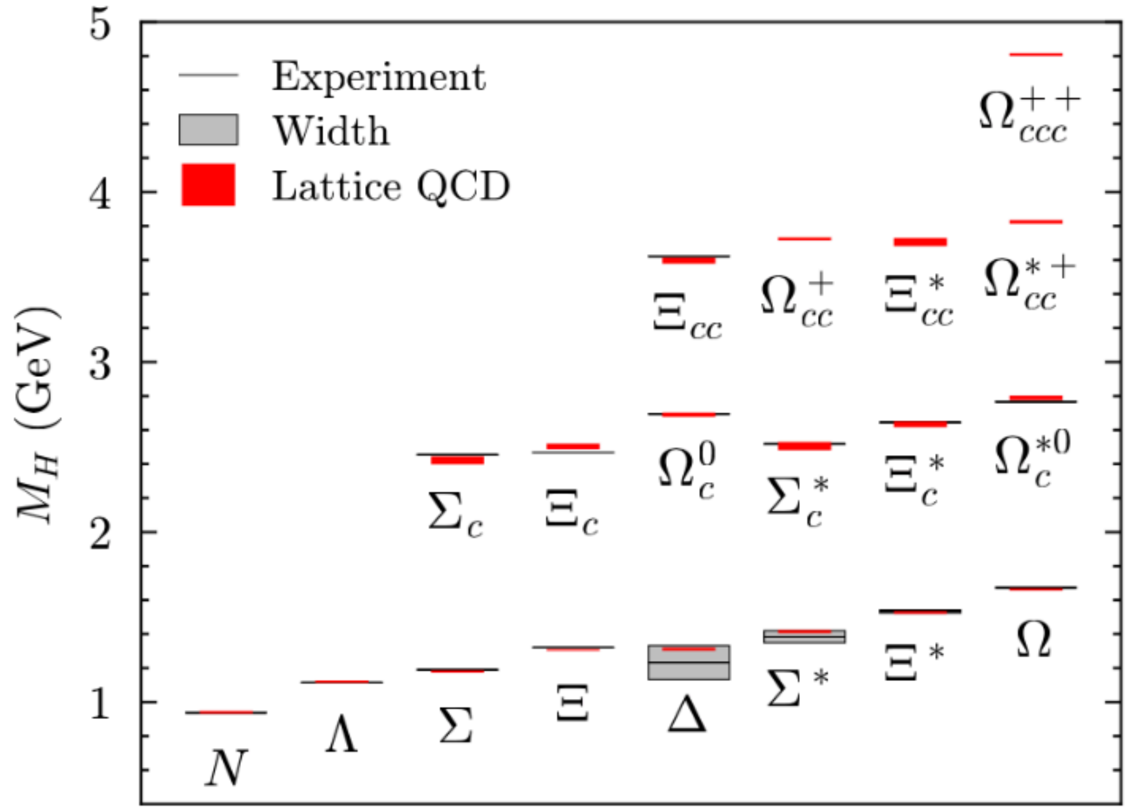


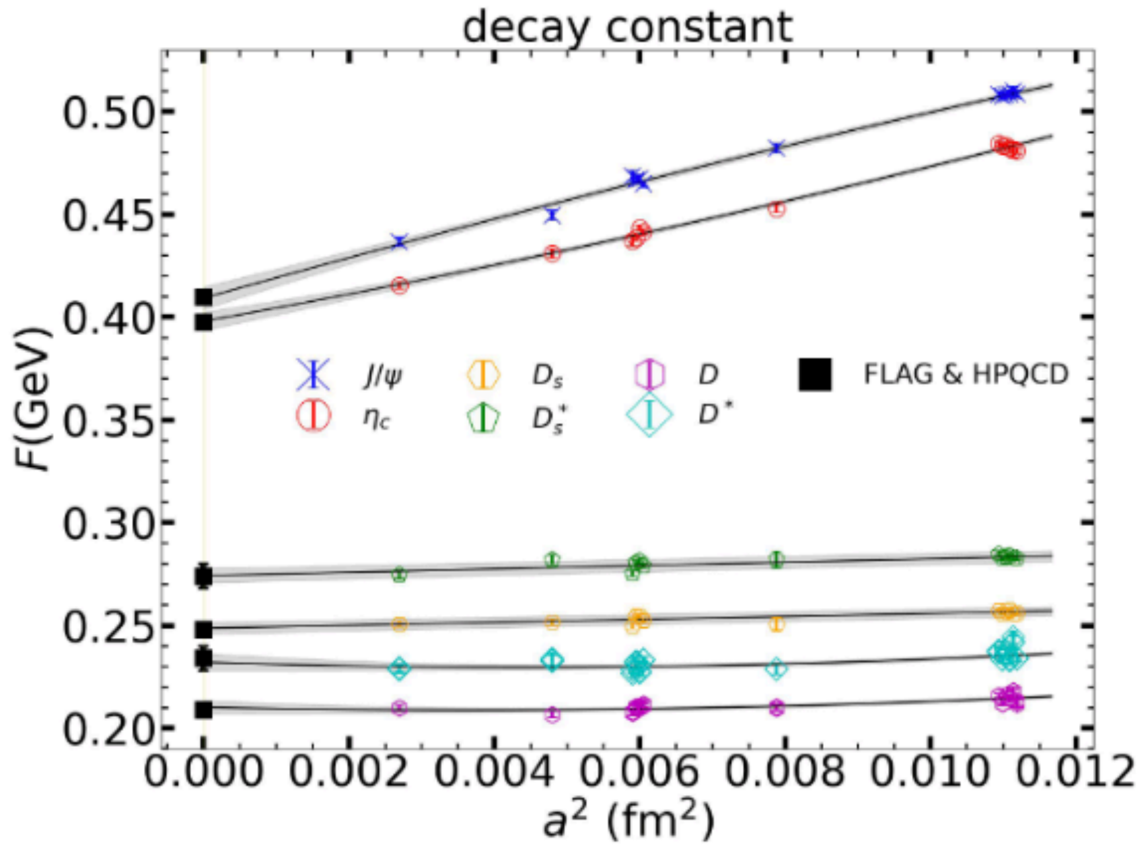




- **Quark mass determinations**

- Hadron masses and decay constants



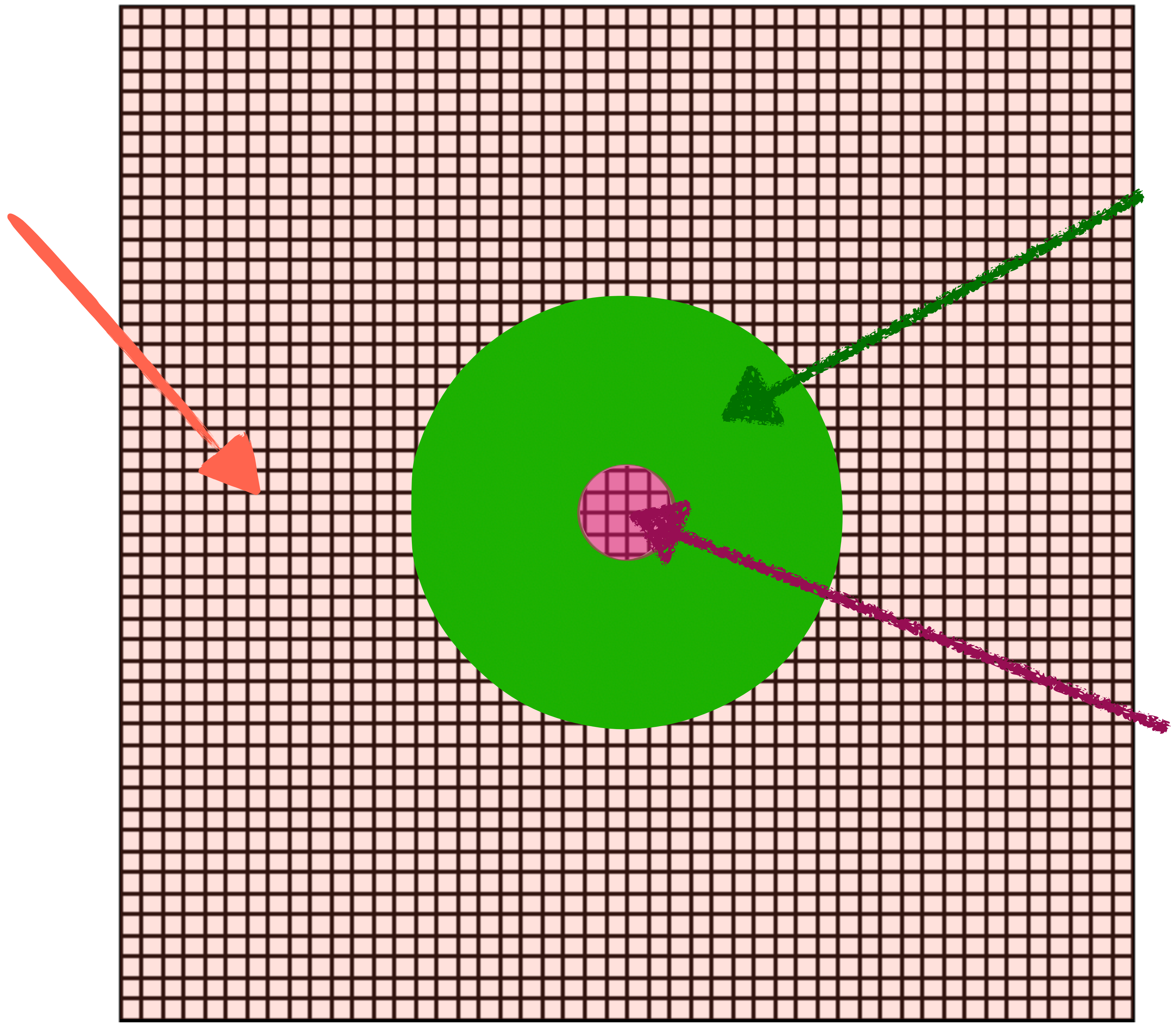


Quark mass

Renormalization through intermediate scheme

$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q,1/a)}{Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^m, \alpha_s^n)$$

- The RI/MOM renormalization targets to cancel the $\alpha_s \log(a)$ divergences using the off-shell quark matrix element;
- **Up to the $\mathcal{O}(a^2p^2)$ correction which can be eliminated by the $a^2p^2 \to 0$ extrapolation.**



Non-perturbative IR region can only be calculate by Lattice QCD

Perturbative region accessible by kinds of the regularizations

UV region with obvious regularization effects

G. Martinelli, et.al, NPB445(1995)81, arXiv: hep-lat/9411010

Quark mass

Non-Perturbative renormalization

- Obtain the regularization independent renormalization constant non-perturbatively:

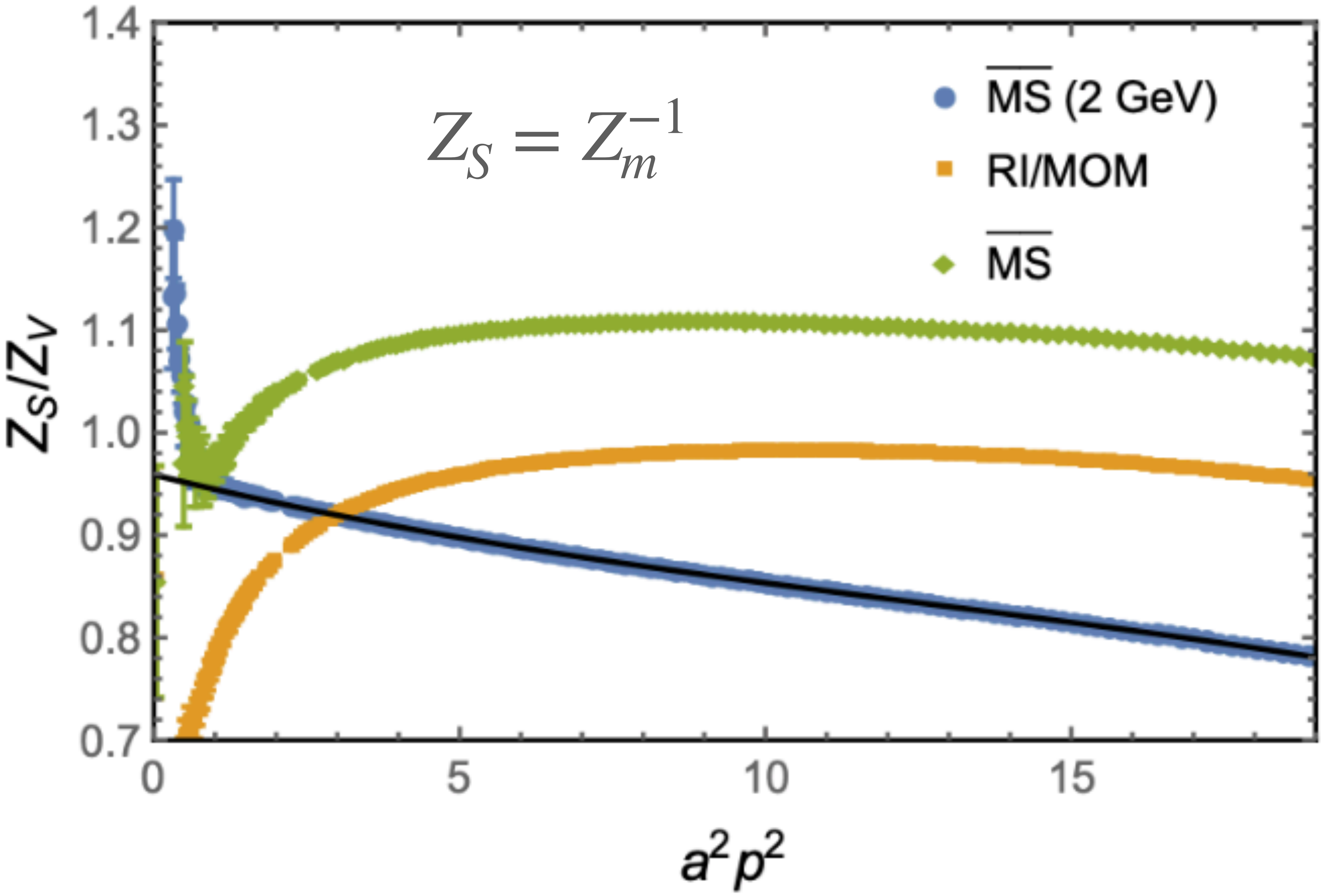
$$Z_S^{\mathrm{MOM}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$

- Calculate the matching coefficient perturbatively and obtain the result at $\overline{\mathrm{MS}}$ scale Q:

$$Z_S^{\mathrm{MS}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$

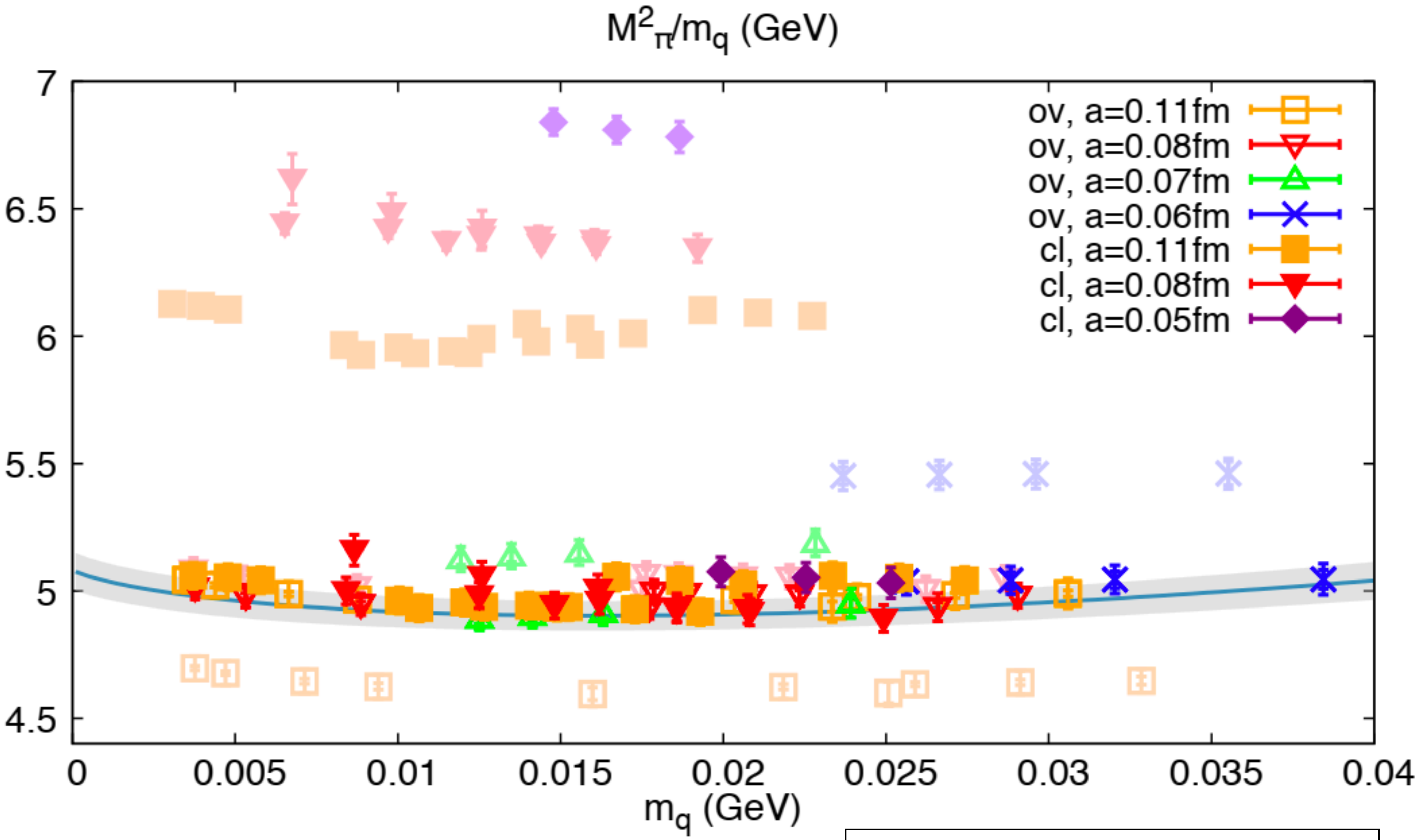
- Obtain the result at $\overline{\mathrm{MS}}$ scale μ with the scale evolution:

$$Z_S^{\overline{\mathrm{MS}}}(\mu,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2\mu^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$



Renormalized quark masses

Impact of the renormalization



Dian-Jun Zhao, et.al., χQCD, in preparation

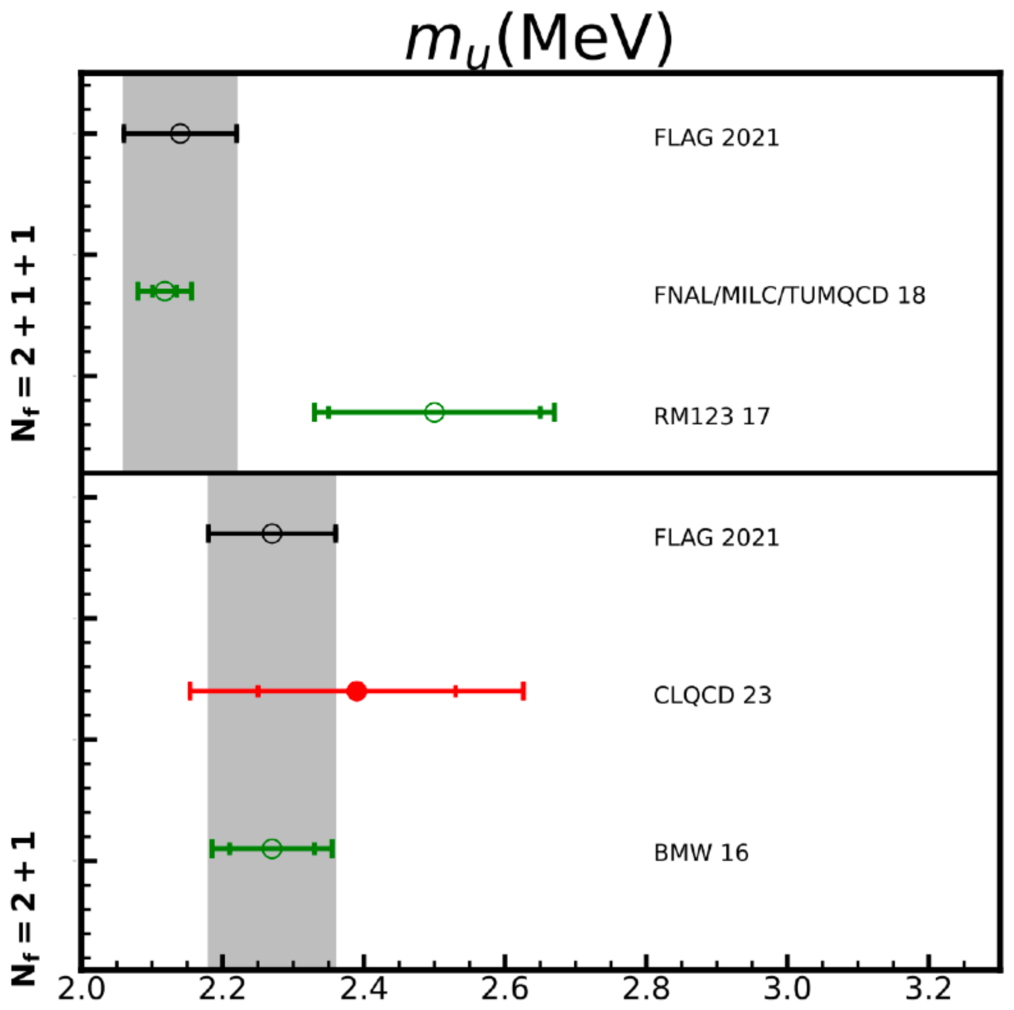
- $m_\pi^2/m_q \sim \Sigma/F^2$ which is insensitive to the quark mass, with the partially quenching effect subtracted;

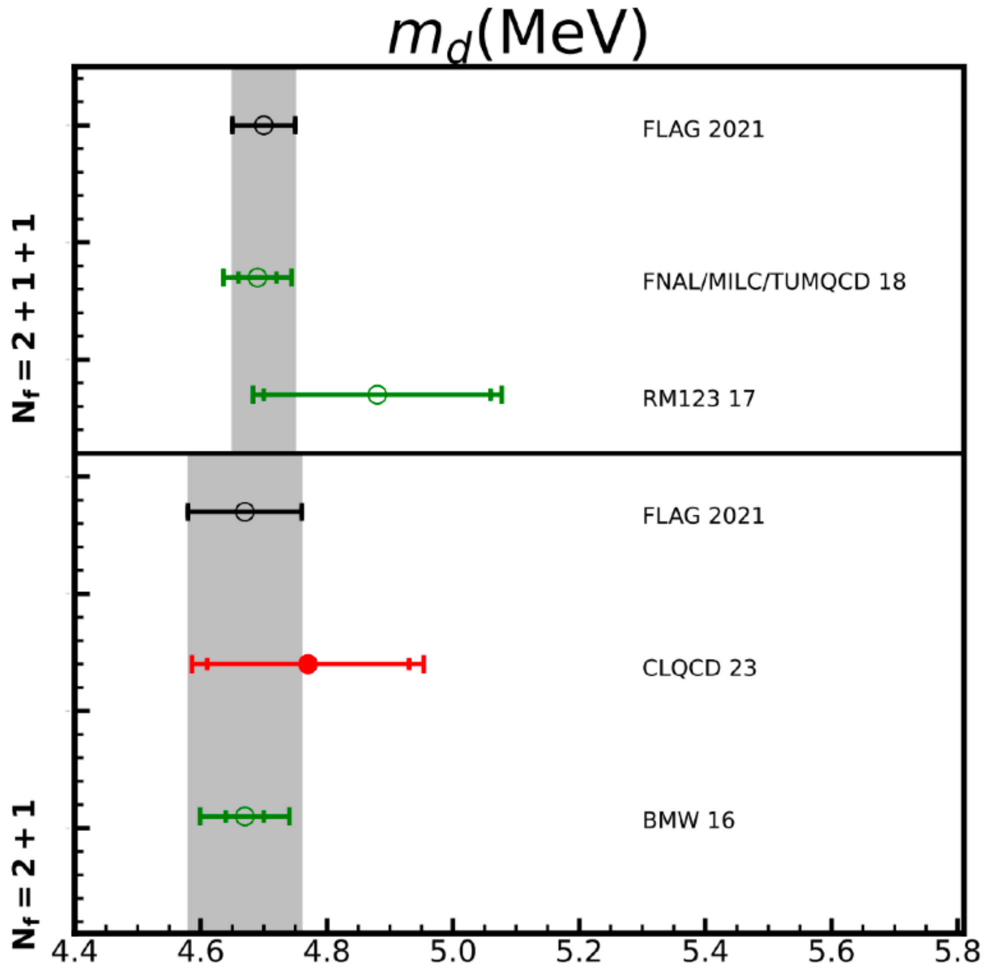
- The PCAC mass $m_q^{\mathrm{PC}} = \frac{\langle 0|\partial_4 A_4|\mathrm{PS}\rangle}{2\langle 0|P|\mathrm{PS}\rangle}$ has obvious 1/a and action dependences:

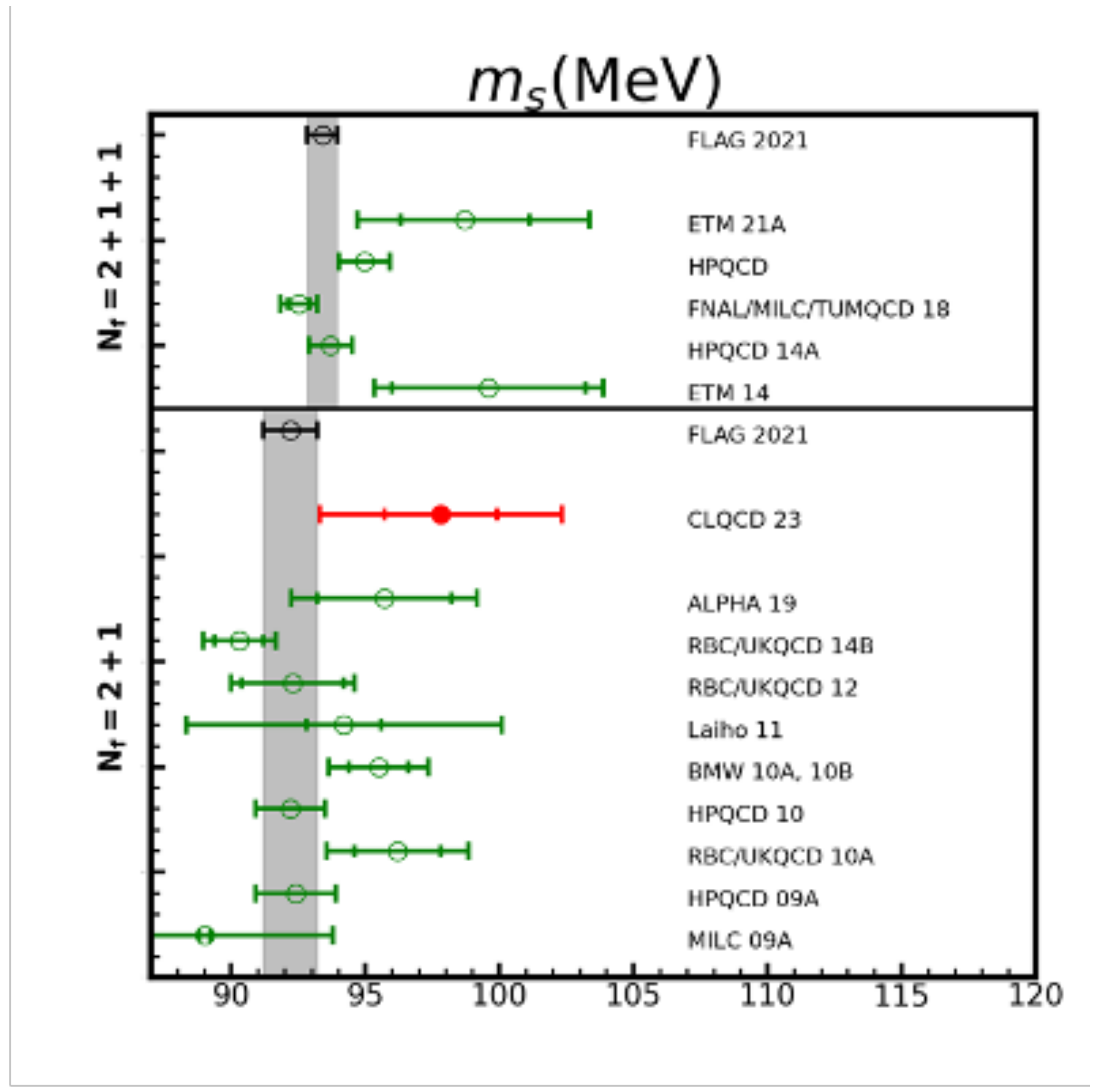
1. Smaller with large intrinsic scale 1/a;
2. Very sensitive to the fermion action.

- RI/MOM renormalization eliminates both the dependences and makes $m_\pi^2/m_q^{\overline{\mathrm{MS}}}$ of all the ensembles on a similar curve.

Quark mass







Determine the pure QCD quark masses

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27\ \text{MeV} = m_{p,\text{QCD}} + 1.00(16)\ \text{MeV} + \ldots;$
- $m_n = 939.57\ \text{MeV};$
- $m_\pi^0 = 134.98\ \text{MeV};$
- $m_\pi^+ = 139.57\ \text{MeV} = m_\pi^0 + 4.53(6)\ \text{MeV} + \ldots;$

X. Feng, et,al. Phys.Rev.Lett.128(2022)062003

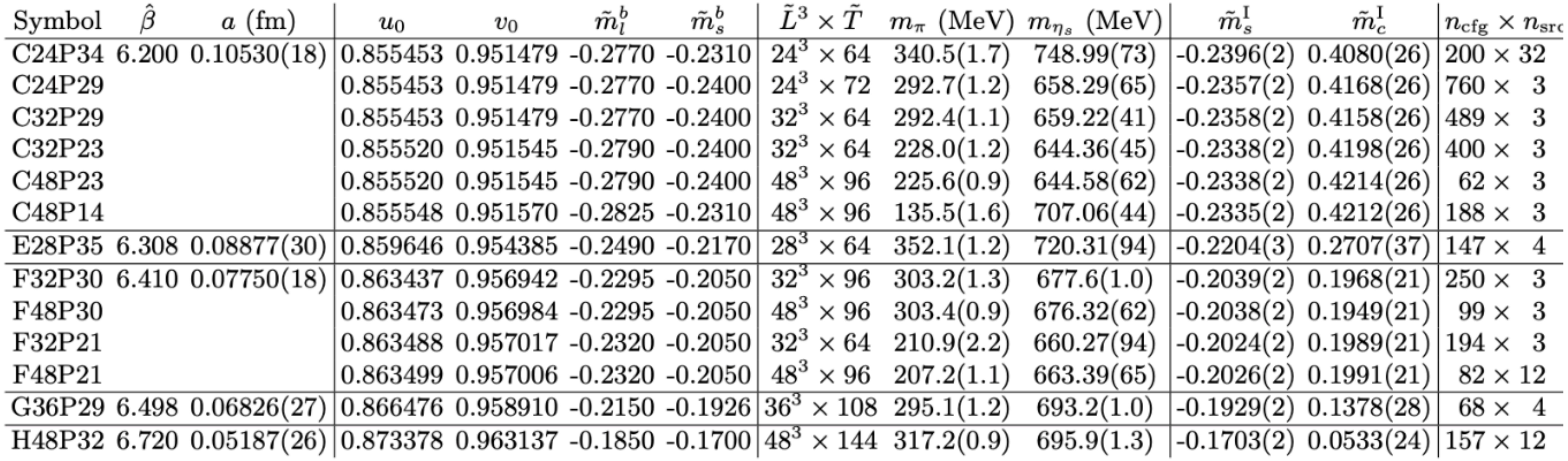
- $m_K^0 = 497.61(1)\ \text{MeV} = m_{K,\text{QCD}}^0 + 0.17(02)\ \text{MeV} + \ldots;$
- $m_K^+ = 493.68(2)\ \text{MeV} = m_{K,\text{QCD}}^+ + 2.24(15)\ \text{MeV} + \ldots$

D. Giusti, et,al. PRD95(2017)114504

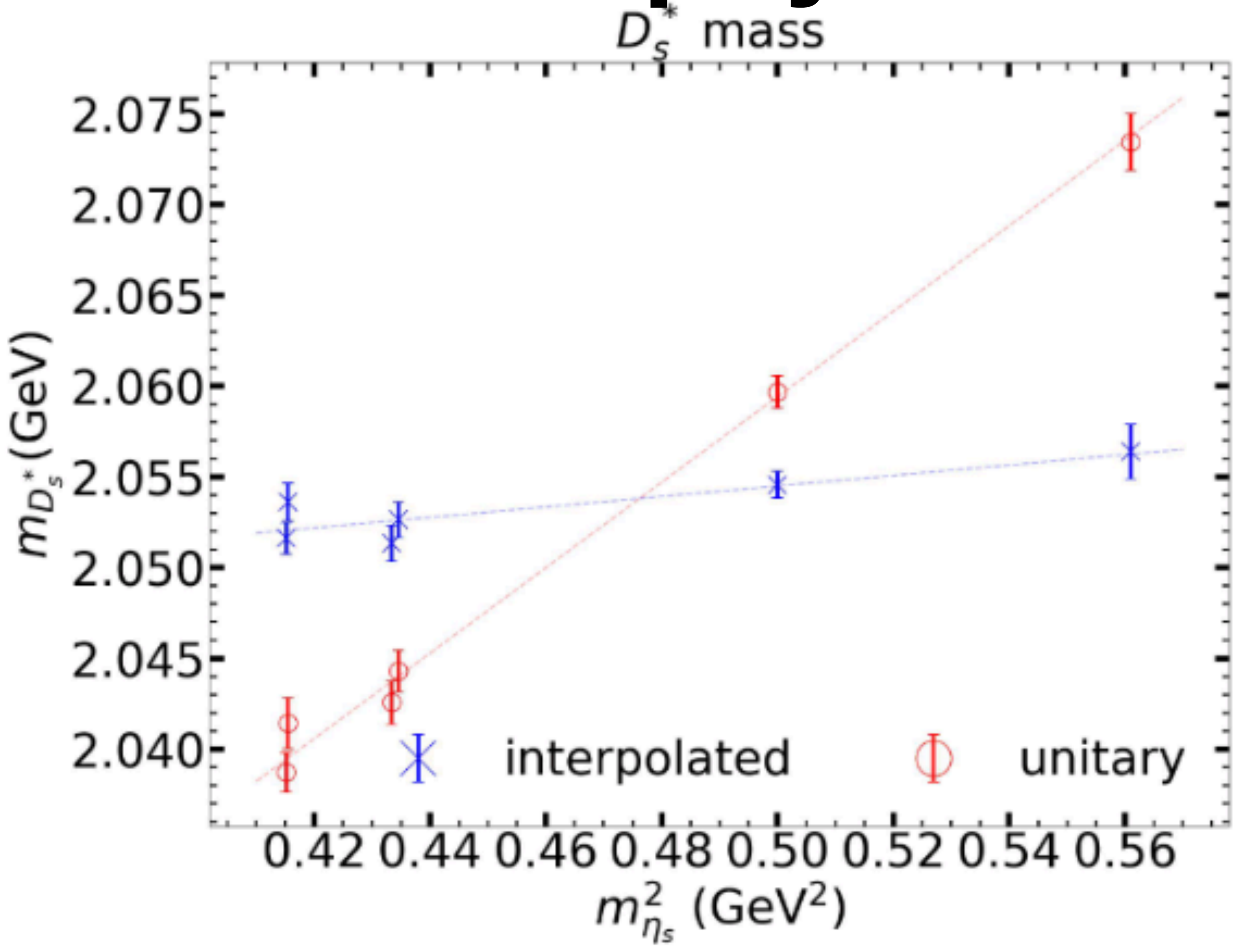
Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

Quark mass

Toward the charm physics



Symbol	$\hat{\beta}$	a (fm)	u_0	v_0	$\tilde{m}_l^b$	$\tilde{m}_s^b$	$\tilde{L}^3 \times \tilde{T}$	m_π (MeV)	m_{η_s} (MeV)	$\tilde{m}_s^{\rm I}$	$\tilde{m}_c^{\rm I}$	$n_{\rm cfg} \times n_{\rm src}$
C24P34	6.200	0.10530(18)	0.855453	0.951479	-0.2770	-0.2310	$24^3 \times 64$	340.5(1.7)	748.99(73)	-0.2396(2)	0.4080(26)	200 × 32
C24P29			0.855453	0.951479	-0.2770	-0.2400	$24^3 \times 72$	292.7(1.2)	658.29(65)	-0.2357(2)	0.4168(26)	760 × 3
C32P29			0.855453	0.951479	-0.2770	-0.2400	$32^3 \times 64$	292.4(1.1)	659.22(41)	-0.2358(2)	0.4158(26)	489 × 3
C32P23			0.855520	0.951545	-0.2790	-0.2400	$32^3 \times 64$	228.0(1.2)	644.36(45)	-0.2338(2)	0.4198(26)	400 × 3
C48P23			0.855520	0.951545	-0.2790	-0.2400	$48^3 \times 96$	225.6(0.9)	644.58(62)	-0.2338(2)	0.4214(26)	62 × 3
C48P14			0.855548	0.951570	-0.2825	-0.2310	$48^3 \times 96$	135.5(1.6)	707.06(44)	-0.2335(2)	0.4212(26)	188 × 3
E28P35	6.308	0.08877(30)	0.859646	0.954385	-0.2490	-0.2170	$28^3 \times 64$	352.1(1.2)	720.31(94)	-0.2204(3)	0.2707(37)	147 × 4
F32P30	6.410	0.07750(18)	0.863437	0.956942	-0.2295	-0.2050	$32^3 \times 96$	303.2(1.3)	677.6(1.0)	-0.2039(2)	0.1968(21)	250 × 3
F48P30			0.863473	0.956984	-0.2295	-0.2050	$48^3 \times 96$	303.4(0.9)	676.32(62)	-0.2038(2)	0.1949(21)	99 × 3
F32P21			0.863488	0.957017	-0.2320	-0.2050	$32^3 \times 64$	210.9(2.2)	660.27(94)	-0.2024(2)	0.1989(21)	194 × 3
F48P21			0.863499	0.957006	-0.2320	-0.2050	$48^3 \times 96$	207.2(1.1)	663.39(65)	-0.2026(2)	0.1991(21)	82 × 12
G36P29	6.498	0.06826(27)	0.866476	0.958910	-0.2150	-0.1926	$36^3 \times 108$	295.1(1.2)	693.2(1.0)	-0.1929(2)	0.1378(28)	68 × 4
H48P32	6.720	0.05187(26)	0.873378	0.963137	-0.1850	-0.1700	$48^3 \times 144$	317.2(0.9)	695.9(1.3)	-0.1703(2)	0.0533(24)	157 × 12



$$m_{\eta_s} = 687.4(2.2)\ \text{MeV}$$

Z.C. Hu, B.L. Hu, J.H. Wang, et. al.,
CLQCD, Phys.Rev.D109 (2024) 054507

$$m_{\eta_s} = 689.89(49)\ \text{MeV}$$

BMWc, Nature 593(2021)51

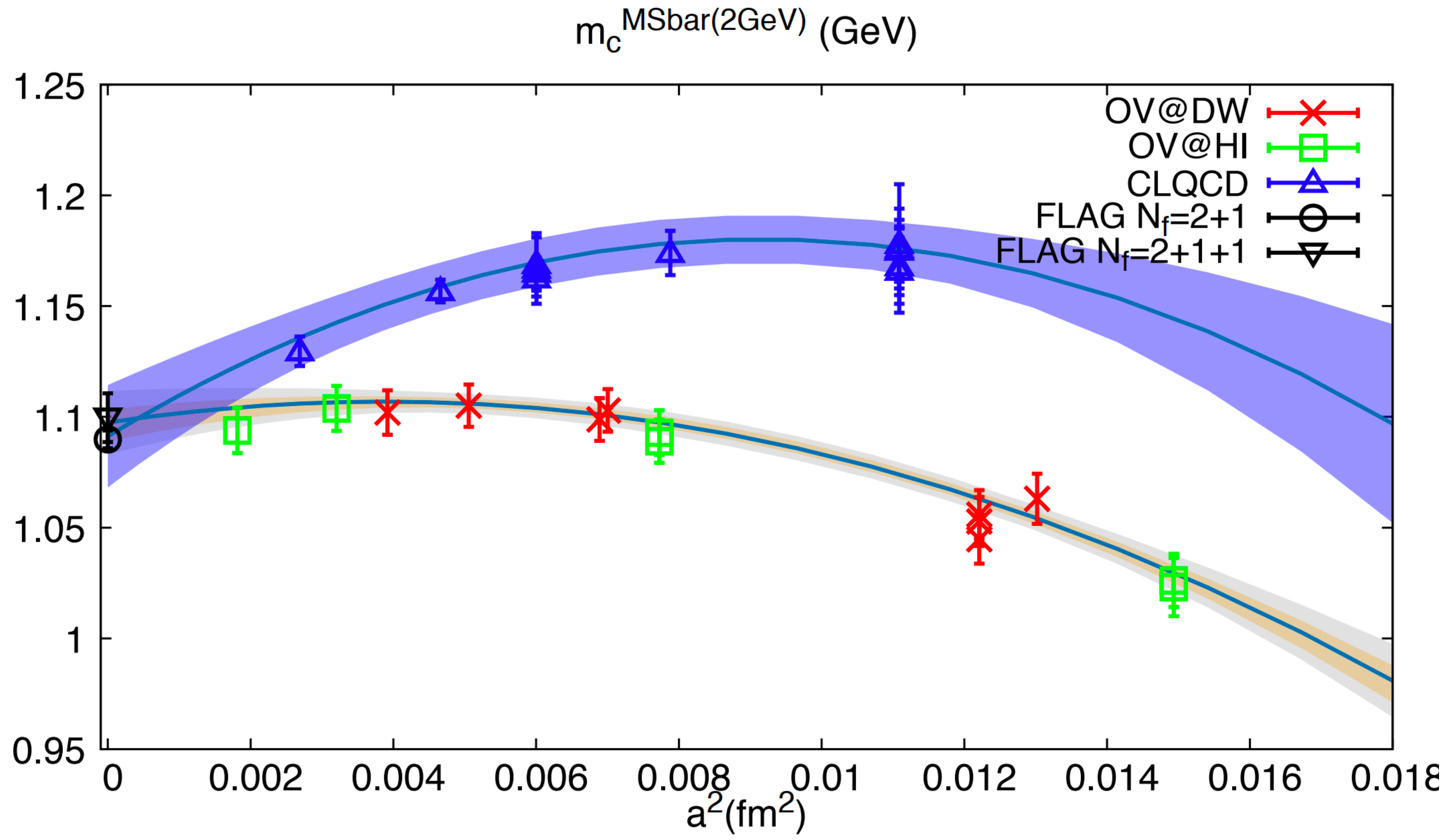
$$m_{D_s}^{\rm QCD} = m_{D_s}^{\rm phys} - \Delta^{\rm QED} m_{D_s} = 1966.7(1.5)\ \text{MeV}\,.$$

RM123, Phys.Rev.D100 (2019)
1904.08731

- Use etas to determine the valence strange quark mass;
  - Significantly suppress the strange quark mass dependence on each ensemble.
- Use QED-subtracted m_{D_s} mass to determine the pure QCD valence charm quark mass;
  - $\Delta^{\rm QED} m_{D_s}$ is determined to be 2.3(4) MeV under the $m_{q,\rm QCD+QED}^{\overline{\rm MS}}(2\text{GeV}) = m_{q,\rm QCD}^{\overline{\rm MS}}(2\text{GeV})$ scheme.
- Eliminate the effects from unphysical light and strange sea quark masses using the joint fit.

Renormalized quark masses

Charm quark mass



Dian-Jun Zhao, et.al., χQCD, in preparation

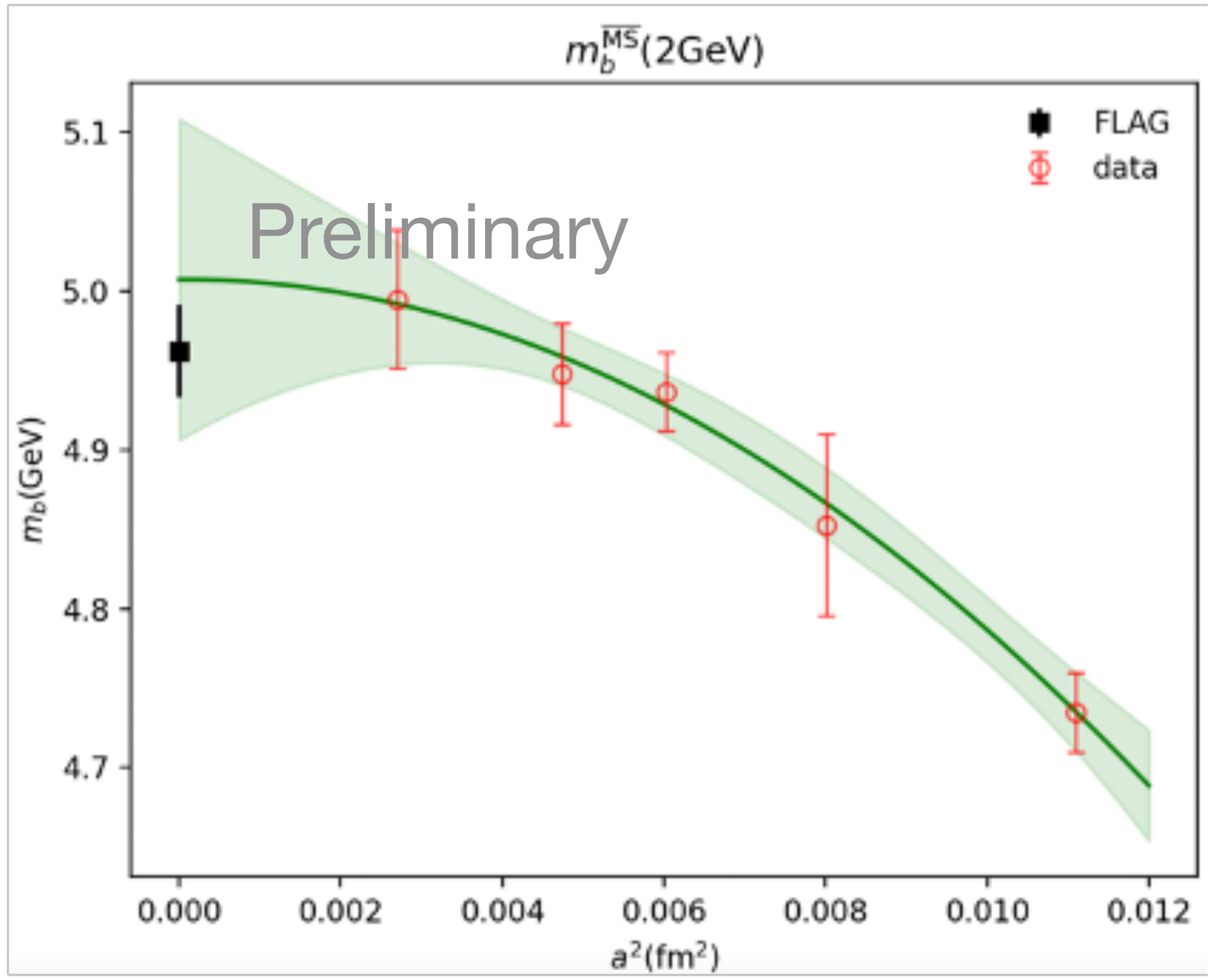
Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

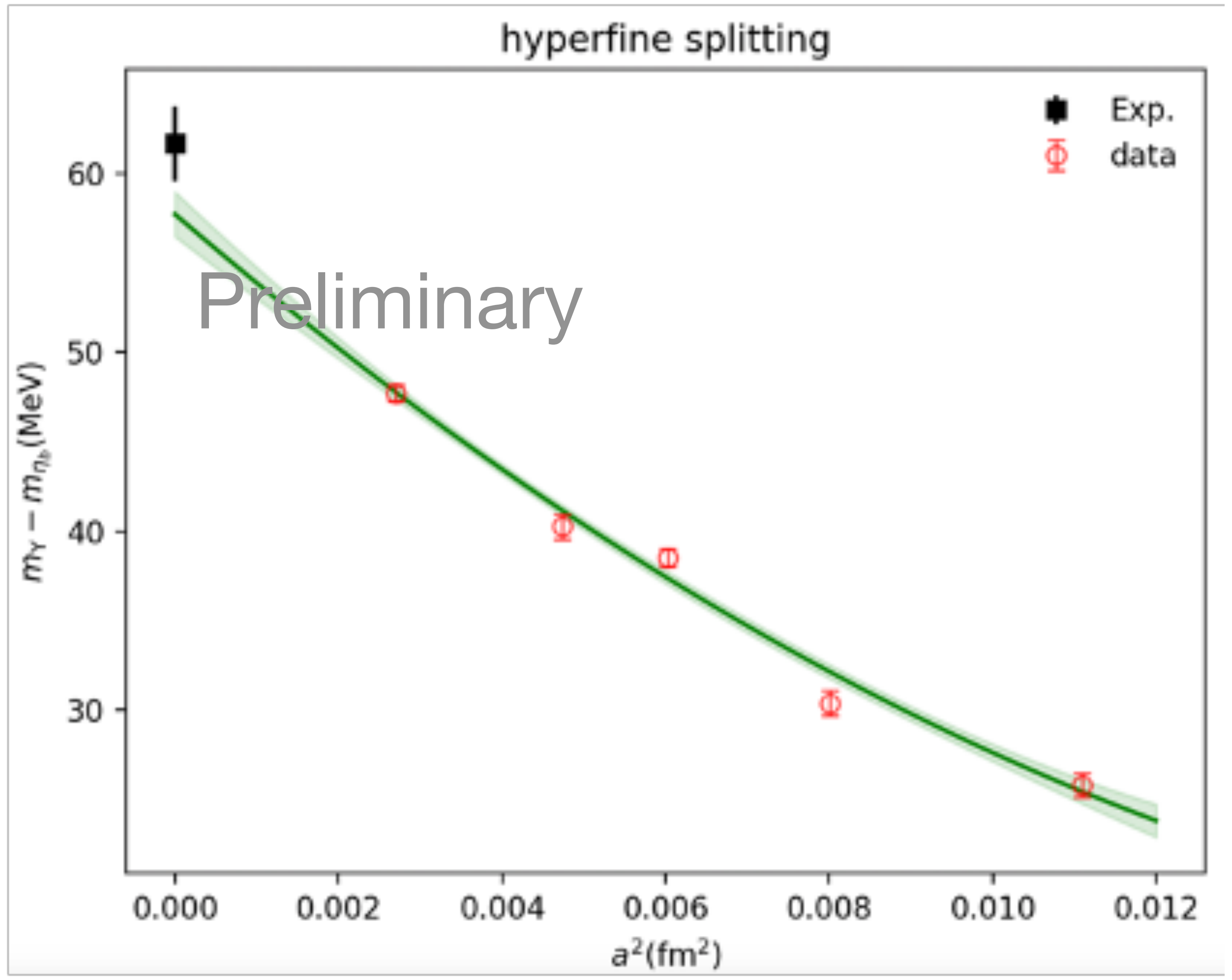
Based on the $a^2 + a^4$ extrapolation:

- The prediction based on the Overlap fermion (χ QCD) and also Clover fermion (CLQCD) agrees within 1-2%.
- Such a value is similar to the current lattice averages within ~1%.

Bottom quark physics

from CLQCD ensemble





H.-Y. Du, et. al., CLQCD, in preparation

- Determine the bare bottom quark mass using the physical Υ mass using the heavy quark improved action;
- The hyperfine splitting $m_\Upsilon - m_{\eta_b}$ and $m_b^{\overline{\text{MS}}}(2\text{ GeV})$ agree with the experimental/FLAG values within 2%.
- More systematic study is in progress.

Outline

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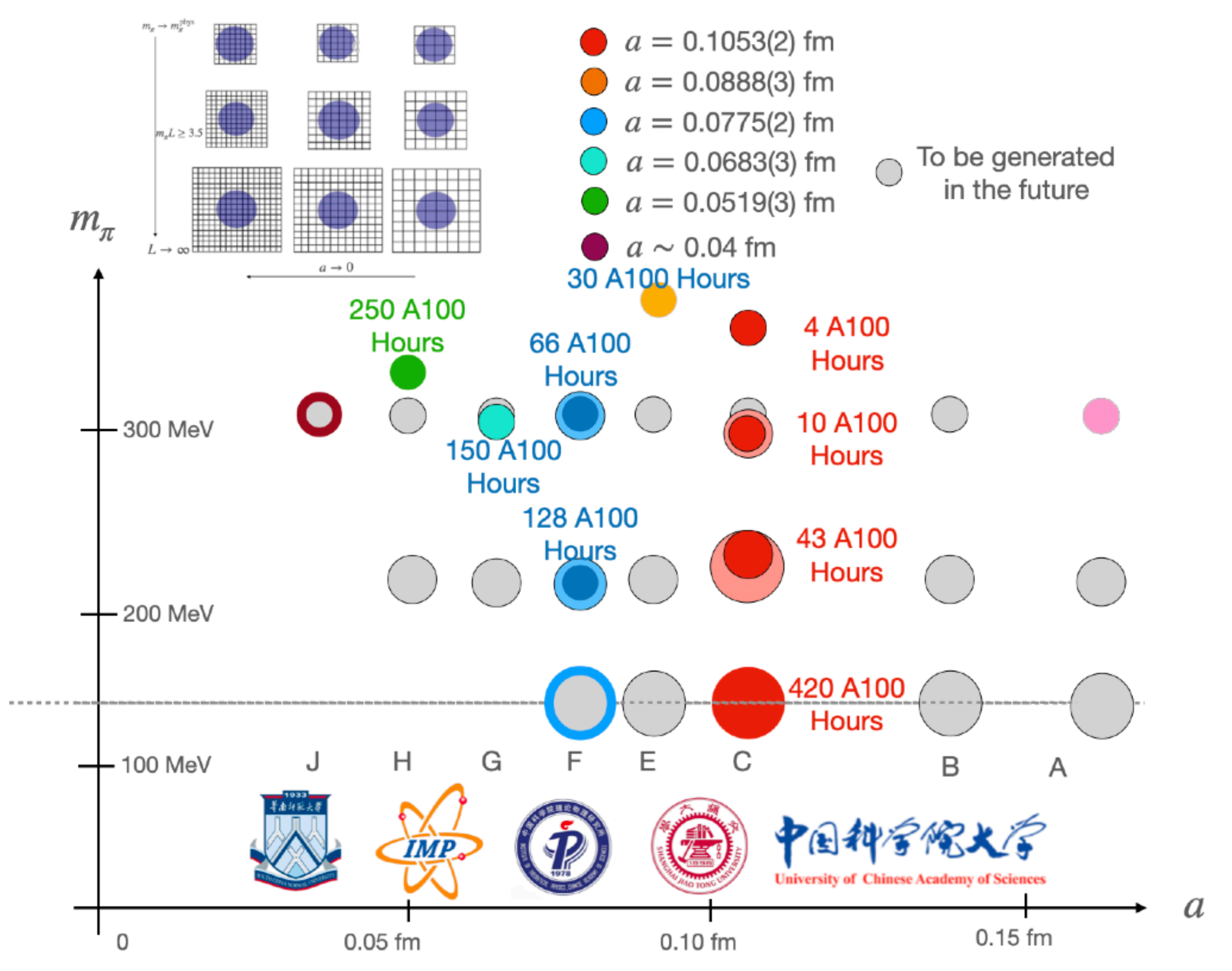
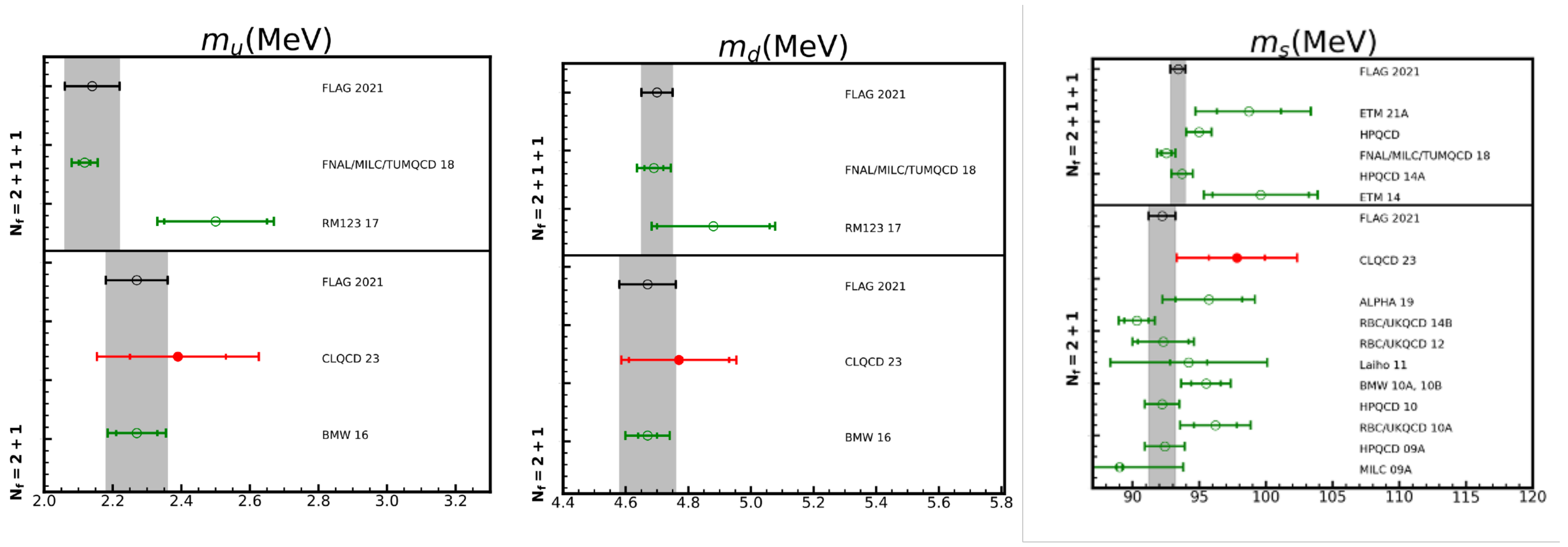
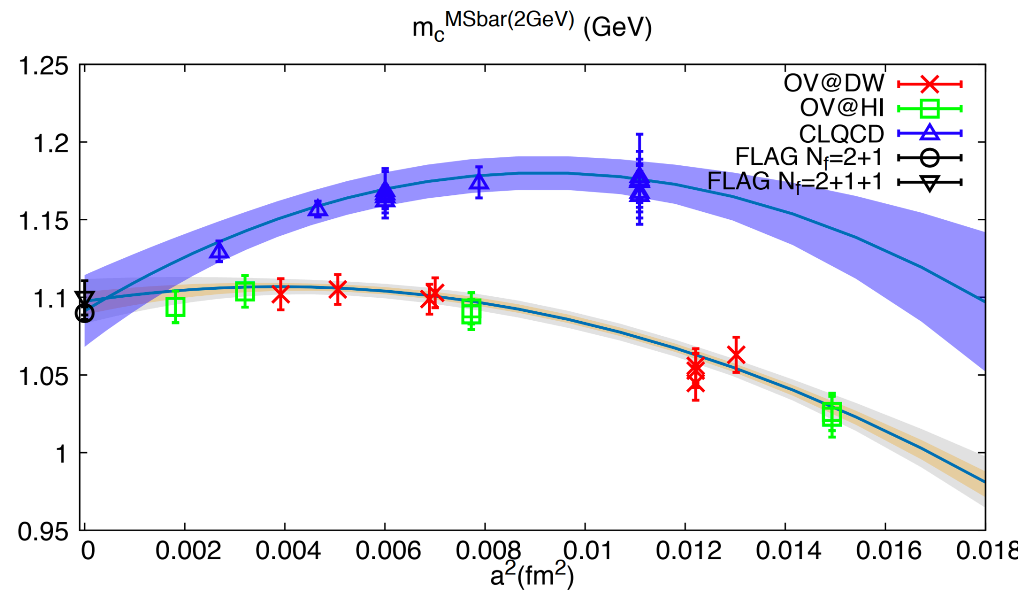
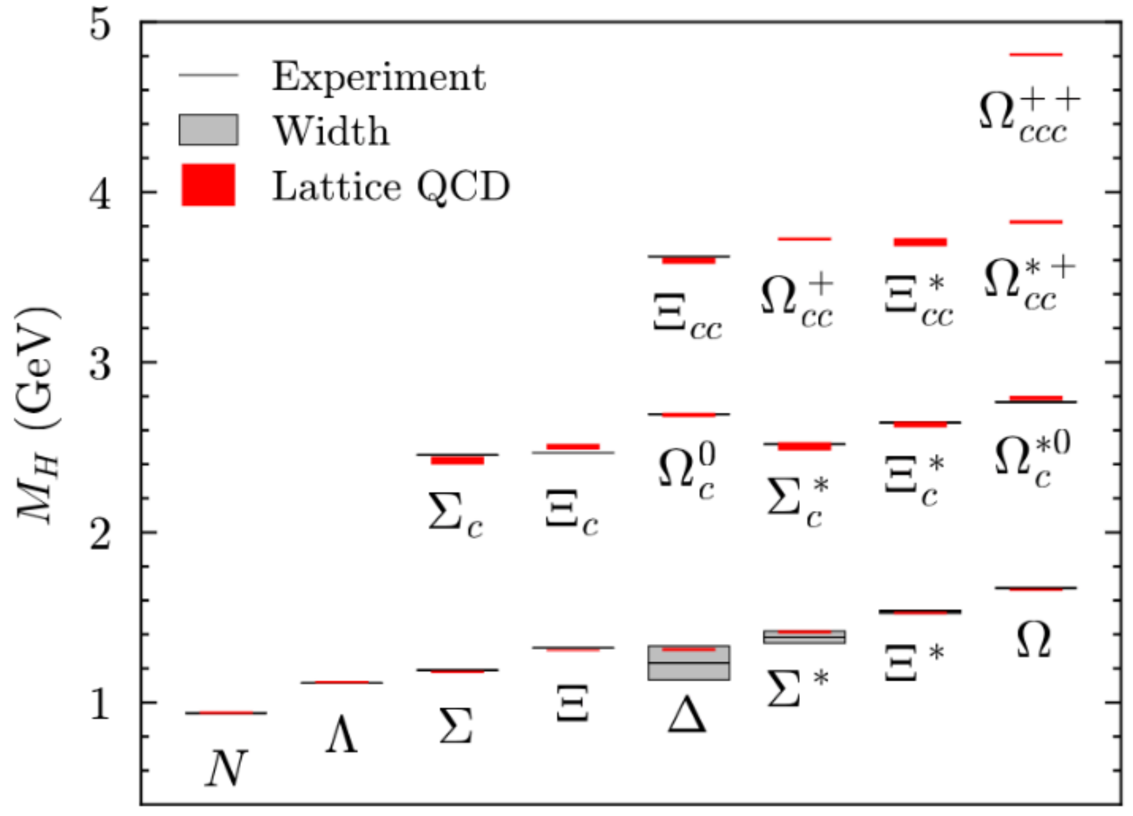


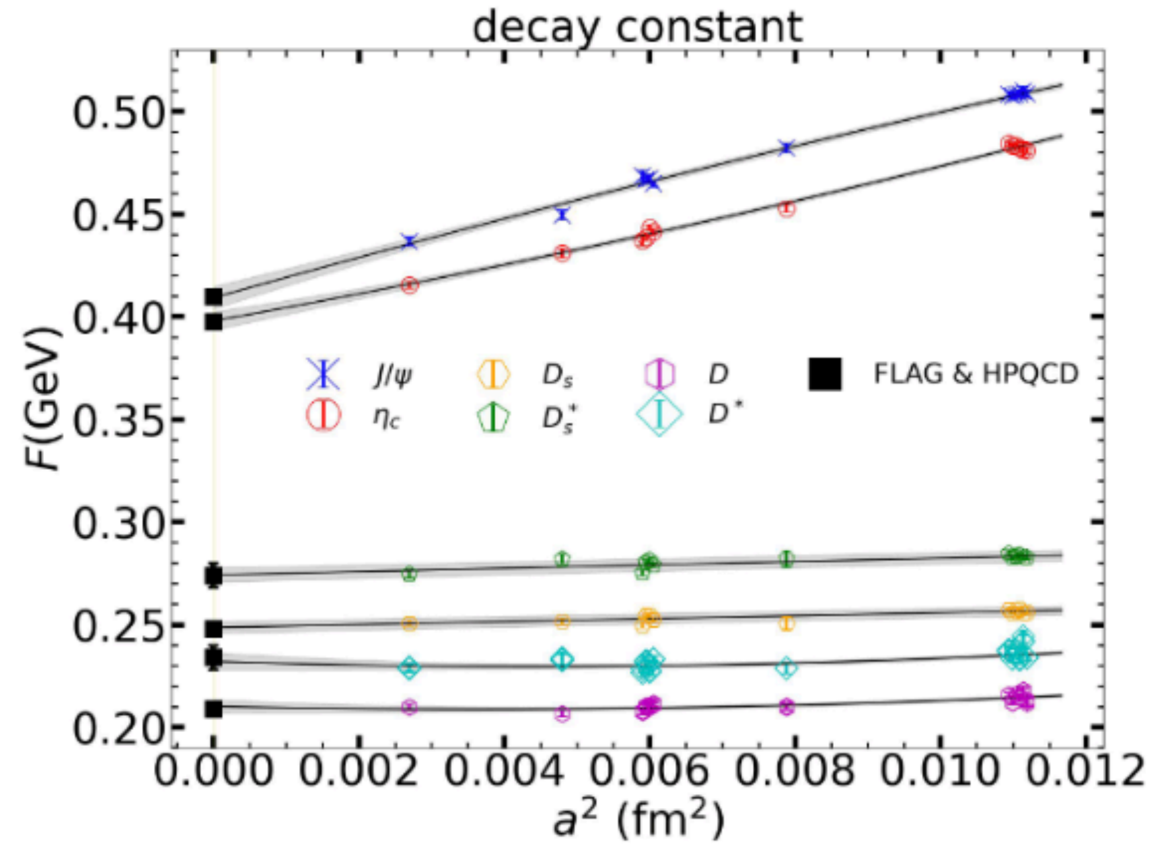




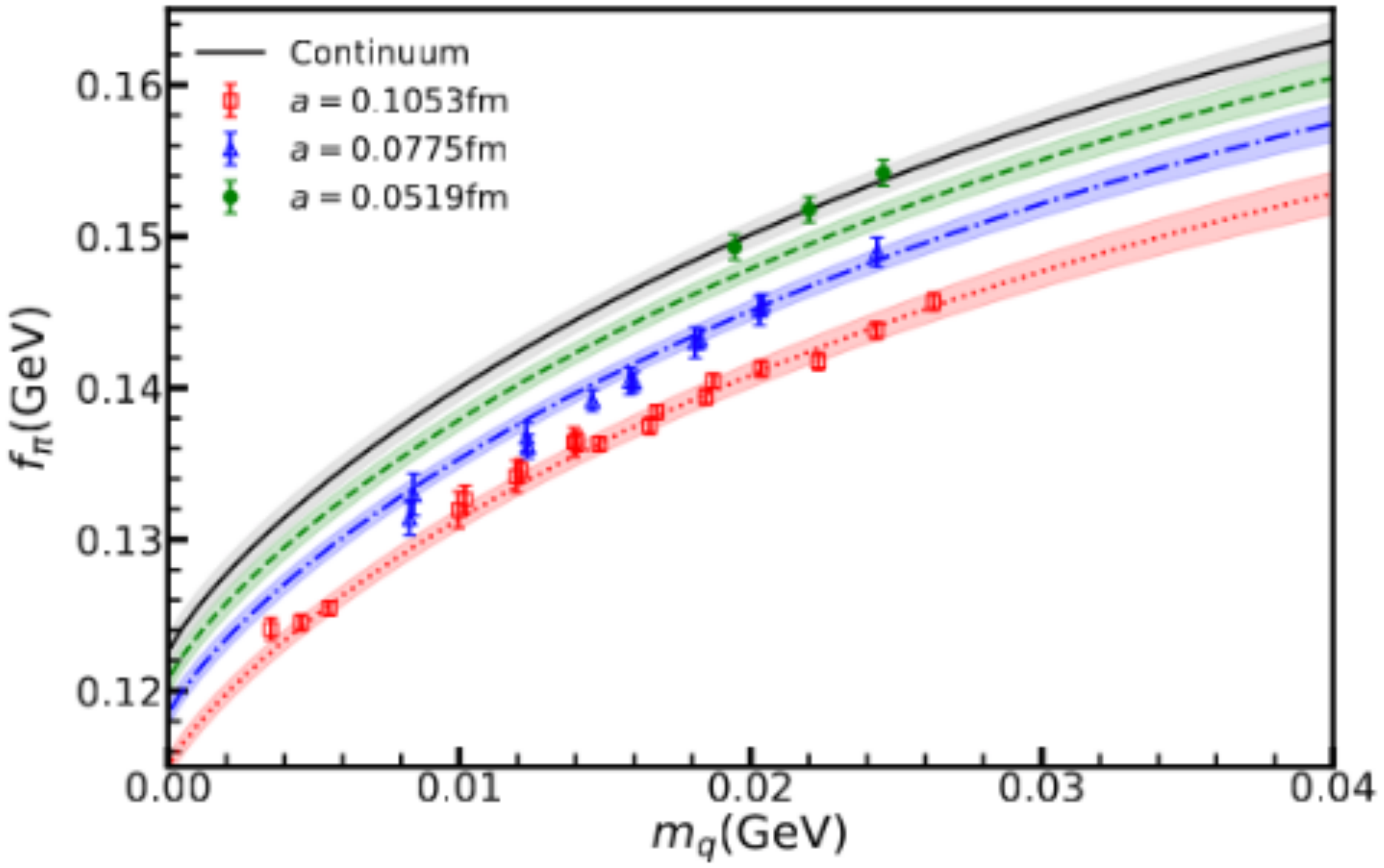
- Quark mass determinations

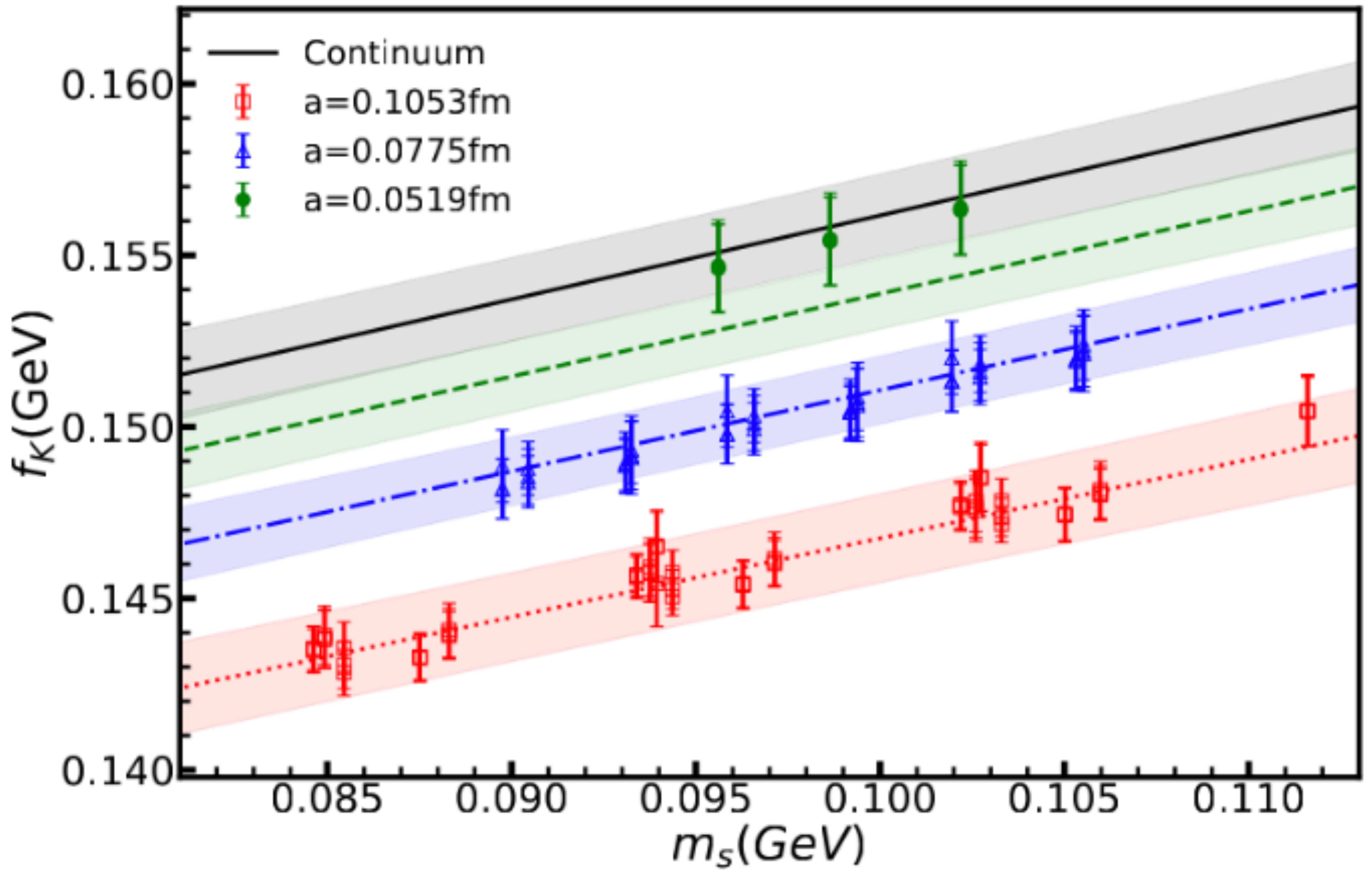
- **Hadron masses and decay constants**





Decay constants





Pion and Kaon cases

$$\frac{f_K}{f_\pi} = 1.1907(76)(03)$$

$$\downarrow$$

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\text{exp}}(20)_{\text{th}} \longrightarrow$$

$$|V_{ud}| = 0.9740(03)_{\text{lat}}(01)_{\text{ph}}$$

$$|V_{us}| = 0.2265(14)_{\text{lat}}(03)_{\text{ph}}$$

$$\uparrow$$

$$|V_{us}| = 0.2243(8)_{\text{PDG}}$$

$$1 = |V_{ud}|^2 + |V^2_{us}| + |V_{ub}|^2 = |V_{ud}|^2 + |V^2_{us}| + 0.0035^2$$

- Additional input likes the form factor of the semileptonic decay $K^0 \rightarrow \pi^- l\nu$ is required to determine $|V_{ud(s)}|$ directly and verify the unitarity of CKM.

Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

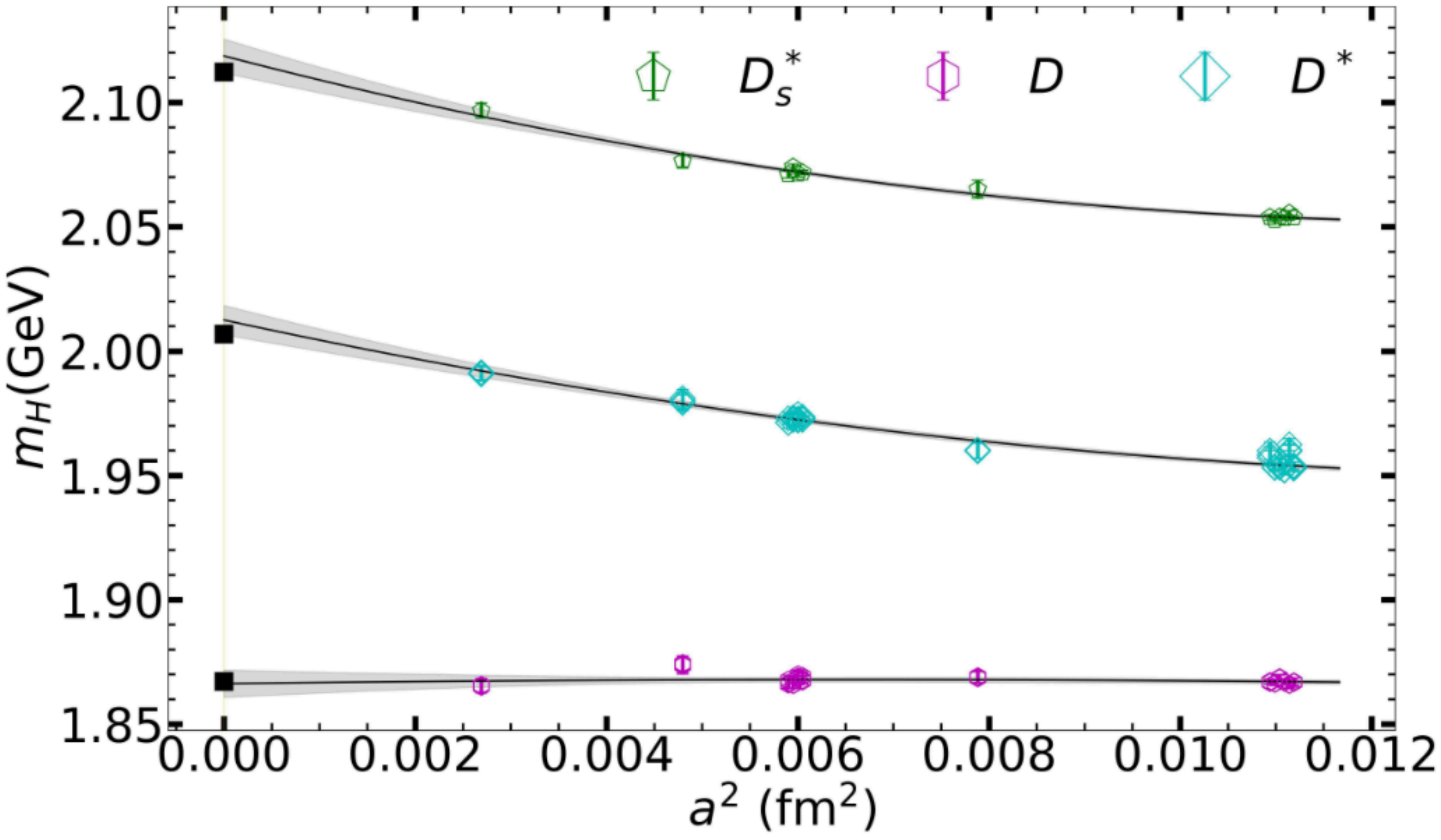
Charmed meson spectrum

Open charm cases

$$m_{D_s}^{\text{QCD}} = m_{D_s}^{\text{phys}} - \Delta^{\text{QED}} m_{D_s} = 1966.7(1.5)\ \text{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- m_D is almost constant at different lattice spacing, with $m_D^{\pm} - m_D^0 = 2.9(3)_{\text{QCD}} + 2.4(5)_{\text{QED}} = 5.3(3)(5)\ \text{MeV}$;

  RM123, Phys.Rev.D95(2017) 114504
- Agree with the PDG value 4.8(1) MeV well.
- Both m_D^* and $m_{D_s}^*$ have obvious lattice spacing dependence and the continuum extrapolated values agree with PDG well.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

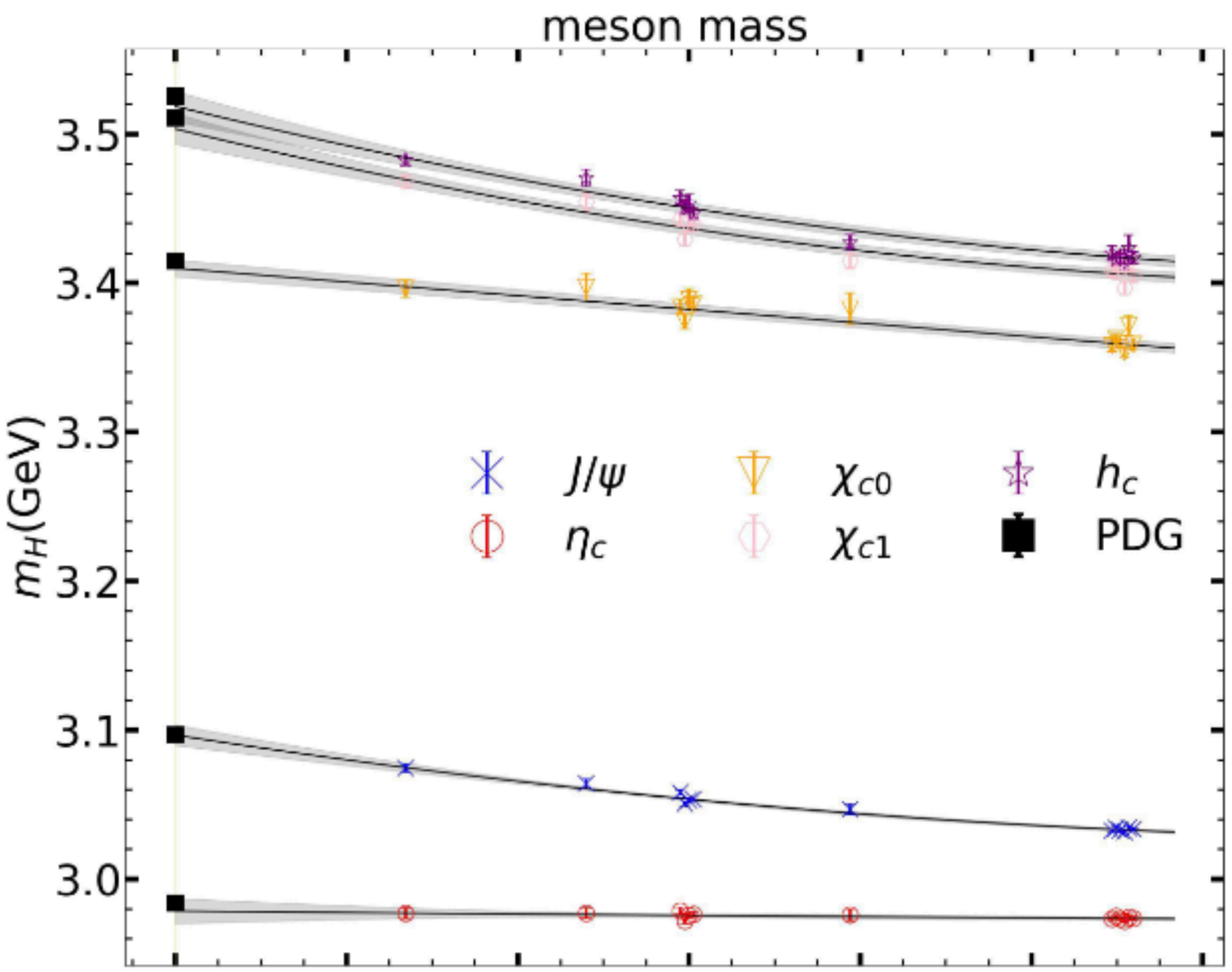
Charmed meson spectrum

charmonium cases

$$m_{D_s}^{\mathrm{QCD}} = m_{D_s}^{\mathrm{phys}} - \Delta^{\mathrm{QED}} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass

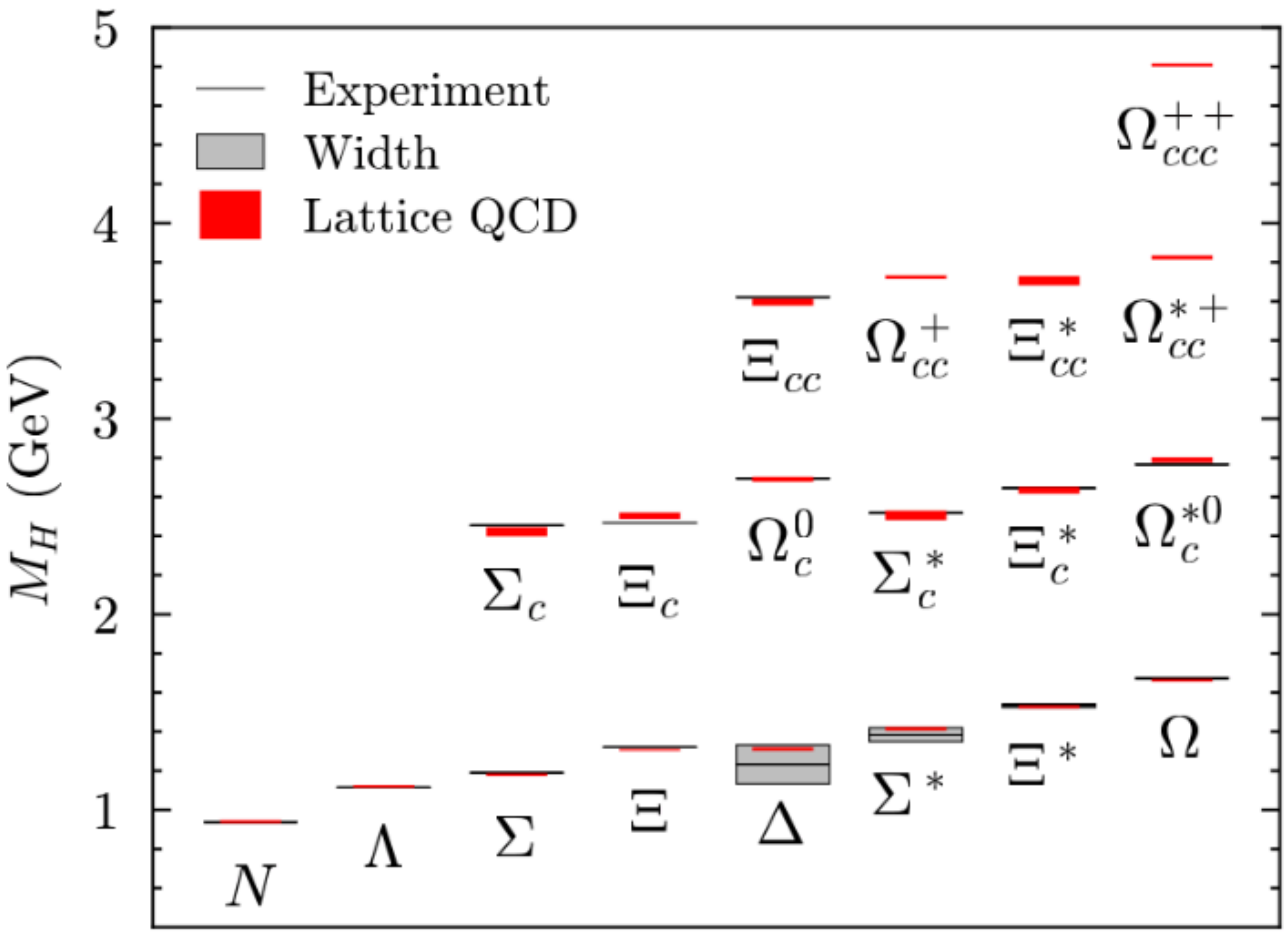


- $m_{J/\psi}$ agrees with PDG well but m_{η_c} is a few MeV lower;
- $m_{J/\psi} - m_{\eta_c} = 118(3)\ \mathrm{MeV}$ agree with previous HPQCD pure QCD prediction 119(1) MeV.
- P-wave charmonium masses also agree with PDG well, with $m_{1P} - m_{1S} = 451(11)\ \mathrm{MeV}$.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

Baryon spectrum

of four light flavors



B.-L. Hu, et. al., CLQCD, in preparation

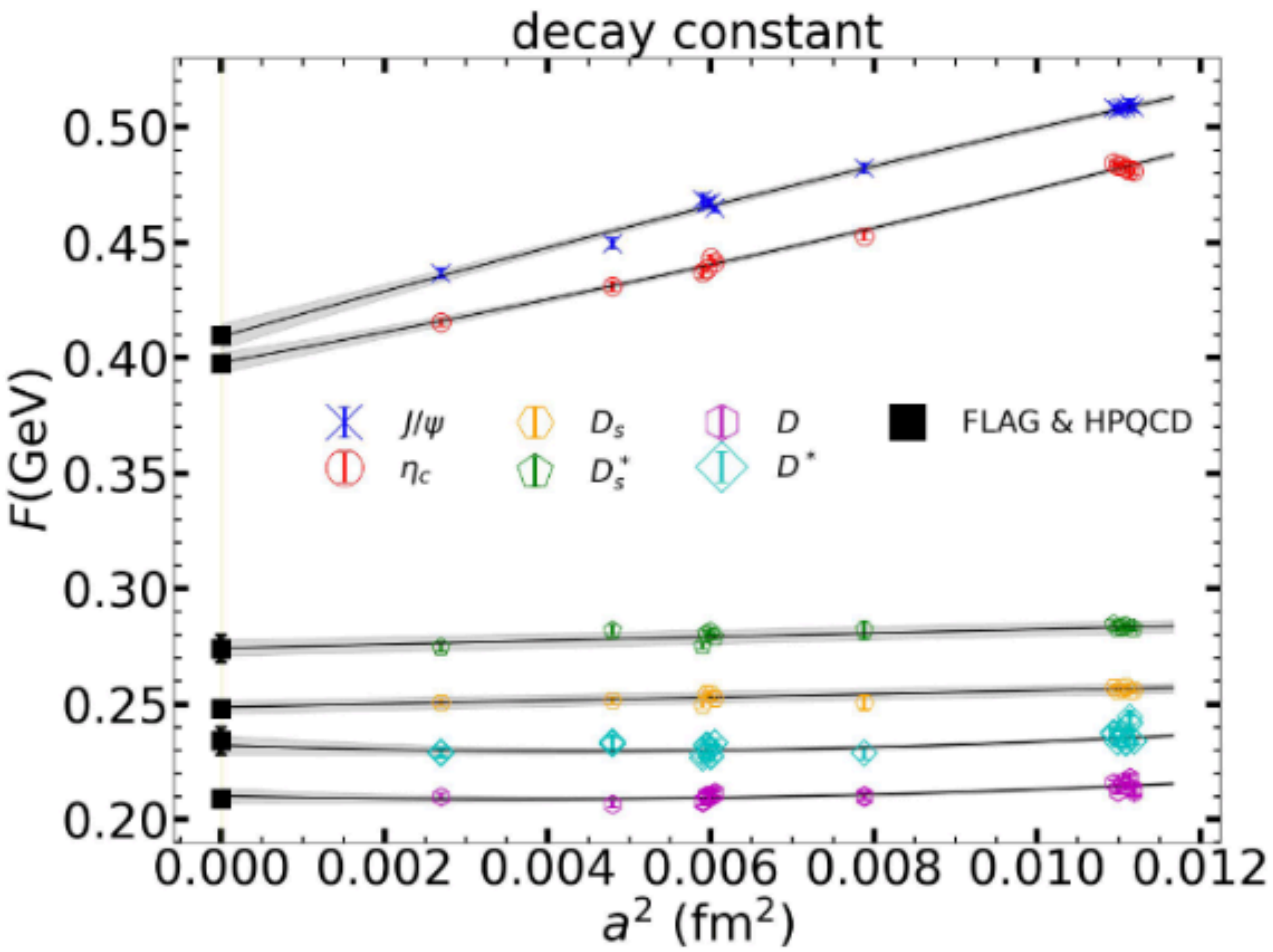
- Generally agree with the PDG values at 1% level;
- Trace anomaly contribution to all the baryon including the charmed one is under investigation:

$$m_H = m\langle\bar{\psi}\psi\rangle_H + \left[\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\langle\bar{\psi}\psi\rangle_H + \left[\left(-\frac{11}{8\pi} + \frac{N_f}{12\pi}\right)\alpha_s + \mathcal{O}(\alpha_s^2)\right]\langle F^2\rangle_H .$$

- The missing QED effect will be investigated in the near future.

Decay constants

Open charm cases



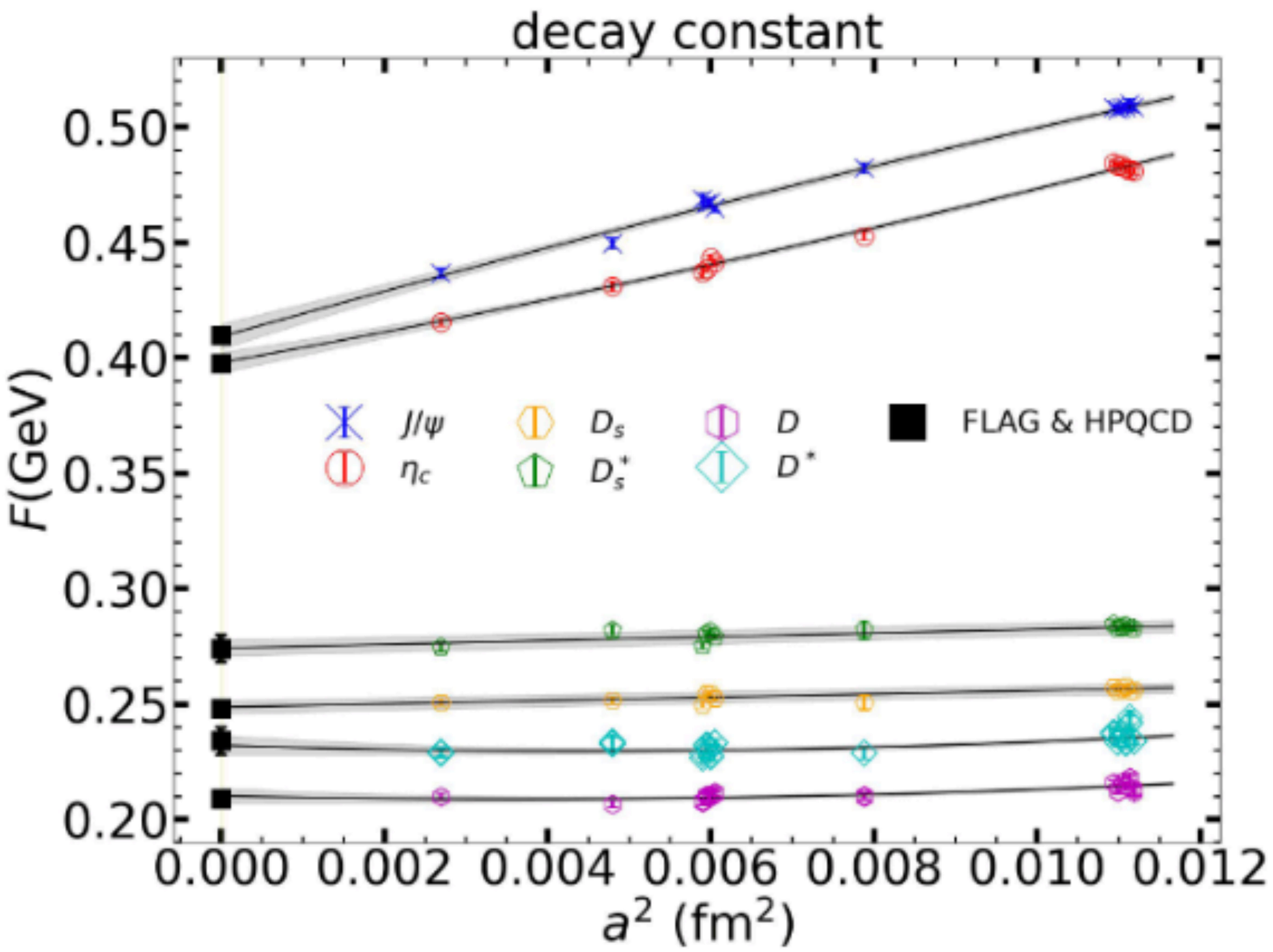
$$\begin{tikzcd}
f_{D^+} = 0.2102(33)_{\text{lat}}\ \text{MeV} \arrow[d] & \\
f_{D^+}|V_{cd}| = 45.8(1.1)_{\text{exp}}\ \text{MeV} \arrow[r] & |V_{cd}| = 0.2179(33)_{\text{lat}}(52)_{\text{exp}} \\
f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\text{exp}}\ \text{MeV} \arrow[r] & |V_{cs}| = 0.979(11)_{\text{lat}}(11)_{\text{exp}} \\
f_{D_s^+} = 0.2487(28)_{\text{lat}}\ \text{MeV} \arrow[u] &
\end{tikzcd}$$

- Verified the unitarity of CKM matrix elements involving the charm quark: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.008(23)(23)$.
- Also provide the most precise f_{D^*} and $f_{D_s^*}$ so far.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

Decay constants

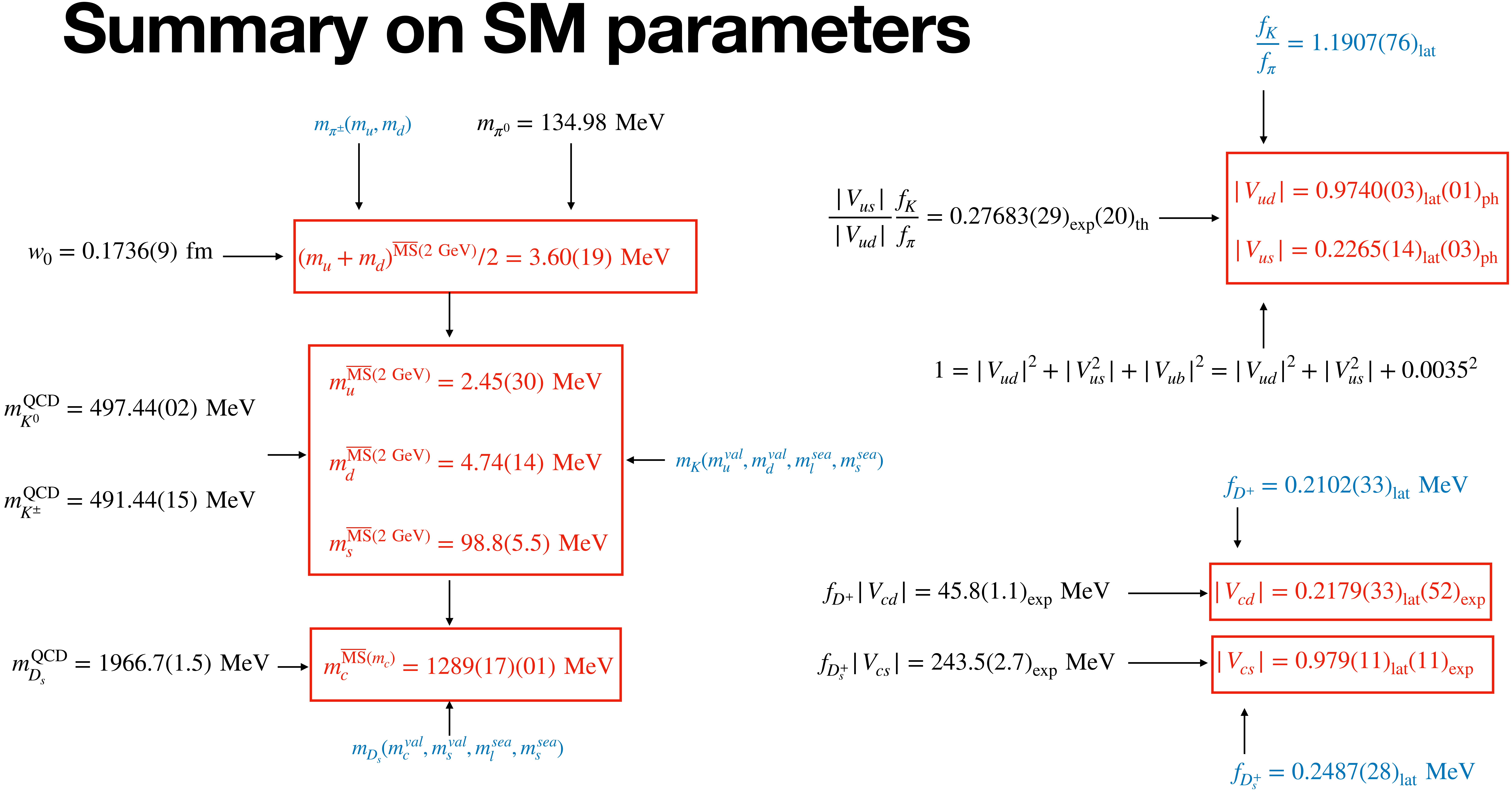
S-wave charmonium



- Our prediction $f_{J/\psi} = 405.9(5.7)$ MeV is consistent with the experimental value 406.5(3.7)(0.5) MeV and also HPQCD prediction 409.6(1.6) MeV;
- We also predict $f_{\eta_c} = 398.1(4.6)$ MeV which is consistent with the HPQCD prediction 397.5(1.0) MeV.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

Summary on SM parameters



$m_{\pi^\pm}(m_u, m_d)$

$m_{\pi^0} = 134.98$ MeV

$w_0 = 0.1736(9)$ fm

$\boxed{(m_u + m_d)^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})}/2 = 3.60(19)\ \mathrm{MeV}}$

$m_{K^0}^{\mathrm{QCD}} = 497.44(02)$ MeV

$m_{K^\pm}^{\mathrm{QCD}} = 491.44(15)$ MeV

$$\boxed{\begin{array}{l} m_u^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 2.45(30)\ \mathrm{MeV} \\ m_d^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 4.74(14)\ \mathrm{MeV} \\ m_s^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 98.8(5.5)\ \mathrm{MeV} \end{array}}$$

$m_K(m_u^{val}, m_d^{val}, m_l^{sea}, m_s^{sea})$

$m_{D_s}^{\mathrm{QCD}} = 1966.7(1.5)$ MeV

$\boxed{m_c^{\overline{\mathrm{MS}}(m_c)} = 1289(17)(01)\ \mathrm{MeV}}$

$m_{D_s}(m_c^{val}, m_s^{val}, m_l^{sea}, m_s^{sea})$

$\frac{f_K}{f_\pi} = 1.1907(76)_{\mathrm{lat}}$

$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\mathrm{exp}}(20)_{\mathrm{th}}$

$$\boxed{\begin{array}{l} |V_{ud}| = 0.9740(03)_{\mathrm{lat}}(01)_{\mathrm{ph}} \\ |V_{us}| = 0.2265(14)_{\mathrm{lat}}(03)_{\mathrm{ph}} \end{array}}$$

$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$

$f_{D^+} = 0.2102(33)_{\mathrm{lat}}$ MeV

$f_{D^+}|V_{cd}| = 45.8(1.1)_{\mathrm{exp}}$ MeV

$\boxed{|V_{cd}| = 0.2179(33)_{\mathrm{lat}}(52)_{\mathrm{exp}}}$

$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\mathrm{exp}}$ MeV

$\boxed{|V_{cs}| = 0.979(11)_{\mathrm{lat}}(11)_{\mathrm{exp}}}$

$f_{D_s^+} = 0.2487(28)_{\mathrm{lat}}$ MeV

Summary

- The state-of-the-arts Lattice QCD ensemble should have enough ensembles to approach the continuum, infinite volume and physical quark masses reliably; and the present CLQCD ensembles have been close to this goal.

- Up, down, strange and charm quark masses have been determined at a few percent level;

- The charmed meson and baryon masses are predicted at ~0.3% uncertainty and agree with the experimental values at 1% level.

- More predictions are in progress.