$\bar{B}^0 \to \bar{K}^{(*)0} X$, $B^- \to K^{(*)-} X$, $\bar{B}_s^0 \to \eta(\eta', \phi) X$ Decays from the molecular picture of $X(3872)$

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Outline

- Introduction and motivation
- Formalism
- Results and discussions
- Summary

Introduction and motivation

$$X(3872) \text{ or } \chi_{c1}(3872): \text{ proximity to the } D^{*0}\bar{D}^0 \text{ threshold}; \quad J^{PC}=1^{++}, I=0$$

- **The nature of *X*(3872) has been the subject of intense debate.**

  - Molecular state $(D^*\bar{D}+c.c.)$

    E. Braaten, L.P. He, J. Jiang, PRD103 (2021)036014;
    M.Z. Liu, L.S. Geng, EPJC81(2021)179;
    M. C. Gordillo, F. De Soto, J. Segovia, PRD104 (2021)054036;
    L. Meng, G.J. Wang, B. Wang, S.L. Zhu, PRD104(2021) 094003;
    Y. Kamiya, T. Hyodo, A. Ohnishi, EPJA58(2022)131;
    T. Ji, X.K. Dong, M. Albaladejo, M.L. Du, F.K. Guo, J. Nieves, PRD106(2022)094002;
    Y. Wang, Q. Wu, G. Li, W.H. Qin, X.H. Liu, C.S. An, J.J. Xie, PRD106(2022)074015; … ...

  - Compact tetraquark state

    P.P. Shi, F. Huang, W.L. Wang, PRD103(2021)094038;
    A. Esposito, L. Maiani, A. Pilloni, A.D. Polosa, V. Riquer, PRD105(2022) L031503;
    H.X. Huang, X. Jin, Y. Tan, J.L. Ping, EPL135(2021)31001;
    A. Sharma, A. Upadhyay, Phys.Scr. 98(2023)095308; … …

  - Mixture of the two structures

    R.F. Lebed, S.R. Martinez, PRD106(2022)074007;
    G.J. Wang, Z. Yang, J.J. Wu, M. Oka, S.L. Zhu, arXiv:2306.12406; … …

Introduction and motivation

- **The reactions of *B* meson decaying into P+X(3872) or V+X(3872) provide a means to learn about the nature of the X(3872).**

$B^- \to K^- X(3872)$ and $\bar{B}^0 \to \bar{K}^0 X(3872)$ were proposed in **[H.N. Wang, L.S. Geng, Q. Wang, J.J. Xie, CPL40(2023)021301].**

➢ With a compact tetraquark nature for the X(3872):

The branching ratio of the two decay modes is expected to be equal.

➢ Experimentally: $$\frac{\mathcal{B}(B^- \to K^- X(3872))}{\mathcal{B}(\bar{B}^0 \to \bar{K}^0 X(3872))} \simeq 2$$

➢ From the molecular nature for the X(3872):

This ratio is tied to the loop functions G_i for the neutral and charged components $D^{*0}\bar{D}^0$, $D^{*+}D^-$.

Our work:

To extend the idea to study *B* meson decays into P+X(3872) and V+X(3872), from the molecular nature of the X(3872), considering both the mechanism of external and internal emission.

Formalism

A. The molecular $X(3872)$ state

The extended local hidden gauge approach is used, with the coupled channels

$$X_{\rm c} = \frac{1}{\sqrt{2}}(D^{*+}D^{-} - D^{*-}D^{+}), \qquad X_{\rm n} = \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^{0} - \bar{D}^{*0}D^{0}).$$

The interaction matrix is

$$V = \begin{pmatrix} \tilde{v} & \tilde{v} \\ \tilde{v} & \tilde{v} \end{pmatrix}, \quad \tilde{v} = -g^2 \frac{4\, m_{D^{*0}}\, m_{D^0}}{M_V^2}; \qquad g = \frac{M_V}{2\, f}; \quad M_V = 800 \ \text{MeV}, \quad f = 93 \ \text{MeV},$$

The isospin states：

$$|X, I=0\rangle = \frac{1}{\sqrt{2}}(X_{\rm c} + X_{\rm n}),$$ (The interaction is attractive, can produce a bound state.)

$$|X, I=1, I_3=0\rangle = \frac{1}{\sqrt{2}}(X_{\rm c} - X_{\rm n}),$$ (The interaction is zero.)

◆ Formalism

The scattering matrix:

$$T = [1 - VG]^{-1} V = \frac{1}{\det} \begin{pmatrix} \tilde{v} & \tilde{v} \\ \tilde{v} & \tilde{v} \end{pmatrix}, \qquad \det = 1 - \tilde{v} G_1 - \tilde{v} G_2,$$

$$G_i = \int_{|\vec{q}|<q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1^{(i)}(\vec{q}) + \omega_2^{(i)}(\vec{q})}{2\,\omega_1^{(i)}(\vec{q})\,\omega_2^{(i)}(\vec{q})} \frac{1}{s - [\omega_1^{(i)}(\vec{q}) + \omega_2^{(i)}(\vec{q})]^2 + i\epsilon}, \qquad (q_{\max} = 420 \text{ MeV})$$

with $\omega_j^{(i)}(\vec{q}) = \sqrt{m_j^2 + \vec{q}^{\,2}}$ the energy of particle j in the i-th channel.

From the molecular point of view, the wave function of X(3872) is given by

$$X = \frac{1}{\sqrt{2}}(X_c + X_n) = \frac{1}{2}(D^{*+}D^- - D^{*-}D^+ + D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0), \qquad (12)$$

with $I = 0$, $C = +$, $J^P = 1^+$ in S-wave. [completing the quantum numbers of the X(3872)]

Formalism

The couplings of the X(3872) to the X_c or X_n components:

$$g_1^2 = \lim_{s \to s_0} (s - s_0) T_{11}, \qquad g_1 g_2 = \lim_{s \to s_0} (s - s_0) T_{12},$$

with s_0 the squared of the energy of the $X(3872)$ state.

$$\Longrightarrow \quad g_1^2 = \frac{\tilde{v}}{-\tilde{v} \frac{\partial}{\partial s} (G_1 + G_2)} = g_1 g_2.$$

$$\Longrightarrow \quad g_1 = g_2 \; (\text{or } g_c = g_n), \qquad g_n / g_c = 1$$

◆ Formalism

B. Internal emission for pseudoscalar production

➢ $\bar{B}^0 \to \bar{K}^0 X(3872), \quad B^- \to K^- X(3872)$

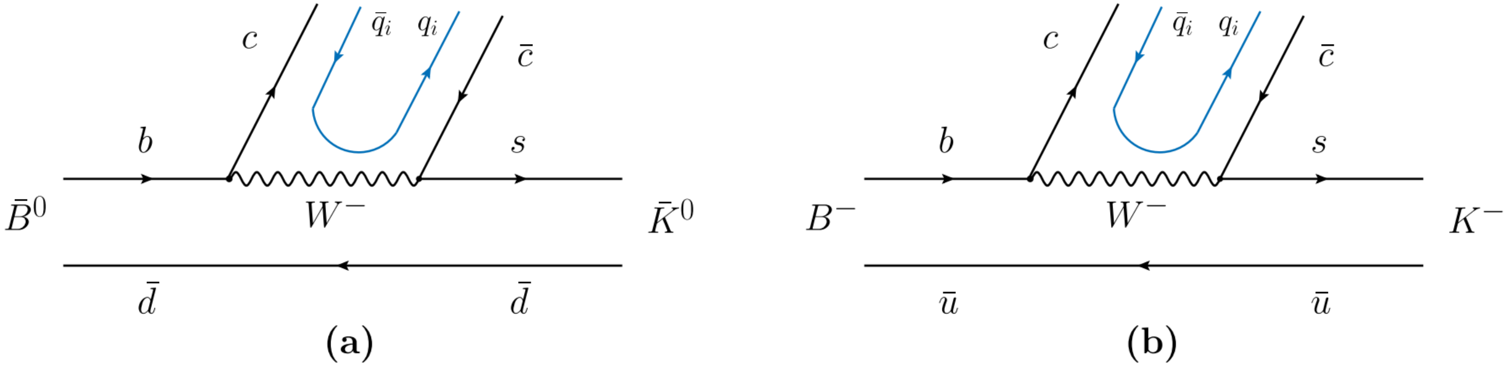


FIG. 1. Internal emission mechanism at the quark level for $\bar{B}^0 \to \bar{K}^0 c\bar{c}$ decay (a) and $B^- \to K^- c\bar{c}$ decay (b).

Hadronization of $c\bar{c}$:

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}, \quad V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}.$$

◆ Formalism

$$\text{a) } PV:\ c\bar{c} \to \sum_i c\bar{q}_i q_i \bar{c} = \sum_i P_{4i} V_{i4} = (PV)_{44} = D^0 \bar{D}^{*0} + D^+ D^{*-} + \cdots,$$

$$\text{b) } VP:\ c\bar{c} \to \sum_i c\bar{q}_i q_i \bar{c} = \sum_i V_{4i} P_{i4} = (VP)_{44} = D^{*0} \bar{D}^0 + D^{*+} D^- + \cdots,$$

$\Longrightarrow$ $(VP)_{44} - (PV)_{44} = D^{*+}D^- + D^{*0}\bar{D}^0 - D^{*-}D^+ - \bar{D}^{*0}D^0$ [A perfect match with W.F. of the X(3872)]

➢ $\bar{B}_s^0 \to \eta(\eta')\, X(3872)$

Hadronization of $c\bar{c}$: Same as before.

$$s\bar{s} \to -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta'.$$

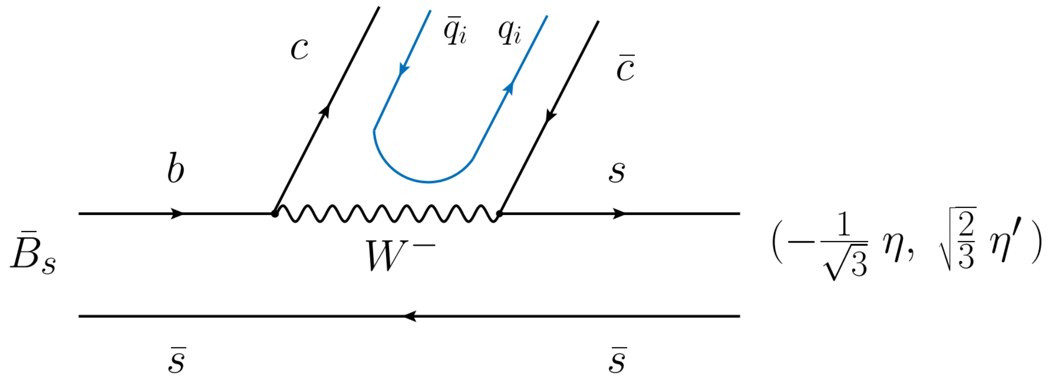


FIG. 2. Internal emission mechanism at the quark level for $\bar{B}_s^0 \to \eta(\eta')c\bar{c}$ decay.

Formalism

C. External emission for pseudoscalar production

- $\bar{B}^0 \to \bar{K}^0\, X(3872), \quad B^- \to K^-\, X(3872)$

Three possibilities upon hadronization:

(a) $\bar{c}s \to PV,\ c\bar{d} \to P(D^+)$:

$$s\bar{c} \to \sum_i s\bar{q}_i q_i \bar{c} = \sum_i P_{3i} V_{i4} = (PV)_{34}.$$

$$\Longrightarrow \quad \bar{c}sc\bar{d} \to (K^- D^{*0} + \bar{K}^0 D^{*-} + \cdots) D^+.$$

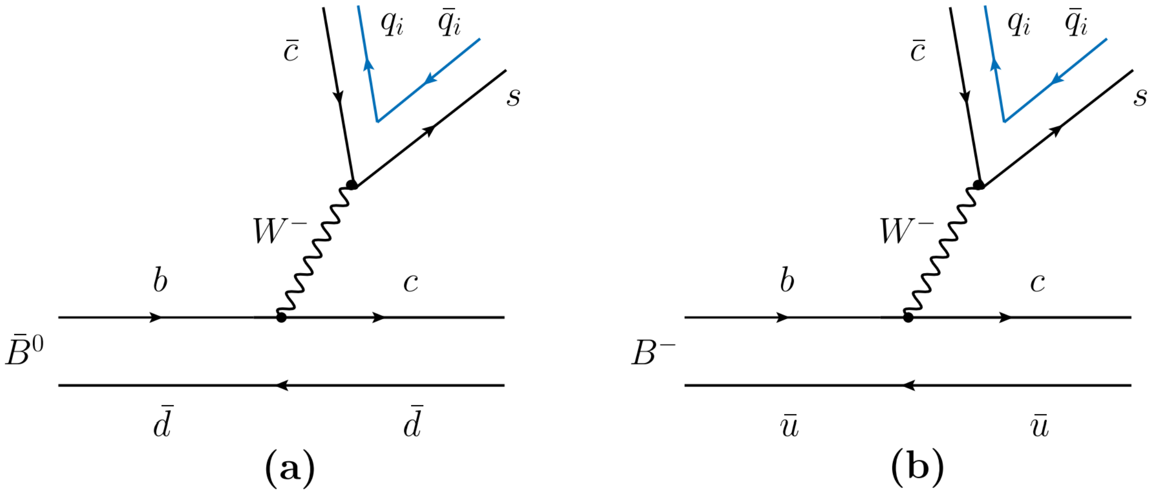


FIG. 3. External emission mechanism for $\bar{B}^0 \to \bar{c}sc\bar{d}$ decay (a) and $B^- \to \bar{c}sc\bar{u}$ decay (b).

(b) $\bar{c}s \to VP,\ c\bar{d} \to P(D^+)$:

$$s\bar{c} \to \sum_i s\bar{q}_i q_i \bar{c} = \sum_i V_{3i} P_{i4} = (VP)_{34}. \quad \Longrightarrow \quad \bar{c}sc\bar{d} \to (K^{*-} \bar{D}^0 + \bar{K}^{*0} D^- + \cdots) D^+.$$

(c) $\bar{c}s \to PP,\ c\bar{d} \to V(D^{*+})$: $\quad s\bar{c} \to \sum_i s\bar{q}_i q_i \bar{c} = \sum_i P_{3i} P_{i4} = (PP)_{34}.$

$$\Longrightarrow \quad \bar{c}sc\bar{d} \to (K^- \bar{D}^0 + \bar{K}^0 D^- + \cdots) D^{*+}.$$

Formalism

The combinations

$$(PP)_{34}D^{*+} - (PV)_{34}D^{+} = (\underline{D^{*+}D^{-} - D^{*-}D^{+}})\bar{K}^{0}, \quad \rightarrow X_c$$

$$(PP)_{34}D^{*0} - (PV)_{34}D^{0} = (\underline{D^{*0}\bar{D}^{0} - \bar{D}^{*0}D^{0}})K^{-}, \quad \rightarrow X_n$$

overlap with the wave function of the X(3872).

- $\bar{B}_s^0 \to \eta\,(\eta')\,X(3872)$

There is no external emission mechanism, proceeding only via internal emission.

- **The rescattering of $D^* \bar{D} + c.c.$:**

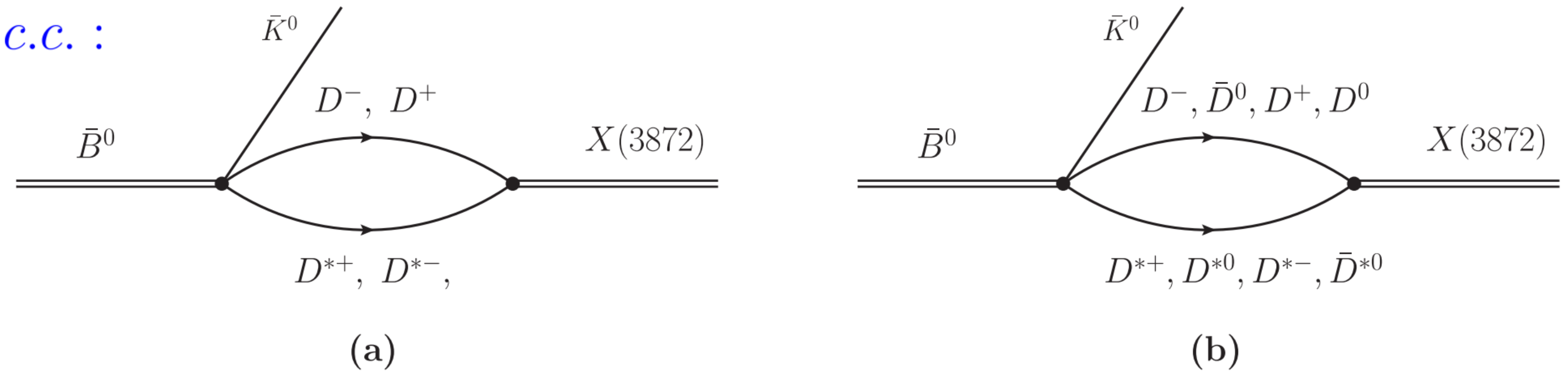


FIG. 4. Propagation of $D\bar{D}^*, \bar{D}D^*$ components to create the $X(3872)$: (a) external emission; (b) internal emission.

- **The amplitudes for the reactions:**

$\bar{B}^0 \to \bar{K}^0\, X(3872)$:

$$t(\bar{K}^0) = CG_{D^{*+}D^-}(M_X)g_c + A[G_{D^{*+}D^-}(M_X)g_c + G_{D^{*0}\bar{D}^0}(M_X)g_n]$$
$$= Cg_c\left\{G_{D^{*+}D^-}(M_X) + A'\left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c}G_{D^{*0}\bar{D}^0}(M_X)\right]\right\}, \qquad (\, A' \equiv A/C\,)$$

$B^- \to K^-\, X(3872)$:

$$t(K^-) = Cg_c\left\{\frac{g_n}{g_c}G_{D^{*0}\bar{D}^0}(M_X) + A'\left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c}G_{D^{*0}\bar{D}^0}(M_X)\right]\right\},$$

$\bar{B}_s^0 \to \eta\, X(3872)$:

$$t(\eta) = Cg_cA'\left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c}G_{D^{*0}\bar{D}^0}(M_X)\right]\left(-\frac{1}{\sqrt{3}}\right),$$

$\bar{B}_s^0 \to \eta'\, X(3872)$:

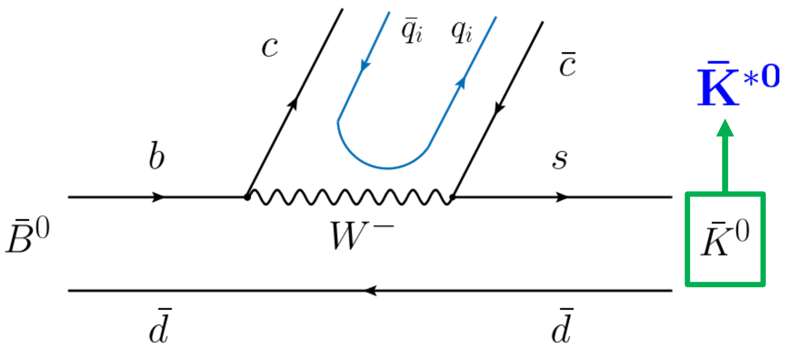
$$t(\eta') = Cg_cA'\left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c}G_{D^{*0}\bar{D}^0}(M_X)\right]\left(\sqrt{\frac{2}{3}}\right).$$

◆ Formalism

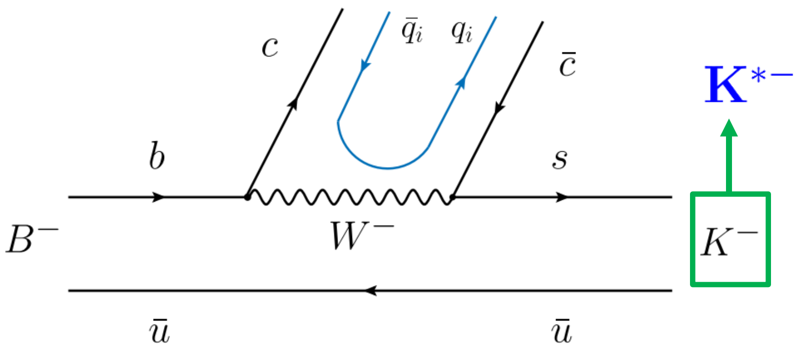
D. Internal emission for vector production

➢ $\bar{B}^0 \to \bar{K}^{*0} X(3872), \quad B^- \to K^{*-} X(3872), \quad \bar{B}_s^0 \to \phi X(3872)$

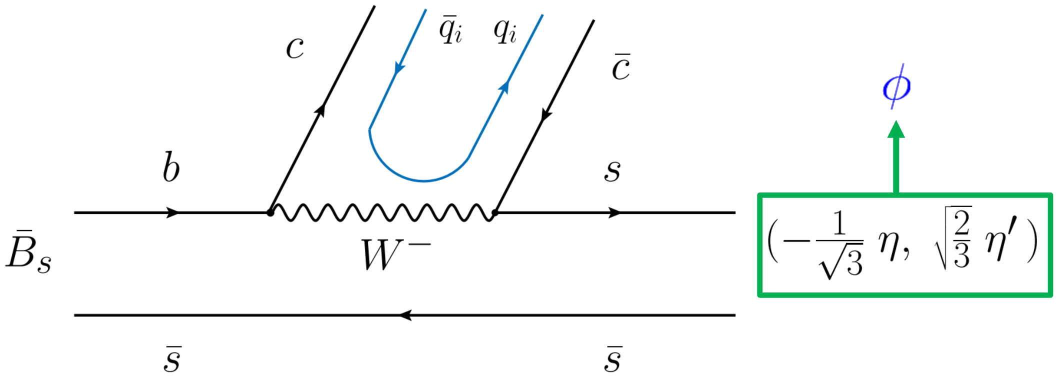
It is identical to the one for pseudoscalar production, replacing $\bar{K}^0$ by $\bar{K}^{*0}$, K^- by K^{*-}, and $s\bar{s}$ by ϕ.



(a)



(b)



◆ Formalism

E. External emission for vector production

$$\bar{B}^0 \to \bar{K}^{*0} X(3872), \quad B^- \to K^{*-} X(3872)$$

(a) $\bar{K}^{*0}$ production:

$$(VV)_{34} D^+ - (VP)_{34} D^{*+} \to \underline{\underline{(-1)}}(D^{*+}D^- - D^{*-}D^+)\bar{K}^{*0};$$

(b) K^{*-} production:

$$(VV)_{34} D^0 - (VP)_{34} D^{*0} \to \underline{\underline{(-1)}}(D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0)K^{*-}.$$

Same as for the pseudoscalar production, replacing K meson with K* meson.

The only difference is the sign, which is relevant to the results obtained.

Formalism

- **The amplitudes:**

$$\bar{B}^0 \to \bar{K}^{*0}\, X(3872): \quad t(\bar{K}^{*0}) = \tilde{C} g_c \left\{ -G_{D^{*+}D^-}(M_X) \; + \tilde{A}' \left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c} G_{D^{*0}\bar{D}^0}(M_X) \right] \right\},$$

$$B^- \to K^{*-}\, X(3872): \quad t(K^{*-}) = \tilde{C} g_c \left\{ -\frac{g_n}{g_c} G_{D^{*0}\bar{D}^0}(M_X) + \tilde{A}' \left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c} G_{D^{*0}\bar{D}^0}(M_X) \right] \right\},$$

$$\bar{B}_s^0 \to \phi\, X(3872): \quad t(\phi) = \tilde{C} g_c \tilde{A}' \left[G_{D^{*+}D^-}(M_X) + \frac{g_n}{g_c} G_{D^{*0}\bar{D}^0}(M_X) \right], \qquad (\tilde{A}' \equiv \tilde{A}/\tilde{C}.)$$

Note : the ratio $A' \equiv A/C$ and $\tilde{A}' \equiv \tilde{A}/\tilde{C}$ indicates the fraction of internal to external emission, relating to the color factor.

We make a reasonable assumption $A' \equiv \tilde{A}'$.

Formalism

- **The decay widths corresponding to these amplitudes are:**

$$\Gamma_{i\to j} = \frac{1}{8\pi}\frac{1}{M_{B_i}^2}|t_{i,j}|^2 p_j F(p_j),$$

$$p_j = \frac{\lambda^{1/2}(M_{B_i}^2, m_j^2, M_X^2)}{2M_{B_i}}, \qquad F(p_j) = \begin{cases} p_j^2, & \text{for pseudoscalar production;} \\ 1, & \text{for vector production.} \end{cases}$$

where B_i is the decaying $B_{(s)}$ meson and j the pseudoscalar or vector produced in addition to the $X(3872)$.

Results and discussions

The present experimental situation:

$$\mathcal{B}(\bar{B}^0 \to \bar{K}^0 X(3872)) = (1.1 \pm 0.4) \times 10^{-4},$$
$$\mathcal{B}(B^- \to K^- X(3872)) = (2.1 \pm 0.7) \times 10^{-4},$$
$$\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0} X(3872)) = (1.0 \pm 0.5) \times 10^{-4},$$
$$\mathcal{B}(B^- \to K^{*-} X(3872)) < 6 \times 10^{-4},$$
$$\mathcal{B}(\bar{B}_s^0 \to \phi X(3872)) = (1.1 \pm 0.4) \times 10^{-4}.$$

$\Longrightarrow$ $A' = 0.2,$

We determine the value of A' to get the ratio between $\mathcal{B}(\bar{B}_s^0 \to \phi X)/\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0} X)$ and then the rest of the ratios are predictions.

Results and discussions

✓ The agreement between our results and the present experimental data is fairly good, supporting the molecular nature of X(3872).

✓ The contribution of the internal emission is important to get these results.

If we neglect the internal emission, then

$$\mathcal{B}(B^- \to K^- X)/\mathcal{B}(\bar{B}^0 \to \bar{K}^0 X) = 2.5,$$

$$\mathcal{B}(\bar{B}_s^0 \to \phi X) = 0.$$

(inconsistent with the experiment)

✓ The external and internal emissions interfere constructively for $\bar{K}^0$ and K^- production, while they interfere destructively for $\bar{K}^{*0}$ and K^{*-} production.

TABLE I. Ratios of $R \equiv \mathcal{B}(B_i \to j)/\mathcal{B}(\bar{B}^0 \to \bar{K}^0 X(3872))$. The different life times τ_{B_i} are considered. In brackets experimental ratio with errors summed in quadrature.

	$B^- \to K^- X$	$\bar{B}_s^0 \to \eta X$	$\bar{B}_s^0 \to \eta' X$
R	$2.07(1.91 \pm 0.94)$	0.043	0.048

TABLE II. Ratios of $R' \equiv \mathcal{B}(B_i \to j)/\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0} X(3872))$. The different life times τ_{B_i} are considered. In brackets experimental ratio with errors summed in quadrature.

	$B^- \to K^{*-} X$	$\bar{B}_s^0 \to \phi X$
R'	$5.2(< 6)$	$1.10(1.10 \pm 0.68)$

✓ The essential ingredient in understanding these results is the fact that $|G_{D^{*0}\bar{D}^0}| > |G_{D^{*+}D^-}|$ at the pole.

Results and discussions

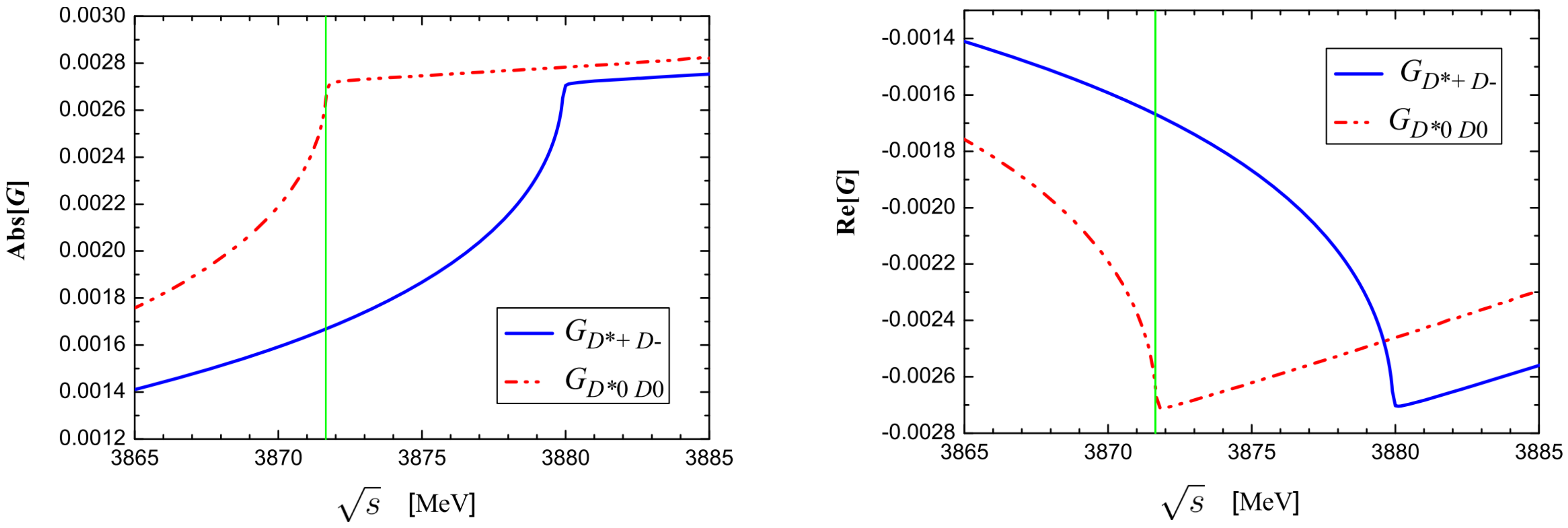


FIG. 5. $G_{D^{*+}D^-}$ and $G_{D^{*0}\bar{D}^0}$ as a function of the energy $\sqrt{s}$. The vertical line indicates the mass of the $X(3872)$.

$G_{D^{*+}D^-}$ looks like $G_{D^{*0}\bar{D}^0}$. Due to the mass difference between the $D^{*+}D^-$ and $D^{*0}\bar{D}^0$ components, $G_{D^{*+}D^-}$ is displaced about 7 MeV to higher energies, leading to

$$\left|\frac{G_n}{G_c}\right|^2 = \left|\frac{G_{D^{*0}\bar{D}^0}}{G_{D^{*+}D^-}}\right|^2 \simeq 2.5. \quad \text{(at the pole energy)}$$

Summary

- From the perspective of the X(3872) being a $D^*\bar{D}+c.c.$ molecular state, we study the decays $\bar{B}^0 \to \bar{K}^0 X,\ B^- \to K^- X,\ \bar{B}_s^0 \to \eta(\eta')X,\ \bar{B}^0 \to \bar{K}^{*0}X,\ B^- \to K^{*-}X,\ \bar{B}_s^0 \to \phi X,$ considering both the external and internal emission mechanisms.

- We find that the internal and external emission mechanisms add constructively in the $\bar{B}^0 \to \bar{K}^0 X$, $B^- \to K^- X$ reactions, while they add destructively in the case of $\bar{B}^0 \to \bar{K}^{*0}X,\ B^- \to K^{*-}X$ reactions.

- The contribution of the internal emission, even if small, has been important to get these results.

- The results obtained are consistent with the present experimental, supporting the molecular nature of X(3872).

- We make predictions for the unmeasured modes of $\bar{B}_s^0 \to \eta(\eta')X(3872)$ and $B^- \to K^{*-}X(3872)$. The future measurement of these decay modes will help us get a better perspective on the nature of the X(3872).

Thank you for your attention!