Occupation-dependent particle separation in non-Hermitian lattices

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Outline

- Some background of non-Hermitian skin effect
- Direction reversal of NHSE

  LLH, Wei Xin Teo, Sen Mu, Jiangbin Gong, Phys. Rev. B 106, 085427 (2022).
- Occupation-dependent particle separation

  Yi Qin, **LLH**, Phys. Rev. Lett. 132, 096501 (2024).
- Summary and outlook

Hermitian Hamiltonian $H = H^\dagger$

Closed systems, real eigenvalues, unitary evolution

non-Hermitian Hamiltonian $H \neq H^\dagger$

Open systems/gain and loss/finite lifetime

- PT symmetry and real spectra

  C. M. Bender and S. Boettcher, Phys. Rev. Lett. 80, 5243 (1998).
  C. M. Bender, Rep. Prog. Phys. 70, 947 (2007).
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- Exceptional degeneracy

  W. D. Heiss and H. L. Harney, Eur. Phys. J. D 17, 149 (2001).
  C. Dembowski, et. al., Phys. Rev. E 69, 056216 (2004).
  M. V. Berry, Czech. J. Phys. 54, 1039 (2004).
  ......

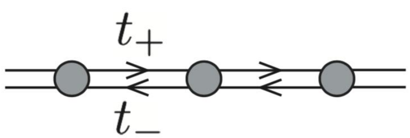
- Non-Hermitian skin effect (2018)

  Massive accumulation of eigenstates at boundaries;
  Modified topological bulk-boundary correspondence;
  Spectral winding topology...

Non-Hermitian skin effect (NHSE)

Yao and Wang, Phys. Rev. Lett. 121, 086803 (2018)

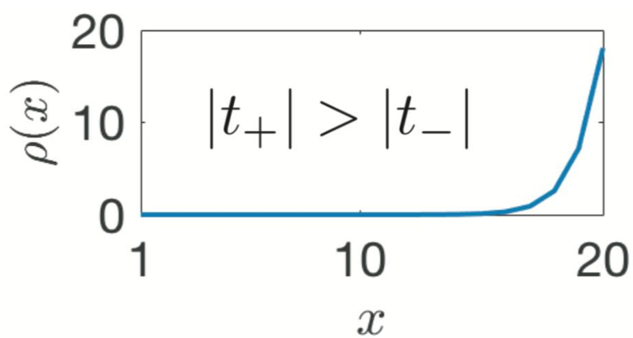
The simplest non-reciprocal/non-Hermitian system (Hatano-Nelson model):



$$H_{\rm 1D\ skin} = \sum_{x=1}^{L-1} t_+ \hat{c}^\dagger_{x+1}\hat{c}_x + t_- \hat{c}^\dagger_x \hat{c}_{x+1} + (t_+ \hat{c}^\dagger_1 \hat{c}_L + t_- \hat{c}^\dagger_L \hat{c}_1) e^{-r}$$

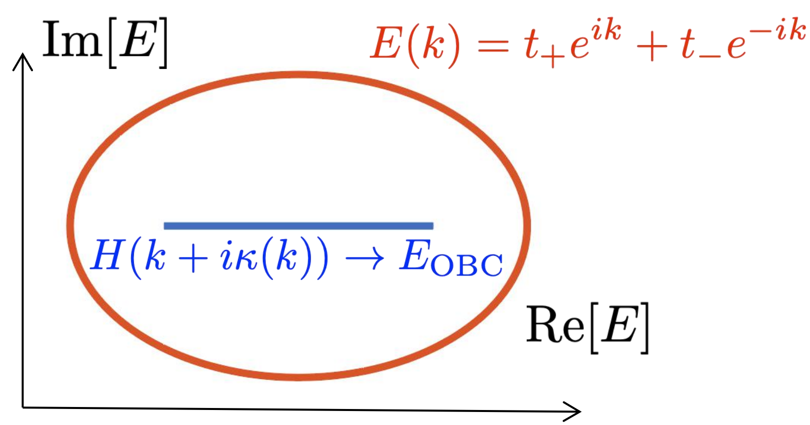
PBCs: $r = 0$ OBCs: $r \to \infty$

OBC density profile:



$$\rho(\mathbf{r}) = \sum_n |\psi_n(\mathbf{r})|^2$$

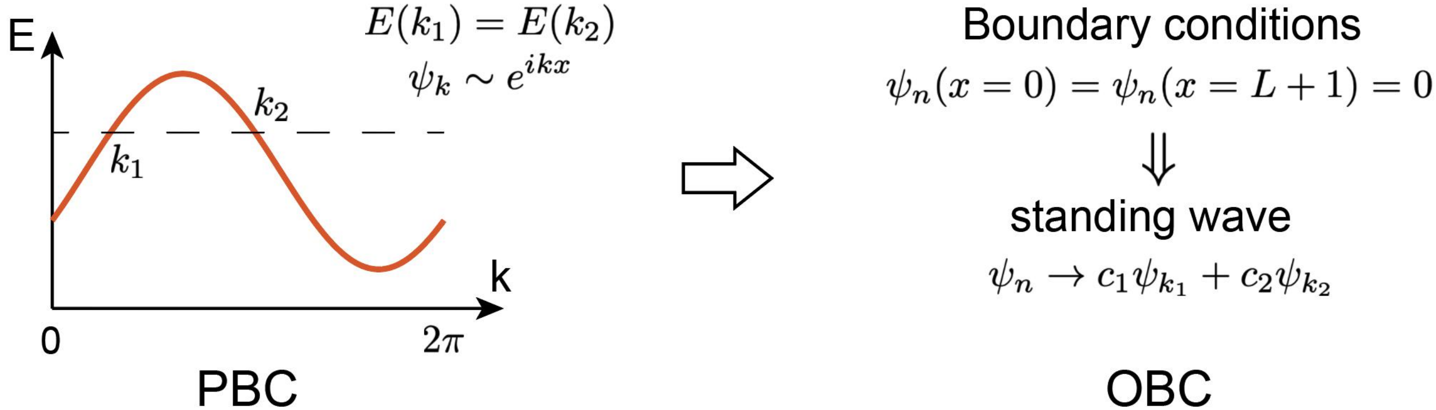
PBC and OBC spectra:



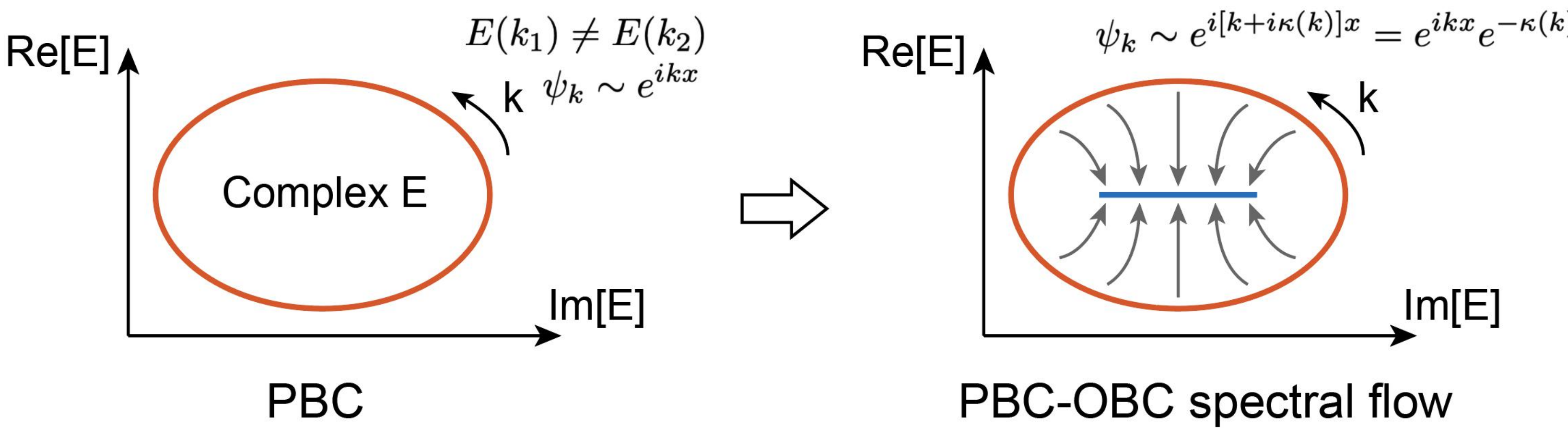
Non-Hermitian skin effect (NHSE)

Yao and Wang, Phys. Rev. Lett. 121, 086803 (2018)

Hermitian Hamiltonian



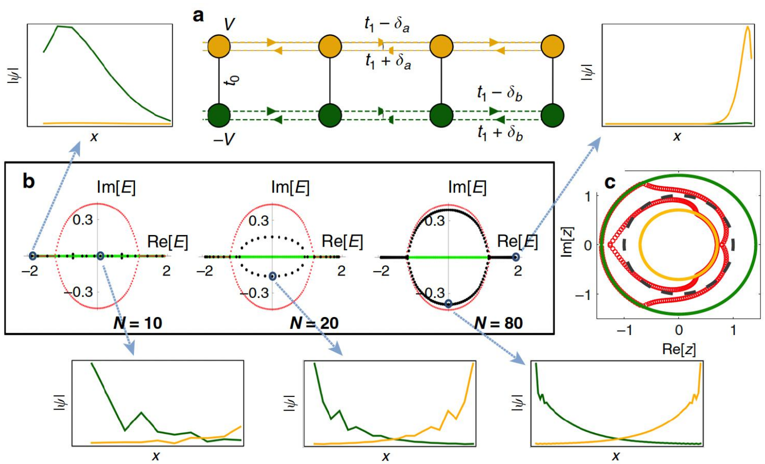
Non-Hermitian Hamiltonian



PRL 123, 066404 (2019), PRB 99, 201103(R) (2019), PRL 124, 086801 (2020), PRL 125, 126402 (2020).

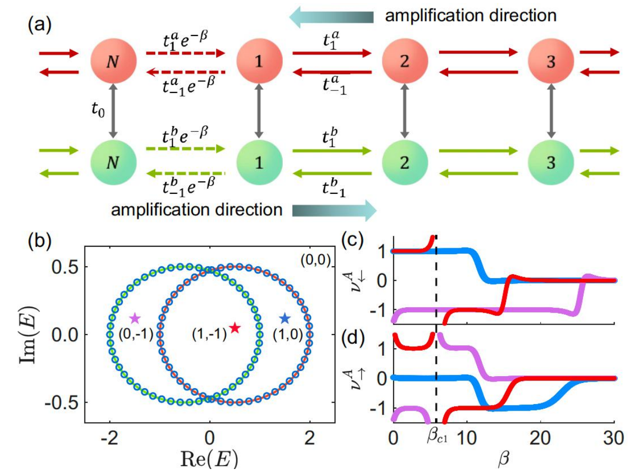
Interplay between multiple NHSE channels

Critical non-Hermitian skin effect



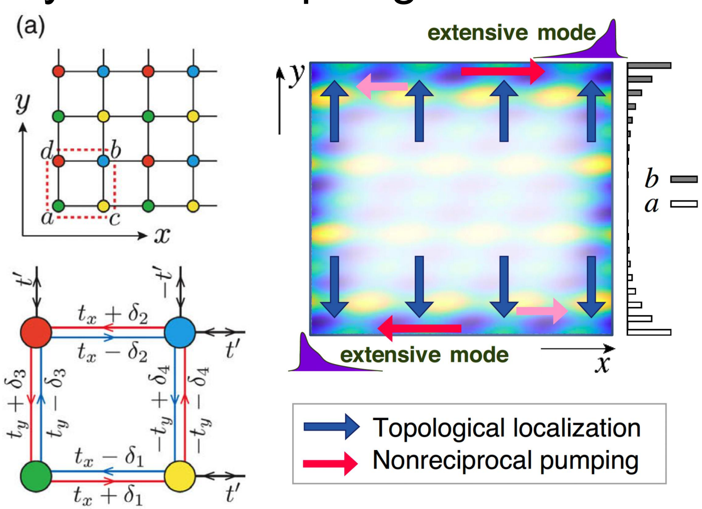
Nat. Commun. 11, 5491 (2020).

Anomalous hybridization



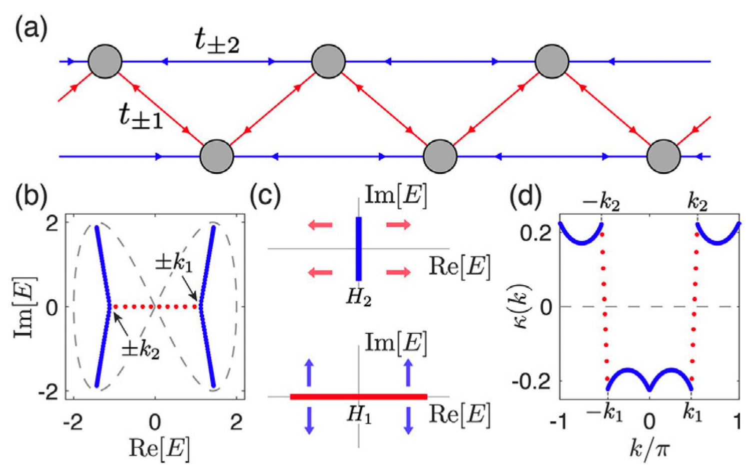
PRB 105, L241402 (2022).

Hybrid skin-topological effect



PRL 123, 016805 (2019); PRL 124, 250402 (2020).

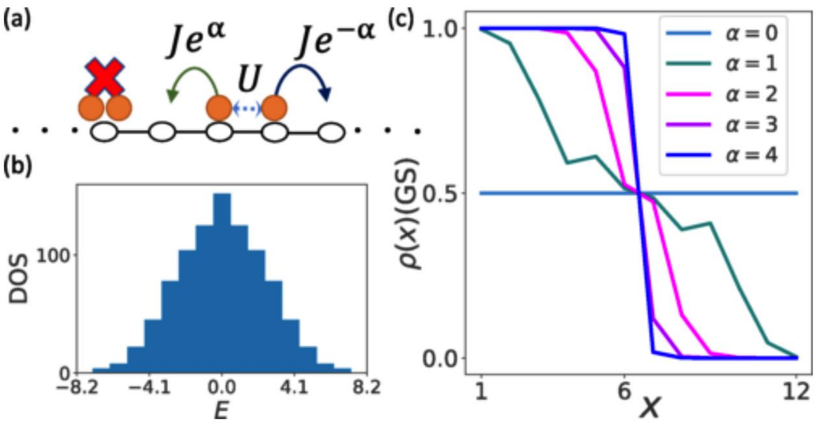
Non-Hermitian pseudo-gaps



Sci. Bull. 67, 685-690 (2022).

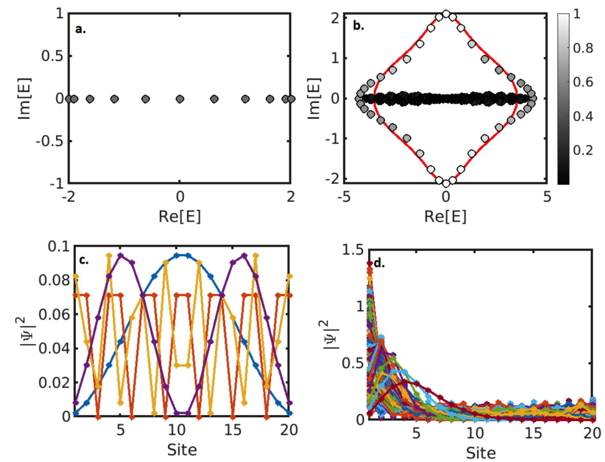
NHSE in many-body systems

Real-space Fermi surface



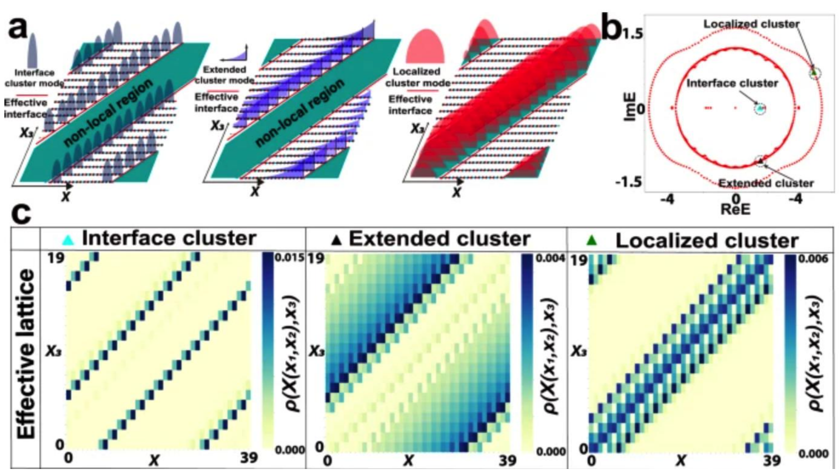
S. Mu, et. al, Phys. Rev. B 102, 081115(R) (2020).

NHSE induced by non-Hermitian interaction



W. N. Faugno and T. Ozawa, PRL 129, 180401 (2022).

Non-Hermitian skin clusters



R. Shen and C. H. Lee, Commun. Phys. 5, 238 (2022).

A recent review of NHSE:

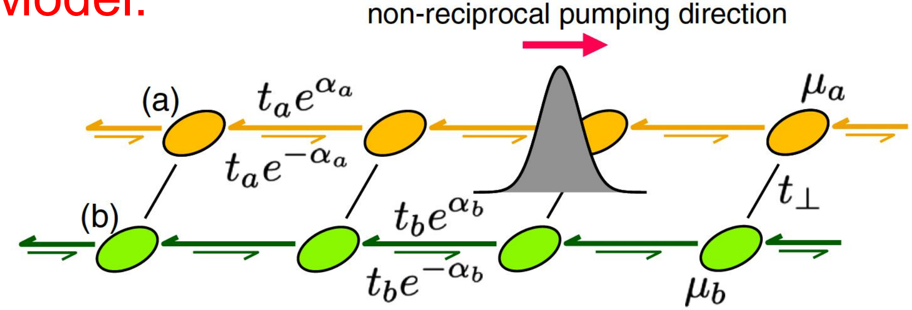
Topological non-Hermitian skin effect

Rijia Lin[1,*], Tommy Tai[2,3,*,†], Linhu Li[1,‡], Ching Hua Lee[3,§]

Frontier of Physics, 5, 53605 (2023).

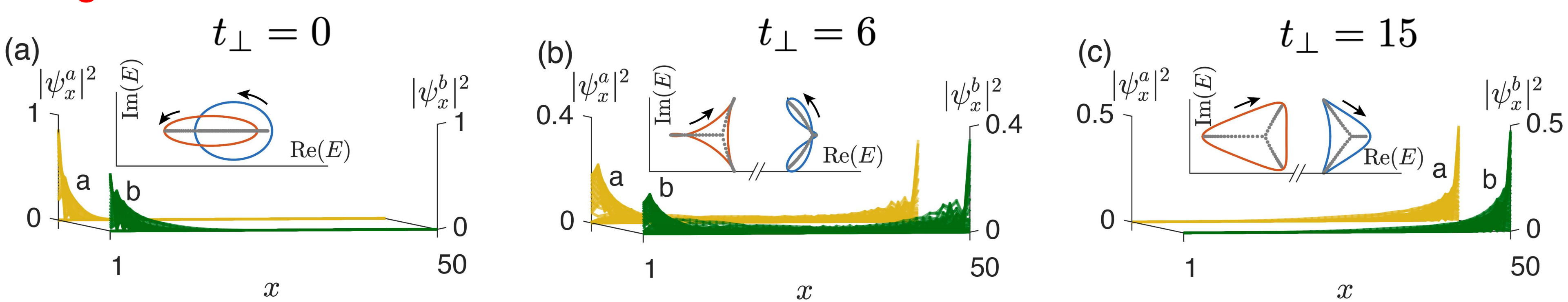
Direction reversal of NHSE

Model:



$$\hat{H} = \sum_{x=1}^{L} \sum_{s=a,b} [t_s e^{\alpha_s} \hat{s}_x^\dagger \hat{s}_{x+1} + t_s e^{-\alpha_s} \hat{s}_x^\dagger \hat{s}_{x-1} + t_\perp (\hat{a}_x^\dagger \hat{b}_x + \hat{b}_x^\dagger \hat{a}_x) + \mu_a \hat{a}_x^\dagger \hat{a}_x + \mu_b \hat{b}_x^\dagger \hat{b}_x,]$$

Eigensolutions:

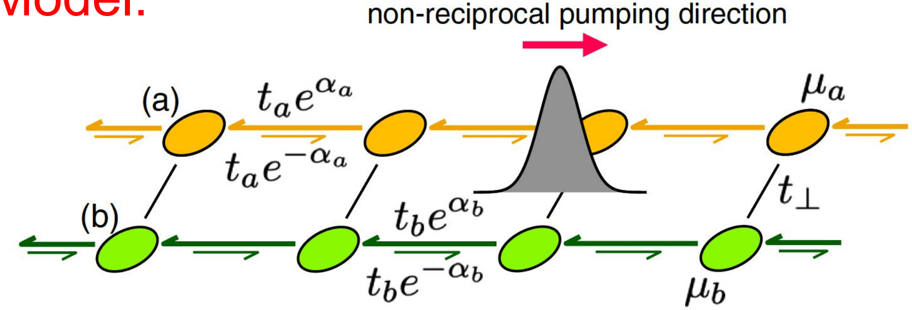


$$t_a = 0.75,\ t_b = -1,\ \alpha_a = 0.5,\ \alpha_b = 0.2,\ \mu_a = -\mu_b = 0.5.$$

- Stronger non-reciprocity to the left, yet NHSE to the right (for large t_perp).

Direction reversal of NHSE

Model:



$$\hat{H} = \sum_{x=1}^{L} \sum_{s=a,b} [t_s e^{\alpha_s} \hat{s}_x^\dagger \hat{s}_{x+1} + t_s e^{-\alpha_s} \hat{s}_x^\dagger \hat{s}_{x-1}$$

$$+ t_\perp (\hat{a}_x^\dagger \hat{b}_x + \hat{b}_x^\dagger \hat{a}_x) + \mu_a \hat{a}_x^\dagger \hat{a}_x + \mu_b \hat{b}_x^\dagger \hat{b}_x,]$$

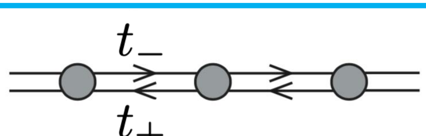
Perturbation treatment:

Unperturbed Hamitonian: $\hat{H}_\perp = t_\perp \sum_x (\hat{a}_x^\dagger \hat{b}_x + \hat{b}_x^\dagger \hat{a}_x)$

First-order perturbation:

$$\hat{H}'_\pm = \sum_x (t_a e^{\alpha_a} + t_b e^{\alpha_b}) \hat{u}_{\pm,x}^\dagger \hat{u}_{\pm,x+1} + (t_a e^{-\alpha_a} + t_b e^{-\alpha_b})$$

$$\times \hat{u}_{\pm,x+1}^\dagger \hat{u}_{\pm,x} + (\mu_a \pm \mu_b) \hat{u}_{\pm,x}^\dagger \hat{u}_{\pm,x}.$$



$$t_+ = t_a e^{\alpha_a} + t_b e^{\alpha_b}$$

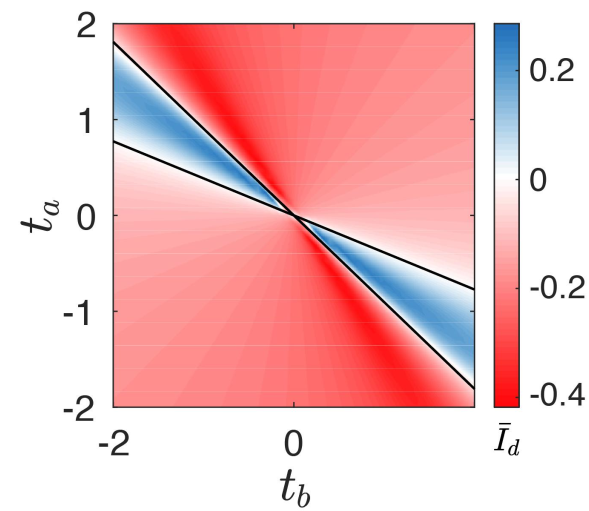
$$t_- = t_a e^{-\alpha_a} + t_b e^{-\alpha_b}$$

Direction reversal of NHSE:

$$|t_s e^{\alpha_s}| > |t_s e^{-\alpha_s}|, \quad \text{but } |t_+| < |t_-|$$

Phase diagram:

$\alpha_a = 0.5,\ \alpha_b = 0.2,\ \mu_a = -\mu_b = 0.5.$



$$\bar{I}_d = \frac{1}{2L} \sum_m \sum_{x=1}^{L} \frac{[x - (L+1)/2](|\psi_{x,m}^a|^4 + |\psi_{x,m}^b|^4)}{(L-1)/2}$$

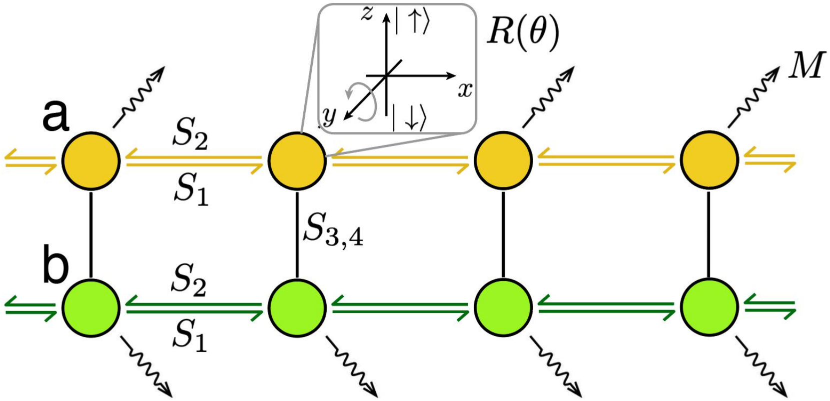
Direction reversal of NHSE - quantum walk

A double-chain non-unitary quantum walk:

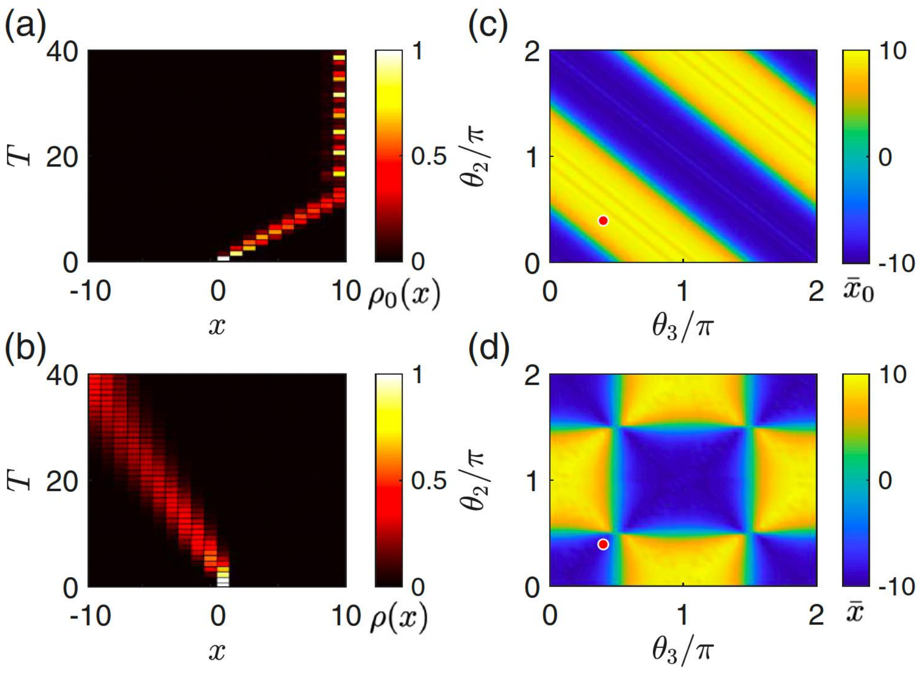
$$U_0 = R(\theta_1)S_2R(\theta_2+\theta_3)MR(\theta_2+\theta_3)S_1R(\theta_1)$$

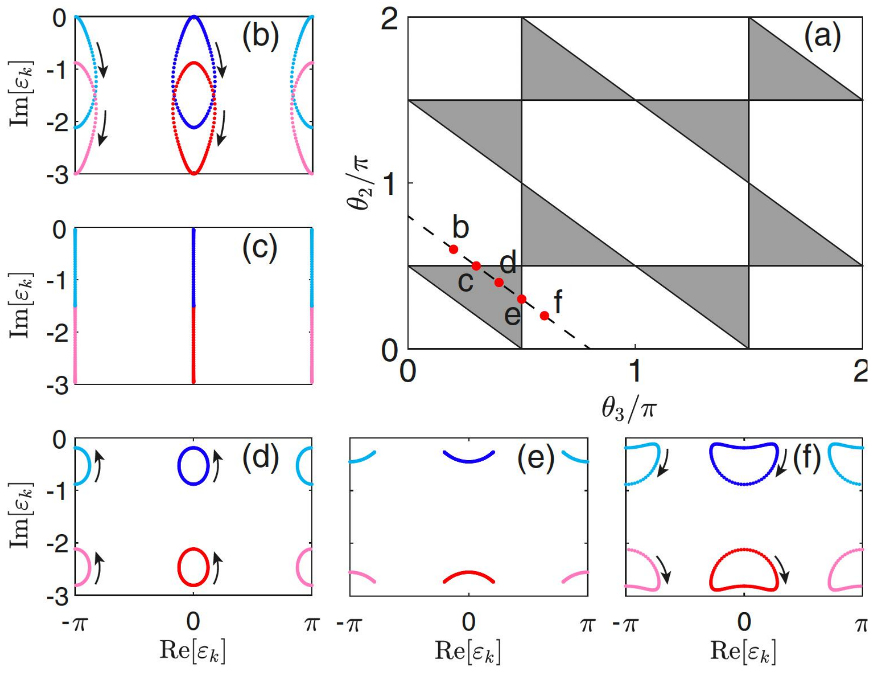
Two copies of the model in Nat. Phys. 16, 761-766 (2020)

$$U = R(\theta_1)S_2R(\theta_2)S_4R(\theta_3)MR(\theta_3)S_3R(\theta_2)S_1R(\theta_1)$$



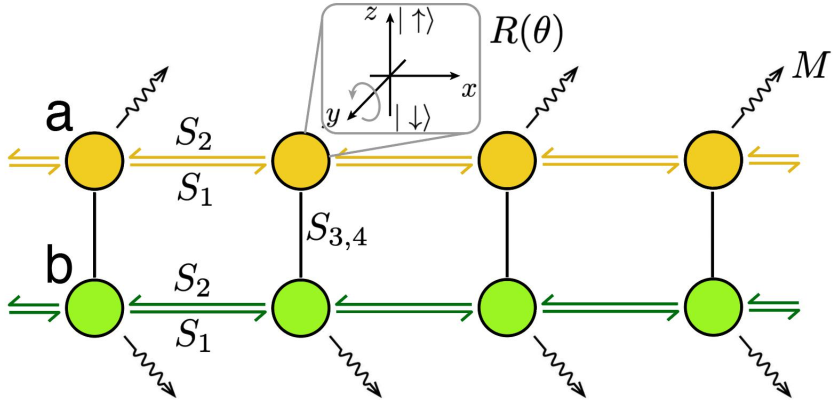
R: rotation of the spin; S_1 and S_2: intrachain shift operators;
S_3 and S_4: interchain shift operators; M: loss operator.





Direction reversal of NHSE - quantum walk

A double-chain non-unitary quantum walk:



$$U_0 = R(\theta_1)S_2R(\theta_2+\theta_3)MR(\theta_2+\theta_3)S_1R(\theta_1)$$

Two copies of the model in Nat. Phys. 16, 761-766 (2020)

$$U = R(\theta_1)S_2R(\theta_2)S_4R(\theta_3)MR(\theta_3)S_3R(\theta_2)S_1R(\theta_1)$$

R: rotation of the spin; S_1 and S_2: intrachain shift operators;
S_3 and S_4: interchain shift operators; M: loss operator.

$$R(\theta) = \sum_{x=-N}^{N}\sum_{s=a,b}|s,x\rangle\langle s,x|\otimes e^{-i\lambda_s\theta\sigma_y/2},\quad M = \sum_{x=-N}^{N}\sum_{s=a,b}|s,x\rangle\langle s,x|\otimes(|\downarrow\rangle\langle\downarrow| + e^{-\alpha_s}|\uparrow\rangle\langle\uparrow|),$$

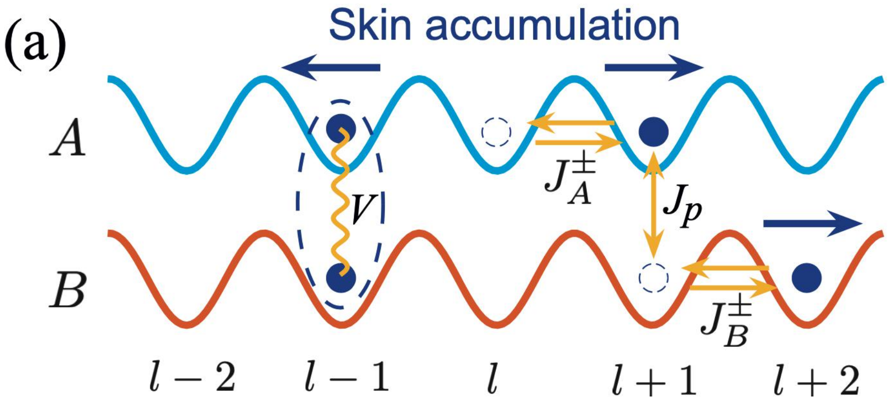
$$S_1 = \sum_{x=-N}^{N}\sum_{s=a,b}(|s,x\rangle\langle s,x|\otimes|\downarrow\rangle\langle\downarrow| + |s,x+1\rangle\langle s,x|\otimes|\uparrow\rangle\langle\uparrow|),\quad S_2 = \sum_{x=-N}^{N}\sum_{s=a,b}(|s,x-1\rangle\langle s,x|\otimes|\downarrow\rangle\langle\downarrow| + |s,x\rangle\langle s,x|\otimes|\uparrow\rangle\langle\uparrow|),$$

$$S_3 = \sum_{x=-N}^{N}\sum_{s,\bar{s}=a,b}(|s,x\rangle\langle\bar{s},x|\otimes|\uparrow\rangle\langle\uparrow| + |s,x\rangle\langle s,x|\otimes|\downarrow\rangle\langle\downarrow|),\quad S_4 = \sum_{x=-N}^{N}\sum_{s,\bar{s}=a,b}(|s,x\rangle\langle\bar{s},x|\otimes|\downarrow\rangle\langle\downarrow| + |s,x\rangle\langle s,x|\otimes|\uparrow\rangle\langle\uparrow|).$$

with $s = a, b$ denoting the two chains, x the site index, and $\lambda_a = 1$ and $\lambda_b = -1$.

multi-particle NHSE in coupled chains

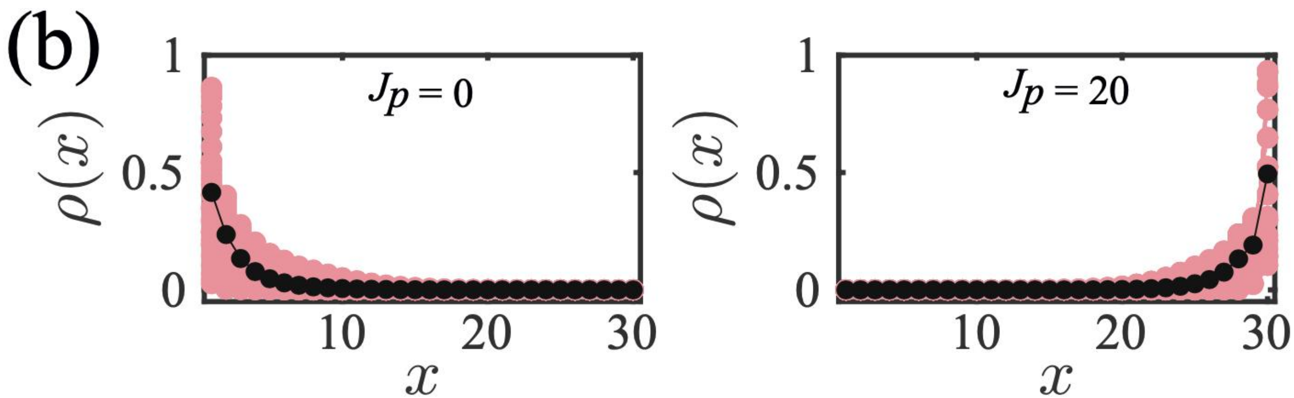
Model (hardcore bosons):



$$\hat{H} = \sum_{l=1}^{L-1} \hat{\psi}_{l+1}^{\dagger} \begin{pmatrix} J_A^{+} & 0 \\ 0 & J_B^{+} \end{pmatrix} \hat{\psi}_l + \hat{\psi}_l^{\dagger} \begin{pmatrix} J_A^{-} & 0 \\ 0 & J_B^{-} \end{pmatrix} \hat{\psi}_{l+1} + \hat{\psi}_l^{\dagger} \begin{pmatrix} 0 & J_p \\ J_p & 0 \end{pmatrix} \hat{\psi}_l + V \sum_{l=1}^{L} \hat{n}_{A,l} \hat{n}_{B,l}$$

$$\hat{\psi}_l = (\hat{c}_{A,l}, \hat{c}_{B,l})^T$$

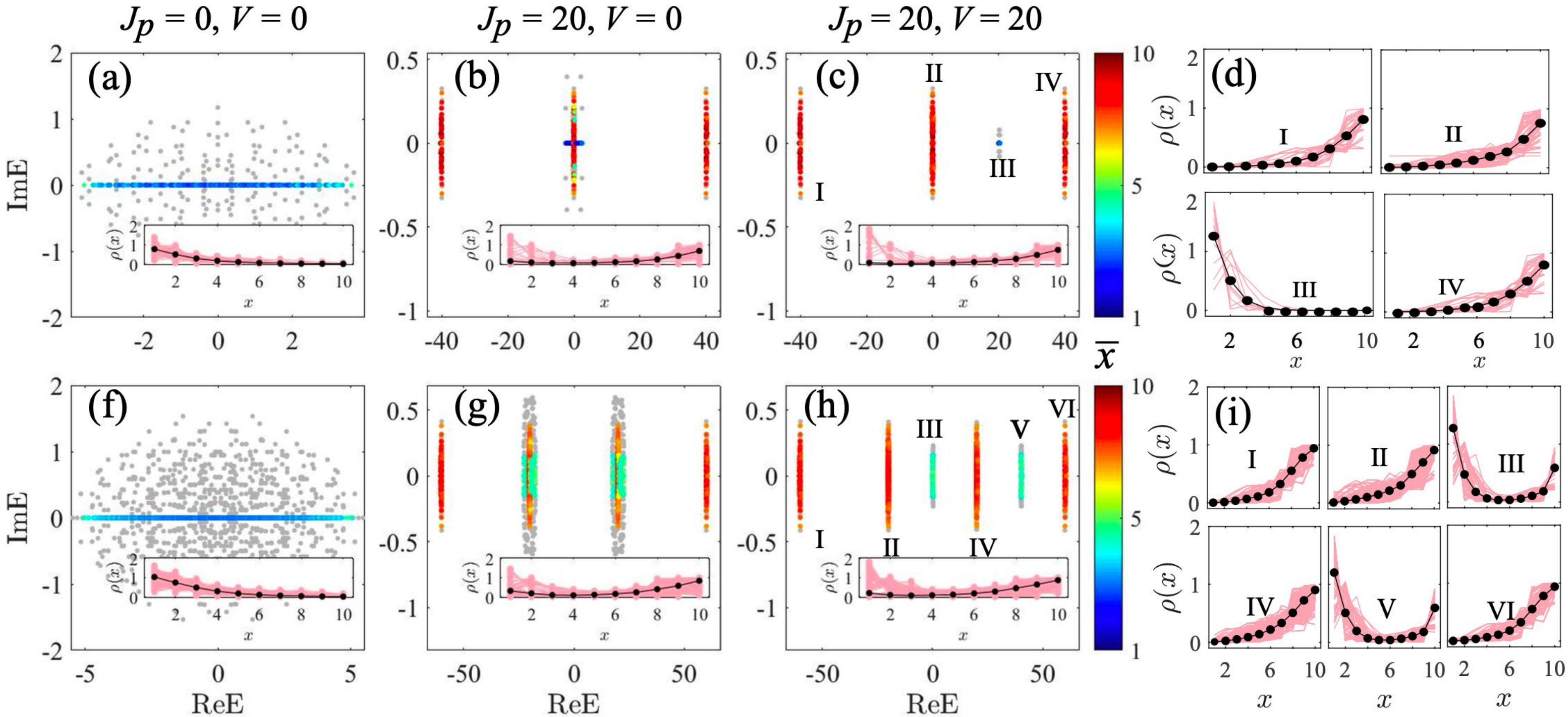
Single-particle eigenstates:



$J_A^{+} = 0.45, J_A^{-} = 1.24, J_B^{+} = -0.82, J_B^{-} = -1.22, L = 30.$

multi-particle NHSE in coupled chains

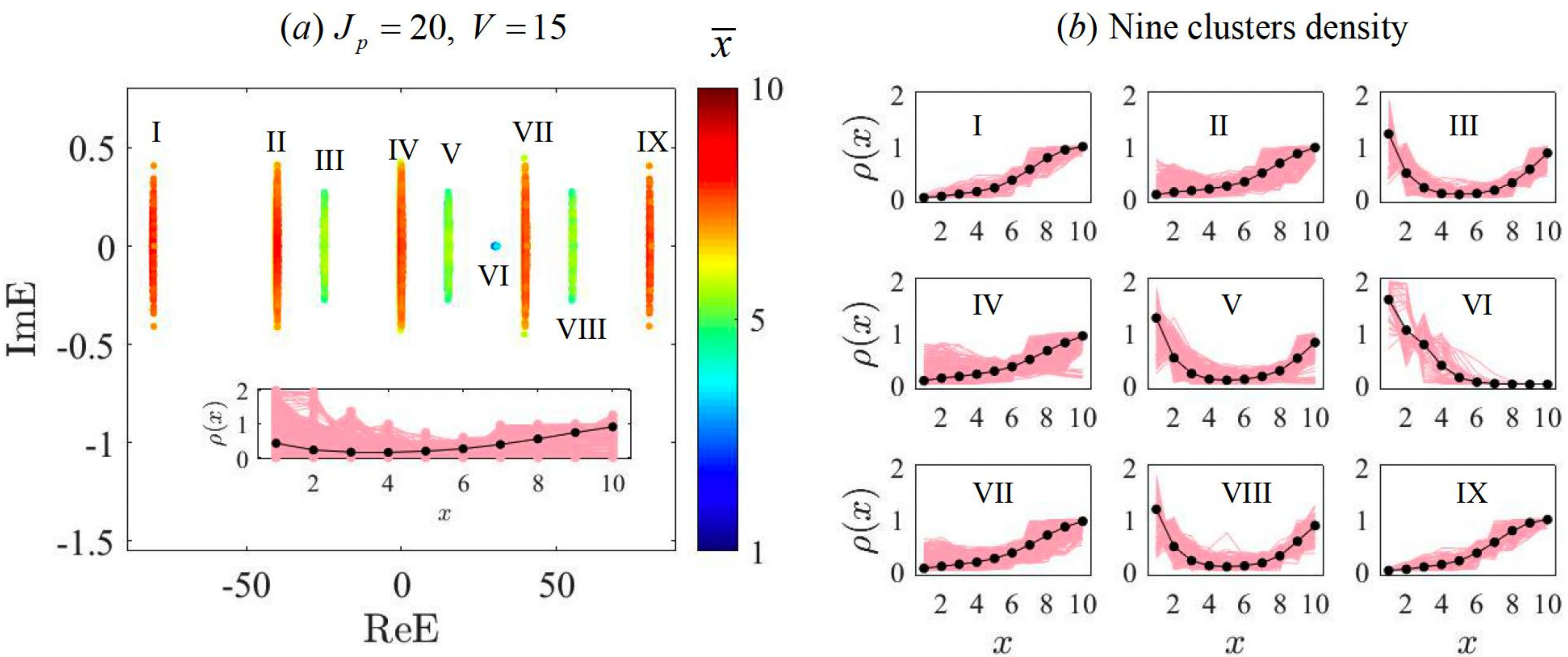
Two (top) and three (bottom) particles:



N	$E\ (V = 0)$	$E\ (V \neq 0)$
1	$-J_p;\ J_p$	$-J_p;\ J_p$
2	$-2J_p;\ 0;\ 2J_p;$	$-2J_p;\ 0;\ V;\ 2J_p$
3	$-3J_p;\ -J_p;\ J_p;\ 3J_p;$	$-3J_p;\ -J_p;\ -J_p+V;\ J_p;\ J_p+V;\ 3J_p$
4	$-4J_p;\ -2J_p;\ 0;\ 2J_p;\ 4J_p;$	$-4J_p;\ -2J_p;\ -2J_p+V;\ 0;\ V;\ 2V;\ 2J_p;\ 2J_p+V;\ 4J_p$
5	$-5J_p;\ -3J_p;\ -J_p;\ J_p;\ 3J_p;\ 5J_p;$	$-5J_p;\ -3J_p;\ -3J_p+V;\ -J_p;\ -J_p+V;\ -J_p+2V;\ J_p;\ J_p+V;\ J_p+2V;\ 3J_p;\ 3J_p+V;\ 5J_p$

multi-particle NHSE in coupled chains

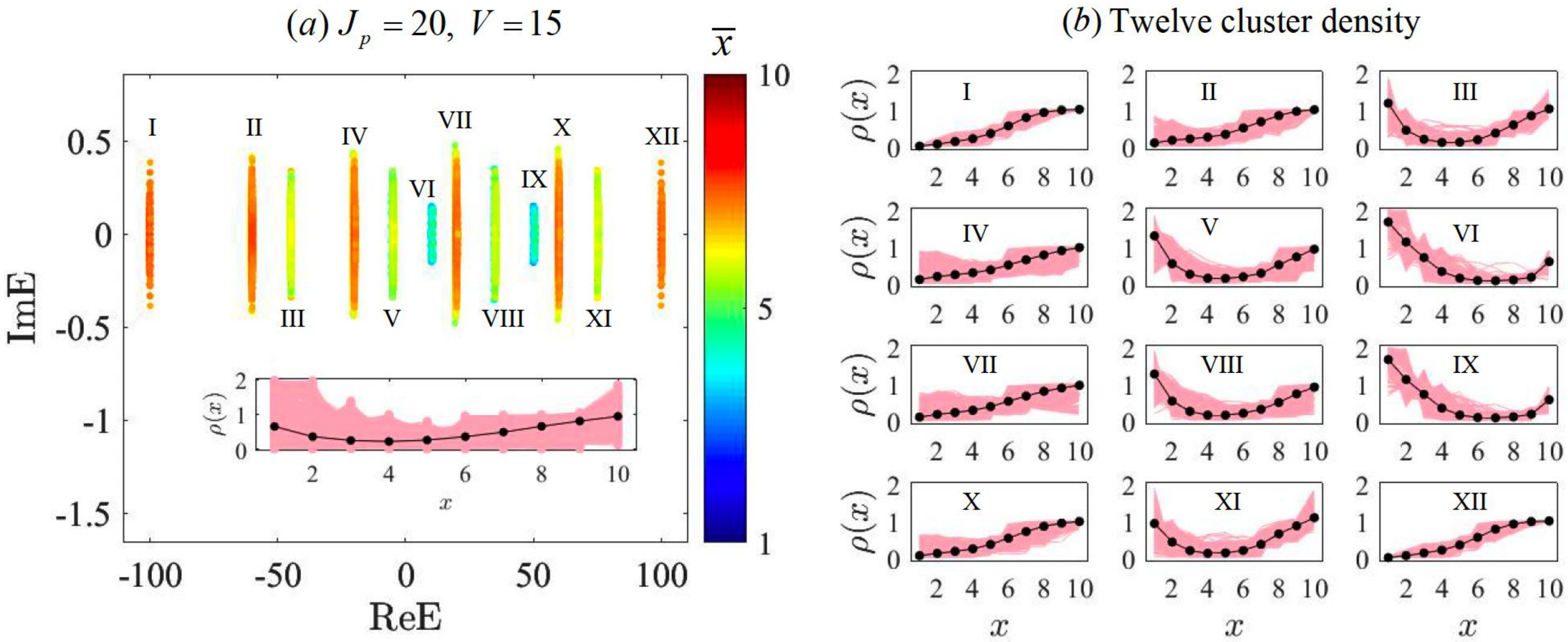
Four particles:



N	$E\ (V=0)$	$E\ (V \neq 0)$
1	$-J_p;\ J_p$	$-J_p;\ J_p$
2	$-2J_p;\ 0;\ 2J_p;$	$-2J_p;\ 0;\ V;\ 2J_p$
3	$-3J_p;\ -J_p;\ J_p;\ 3J_p;$	$-3J_p;\ -J_p;\ -J_p+V;\ J_p;\ J_p+V;\ 3J_p$
4	$-4J_p;\ -2J_p;\ 0;\ 2J_p;\ 4J_p;$	$-4J_p;\ -2J_p;\ -2J_p+V;\ 0;\ V;\ 2V;\ 2J_p;\ 2J_p+V;\ 4J_p$
5	$-5J_p;\ -3J_p;\ -J_p;\ J_p;\ 3J_p;\ 5J_p;$	$-5J_p;\ -3J_p;\ -3J_p+V;\ -J_p;\ -J_p+V;\ -J_p+2V;\ J_p;\ J_p+V;\ J_p+2V;\ 3J_p;\ 3J_p+V;\ 5J_p$

multi-particle NHSE in coupled chains

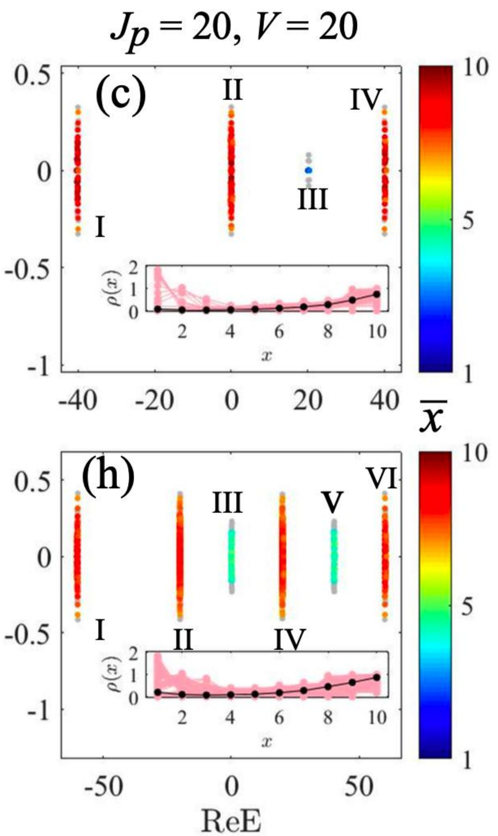
Five particles:

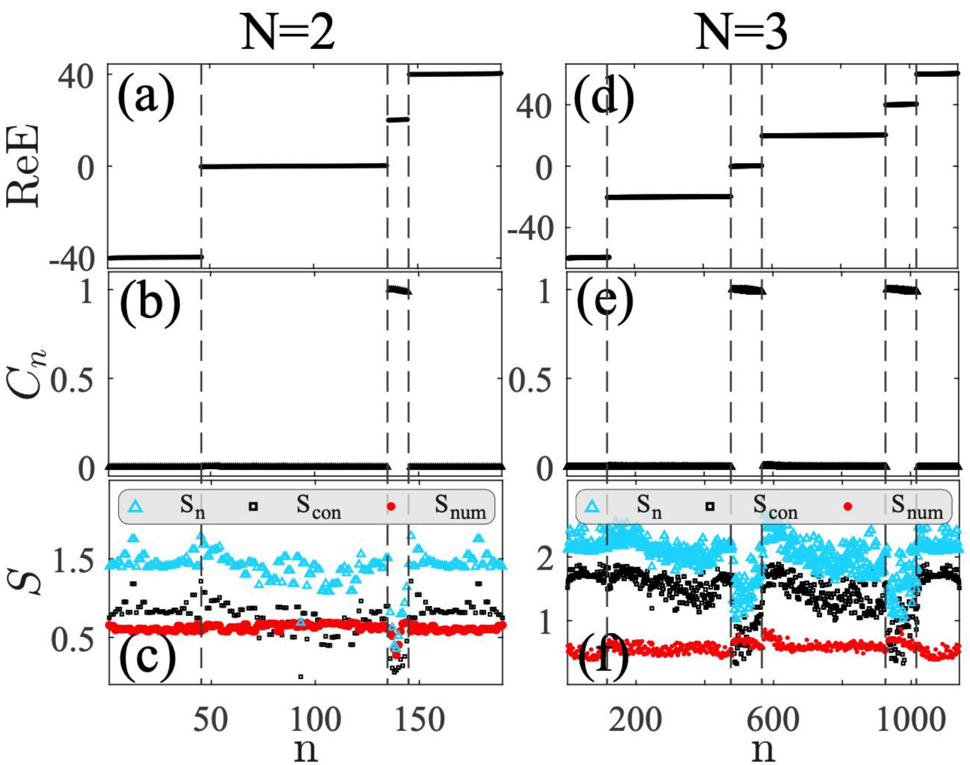


N	$E\ (V=0)$	$E\ (V \neq 0)$
1	$-J_p; J_p$	$-J_p; J_p$
2	$-2J_p; 0; 2J_p;$	$-2J_p; 0; V; 2J_p$
3	$-3J_p; -J_p; J_p; 3J_p;$	$-3J_p; -J_p; -J_p+V; J_p; J_p+V; 3J_p$
4	$-4J_p; -2J_p; 0; 2J_p; 4J_p;$	$-4J_p; -2J_p; -2J_p+V; 0; V; 2V; 2J_p; 2J_p+V; 4J_p$
5	$-5J_p; -3J_p; -J_p; J_p; 3J_p; 5J_p;$	$-5J_p; -3J_p; -3J_p+V; -J_p; -J_p+V; -J_p+2V; J_p; J_p+V; J_p+2V; 3J_p; 3J_p+V; 5J_p$

multi-particle NHSE in coupled chains

Sublattice correlation and entanglement entropy:





$$C_n = \sum_{l=1}^{L} \langle \psi_n | \hat{n}_{A,l} \hat{n}_{B,l} | \psi_n \rangle$$

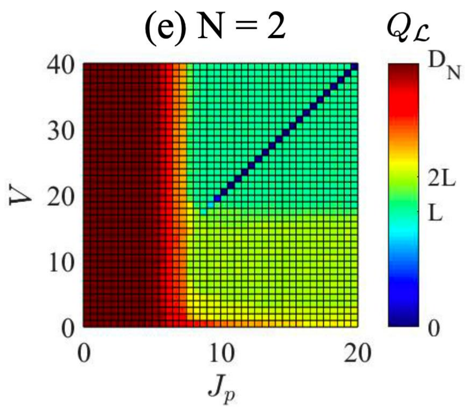
$$S_n = -\mathrm{Tr}\rho_{n,A} \ln \rho_{n,A} = S_{\mathrm{num}} + S_{\mathrm{con}}$$

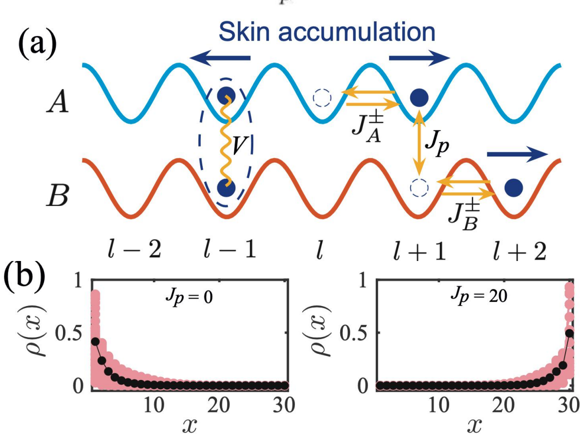
A. Lukin, et. al., Science 364, 256 (2019).

multi-particle NHSE in coupled chains

Phase diagram:

$Q_{\mathcal{L}}$: the number of state with left NHSE





large V and J_p:

$$\hat{H}_{\rm eff} \simeq V + \frac{2}{V}\left(J_A^+ J_A^- + J_B^+ J_B^-\right) + \sum_{i=1}^{L}\left[\frac{2}{V} J_B^+ J_A^+ \left|\alpha_{i+1}\right\rangle\!\rangle\langle\!\langle\alpha_i\right| + \frac{2}{V} J_A^- J_B^- \left|\alpha_i\right\rangle\!\rangle\langle\!\langle\alpha_{i+1}\right|\right]$$

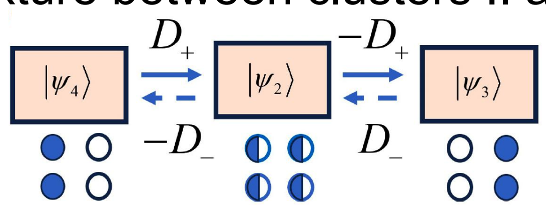
$$|(AB)_i\rangle = \hat{c}^\dagger_{A,i}\hat{c}^\dagger_{B,i}|\text{vac}\rangle \equiv |\alpha_i\rangle\!\rangle, (i = 1, 2, ..., L)$$

small J_p:

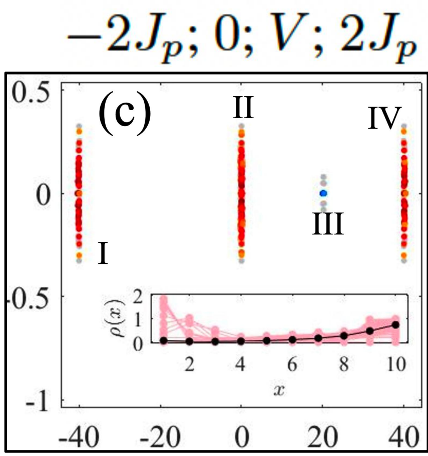
Nearly decoupled chains with left NHSE only.

small V, large J_p:

Mixture between clusters II and III.

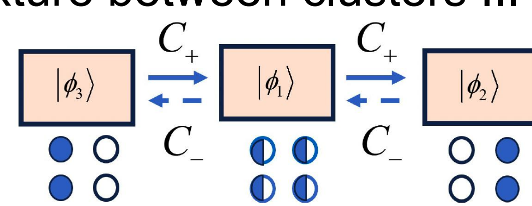


$$D_\pm = \pm(J_A^\pm - J_B^\pm)/\sqrt{2}$$



large V ~ 2J_p:

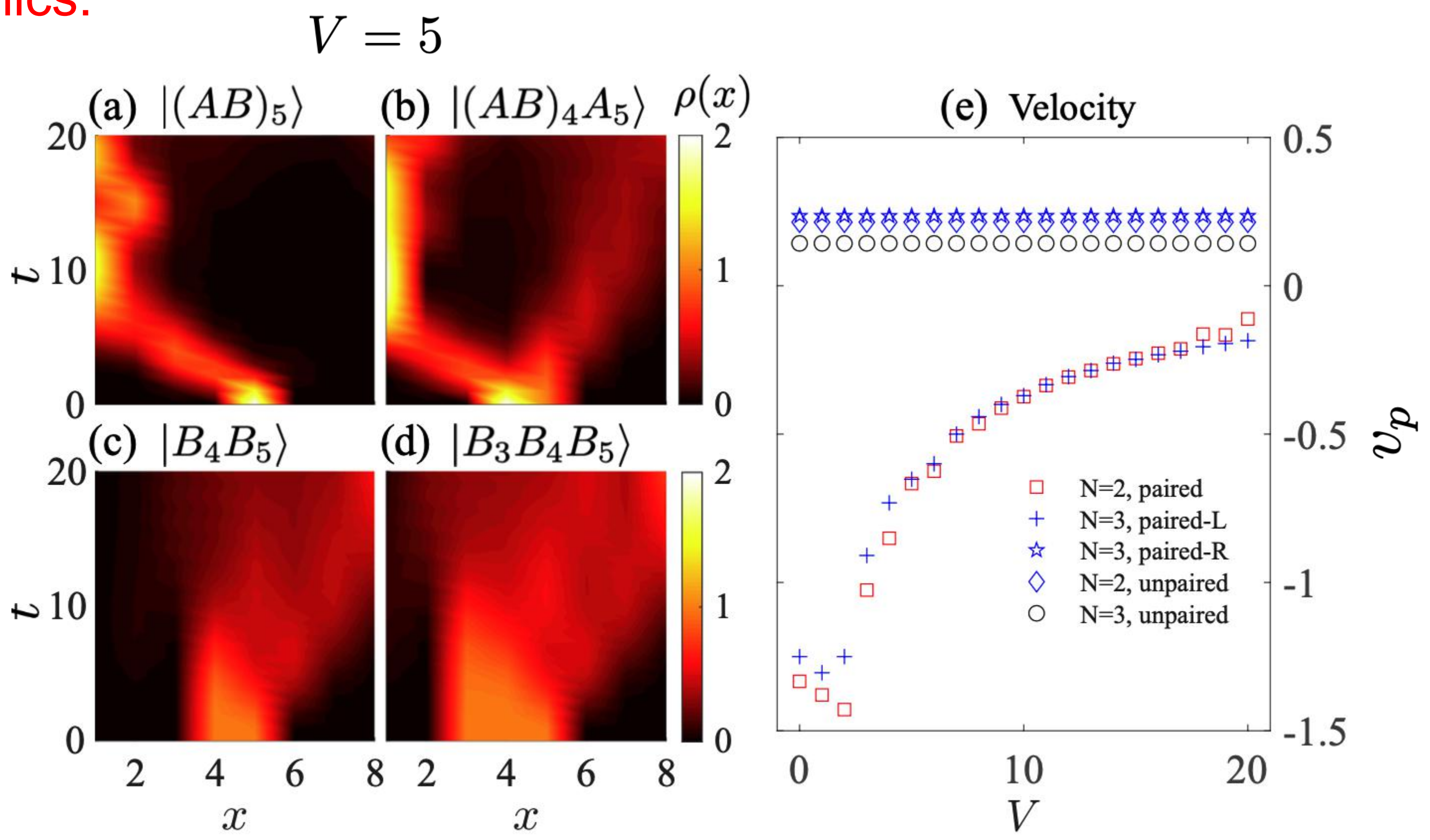
Mixture between clusters III and IV.



$C_\pm = (J_A^\pm + J_B^\pm)/2$ (as for the single-particle case)

multi-particle NHSE in coupled chains

Dynamics:



$$v_p = [x_p(t) - x_p(0)]/t$$

$x_p(t)$: the position with maximal density.

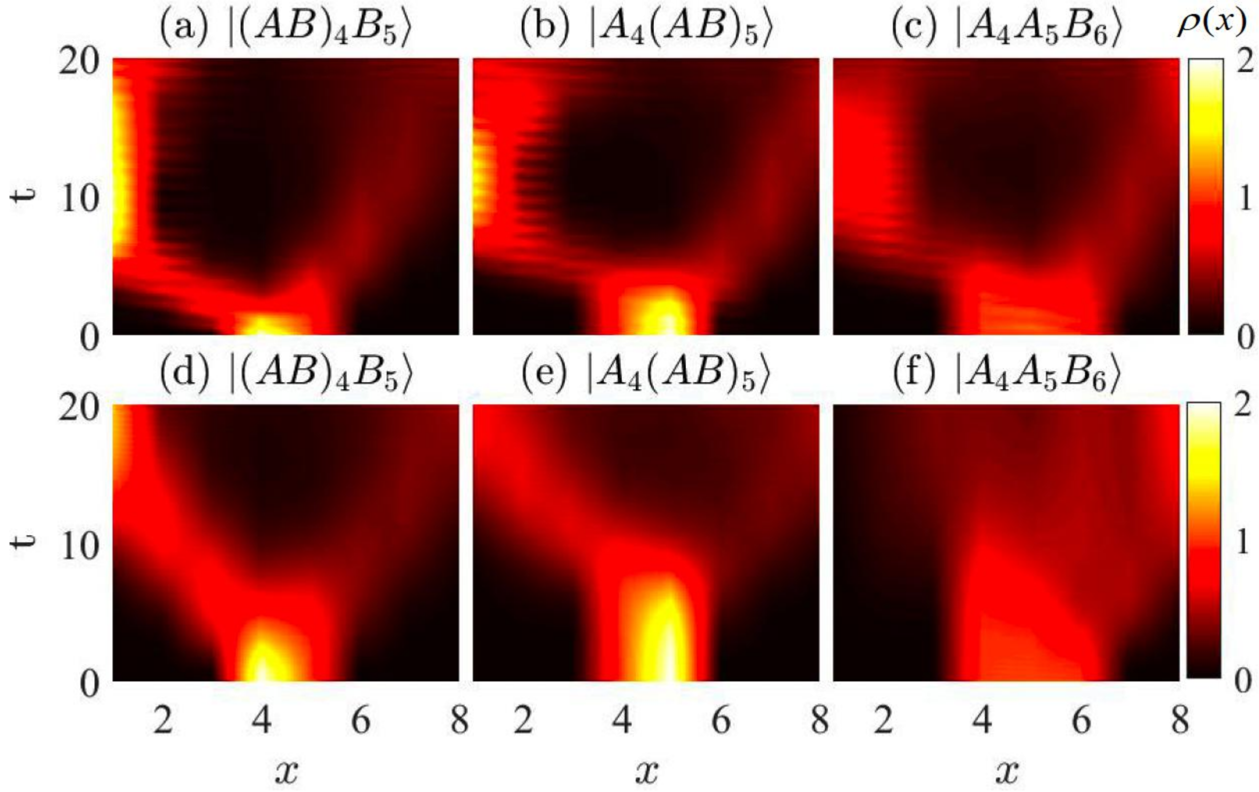
t is chosen to be the time when the density peak reaches the boundary.

multi-particle NHSE in coupled chains

Dynamics:

$V = 5$

$V = 15$

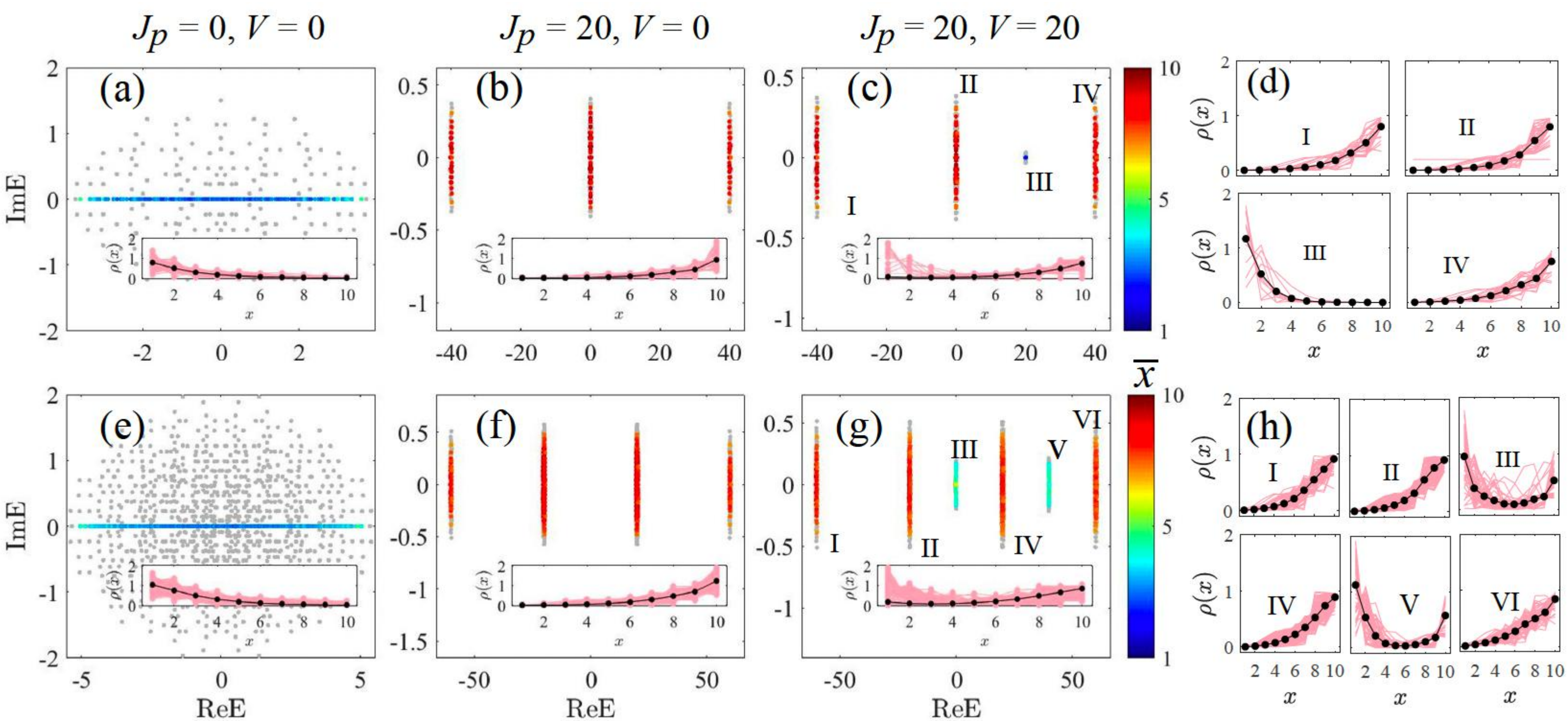


Weak V: faster left-propagation for “paired” particles.
Strong V: better separation of “paired” and “unpaired” particles.

multi-particle NHSE in coupled chains

The same model, but for fermions:

$$\hat{H}_{\text{fermions}} = \sum_{l=1}^{L-1} \hat{\psi}_{l+1}^{\dagger} \begin{pmatrix} J_{\uparrow}^{+} & 0 \\ 0 & J_{\downarrow}^{+} \end{pmatrix} \hat{\psi}_{l} + \hat{\psi}_{l}^{\dagger} \begin{pmatrix} J_{\uparrow}^{-} & 0 \\ 0 & J_{\downarrow}^{-} \end{pmatrix} \hat{\psi}_{l+1} + \hat{\psi}_{l}^{\dagger} \begin{pmatrix} 0 & J_p \\ J_p & 0 \end{pmatrix} \hat{\psi}_{l} + V \sum_{l=1}^{L} \hat{n}_{\uparrow,l} \hat{n}_{\downarrow,l}.$$



No left-NHSE (i.e., deactivation of direction reversal due ot “pairing”) when V=0.

multi-particle NHSE in coupled chains

The same model, but for fermions:

N=2, paired

$|(\uparrow\downarrow)_5\rangle$

N=3, paired

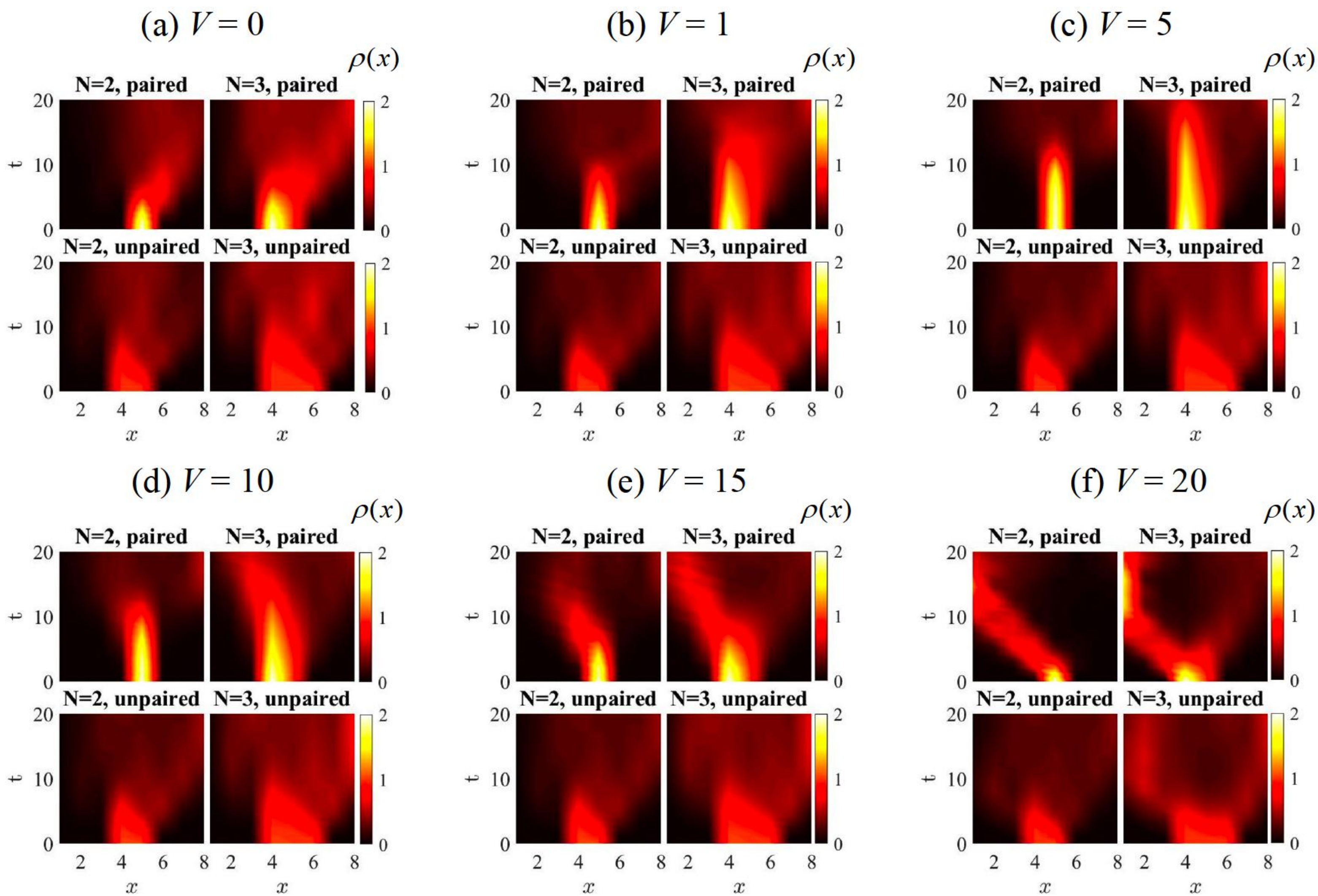
$|(\uparrow\downarrow)_4\ \uparrow_5\rangle$

N=2, unpaired

$|\ \downarrow_4\downarrow_5\rangle$

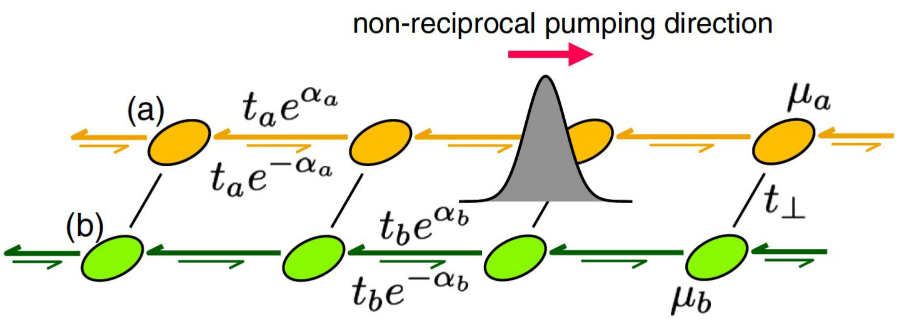
N=3, unpaired

$|\ \downarrow_4\downarrow_5\downarrow_6\rangle$

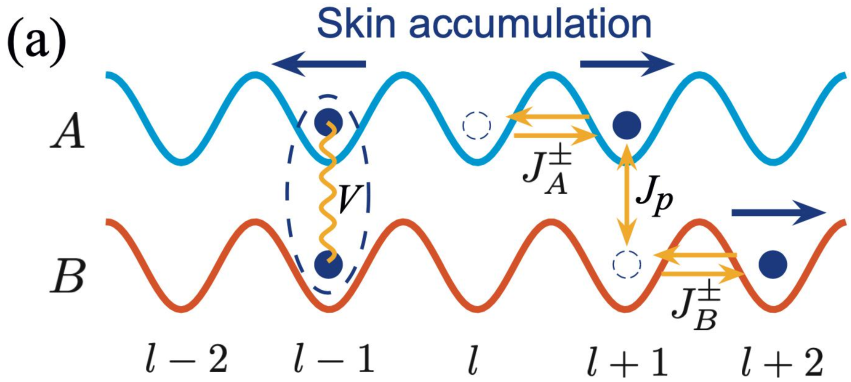


Summary and outlook

Take home message:



Phys. Rev. B 106, 085427 (2022).



Phys. Rev. Lett. 132, 096501 (2024).

Summary and outlook

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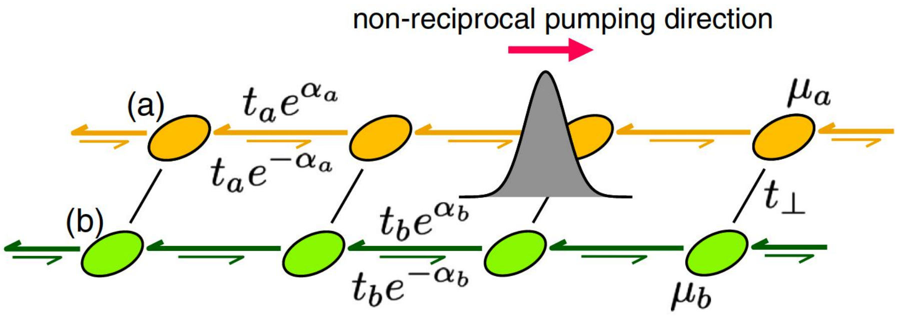

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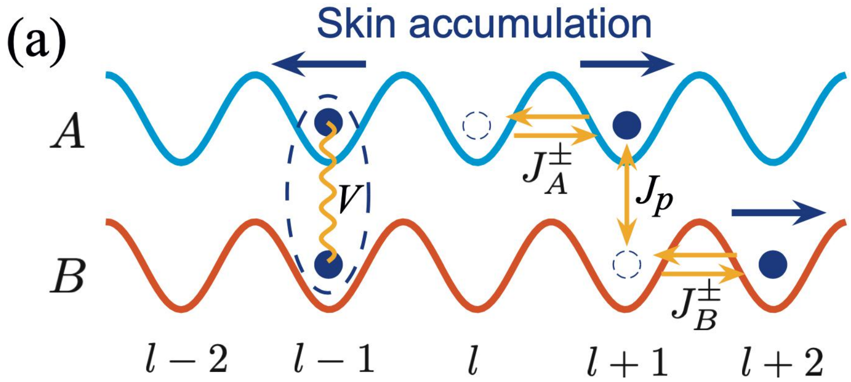

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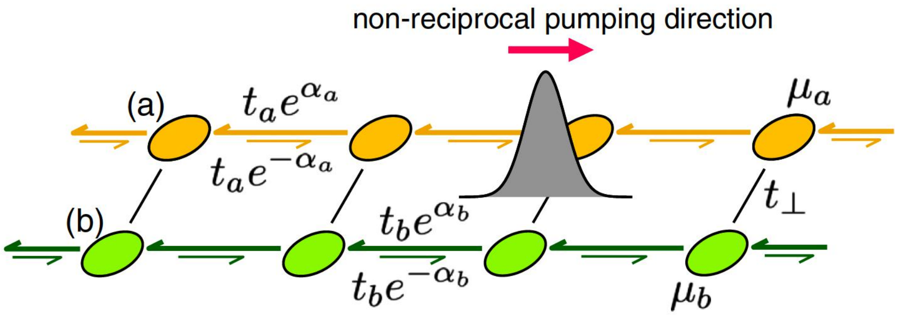
Questions and outlooks:

- Why fermions and bosons behave so differently? How about anyons?
- Richer particle configurations induced by different types of interactions (hilbert space fragmentation).
- More sophisticated non-reciprocal pumping channels on different bulk and boundaries of higher-dimensional systems.

Thank you!

Summary and outlook

Take home message:



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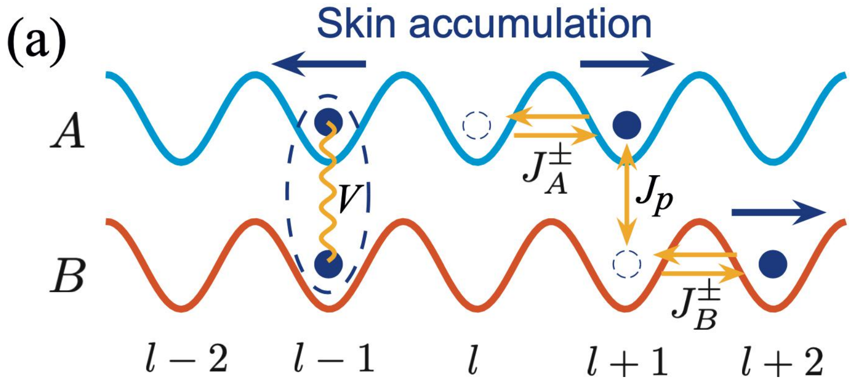

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