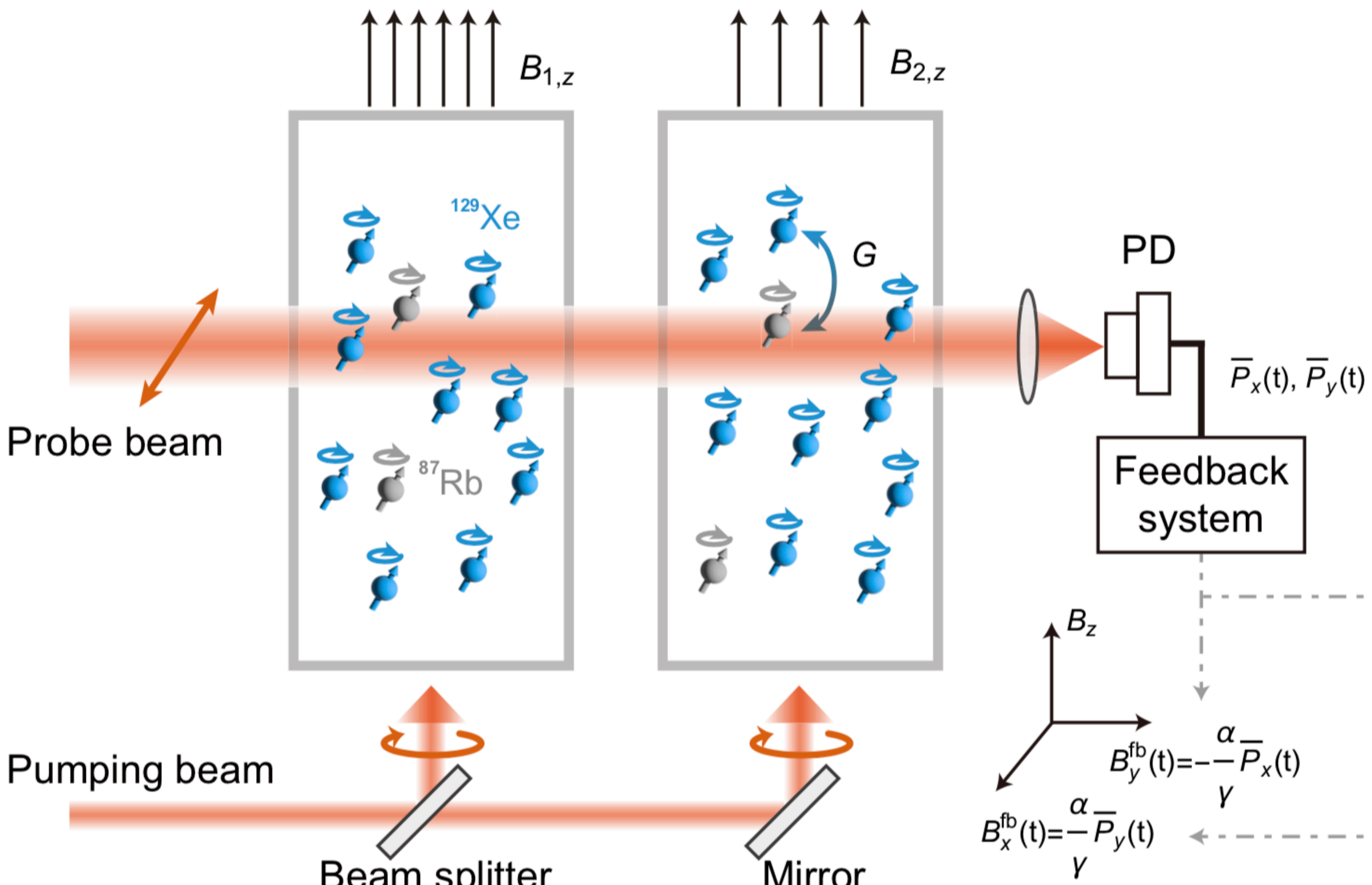
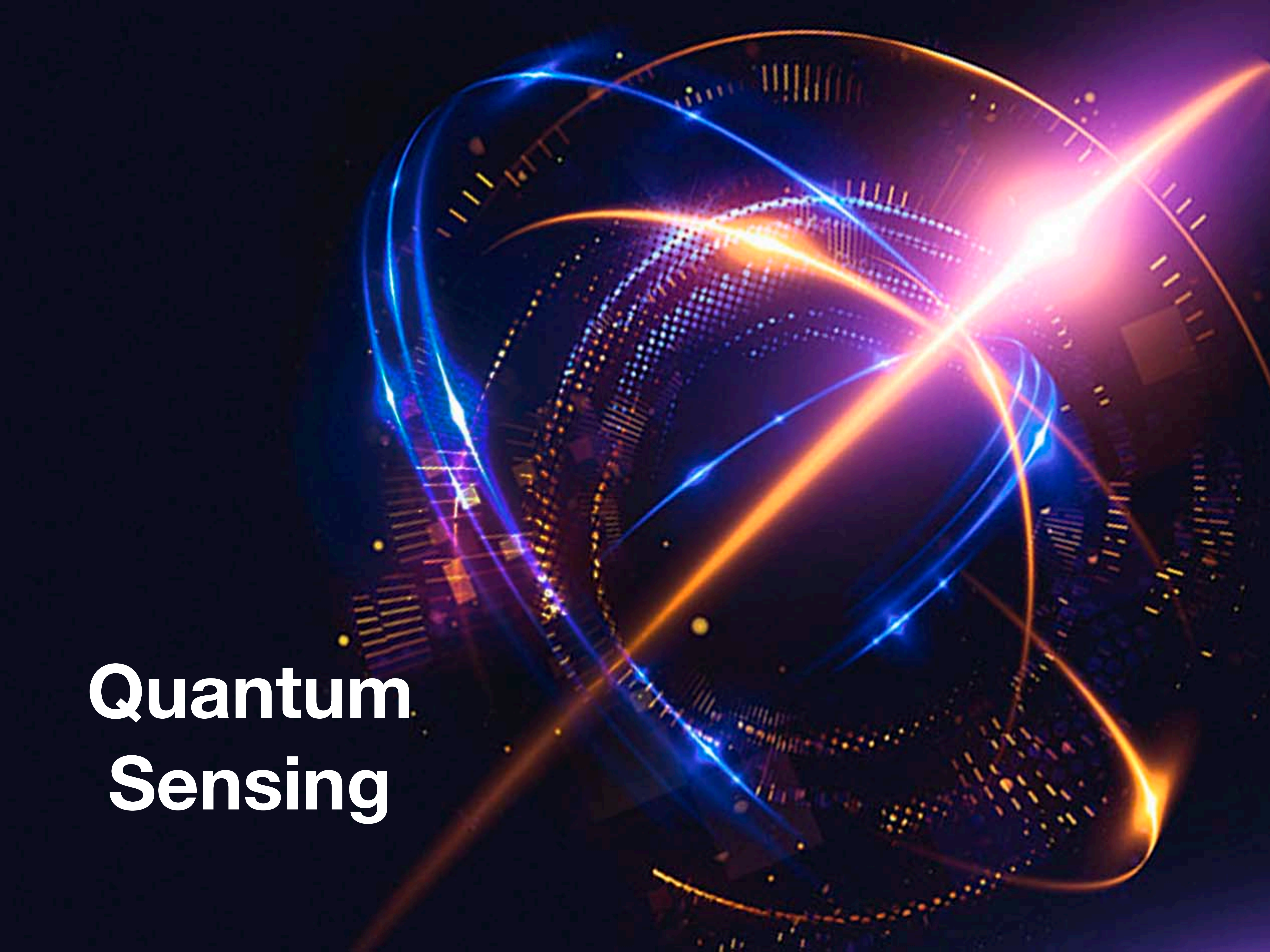
Atomic Magnetometers: Nonlinear Dynamics & Applications

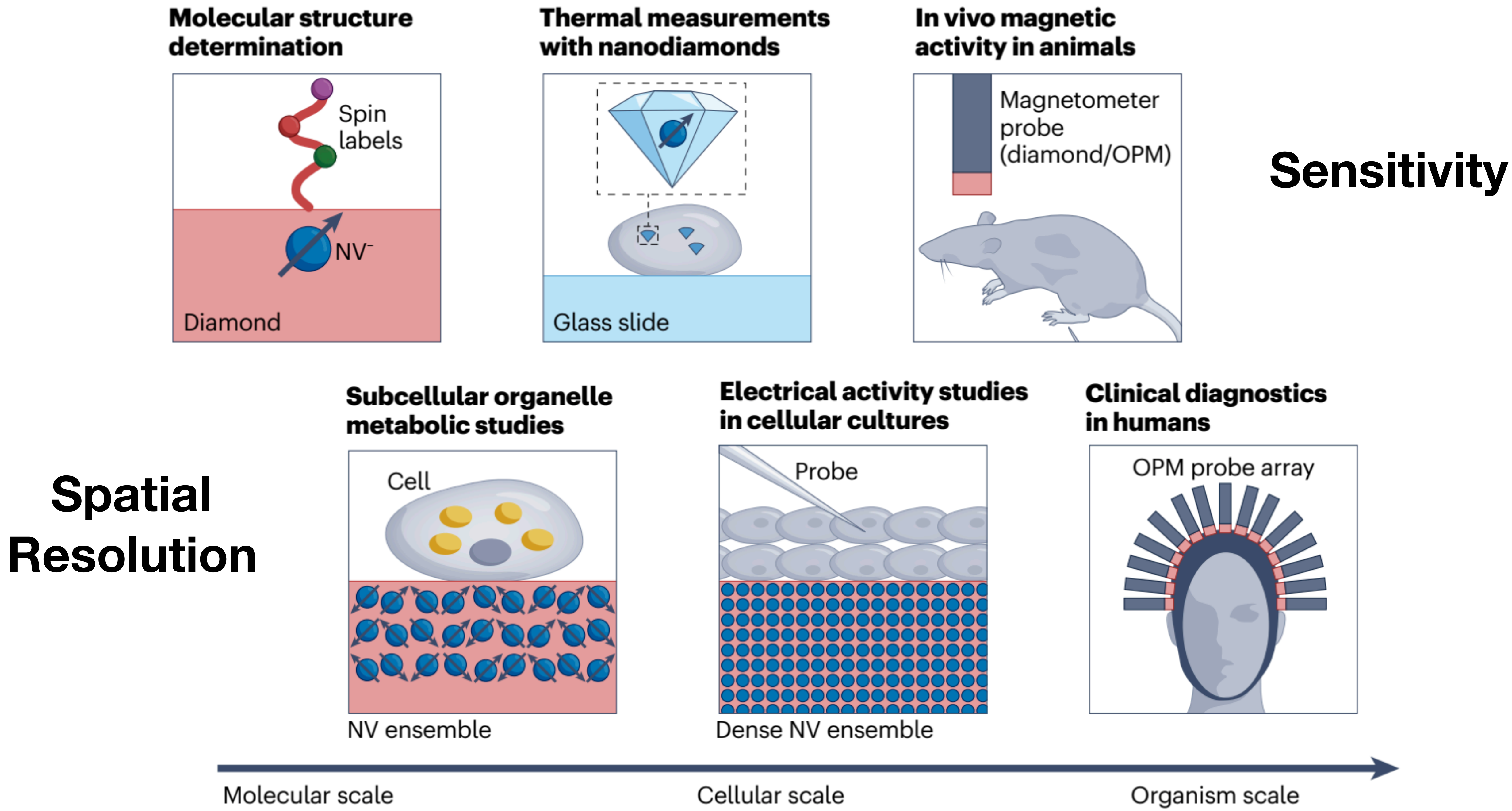
Zhenhua Yu



FB23, Sept 23, 2024

Quantum Sensing





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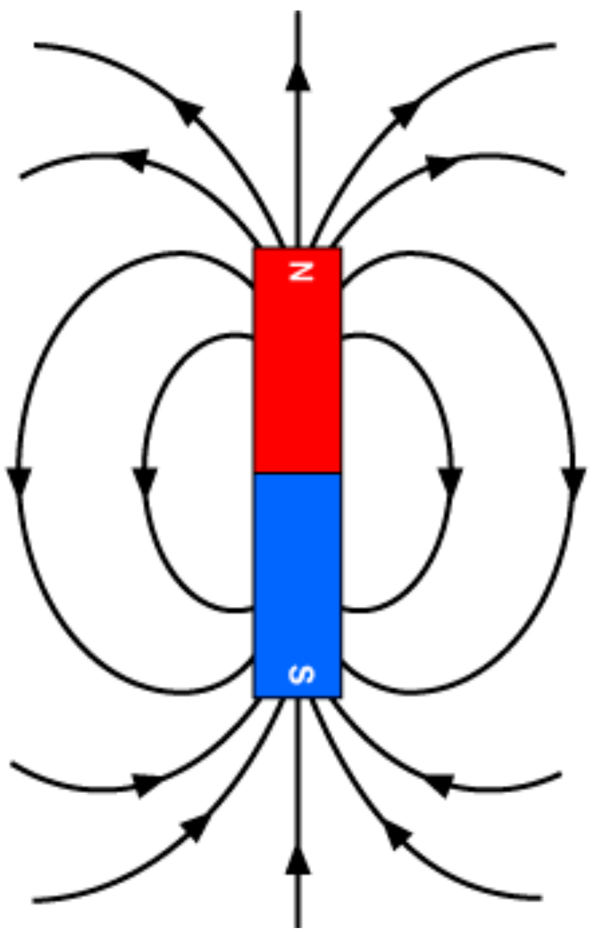
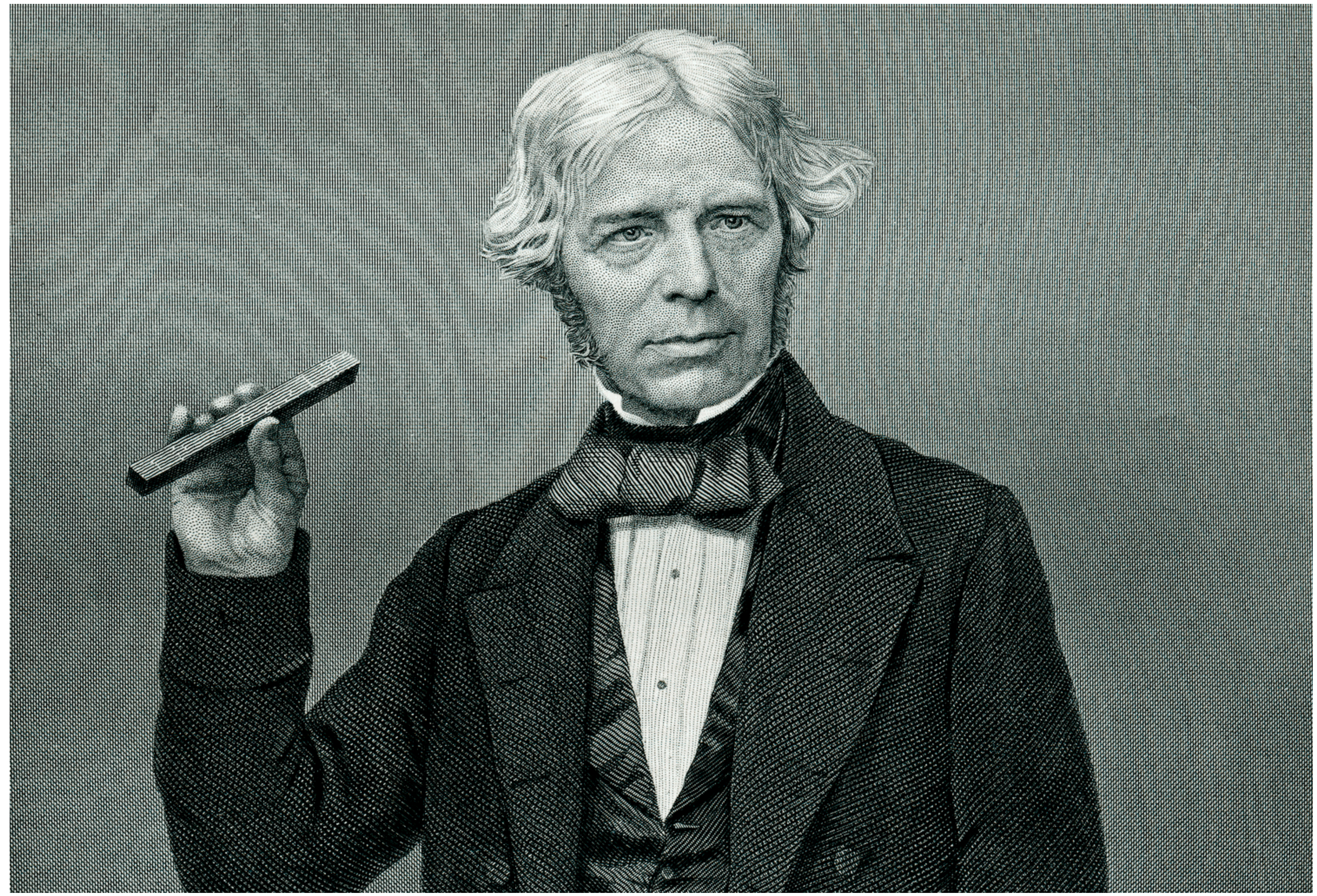
Concept of Fields

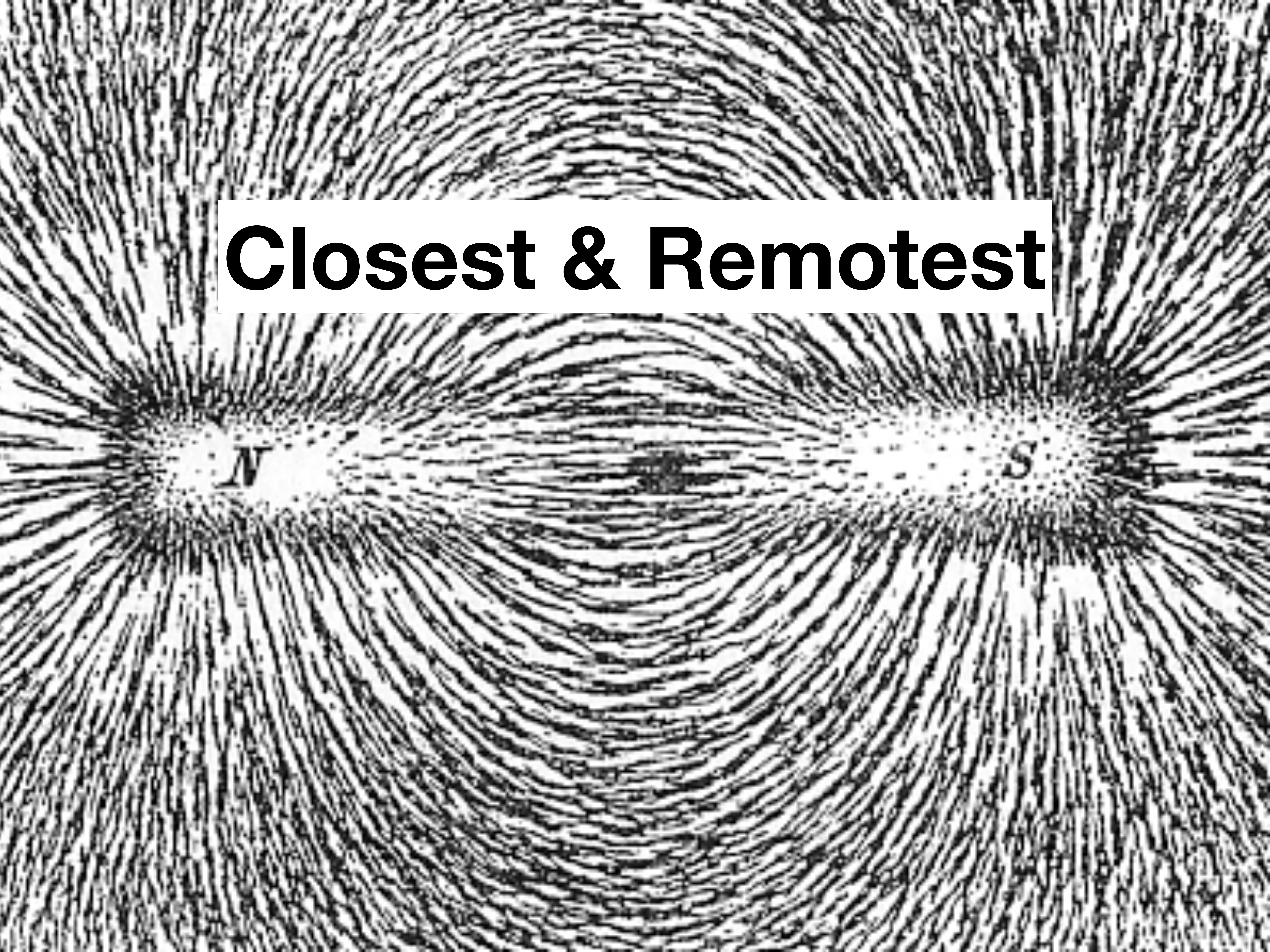




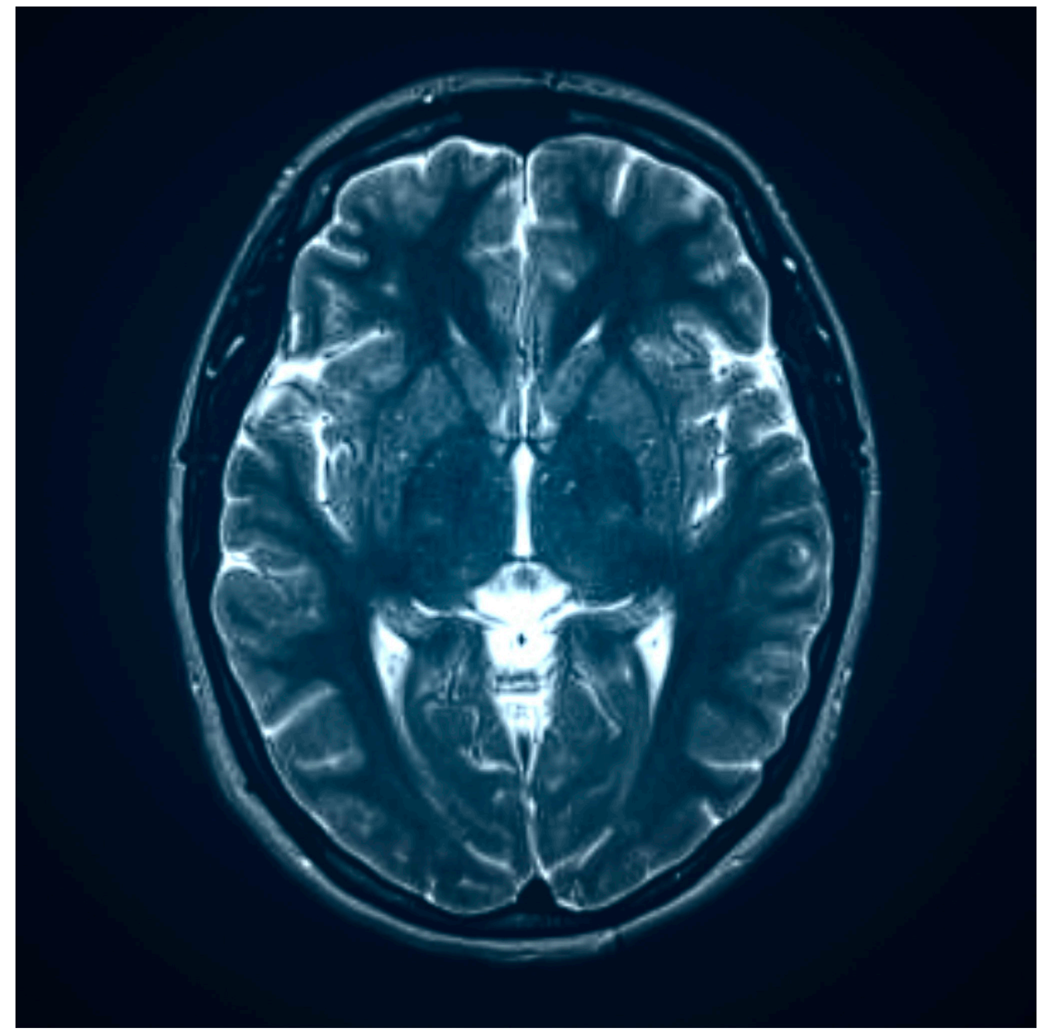
***On New Magnetic Actions* in 1846**

Michael Faraday

Closest & Remotest

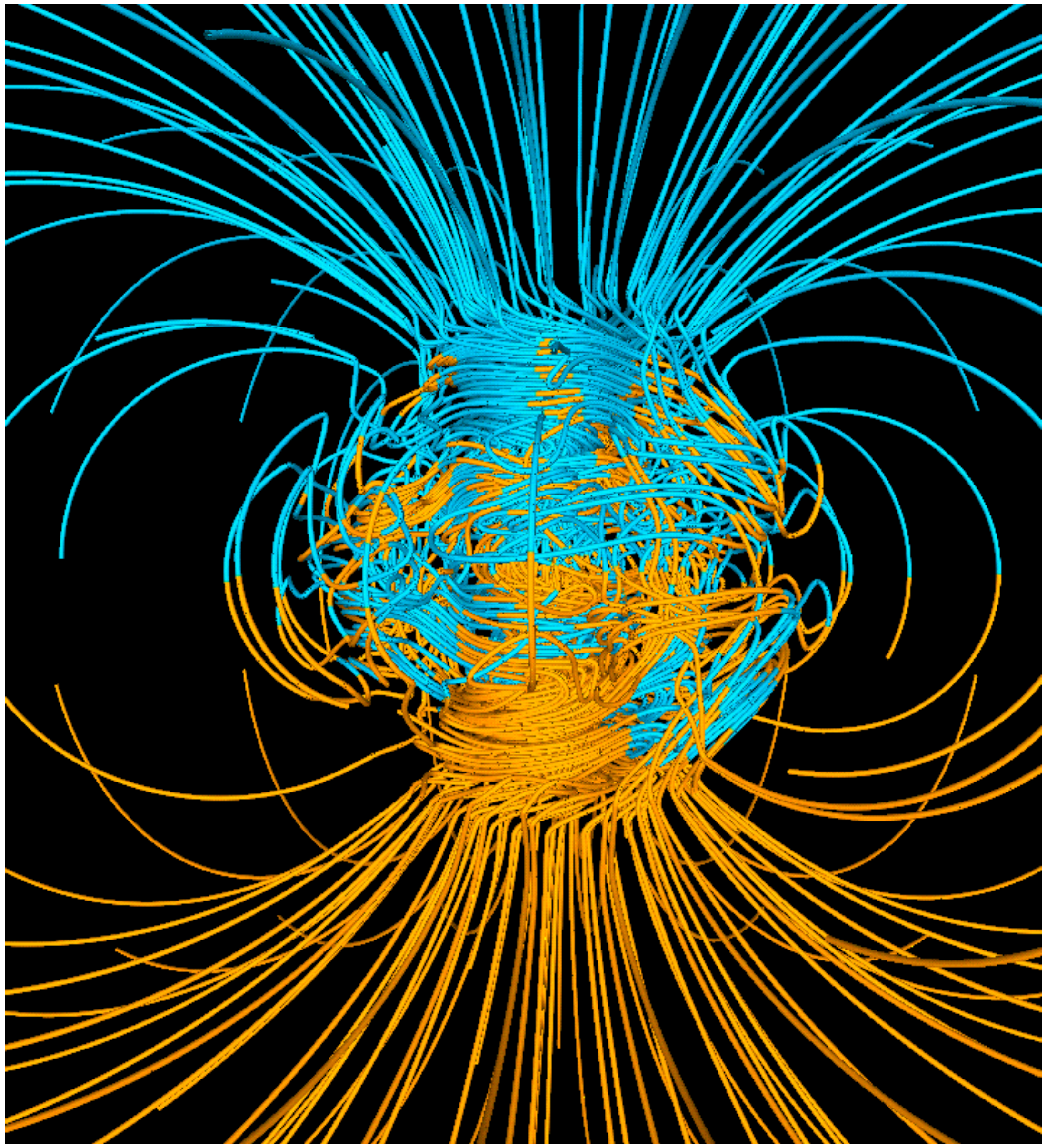


Measurement of Magnetic Fields

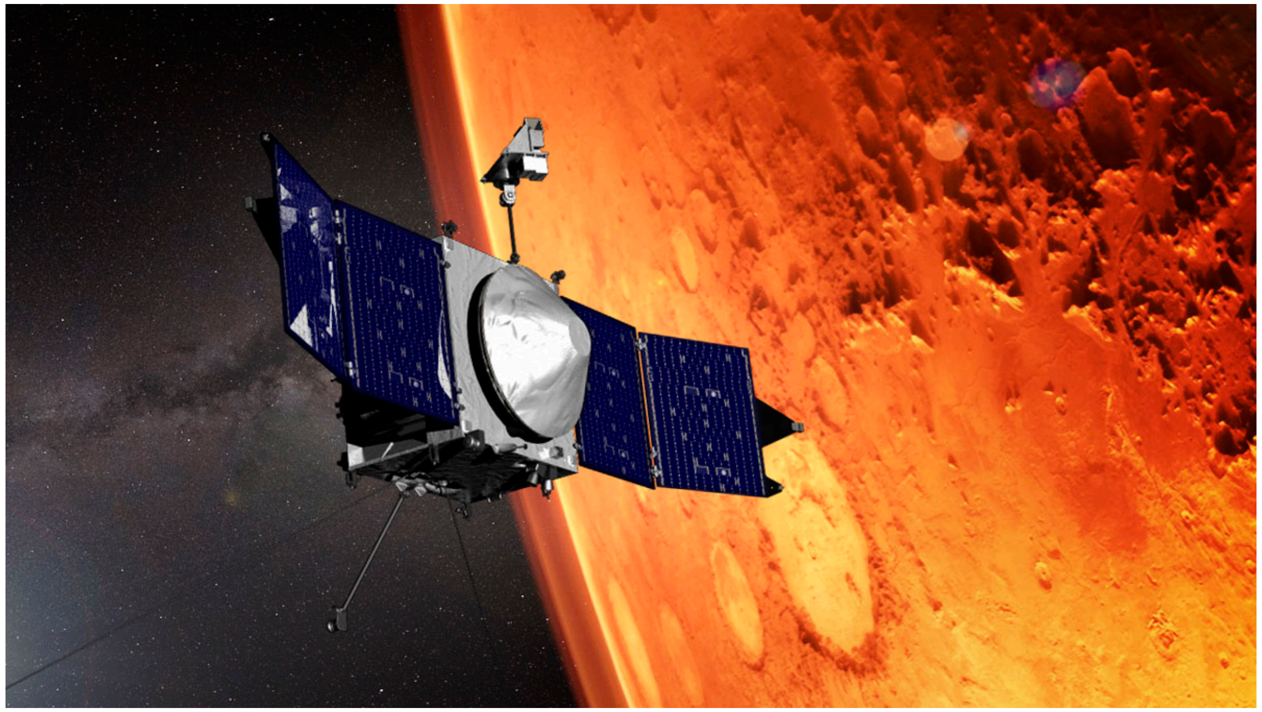


Magnetic Resonance Imaging (MRI)

Measurement of Magnetic Fields



Earth Magnetic Field



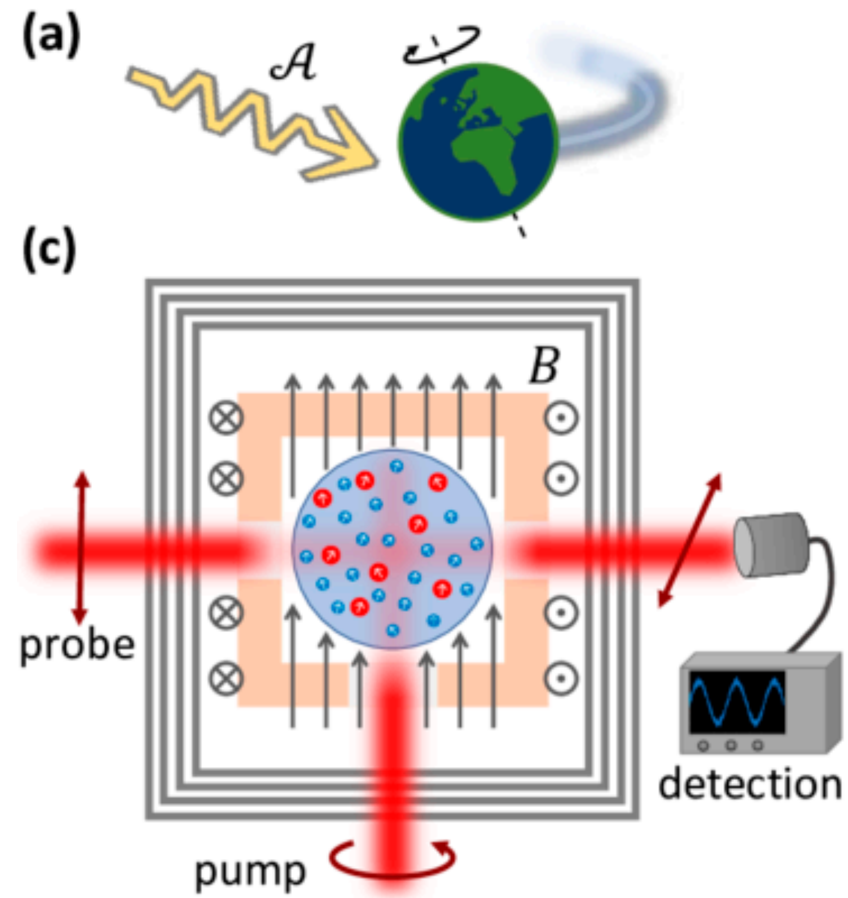
Mars Maven Mission

Mission Facts

Mission Status	Currently Operating
Mars Orbit Insertion	September 21, 2014
Launched	November 18, 2013
Launch Site	Cape Canaveral Air Force Station, Florida

Measurement of Magnetic Fields

Axions



$$H_{\text{spin}} = H_{\text{mag}} + H_{\text{BSM}} = -\vec{\mu} \cdot \vec{B}_{ex} - \vec{\mu} \cdot \underline{\vec{\beta}}$$

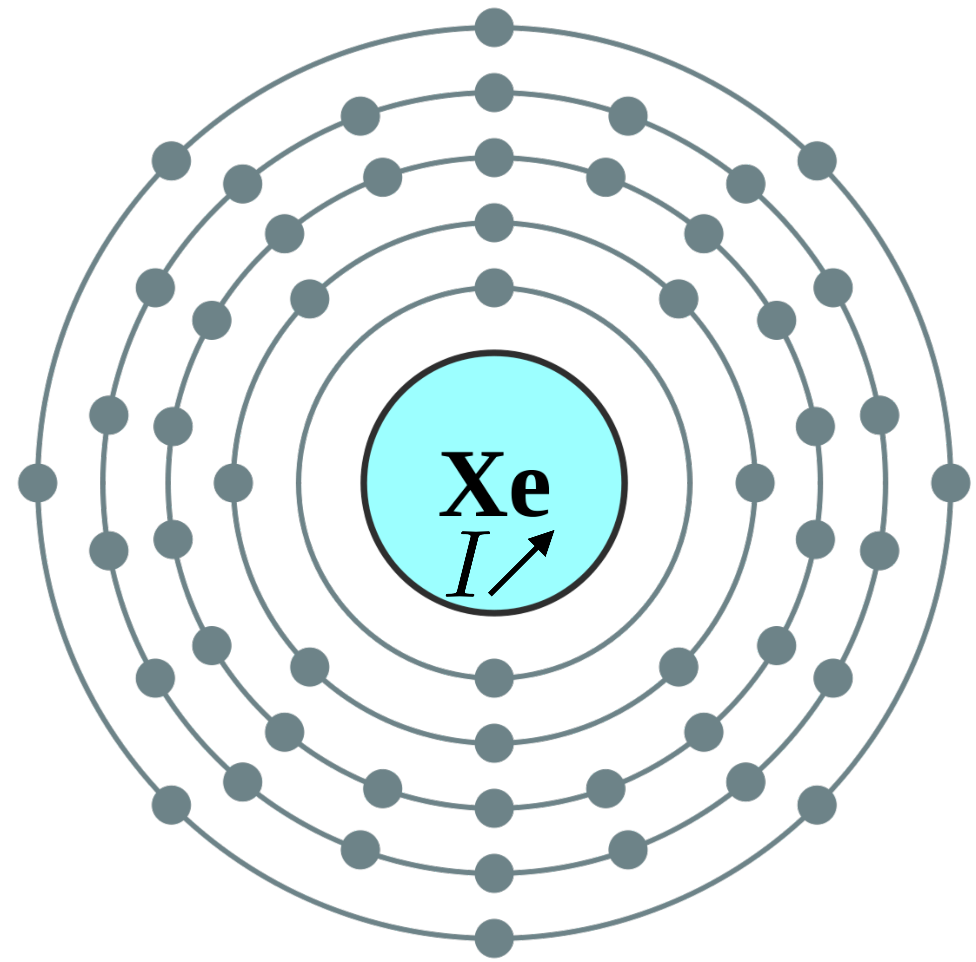
Physics beyond the Standard Model

Noble Atom Based Magnetometers

$$S = 0,\ L = 0$$

$$I_{\mathrm{Xe-129}} = 1/2,\ I_{\mathrm{Xe-131}} = 3/2$$

Noble Atom Based Magnetometers

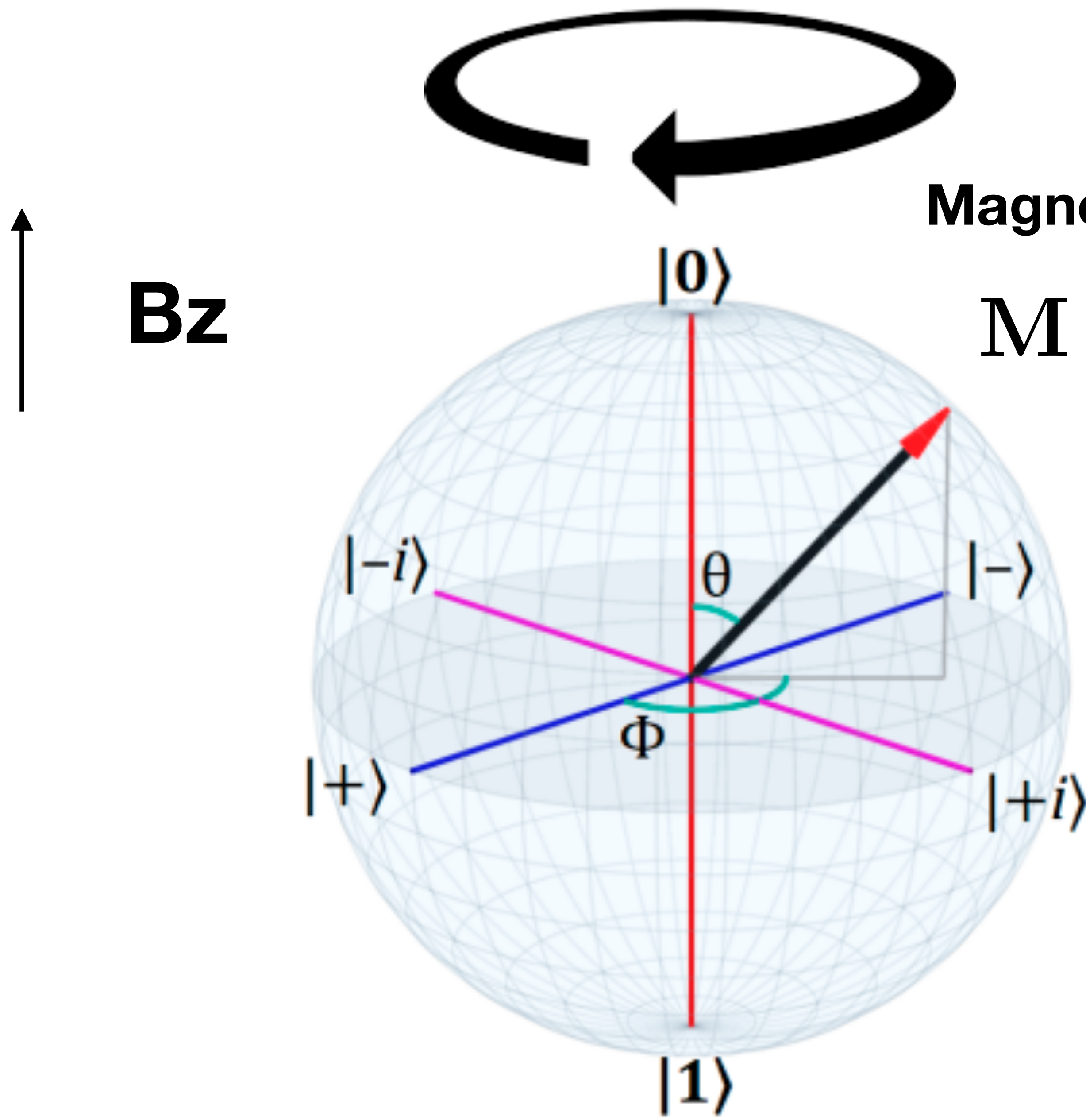


$$S = 0,\ L = 0$$

$$I_{\mathrm{Xe-129}} = 1/2,\ I_{\mathrm{Xe-131}} = 3/2$$

Long nuclear spin coherence time

Noble Atom Based Magnetometers



Magnetic dipole moment

$$\mathrm{M} = \gamma \langle \mathbf{I} \rangle$$

Xe-129

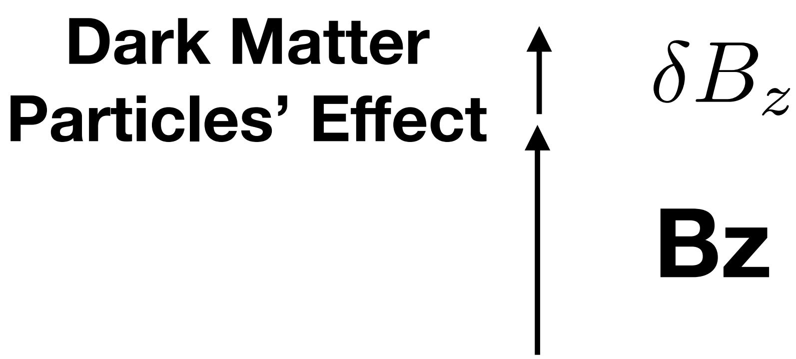
S=0

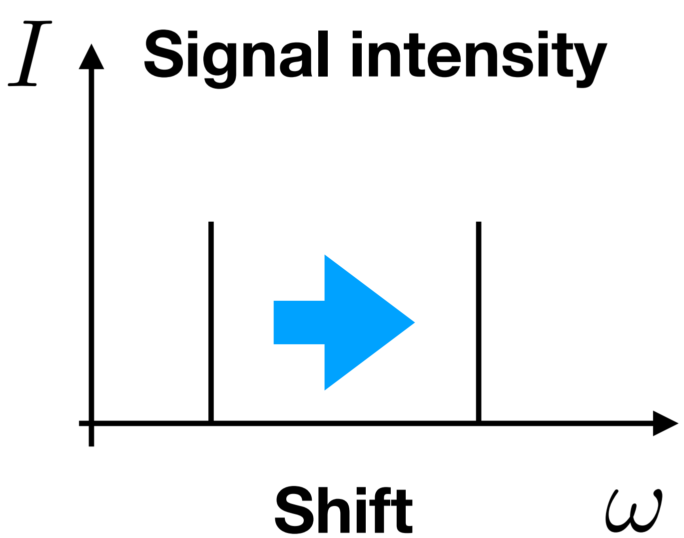
L=0

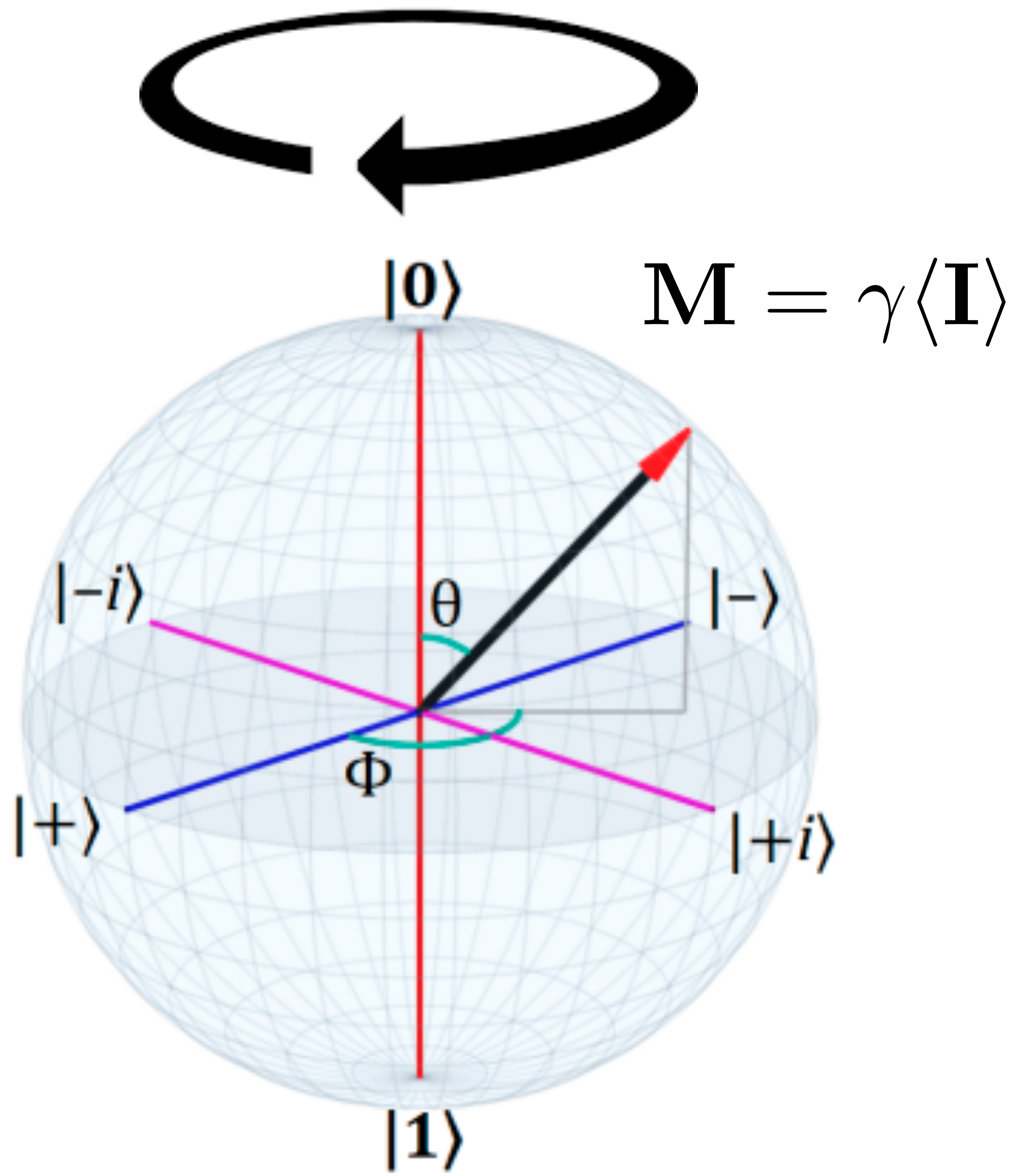
I=1/2

Larmor Precession $\omega_L = \gamma B_z$

Noble Atom Based Magnetometers



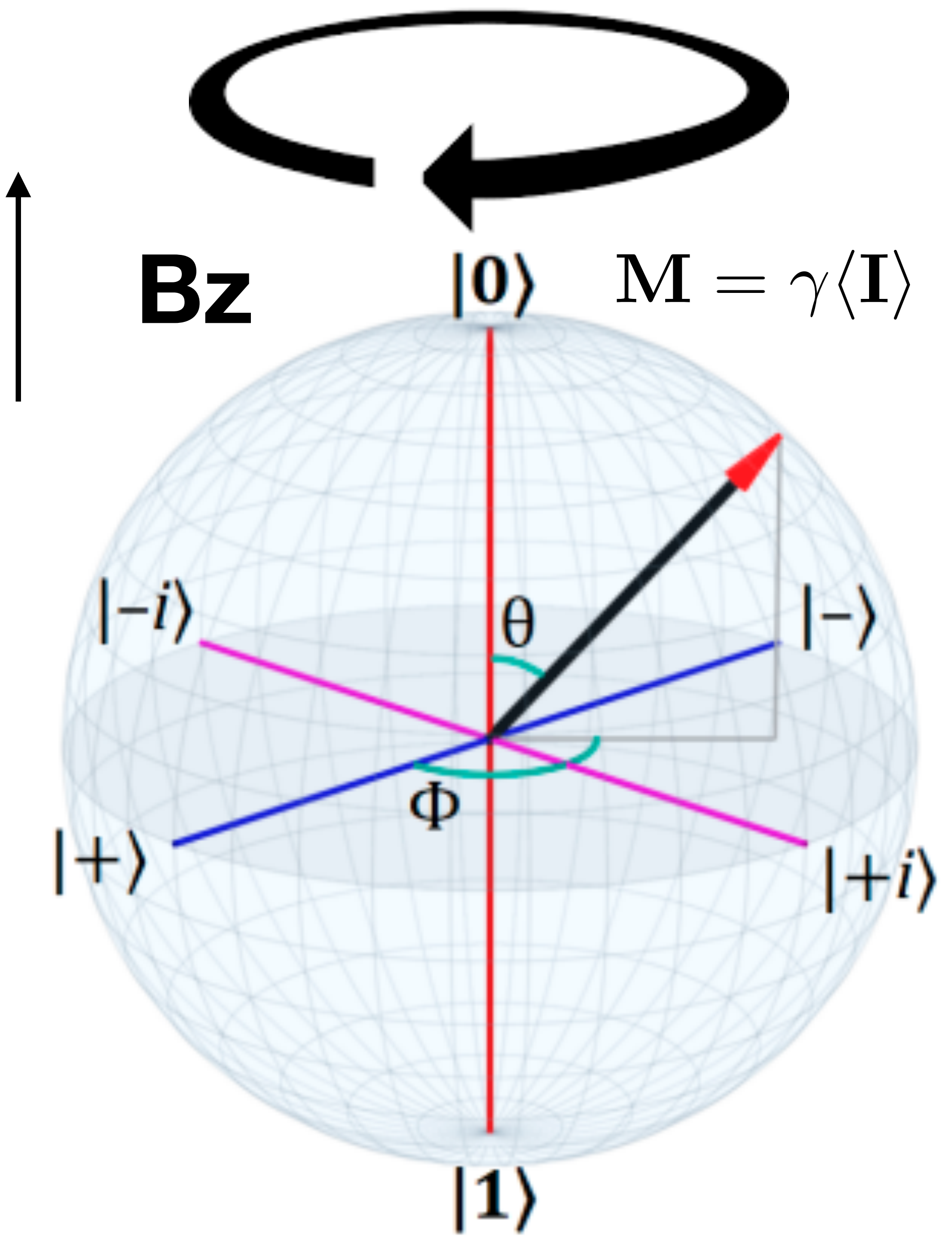




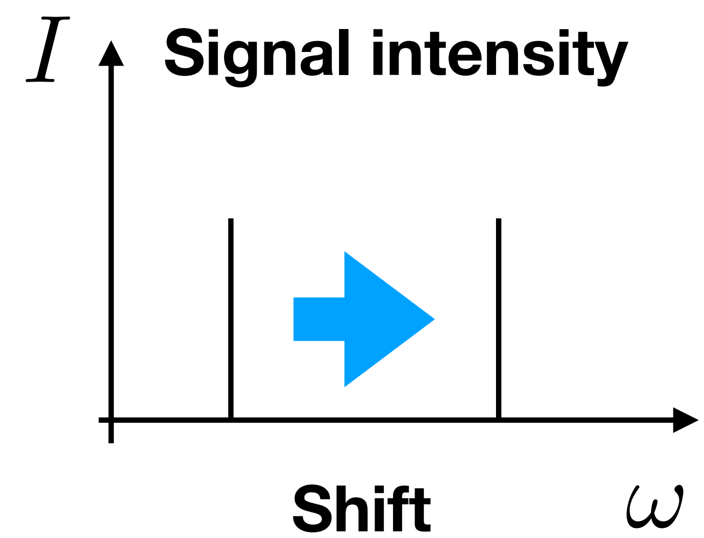
Xe-129
S=0
L=0
I=1/2

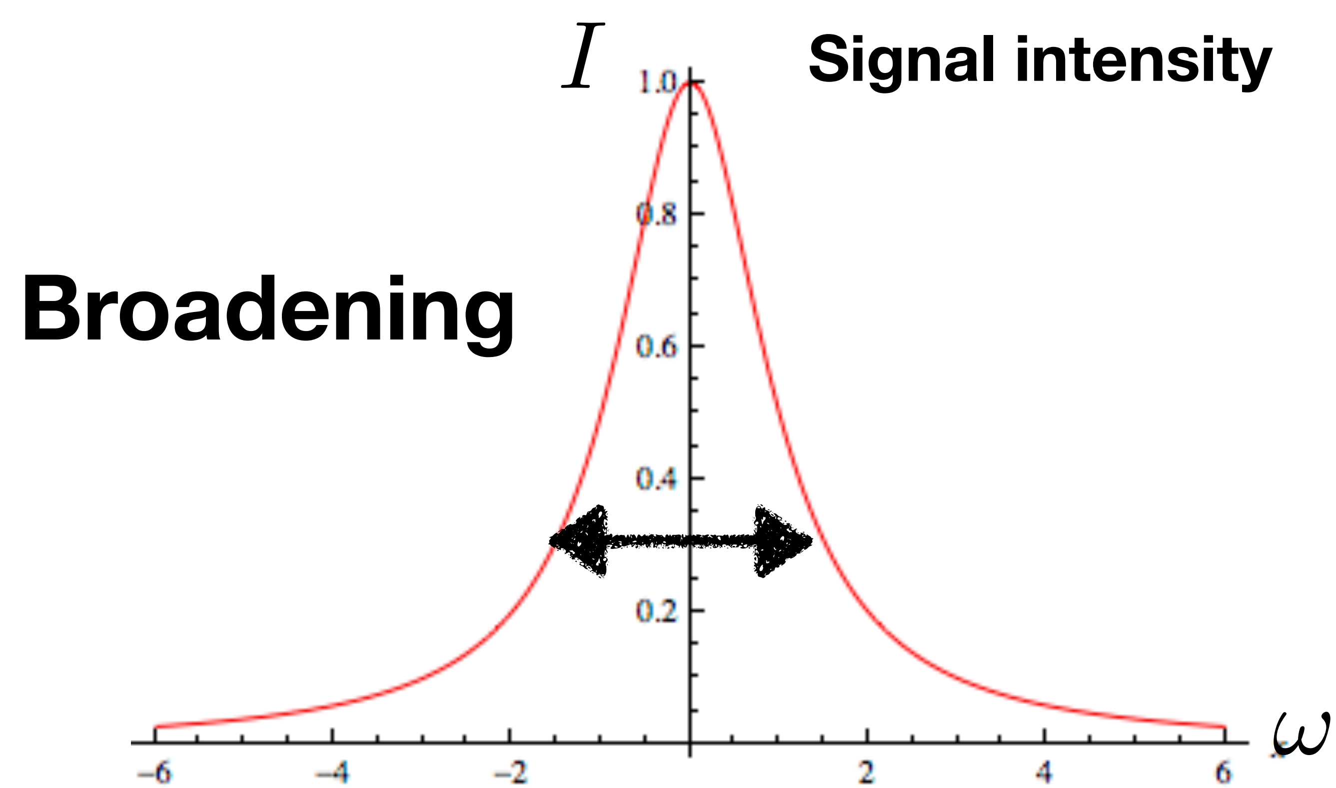
Larmor Precession $\omega_L = \gamma B_z$

Obstacle to Precision Measurement: Dissipation



Xe-129
S=0
L=0
I=1/2

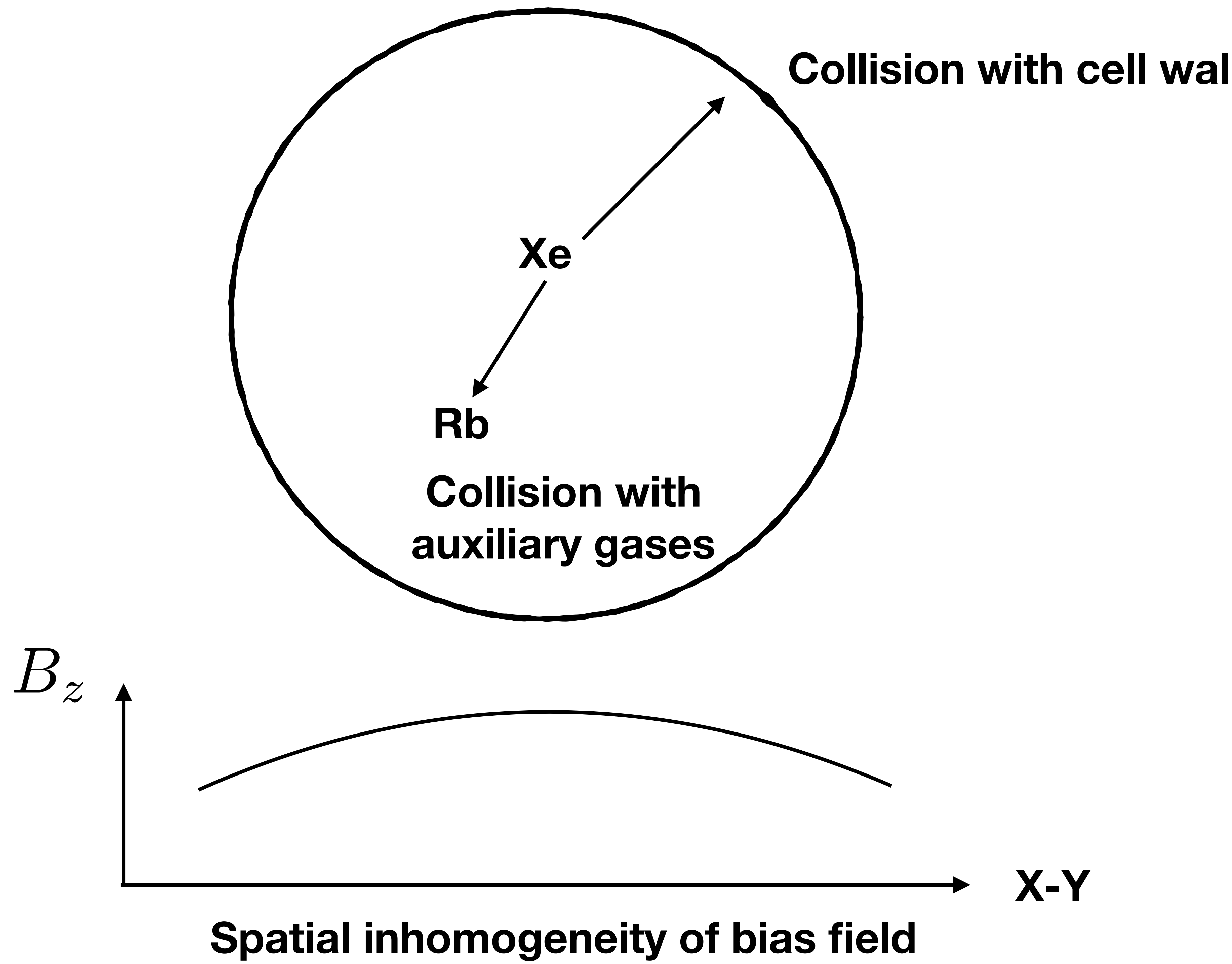




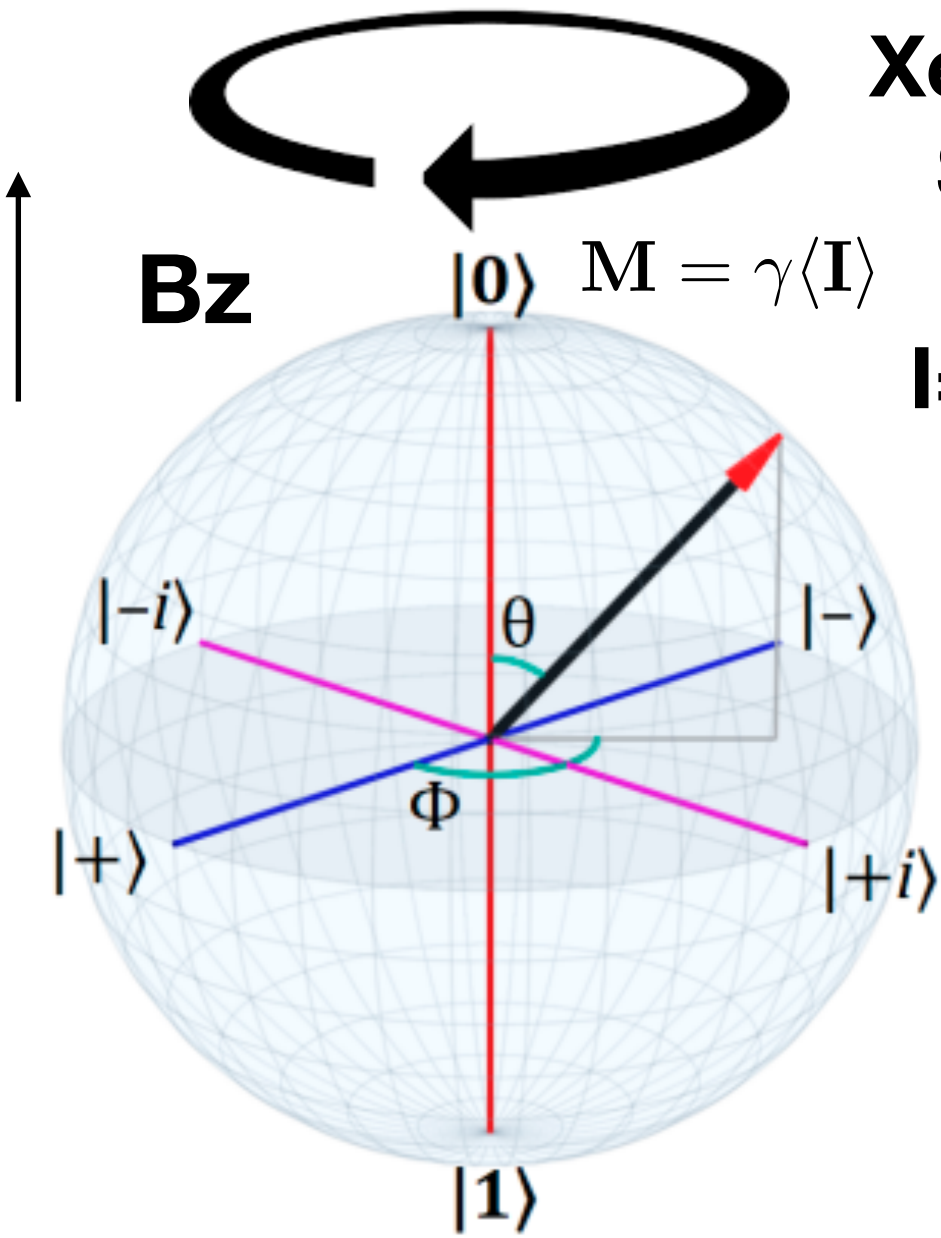
Larmor Precession

$$\omega_L = \gamma B_z$$

Where comes dissipation?



Obstacle to Precision Measurement: Dissipation



$\mathbf{M} = \gamma\langle\mathbf{I}\rangle$

Xe-129
S=0
L=0
I=1/2

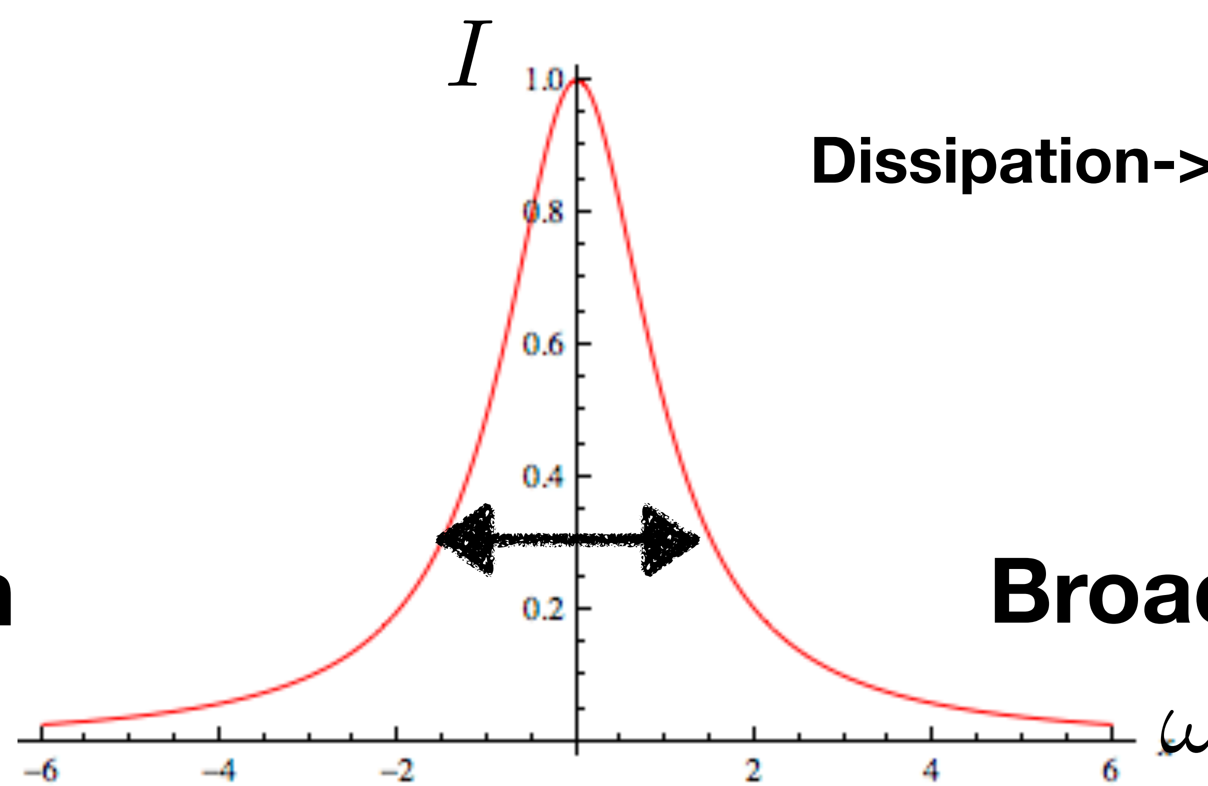
Bloch Equations:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_0 + \begin{bmatrix} -M_x/T_2 \\ -M_y/T_2 \\ -(M_z - M_0)/T_1 \end{bmatrix}$$

Magnetic dipole moment M

Pumping

Dissipation-> T_1, T_2

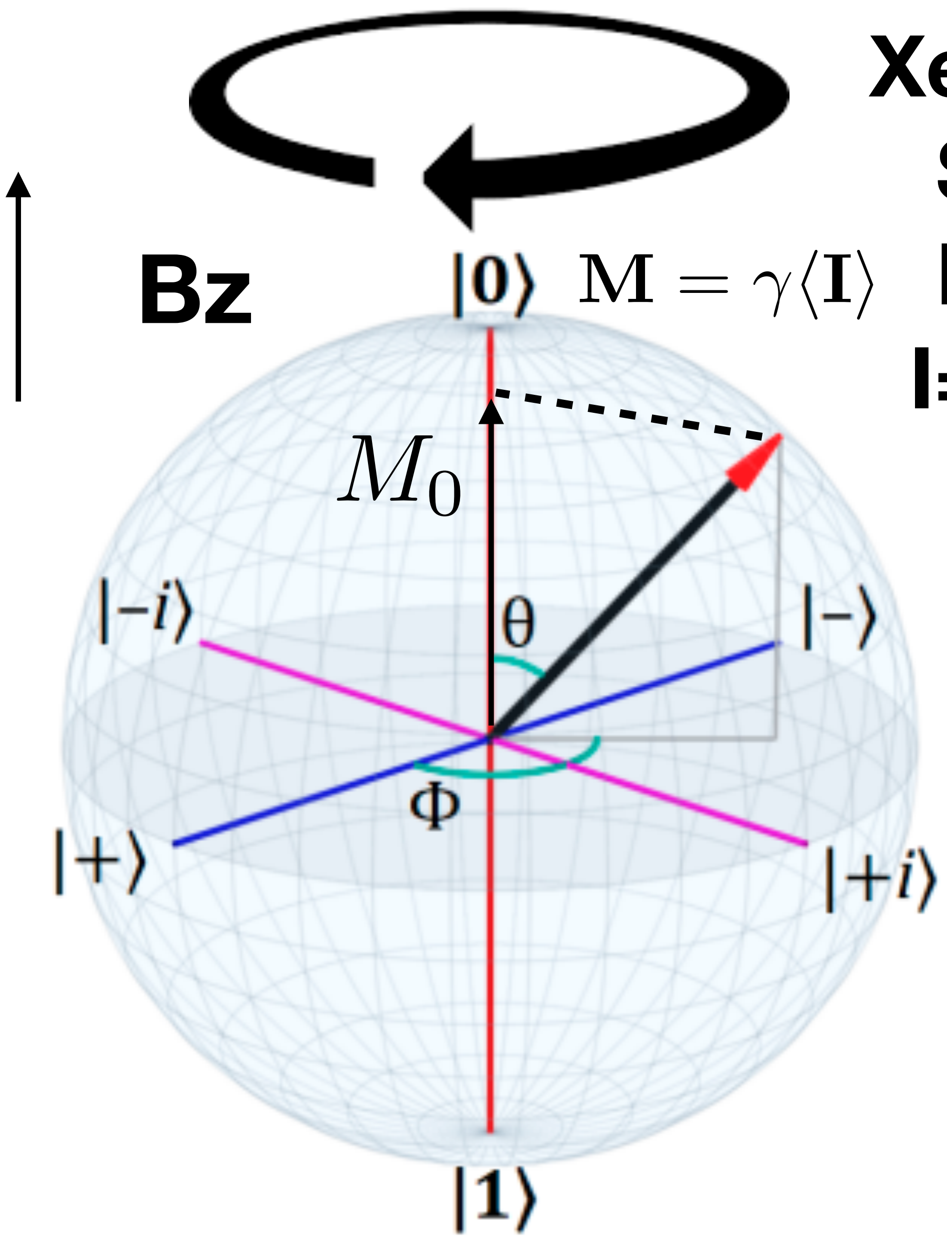


Larmor Precession

$$\omega_L = \gamma B_z$$

Broadening

Obstacle to Precision Measurement: Dissipation



Xe-129
S=0
L=0
I=1/2

Larmor Precession

$$\omega_L = \gamma B_z$$

Bloch Equations:

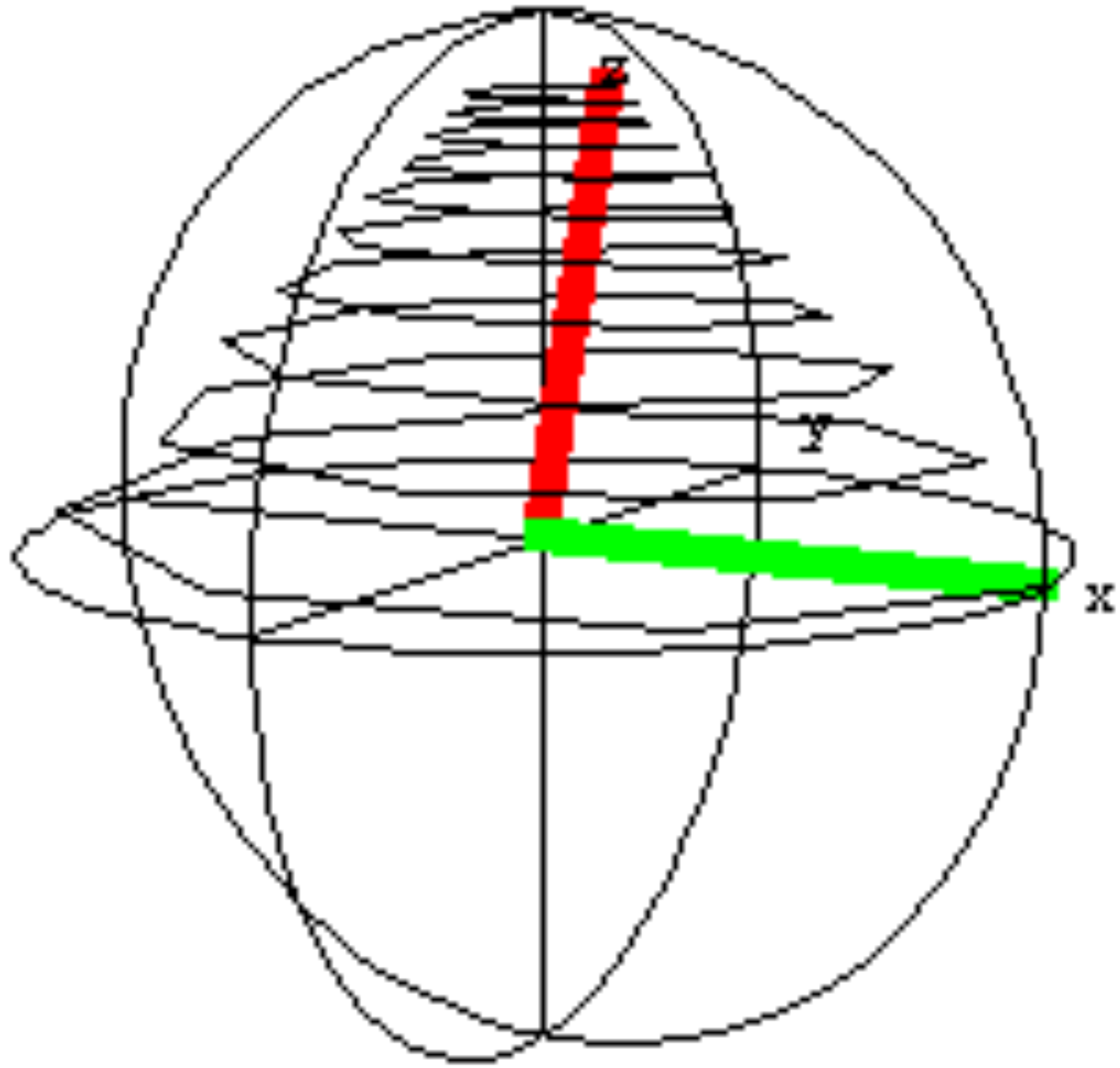
$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_0 + \begin{bmatrix} -M_x/T_2 \\ -M_y/T_2 \\ -(M_z - M_0)/T_1 \end{bmatrix}$$

Magnetic dipole moment M

Pumping

Long time steady state:

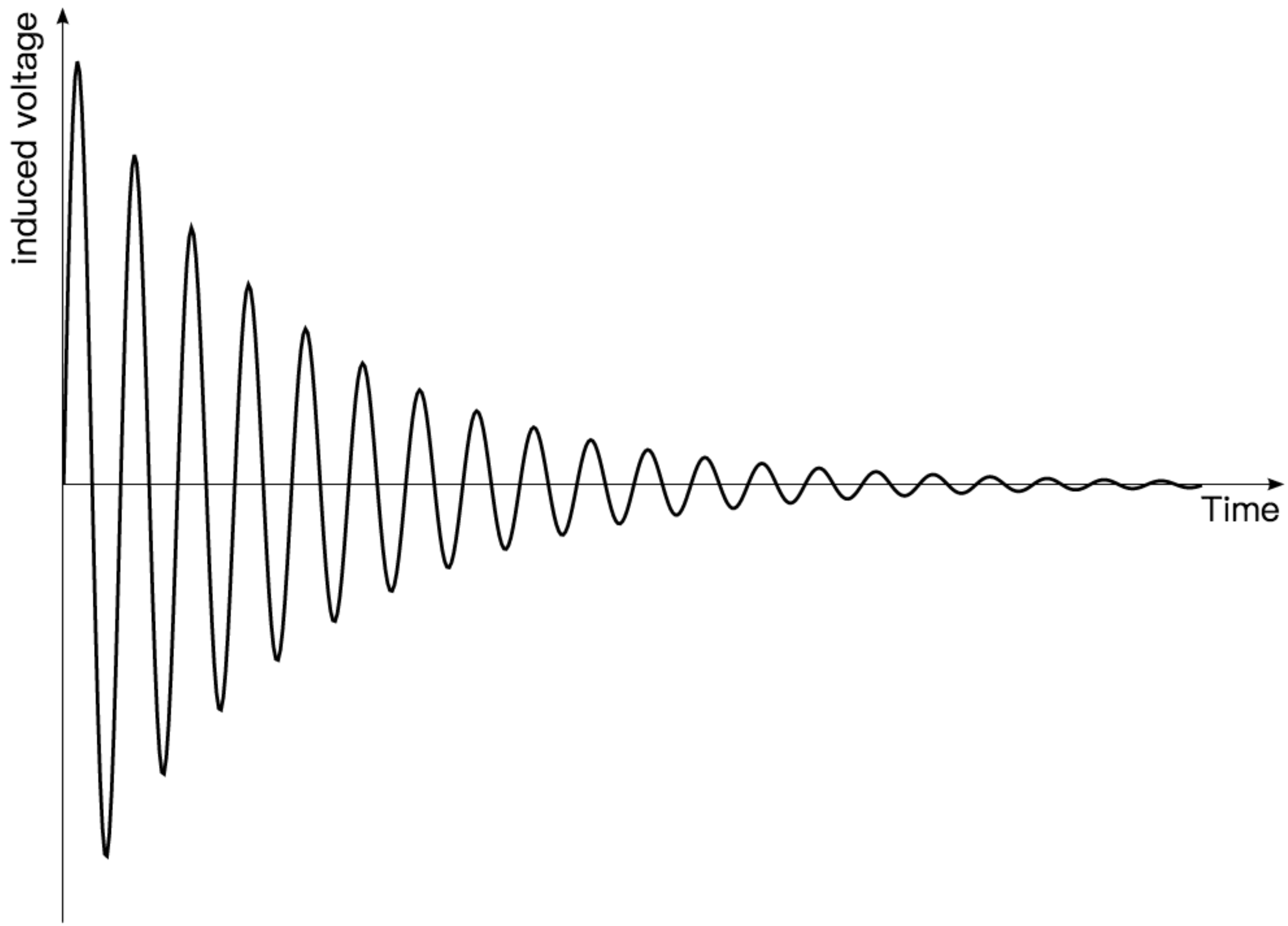
$$M_x = M_y = 0, \; M_z = M_0$$



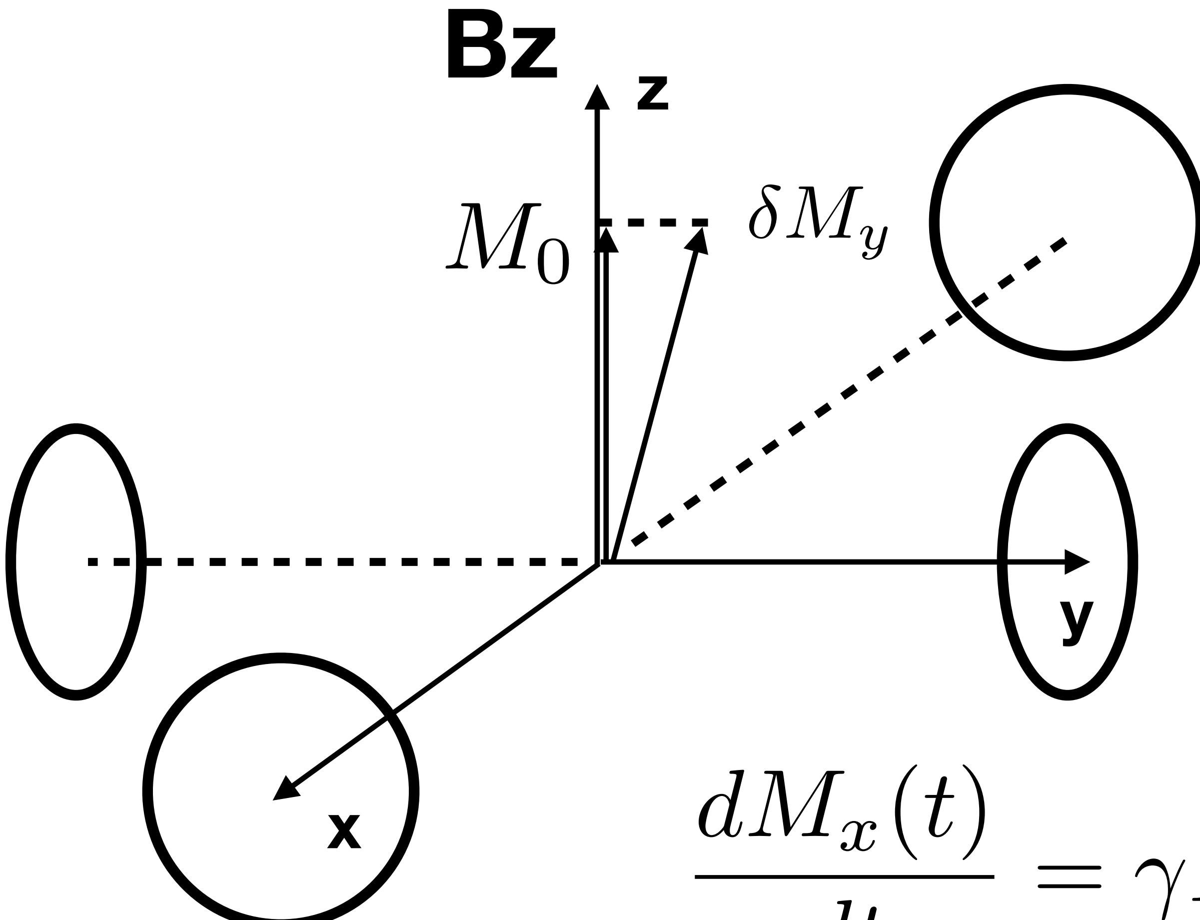
Long time steady state: $M_x = M_y = 0, \ M_z = M_0$

Free Induction Decay

M_x



Another Scheme : Feedback

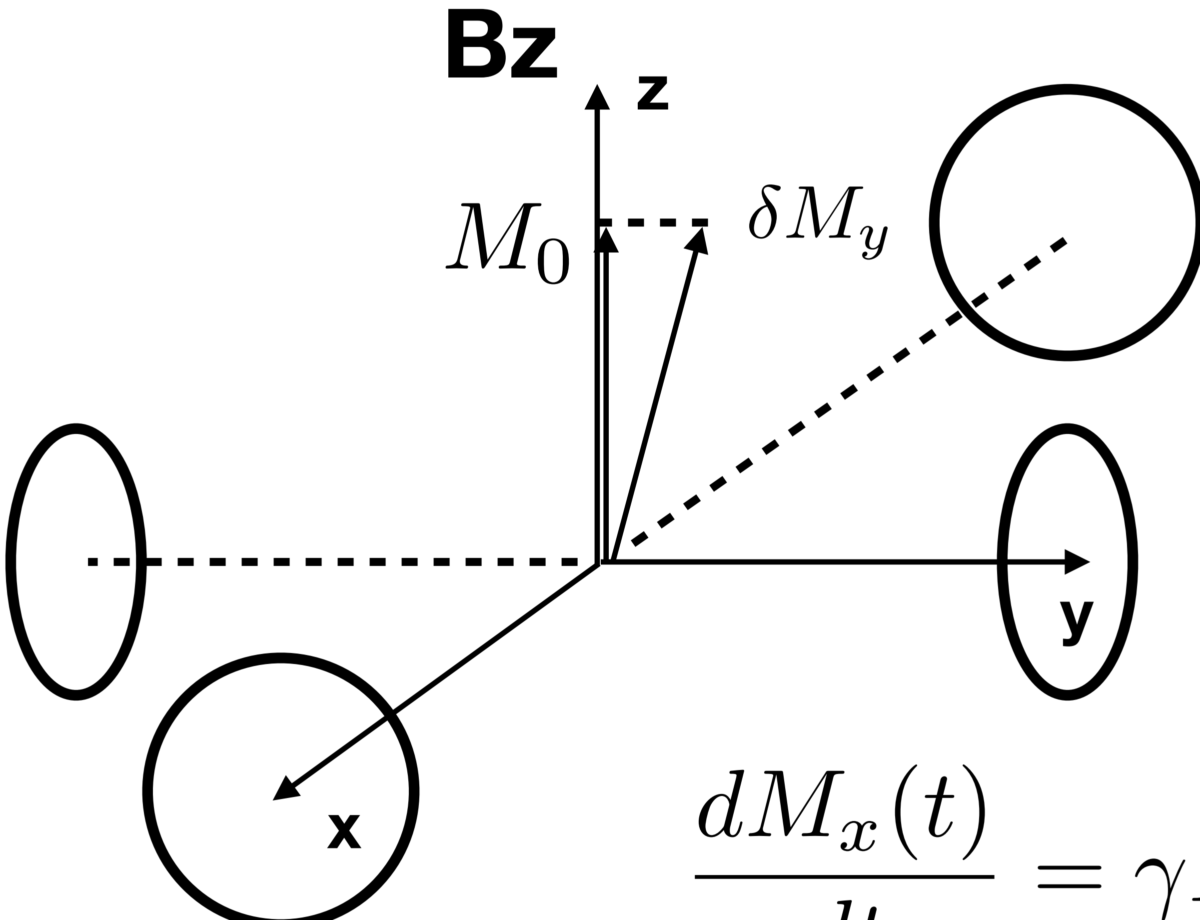


$$B_x(t) = \alpha M_y(t)/\gamma$$
$$B_y(t) = -\alpha M_x(t)/\gamma$$

$$\frac{dM_x(t)}{dt} = \gamma B_z M_y + \alpha M_z M_x - \frac{M_x}{T_2}$$
$$\frac{dM_y(t)}{dt} = -\gamma B_z M_x + \alpha M_z M_y - \frac{M_y}{T_2}$$

$$\frac{dM_z(t)}{dt} = \alpha(M_x + M_y)^2 - \frac{M_0 - M_z}{T_1}$$

Another Scheme : Feedback



$$B_x(t) = \alpha M_y(t)/\gamma$$
$$B_y(t) = -\alpha M_x(t)/\gamma$$

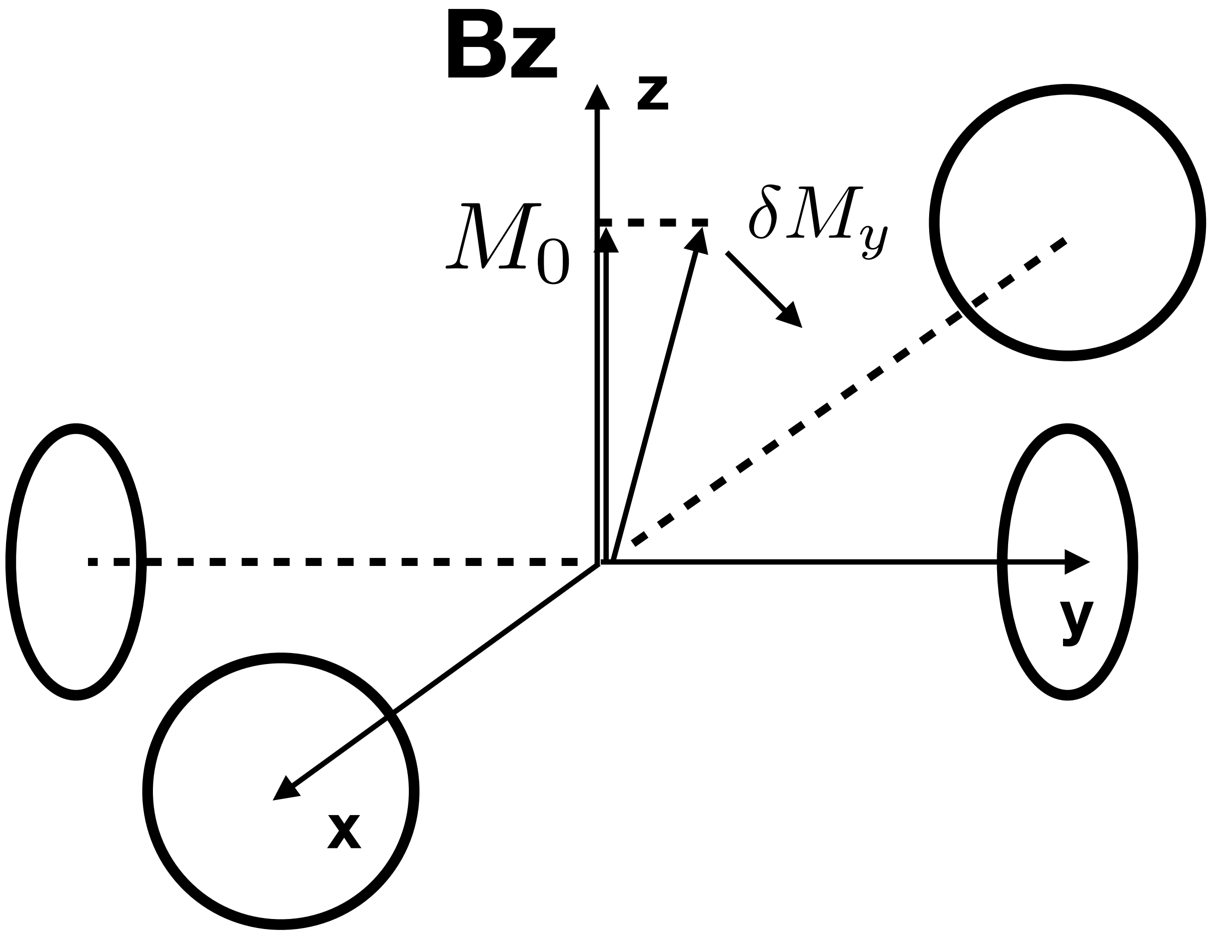
Deviate from the steady state (0,0,M_0)

$$\frac{dM_x(t)}{dt} = \gamma B_z M_y + \alpha M_z M_x - \frac{M_x}{T_2}$$

$$\frac{dM_y(t)}{dt} = -\gamma B_z M_x + \alpha M_z M_y - \frac{M_y}{T_2}$$

$$\frac{dM_z(t)}{dt} = \alpha (M_x + M_y)^2 - \frac{M_0 - M_z}{T_1}$$

Another Scheme : Feedback



$$\frac{dM_x(t)}{dt} = \gamma B_z M_y + \alpha M_z M_x - \frac{M_x}{T_2}$$

$$\frac{dM_y(t)}{dt} = -\gamma B_z M_x + \alpha M_z M_y - \frac{M_y}{T_2}$$

$$\frac{dM_z(t)}{dt} = \alpha (M_x + M_y)^2 - \frac{M_0 - M_z}{T_1}$$

$$B_x(t) = \alpha M_y(t)/\gamma$$

$$B_y(t) = -\alpha M_x(t)/\gamma$$

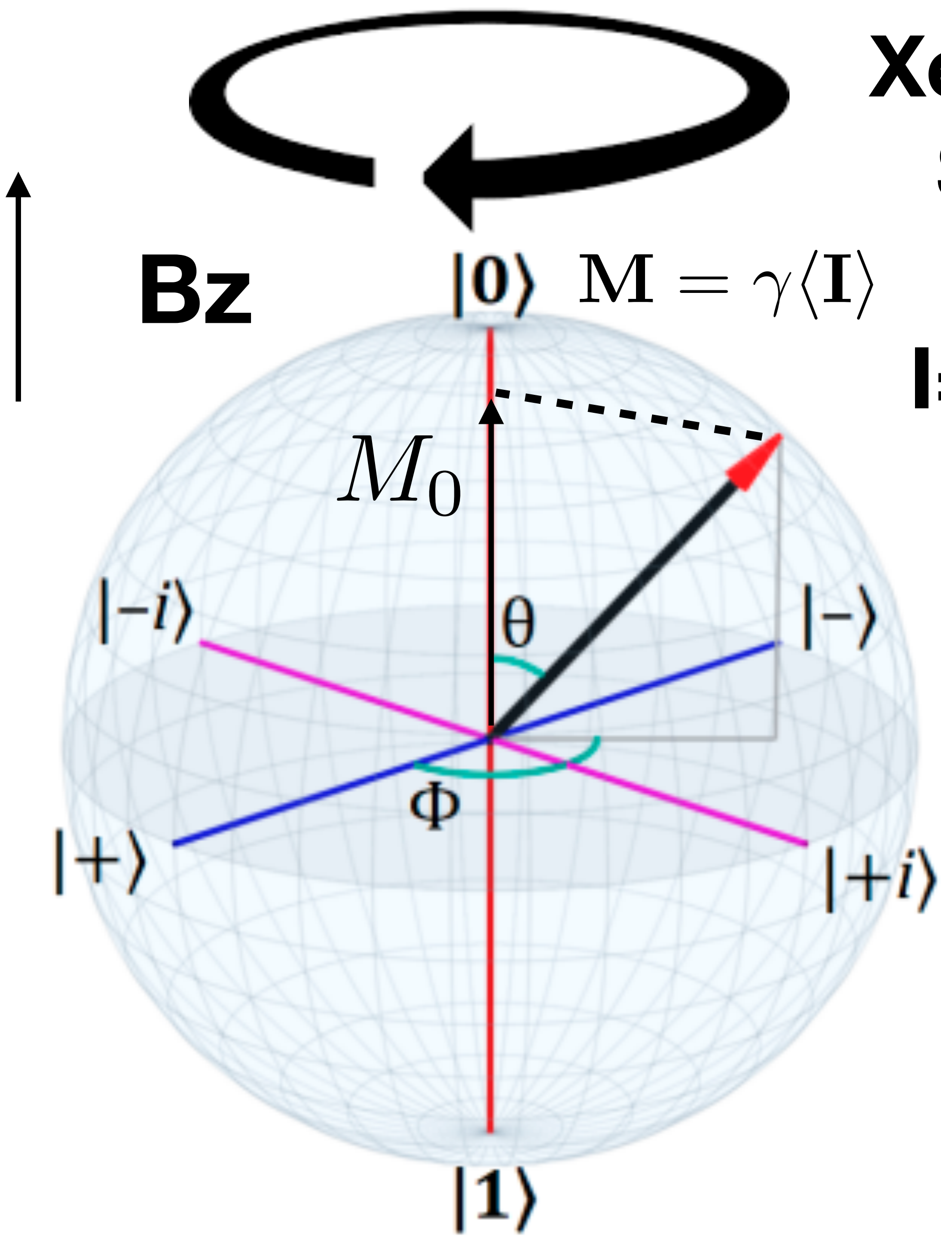
Deviate from the steady state (0,0,M_0)

If

$$\frac{d\delta M_y}{dt} = (\alpha M_0 - 1/T_2)\delta M_y$$

$\alpha M_0 - 1/T_2 > 0$ **unstable**

Steady Precession - Limit Cycle



Xe-129
S=0
L=0
I=1/2

Bloch Equations:

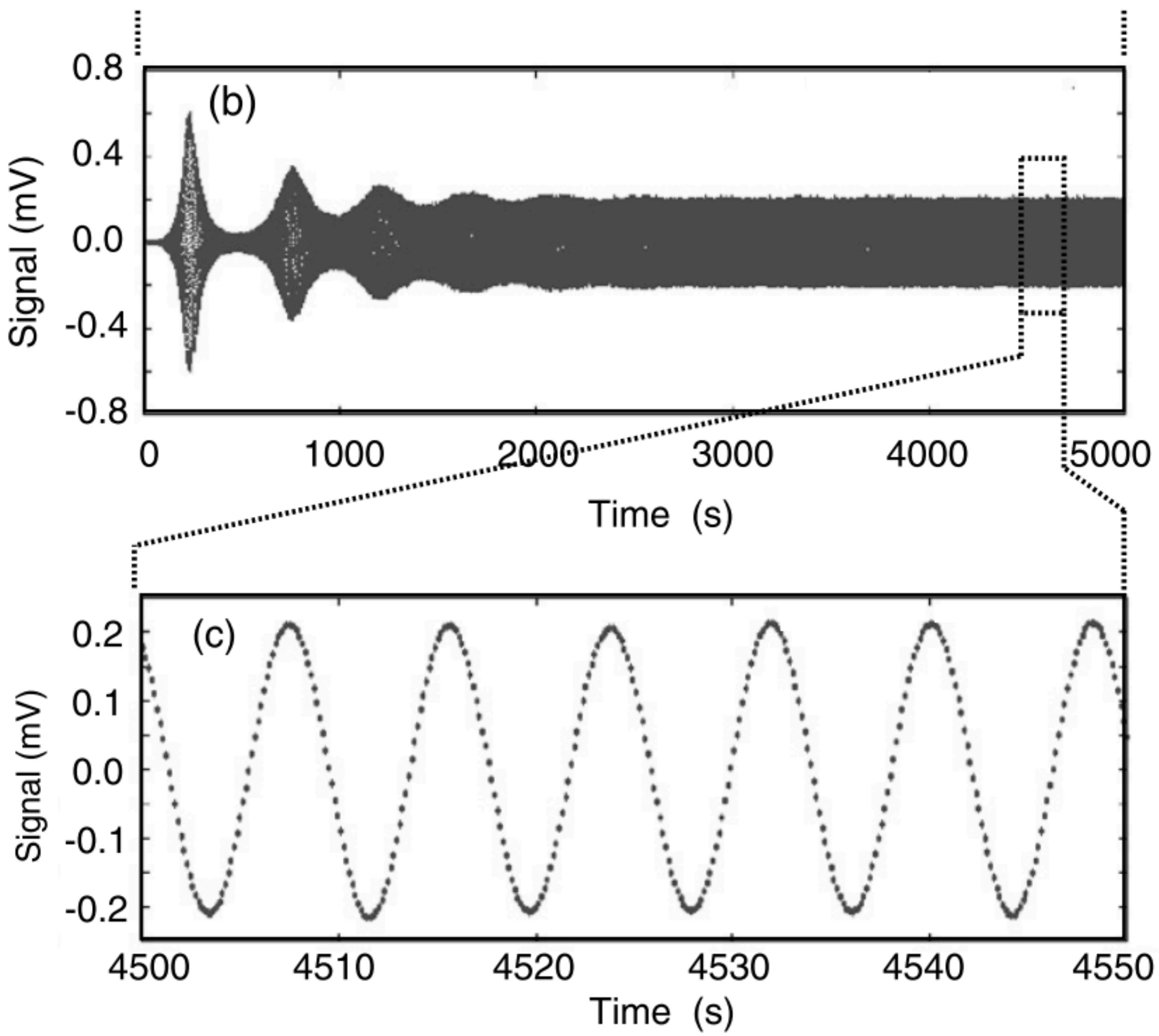
$$\frac{dM_x(t)}{dt} = \gamma B_z M_y + \alpha M_z M_x - \frac{M_x}{T_2}$$

$$\frac{dM_y(t)}{dt} = -\gamma B_z M_x + \alpha M_z M_y - \frac{M_y}{T_2}$$

$$\frac{dM_z(t)}{dt} = \alpha (M_x + M_y)^2 - \frac{M_0 - M_z}{T_1}$$

Larmor Precession

$$\omega_L = \gamma B_z$$



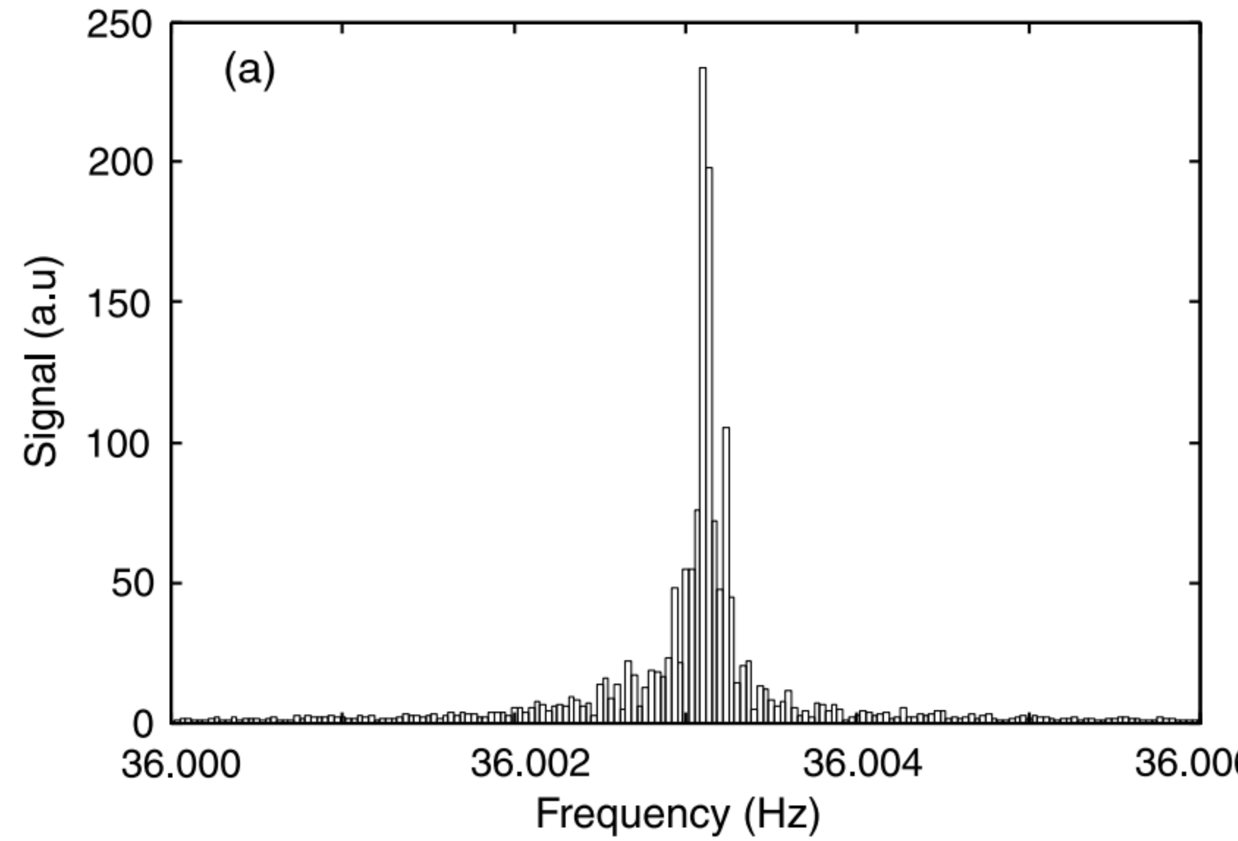


Fig. 3. (a) Spin maser oscillation signal observed in a time span of 24 hours. (b) Transient pattern in the initial spin maser oscillation. (c) Steady state oscillation after the transient settled. The signals shown in the ordinates represent the beat between the spin detection signal and a 36.12 Hz fixed frequency reference signal for a lock-in amplifier.

$$B_0 = 3.04\ \mu T,\ T_m = 3 \times 10^4\ s$$

$$\Delta\nu_{FT} = 50\ \mu Hz$$

A. Yoshimi, et al, Physics Letters A 376, 1924 (2012)

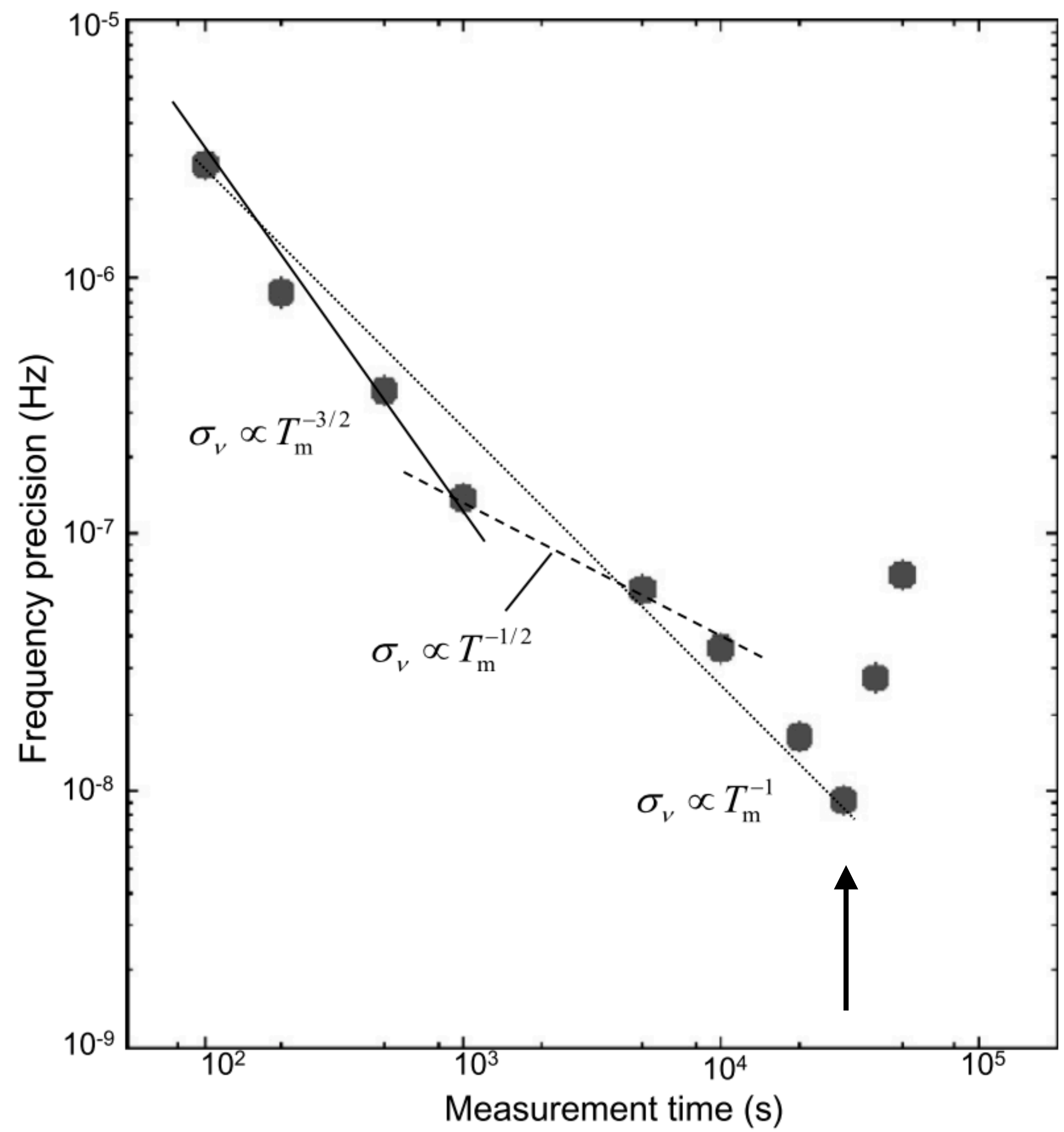
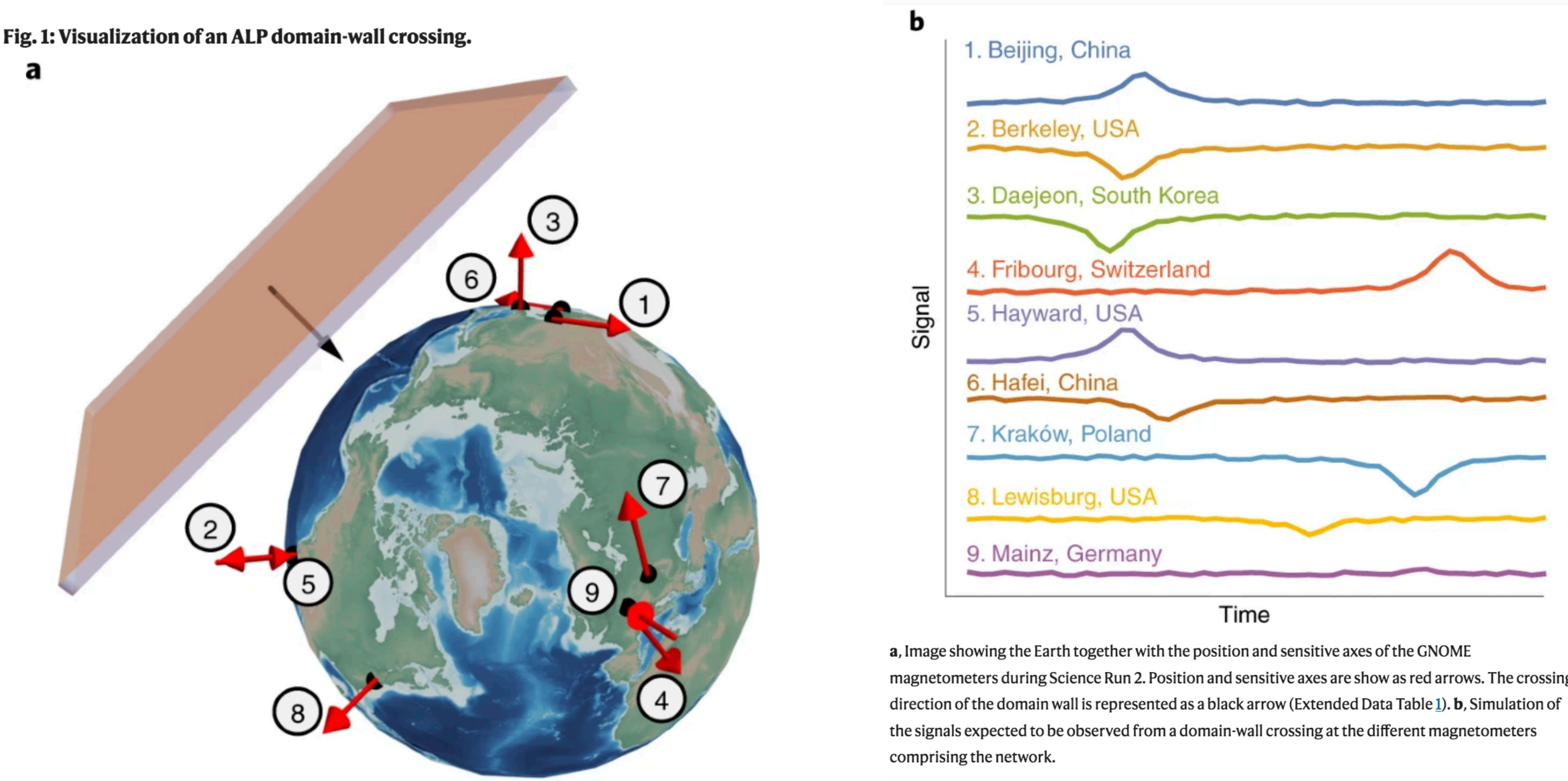


Fig. 7. Frequency precision of the spin oscillation. The abscissa represents the standard deviation of the frequency ν determined by fitting a function $\phi(t) = 2\pi\nu t + \phi_0$ to the observed precession phases ϕ from $t = 0$ to $t = T_m$. Solid, dotted and dashed lines are the presentation of three cases with power laws $\sigma_\nu \propto T_m^{-3/2}$, $\sigma_\nu \propto T_m^{-1}$, and $\sigma_\nu \propto T_m^{-1/2}$ respectively.

A. Yoshimi, et al, Physics Letters A 376, 1924 (2012)

Co-magnetometers

Fig. 1: Visualization of an ALP domain-wall crossing.



a, Image showing the Earth together with the position and sensitive axes of the GNOME magnetometers during Science Run 2. Position and sensitive axes are show as red arrows. The crossing direction of the domain wall is represented as a black arrow (Extended Data Table 1). **b**, Simulation of the signals expected to be observed from a domain-wall crossing at the different magnetometers comprising the network.

Axion-like particles for dark matter

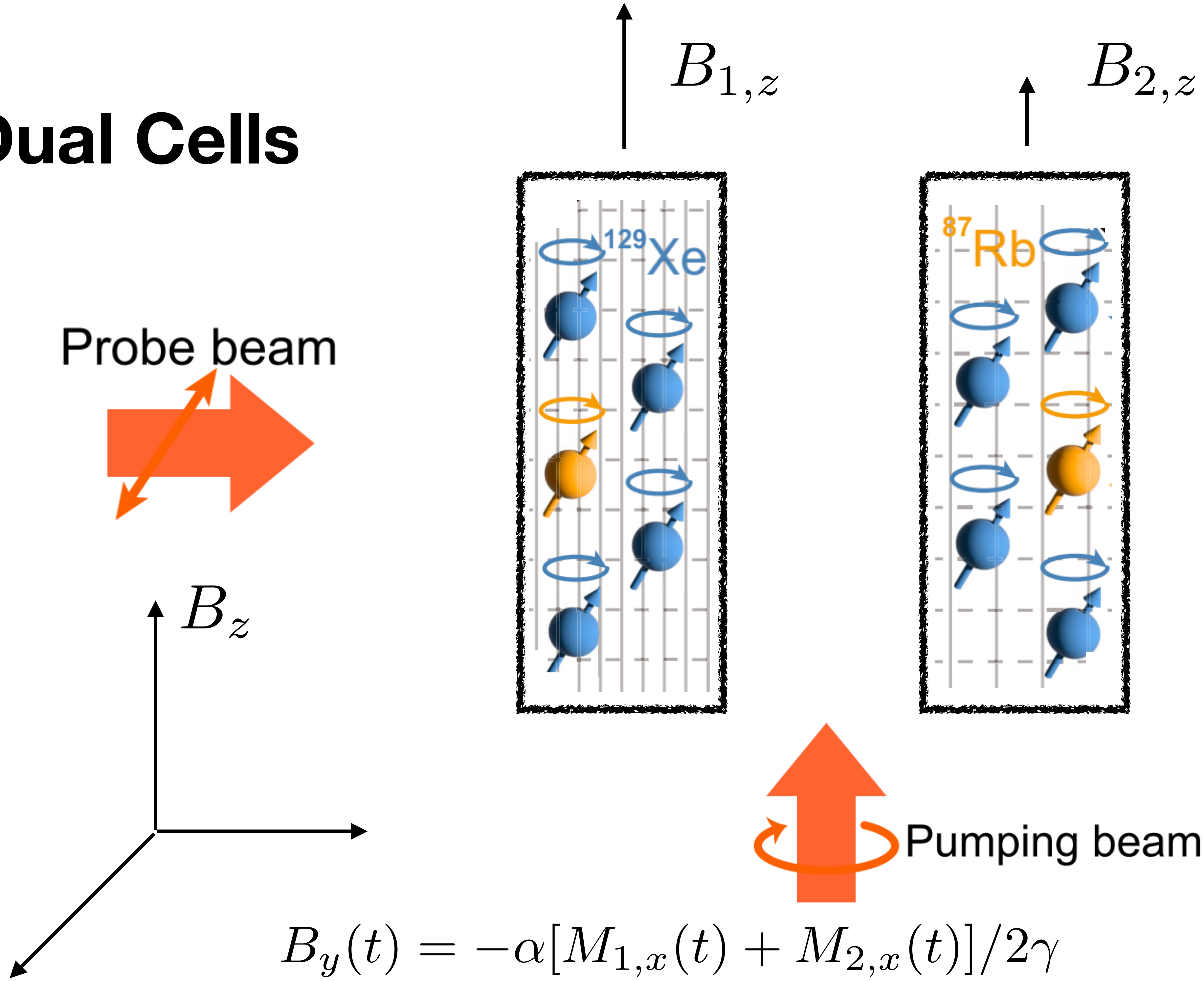
$$H_{\rm lin} = -(\hbar c)^{3/2}\frac{\xi}{f_{\rm SB}}\frac{\mathbf{S}}{\|S\|}\cdot\nabla a(\mathbf{r},t)\,,\qquad H_{\rm Z} = -\gamma\mathbf{S}\cdot\mathbf{B}\,,$$

The global network of optical magnetometers for exotic (GNOME) physics searches

S. Afach, *et. al*, Nature Physics 17, 1396-1401 (2021)

A New Coupled Set-up

Dual Cells

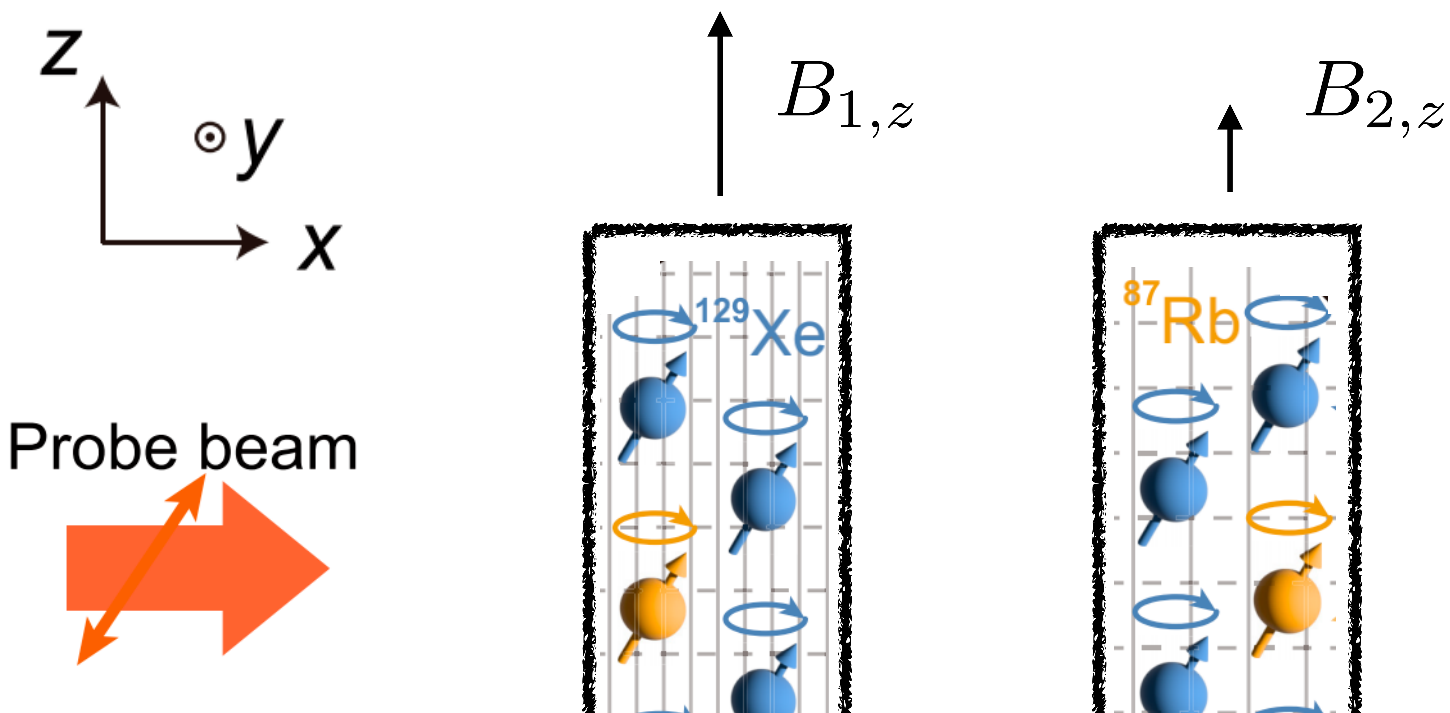


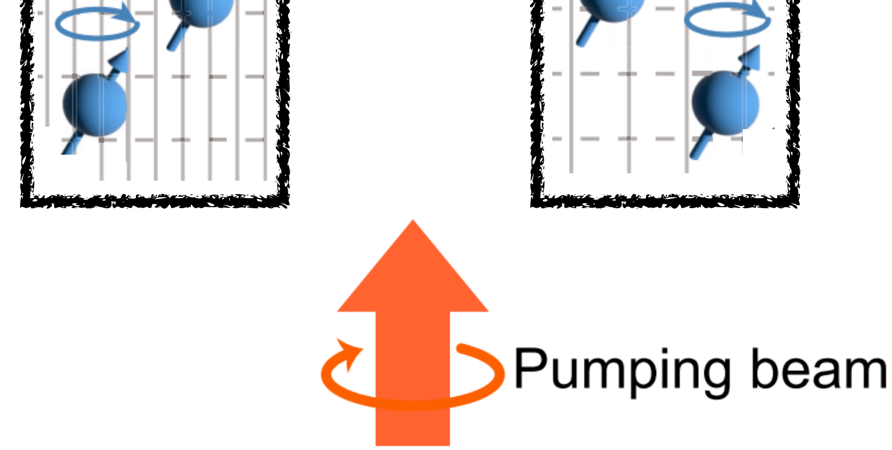
$$B_y(t) = -\alpha[M_{1,x}(t) + M_{2,x}(t)]/2\gamma$$

$$B_x(t) = \alpha[M_{1,y}(t) + M_{2,y}(t)]/2\gamma$$

arXiv:2302.05264

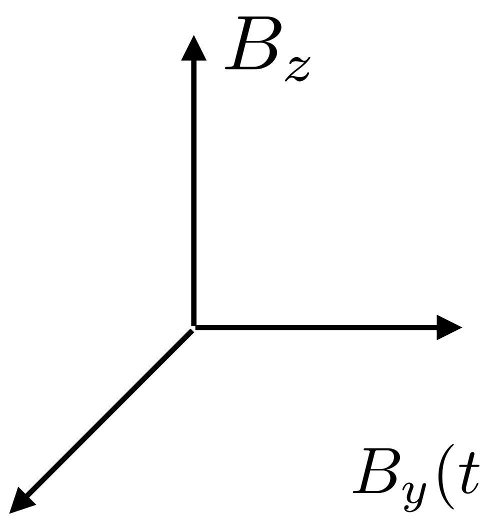
Bloch Equations





$$\omega_{1,L} = \gamma B_{1,z}$$
$$\omega_{2,L} = \gamma B_{2,z}$$

$$\omega_{1,L} = \omega_c + \epsilon/2$$
$$\omega_{2,L} = \omega_c - \epsilon/2$$

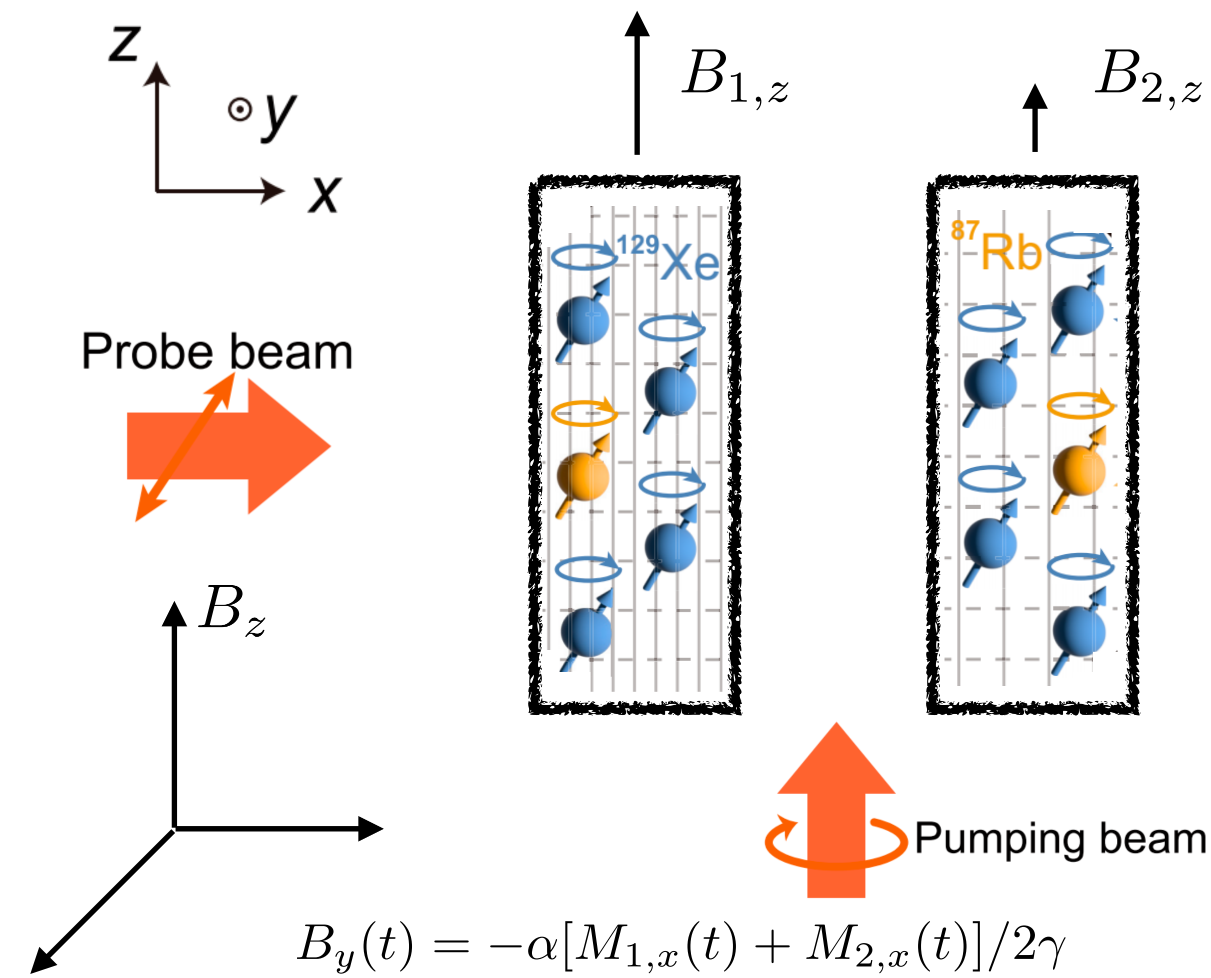


$$B_y(t) = -\alpha[M_{1,x}(t) + M_{2,x}(t)]/2\gamma$$

$$B_x(t) = \alpha[M_{1,y}(t) + M_{2,y}(t)]/2\gamma$$

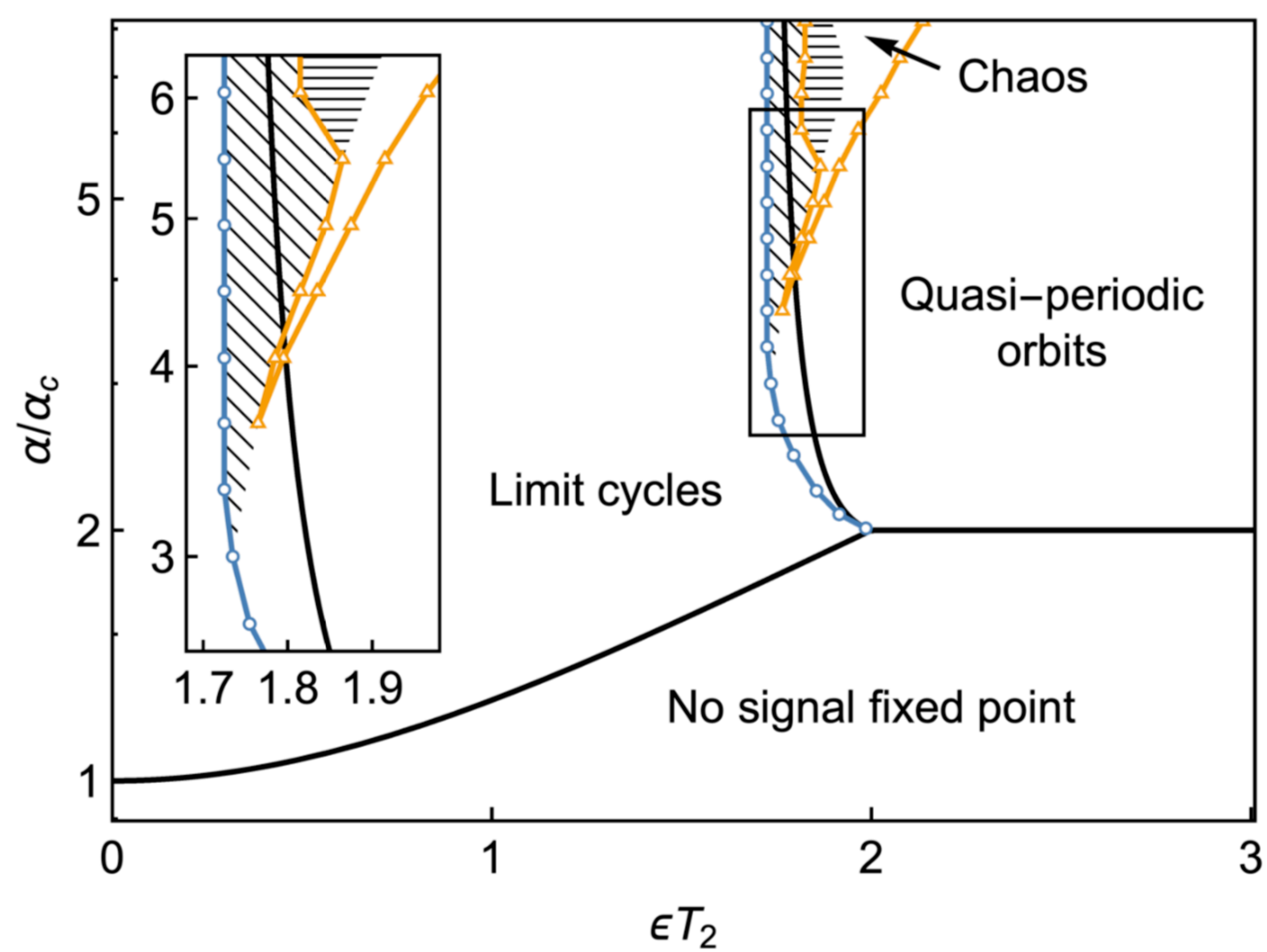
$$\frac{d\mathbf{M}_j}{dt} = \gamma \mathbf{M}_j \times \mathbf{B}_j + \begin{bmatrix} -M_{j,x}/T_2 \\ -M_{j,y}/T_2 \\ -(M_{j,z} - M_0)/T_1 \end{bmatrix}$$

Stability Diagram



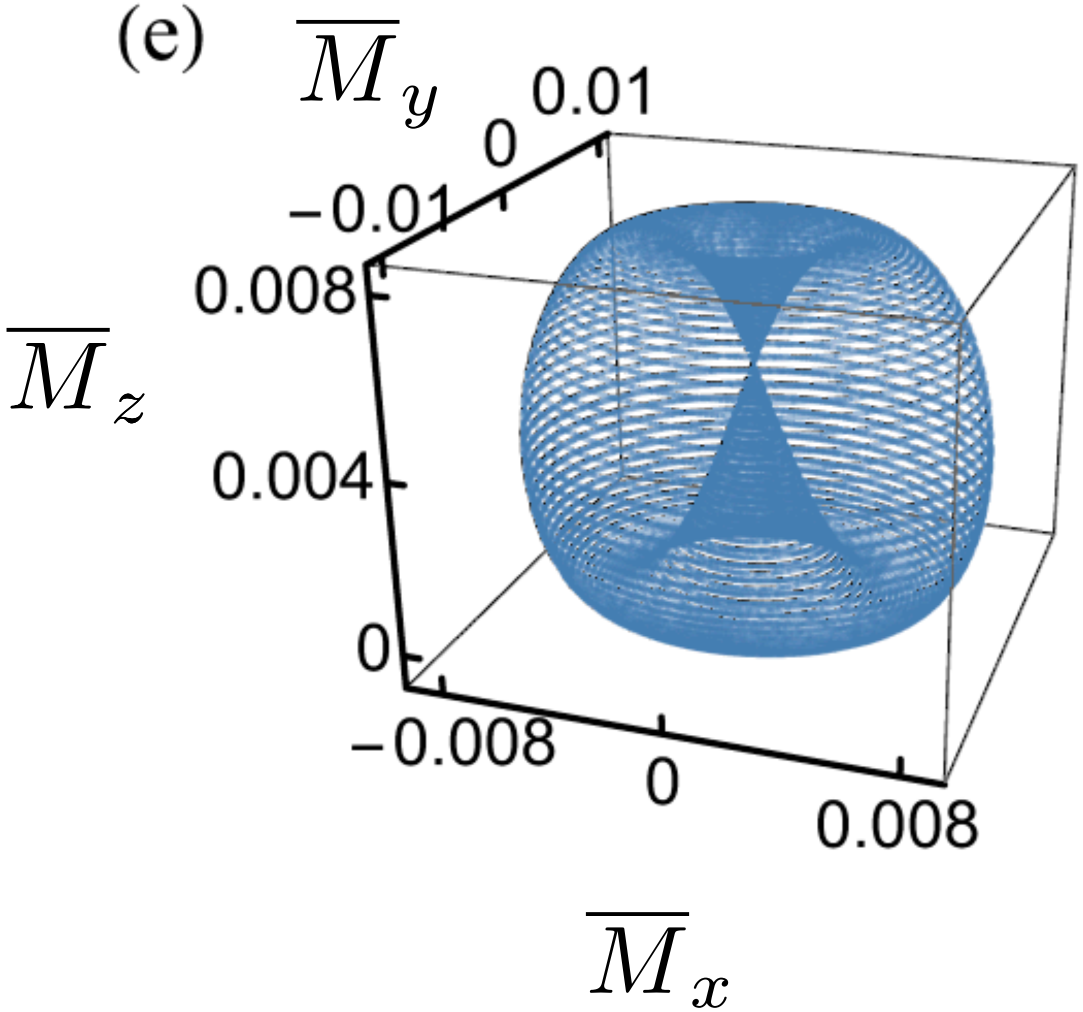
$$B_y(t) = -\alpha[M_{1,x}(t) + M_{2,x}(t)]/2\gamma$$

$$B_x(t) = \alpha[M_{1,y}(t) + M_{2,y}(t)]/2\gamma$$

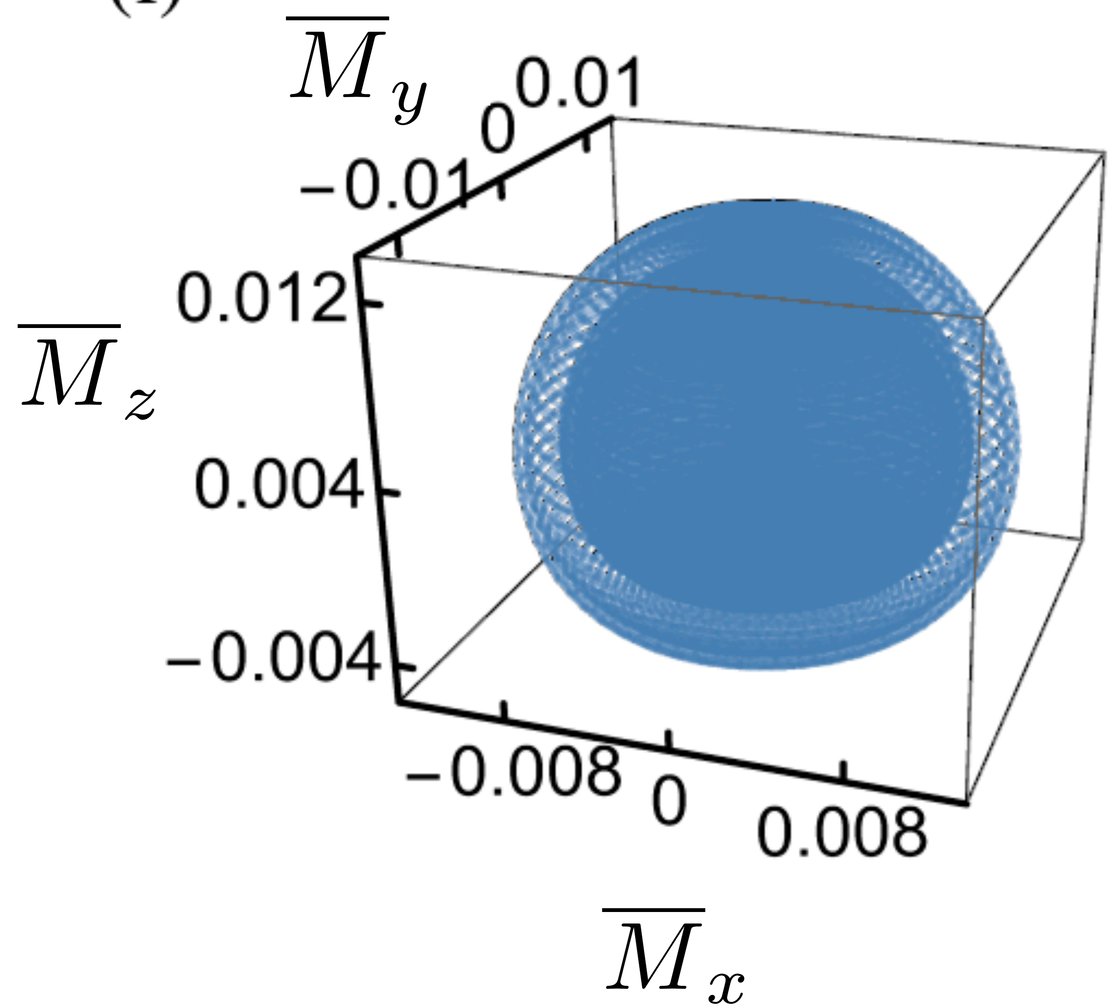


Spot by Sight

(e)

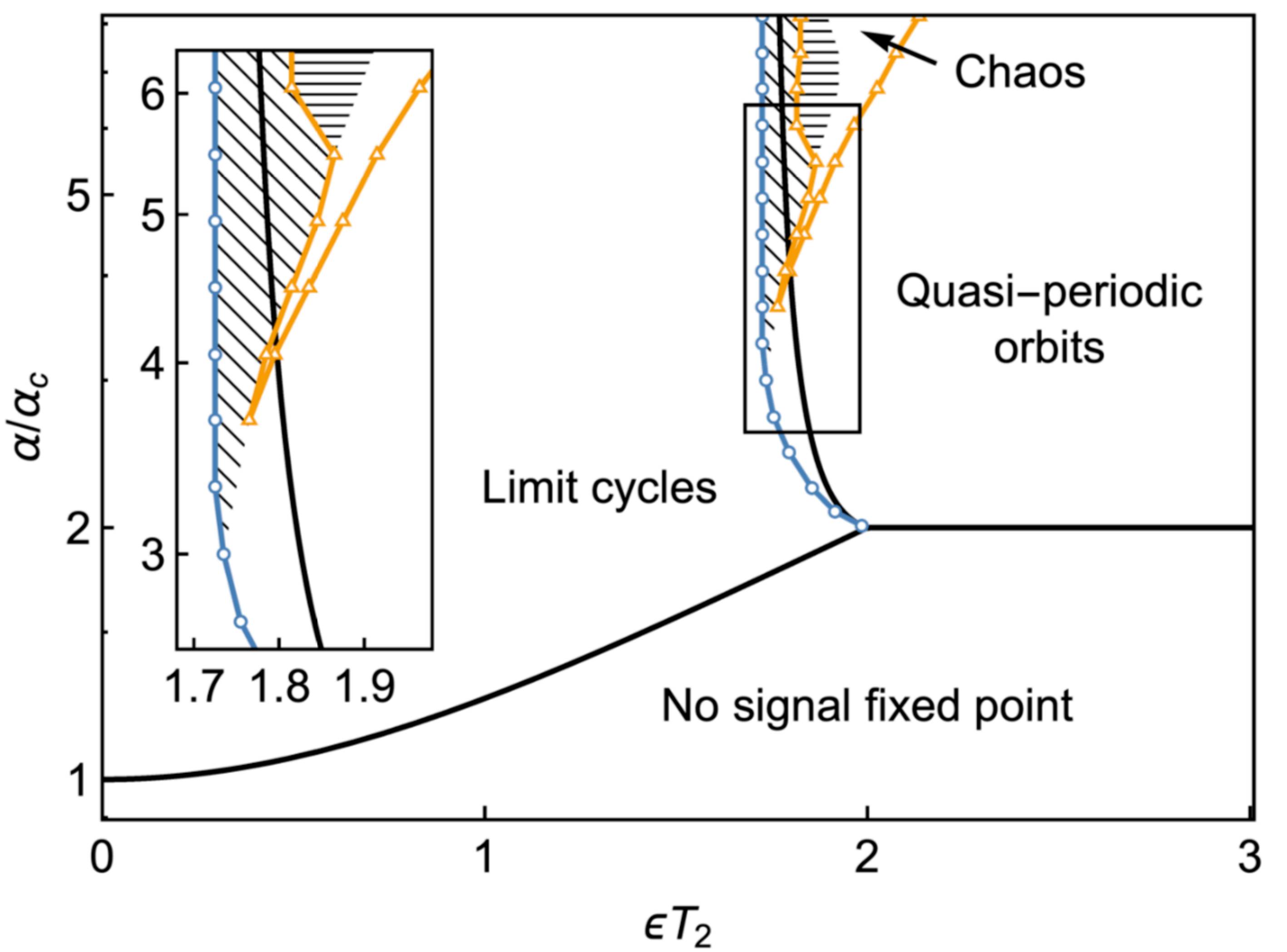


(i)



Which is which?

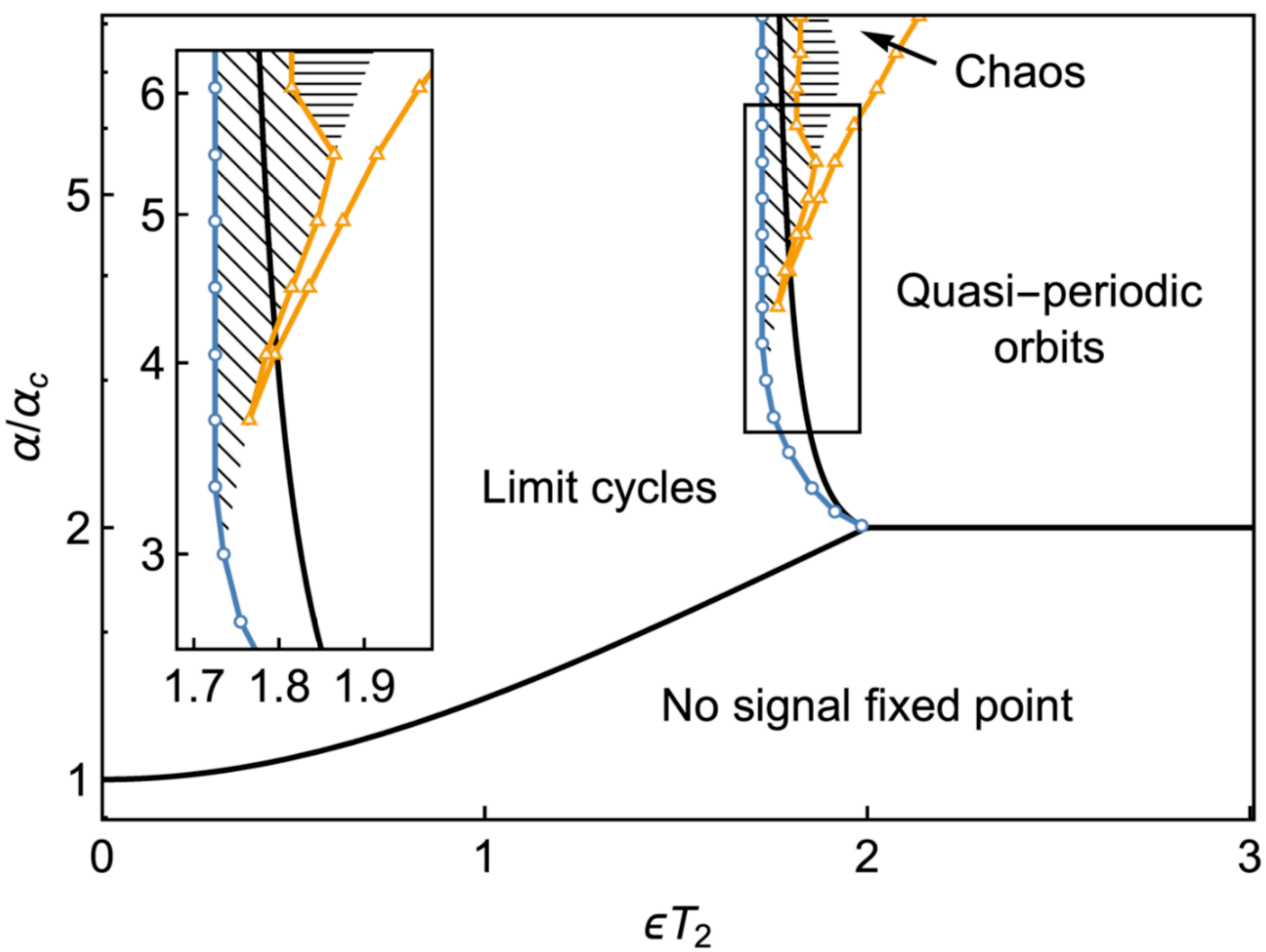
Dimension Reduction



$$M_{1,z} = M_{2,z}$$

$$M_{1,x}^2 + M_{1,y}^2 = M_{2,x}^2 + M_{2,y}^2$$

Dimension Reduction



$$M_{1,z} = M_{2,z}$$
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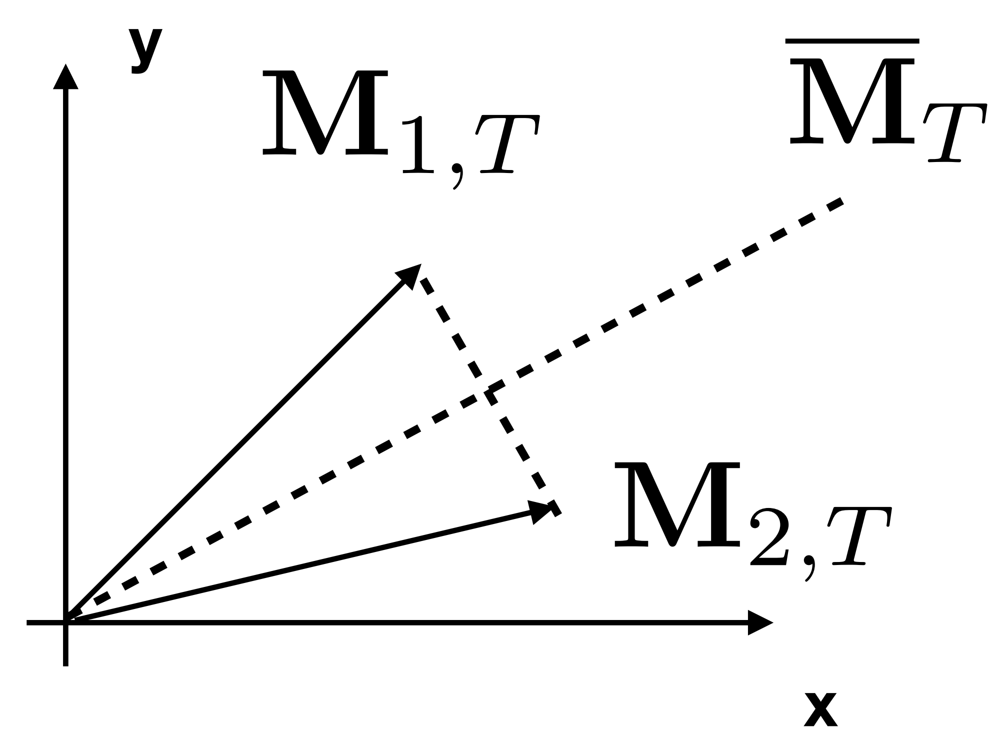
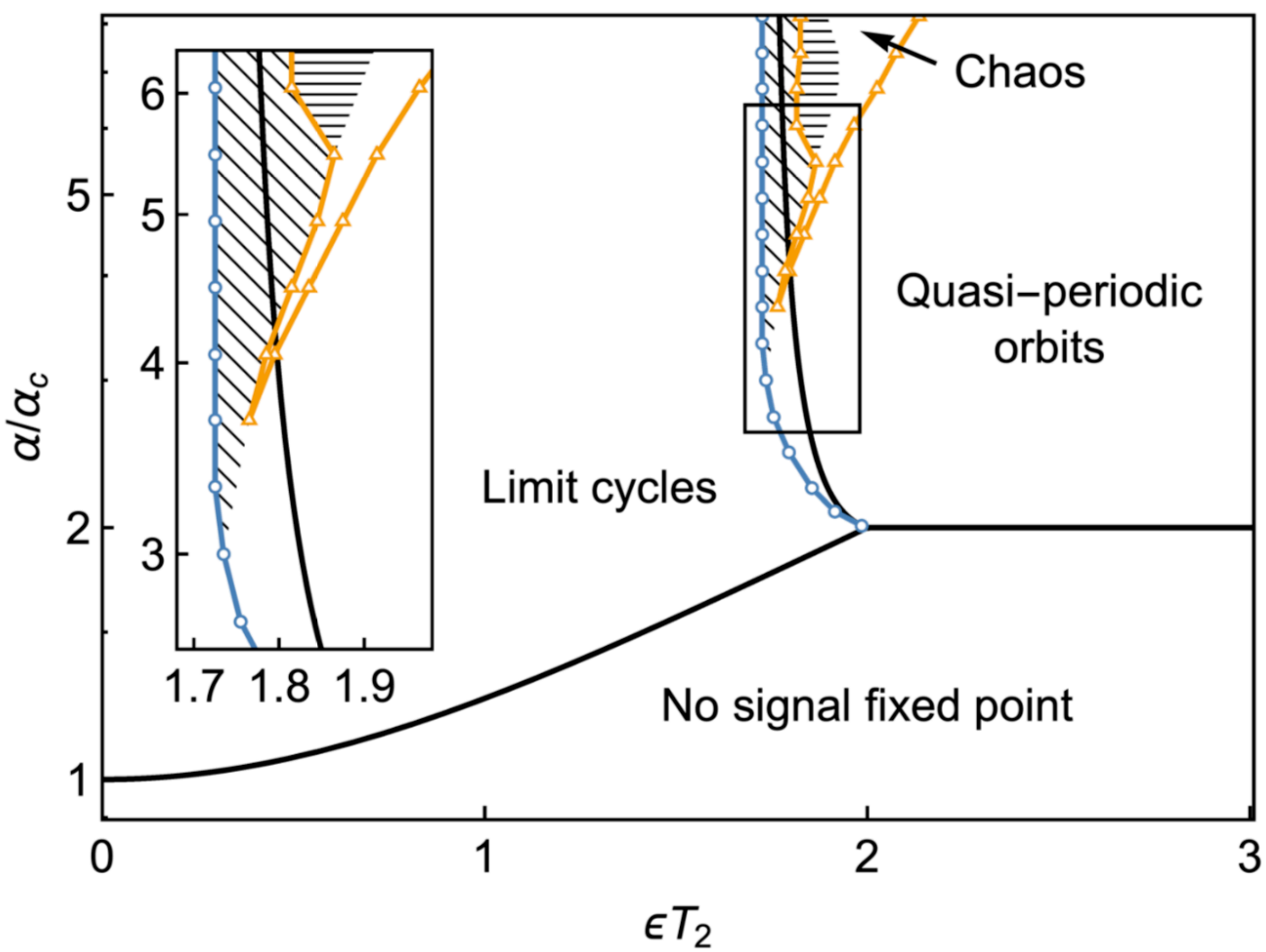
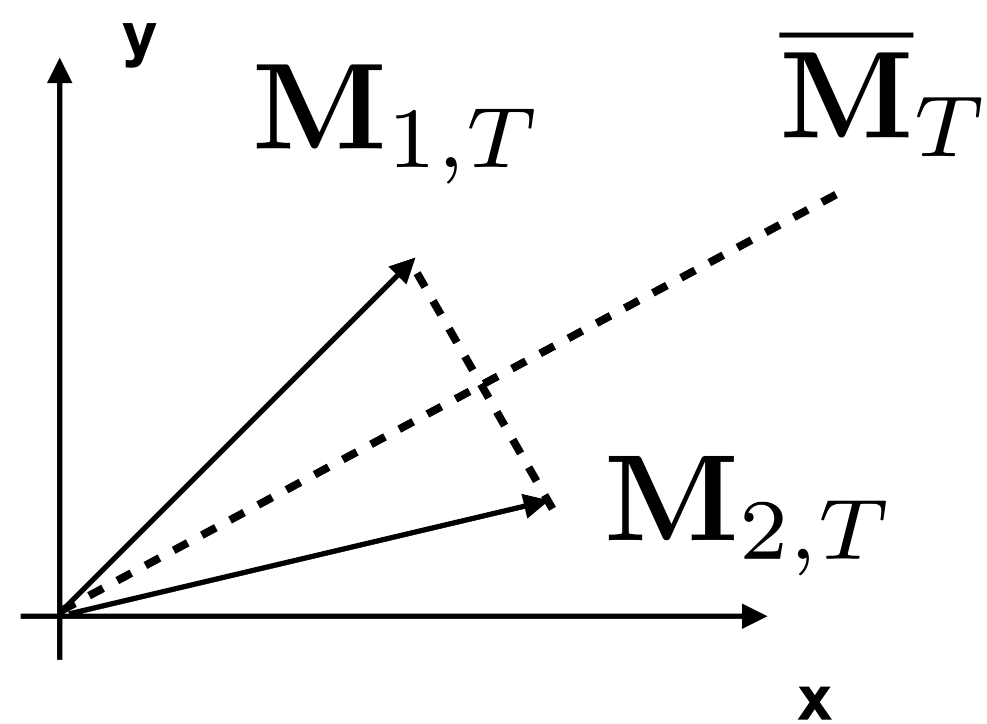


Dimension Reduction

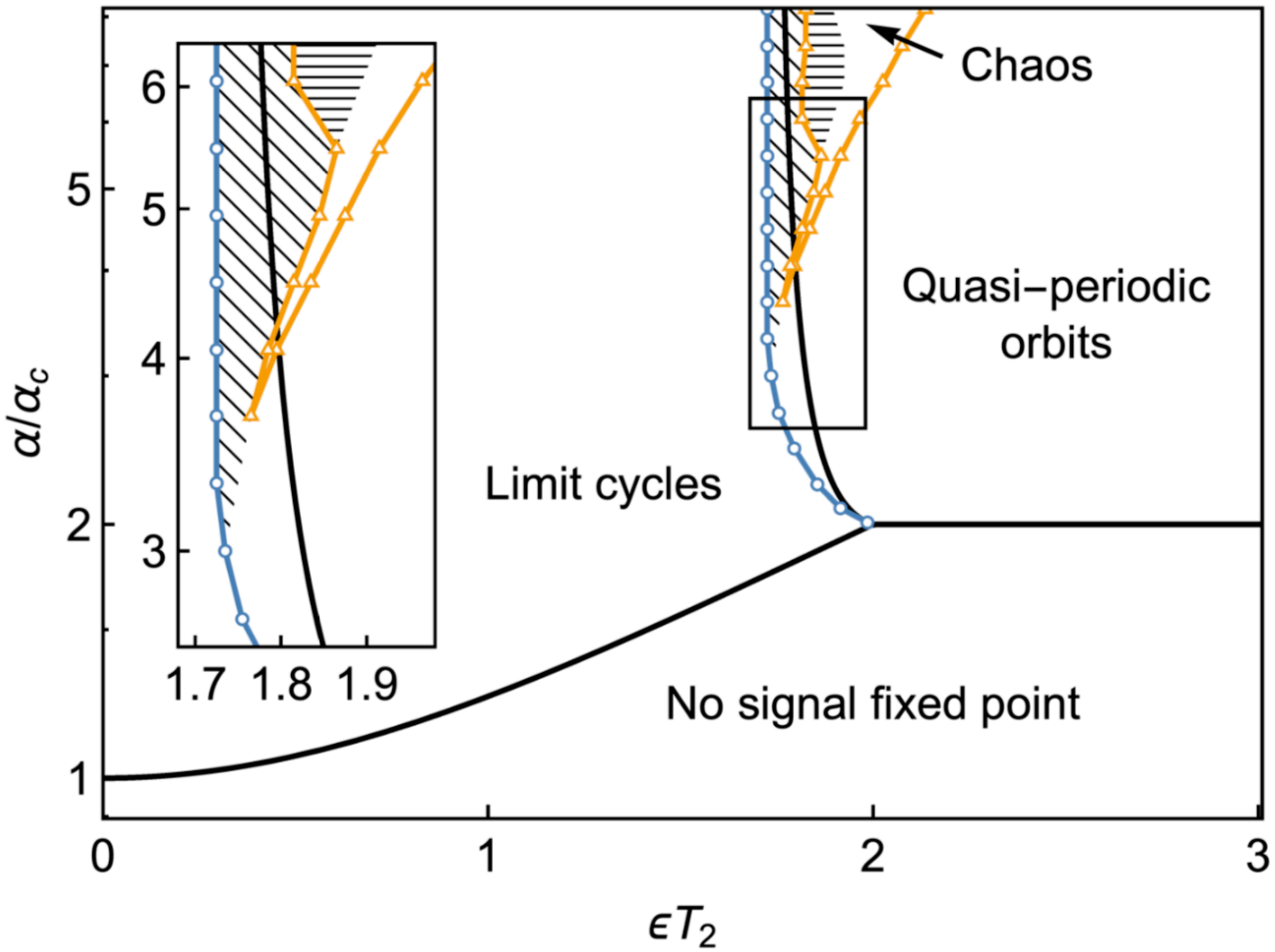


$$M_{1,z} = M_{2,z}$$
$$M_{1,x}^2 + M_{1,y}^2 = M_{2,x}^2 + M_{2,y}^2$$



$$\overline{M}_T \equiv \overline{M}_x + i\overline{M}_x \equiv Ae^{i\theta}/2$$
$$\Delta M_T \equiv M_{1,T} - M_{2,T} \equiv Be^{i(\theta+\pi/2)}$$

Dimension Reduction



$$\frac{dA}{dt} =\alpha\overline{M}_z A + \epsilon B/2 - A/T_2$$

$$\frac{dB}{dt} = -\epsilon A/2 - B/T_2$$

$$\frac{d\overline{M}_z}{dt} = -\alpha A^2/4 - (\overline{M}_z - M_0)/T_1$$

$$\frac{d\theta}{dt} = -\omega_c$$

$$\overline{M}_T \equiv \overline{M}_x + i\overline{M}_x \equiv Ae^{i\theta}/2$$

$$\Delta M_T \equiv M_{1,T} - M_{2,T} \equiv Be^{i(\theta+\pi/2)}$$

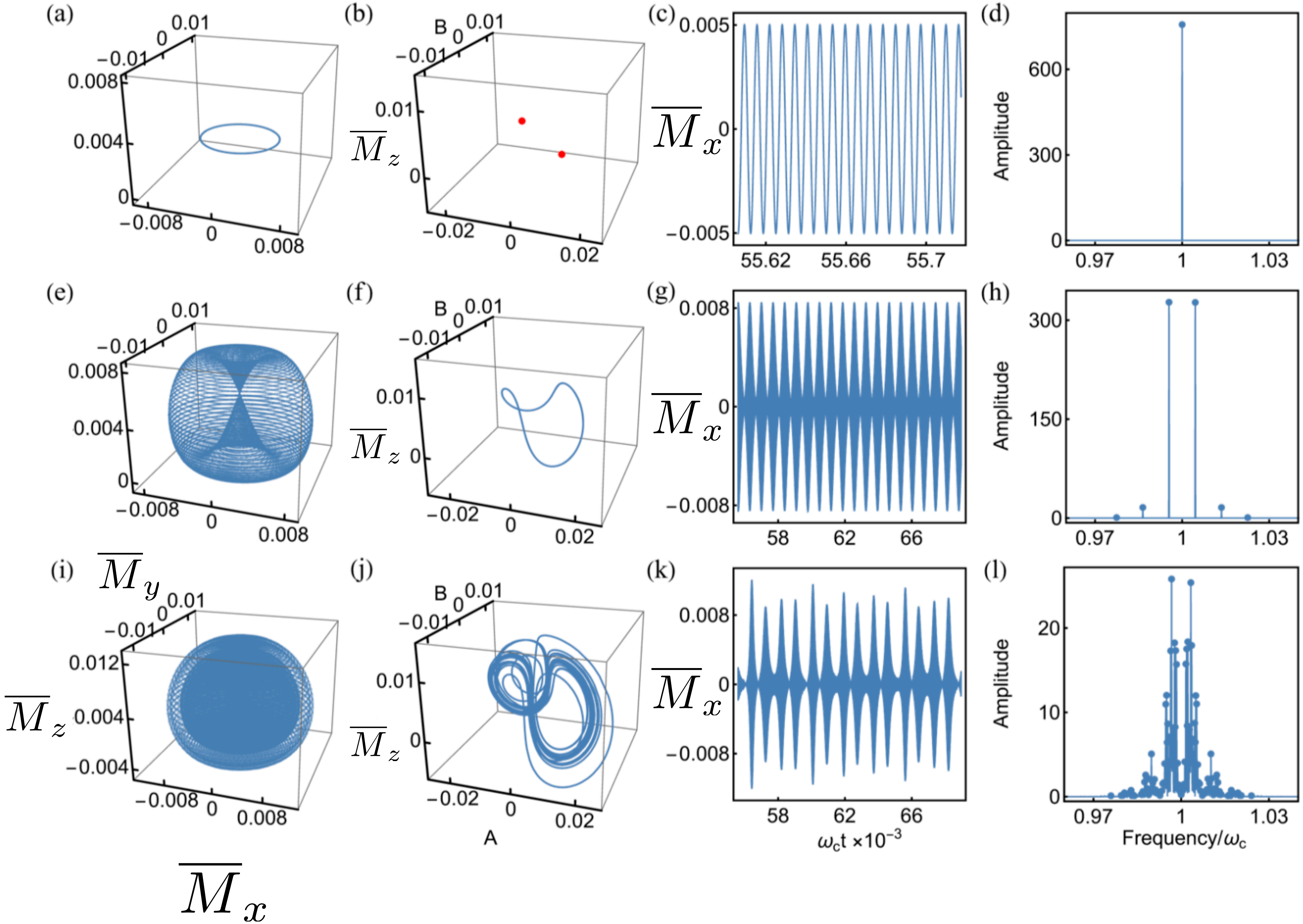
6 -> 3

Phase Portraits

Limit Cycles

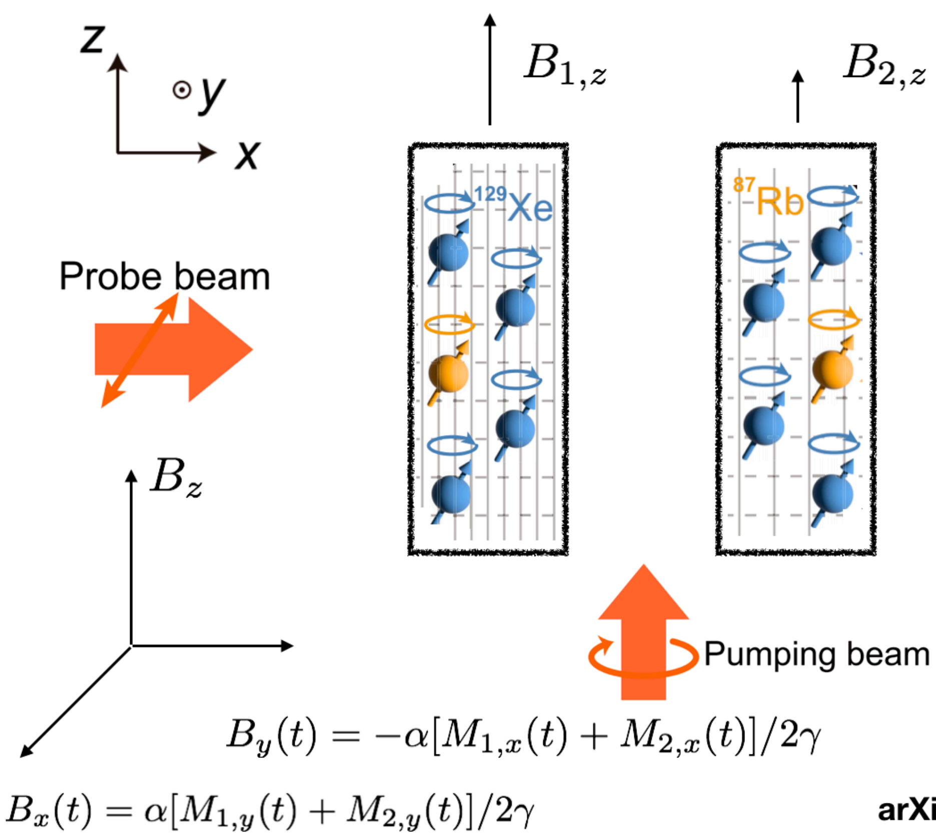
Quasi-periodic Orbits

Chaos

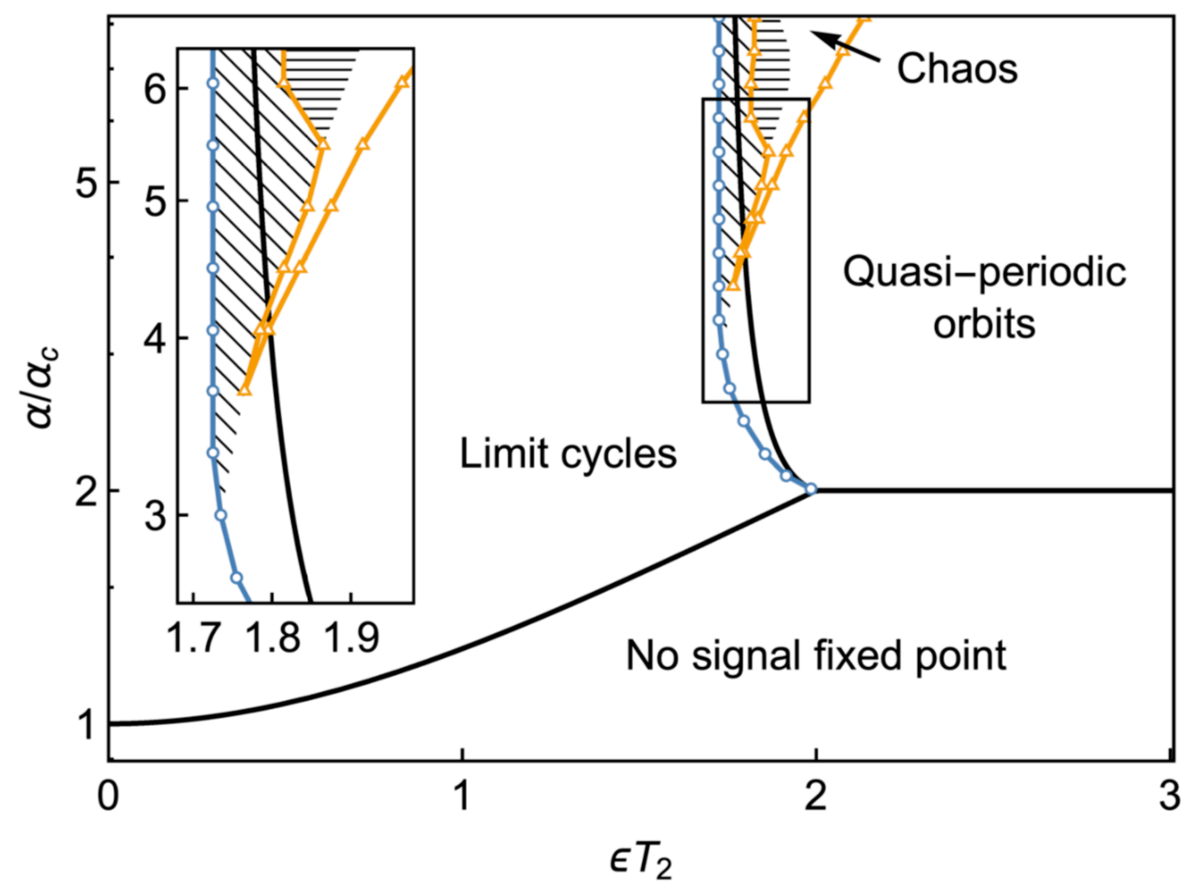


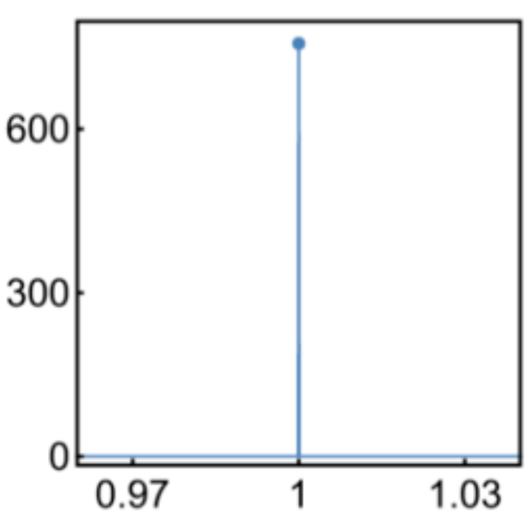
Summary

A New Coupled Set-up



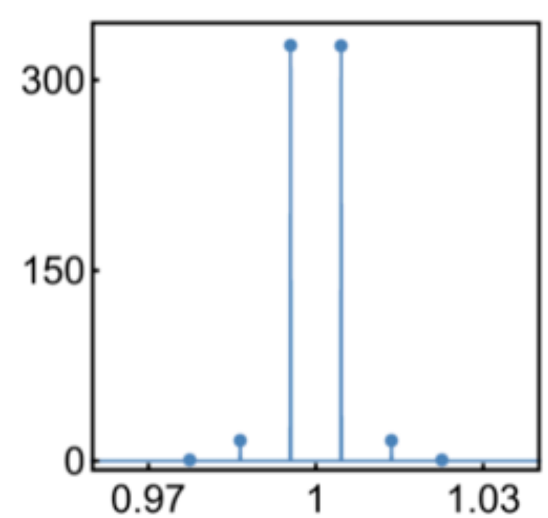
arXiv:2302.05264





Limit Cycle

$$\omega_s = \frac{\omega_{1,L} + \omega_{2,L}}{2}$$

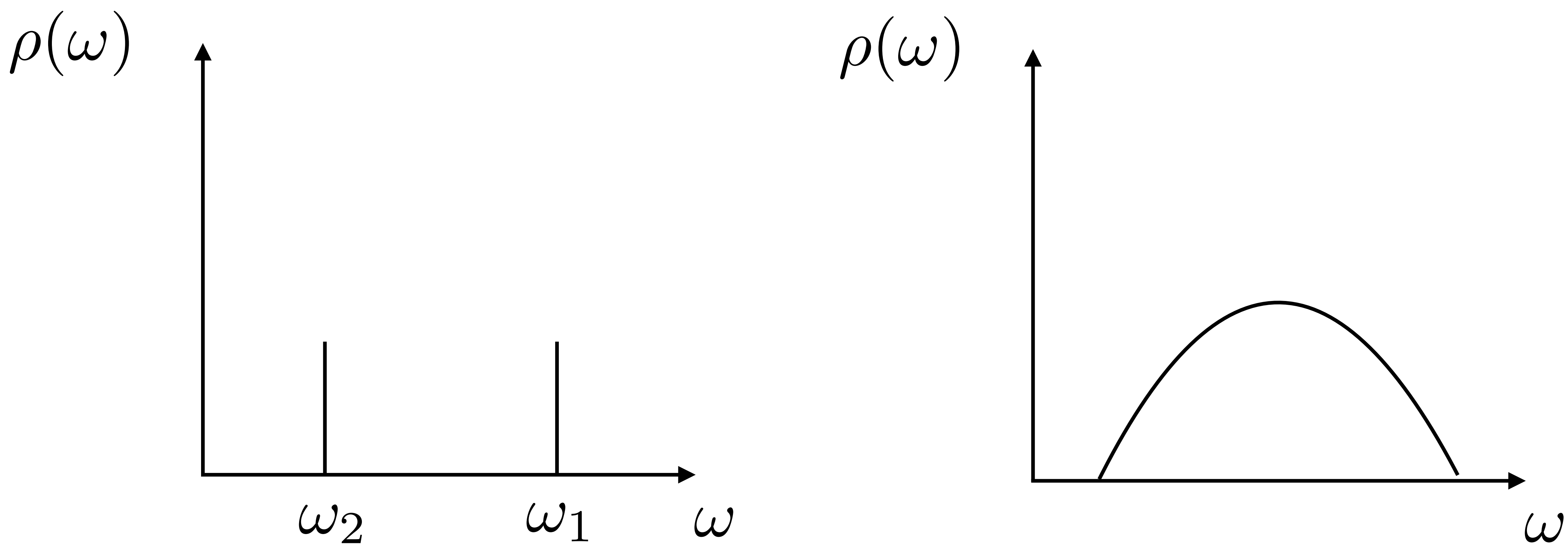


QPO

Multi-band Excitation

Thanks for Your Attention

Continuum Limit



Continuum Limit: Stability Diagram