中国科学院高能物理研究所
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The 23rd International Conference on Few-Body Problems in Physics

Correlation function and the inverse problem in the two-body interactions

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arXiv: 2409.05787

2024.9. Beijing

Outline

§1. Introduction

A $\Sigma^*(1/2-)$ state with mass 1430 MeV near $\bar{K}N$ threshod was **predicted**:

N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A 594, 325 (1995)

E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998)

Isospin $I = 1$

J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001)

D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)

→ **two-poles structure of Λ(1405) was found**

But, still NOT found yet……..

A recent review on the $\Sigma*(1/2-)$ state:

E. Wang, L.-S. Geng, J.-J. Wu, J.-J. Xie, and B.-S. Zou, arXiv:2406.07839

There are some proposals to search for this **predicted** state:

Y.-H. Lyu, H. Zhang, N.-C. Wei, B.-C. Ke, E. Wang, and J.-J. Xie, Chin. Phys. C 47, 053108 (2023)

$$\gamma n \to K^+ \Sigma^{*-}_{1/2^-}$$

X.-L. Ren, E. Oset, L. Alvarez-Ruso, and M. J. Vicente Vacas, Phys. Rev. C 91, 045201 (2015)
J.-J. Wu and B.-S. Zou, Few Body Syst. 56, 165 (2015)

$$\bar{\nu}_l p \to l^+ \Phi B$$

E. Wang, J.-J. Xie, and E. Oset, Phys. Lett. B 753, 526 (2016)

$$\chi_{c0}(1P) \to \bar{\Sigma}\Sigma\pi$$

L.-J. Liu, E. Wang, J.-J. Xie, K.-L. Song, and J.-Y. Zhu, Phys. Rev. D 98, 114017 (2018)

$$\chi_{c0} \to \bar{\Lambda}\Sigma\pi$$

J.-J. Xie and E. Oset, Phys. Lett. B 792, 450 (2019)

$$\Lambda_c^+ \to \pi^+\pi^0\pi^-\Sigma^+ \qquad \Lambda_c^+ \to \pi^+\pi^+\pi^-\Lambda$$

A recent **evidence** of the $\Sigma*(1/2-)$ state:

Y. Ma et al. (Belle), Phys. Rev. Lett. 130, 151903 (2023)

$$\Lambda_c^+ \to \Lambda\pi^+\pi^+\pi^-$$

But, from their analysis, they can NOT discriminate from the peak being due to a resonance or to a cusp in the $\bar{K}N$ threshold.

$\Sigma^*(1430)$?

§2. Two-body interaction

(1) Coupled channel interaction from the chiral unitary approach

$\bar{K}^0 p, \pi^+\Sigma^0, \pi^0\Sigma^+, \pi^+\Lambda$, and $\eta\Sigma^+$

Without the Coulomb interaction

$$|\pi^+\Sigma^0\rangle = -\frac{1}{\sqrt{2}}\left(|\pi\Sigma, I=2, I_3=1\rangle + |\pi\Sigma, I=1, I_3=1\rangle\right)$$

$$|\pi^0\Sigma^+\rangle = -\frac{1}{\sqrt{2}}\left(|\pi\Sigma, I=2, I_3=1\rangle - |\pi\Sigma, I=1, I_3=1\rangle\right)$$

$$|\bar{K}^0 p\rangle = |\bar{K}N, I=1, I_3=1\rangle,$$

$$|\pi^+\Lambda\rangle = -|\pi\Lambda, I=1, I_3=1\rangle$$

$$|\eta\Sigma^+\rangle = -|\eta\Sigma, I=1, I_3=1\rangle$$

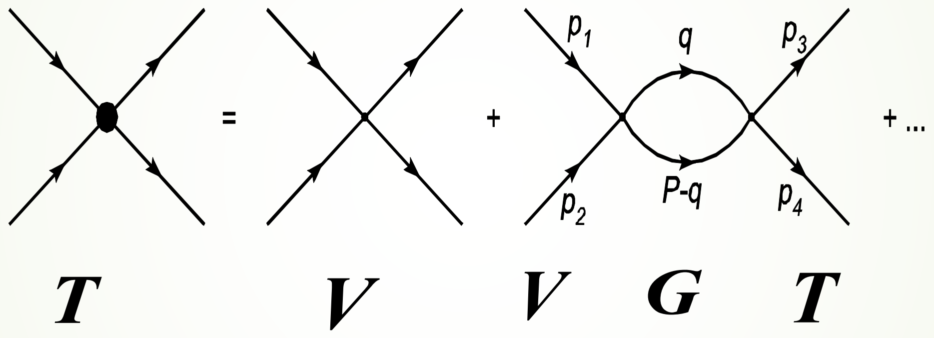
$$V_{ij} = -\frac{1}{4f^2} C_{ij}(k_i^0 + k_j^0)$$

$f = 93$ MeV.

C_{ij}	$\bar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0\Sigma^+$	$\pi^+\Lambda$	$\eta\Sigma^+$
$\bar{K}^0 p$	1	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$
$\pi^+\Sigma^0$		0	-2	0	0
$\pi^0\Sigma^+$			0	0	0
$\pi^+\Lambda$				0	0
$\eta\Sigma^+$					0

➨ **Coupled Channel Unitary Approach**： solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



where V matrix (potentials) can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99
J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i\int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\mathrm{Im}\, T_{ij} = T_{in}\, \sigma_{nn}\, T_{nj}^{*}$$

$$\sigma_{nn} \equiv \mathrm{Im}\, G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s-(m_1+m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i\,\boxed{\frac{i}{2\pi}\frac{M_l q_{cml}(s)}{\sqrt{s}}}$$

(2) Correlation functions

$$\mathcal{C}_{\bar{K}^0 p}(p_{\bar{K}^0}) = 1 + 4\pi\theta(q_{\max} - p_{\bar{K}^0}) \int \mathrm{d}r r^2 S_{12}(r) \times \Big\{ \Big| j_0(p_{\bar{K}^0} r) + T_{11}(E)\tilde{G}_1(r,E) \Big|^2 + \Big| T_{21}(E)\tilde{G}_2(r,E) \Big|^2 + \Big| T_{31}(E)\tilde{G}_3(r,E) \Big|^2 + \Big| T_{41}(E)\tilde{G}_4(r,E) \Big|^2 + \Big| T_{51}(E)\tilde{G}_5(r,E) \Big|^2 - j_0^2(p_{\bar{K}^0} r) \Big\}, \quad (7)$$

$$\mathcal{C}_{\pi^+\Sigma^0}(p_{\pi^+}) = 1 + 4\pi\theta(q_{\max} - p_{\pi^+}) \int \mathrm{d}r r^2 S_{12}(r) \times \Big\{ \Big| j_0(p_{\pi^+} r) + T_{22}(E)\tilde{G}_2(r,E) \Big|^2 + \Big| T_{12}(E)\tilde{G}_1(r,E) \Big|^2 + \Big| T_{32}(E)\tilde{G}_3(r,E) \Big|^2 + \Big| T_{42}(E)\tilde{G}_4(r,E) \Big|^2 + \Big| T_{52}(E)\tilde{G}_5(r,E) \Big|^2 - j_0^2(p_{\pi^+} r) \Big\}, \quad (8)$$

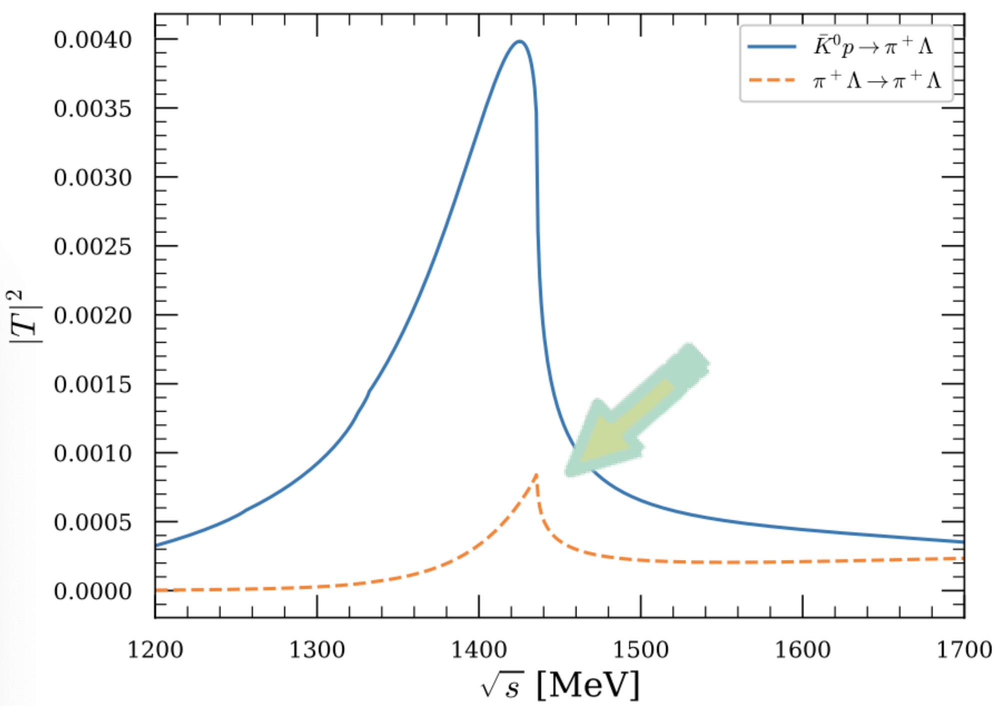
source function

$$S_{12}(r) = \frac{1}{(\sqrt{4\pi}R)^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

$$\tilde{G}_i = 2M_i \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{w_1(\vec{q}) + w_2(\vec{q})}{2\, w_1(\vec{q})\, w_2(\vec{q})} \times \frac{j_0(|\vec{q}|r)}{s - [w_1(\vec{q}) + w_2(\vec{q})]^2 + i\epsilon}$$

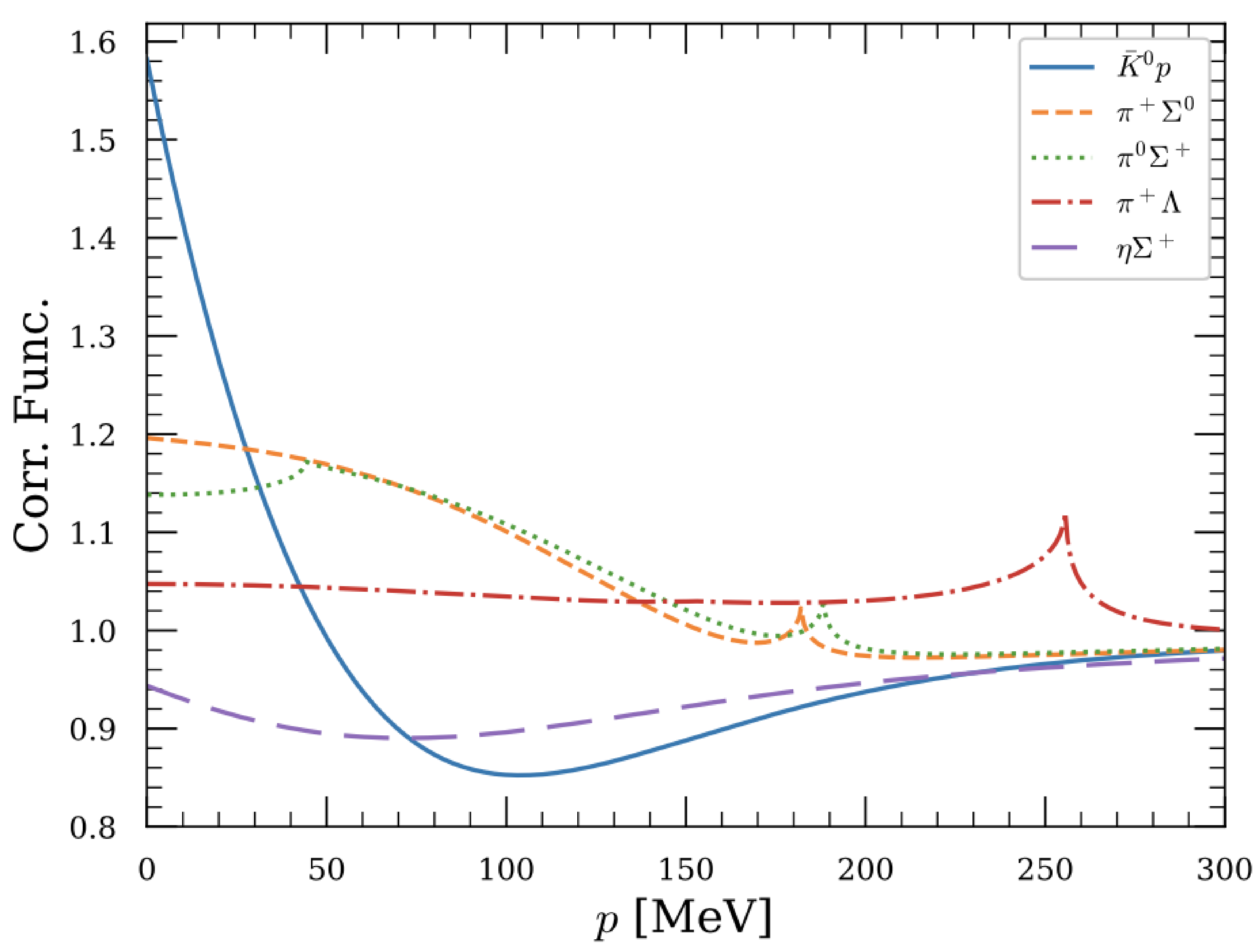
The zeroth order spherical Bessel function

(3) results



$\sqrt{s_0}$ (MeV)	g_1	g_2	g_3	g_4	g_5
(1431.83, 104.75)	(3.03, 2.71)	(1.98, 1.45)	(1.98, 1.47)	(0.19, 1.21)	(0.21, 1.27)
Probabilities	P_1	P_2	P_3	P_4	P_5
	(−0.60, 0.19)	(0.24, −0.41)	(0.25, −0.41)	(0.09, 0.05)	(−0.02, −0.00)
Scattering	a_1	a_2	a_3	a_4	a_5
lengths (fm)	(0.45, −1.13)	(−0.15, −0.03)	(−0.12, −0.00)	(−0.05, −0.00)	(0.08, −0.15)
Effective	r_1	r_2	r_3	r_4	r_5
ranges (fm)	(0.04, −0.45)	(−35.24, −16.61)	(−67.00, 0.39)	(−66.12, 0.00)	(0.31, 0.32)

Correlation functions



Generating pseudo data

Assuming ± 0.02 error

§3. Inverse problem

Assume an energy dependence interaction potential

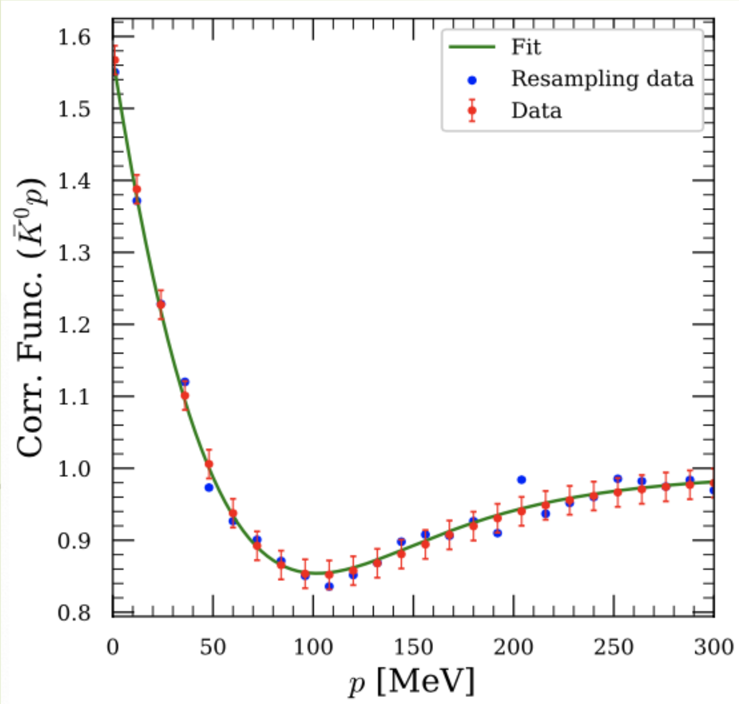
$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}(k_i^0 + k_j^0)$$

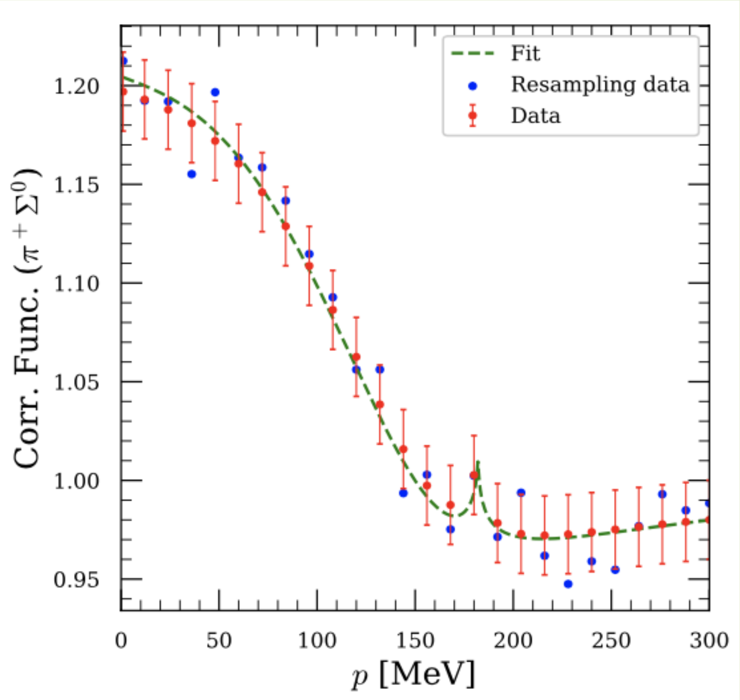
$\tilde{C}_{ij}$	$\bar{K}^0 p$	$\pi^+\Sigma^0$	$\pi^0\Sigma^+$	$\pi^+\Lambda$	$\eta\Sigma^+$
$\bar{K}^0 p$	$\tilde{C}_{11}$	$-\frac{1}{\sqrt{2}}\tilde{C}_{12}$	$\frac{1}{\sqrt{2}}\tilde{C}_{12}$	$-\tilde{C}_{14}$	$-\tilde{C}_{15}$
$\pi^+\Sigma^0$		$\frac{1}{2}(\tilde{C}_{22}+\tilde{C}'_{22})$	$\frac{1}{2}(-\tilde{C}_{22}+\tilde{C}'_{22})$	$\frac{1}{\sqrt{2}}\tilde{C}_{24}$	$\frac{1}{\sqrt{2}}\tilde{C}_{25}$
$\pi^0\Sigma^+$			$\frac{1}{2}(\tilde{C}_{22}+\tilde{C}'_{22})$	$-\frac{1}{\sqrt{2}}\tilde{C}_{24}$	$-\frac{1}{\sqrt{2}}\tilde{C}_{25}$
$\pi^+\Lambda$				$\tilde{C}_{44}$	$\tilde{C}_{45}$
$\eta\Sigma^+$					$\tilde{C}_{55}$

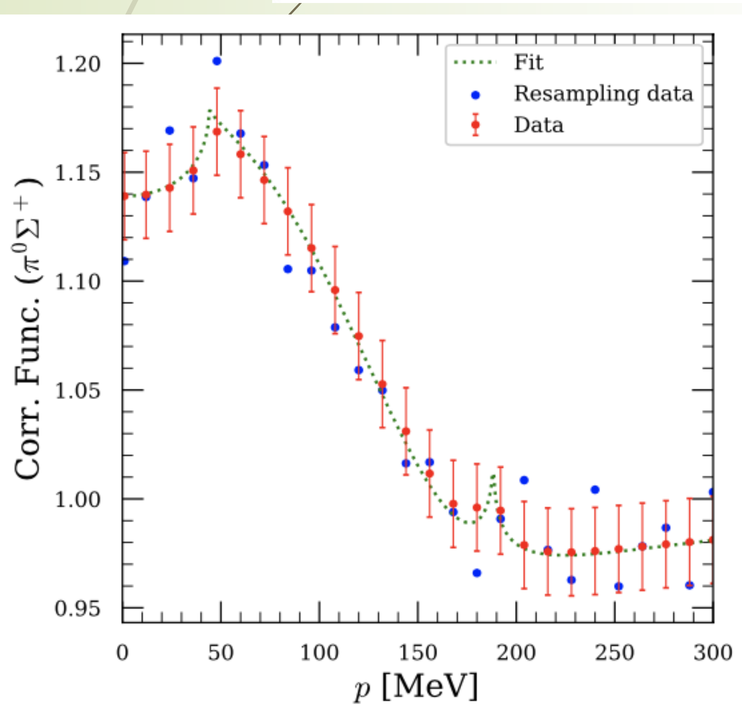
11 free parameters + R + q_{max} : 13 totally

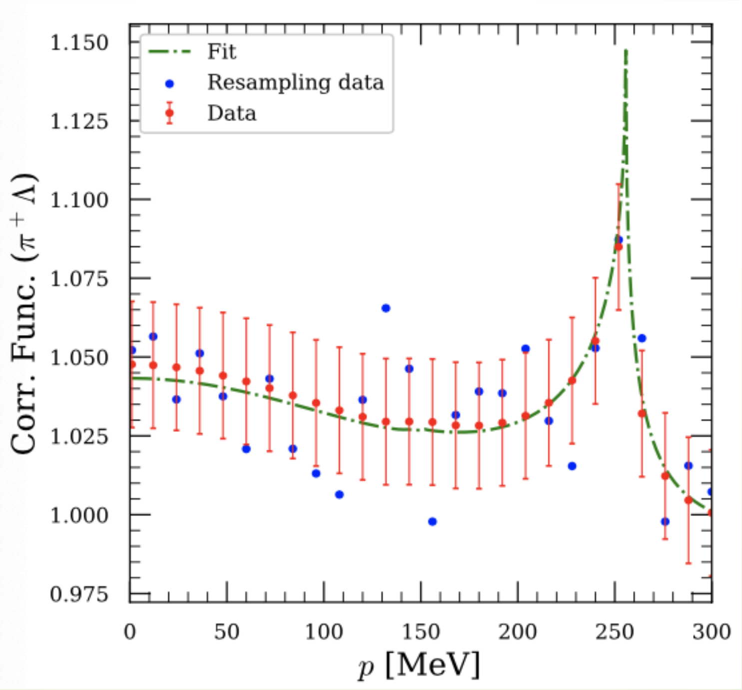
Using the bootstrap or resampling method

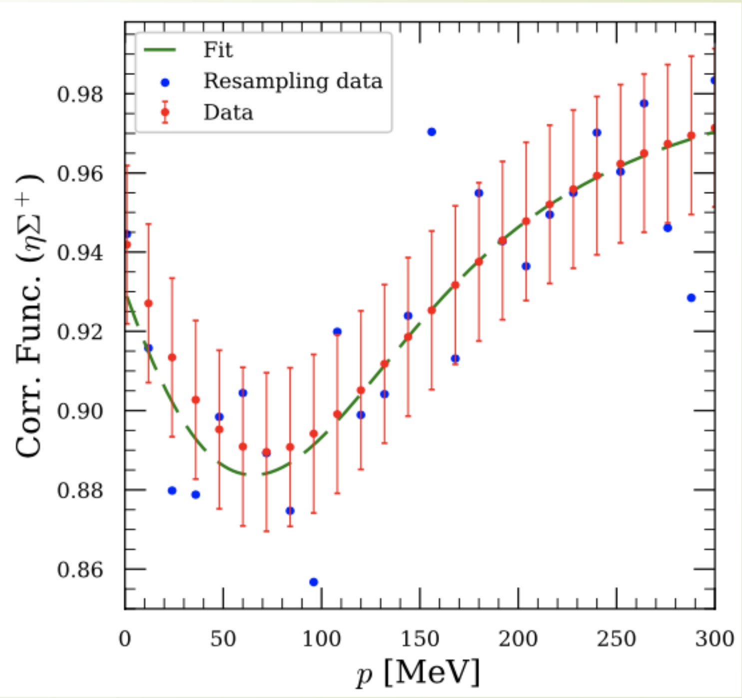
Generating random centroids of the data


Fit
Resampling data
Data
Corr. Func. $(\bar{K}^0 p)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\pi^+ \Sigma^0)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\pi^0 \Sigma^+)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\pi^+ \Lambda)$
p [MeV]


Fit
Resampling data
Data
Corr. Func. $(\eta \Sigma^+)$
p [MeV]

Average of the fitted parameters

$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{14}$	$\tilde{C}_{15}$	$\tilde{C}_{22}$
1.036 ± 0.261	-0.985 ± 0.138	-1.204 ± 0.220	-0.829 ± 0.406	1.924 ± 0.147
$\tilde{C}'_{22}$	$\tilde{C}_{24}$	$\tilde{C}_{25}$	$\tilde{C}_{44}$	$\tilde{C}_{45}$
-2.136 ± 0.465	-0.057 ± 0.342	-0.028 ± 0.571	-0.053 ± 0.141	-0.066 ± 0.706
$\tilde{C}_{55}$	$q_{\max}$(MeV)	R(fm)		
0.043 ± 0.447	653.468 ± 63.802	0.995 ± 0.029		

Observables： the scattering length and effective range

$$\frac{1}{a_i} = \frac{8\pi\sqrt{s}}{2M_i}(T_{ii})^{-1}\bigg|_{\sqrt{s}_{\mathrm{th},i}}$$

$$r_i = \frac{1}{\mu_i}\frac{\partial}{\partial\sqrt{s}}\left[\frac{-8\pi\sqrt{s}}{2M_i}(T_{ii})^{-1} + ik_i\right]_{\sqrt{s}_{\mathrm{th},i}}$$

$$\sqrt{s}_p = (1420 \pm 10) - i(101 \pm 19)\ \text{MeV}$$

J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001)
D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)

Average of the scattering lengths

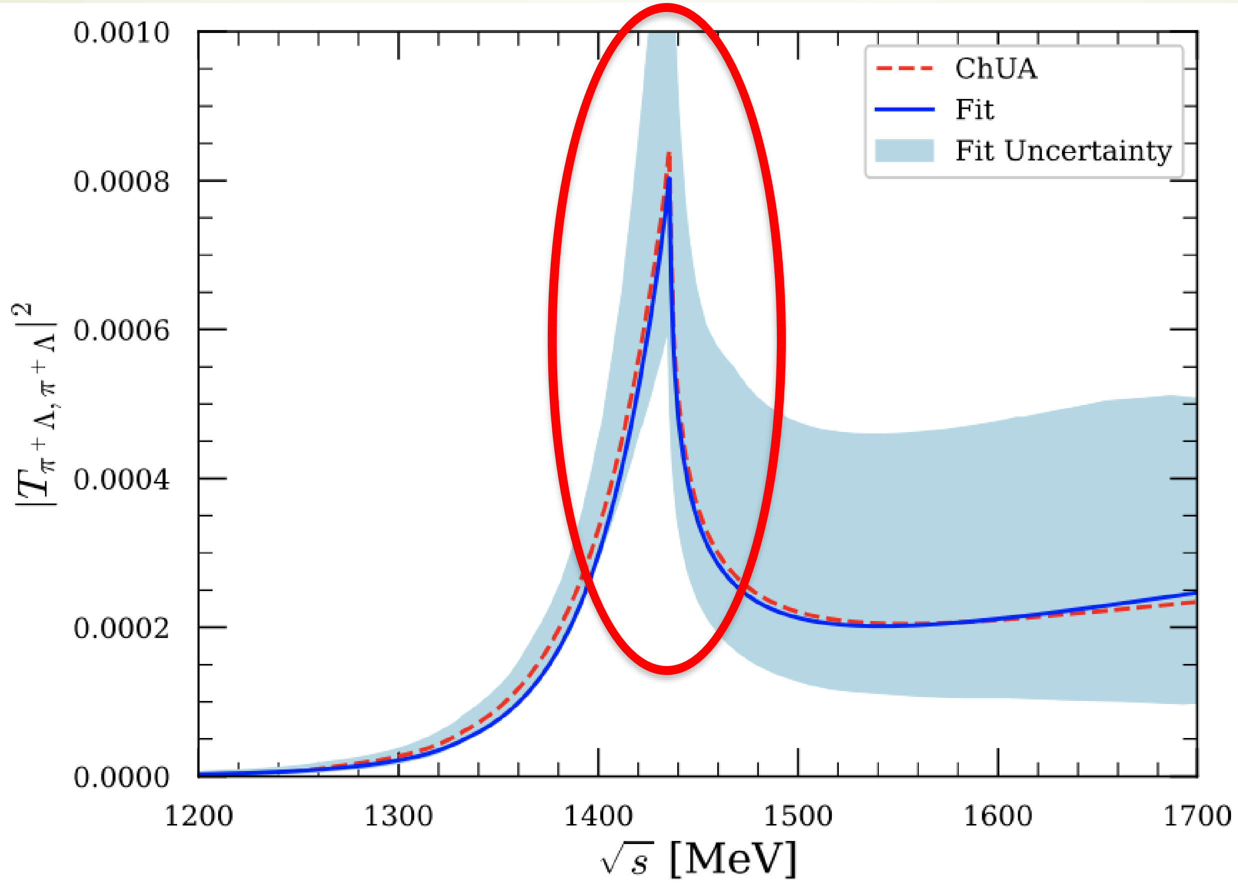
a_1	a_2	a_3
$(0.468 \pm 0.088) - i(1.130 \pm 0.041)$	$-(0.148 \pm 0.010) - i(0.030 \pm 0.004)$	$-(0.113 \pm 0.010) - i(0.004 \pm 0.003)$
a_4	a_5	
$-(0.045 \pm 0.008)$	$(0.083 \pm 0.010) - i(0.161 \pm 0.026)$	

Average of the effective ranges

r_1	r_2	r_3
$(0.025 \pm 0.150) - i(0.452 \pm 0.089)$	$-(38.019 \pm 6.345) - i(16.534 \pm 1.932)$	$-(75.053 \pm 17.150) + i(1.143 \pm 1.456)$
r_4	r_5	
$-(75.035 \pm 19.508)$	$(0.334 \pm 0.761) + i(0.380 \pm 0.947)$	

Consistent with the theoretical results before

A cusp- like structure



§4. Summary

- We use the chiral unitary approach to dynamically generate the state $\Sigma^*(1430)$
- Taking the pseudo data from theory, we use the resampling method for the inverse problem in the fitting of the correlation functions.
- The existing of this resonance can be tested by the information from the correlation functions.

Hope future experiments bring more clarifications on these issues.......

Thanks for your attention!

感谢大家的聆听！