Relativistic three-body scattering and the DD^* system

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Based on : Xu Zhang, Phys. Rev. D 109, 094010

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Outlines

- Some backgrounds
- The $D^0 D^{*+} - D^+ D^{*0}$ scattering
- Analytic structure of the $D^0 D^{*+} - D^+ D^{*0}$ scattering amplitude
- The $D^0 D^0 \pi^+$ decay and the pole position

S-matrix theory

Scattering theory, John R. Taylor

Two-body scattering

A particle that enters the collision with in asymptote $|\phi\rangle$
be observed to leave with out asymptote $|\chi\rangle$

1. Stable particle
2. Does not include bound state

- The probability

  $w(\chi \leftarrow \phi) = |\langle\chi|\, S\, |\phi\rangle|^2$

- The scattering amplitude

$$\langle\vec{p}\,'|S|\vec{p}\,\rangle = \delta^3(\vec{p} - \vec{p}\,') - i2\pi\delta\left(E_{p'} - E_p\right)t(\vec{p} \leftarrow \vec{p}\,')$$

- Bound state, virtual state and resonance ⟷ Pole in the *S*-matrix

$$s_l(p) = \frac{f_l(-p)}{f_l(p)}$$

$f_l(p)$ is the Jost function.

The asymptotic behavior of the wave function ($r \to \infty$)

$$\varphi_{l,p}(r) \to \frac{i}{2}\left[f_l(p)h_l^-(pr) - f_l(-p)h_l^+(pr)\right] \qquad h_l^\pm(pr) \to e^{\pm ipr}$$

Three-body scattering

1. G. Skorniakov and K. Ter-Martirosian, Three body problem for short range forces 1. Scattering of low energy neutrons by deuterons, Sov. Phys. JETP 4, 648 (1957).
2. L. Faddeev, Scattering theory for a three particle system, Sov. Phys. JETP 12, 1014 (1961).
3. G. Danilov, On the three-body problem with short-range forces, Sov. Phys. JETP 13, 349 (1961).
4. L. F. R.A. Minlos, On the three-body problem with short-range forces, Sov. Phys. JETP 14, 1315 (1962).
5. G. Danilov, On the three-body problem with short-range forces, Sov. Phys. JETP 17, 1015 (1963).
6. R. Aaron, R. D. Amado, and J. E. Young, Relativistic three-body theory with applications to pi-minus n scattering, Phys. Rev. 174, 2022 (1968).
7. P. F. Bedaque, H. W. Hammer, and U. van Kolck, Renormalization of the three-body system with short range interactions, Phys. Rev. Lett. 82, 463 (1999).
8. K.P. Khemchandani, A. Martinez-Torres, E. Oset, Eur. Phys. J. A **37**, 233 (2008).

Faddeev equations

Faddeev, Sov. Phys. JETP 12, 1014 (1961)

$$\begin{bmatrix} T_{1\alpha} \\ T_{2\alpha} \\ T_{3\alpha} \end{bmatrix} = \bar{\delta}_{\gamma\alpha} G_0 + \begin{bmatrix} 0 & t_{12} & t_{13} \\ t_{21} & 0 & t_{23} \\ t_{31} & t_{32} & 0 \end{bmatrix} \begin{bmatrix} G_0 & 0 & 0 \\ 0 & G_0 & 0 \\ 0 & 0 & G_0 \end{bmatrix} \begin{bmatrix} T_{1\alpha} \\ T_{2\alpha} \\ T_{3\alpha} \end{bmatrix}$$

$$\bar{\delta}_{\gamma\alpha} = 1 - \delta_{\gamma\alpha}$$

Satisfying unitarity relation

$$i \left[T^{\dagger} - T \right] = T^{\dagger} T$$

The Quantum Mechanical Few-Body Problem, Walter Glöckle

- The subsystem scattering amplitude

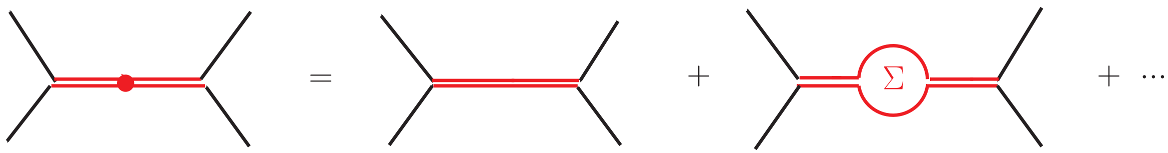
$$t = v + vGt$$

Lippmann-Schwinger equation

Relativistic three-body scattering

R. Aaron at al., PR 174, 2022 (1968).
R. Aaron et al., Modern three-hadron physics, 1977.

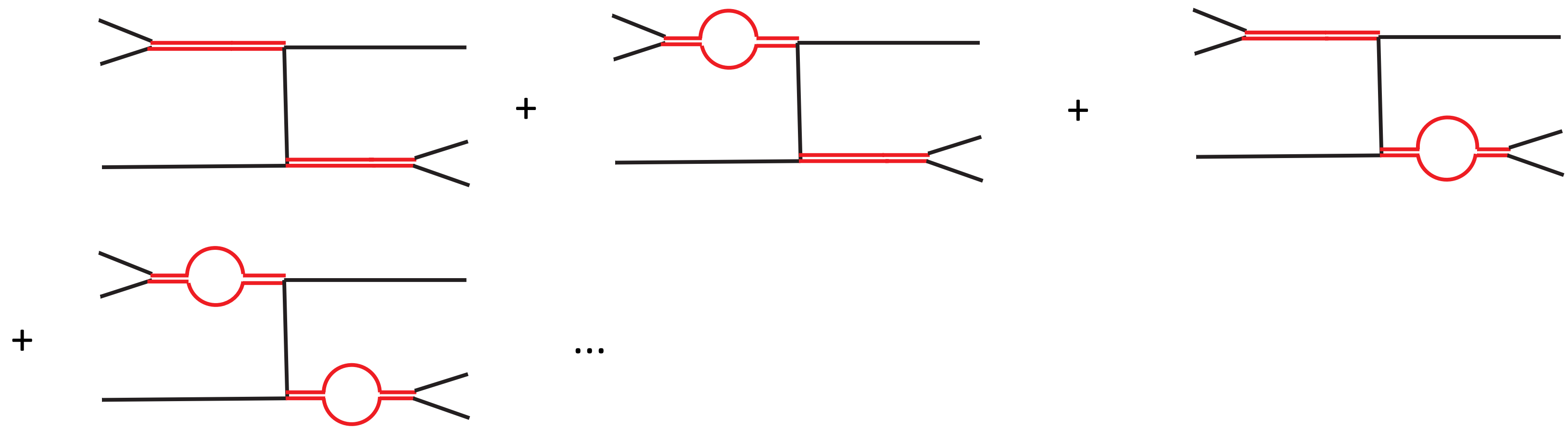
❖ Two-body subsystem interaction



$$A = \frac{v^\dagger v}{\sigma - m^2 - \Sigma(\sigma) + i\varepsilon}$$

Relativistic variable
Separable two-body interaction

❖ Three-body interaction



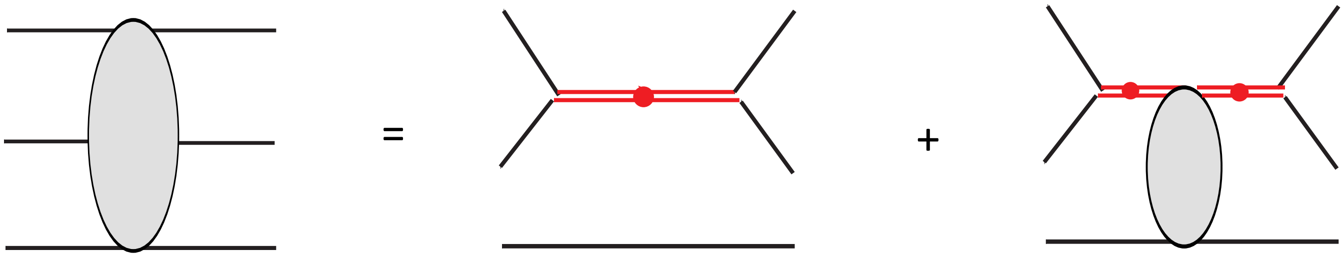
$$T = V + V\,\tau\,T$$

where $\tau = \frac{1}{\sigma - m^2 - \Sigma(\sigma) + i\varepsilon}$, V is the Bonn potential.

Unitarity relation

R. Aaron at al., PR 174, 2022 (1968).
R. Aaron et al., Modern three-hadron physics, 1977.

❖ Three-body interaction



Disconnected part

Connected part

❖ Unitarity relation

$$i\,[\,M^{\dagger}\ -M\,] = M^{\dagger}\,M \quad \Longrightarrow \quad \Sigma \sim \int_0^{\infty} cof.\, \frac{k^2\,dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)\,]^2 + i\varepsilon}$$

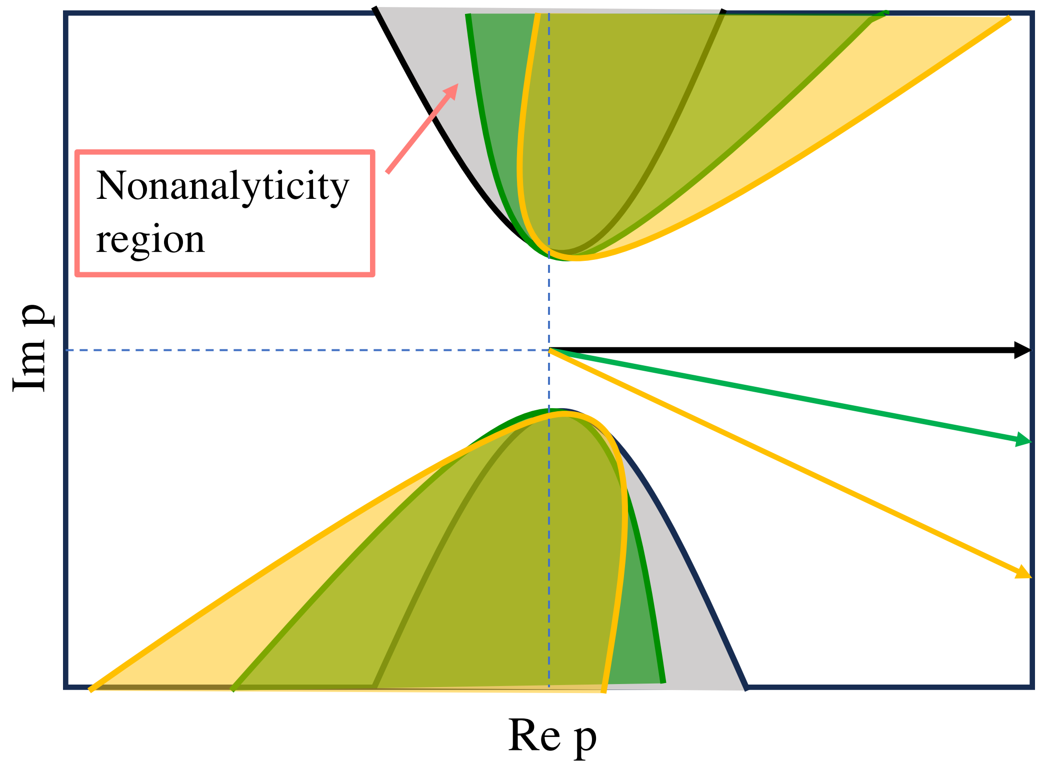
Analytic properties

- The domains of nonanalyticity

$V(p', p) \sim$

$$\int_{-1}^{1} \frac{dx}{\sqrt{s} - [\omega_D(p') + \omega_D(p) + \omega_\pi(q)] + i\varepsilon}$$

Denominator = 0 ➡ Nonanalyticity region



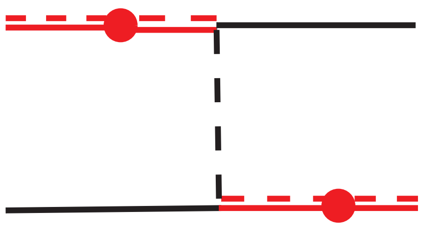
W. Glöckle, PRC 18, 564 (1978).

- Contour deformation

$$p \to p\, e^{-i\theta}$$
$$p' \to p'\, e^{-i\theta}$$

DD^* scattering

❖ π-exchange

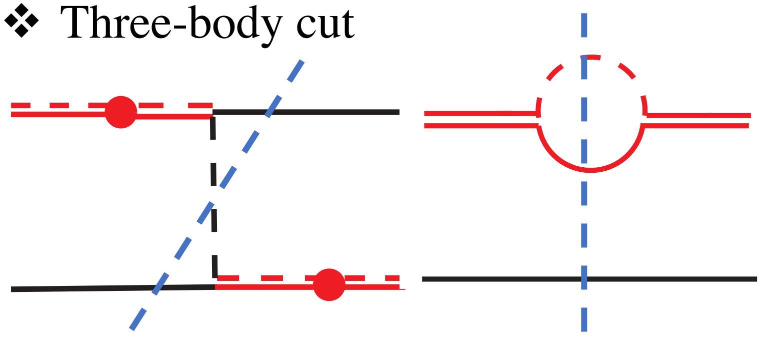


$$V \sim \frac{1}{q^2 - m_\pi^2 + i\varepsilon}$$

$$q^2 - m_\pi^2 \simeq -2m_\pi\Delta + \left(1 + \frac{m_\pi^2}{4m_D m_{D^*}}\right)\boldsymbol{q}^2$$

$$\Delta = m_{D^*} - m_D - m_\pi$$

❖ Three-body cut



zero appears !

Different from the NN scattering

D. B. Kaplan, M. J. Savage, and M. B. Wise, PLB 424, 390 (1998).

D. B. Kaplan, M. J. Savage, and M. B. Wise, NPB 534, 329 (1998).

Three-body dynamics

M.-L. Du, V. Baru, X.-K. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, PRD 105, 014024 (2022).

L. Qiu, C. Gong, and Q. Zhao, PRD 109, 076016 (2024).

J.-Z. Wang, Z.-Y. Lin, and S.-L. Zhu, PRD 109, L071505 (2024).

Similar phenomenon happens in $D\bar{D}^*$

S. Fleming, M. Kusunoki, T. Mehen, and U. van Kolck, PRD 76, 034006 (2007).

M. Suzuki, PRD 72, 114013 (2005).

V. Baru, A. A. Filin, C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, PRD 84, 074029 (2011).

M. Schmidt, M. Jansen, and H. W. Hammer, PRD 98, 014032 (2018).

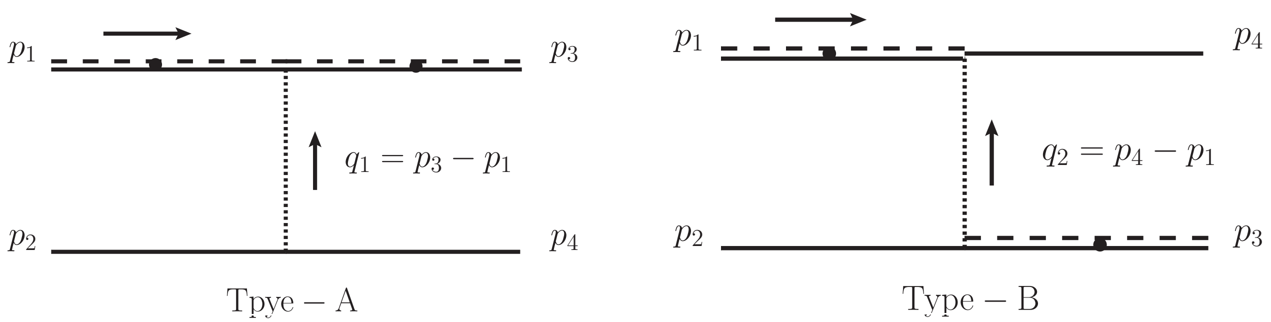
DD^* scattering

❖ Scattering equation

$$T(s,p',p) = V(s,p',p) + \int_0^\Lambda \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\, \tau(\sigma_k)\, T(s,k,p)$$

$$\sigma_k = s - 2\sqrt{s}\,\omega_D(k) + m_D^2$$

The interaction potential (including the π, σ , ρ and ω-exchange).



The isospin factors

	Type-A			Type-B				
	ρ^0	$\rho^\pm$	ω	ρ^0	$\rho^\pm$	ω	π^0	$\pi^\pm$
1→ 1 2→ 2	1/2	…	-1/2	…	1	…	…	1
1→ 2	…	-1	…	-1/2	…	1/2	-1/2	…

DD^* scattering

- The interaction potential (including the π, σ , ρ and ω-exchange).

$$
\begin{aligned}
\langle \vec{p}'\lambda'|V_{A-V}(E)|\vec{p}\lambda\rangle &= -g_{\mathrm{DDV}}\cdot g_{\mathrm{D^*D^*V}}\cdot IF\cdot(p_3+p_1)^{\mu}\frac{-g_{\mu\nu}+q_{1\mu}q_{1\nu}/m_V^2}{q_1^2-m_V^2+i\epsilon}(p_4+p_2)^{\nu}\epsilon^{*\alpha}_{\lambda'}(p_3)\epsilon_{\lambda\alpha}(p_1)\\
&\quad +2g_{\mathrm{DDV}}\cdot g'_{\mathrm{D^*D^*V}}\cdot IF\cdot[\epsilon^{*\mu}_{\lambda'}(p_3)\epsilon^{\alpha}_{\lambda}(p_1)q_{1\alpha}-\epsilon^{*\alpha}_{\lambda'}(p_3)\epsilon^{\mu}_{\lambda}(p_1)q_{1\alpha}]\frac{-g_{\mu\nu}}{q_1^2-m_V^2+i\epsilon}(p_4+p_2)^{\nu},\\
\langle \vec{p}'\lambda'|V_{B-V}(E)|\vec{p}\lambda\rangle &= g^2_{\mathrm{DD^*V}}\cdot IF\cdot\varepsilon_{\alpha'\beta'\mu'\nu'}(p_3+p_2)^{\alpha'}q_2^{\beta'}\epsilon^{*\nu'}_{\lambda'}(p_3)\frac{-g_{\mu'\mu}}{q_2^2-m_V^2+i\epsilon}\varepsilon_{\alpha\beta\mu\nu}(p_4+p_1)^{\alpha}q_2^{\beta}\epsilon^{\nu}_{\lambda}(p_1),\\
\langle \vec{p}'\lambda'|V_{B-P}(E)|\vec{p}\lambda\rangle &= g^2_{\mathrm{DD^*P}}\cdot IF\cdot\epsilon^{*\nu}_{\lambda'}(p_3)q_{2\nu}\frac{\omega_{i,2}(p)+\omega_{i',2}(p')+\omega_P(q)}{\omega_P(q)[s-(\omega_{i,2}(p)+\omega_{i,2'}(p')+\omega_P(q))^2+i\epsilon]}\epsilon^{\mu}_{\lambda}(p_1)q_{2\mu},\\
\langle \vec{p}'\lambda'|V_{A-S}(E)|\vec{p}\lambda\rangle &= g_{\mathrm{D^*D^*}\sigma}g_{\mathrm{DD}\sigma}\epsilon^{*\mu}_{\lambda'}(p_3)\epsilon_{\lambda\mu}(p_1)\frac{1}{q_1^2-m_S^2+i\epsilon},
\end{aligned}
$$

- Spin-1 helicity polarization vectors

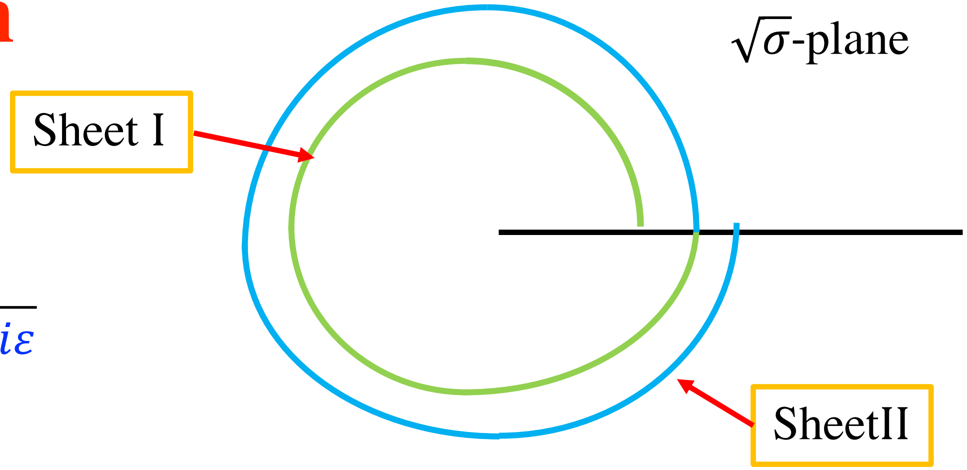
$$
\epsilon^{\mu}_{0}(p)=\frac{1}{m}\begin{pmatrix}|\vec{p}|\\ E\sin\theta\cos\phi\\ E\sin\theta\sin\phi\\ E\cos\theta\end{pmatrix},\qquad \epsilon^{\mu}_{\pm1}(p)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \mp\cos\theta\cos\phi+i\sin\phi\\ \mp\cos\theta\sin\phi-i\cos\phi\\ \pm\sin\theta\end{pmatrix}
$$

- The partial wave interaction potentials

$$
V^{J}_{\lambda'\lambda}(s,p',p)=2\pi\int_{-1}^{+1}d^{J}_{\lambda\lambda'}(\cos\theta)V_{\lambda'\lambda}(s,\vec{p}',\vec{p})\,d\cos\theta
$$

$$
V^{J}_{L'L}(s,p',p)=\sum_{\lambda'\lambda}\langle JL'S|J\lambda'\rangle V^{J}_{\lambda'\lambda}(s,p',p)\langle J\lambda|JLS\rangle
$$

Analytic continuation



Multi-Riemann sheets

- The self-energy

$$\Sigma \sim \int_0^{\infty} cof.\ \frac{k^2\, dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)]^2 + i\varepsilon}$$

- The effective BS equation

$$\sqrt{\sigma} = (m_D + m_\pi) + re^{i\theta}$$

$$T(s,p',p) = V(s,p',p) + \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\, \tau(\sigma_k)\, T(s,k,p)$$

Analytic continuation

- Calculating the discontinuity

$$\Sigma^{II}(\sqrt{\sigma} + i\varepsilon)$$
$$= \Sigma^{I}(\sqrt{\sigma} + i\varepsilon) - i2\mathrm{Im}\Sigma^{I}(\sqrt{\sigma} + i\varepsilon)$$

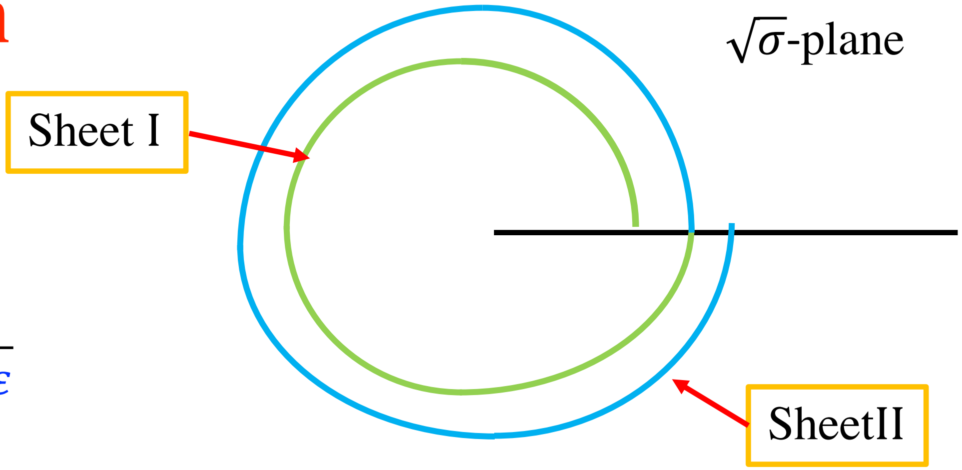
- Contour deformation

B. C. Pearce and I. R. Afnan, PRC 30, 2022 (1984).
D. Sadasivan, A. Alexandru, H. Akdag, F. Amorim, R. Brett, C. Culver, M. Döring, F. X. Lee, and M. Mai, PRD 105, 054020 (2022).
S. M. Dawid, M. H. E. Islam, and R. A. Briceño, Phys. Rev. D 108, 034016 (2023).

Analytic continuation

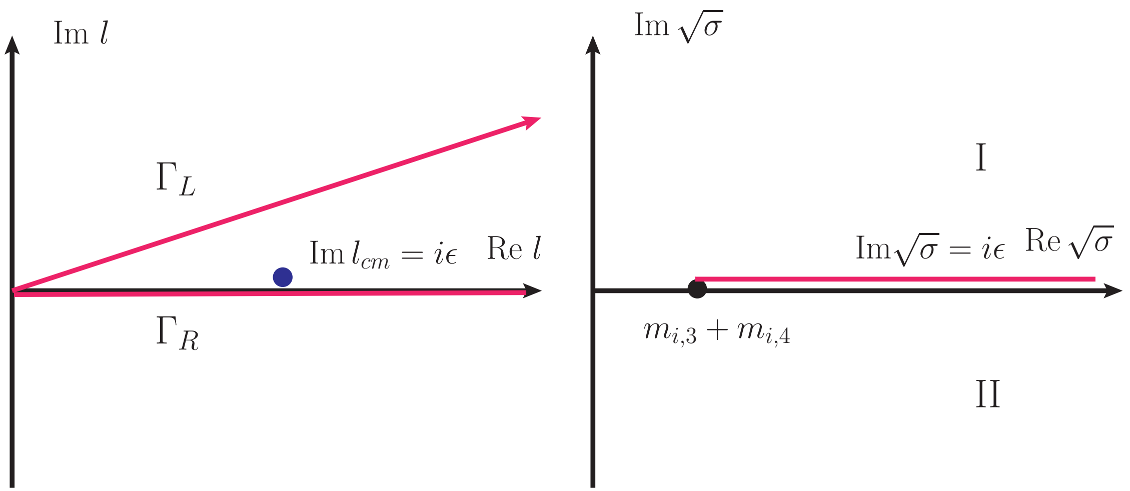
- Self-energy
  - First Riemann sheet

$$\Sigma \sim \int_0^\infty cof.\ \frac{k^2\,dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)\,]^2 + i\varepsilon}$$



$$\sqrt{\sigma} = (\,m_D + m_\pi) + re^{i\theta}$$

Integral contour



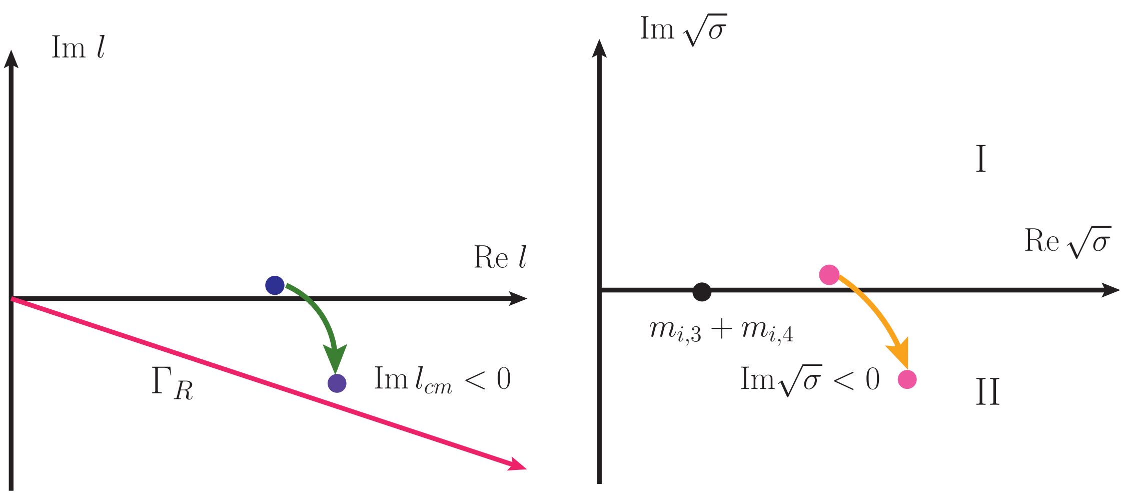
Analytic continuation

❖ Self-energy

$$\Sigma \sim \int_0^\infty cof. \frac{k^2\, dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)]^2 + i\varepsilon}$$

- Second Riemann sheet (resonance)

Integral contour

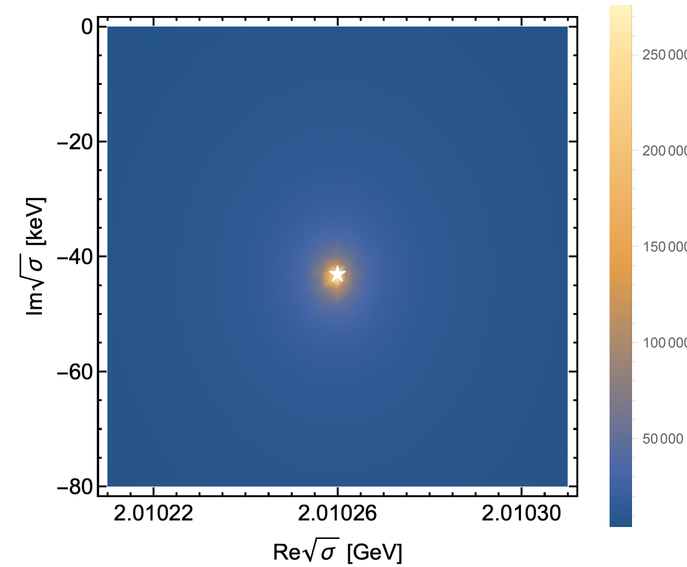


$$\tau = \frac{1}{\sigma - m^2 - \Sigma_1^R(\sigma) - \Sigma_2^R(\sigma) + i\varepsilon}$$

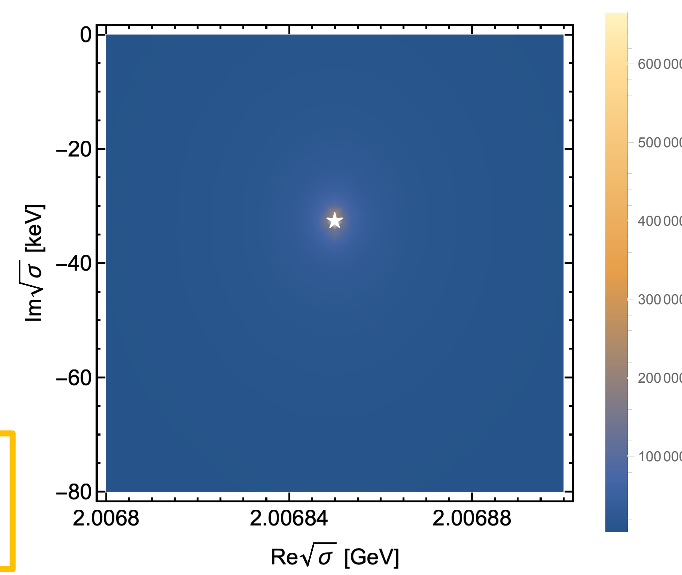
$$\Sigma_i^R(\sigma) = \Sigma_i(\sigma) - \Sigma_i^{\text{sub}}(\sigma)$$

$D\pi$ interaction in p -wave
Twice subtraction

D^{*+} pole position



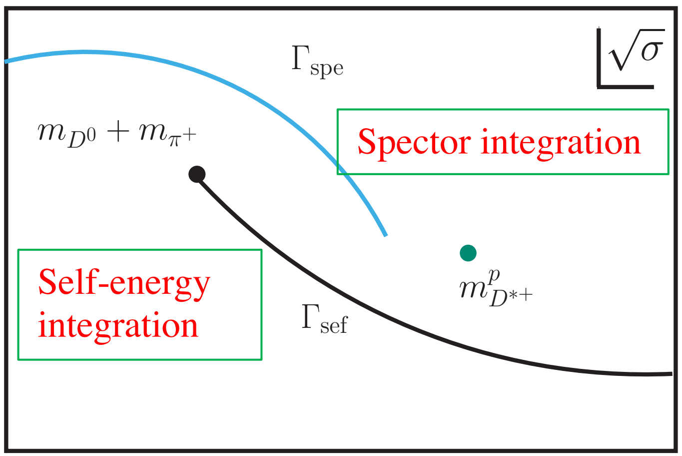
D^{*0} pole position

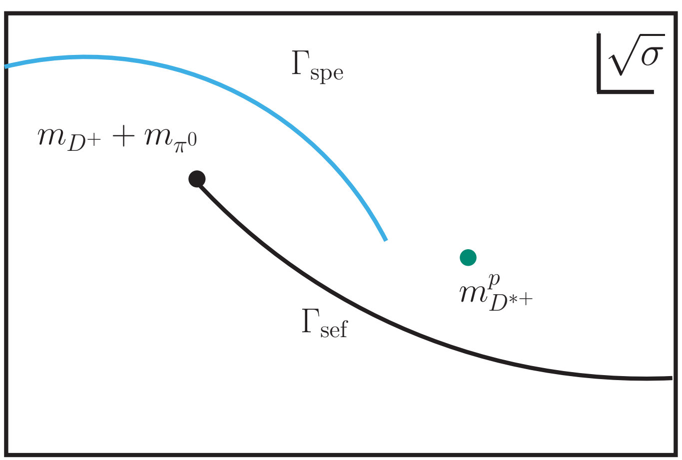


Analytic continuation

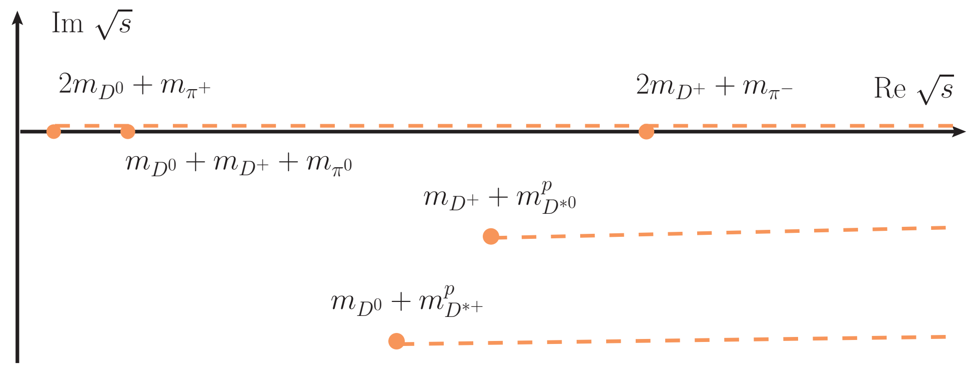
$$T(s,p',p) = V(s,p',p) + \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\, \tau(\sigma_k)\, T(s,k,p)$$

- Contour mapped to the $\sqrt{\sigma}$ plane.

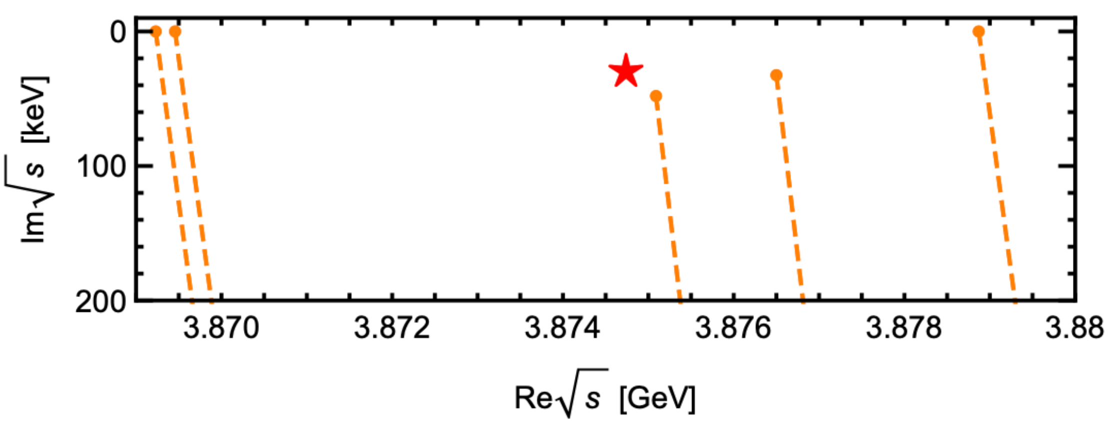




- The cut structure in complex $\sqrt{s}$ plane

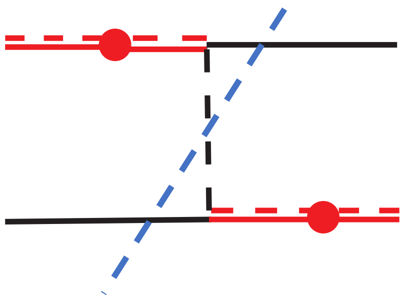


- The cut structure after contour deformation



Solution for real momentum

❖ Three-body singularities

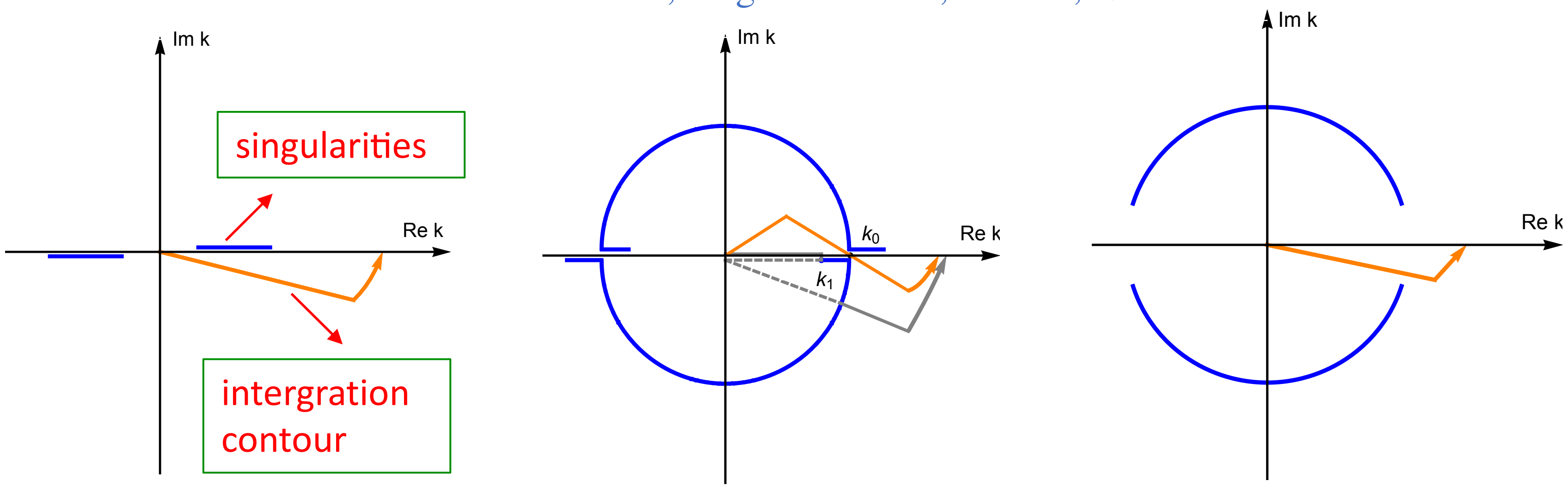


$$V \sim \int_{-1}^{1} \frac{dx}{\sqrt{s} - [\omega_D(p') + \omega_D(p) + \omega_\pi(q)] + i\varepsilon} \Rightarrow \text{Zero}$$

$$x = \vec{p}\,\vec{p}'/pp'$$

❖ Contour deformation

E. Schmid, H. Ziegelmann, The Quantum Mechanical Three-Body Problem, Pergamon Press, Oxford, 1974



Solution for real momentum

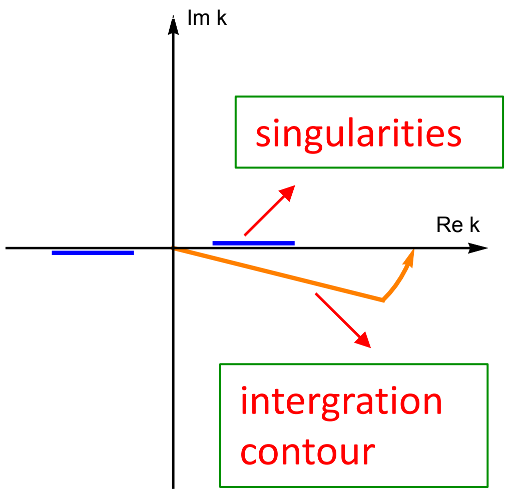
❖ Contour deformation

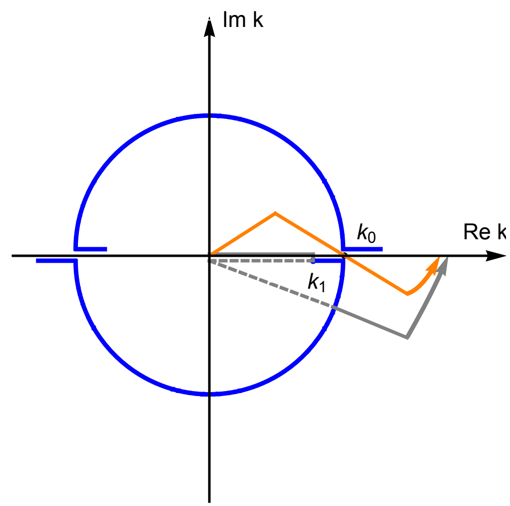
$$T(s,p',p) = V(s,p',p) + \int_C \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\, \tau(\sigma_k)\, T(s,k,p)$$

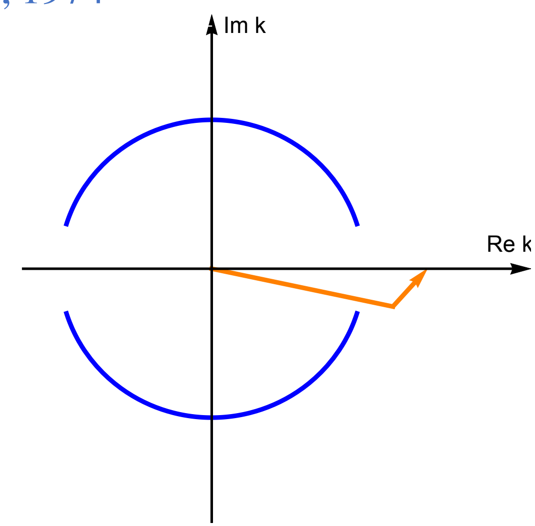
The effective BS equation is analytically continued into the complex momentum plane using Cauchy's theorem.

$$p' \rightarrow p' e^{-i\theta}$$
$$k \rightarrow k\, e^{-i\theta}$$

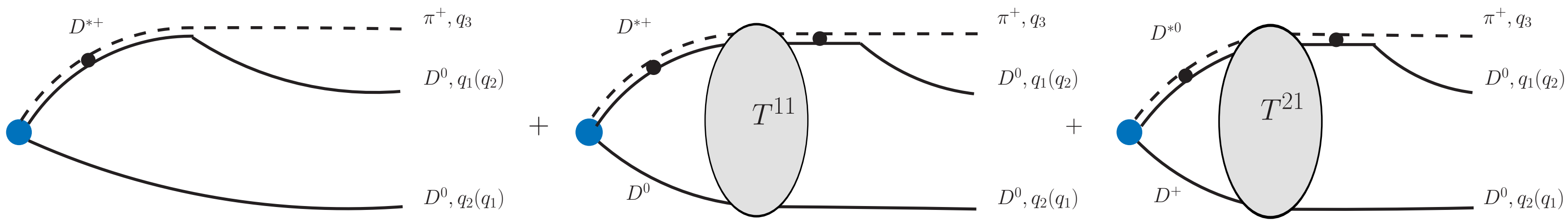
E. Schmid, H. Ziegelmann, The Quantum Mechanical Three-Body Problem, Pergamon Press, Oxford, 1974







Three-body $D^0 D^0 \pi^+$ dacay



- The mass distribution

$$\frac{d\Gamma(\sqrt{s})}{d\sqrt{s}} = \int \frac{1}{(2\pi)^5} \frac{1}{16\, s} \left(\frac{1}{3} \sum_{\Lambda} \left| \sum_{\lambda} M_{\Lambda\lambda}(\vec{q}_1, \vec{q}_2, \vec{q}_3) \right|^2 \right) q_3^* \, q_1 \, dm_{23} \; d\Omega_3^* \, d\Omega_1$$

Decay amplitude

$$M_{\Lambda\lambda}(\vec{q}_1, \vec{q}_2, \vec{q}_3) = \frac{\mathcal{F}}{\sqrt{2}} \left[\sqrt{\frac{3}{4\pi}} \, D^{1*}_{\Lambda\lambda}(\phi_1, \theta_1, 0) M_L(q_1) \, U_{L\lambda} \, v_\lambda(\vec{q}_2, \vec{q}_3) + \vec{q}_1 \leftrightarrow \vec{q}_2 \right]$$

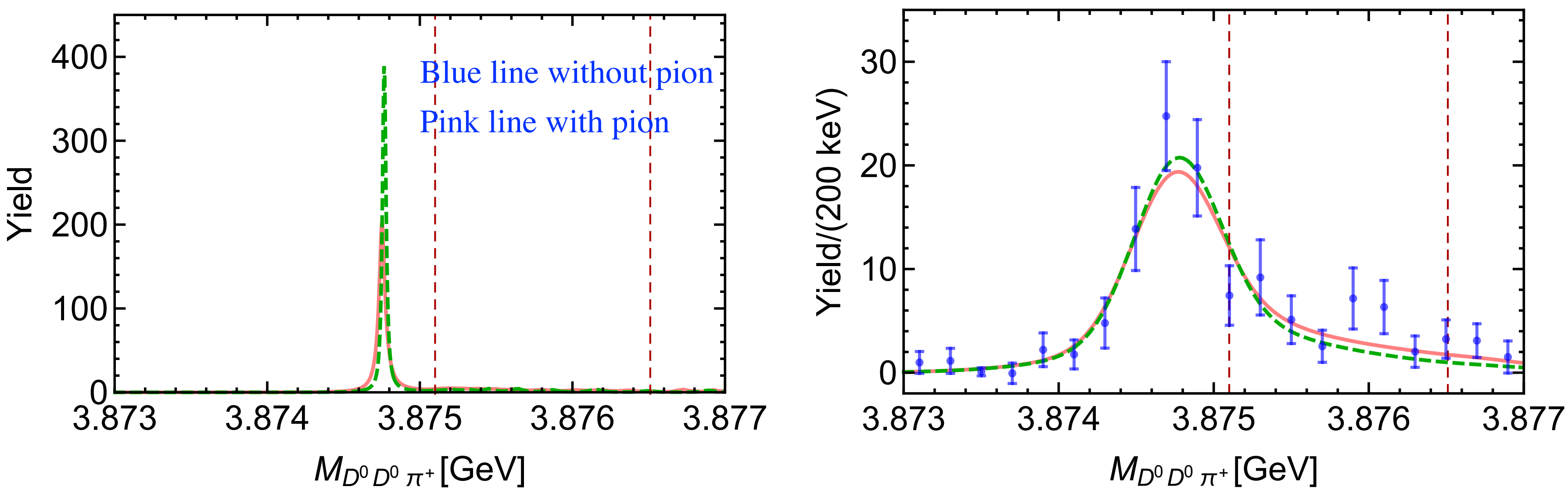
- Short-distance interaction is absorbed into $\mathcal{F}$.

Numerical results

Cut off Λ and the pole positions from fitting of the $D^0D^0\pi^+$ line shape obtained by the LHCb Collaboration.

Scheme	$\chi^2/d.o.f$	Λ GeV	$\sqrt{s_{pole}^{thr}}$ keV
I	18.11/(20−1) = 0.95	0.4551 ± 0.0018	$-332^{+37}_{-36} - i(18 \pm 1)$
II	14.47/(20−1) = 0.76	0.3701 ± 0.0017	$-351^{+37}_{-35} - i(28 \pm 1)$

Scheme I: without pion exchange   Scheme II : with pion exchange



Fitting results of the $D^0D^0\pi^+$ line shapes before (left panel) and after (right panel) convolution with the energy resolution function.

Numerical results

The wave function

$$|DD^*, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),$$

$$|DD^*, I=1\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+).$$

The effective coupling constants

$$g^{i'}g^{i} = \lim_{s\to s_{pole}} \frac{1}{4\pi}(s - s_{pole})T^{i'i}(s, k_b, k_b)$$

Scheme	g^1	g^2	$g^{I=0}$	$g^{I=1}$
I	$3.90^{+0.09}_{-0.09} - i0.04^{+0.00}_{-0.00}$	$-4.11^{+0.09}_{-0.09} + i0.04^{+0.00}_{-0.00}$	$-5.66^{+0.13}_{-0.13} + i0.06^{+0.00}_{-0.00}$	$0.15^{+0.00}_{-0.00} + i0.00^{+0.00}_{-0.00}$
II	$4.00^{+0.09}_{-0.09} + i0.04^{+0.00}_{-0.00}$	$-4.13^{+0.09}_{-0.09} + i0.05^{+0.00}_{-0.00}$	$-5.75^{+0.13}_{-0.13} + i0.01^{+0.00}_{-0.00}$	$0.09^{+0.00}_{-0.00} - i0.07^{+0.00}_{-0.00}$

We find that the coupling constants g^1 and g^2 are very close to each other with an opposite sign. This indicates that we have basically a state with an isospin I = 0.

Summary

- The analytic continuation of the coupled-channel $D^0D^{*+} - D^+D^{*0}$ scattering amplitude is studied.
- The π-exchange term has a signification on the pole position of the T_{cc}^+. Including the π-exchange term, the width of T_{cc}^+ will be increased by a factor of 1.5.
- We will extend our framework to calculate $3\pi - K\bar{K}\pi$ coupled system.

Thank you for your attention!