Neutron scattering off one-neutron halo nuclei in halo effective field theory

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Based on
X. Zhang, H.-L. Fu, Feng-Kun Guo, H.-W. Hammer, Phys.Rev.C 108, 044304(2023).

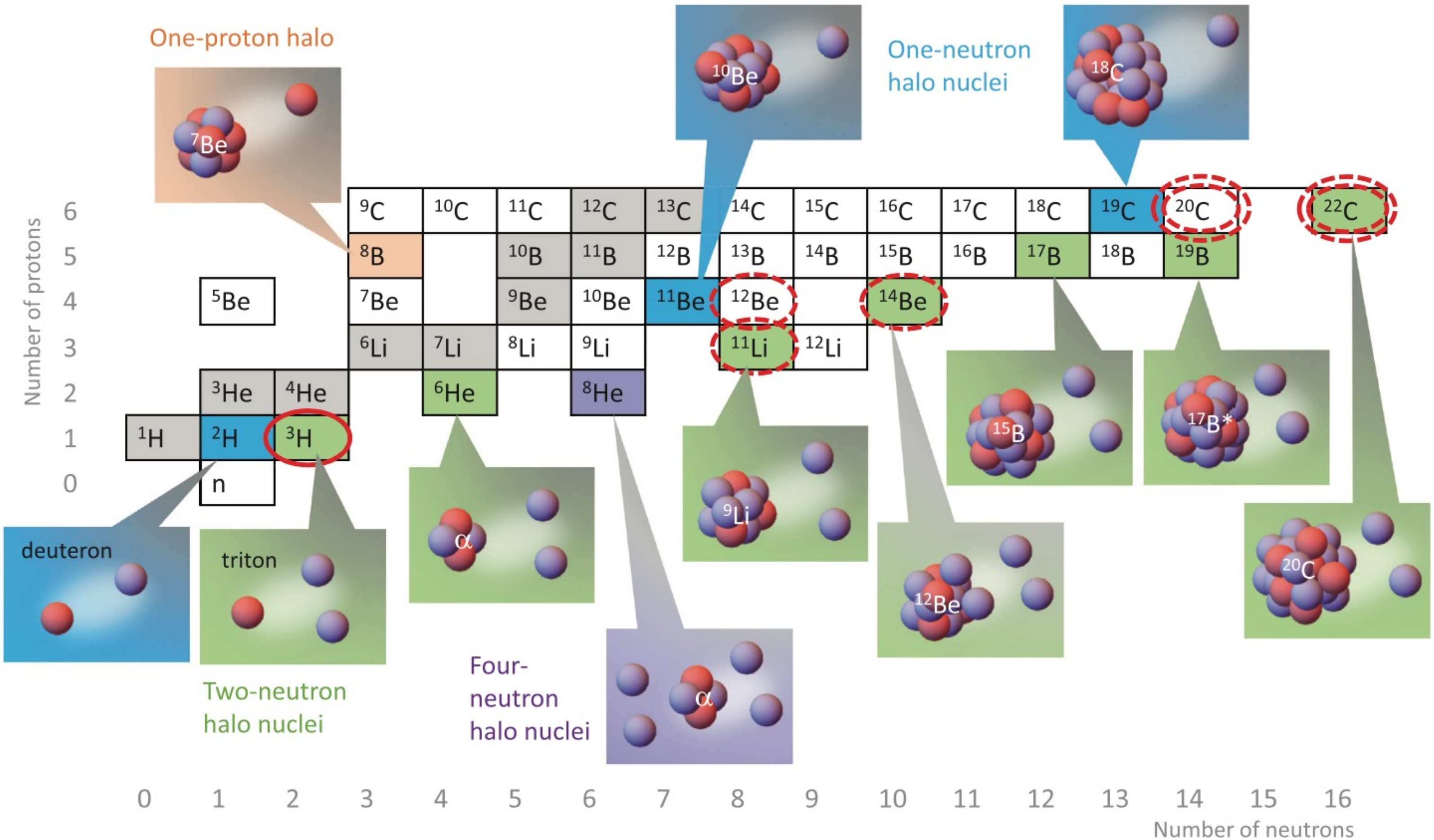
The 23rd International Conference on Few-Body Problems in Physics (FB23)

Beijing, September 22-27, 2024

Outlines

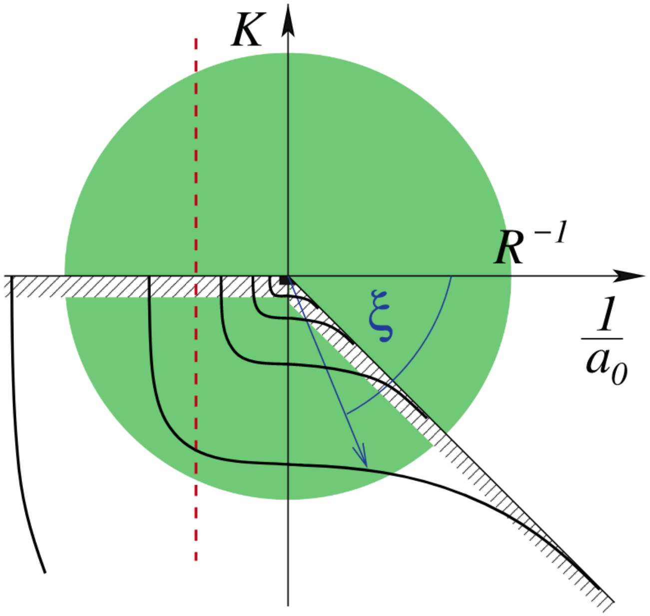
- ❖ Background
- ❖ Halo effective field theory
- ❖ Numerical results and discussions
- ❖ Summary

Halo nuclei



Pascal Naidonand Shimpei Endo,
Rep. Prog. Phys. 80, 056001(2017)

Efimov state in three-body system



Bound-state energies

$$B_3 = -\frac{1}{m a_0^2} + \left[e^{-2\pi n} f(\xi)\right]^{1/s_0} \frac{\kappa_*^2}{m}$$

where n is an integer labelling the states, and κ_* is the binding wave number at unitarity of the state $n = 0$.

Figure 1. Illustration of the Efimov spectrum: the energy variable $K = \mathrm{sgn}(E)\sqrt{m|E|}$ is shown as a function of the inverse scattering length $1/a_0$. The shaded circular region exhibits the window of universality. The solid lines indicate the Efimov states, while the hashed areas give the scattering thresholds and the dashed vertical line illustrates an exemplary system with fixed scattering length.

H.-W. Hammer, C. Ji, D. R. Phillips, JPG 44 (2017) 103002

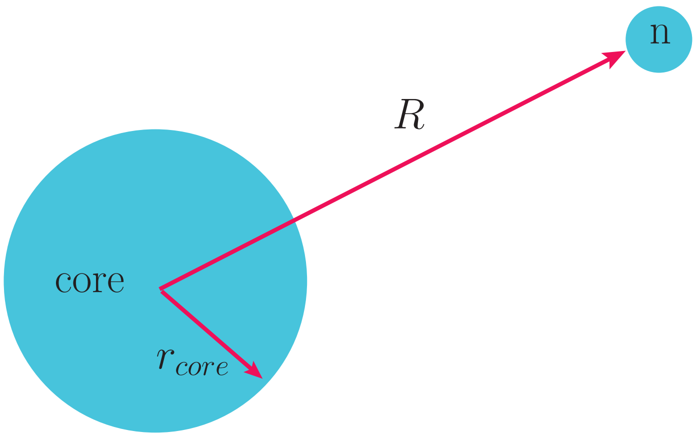
Halo effective field theory

❑ Halo effective field theory (Halo EFT)

C. A. Bertulani, H.-W. Hammer, and U. van Kolck, NPA 712 (2002) 37; P. F. Bedaque, H. W. Hammer, and U. van Kolck, PLB 569 (2003) 159; … Reviews: H.-W. Hammer, C. Ji, D. R. Phillips, JPG 44 (2017) 103002; H.-W. Hammer, arXiv:2203.13074 [nucl-th]; …

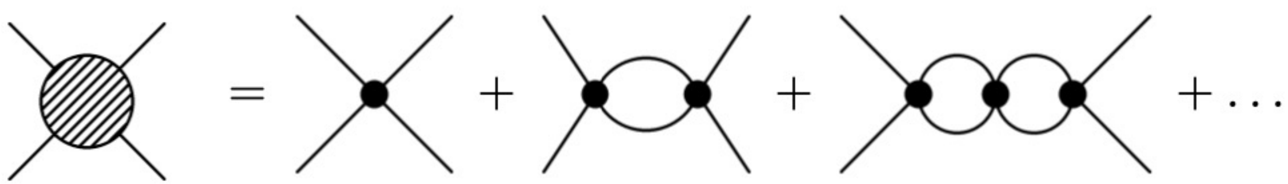
- Scale separation

$$R \sim \frac{1}{Q} \sim \frac{1}{\sqrt{2\mu E_n}} \gg r_{\text{core}} \sim \frac{1}{\Lambda} \sim \frac{1}{\sqrt{2\mu E_c^*}}$$



- derivative expansion
  - ➢ approximation with controlled uncertainty
  - ➢ systematically improvable
- low-energy constants unknown
  - ➢ fix to experimental inputs

Two-body interaction at LO



$$\langle \boldsymbol{k}' | t_x(E) | \boldsymbol{k} \rangle = C_{0,x} + C_{0,x} I C_{0,x} + C_{0,x} I C_{0,x} I C_{0,x} + \ldots = [1/C_{0,x} - I]^{-1},$$

$$I = -\frac{\mu_x}{2\pi}\left(ik + \frac{2}{\pi}\Lambda + \mathscr{O}(k^2/\Lambda)\right), \quad C_{0,x}(\Lambda) = \frac{2\pi}{\mu}\left[\frac{1}{a} - \frac{2}{\pi}\Lambda\right]^{-1}$$

One-neutron halo nuclei in Halo EFT

Properties of s-wave one-neutron halos.

	^{11}Be	^{15}C	^{19}C
	Experiment		
J^P	$1/2^+$	$1/2^+$	$1/2^+$
$S_{1n}[MeV]$	0.50164(25)	1.2181(8)	0.58(9)
$E_c^*[MeV]$	3.36803(3)	6.0938(2)	1.62(2)
$\langle r^2\rangle_{nc}^{1/2}[fm]$	6.05(23)	4.15(50)	6.6(5)
	Halo EFT		
Q/Λ	0.39	0.45	0.6
r_{nc}/a_{nc}	0.38	0.43	0.33
$\langle r^2\rangle_{nc,\text{ theo}}^{1/2}\ [fm]$	6.85	4.93	5.72

H.-W. Hammer, C. Ji, D. R. Phillips, JPG 44 (2017) 103002

nh scattering: motivation

- **Neutron scattering off one-neutron halo nuclei in Halo EFT**
  - Providing information about the internal structure of the nuclei.
  - Efimov states in halo nuclei.

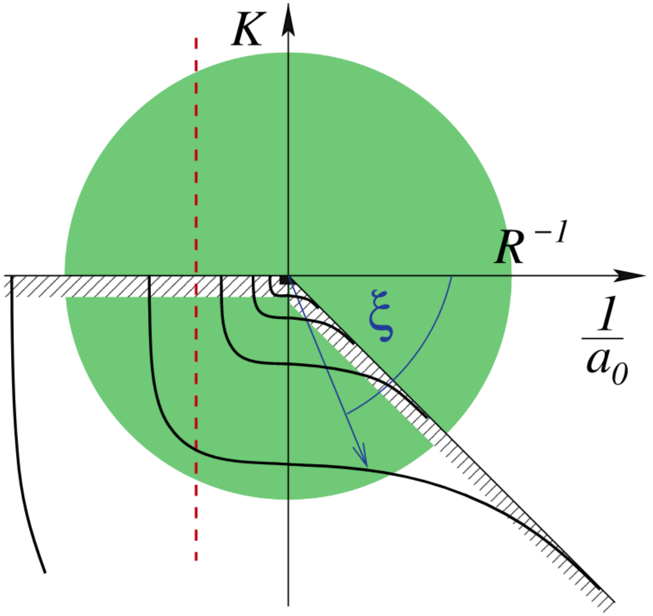


Figure 1. Illustration of the Efimov spectrum: the energy variable $K = \mathrm{sgn}(E)\sqrt{m|E|}$ is shown as a function of the inverse scattering length $1/a_0$. The shaded circular region exhibits the window of universality. The solid lines indicate the Efimov states, while the hashed areas give the scattering thresholds and the dashed vertical line illustrates an exemplary system with fixed scattering length.

H.-W. Hammer, C. Ji, D. R. Phillips, JPG 44 (2017) 103002

nh scattering

❑ n-^{11}Be, n-^{15}C, n-^{19}C scattering, 3-body system

Halo nuclei	^{11}Be	^{15}C	^{19}C
B_σ(MeV)	0.502	1.218	0.58
a_σ(fm)	6.741	4.27	6.142

- Unnaturally large scattering lengths:
  - nn: $a_s = -18.6$ fm, virtual state
  - nc: one-neutron halo
- LO Lagrangian, the dimer formalism

D.B. Kaplan, NPB 494 (1997) 471

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

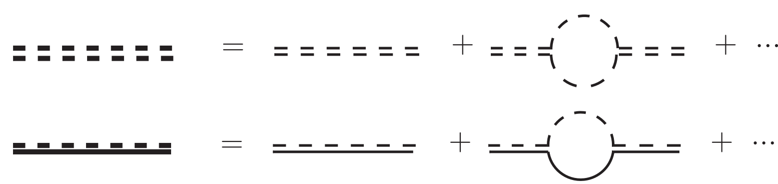
$$\mathcal{L}_1 = \vec{n}^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) \vec{n} + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c$$

P.F. Bedaque et al., NPA 676 (2000) 357
H.-W. Hammer et al., NPA 865 (2011) 17

$$\mathcal{L}_2 = \Delta_s s^\dagger s + \Delta_\sigma \sigma_i^\dagger \sigma_i - g_s C^{00}_{1/2\alpha,1/2\beta} [s^\dagger n_\alpha n_\beta + \text{H.c.}] - g_\sigma [\sigma_i^\dagger n_i c + \text{H.c.}]$$

$$\mathcal{L}_3 = g_s^2 D_0 (sc)^\dagger (sc)$$

- Dimer propagators



pole at

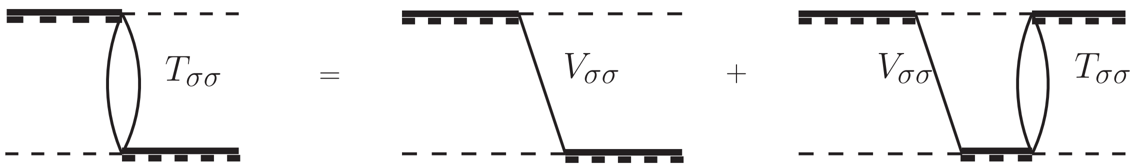
$$p_0 - \frac{p^2}{2m_\sigma} = -\frac{1}{2\mu_{nc} a_\sigma^2} = -B_\sigma$$

$$iD_s(p_0, \vec{p}) = \frac{2\pi}{m_n g_s^2} \frac{-i}{-1/a_s + \sqrt{m_n(\vec{p}^2/4m_n - p_0 - i\epsilon)}}$$

$$iD_\sigma(p_0, \vec{p}) = \frac{2\pi}{\mu_{nc} g_\sigma^2} \frac{-i}{-1/a_\sigma + \sqrt{2\mu_{nc}(\vec{p}^2/2m_\sigma - p_0 - i\epsilon)}}$$

nh scattering with total spin $J = 1$

- Faddeev equation with total spin $J = 1$



$$iT_{\sigma\sigma}(k,p,E) = iV_{\sigma\sigma}(k,p,E) - \int_0^{\Lambda} \frac{q^2 dq}{2\pi^2} V_{\sigma\sigma}(k,q,E)\, Z_\sigma^{-1} D_\sigma\left(E - \frac{q^2}{2m_n}, q\right) iT_{\sigma\sigma}(q,p,E)$$

- Inputs at LO: separation energy between the core and the valence n

Halo nuclei	^{11}Be	^{15}C	^{19}C
B_σ(MeV)	0.502	1.218	0.58
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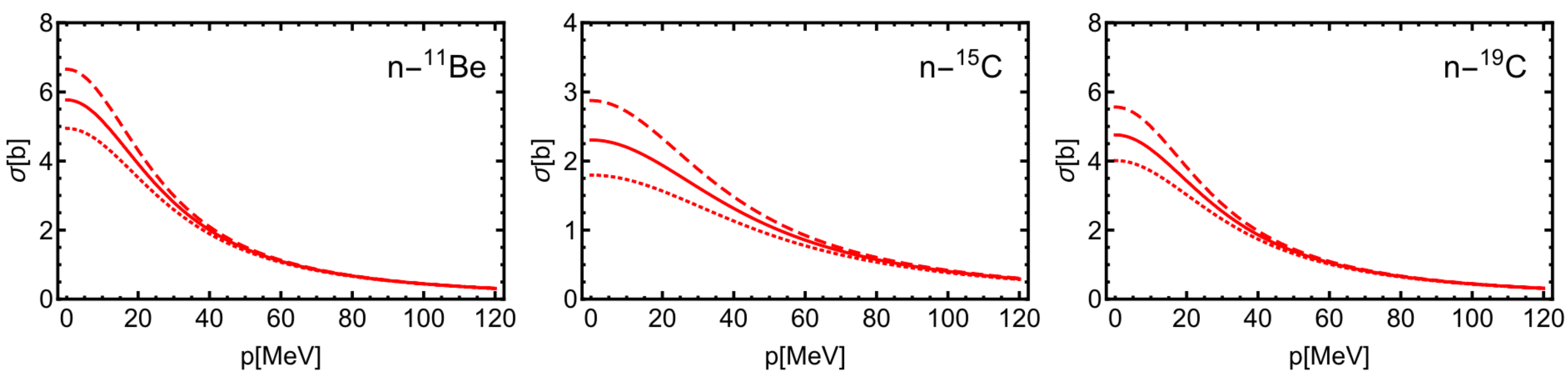
- s-wave cross sections



dashed/dotted: $a_\sigma \pm 0.5$ fm

nh scattering with total spin $J = 1$

- Inputs at LO: separation energy between the core and the valence n

Halo nuclei	^{11}Be	^{15}C	^{19}C
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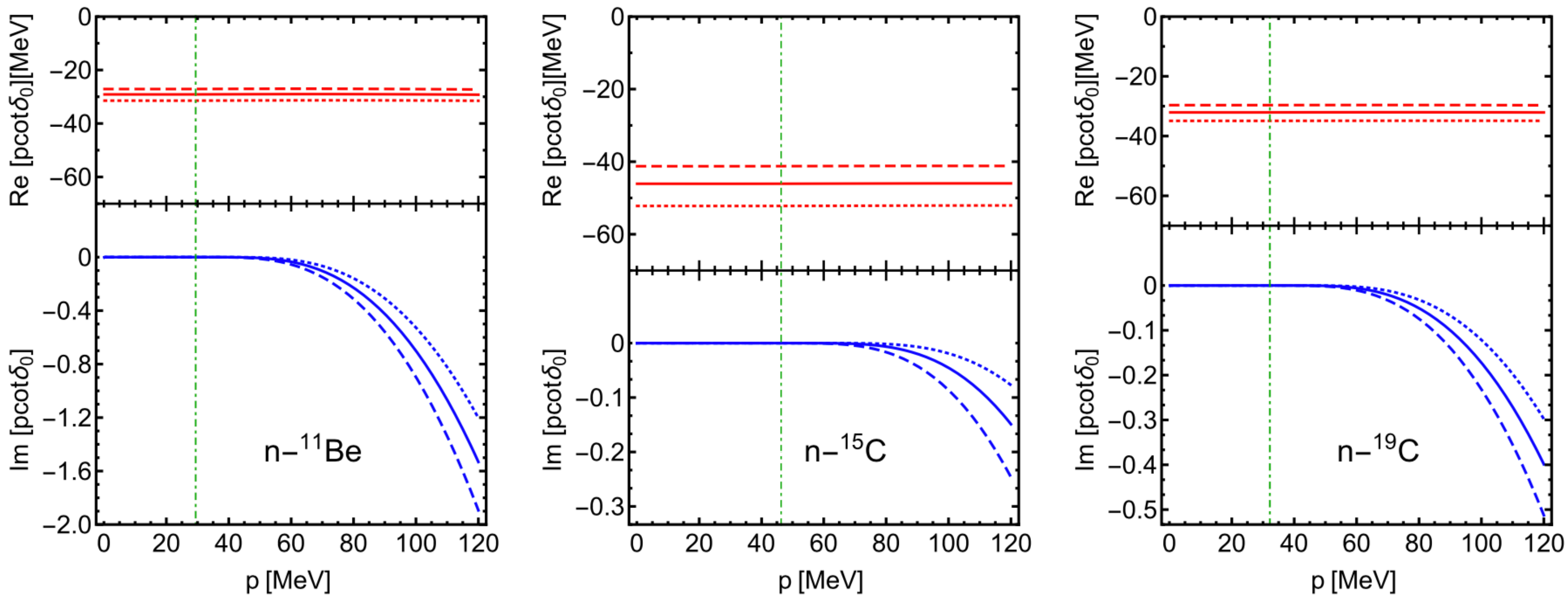
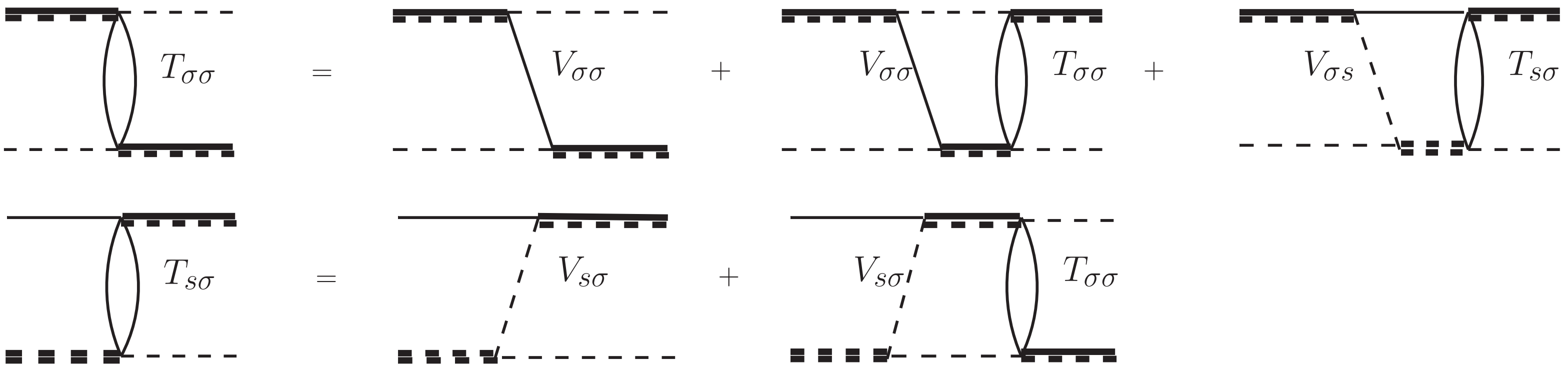
- The $p \cot \delta_0^R(p)$



dashed/dotted: $a_\sigma \pm 0.5$ fm

nh scattering with total spin $J = 0$

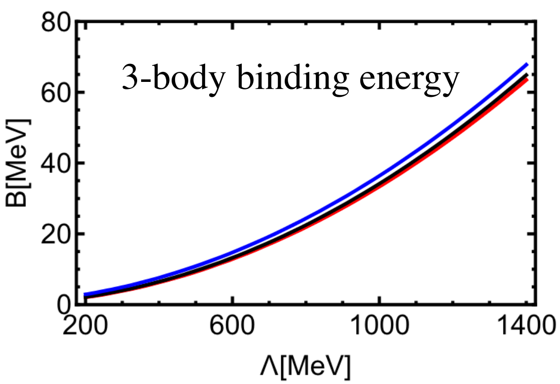
- Faddeev equation for $J = 0$

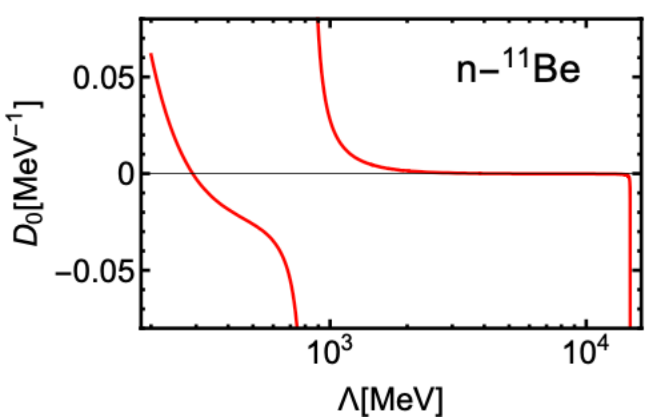


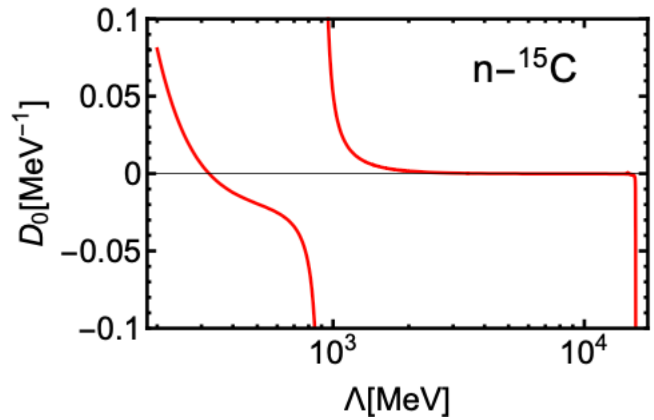
$$\begin{pmatrix} iT^{l}_{\sigma\sigma}(k,p,E) \\ iT^{l}_{s\sigma}(k,p,E) \end{pmatrix} = \begin{pmatrix} -iV^{l}_{\sigma\sigma}(k,p,E) \\ i2V^{l}_{s\sigma}(k,p,E) \end{pmatrix}$$

$$-\int_0^{\Lambda} \frac{q^2 dq}{2\pi^2} \begin{pmatrix} -V^{l}_{\sigma\sigma}(k,q,E) & 2V^{l}_{\sigma s}(k,q,E) \\ 2V^{l}_{s\sigma}(k,q,E) & 0 \end{pmatrix} \begin{pmatrix} Z_{\sigma}^{-1} D_{\sigma}(E - \frac{q^2}{2m_n}, q) & 0 \\ 0 & Z_s^{-1} D_s(E - \frac{q^2}{2m_c}, q) \end{pmatrix} \begin{pmatrix} iT^{l}_{\sigma\sigma}(q,p,E) \\ iT^{l}_{s\sigma}(q,p,E) \end{pmatrix}$$

- Cutoff dependence => renormalization, absorbed by the 3-body contact term $D_0(\Lambda)$





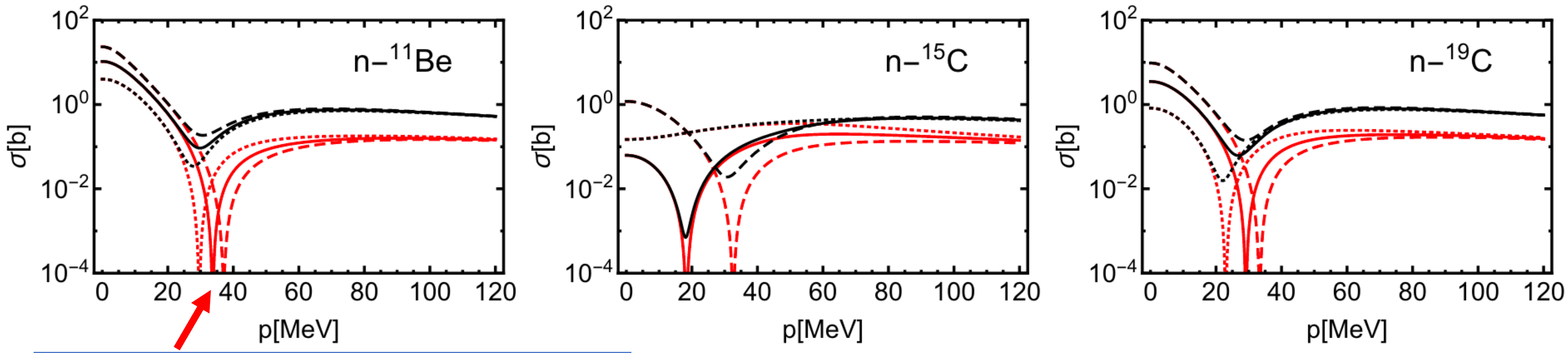


nh scattering with total spin $J = 0$

- Inputs: a_s (nn sca. length), B_σ (one neutron halo), B_{2n} (2-neutron separation energy)

Nuclei	^{12}Be	^{16}C	^{20}C
B_{2n}(MeV)	3.672	5.468	3.560

- Total cross sections [red: s-wave, black: $l \le 4$]

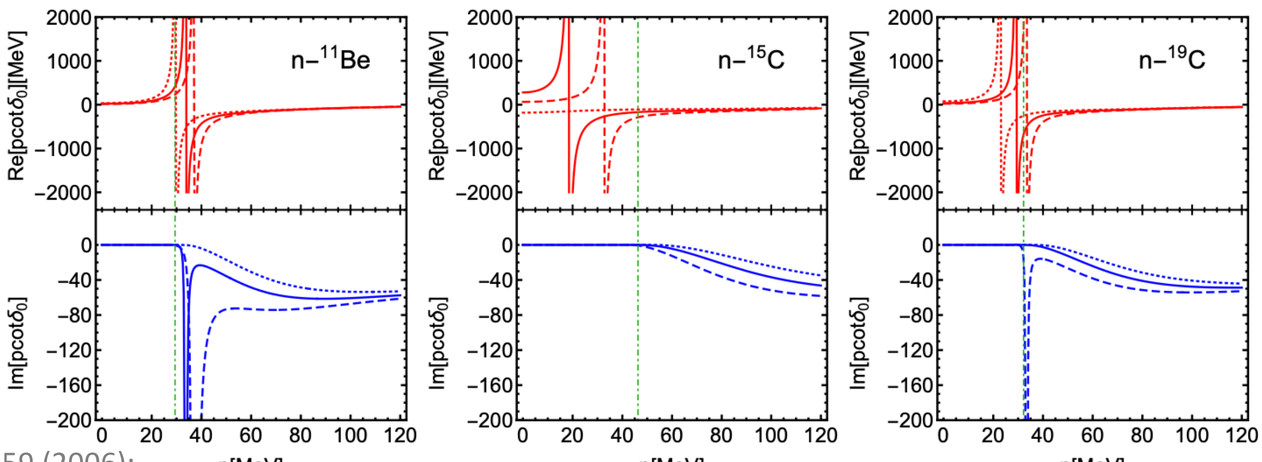


Zero in the s-wave cross section!

$$T^{l}_{\sigma\sigma}(p,p,E) = \frac{2\pi}{\mu_{n\sigma}} \frac{\eta e^{2i\delta_l^R(p)} - 1}{2ip}$$

pole in $p\cot\delta_0^R(p)$!

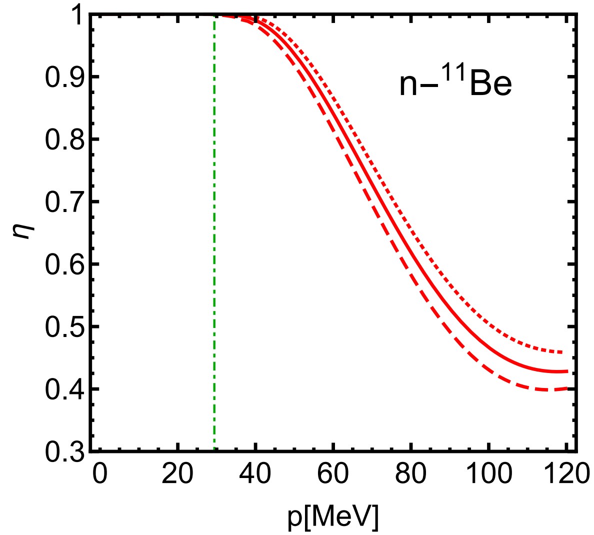
Efimov state

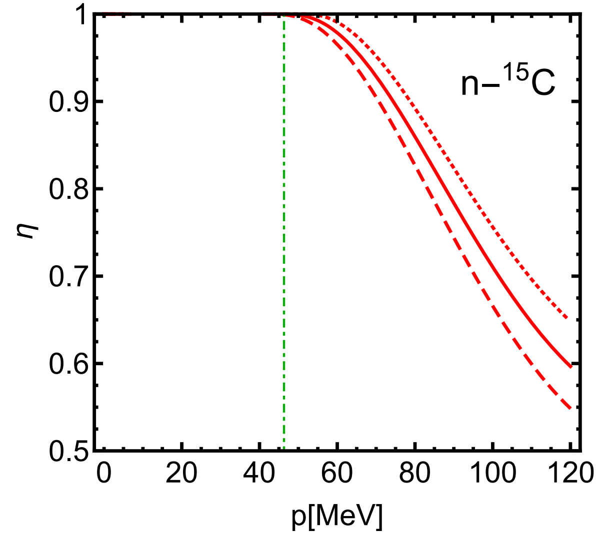


E. Braaten and H.-W. Hammer, Phys. Rep. 428, 259 (2006);..

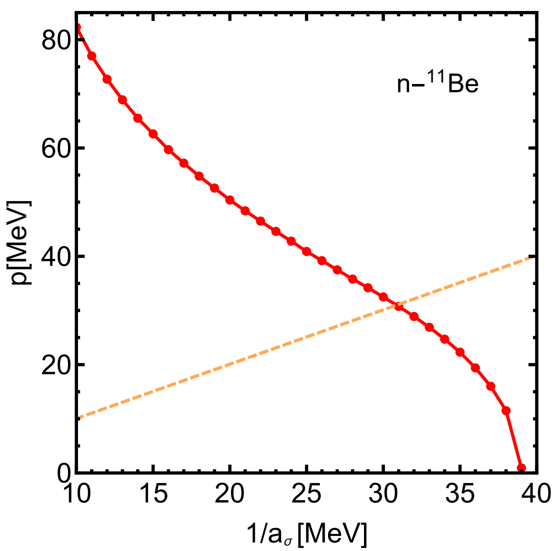
nh scattering with total spin $J = 0$

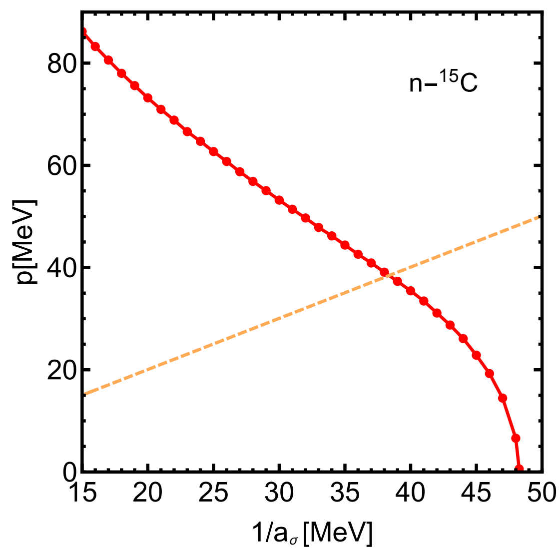
- inelasticity factor





- Location of the pole in $p\cot\delta(p)$





Zero of the scattering amplitude

- Found before for n-d, n-^{19}C scattering

W.T.H. van Oers, J.D. Seagrave, PLB 24 (1967) 562; …
M. T. Yamashita, T. Frederico, L. Tomio, PLB 670 (2008) 49; …

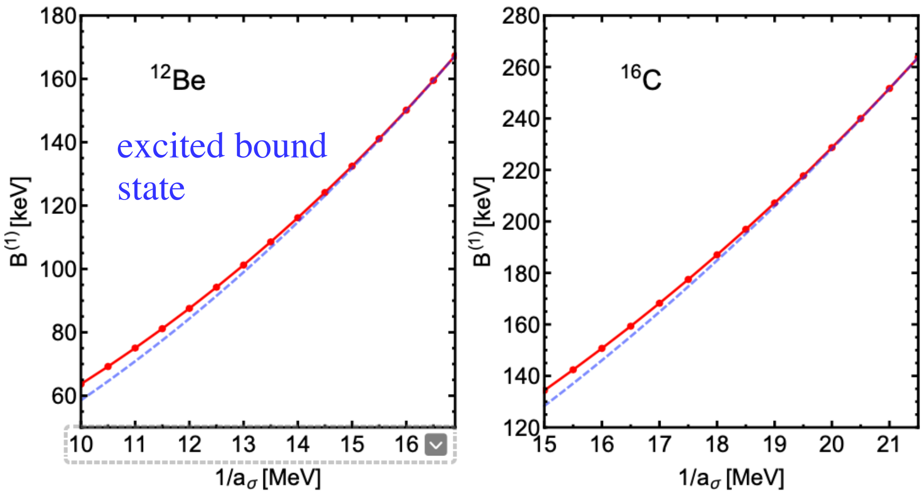
  - ➢ Leads to a modified effective range expansion

$$k \cot \delta_0 = -A + Bk^2 - \frac{C}{1 + Dk^2}$$

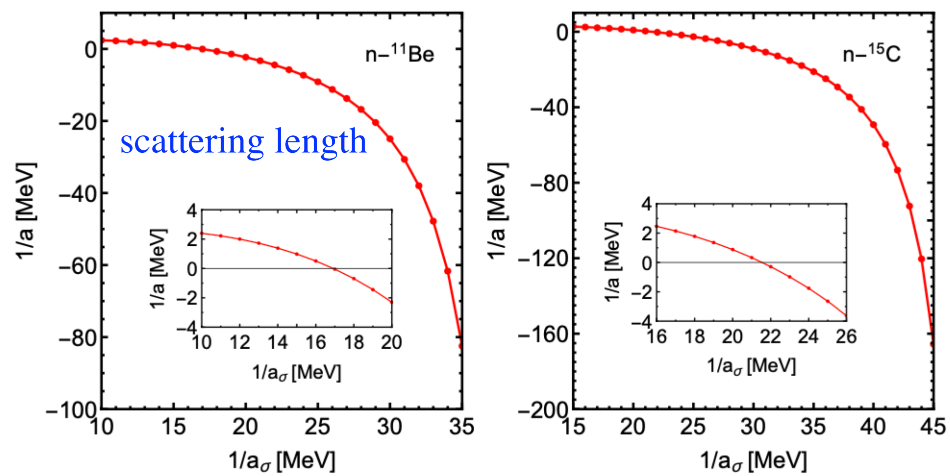
- Manifestation of a 3-body virtual state

B. A. Girard, M. G. Fuda, PRC 19 (1979) 579; …
E. Braaten and H.-W. Hammer, Phys. Rep. 428, 259 (2006);..

Increasing the nc scattering length a_σ
=> excited Efimov state



nh scattering length changes sign



Summary

- The cross section of neutron-halo scattering is around few barns
- Efimov states manifested in neutron-halo scattering as a virtual state for the halo being ^{11}Be, ^{15}C, ^{19}C
- Scattering of deuteron off neutron halo => realistic predictions for experiments

Thank you for your attention!