Resonances in the quark model

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Based on recent or past work with J.-P. Ader, P. Taxil, J. Vijande, A. Valcarce, Cafer Ay, Hyam Rubinstein, S. Zouzou, C. Gignoux, B. Silvestre-Brac, S. Fleck, M. Genovese, Fl. Stancu, J.-L. Ballot, E. Hernandez, E. Hiyama, M. Oka, A. Hosaka . . .

Introduction

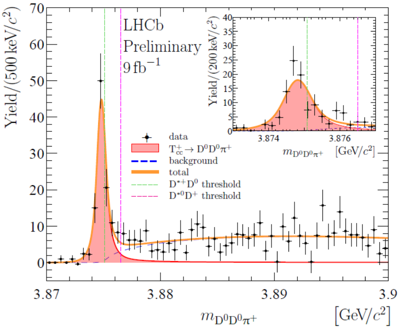
- Many candidates for exotic hadrons along the years
- Z-baryons in the 60s
- Baryonium in the 70s and 80s
- Θ^+ pentaquark, etc.
- not confirmed

More recently

- XYZ hidden-flavor tetraquarks
- Anticharmed pentaquarks
- Fully-charmed tetraquarks $cc\bar{c}\bar{c}$
- etc.

Introduction

T_{cc}



- Stable with respect to DD^*
- Slightly unstable due to the D^* width $T_{cc} \to DD\pi$

Introduction

- Quark model suited for bound states
- For instance, early prediction of $QQ\bar{q}\bar{q}$
- Is quark model also applicable for resonances?
- Outline
  - Brief review on QM bound states
  - Chromomagnetic vs. chromoelectic binding
  - Brief review on resonances in toy models or atomic physics
  - Application of methods to QM

Binding or not?

- Binding is rarely obvious
- Consider for instance

$$H = \frac{\boldsymbol{p}_1^2}{2\,m} + \cdots + \frac{\boldsymbol{p}_4^2}{2\,m} + \sum g_{ij}\, v(r_{ij})\,, \qquad G = \sum_{i<j} g_{ij} = 2 \quad v(r)\,\text{attractive}$$

$$\text{threshold} \qquad \{g_{ij}\} = \{0, 1, 0, 1, 0\} \qquad \sum g_{ij} = 2$$

- An example is Ps_2 with $v(r) = -1/r$ weakly bound
- But for $QQ\bar{Q}\bar{Q}$ with $v(r) = r$ and

$$\{g_{ij}\} = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right\}$$

  a tetraquark with color $\bar{3}3$, it is above the threshold.
- Way out to bind tetraquarks?
  - Better $\{g_{ij}\}$ (CM)
  - Unequal masses (CE)

Chromomagnetic binding

$$H = ssuudd \qquad \text{vs.} \qquad \Lambda(sud) + \Lambda(sud)$$

1. In the SU(3) limit, $g_{ij} \propto \tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\, v(r_{ij})$ (spin-color)
2. $$\sum_H g_{ij} = \frac{3}{2}\left[\sum_\Lambda g_{ij} + \sum_\Lambda g_{ij}\right]$$
3. If short-range corr. $\langle v(r_{ij})\rangle$ taken the same as for baryons
4. 150 MeV extrabinding for H vs. $\Lambda\Lambda$

But, if

- SU(3) breaking
- 6-body problem solved (with kin. ener. and spin-indep. terms)
- In particular $\langle v(r_{ij})\rangle$ calculated, not guessed

H unbound in potential models.

Similar scenario: $P = \bar{c}uuds$, $\bar{c}ddsu$, $\bar{c}ssud$. (Lipkin, Gignoux et al.)

Introduction

- *H* dibaryon with $S = -2$ actively searched in several experiments
- Never identified, including in double hypernuclei

Nuclear Physics A553 (1993) 667c–674c
North-Holland, Amsterdam

NUCLEAR PHYSICS A

Search for Double Strangeness Systems

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Abstract

Recently, the double strangeness systems, an H-dibaryon and double hypernuclei, have been searched for at various laboratories. Evidence of the weak decay of double hypernuclei was found by an emulsion-counter hybrid experiment and the lower limit of the H-dibaryon mass was provided as 2200 MeV/c^2. The results of this experiment are reported together with the current status of on-going experiments at KEK and BNL.

Chromoelectric binding

Unequal masses

$$H = \frac{\boldsymbol{p}_1^2}{2\, m_1} + \cdots + \frac{\boldsymbol{p}_4^2}{2\, m_4} + \sum_{i<j} g_{ij}\, v(r_{ij})\ .$$

- In the case of QED, it was observed that H_2 more deeply bound than Ps_2 (as compared to respective threshold energy)
- Due to favorable *C*-symmetry breaking (Adamowski et al., Richard et al.)
- Other symmetry breakings starting from Ps_2 do not work

Breaking charge conjugation

- $(M^+M^+m^-m^-)$ vs. $(\mu^+\mu^+\mu^-\mu^-)$ with $2\,\mu^{-1} = M^{-1} + m^{-1}$
- Same threshold
- The decomposition $H = H_{\text{even}} + H_{\text{odd}}$

$$\frac{\boldsymbol{p}_1^2}{2M} + \frac{\boldsymbol{p}_2^2}{2M} + \frac{\boldsymbol{p}_3^2}{2m} + \frac{\boldsymbol{p}_4^2}{2m} + V = \left[\sum \frac{\boldsymbol{p}_i^2}{2\mu} + V\right] + \left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]$$

- Implies $E(H) < E(H_{\text{even}})$
- This explains why H_2 is more stable than Ps_2.
- Same reasoning holds for $QQ\bar{q}\bar{q}$ in a central interaction:
  - It starts unstable for $M = m$
  - It becomes stable if M/m large enough

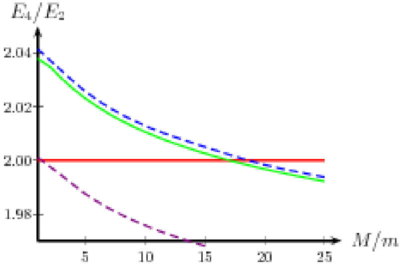
Application to $QQ\bar{q}\bar{q}$

- In practice, the critical M/m at which purely chromoelectric binding occurs is rather elusive.
- But $QQ\bar{u}\bar{d}$ benefits from the help of a favorable CM interaction in the light sector.
- Transition $\rightarrow$ stability near $Q = c$, as observed exp.

Remarks about bound states in QM

- Diquark approximation dangerous, as antivariational

$$V(3-a)+V(3-b) > 2\,V(3-\tfrac{a+b}{2})$$



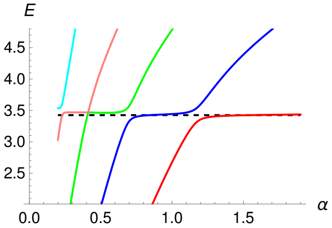
- Color mixing crucial
- Born-Oppenheimer fine
- $V_{\rm BO}(QQ\bar{q}\bar{q})$ and $V_{\rm BO}(QQq)$ similar

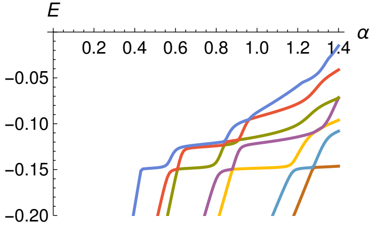
Resonances: Introduction

- All candidates, X, Y, Z, pentaquarks, are resonances
- Recently in the all-heavy sector: $J/\psi\, J/\psi$ resonances at LHC
- Challenge: estimate masses and widths in the quark model
- Strategy: learn from toy models and try to apply to quark physics
- In particular:
  - Real scaling
  - Complex scaling
  - External absorption

Real scaling (stabilization)

- Initially $\psi = \sum_{i=1}^{N} \gamma_i\, \varphi(\underline{\alpha}; \underline{r})$ as a function of N.
- More often, $\Psi(\underline{\alpha}; \underline{r}) \to \Psi(\underline{\alpha}; \lambda\, \underline{r})$ as a function of λ
- Left: Bardsley potential $V(r) = 7.5\, r^2 \exp(-r)$, $m = 1$
- Right: H^- resonance

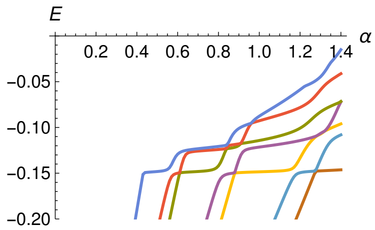


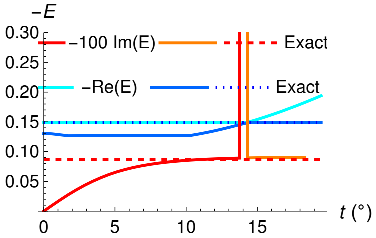


Complex scaling, variational-like variant

$$r \to r\exp(i\,t) \quad \Leftrightarrow \quad a_i \to a_i\,\exp(-n\,i\,t)$$

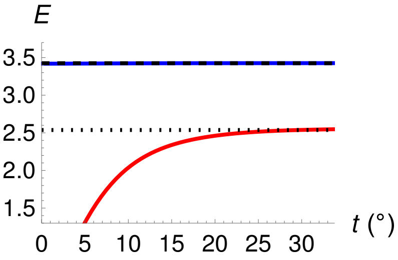
in a exp. ($n=1$) or G. ($n=2$) expansion $\sum_i \gamma_i\,\exp(-a_i x^n - \cdots)$
Example: H^-
Basis = a few exponentials (that fit the ground state)





Surprisingly efficient, even with a small basis

Complex scaling, Bardsley potential



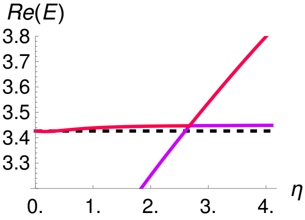
real part, −200 imaginary part

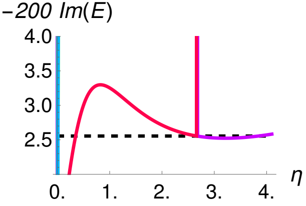
Note: real + complex scaling $r \to r\,\lambda\,\exp(i\,t)$
and search for stationary energies by varying λ and t.

Complex absorption

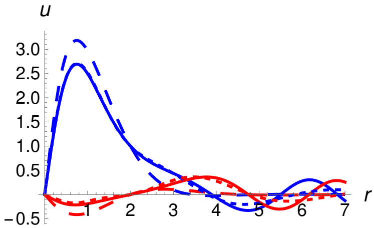
- Supplement
$$H \quad \to \quad H' = H - i\,\eta\,W$$
- For instance
$$W = \sum_{i<j} (r_{ij} - r_0)^2 \underbrace{\Theta(r_{ij} - r_0)}_{\text{Heaviside}}$$
- Limit $\eta \to 0$ if one uses a large basis otherwise stationary energies when η is varied

Complex absorption: example



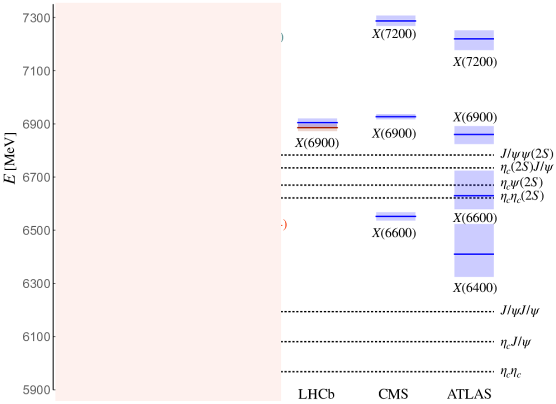


Comparison of w.f.



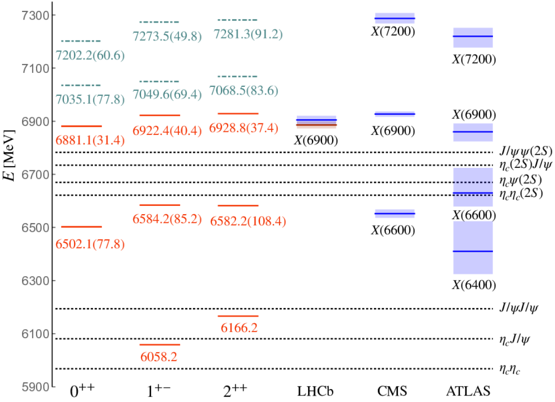
Blue= Re Red=Im, solid=Exact, Dashed= Complex scaling, Dotted= Complex absorption

Resonances: example ($J/\psi\, J/\psi$)



From Wang et al.

Resonances: example ($J/\psi\, J/\psi$)



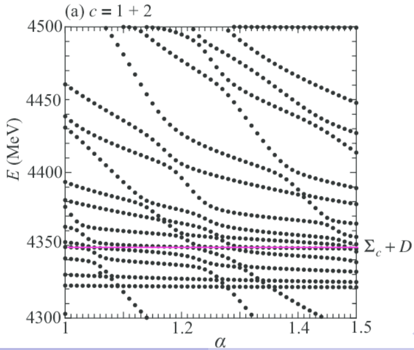
From Wang et al.

Quark model resonances: Introduction

- A very long way!
- No simple ansatz to get an approximate estimate (unlike the case of bound states)
- One always get energies above the ground state
- The problem is to separate genuine resonances from states paving the continuum.
- Up to now, big computations
- Attempts to elaborate laptop-size methods not yet successful

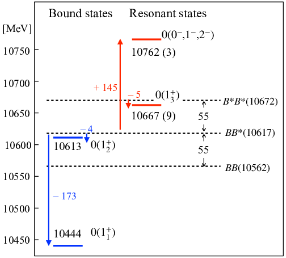
Quark model resonances: Real scaling

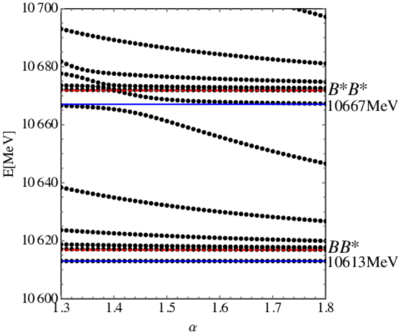
- Hiyama et al. applied real scaling to various pentaquark configurations
- Further studies by Japanese and/or Chinese teams
- Now considered as a seminal work
- Demonstrate the possibility of separating resonances from the discrete states mimicking the continuum
- Not very accurate for the widths (Simons formula)



Quark model resonances: Real scaling-2

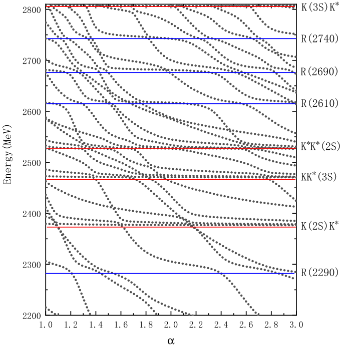
- Also applied to $bb\bar{u}\bar{d}$
- Qi Meng et al., Phys.Lett.B 824 (2022) 136800

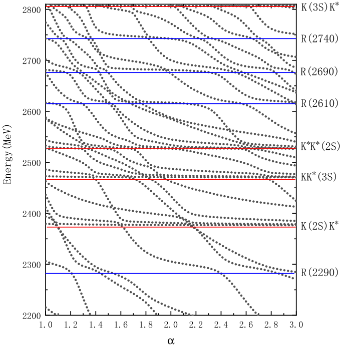




Quarl model resonances: Real scaling-3

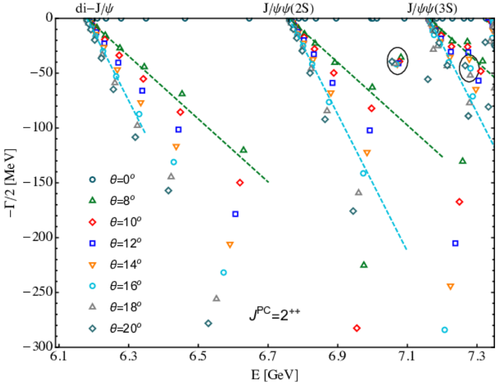
Very recent example. T_{ss} in a chiral quark model
Jiazheng Ji, Yuheng Xing, Xinxing Wu, Ning Xu, and Yue Tan





Complex scaling applied to the quark model

- Wang, Meng and Oka, for instance, used a standard quark model combined to Gaussian expansion, to study $cc\bar{c}\bar{c}$.
- Interesting spectrum of resonances
- For instance, for spin $S = 2$, two states near $7 - i\,0.04\,\mathrm{GeV}$



Complex scaling applied to the quark model-2

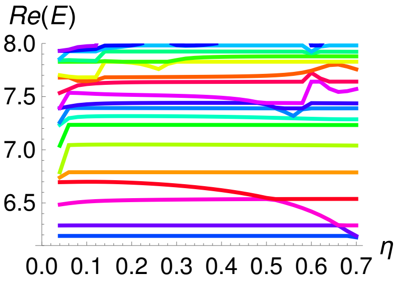
- Such calculations are technically delicate
- In some papers, crucial details are omitted
- For instance, if $\psi = \sum_{i=1}^{N} \gamma_i \exp(-a_i x^2 - 2b_i x.y - \cdots)$
- Real scaling $(a_i, b_i,) \ldots \to \lambda (a_i, b_i, \ldots)$
- Complex scaling $(a_i, b_i,) \ldots \to \exp(-2 i \theta) (a_i, b_i, \ldots)$
- Are well explained, but the starting point is elusive
- If N is large, the eigenvalue equation

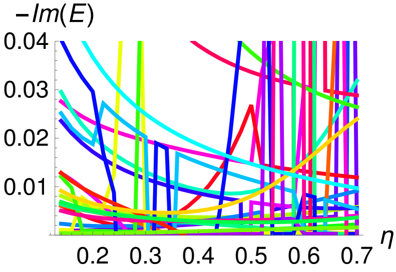
$$\mathbb{H} \{\gamma_i\} = E \, \mathbb{N} \{\gamma_i\}$$

  becomes singular!
- And requires a careful handling. Similar pb. in large shell-model calculations in NP

Complex absorption applied to quark model

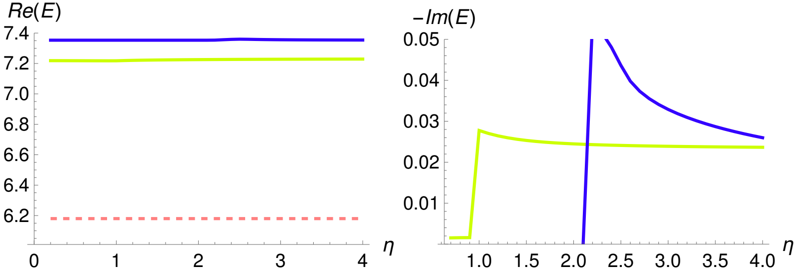
- Same model as in the previous slide
- $-i\,\eta\,(r_{ij}-r_0)^2\,\Theta(r_{ij}-r_0)$ added to each pair
- Spectrum as a function of η and r_0
- At first glance, slightly confusing





Complex absorption applied to quark model

- After some adjustments
- And some filtering
- One confirms the results of the Sino-Japanese collaboration



Outlook-1

- The calculation of multiquark resonances is rather delicate
- Especially the width
- Already significant progress done, and published using
  - Real scaling
  - Complex scaling (variational-like version)
- The method of imaginary external absorption seems promising
- A strategy for starting values of the range parameters in the wf is needed
- Not satisfactory to start with thousands of states, of which only a few dozens are effective

Outlook-2

- Binding? Strong competition between
  - Collective 4-body
  - $2 + 2$ threshold
- Interesting analogies between
  - 4 unit charges in atomic physics
  - tetraquarks in the quark model
- Better understanding of symmetry breaking in quark models
- This is a new effect (chromo-electric) atop the more advertised chromomagnetic effect of Jaffe, ...
- T_{bb} ... new chapter of weak interaction
- Hopefully T_{bc} and T_{bb} actively searched for at LHC (LHCb, ..., ALICE) and elsewhere.
- If properly treated, the quark model offers a comprehensive picture including compact states and hadron-hadron resonances

The End