Heteronuclear Efimov universality with positive intraspecies scattering length

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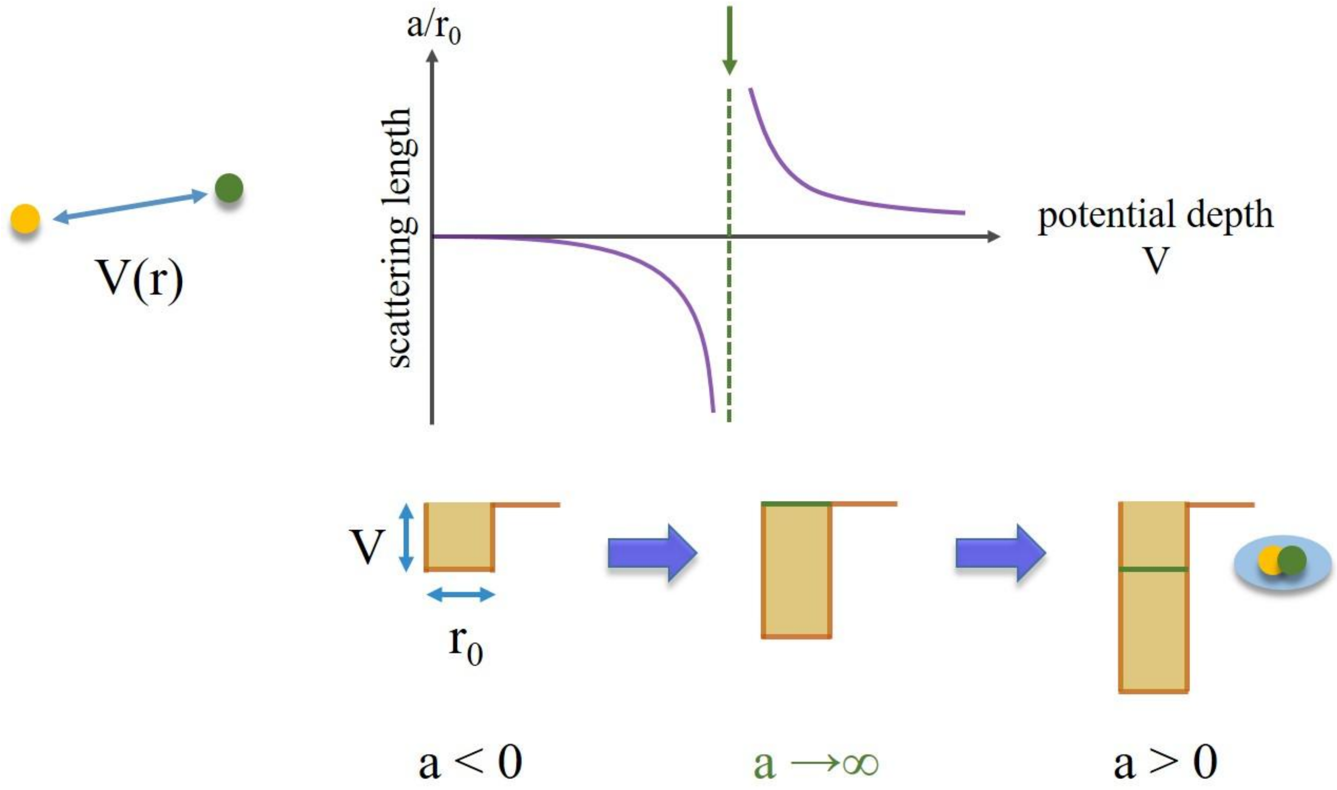
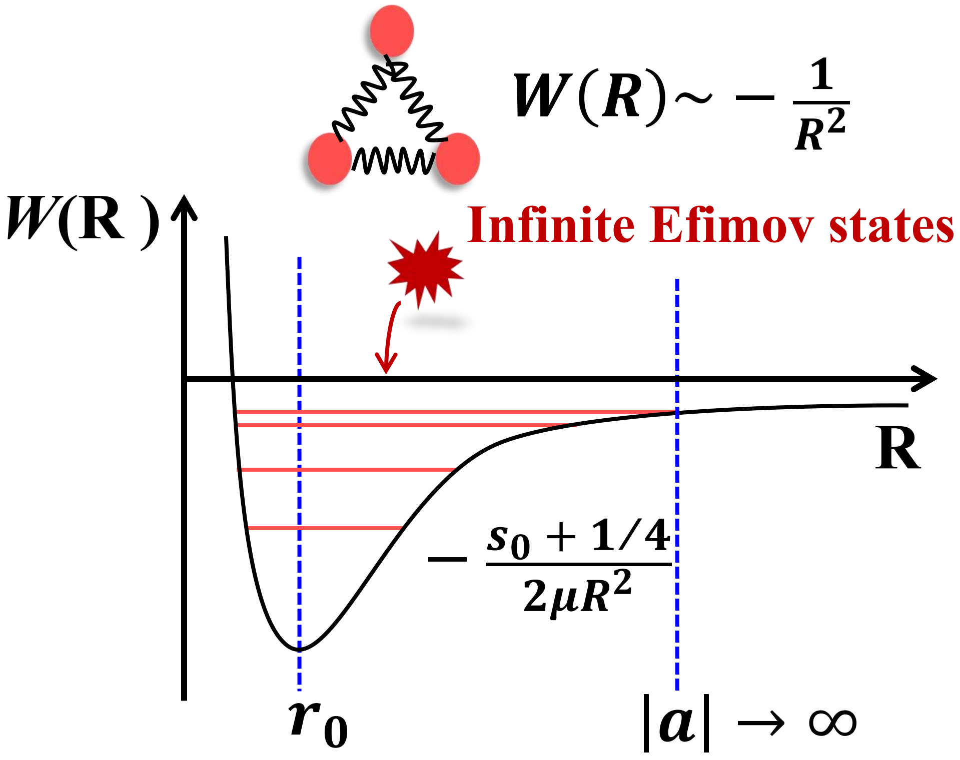
Outline

- Efimov physics
- Efimov features in three-body collision
- Theoretical investigation of three-body collision using R-matrix propagate method in the hyperspherical coordinates： ^{87}Rb-^{87}Rb-^{40}K, ^{133}Cs-^{133}Cs-^{6}Li
- Universality of Efimov features in heteronuclear systems
- Summary

What's the Efimov effects?

☐ Condition for Efimov effects: $a \gg r_0$



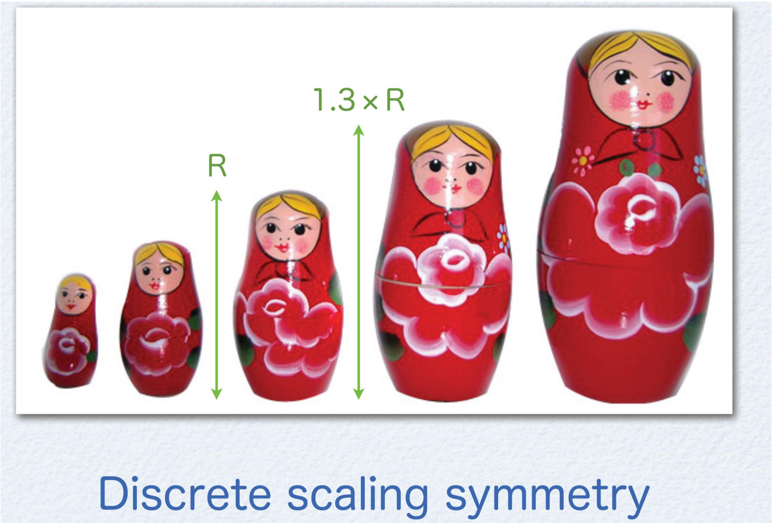


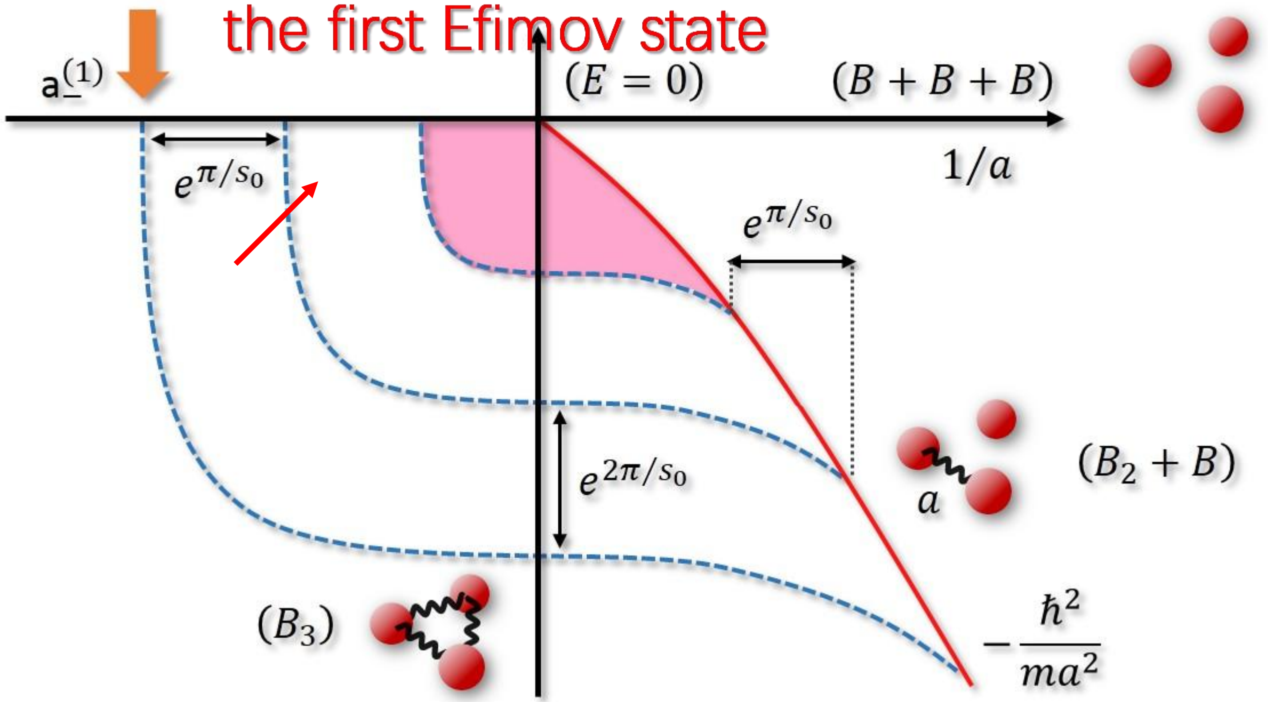
Three-body effective potential

[1] V. Efimov, Physics Letters B 33, 563 (1970).
[2] P. Naidon and S. Endo, Reports on Progress in physics 80, 056001 (2017).

Universality of Efimov effects

Discrete scaling symmetry





scaling parameter $\lambda = e^{\pi/s_0}$

$$\mathbf{E^{(N)}} = \lambda^2 \mathbf{E^{(N-1)}}$$

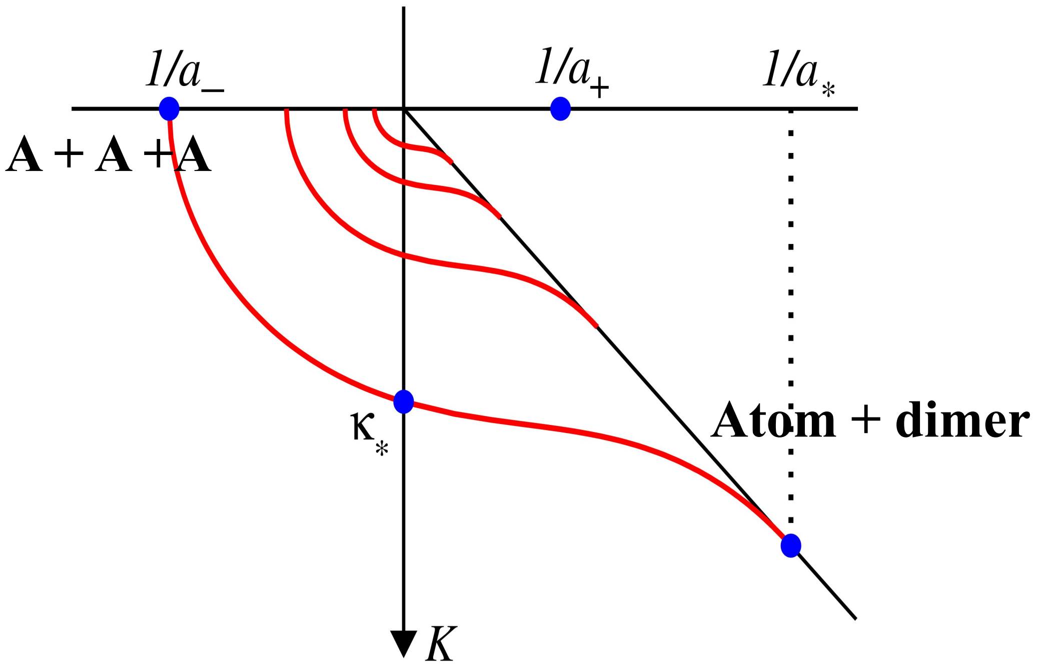
$$\mathbf{a_-^{(N)}} = \lambda \mathbf{a_-^{(N-1)}}$$

$s_0$determined by the mass ratio and resonant condition of the system; For three identical particles, $s_0 = 1.00624$, $\lambda = 22.7$(very large scaling factor, the factor of Russian doll is 1.3)

[1]P. Naidon and S. Endo, Rep. Prog. Phys. 80, 056001 (2017).

Signature of Efimov features

- a_- : Efimov states approach the three-body threshold
- a_+ :minima in the three-body recombination rate caused interference effects
- a_*: Efimov trimer state degenerates with the atom-molecule threshold



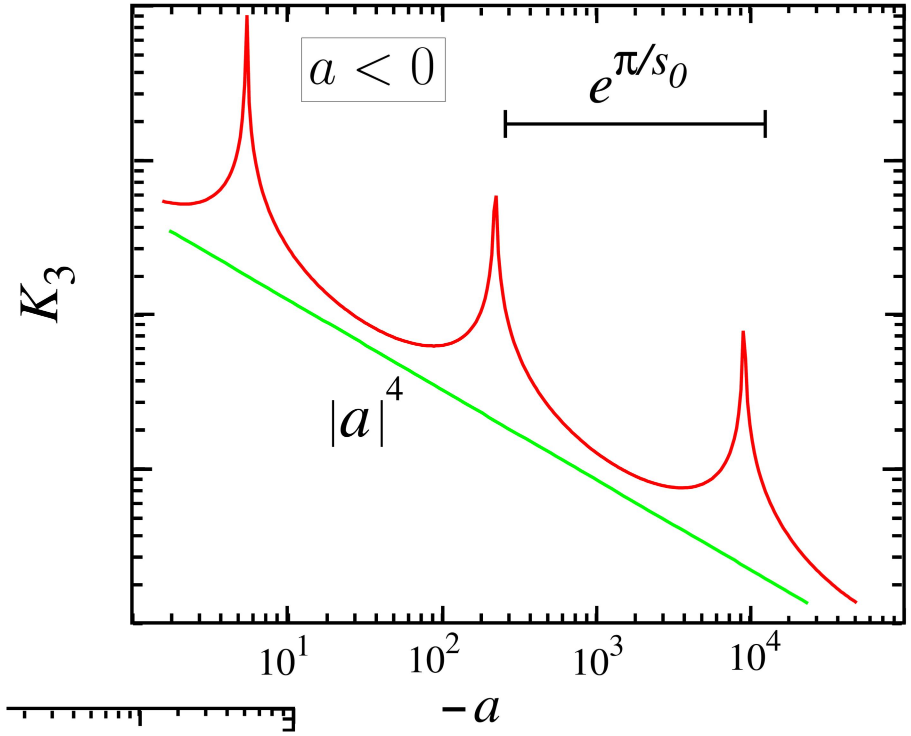
Schematic showing the location of Efimov features.

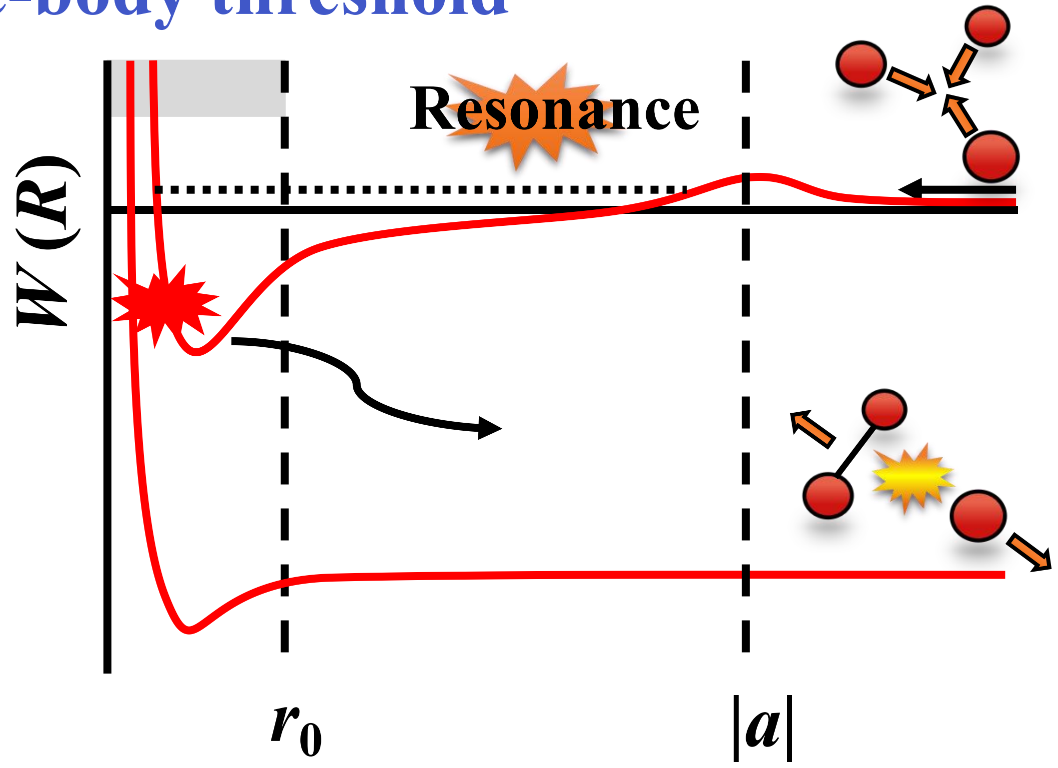
[1] V. Efimov, Physics Letters B 33, 563 (1970).
[2] P. Naidon and S. Endo, Reports on Progress in physics 80, 056001 (2017).

Efimov features in three-body collision ($a < 0$)

- **Three-body recombination in a<0 : B + B +B → B_2 + B**

  a_- : Efimov states approach the three-body threshold



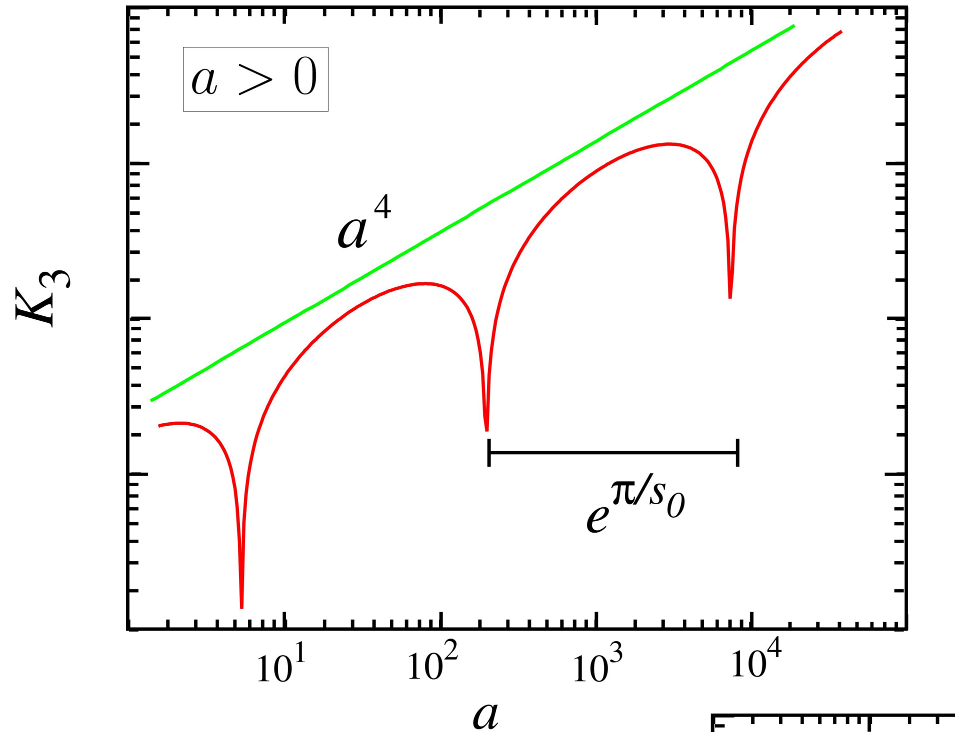


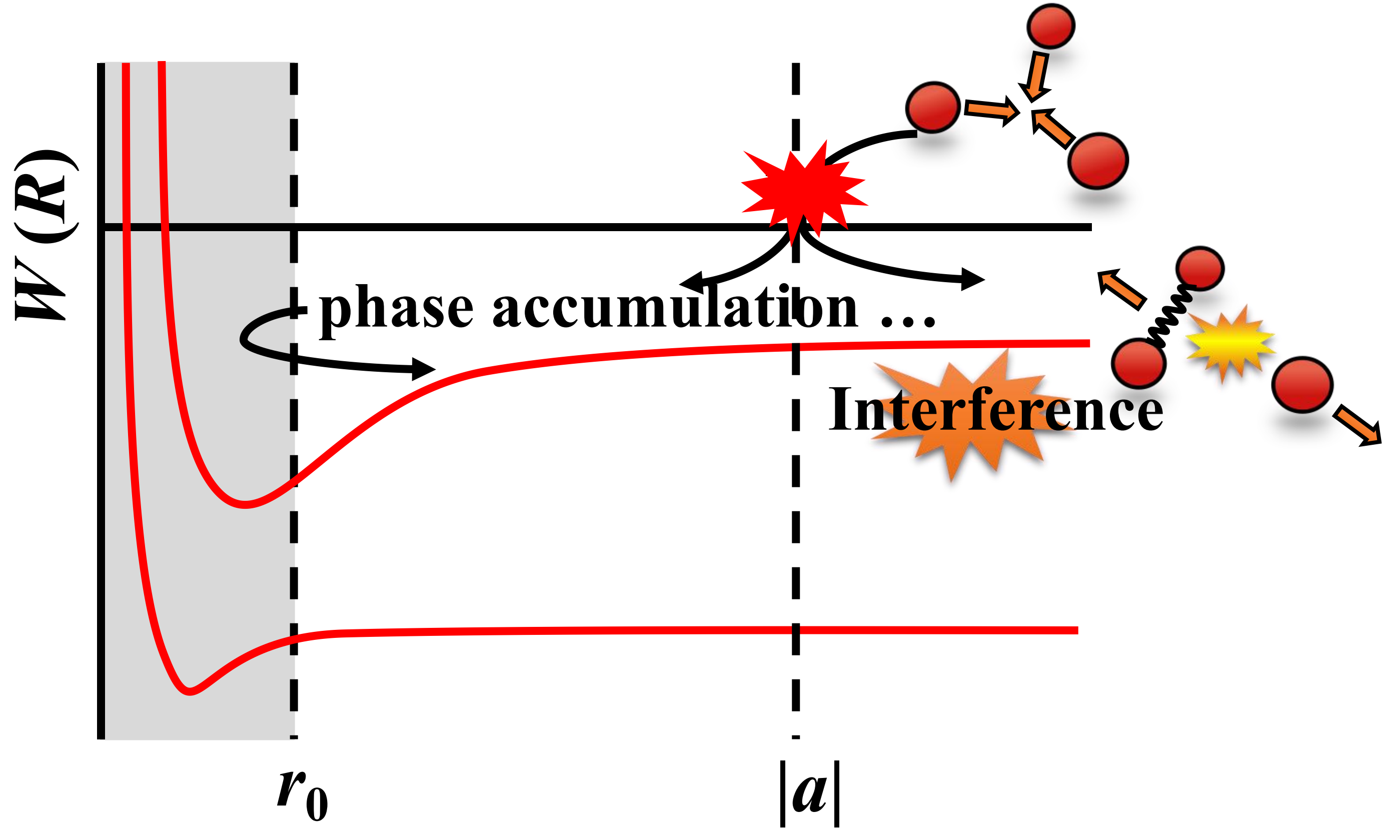
[1] V. Efimov, Physics Letters B 33, 563 (1970).
[2] P. Naidon and S. Endo, Reports on Progress in physics 80, 056001 (2017).

Efimov features in three-body collision ($a > 0$)

- **Three-body recombination in $a>0$: B + B +B → B_2 + B recombination into weakly bound molecules.**

  a_+ : minima in the three-body rate coefficient.

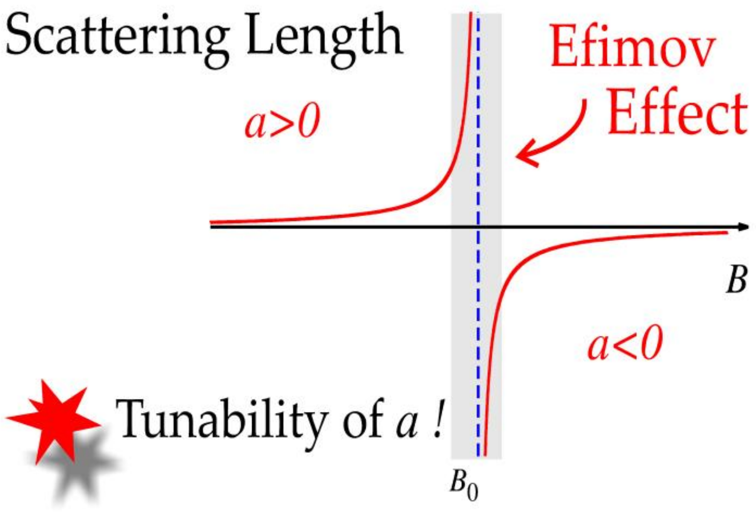


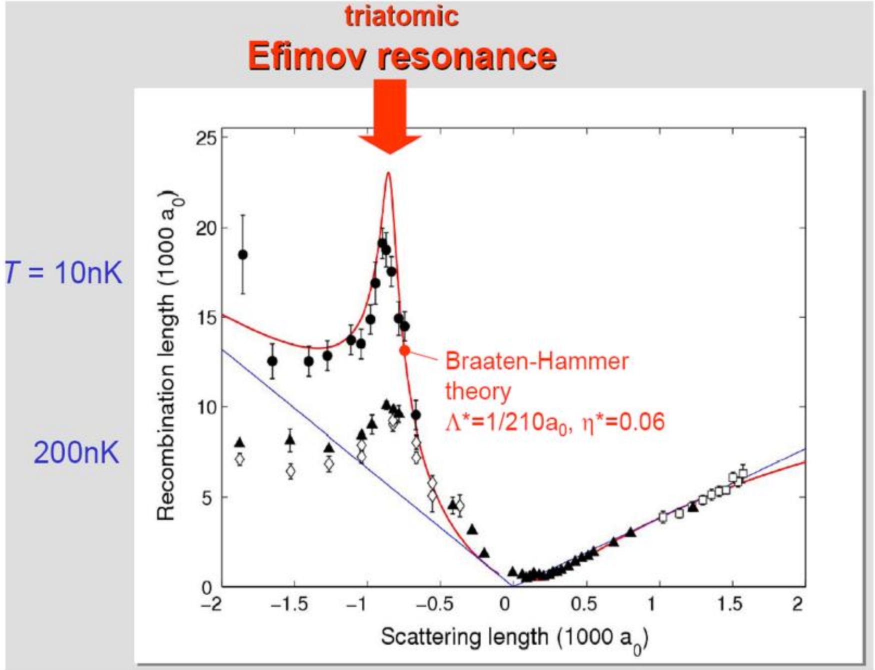


[1] V. Efimov, Physics Letters B 33, 563 (1970).
[2] P. Naidon and S. Endo, Reports on Progress in physics 80, 056001 (2017).

Observe efimov effects in ultracold quantum gas

- ultracold atomic gas + feshbach resonance = ideal test bed for Efimov physics





[1]T. Kraemer et al, Nature 440, 315-318 (2006) [2]Bo Huang (黄博),and Rudolf Grimm,PRL 112, 190401 (2014)

Outline

- Efimov physics
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- **Theoretical investigation of three-body collision using R-matrix propagate method in the hyperspherical coordinates： ^{87}Rb-^{87}Rb-^{40}K, ^{133}Cs-^{133}Cs-^{6}Li**
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Three-body collision in heavy-heavy-light system

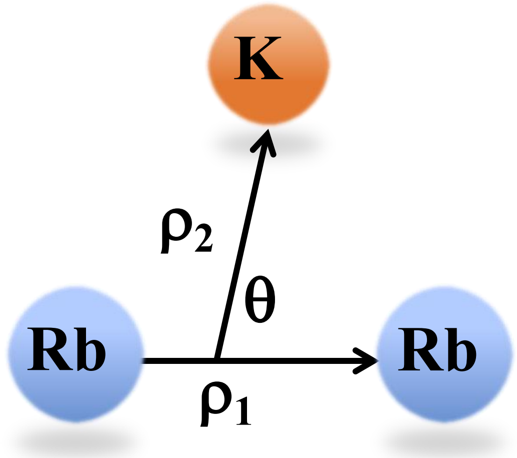
□ Theoretical method: R-matrix propagation in hyperspherical coordinates

- **Two-body model potential:** $v(r_{ij}) = -\frac{C_{6,ij}}{r_{ij}^6}\left[1 - \frac{1}{2}\left(\frac{\lambda_{ij}}{r_{ij}}\right)^6\right]$
- **Three-body interaction potential:**

$$V(R;\theta,\phi) = v(r_{12}) + v(r_{13}) + v(r_{23})$$

- **The Schrodinger equation is:**

$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2} + \left(\frac{\Lambda^2 - \frac{1}{4}}{2\mu R^2} + V(R;\theta,\phi)\right)\right]\psi_{v'}(R;\Omega) = E\psi_{v'}(R;\Omega)$$



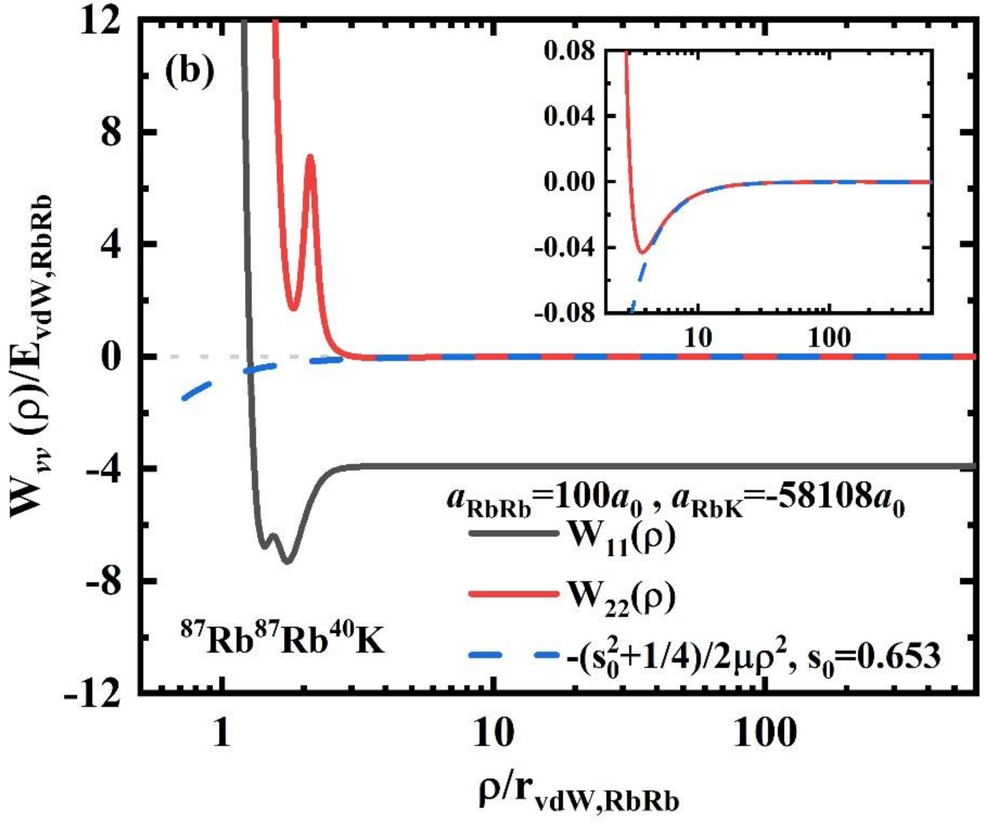
- **R-matrix propagate**

$$\underline{\mathcal{R}}(R) = \underline{F}(R)\left[\underline{\widetilde{F}}(R)\right]^{-1} \quad \underline{\mathcal{R}}(a_2) = \underline{\mathcal{R}}_{22} - \underline{\mathcal{R}}_{21}\left[\underline{\mathcal{R}}_{11} + \underline{\mathcal{R}}(a_1)\right]^{-1}\underline{\mathcal{R}}_{12}.$$

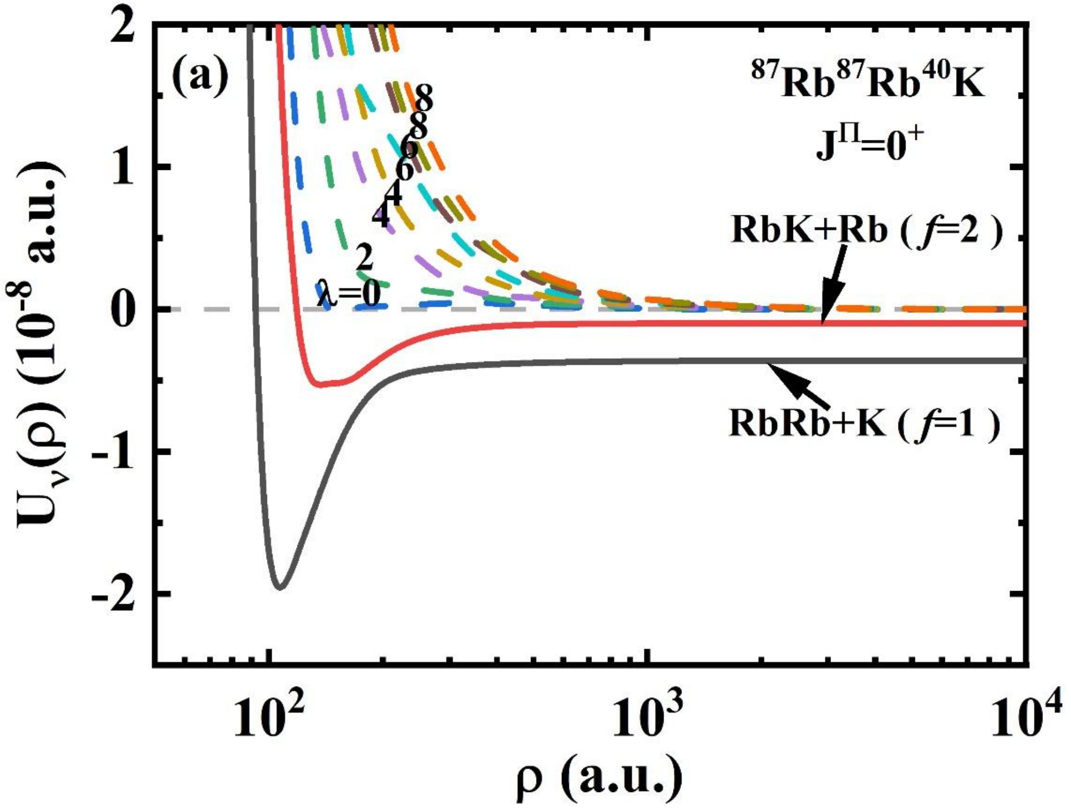
Three-body collision in heavy-heavy-light system

- Hypersperical potential curves for positive intraspecify scattering length: $a_{HH} > 0$

$a_{HL} < 0$



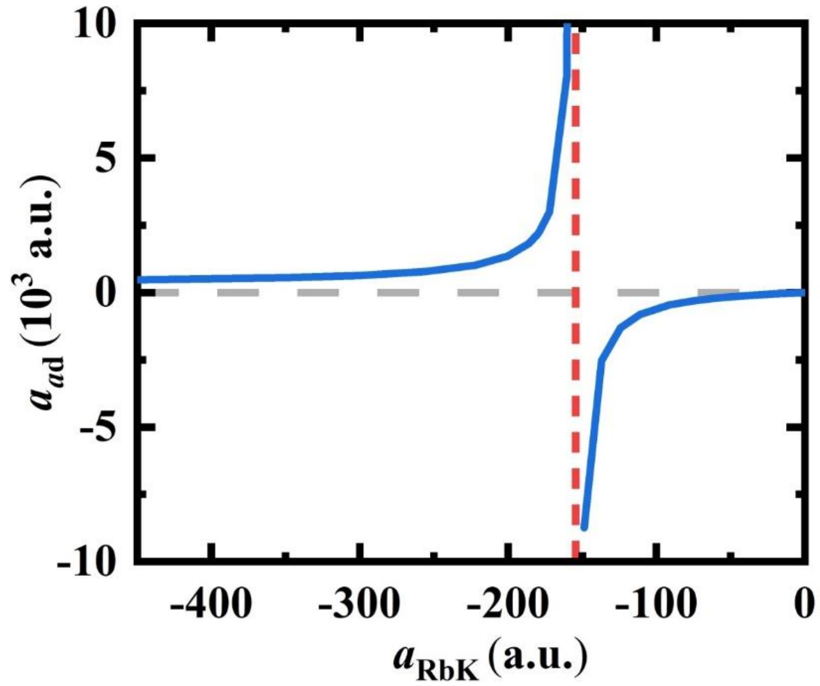
$a_{HL} > 0$

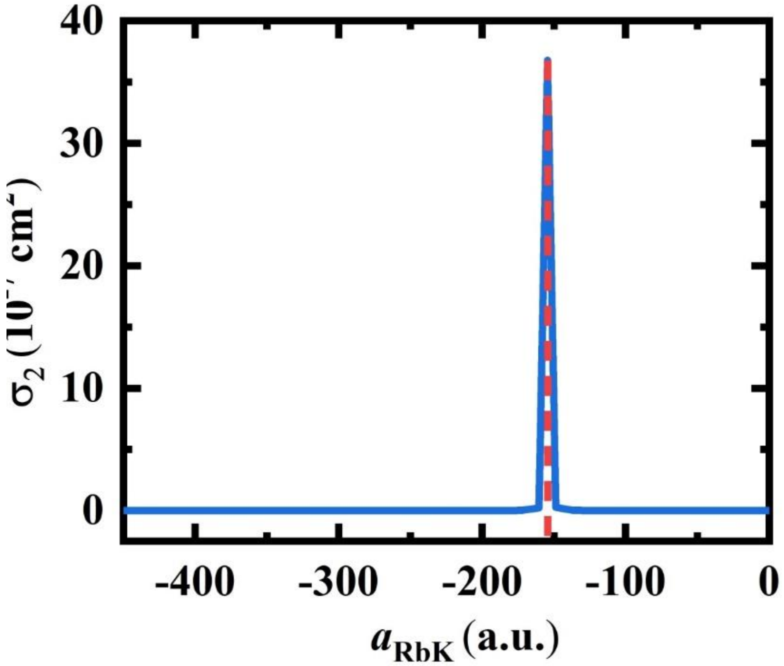


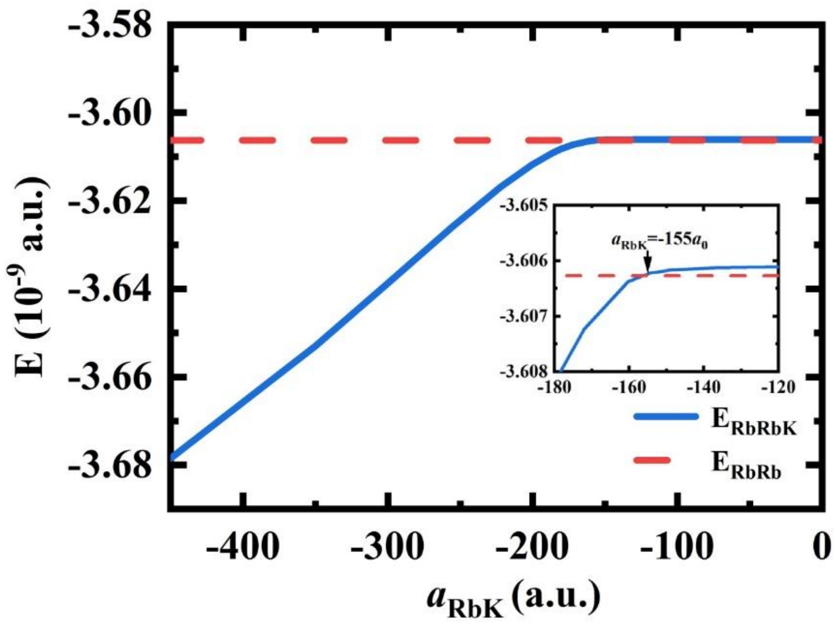
Three-body collision in Rb-Rb-K system

- **On the negative side of interspecifies scattering length: $a_{\mathrm{RbK}} < 0$**

One normal three-body bound state is found at $a_{\mathrm{RbK}} = -155\ a_0$





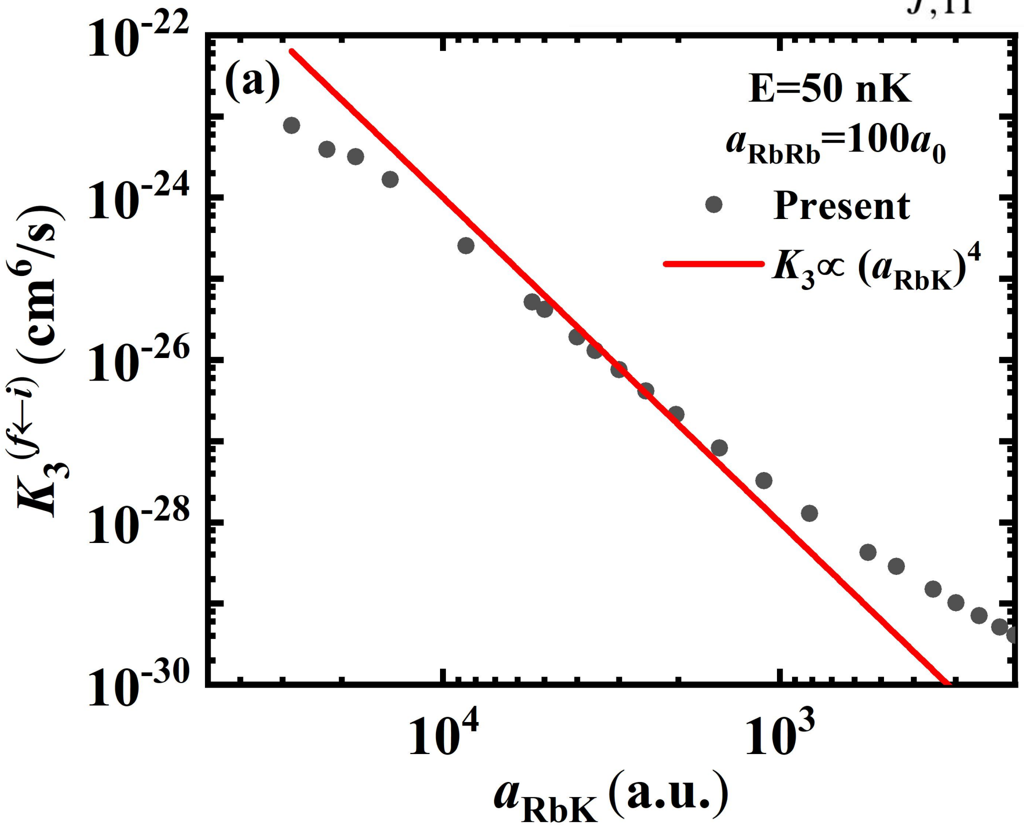


a_{ad} is the ^{40}K - ^{87}Rb^{87}Rb scattering length σ_2 is the elastic scattering cross section

$r_{\mathrm{vdw,RbRb}} = 83\ a_0$, $E_{\mathrm{vdw,RbRb}} = 9.25 \times 10^{-10}$a.u. $|a_{\mathrm{RbK}}| = 155\ a_0 < 2 \times r_{\mathrm{vdw,RbRb}}$, $E > E_{\mathrm{vdw,RbRb}}$

Three-body collision in Rb-Rb-K system

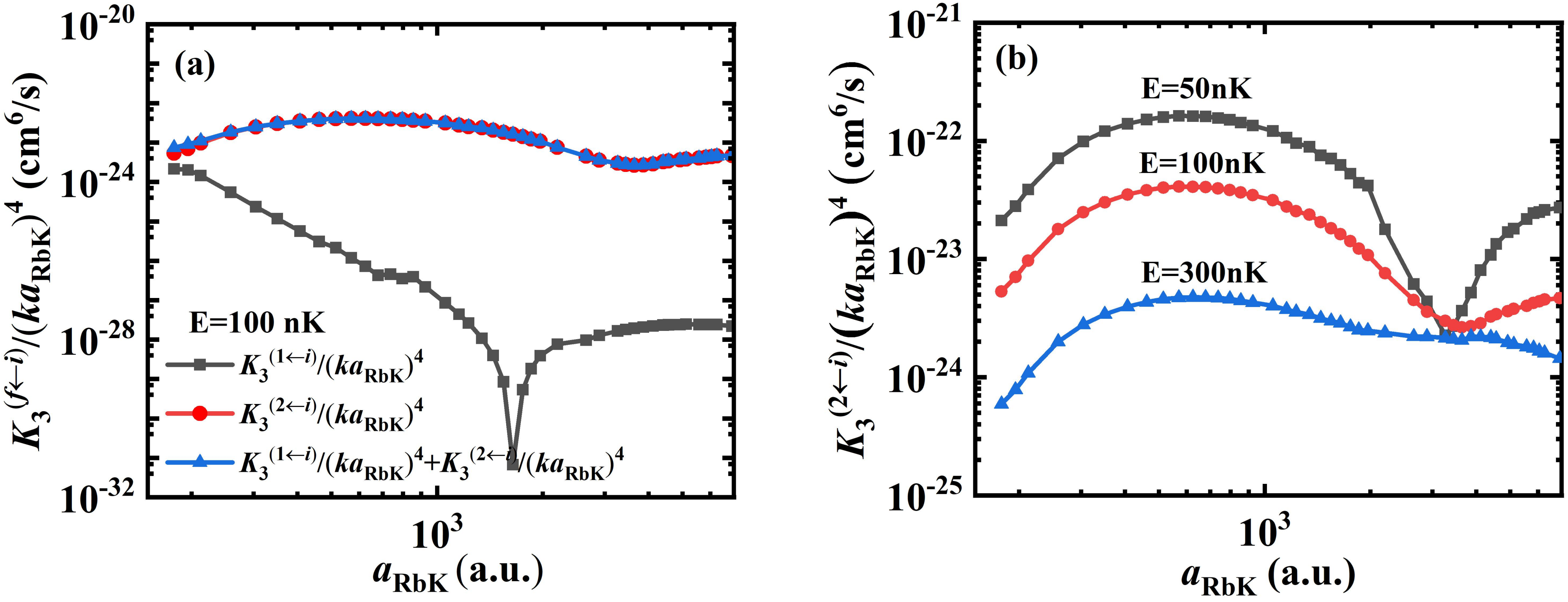
- **On the negative side of interspecifies scattering length: $a_{\mathrm{RbK}} < 0$**
  - **Three-body recombination rates K_3:** $K_3 = \frac{k}{\mu}\sigma_3 = \sum_{J,\Pi} K_3^{J,\Pi} = 2! \sum_{J,\Pi} \sum_{f,i} \frac{32(2J+1)\pi^2}{\mu k^4} \left|S_{i\to f}^{J,\Pi}\right|^2$



No Efimov features between -200 a_0 — 40000 a_0

Three-body collision in Rb-Rb-K system

- On the positive side of interspecifies scattering length: $a_{\mathrm{RbK}} > 0$

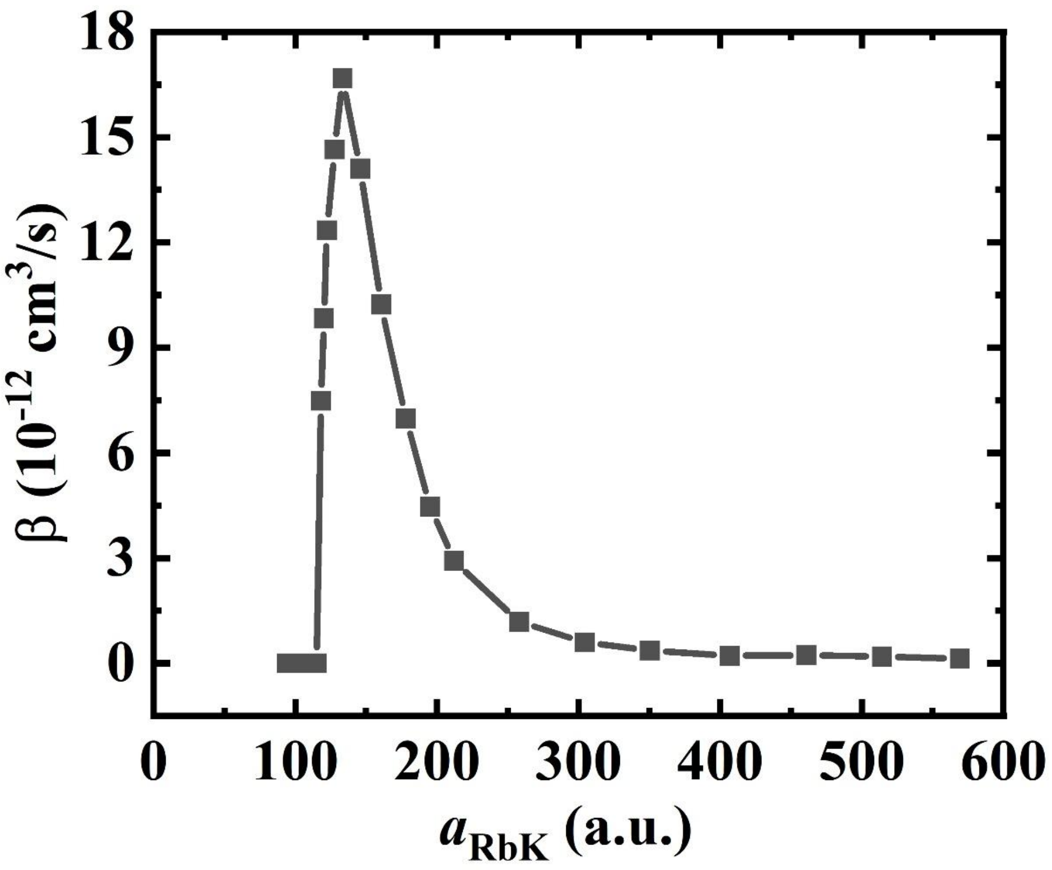


An Efimov recombination minima on the total rates is shown at $a_{\mathrm{RbK}} = 3638\ a_0$.

Three-body collision in Rb-Rb-K system

□ On the positive side of interspecifies scattering length: $a_{\mathrm{RbK}} > 0$

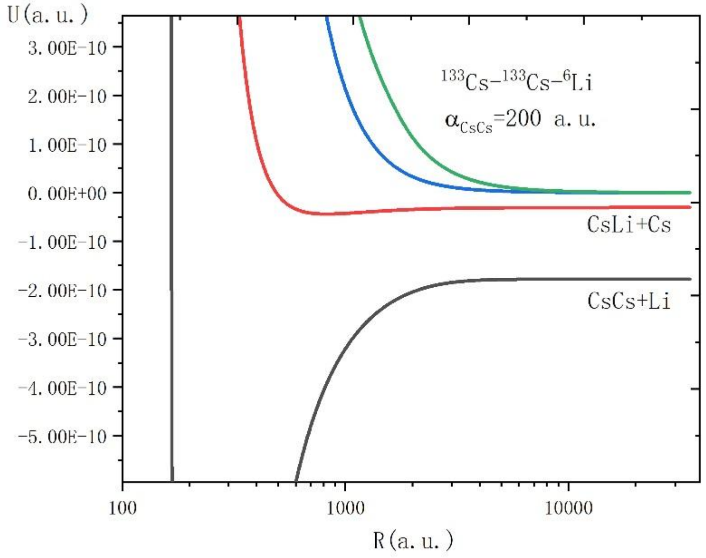
➢ **Vibrational relaxtion resonance**



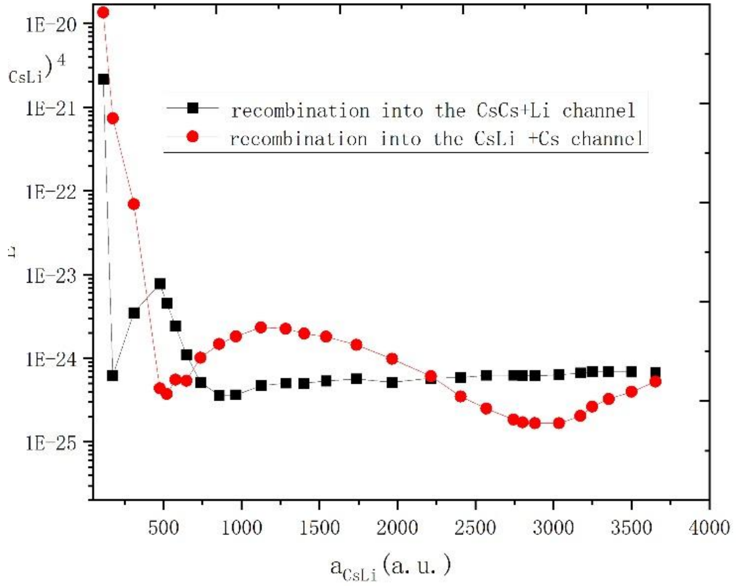
The vibrational relaxtion resonance is shown at $a_{\mathrm{RbK}} = 133\ a_0$

Three-body collision in Cs-Cs-Li system

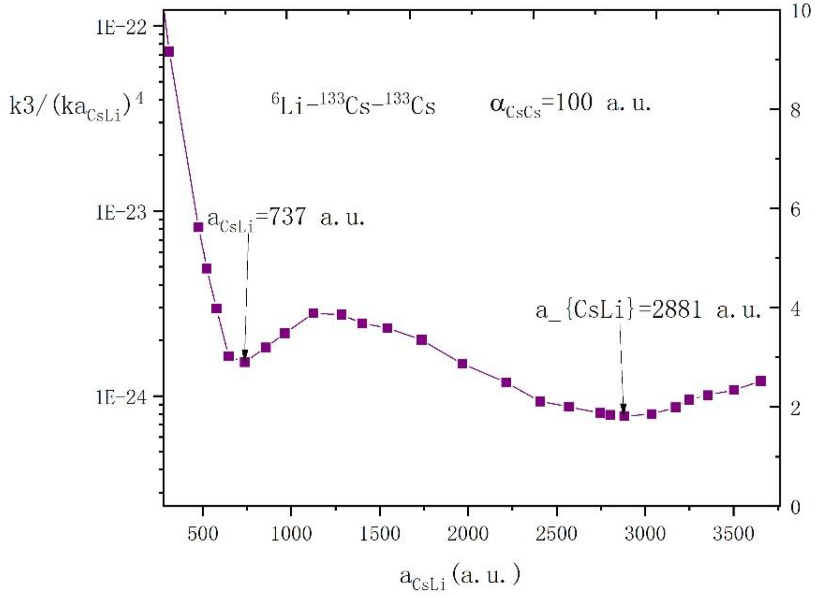
- **Cs + CsLi collision on the positive interspecifies scattering length a_{CsCs} =200 a.u. E=100 nK**



Hyperspherical potential curves



The partial K_3 rates



The total K_3 rates

Two minima are found: a_{CsLi} =737 a.u. and a_{CsLi} =2881 a.u.

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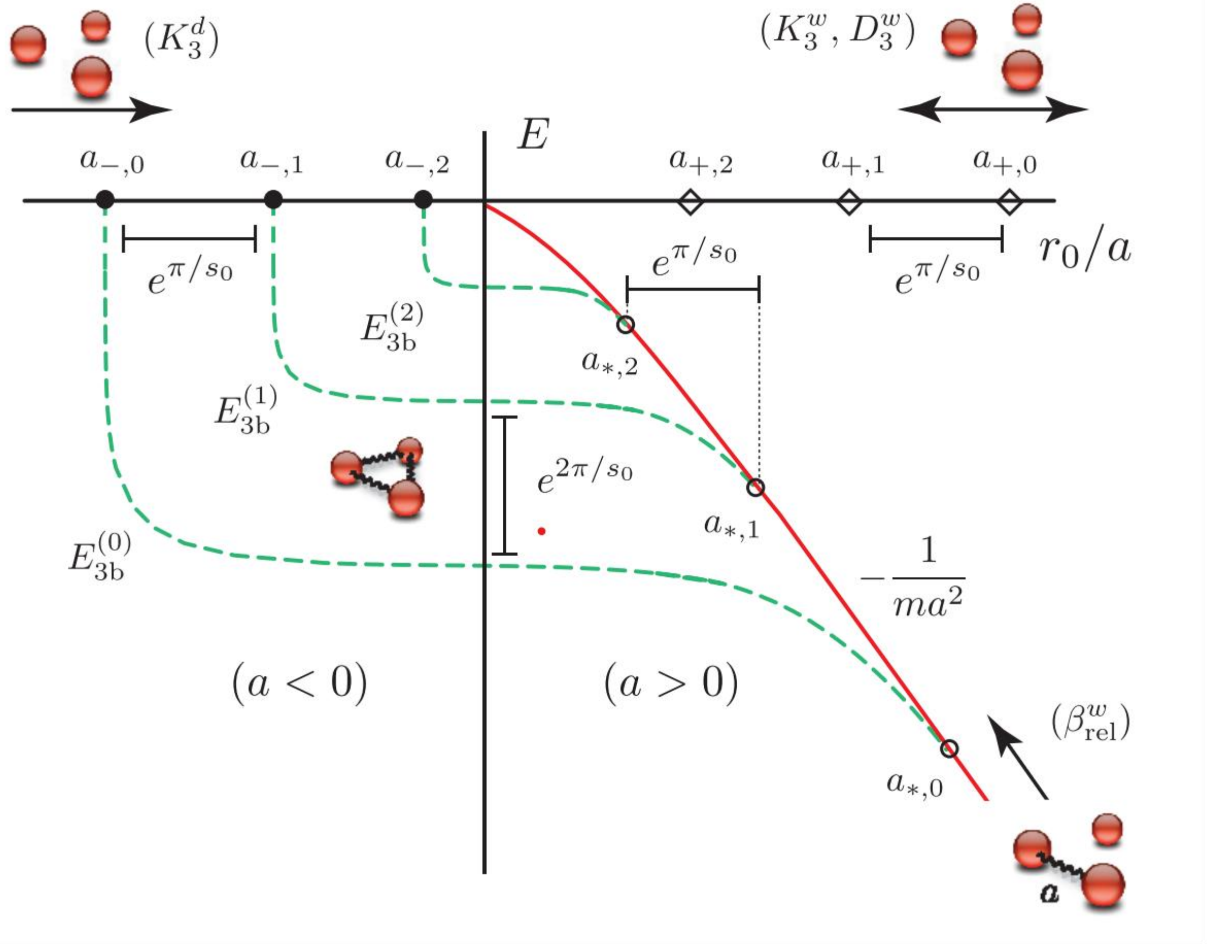
Universal relations between Efimov features

- **The universal relations between Efimov features from same scattering observables.**



[1] J. P. D'Incao, J. Phys. B: At. Mol. Opt. Phys. 51 043001 (2018).

Universal relations between Efimov features

☐ The universal relations between Efimov features from different scattering observables.

$$a_{\alpha,i}/a_{\beta,j} = \theta_\beta^\alpha \left(e^{\pi/s_0}\right)^{i-j}$$

where α and β assuming the values "–", "+" and "* ", and i and j running over the index labeling the Efimov states

➢ **For homonuclear system, θ_β^α is as following**

$$\theta_-^+ = a_+/a_- \approx -0.209914,$$
$$\theta_-^* = a_*/a_- \approx -0.046938,$$
$$\theta_+^* = a_*/a_+ \approx 0.223604.$$

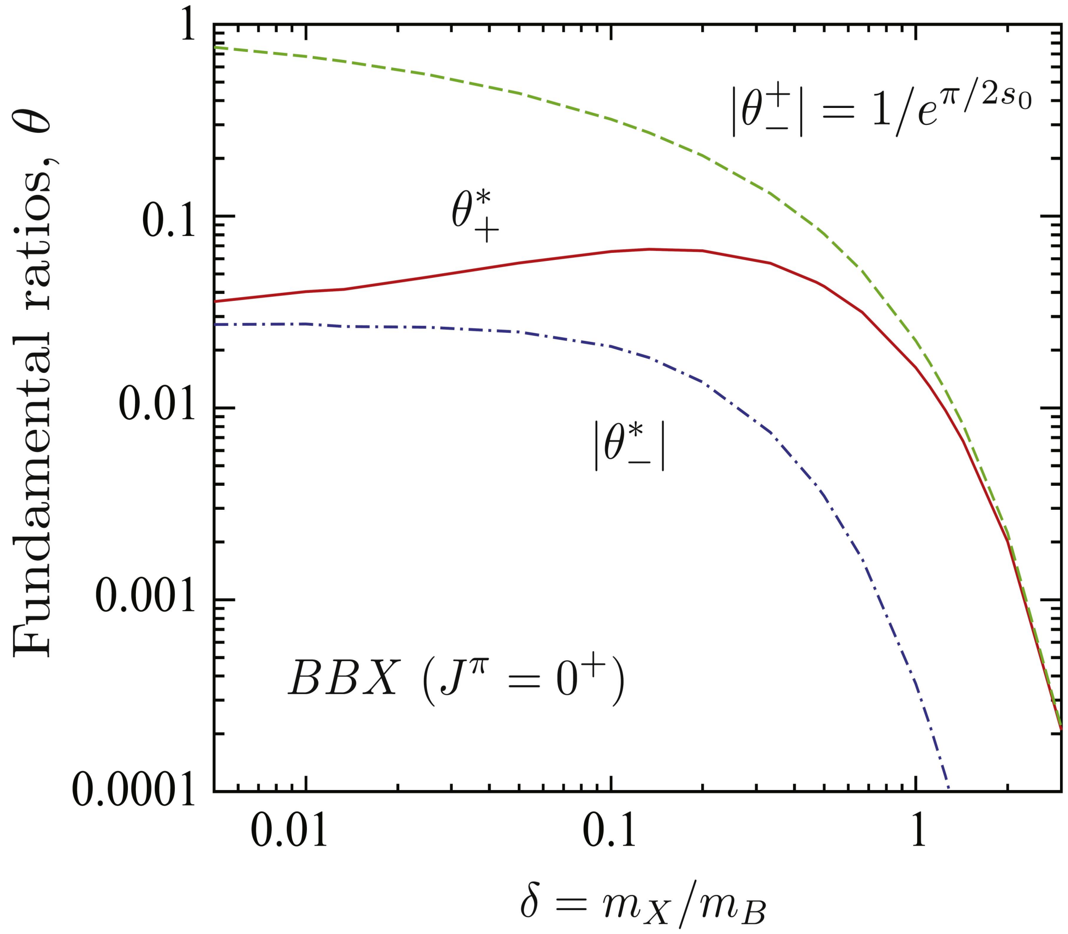
[1] J. P. D'Incao, J. Phys. B: At. Mol. Opt. Phys. 51 043001 (2018).

Universal relations between Efimov features

- **The universal relations between Efimov features from different scattering observables.**

$$a_{\alpha,i}/a_{\beta,j} = \theta_{\beta}^{\alpha}\ \left(e^{\pi/s_0}\right)^{i-j}$$

For heteronuclear system, θ_{β}^{a} is related with mass ratio



[2] K. Helfrich, H.-W. Hammer, and D. S. Petrov, Phys. Rev. A 81, 042715 (2010).
[3] B. Acharya, C. Ji, and L. Platter, Phys. Rev. A 94, 032702 (2016).

Universal relations between Efimov features

- **Efimov features in ^{87}Rb-^{87}Rb-^{40}K system**

Universal relation predicted by zero-range model:

System	$\delta=m_X/m_B$	$\lvert\theta_-^*\rvert$	θ_+^*	$\lvert\theta_-^+\rvert$	s_0	$\exp(\pi/s_0)$	$1/\exp(\pi/2s_0)$
$^{87}Rb^{87}Rb^{40}K$	0.459836	0.0041	0.0458	0.0894	0.653	122.8562	0.090

---Experiment values[3]: $a_* = 230\ a_0$

---Present results : $a_* = 133\ a_0$, $a_+ = 3638\ a_0$

---Predict Efimov resonance through universal relationship:

$a_- = 40693\ a_0$ with $a_+ = 3638\ a_0$ and $a_- = 56097\ a_0$ with $a_* = 230\ a_0$

[2] K. Helfrich, H.-W. Hammer, and D. S. Petrov, Phys. Rev. A 81, 042715 (2010).
[3]R. S. Bloom, M.-G. Hu, T. D. Cumby, and D. S. Jin, Phys. Rev. Lett. 111, 105301 (2013).

Universal relations between Efimov features

- **Efimov features in $^{133}Cs^{133}Cs^{6}Li$ system**

Universal relation predicted by zero-range method:

System	$\delta=m_X/m_B$	$\lvert\theta_-^*\rvert$	θ_+^*	$\lvert\theta_-^+\rvert$	s_0	$\exp(\pi/s_0)$	$1/\exp(\pi/2s_0)$
$^{133}Cs^{133}Cs^{6}Li$	0.045113	0.0251	0.0558	0.4551	1.983	4.8757	0.4528

---Experiment values[4]: when a_{CsCs}= 200 a_0, $a_-^{(1)}$ = -2130 a_0, $a_-^{(1)}$= -8500 a_0

---Present results : a_{CsCs}= 200 a_0 , $a_+^{(1)}$ = 737 a_0, $a_+^{(2)}$ = 2881 a_0

$a_-^{(1)}$ = - 1476 a_0, $a_-^{(2)}$= -7264 a_0

---Universal relationship with present results is: $\lambda = a_+^{(2)}/\, a_+^{(1)} = 3.91$, $\lvert\theta_-^+\rvert$=0.499

---Efimov scaling λ with experiment results is : $\lambda = a_-^{(2)}/\, a_-^{(1)} = 3.99$

[2] K. Helfrich, H.-W. Hammer, and D. S. Petrov, Phys. Rev. A 81, 042715 (2010).
[4] J. Ulmanis, S. H¨afner, R. Pires, E. D. Kuhnle, Y. Wang, C. H. Greene, and M. Weidem¨uller, Phys. Rev. Lett. 117, 153201 (2016)

Summary

- **We investigated the three-body collisions in Rb-Rb-K and CsCsLi system**
- **Using the calculated minima of three-body recombination, We explored the universality of Efimov features in these systems.**

Thank you for your attention!