FB23 THE 23rd INTERNATIONAL CONFERENCE ON FEW-BODY PROBLEMS IN PHYSICS (FB23)

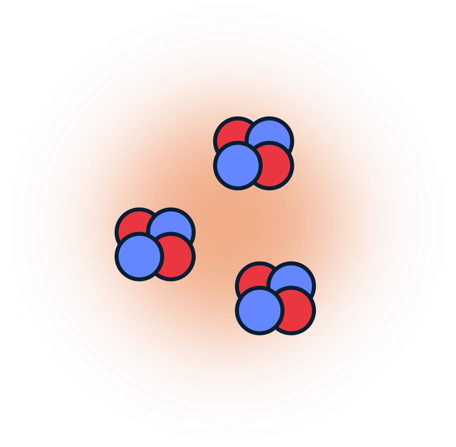
Sept. 22 -27, 2024 • Beijing, China

Host Institute of High Energy Physics, Chinese Academy of Sciences   Institute for Advanced Study, Tsinghua University   University of Chinese Academy of Sciences
China Center of Advanced Science and Technology   Institute of Theoretical Physics, Chinese Academy of Sciences   South China Normal University
Co-host Chinese Physical Society (CPS)   High Energy Physics Branch of CPS

Searching for Hoyle-analog States in light nuclei

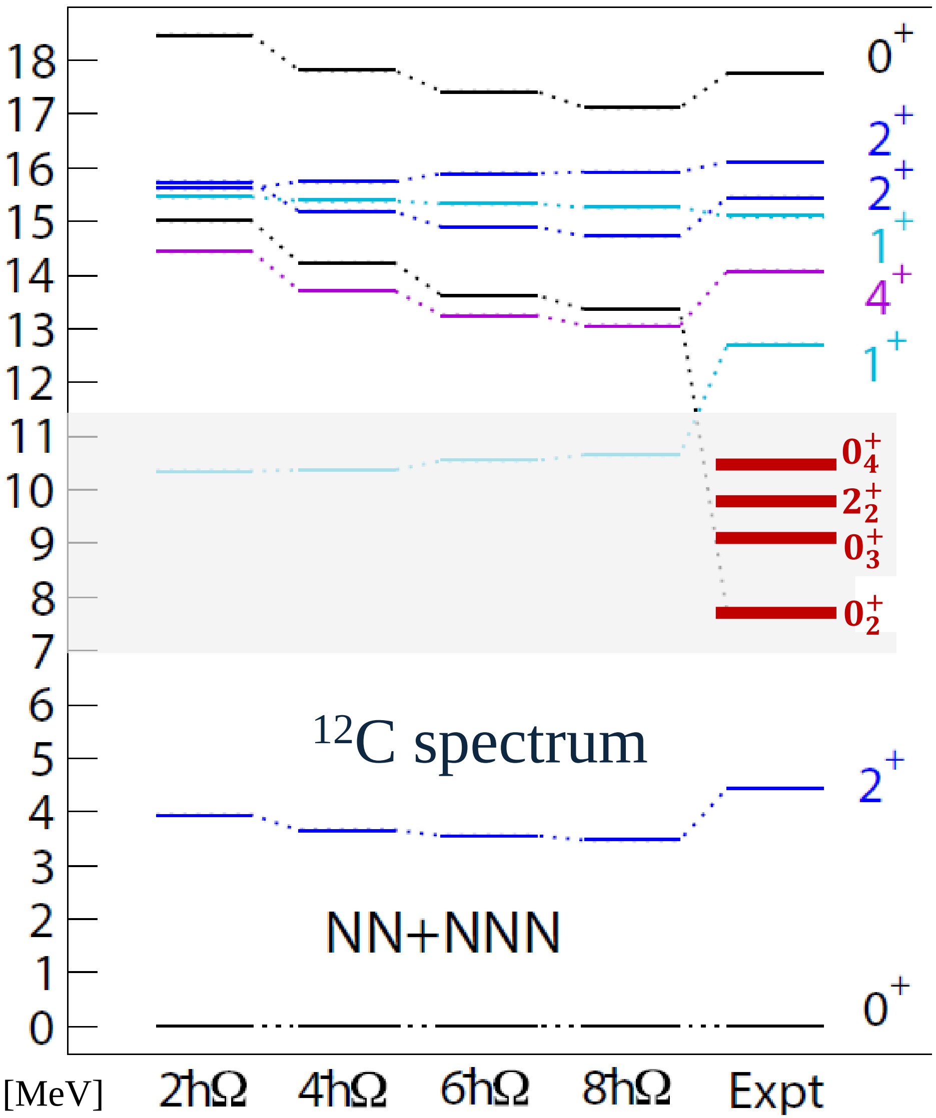
Bo Zhou (周波)

Fudan University



2024.09.26@Beijing

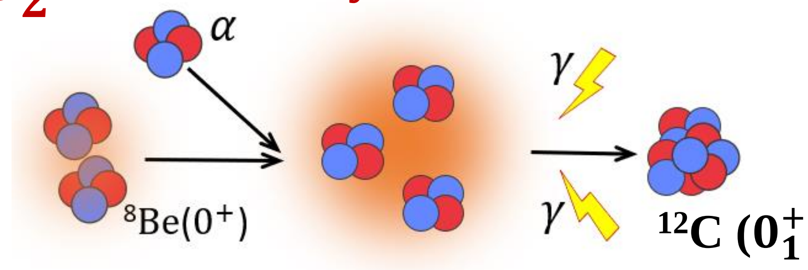
Cluster states of ^{12}C



Recent No-Core-Shell-Model calculations

V.Somà,P.Navrátil, *et al.* PRC,101,014318 (2020)

0_2^+ Hoyle State



The 3α gas-like state and Bose-Einstein Condensate

Rev. Mod. Phys. **89**, 011002 (2017)

$0_{3,4}^+$

Two broad resonance states with large decay width

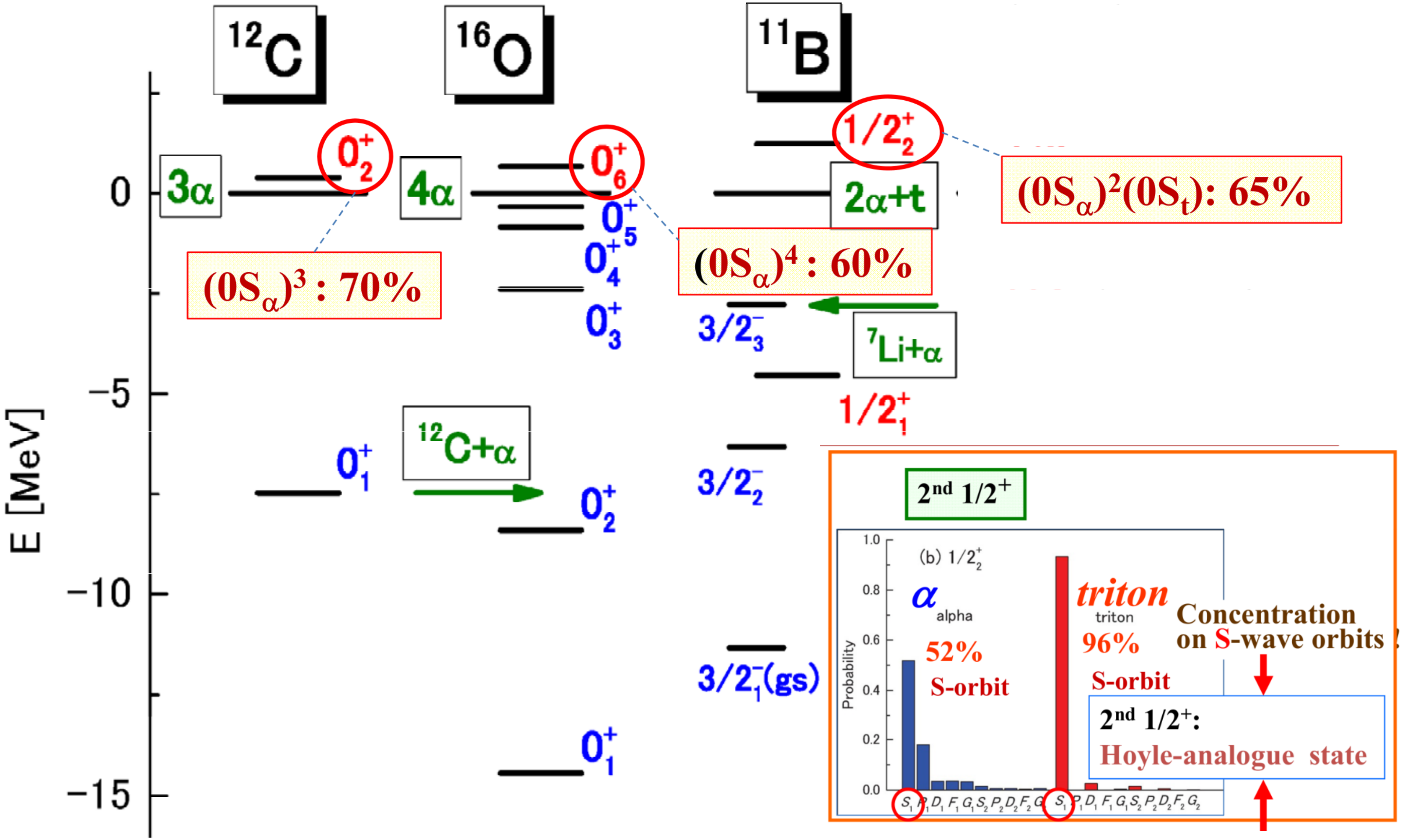
Phys. Rev. C **84**, 054308 (2011)

2_2^+

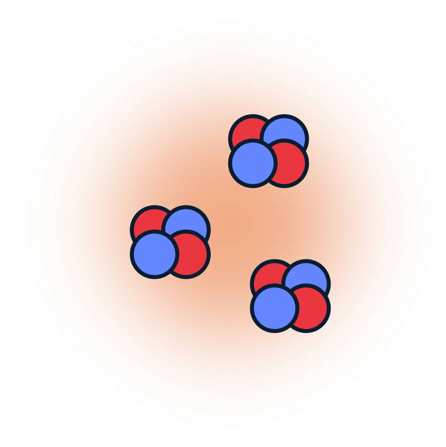
Long puzzle and it now has been confirmed for its existence.

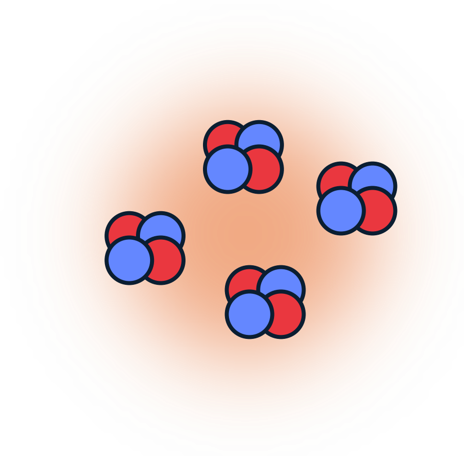
Phys. Rev. Lett. **110**, 152502 (2013)

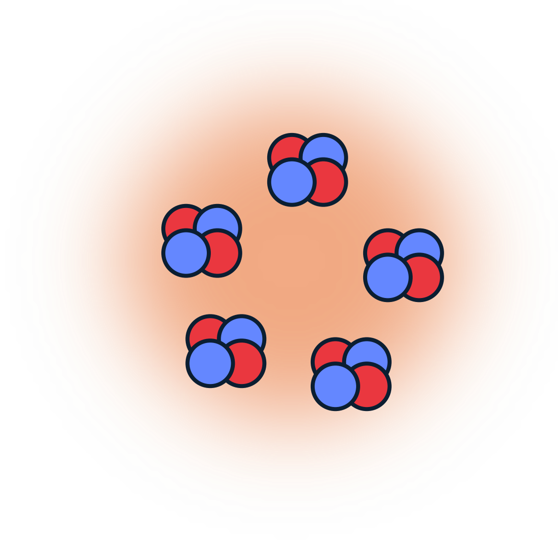
Hoyle-analog states



T.Yamada







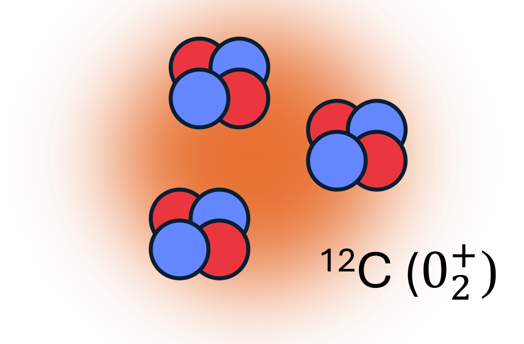
Search for the 5α condensate state

B. Zhou, Y. Funaki, H.Horiuchi, Y-G. Ma,
G. Röpke,P. Schuck, A. Tohsaki & T. Yamada

3α condensate	4α condensate	5α condensate
(Hoyle state)	(0_6^+ state)	(?)
2001 (THSR)	2008~ (OCM,THSR)	2019~

study of alpha condensate in finite nuclei

Hoyle states of ^{12}C



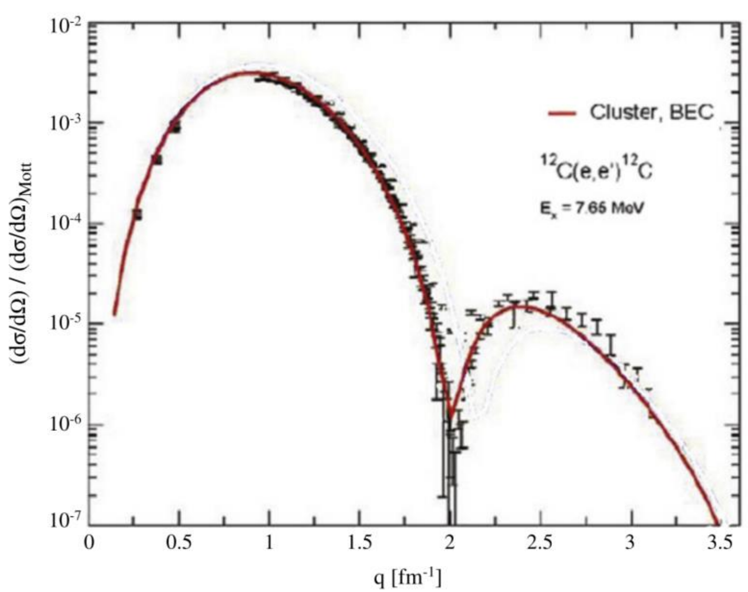
3α Bose-Einstein state

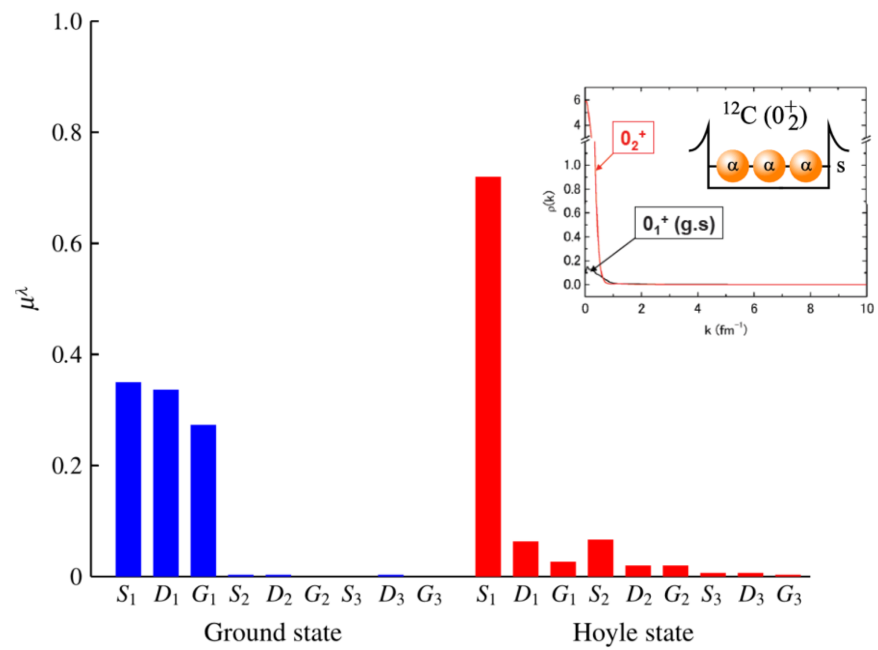
$$\Psi_{3\alpha}^{\rm THSR} = \mathcal{A}\left\{\exp\left[-\frac{2}{B^2}(\boldsymbol{X}_1^2+\boldsymbol{X}_2^2+\boldsymbol{X}_3^2)\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}$$
$$= \exp\left(-\frac{6}{B^2}\boldsymbol{\xi}_3^2\right)\mathcal{A}\left\{\exp\left(-\frac{4}{3B^2}\boldsymbol{\xi}_1^2-\frac{1}{B^2}\boldsymbol{\xi}_2^2\right)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\},$$

$$\boldsymbol{\xi}_1 = \boldsymbol{X}_1 - \frac{1}{2}(\boldsymbol{X}_2+\boldsymbol{X}_3), \qquad \boldsymbol{\xi}_2 = \boldsymbol{X}_2 - \boldsymbol{X}_3, \qquad \boldsymbol{\xi}_3 = \frac{1}{3}(\boldsymbol{X}_1+\boldsymbol{X}_2+\boldsymbol{X}_3)$$

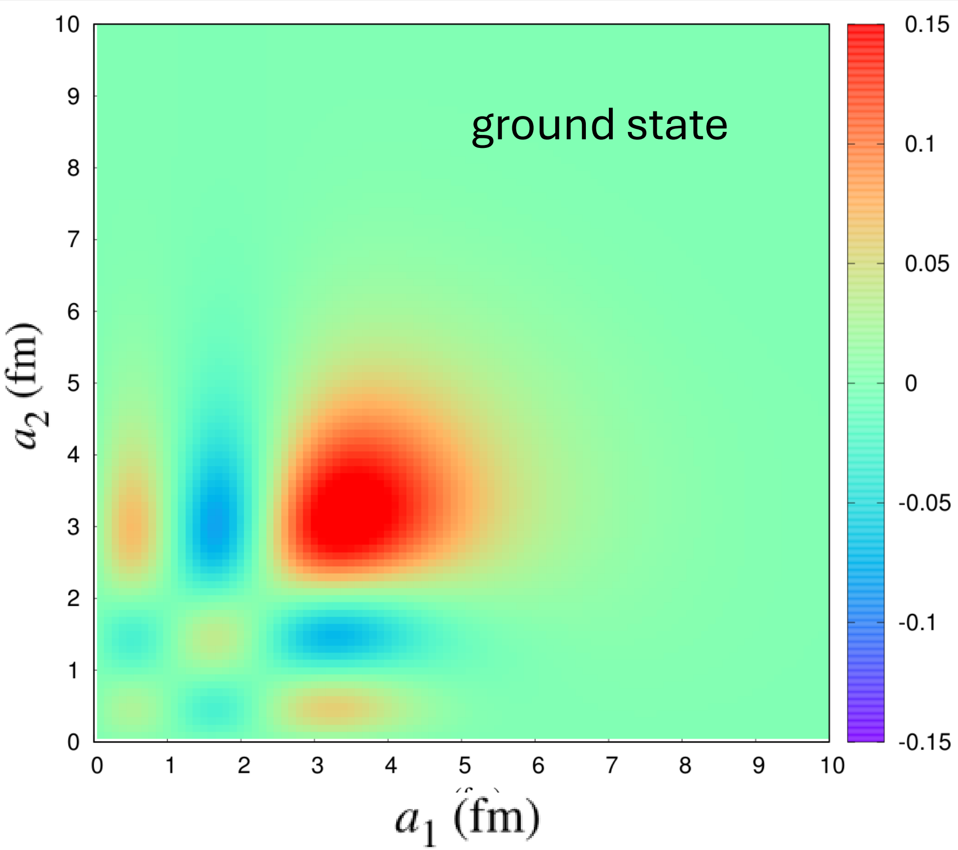
Rev. Mod. Phys. **89**, 011002 (2017)

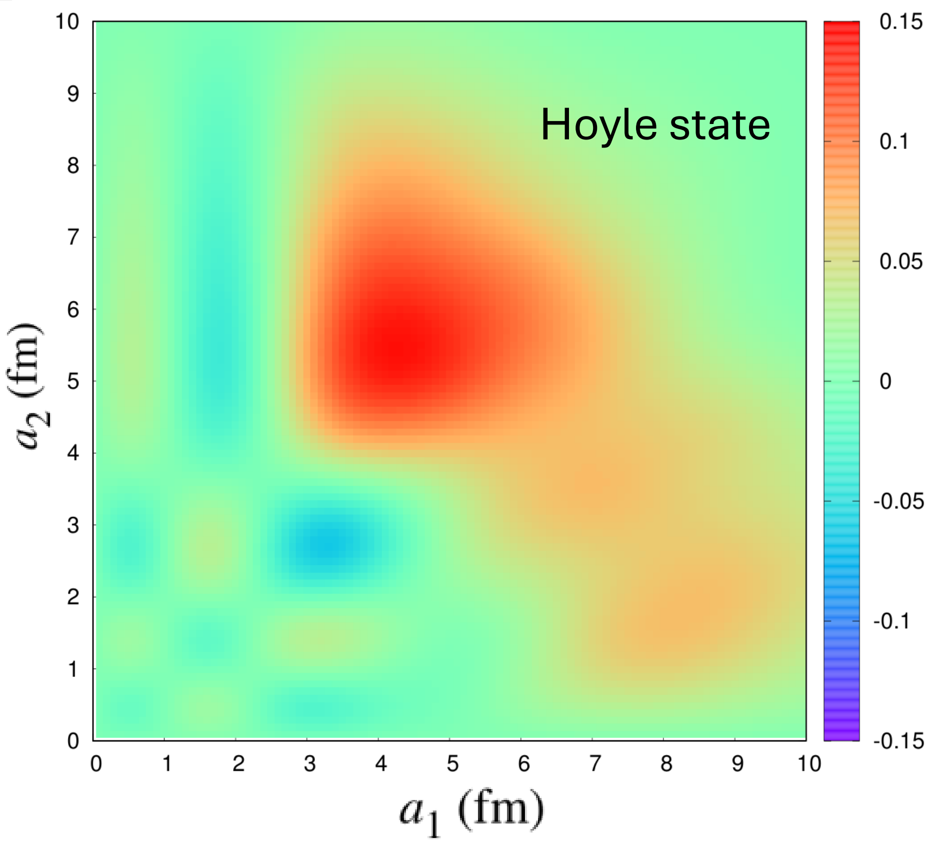
Y. Funaki et al. / Progress in Particle and Nuclear Physics 82 (2015) 78–132





Ground state and Hoyle state



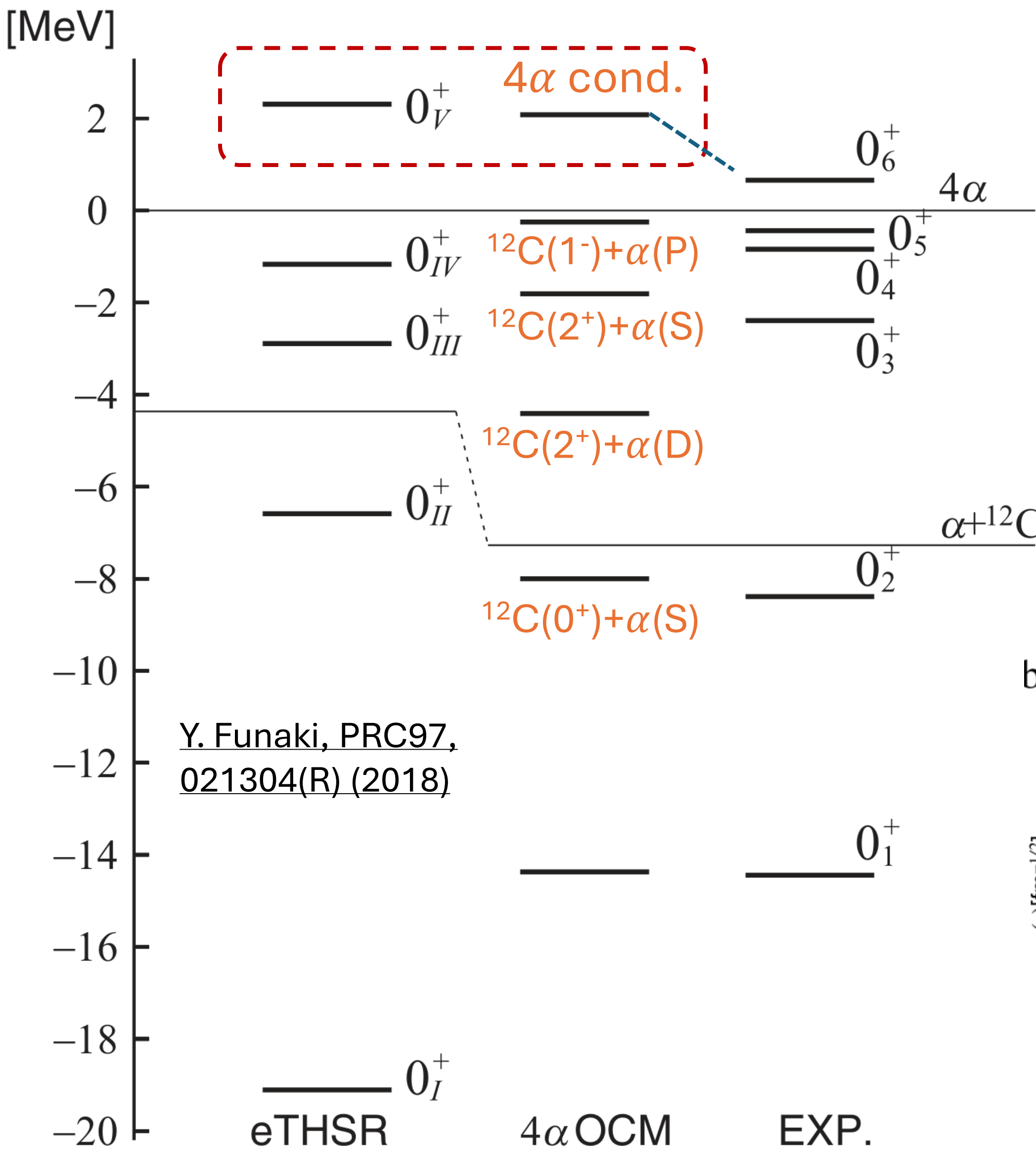


$$[\alpha \otimes [\alpha \otimes \alpha]_0]_0 \otimes [0 \otimes 0]_0$$

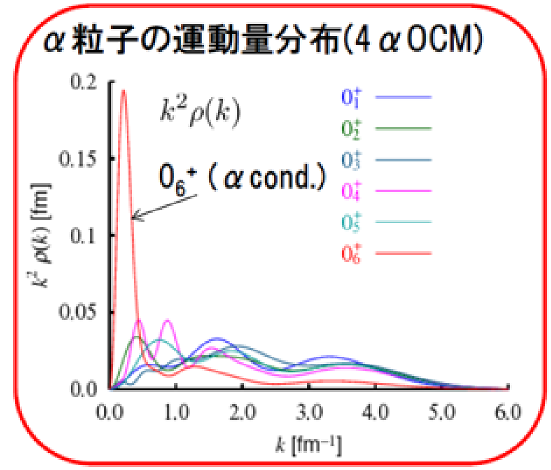
$$\mathcal{Y}_c^{J\pi}(a_1, a_2) = \sqrt{\frac{A!}{C_1!C_2!C_3!}} \left\langle \frac{\delta(r_1 - a_1)\delta(r_2 - a_2)}{r_1^2 r_2^2} \left[[Y_{l_1}(\hat{r}_1) \otimes Y_{l_2}(\hat{r}_2)]_L \otimes \left[\Phi_{C_1}^{j_1\pi_1} \otimes \left[\Phi_{C_2}^{j_2\pi_2} \otimes \Phi_{C_3}^{j_3\pi_3} \right]_{j_{23}} \right]_{j_{123}} \right]_{JM} \middle| \Psi_M^{J\pi} \right\rangle$$

De-ye Tao

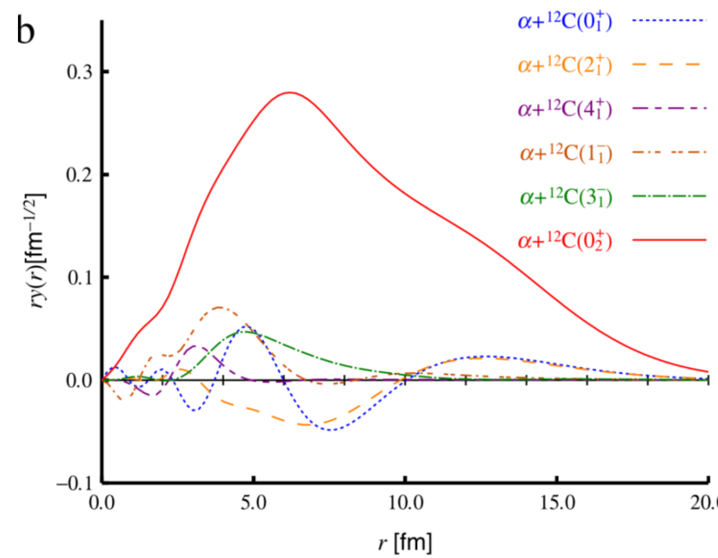
Alpha condensate in ^{16}O



Y. Funaki, PRC97, 021304(R) (2018)



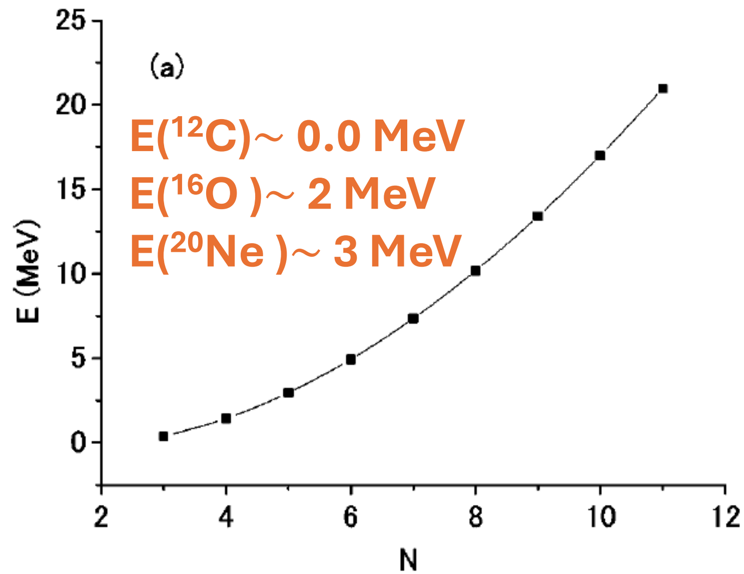
4αOCM
Y. F. et al., PRL **101**, 082502 (2008).
4αTHSR
Y. F. et al., PRC **82**, 024312 (2010).



Multi-alpha condensation

Dilute multi-α cluster condensed states with spherical and axially deformed shapes are studied with the Gross-Pitaevskii equation and Hill-Wheeler equation where the α cluster is treated as a structureless boson, **it is predicted to exist in heavier self-conjugate 4*N* nuclei up to *N*=10.**

T. Yamada and P. Schuck, Phys. Rev. C 69, 024309 (2004).



Some candidates for α condensate were found from experiments for ^{12}C and ^{16}O.

Rev. Mod. Phys. **89**, 011002 (2017).

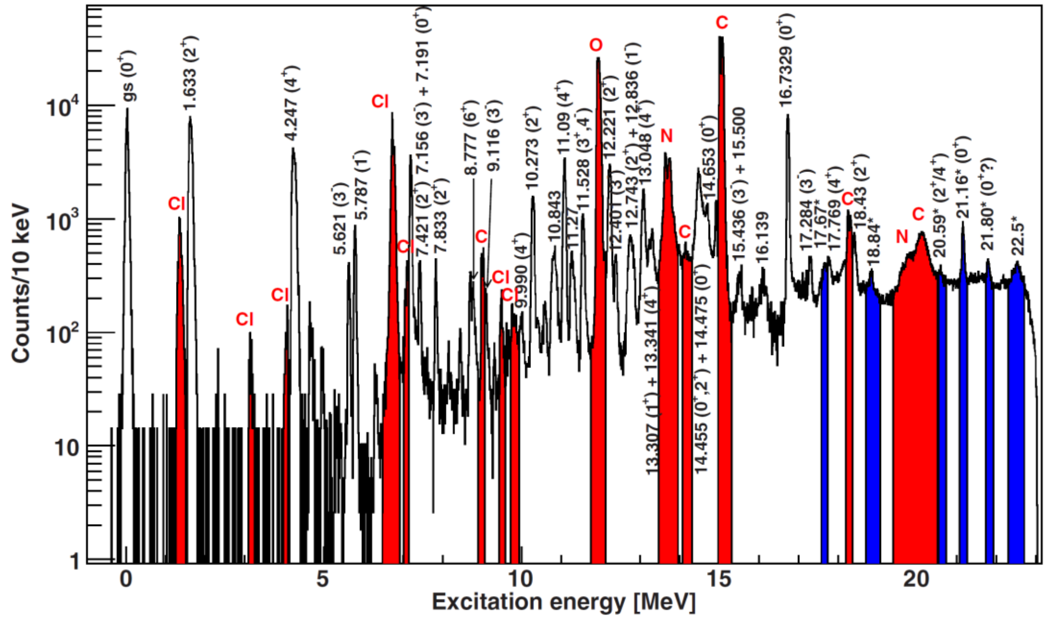
No experimental signatures for α condensation were observed

Phys. Rev. C **100**, 034320 (2019)

An experimental way of testing Bose-Einstein condensation of clusters in the atomic nucleus is reported. The enhancement of cluster emission and the multiplicity partition of possible emitted clusters could be direct signatures for the condensed states.

PRL **96,** 192502 (2006)

Recent experiment for 5α condensation



$^{22}Ne(p,t)^{20}Ne$

E_x calculated MeV	Model	E_x measured [MeV]
6.05	$2\hbar\omega$	6.725
6.70	USDB	7.191
12.58	$2\hbar\omega$	13.642
14.43	USDB	14.455/14.475
14.67	USDB	14.653
16.76	USDB	16.73
18.06	USDB	17.90
19.02	USDB	18.840(56)
20.47	$2\hbar\omega$	21.160(53)
22.14	5-α [31]	22.500(52)

PHYSICAL REVIEW C **91**, 034317 (2015)

Spectroscopy of narrow, high-lying, low-spin states in ^{20}Ne

J. A. Swartz,[1,2,*] B. A. Brown,[3,4] P. Papka,[1,2] F. D. Smit,[2] R. Neveling,[2] E. Z. Buthelezi,[2] S. V. Förtsch,[2] M. Freer,[5] Tz. Kokalova,[5] J. P. Mira,[1,2] F. Nemulodi,[1,2] J. N. Orce,[6] W. A. Richter,[2,6] and G. F. Steyn[2]
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[4] *Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
[5] *School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom*
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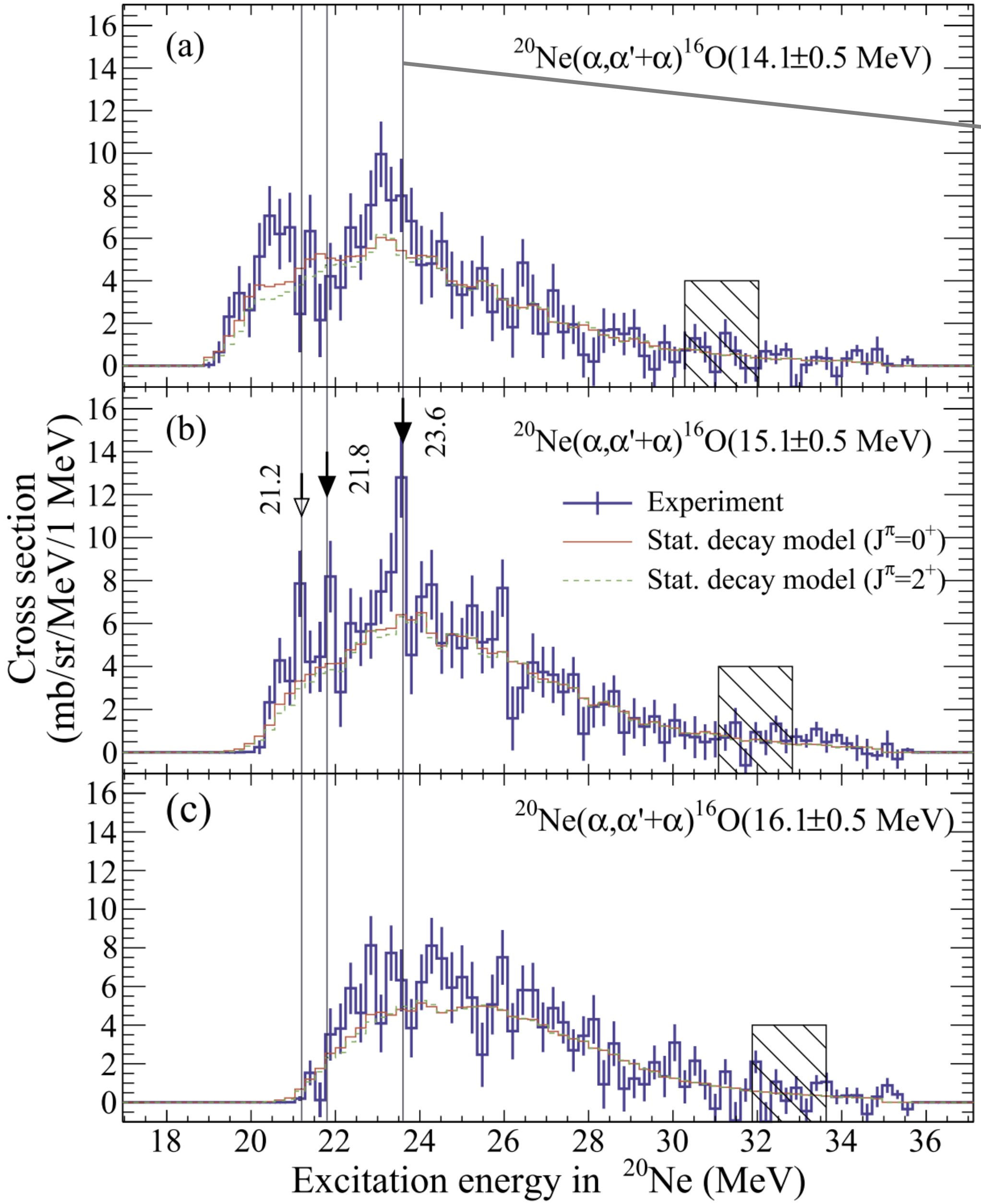
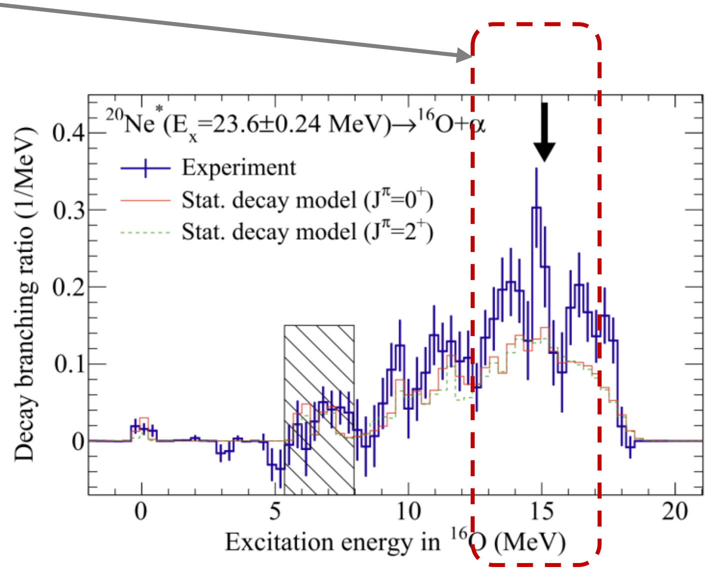
(Received 29 August 2014; revised manuscript received 18 February 2015; published 16 March 2015)

The state at ***Ex=22.5 MeV***, which could not be interpreted by the shell-model calculations, is a tentative candidate for the 5α cluster state.

Recent experiment for 5α condensation



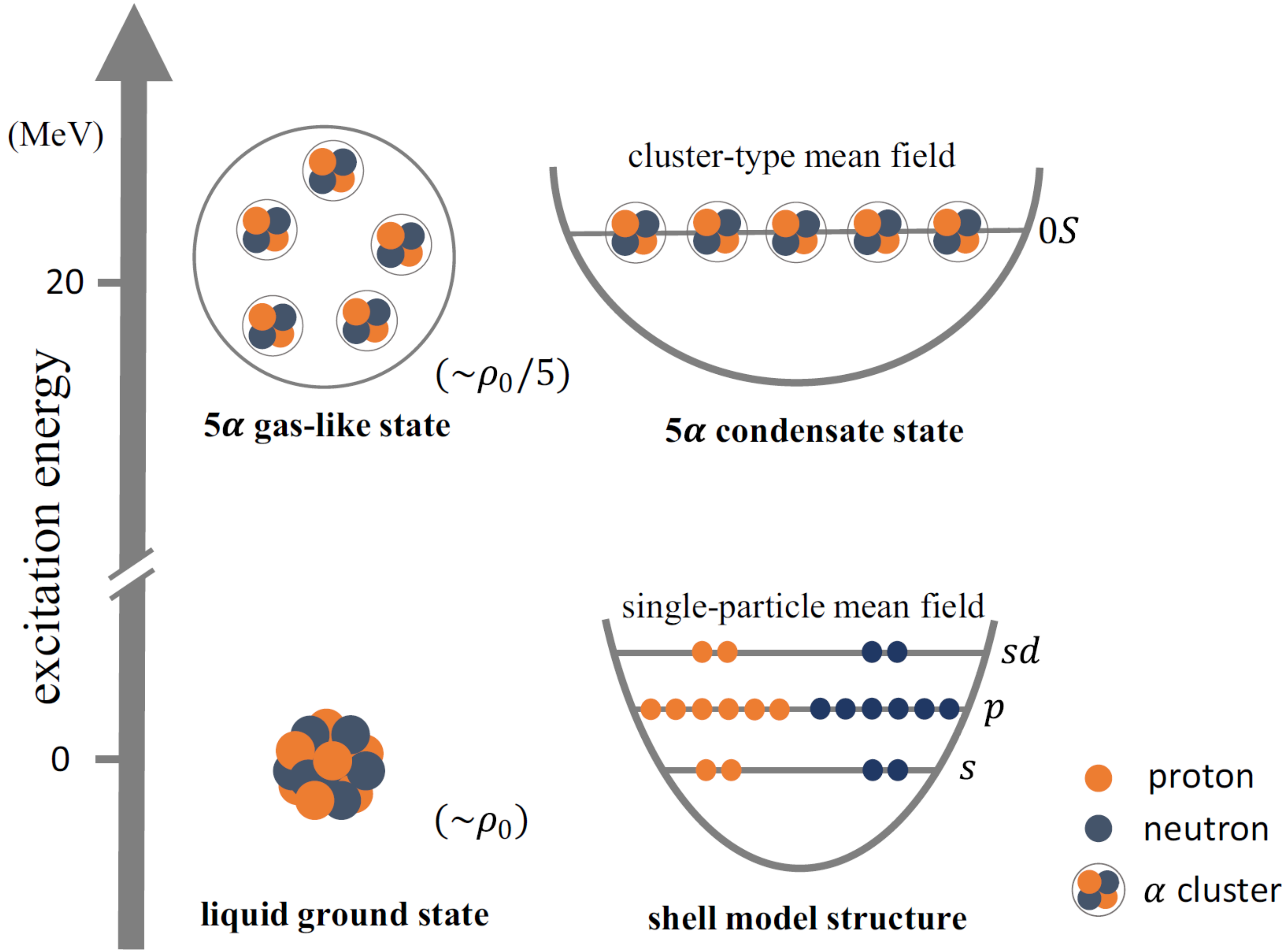
23.6-MeV state enhances in the decay to the ^{16}O (0_6^+) +α channel



- 3.3 MeV above the 5α threshold
- strongly coupled to ^{16}O(0_6^+) state

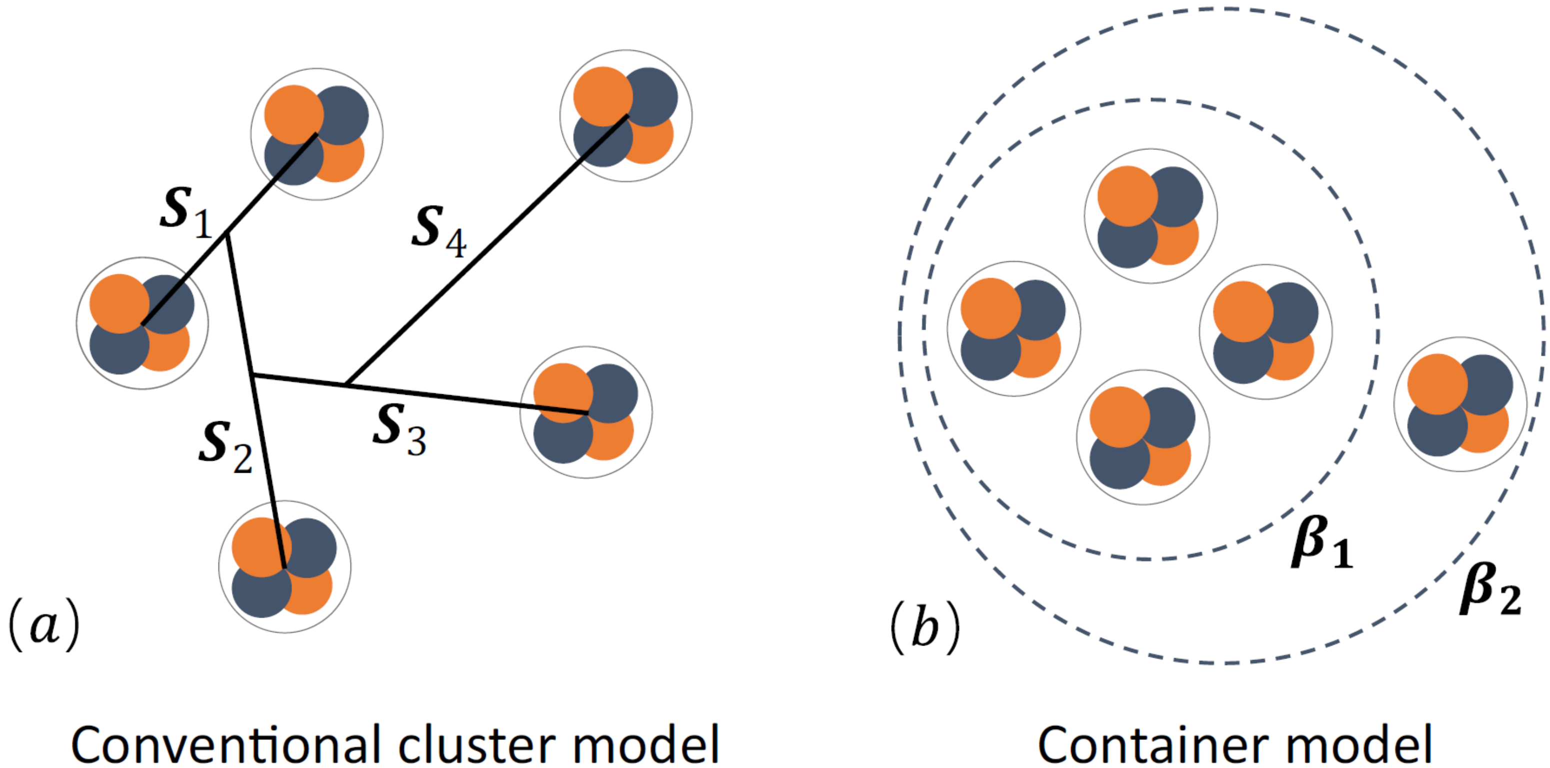
S. Adachi et al., *Candidates for the 5α condensed state in ^{20}Ne*, PLB **819**,136411 (2021).

Search for the 5alpha condensate state



Theoretical Framework

To solve the configurations problem:



Schematic illustrations of two distinct microscopic cluster models.

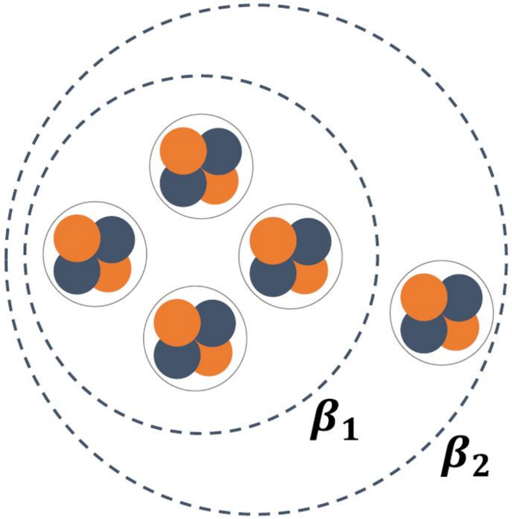
a The conventional cluster model of Φ^B, in which the inter-cluster variables $\{S_i\}$ are the Jacobi coordinates of $\{R_i\}$. **b** Container picture for $4\alpha + \alpha$ cluster structure of ^{20}Ne. The β_1 is the size variable for the description of 4α and β_2 for the description of the relative motion between 4α and α clusters.

Theoretical Framework

To solve the configurations problem:

$$\Psi(\beta_1,\beta_2) = \int d^3R_1 d^3R_2 d^3R_3 d^3R_4 d^3R_5 \times \exp\left[-\frac{1/2S_1^2+2/3S_2^2+3/4S_3^2}{\beta_1^2} - \frac{4/5S_4^2}{\beta_2^2}\right]\Phi^{\mathrm{B}}(R_1,R_2,R_3,R_4,R_5)$$

$$= n_0\,\mathscr{A}\left\{\exp\left[-\frac{2\xi_1^2+8/3\xi_2^2+3\xi_3^2}{2(b^2+2\beta_1^2)}\right]\exp\left[-\frac{16/5\xi_4^2}{2(b^2+2\beta_2^2)}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\},$$



$$\Phi^{\mathrm{B}}(R_1,R_2,R_3,R_4,R_5) = \frac{1}{\sqrt{20!}}\mathscr{A}[\phi_1(R_1)\ldots\phi_5(R_2)\ldots\phi_{20}(R_5)]$$

$$\propto \phi_g\,\mathscr{A}\left\{\exp\left[-\frac{2(\xi_1-S_1)^2+8/3(\xi_2-S_2)^2+3(\xi_3-S_3)^2}{2b^2}\right] \times \exp\left[-\frac{16/5(\xi_4-S_4)^2}{2b^2}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\},$$

with the single-nucleon wave function,

$$\phi_i(R_k) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}}\exp\left[-\frac{1}{2b^2}(r_i-R_k)^2\right]\chi_i\tau_i.$$

Three-body interaction

To solve the interaction problem:

The Hamiltonian for ^{20}Ne in this work can be written as:

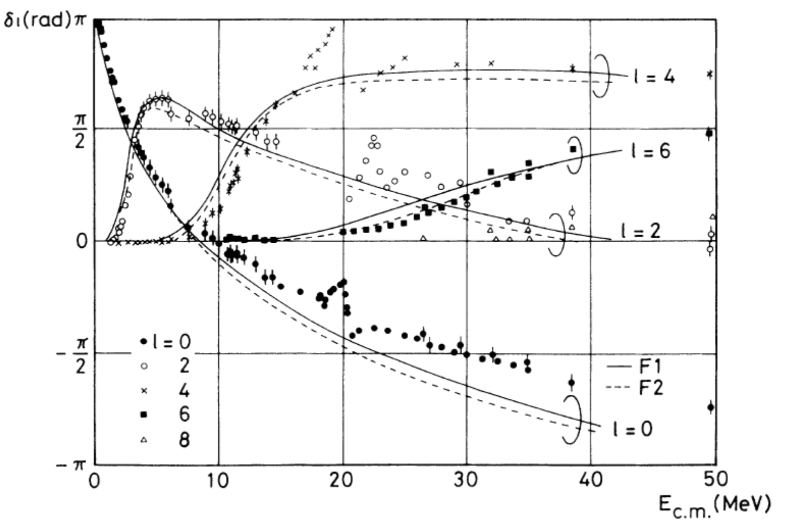
$$\mathcal{H} = -\frac{\hbar^2}{2M}\sum_i \nabla_i^2 - T_G + \sum_{i<j} V_{ij}^C + \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)},$$

The effective nucleon-nucleon potential part is taken a Gaussian form, which is expressed as:

$$V_{ij}^{(2)} = \sum_n v_n^{(2)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(2)}}\right)^2\right\}(W_n^{(2)} + M_n^{(2)} P_{ij})$$

and

$$V_{ijk}^{(3)} = \sum_n v_n^{(3)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(3)}}\right)^2 - \left(\frac{r_{jk}}{r_n^{(3)}}\right)^2\right\}$$
$$\times (W_n^{(3)} + M_n^{(3)} P_{ij})(W_n^{(3)} + M_n^{(3)} P_{jk}),$$



Tohsaki F1 three-body interaction was used.

A. Tohsaki,Phys. Rev. C **49**, 1814 (1994).

Radius-Constraint Method

To solve the resonance problem:

Radius-Constraint Method for treating the resonance states,

$$\sum_{\boldsymbol{\beta}'_1,\boldsymbol{\beta}'_2} \langle \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2)| \sum_{i=1} \frac{1}{20}(\boldsymbol{r}_i - \boldsymbol{X}_G)^2 |\hat{\Phi}^{0^+}_{4\alpha+\alpha}(\boldsymbol{\beta}'_1,\boldsymbol{\beta}'_2)\rangle \times g^{(\gamma)}(\boldsymbol{\beta}'_1,\boldsymbol{\beta}'_2)$$

$$= \{R^{(\gamma)}\}^2 g^{(\gamma)}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2)\langle \hat{\Phi}^{0^+}_{4\alpha+\alpha}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2)|\hat{\Phi}^{0^+}_{4\alpha+\alpha}(\boldsymbol{\beta}'_1,\boldsymbol{\beta}'_2)\rangle$$

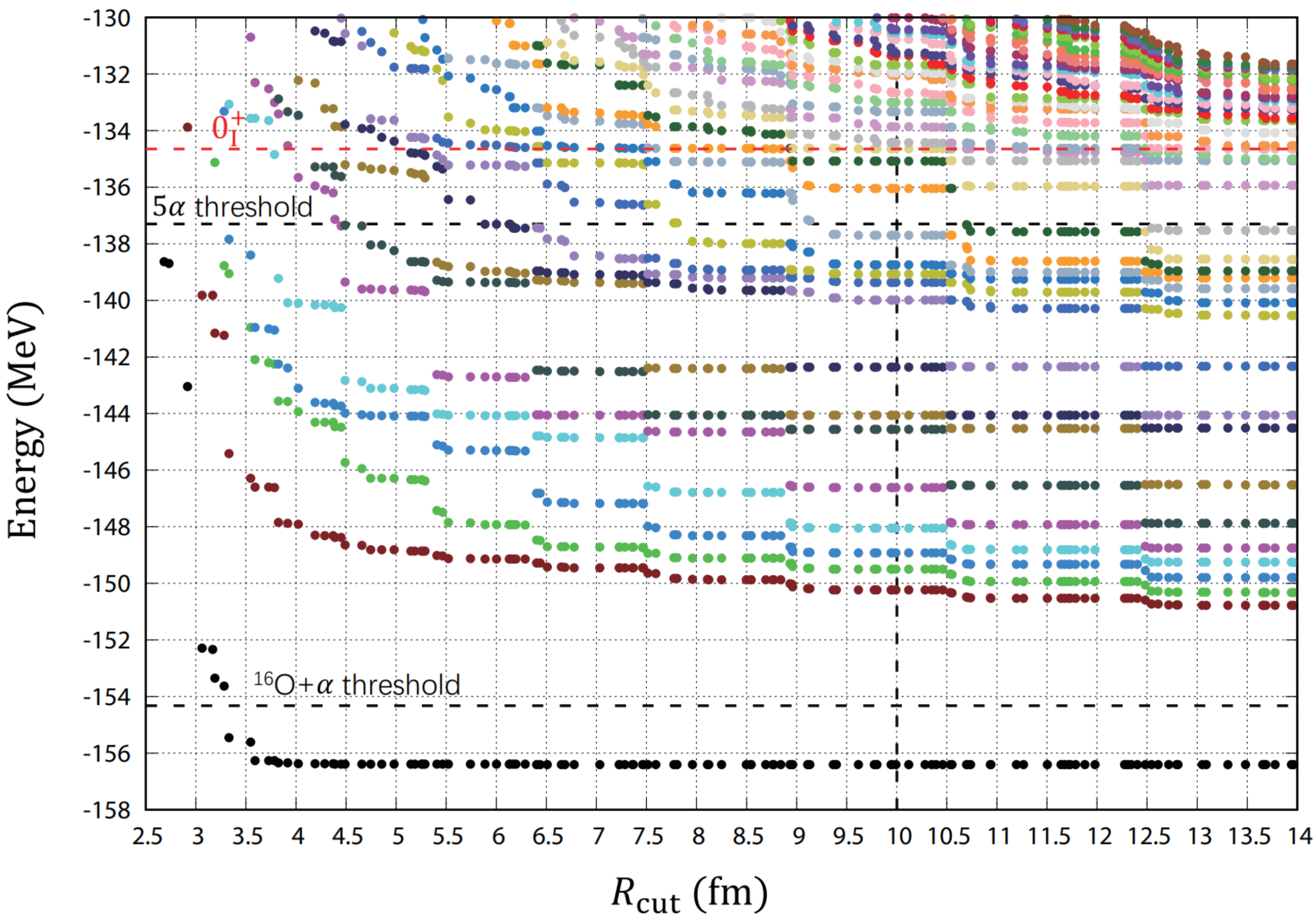
$$\Psi^{0^+}_{\mathrm{GCM}} = \sum_{\boldsymbol{\beta}_1,\boldsymbol{\beta}_2} g^{(\gamma)}(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2)\hat{\Phi}^{0^+}_{4\alpha+\alpha}(\boldsymbol{\beta}'_1,\boldsymbol{\beta}'_2)$$

Here, $R^{(\gamma)} \le R_{\mathrm{cut}}$ and R_{cut} is the radius cut-off parameter.

Y. Funaki, et al., Prog.Theor.Phys.115,115(2006).

Radius-Constraint Method + Stabilization Method

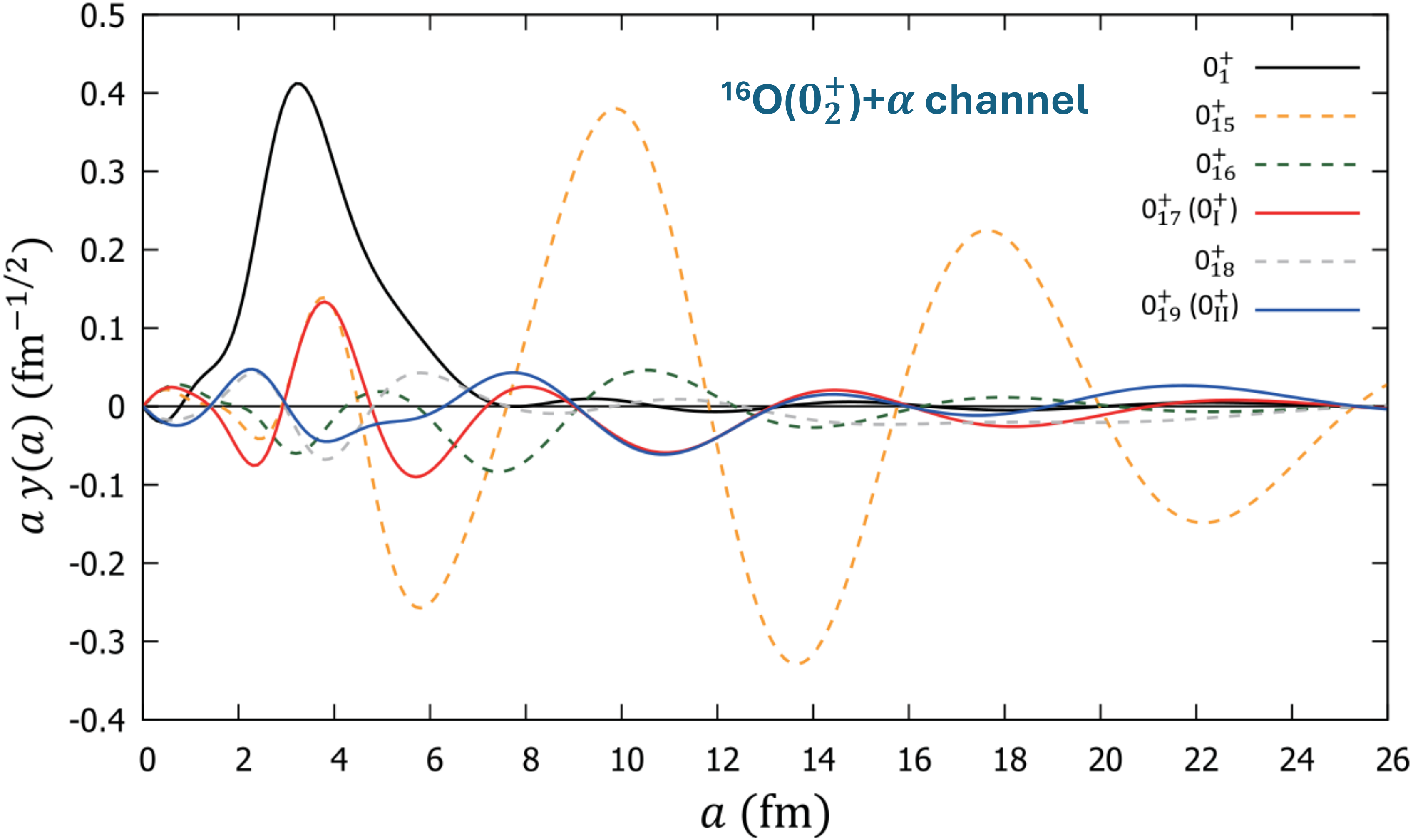
To solve the resonance problem



Above the 5alpha threshold: $0^+_{15} \sim 0^+_{19}$

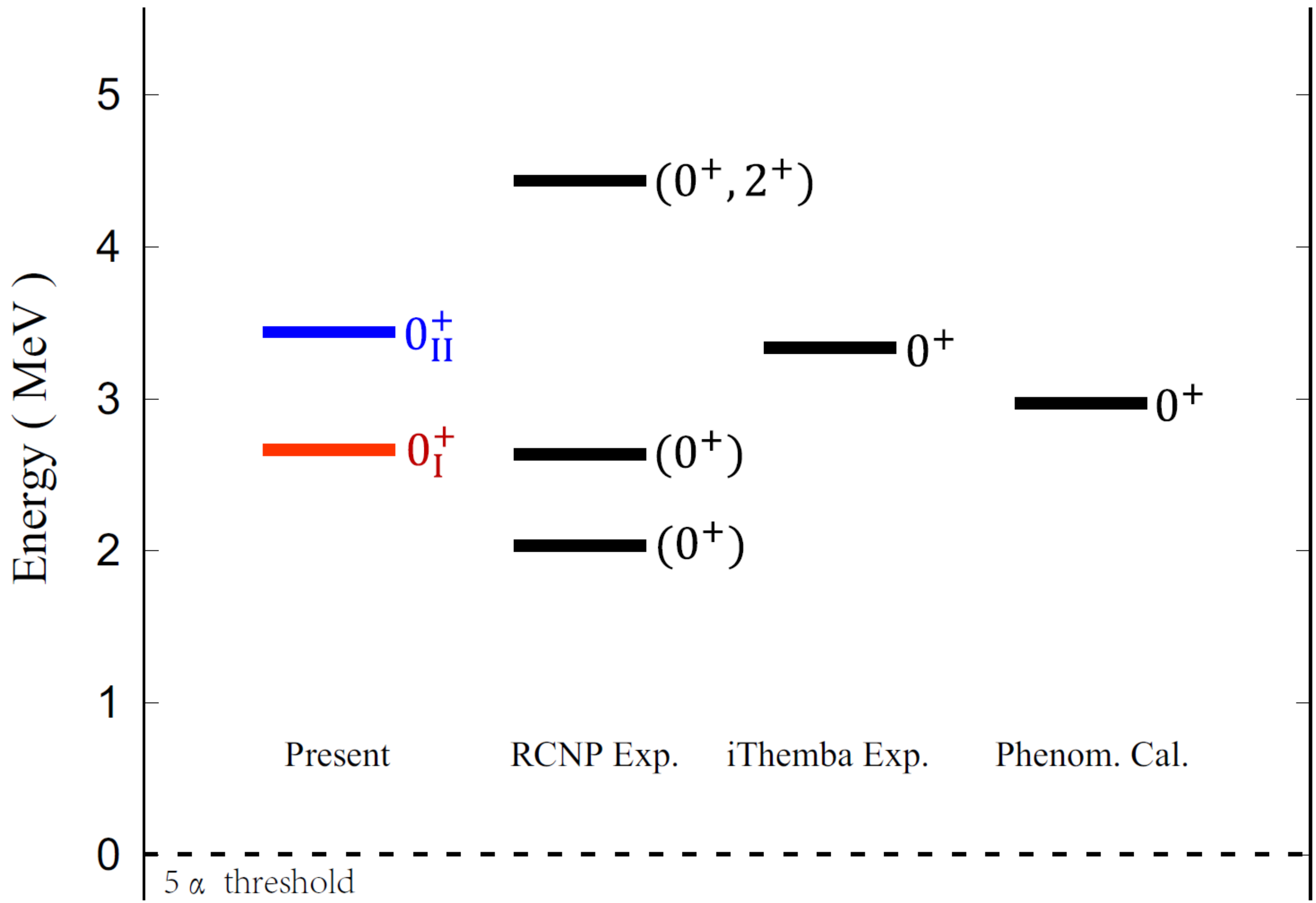
Five 0^+ states above the threshold

To solve the resonance problem



$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[\left[\Psi_{\rm gcm}^{0_s^+}(^{16}{\rm O}) \varphi_5(\alpha) \right]_{0^+} Y_{00}(\hat{\xi}_4) \right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi_{\rm gcm}^{0_\lambda^+}(^{20}{\rm Ne}) \right\rangle,$$

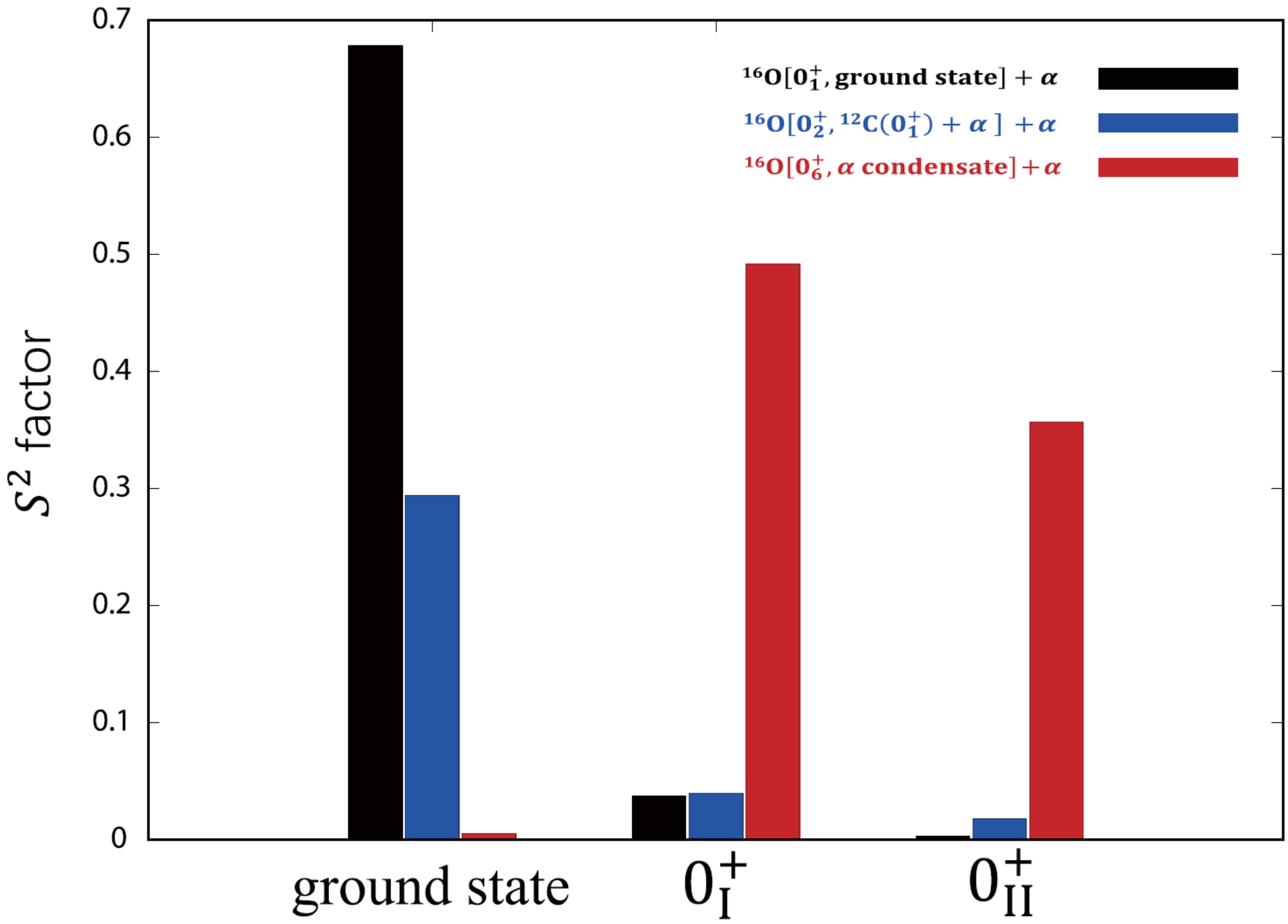
Energy level above the threshold



Two 0^+ states around 3 MeV are found in our calculations.

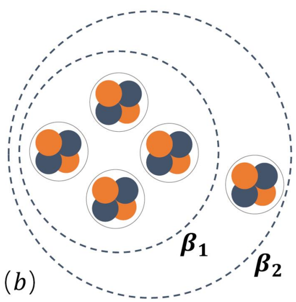
B.Zhou, *et al. Nat Commun* **14**, 8206 (2023).

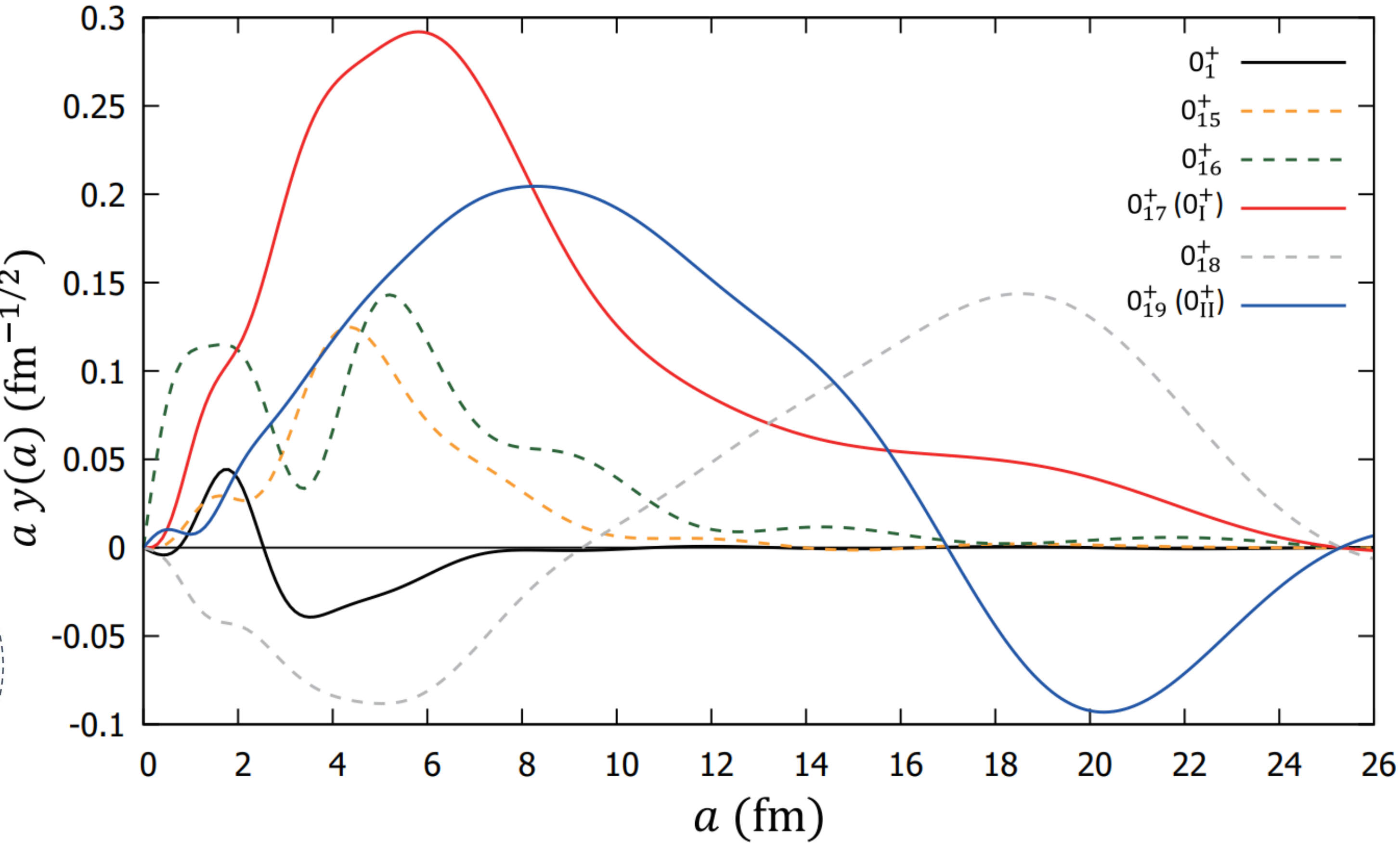
S^2 factor of different channels



B.Zhou, *et al. Nat Commun* **14**, 8206 (2023).

Reduced width amplitude



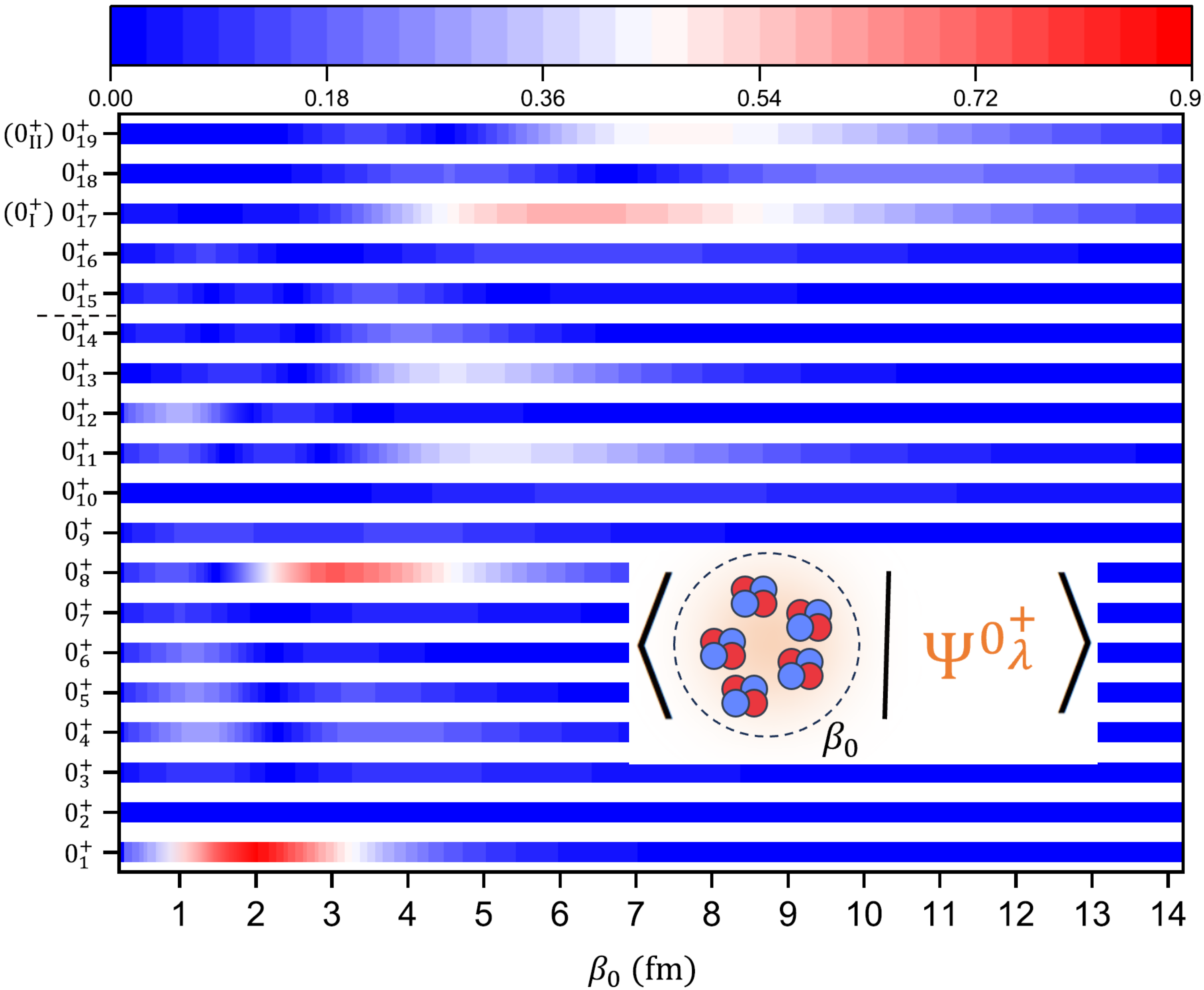


The reduced width amplitudes of the ground state and excited states above 5α threshold in ^{20}Ne in the channel of ^{16}O (0^+_6) + α. The ground state 0^+_1, 0^+_{17} $(0^+_{\rm I})$, and 0^+_{19} $(0^+_{\rm II})$ states are shown in solid lines. The 0^+_{15}, 0^+_{16}, and 0^+_{18} states are shown in dashed lines.

$$y(a) = \sqrt{\frac{20!}{4!16!}}\left\langle \left[\left[\Psi^{0^+_s}_{\rm gcm}(^{16}{\rm O})\varphi_5(\alpha)\right]_{0^+} Y_{00}(\hat{\xi}_4)\right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi^{0^+_\lambda}_{\rm gcm}(^{20}{\rm Ne}) \right\rangle,$$

B.Zhou, *et al. Nat Commun* **14**, 8206 (2023).

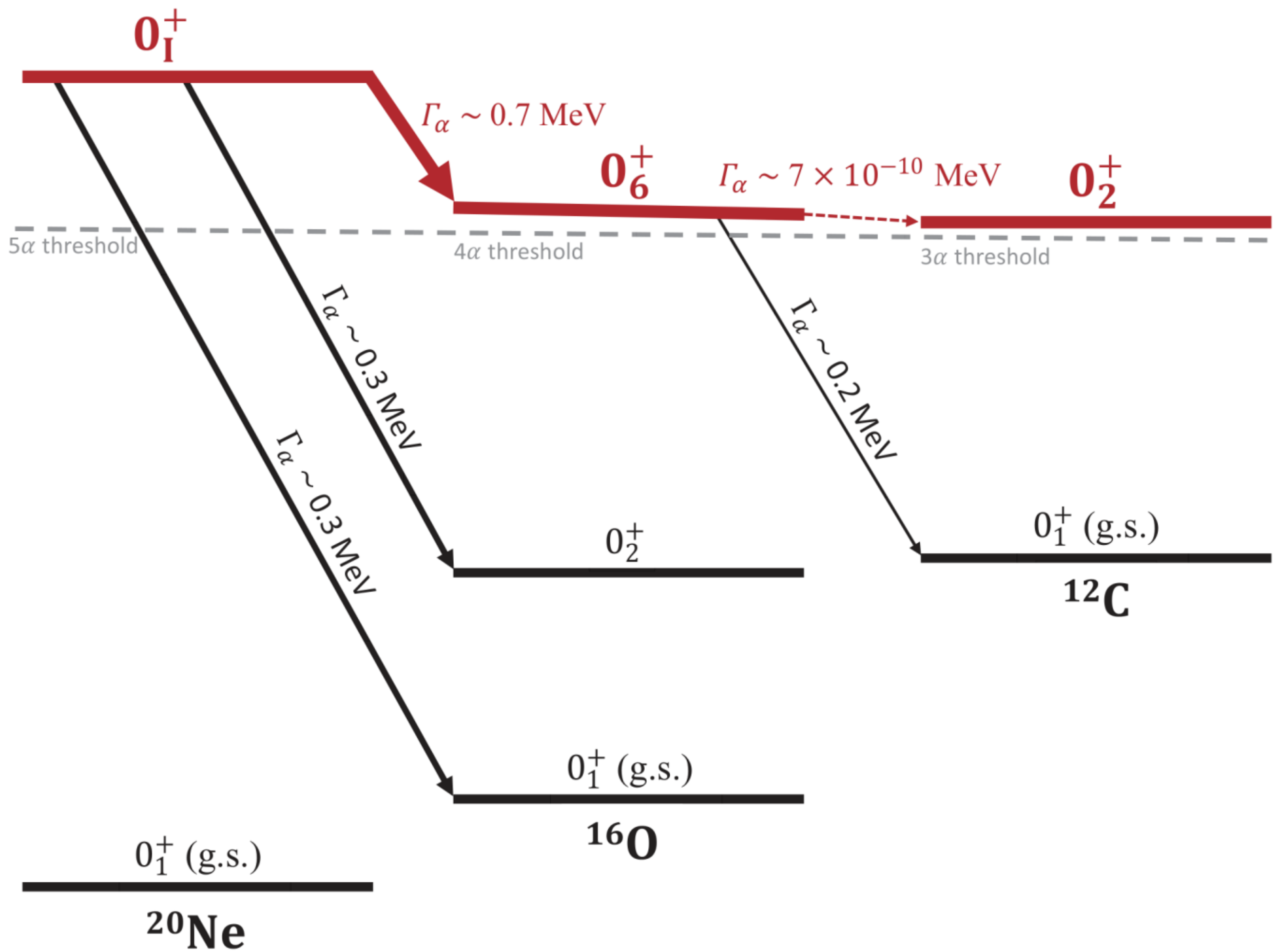
Another simple way to confirm the condensate state



B.Zhou, *et al. Nat Commun* **14**, 8206 (2023).

The decay scheme and connections

Exotic clustering structure ?



B.Zhou, *et al. Nat Commun* **14**, 8206 (2023).

The 6α THSR calculations

A. Tohsaki et al. / Nuclear Physics A738 (2004) 259–263 261

Table 1
The independent number of permutations for each kernel. Here, the case of the norm kernel for ^{24}Mg is added. The final row shows a full number of permuations without any reduction for the norm kernel.

	^{8}Be(2α)	^{12}C(3α)	^{16}O(4α)	^{20}Ne(5α)	^{24}Mg(6α)
norm	3	9	35	185	1614
kinetic	7	34	242	2546	
two-body	9	58	669	10912	
three-body	40	366	6773	156617	
$(n!)^4$	16	1296	3.32×10^5	2.07×10^8	2.79×10^{11}

$$\left\langle \Psi_{n\alpha}^{\mathrm{THSR}}(\beta) \left| \mathcal{O} \right| \Psi_{n\alpha}^{\mathrm{THSR}}(\beta') \right\rangle = \sum_{p=0}^{m_p^{(1)}-1} W_p^{(1)} I_p^{(1)} = \left(a_0 a_0'\right)^{-3n/2} \sum_{l=0} t^l \sum_{m=n_p} \gamma_l^{(1)} x^m$$

Neutron Pairs Condense in Excited Helium-8

Yoshiko Kanada-En'yo originally proposed the dineutron condensate of ^{8}He (Phys. Rev. C **88**, 034321 (2013))

PHYSICAL REVIEW LETTERS **131,** 242501 (2023)

Editors' Suggestion | Featured in Physics

Observation of the Exotic 0_2^+ Cluster State in ^{8}He

Z. H. Yang,[1,2,*,†] Y. L. Ye,[1,*,‡] B. Zhou,[3,4,5] H. Baba,[2] R. J. Chen,[6] Y. C. Ge,[1] B. S. Hu,[1] H. Hua,[1] D. X. Jiang,[1] M. Kimura,[2,5,7] C. Li,[2] K. A. Li,[6] J. G. Li,[1] Q. T. Li,[1] X. Q. Li,[1] Z. H. Li,[1] J. L. Lou,[1] M. Nishimura,[2] H. Otsu,[2] D. Y. Pang,[8] W. L. Pu,[1] R. Qiao,[1] S. Sakaguchi,[2,9] H. Sakurai,[2] Y. Satou,[10] Y. Togano,[2] K. Tshoo,[10] H. Wang,[2,11] S. Wang,[2] K. Wei,[1] J. Xiao,[1] F. R. Xu,[1] X. F. Yang,[1] K. Yoneda,[2] H. B. You,[1] and T. Zheng[1]

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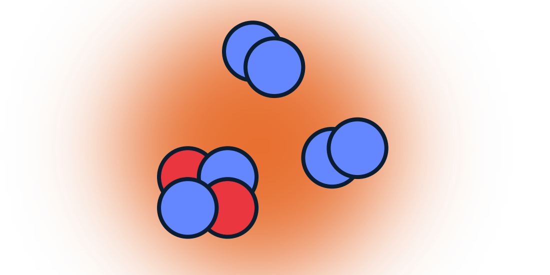

0_2^+ state

$$\Phi(\boldsymbol{B}, b_n) \propto \mathcal{A}\left\{\exp\left[-\frac{4\xi_1^2}{3\boldsymbol{B}^2} - \frac{3\xi_2^2}{2\boldsymbol{B}^2}\right] \times \phi_\alpha(b_\alpha)\phi_{2_1 n}(b_n)\phi_{2_2 n}(b_n)\right\},$$

(new trial wave function)

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Letter

New trial wave function for the nuclear cluster structure of nuclei

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A new trial wave function is proposed for nuclear cluster physics, in which an exact solution to the long-standing center-of-mass problem is given. In the new approach, the widths of the

$$\Psi(\mathbf{r}) = \Phi_{\rm g}(\mathbf{r}_g)\Phi_{\rm int}(\mathbf{r}_i - \mathbf{r}_j)$$

$$\begin{aligned}\Psi_{\rm new} &= \widehat{L}_{n-1}(\beta)\widehat{G}_n(\beta_0)\widehat{D}(\boldsymbol{Z})\Phi_0(r)\\ &= \int d^3\widetilde{T}_1\cdots d^3\widetilde{T}_{n-1}\exp[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}]\int d^3R_1\cdots d^3R_n\exp[-\sum_{i=1}^{n}(\frac{A_i}{\beta_0^2-2b_i^2})(\boldsymbol{R}_i-\boldsymbol{Z}_i-\boldsymbol{T}_i)^2]\Phi_0(\boldsymbol{r}-\boldsymbol{R})\\ &= n_0\exp[-\frac{A}{\beta_0^2}X_g^2]\,\mathcal{A}\{\prod_{i=1}^{n-1}\exp[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i-\boldsymbol{S}_i)^2]\prod_{i=1}^{n}\phi_i^{\rm int}(b_i)\}.\end{aligned}$$

a tool for studying the cluster correlations

Cluster structure of $3\alpha + p$ states in ^{13}N

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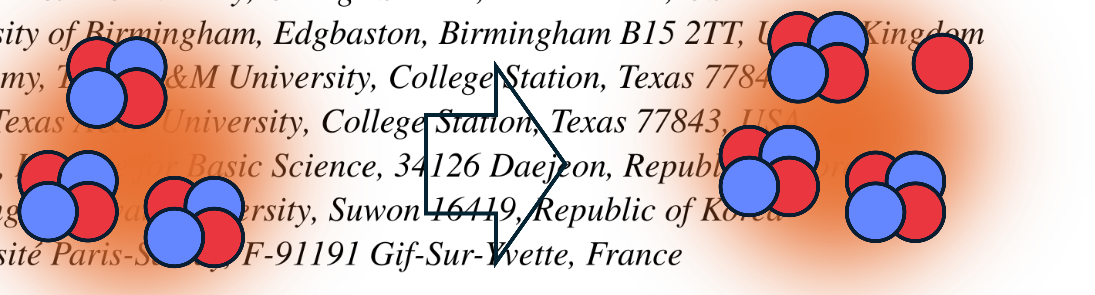
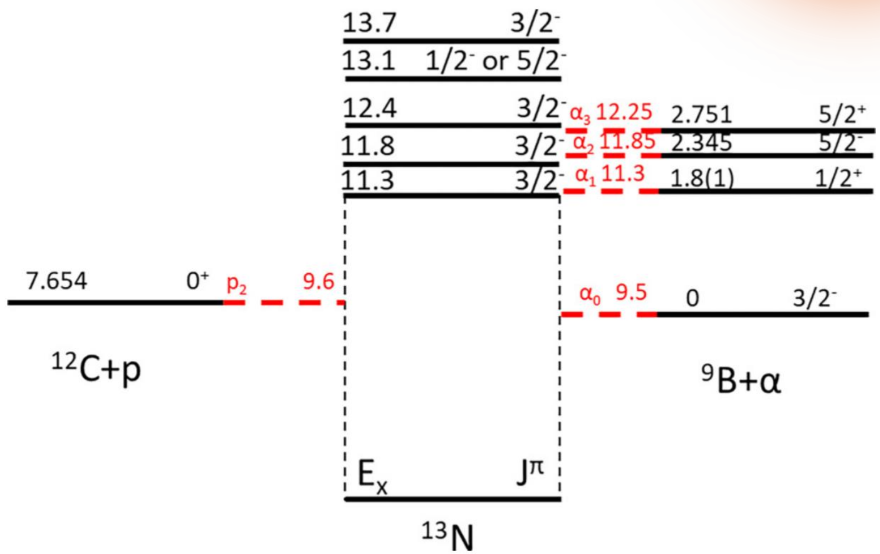
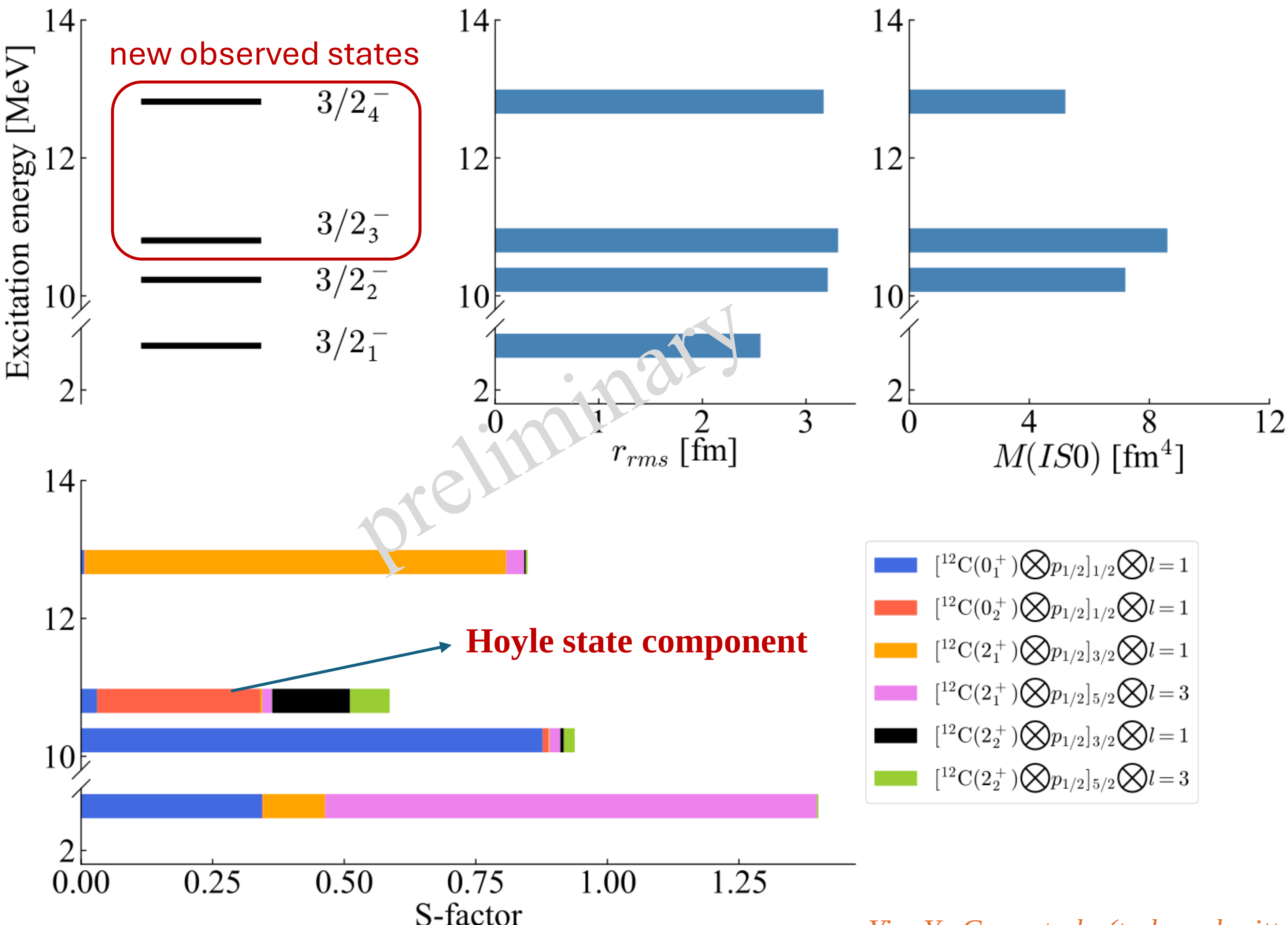




State		Counts					
E_x	J^π	α_0	α_1	α_3	p_0 [13]	p_1 [13]	p_2
11.3(1)	$(3/2^-)$	18(4.4)	0	0	6(2.6)	<3	7(2.8)
11.8(1)	$(3/2^-)$	<1.8	0	0	28(14)	<4	4(2.2)
12.4(1)	$(3/2^-)$	22(4.8)	4(2.2)	0	<3	<10	5(2.5)
13.1	$(1/2^-)$ $(5/2^-)$	0	3(2)	5(2.5)	21(6)	<10	0
13.7(1)	$(3/2^-)$	1(1.4)	3(2)	4(2.2)	<3	<10	6(2.7)

Conclusions: These states are seen to have a $[^9\mathrm{B(g.s)} \otimes \alpha / p + {}^{12}\mathrm{C}(0_2^+)]$, $[^9\mathrm{B}(\frac{1}{2}^+) \otimes \alpha]$, $[^9\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$, and $[^9\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$ structure, respectively. A previously seen state at 11.8 MeV was also determined to have a $[p + {}^{12}\mathrm{C(g.s.)} / p + {}^{12}\mathrm{C}(0_2^+)]$ structure. The overall magnitude of the clustering is not able to be extracted, however,

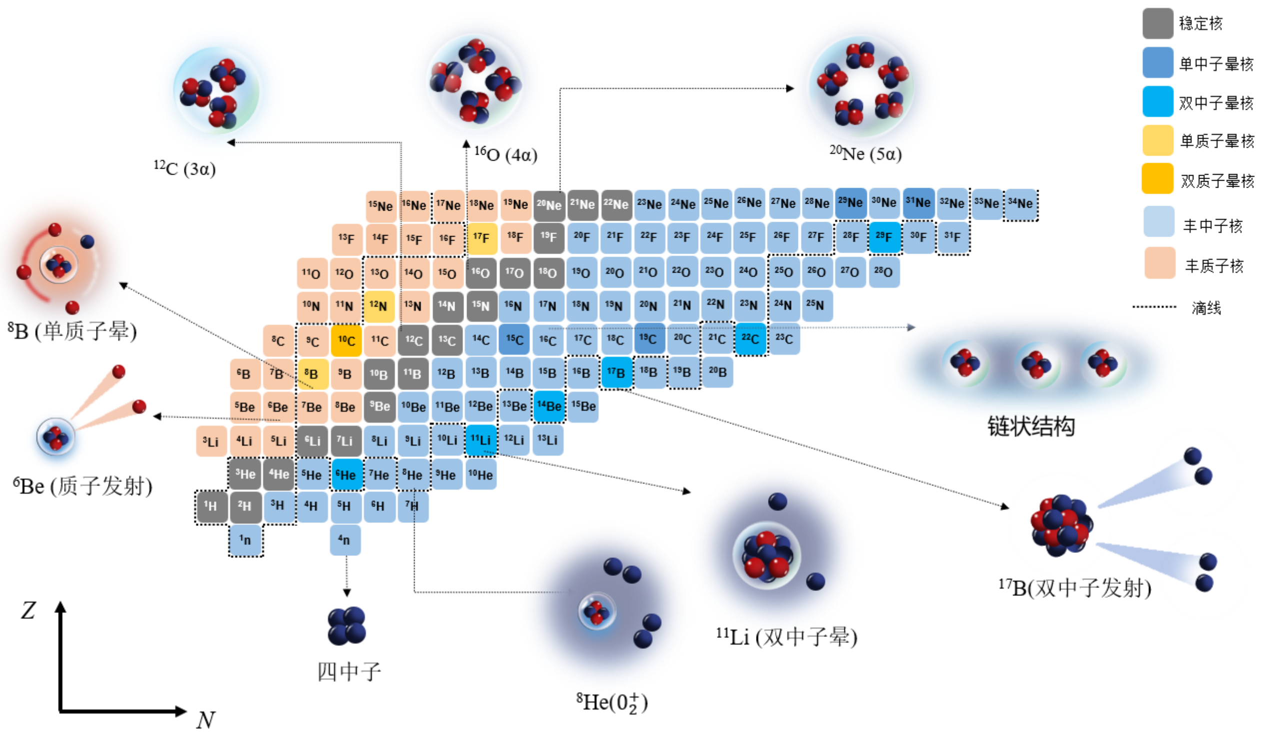
The 3/2⁻ states of ^{13}N



YingYu Cao et al., (to be submitted)

Summary and Prospect

- The Nα condensate problem. Study of ^{24}Mg is in progress.
- Search for the novel clustering states in Nα+X system.



Thanks for my collaborators and your attention.