

$$B_y(t) = (\partial_t \eta) \sin \phi \sin \theta + (\partial_t \theta) / 2 \cos \phi$$