

$$s_0(\varphi) = (\partial_\eta \cosh \rho + (\partial_\rho)^2$$