

$$\eta_{\text{opt}}(t) = - \int_{t_i}^t \frac{\cosh \rho(\tau)}{2 \cosh 2 \rho(\tau)} \left[\partial_\tau \phi(\tau) \right] d\tau$$