Theoretical prediction of low energy neutrino-nucleon interactions

(nucleon as the simplest nucleus)

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The 23th International Conference on Few-Body Problems in Physics (FB23)

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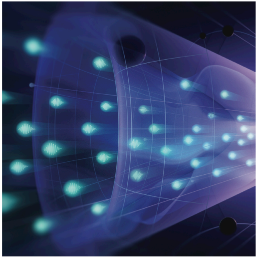
Outline

1. Introduction
2. Neutral current elastic neutrino-nucleon scattering
3. Weak single pion production off the nucleon
4. Summary and Outlook

I. Introduction

Neutrino physics

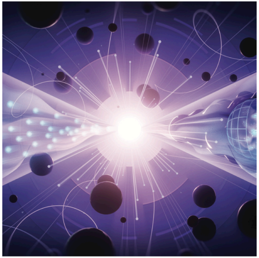
❑ Report of the 2023 P5 [https://www.usparticlephysics.org/2023-p5-report/]



Decipher the Quantum Realm

Elucidate the Mysteries of Neutrinos

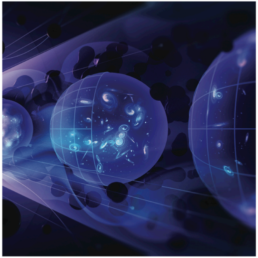
Reveal the Secrets of the Higgs Boson



Explore New Paradigms in Physics

Search for Direct Evidence of New Particles

Pursue Quantum Imprints of New Phenomena



Illuminate the Hidden Universe

Determine the Nature of Dark Matter

Understand What Drives Cosmic Evolution

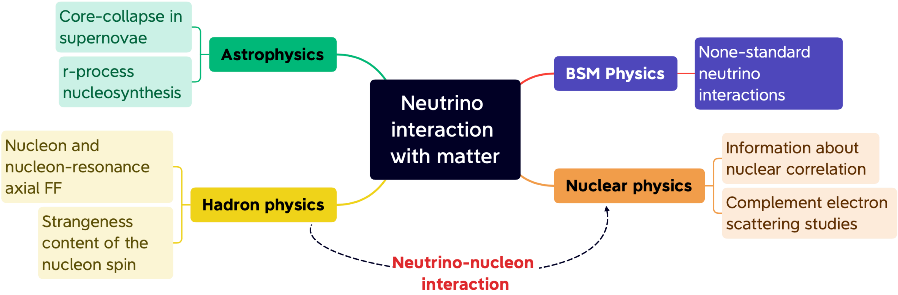
❑ Elucidate the mysteries of neutrinos

- ☞ US, etc: DUNE, IceCube-Gen2, NOνA, T2K, LEGEND , XLZD, nEXO, ...
- ☞ In China: JUNO, JUNO-TAO, PandaX-xT, CDEX-1T, CNUF ...

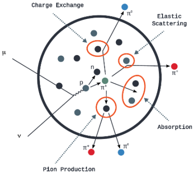
Neutrino interaction with matter

- At the heart of many interesting and relevant physical phenomena

[Neutrinos in particle physics , astronomy and cosmology, Z-Z. Xing and S. Zhou, 2010]



- Neutrino-nucleon scattering:



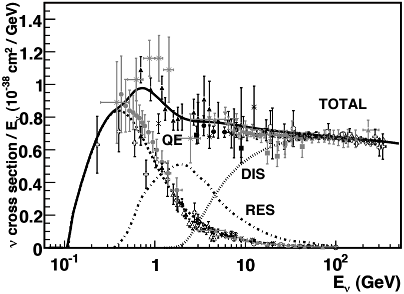
$\rightarrow$ a bridge connecting hadron physics and nuclear physics

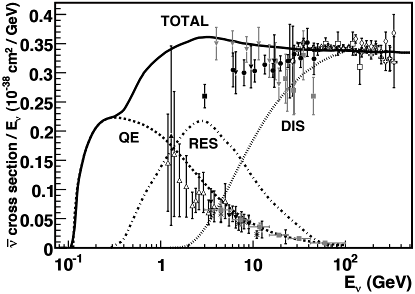
$\rightarrow$Important contribution to the inclusive neutrino-nuclei (νA) cross section

Processes of neutrino-nucleon scattering

- Two categories of processes:
  Charged-Current (CC) & Neutral-Current (NC) induced.

- Different processes in different energy regions





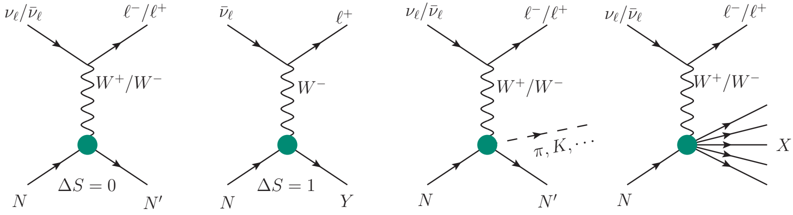
Total cross section per nucleon (Prediction by NUANCE generator).

☞ Charged current: **CCQE** $\Longrightarrow$ **IS** (RES,CC1$\pi, \cdots$) $\Longrightarrow$ **DIS**

☞ Neutral current : **NCE** $\Longrightarrow$ **IS** (RES,NC1$\pi, \cdots$)$\Longrightarrow$ **DIS**

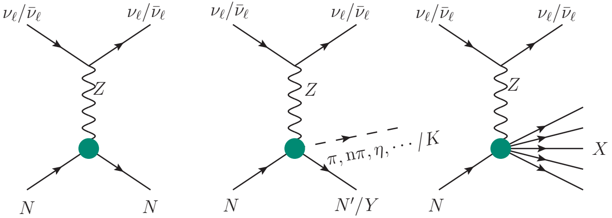
Processes of neutrino-nucleon scattering

- CC processes:



CCQE IE DIS

- NC processes:



NCE IE DIS

II. Neutral current elastic νN scattering

J. M. Chen, Z. R. Liang and **DLY**, Front.Phys.(Beijing) 19 (2024) 6, 64202

NCE νN scattering

- Low energies:

$$\text{NCE}: \ \nu + N \to \nu + N \ , \quad \text{CCQE}: \ \nu_\ell + N \to \ell + N$$

NCE processes are sensitive both to isovector and isoscalar weak current!

- Strangeness contribution to the nucleon spin $\Delta s = G_A^s(Q^2 = 0)$
  - 1980s, the E734 experiment at BNL: $0.45 \leq Q^2 \leq 1.05$ GeV^2

    [Ahrens et al., PRD1985]
  - 2010 & 2015, the MiniBooNE experiment at FNAL: $Q^2 \leq 2$ GeV^2

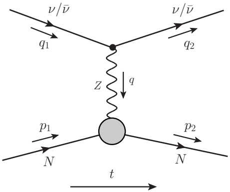
    [Aguilar-Arevalo et al., PRD2010 & PRD2015)]
  - The future MicroBooNE experiment in Argon: $0.1 \leq Q^2 \leq 1$ GeV^2

    [Ren, JPS Conf. Proc. 37, 020309 (2022).]
- Various parametrizations for form factors
  - Dipole parametrization
  - z expansion
  - ...

A model-independent and systematical study is needed!

Kinematics & amplitude structure

❑ Kinematics: $\nu(q_1) + N(p_1) \to \nu(q_2) + N(p_2)$



$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} L_\mu H^\mu \ ,$$

Leptonic part: $L_\mu = \bar{\nu}(q_2)\gamma_\mu(1-\gamma_5)\nu(q_1)$,

Hadronic part: $H^\mu = \langle N(p_2)|\mathcal{J}^\mu_{NC}(0)|N(p_1)\rangle$.

❑ Hadronic amplitude → 6 form factors (FFs)

☞ Isospin structure: isovector (V) & isoscalar (S)

$$H^\mu = \chi_f^\dagger \left[\frac{\tau_a}{2} H_V^\mu + \frac{\tau_0}{2} H_S^\mu\right] \chi_i \ , \quad a = 3,$$

$$H_V^\mu = (1 - 2\sin^2\theta_W) V_V^\mu - A_V^\mu \ , \quad H_S^\mu = -2\sin^2\theta_W V_S^\mu \ .$$

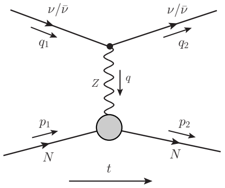
☞ Lorentz decomposition:

$$V^\mu_{V,S} = \bar{\mathbf{u}}(p_2)\left[\gamma^\mu F_1^{V,S}(t) + \frac{i}{2m_N}\sigma^{\mu\nu} q_\nu F_2^{V,S}(t)\right]\mathbf{u}(p_1),$$

$$A^\mu_V = \bar{\mathbf{u}}(p_2)\left[\gamma^\mu\gamma_5 G_A(t) + \frac{q^\mu}{m}\gamma_5 G_P(t)\right]\mathbf{u}(p_1) \ .$$

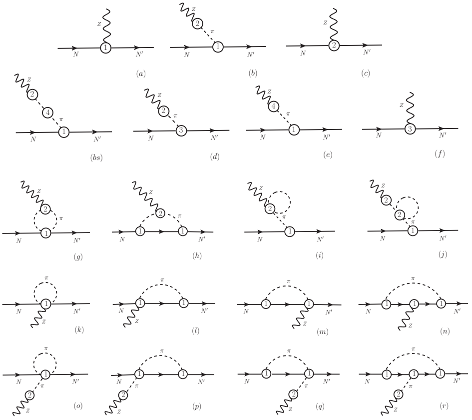
Form factors from BChPT

- Calculation up to $\mathcal{O}(p^3)$



black box

BChPT as a key



Feynman Diagrams

Form factors from BChPT

❑ Form factors in a chiral series

$$
\begin{aligned}
F_1^V(t) =& 1 - 2d_6 t + F_1^{V,\text{loops}} + F_1^{V,\text{wf}} \ , \\
F_2^V(t) =& c_6 + 2d_6 t + F_2^{V,\text{loops}} + F_2^{V,\text{wf}} \ , \\
F_1^S(t) =& 1 - 4d_7 t + F_1^{S,\text{loops}} + F_1^{S,\text{wf}} \ , \\
F_2^S(t) =& (c_6 + 2c_7) + 4d_7 t + F_2^{S,\text{loops}} + F_2^{S,\text{wf}} \ , \\
G_A(t) =& g + (4d_{16} M^2 + d_{22} t) + G_A^{\text{loops}} + G_A^{\text{wf}} \ , \\
G_P(t) =& \frac{2g m_N^2}{M^2 - t} + \frac{4m_N^2 M^2 (2d_{16} - d_{18})}{M^2 - t} + \frac{4g m_N^2 M^2 \ell_4}{F^2(M^2 - t)} - 2m_N^2 d_{22} \\
& - \frac{4g m_N^2 M^2 \left[M^2 \ell_3 + (M^2 - t)\ell_4\right]}{F^2 (M^2 - t)^2} + G_P^{\text{loops}} + G_P^{\text{wf}} \ ,
\end{aligned}
$$

Remarks:

- ☞ Wave function renormalization
- ☞ UV divergences: dimensional regularizaion (DR) with $\overline{\text{MS}}$-1 ($\widetilde{\text{MS}}$) subtraction
- ☞ **PCB terms**: EOMS scheme

NCE scattering within EOMS scheme

- **Essence:** two-step renormalization ($\widetilde{\mathrm{MS}}$+finite)

1. UV subtraction:

$$m = m^r(\mu) + \beta_m \frac{R}{16\pi^2 F^2} \ ,$$

$$g = g^r(\mu) + \beta_g \frac{R}{16\pi^2 F^2} \ ,$$

$$c_i = c_i^r(\mu) + \beta_{c_i} \frac{R}{16\pi^2 F^2} \ ,$$

$$d_j = d_j^r(\mu) + \beta_{d_j} \frac{R}{16\pi^2 F^2} \ .$$

2. Finite subtraction:

$$m^r(\mu) = \widetilde{m} + \frac{\widetilde{\beta}_m}{16\pi^2 F^2} \ ,$$

$$g^r(\mu) = \widetilde{g} + \frac{\widetilde{\beta}_g}{16\pi^2 F^2} \ ,$$

$$c_i^r(\mu) = \widetilde{c}_i + \frac{\widetilde{\beta}_{c_i}}{16\pi^2 F^2} \ .$$

- **Advantages:**
  - ☞ Power counting is restored $\longrightarrow$ predictive power
  - ☞ Respect original analytic properties $\longrightarrow$ spectroscopy (poles and cuts), chiral extrapolation, finite volume corrections
  - ☞ Fast convergency behaviour in many cases, w.r.t. IR, HB, etc

Observables for physical processes

❑ Differential cross sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 m_N^2}{8\pi E_\nu^2}\left[A(Q^2) \pm \frac{(s-u)}{m_N^2}B(Q^2) + \frac{(s-u)^2}{m_N^4}C(Q^2)\right]$$

☞ Convenient scalar functions A, B and C: ($\eta = Q^2/4m_N^2$ & No G_P for NCE)

$$A(Q^2) \equiv 4\eta\left[\mathcal{G}_A^2(Q^2)\left(1+\eta\right) + 4\eta\mathcal{F}_1(Q^2)\mathcal{F}_2(Q^2) - \left(\mathcal{F}_1^2(Q^2) - \eta\mathcal{F}_2^2(Q^2)\right)\left(1-\eta\right)\right]$$

$$B(Q^2) \equiv 4\eta\ \mathcal{G}_A(Q^2)\left(\mathcal{F}_1(Q^2) + \mathcal{F}_2(Q^2)\right)$$

$$C(Q^2) \equiv \frac{1}{4}\left[\mathcal{G}_A^2(Q^2) + \mathcal{F}_1^2(Q^2) + \eta\mathcal{F}_2^2(Q^2)\right]$$

☞ Relationship between isospin and physical bases

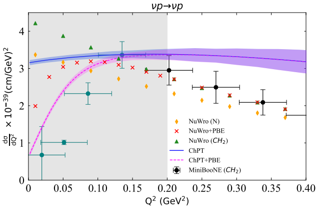
$$\mathcal{F}_i(t) = \cos 2\theta_W F_i^V(t)\frac{\mathcal{C}_3}{2} - 2\sin^2\theta_W F_i^S(t)\frac{\mathcal{C}_0}{2}\ , \quad i = 1, 2,$$

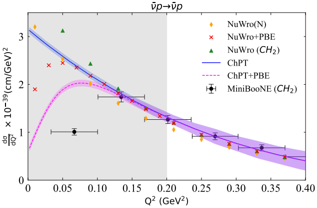
$$\mathcal{G}_j(t) = G_j(t)\frac{\mathcal{C}_3}{2}\ , \quad j = A, P,$$

physical process	$\mathcal{C}_3$	$\mathcal{C}_0$	physical process	$\mathcal{C}_3$	$\mathcal{C}_0$
$\nu + p \to \nu + p$	1	1	$\nu + n \to \nu + n$	-1	1
$\bar\nu + p \to \bar\nu + p$	1	1	$\bar\nu + n \to \bar\nu + n$	-1	1

Differential cross section

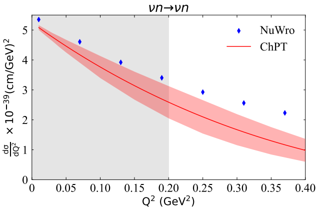
❑ Proton channels

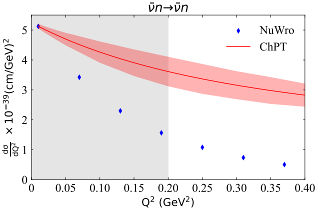




- ☞ Sizeable Pauli blocking effects
- ☞ Contribution of strangeness axial form factor?

❑ Neutron channels

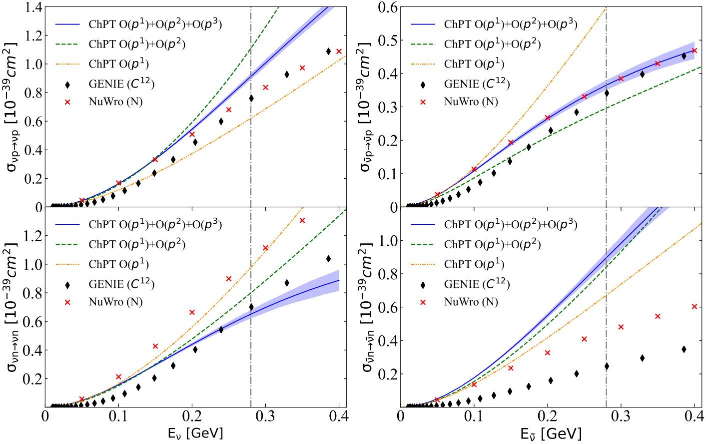




- ☞ No experimental data
- ☞ Large deviation from the NuWro results

Total cross section

❑ Order by order



- ☞ Our ChPT results deviate from the NuWro ones for neutron channels
- ☞ Large difference between NuWro and GENIE due to nuclear effects

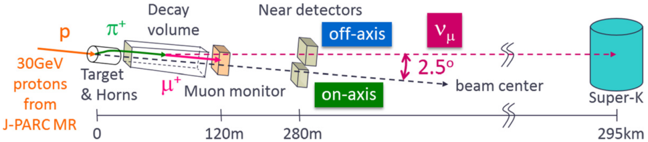
III. Weak single pion production

[**DLY**, L. Alvarez-Ruso, A. N. Hiller Blin and M. J. Vicente Vacas, PRD2018]
[**DLY**, L. Alvarez-Ruso and M. J. Vicente Vacas, PLB2019]

Weak single pion production

❑ Oscillation experiments (e.g. T2K)

- survival probability of ν_μ: $P(\nu_\mu) = 1 - \sin^2 2\theta_{\mu\tau} \cdot \sin^2 \frac{\Delta m_{23} L}{E_\nu}$

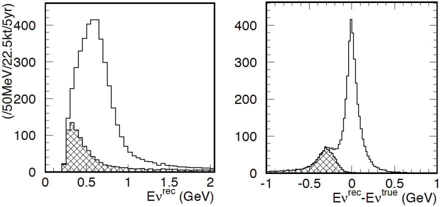


❑ Source of experimental uncertainties

CC 1π:

☞ CCQE-like events: misiden. of pion

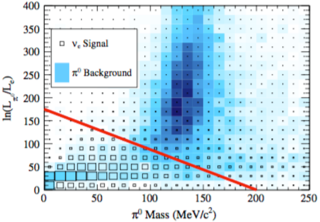
☞ to be subtracted for a good E_ν



NC 1π:

☞ e-like background to $\nu_\mu \to \nu_e$ searches

☞ improved at T2K with a π^0 rejection cut



Theoretical status

Isobar Models

- Δ and heavier resonances $\rightarrow$ nucleon-to-resonance form factors:
  [e.g., Llewellyn Smith, Phys. Rep. 3 (1972)] [Fogli and Nardulli, Nucl. Phys. B160 (1979)] [Rein and Sehgal, Ann. Phys. (1981)]
  - Real form factor from quark models
  - Conserved vector current $\rightarrow$ related to electromagnetic ones extracted from electron scattering data
  - PCAC $\rightarrow$ off-diagonal Goldberger-Treiman (GT) relation for the axial couplings
- Nonresonant mechanisms
  [Fogli and Nardulli, Nucl. Phys. B160 (1979)] [Bijtebier, Nucl. Phys. B21 (1970)] [Alevizos et al., J. Phys. G 3(1977)]

Hernandez-Nieves-Valverde (HNV) Model

- Δ resonances & non-resonant terms $\rightarrow$ constrained by chiral symmetry at threshold [Hernandez, Nieves and Valverde, Phys. Rev. D (2007)]
- Final state interaction: imposing Watson's theorem [Alvarez-Ruso et al., Phys. Rev. D 93 (2016)]
- Unphysicsal spin-1/2 components: adding new contact terms
  [Hernandez and Nieves, Phys. Rev. D (2017)]

Theoretical status: χEFT

- Other Models:
  - ☞ Dynamical model: coupled-channel Lippmann Schwinger equation
    - Fulfilling Watson's theorem
    - PCAC $\rightarrow$ partially constrain the axial current in terms of πN scattering amplitude fitted to data [Nakamura, Kamano and Sato, Phys. Rev. D (2015)]
  - ☞ Chiral effective model with π, N, Δ together with σ, ρ, ω
    - Power counting only for tree diagrams [Serot and Zhang, Phys. Rev. C (2012)]
  - ☞ etc.

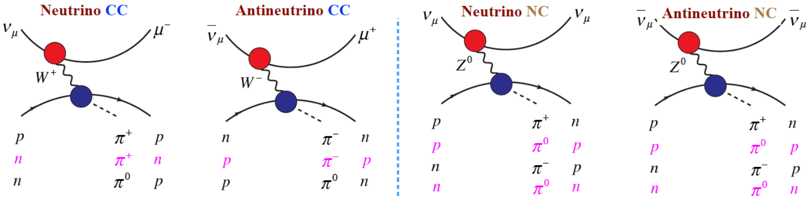
- Low energy regime:

  Chiral symmetry + Power counting + Perturbative Unitarity

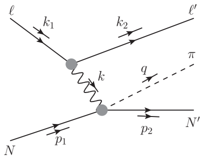
- Baryon Chiral Perturbation Theory **(BChPT)**
  - ☞ Low-Energy theorems (axial only) at threshold using heavy baryon formalism [Bernard, Kaiser and Meißner, Phys. Lett. B (1994)]
  - ☞ Our work: One-loop analyses in relativistic BChPT with explicit Δs [DLY, Alvarez-Ruso, Hiller-Blin and Vicent-Vacas, Phys. Rev. D (2018)] [DLY, Alvarez-Ruso and Vicent-Vacas, Phys. Lett. B (2019)]

Leptonic and Hadronic parts

- Physical channels (3 for CC & 4 for NC)



- Amplitude structure:
  - One-boson approximation and $k^2 \ll M_B^2$
  - Leptonic part L_ν is well-known; Hadronic part H_μ needs to be investigated.



$$= i(2\pi)^4\delta^{(4)}(k_1+p_1-k_2-p_2-q)\frac{iN^2}{M_B^2}\underbrace{\langle \ell'|J_\nu(0)|\ell\rangle}_{L^\mu}\underbrace{\langle \pi N'|J_\mu(0)|N\rangle}_{H_\mu}$$

Convenient isospin decomposition

- Isospin even $(+)$, isospin odd $(-)$, isoscalar (0)

$$\langle \pi^b N' | J^a_\mu(0) | N \rangle = \chi^\dagger_f \left[\delta^{ba} H^+_\mu + i\epsilon^{bac}\tau^c H^- + \tau^b H^0_\mu \right] \chi_i$$

- The physical amplitudes constructed from the isospin amplitudes

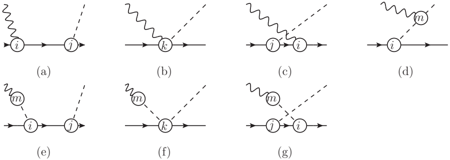
$$H_\mu(\text{physical process}) \quad = \quad a_+ H^+_\mu + a_- H^-_\mu + a_0 H^0_\mu$$

	Physical Process	a_+	a_-	a_0
NC	$Z^0 p \to p\pi^0$	1	0	1
	$Z^0 n \to n\pi^0$	1	0	-1
	$Z^0 n \to p\pi^-$	0	$-\sqrt{2}$	$\sqrt{2}$
	$Z^0 p \to n\pi^+$	0	$\sqrt{2}$	$\sqrt{2}$
CC	$W^+ p \to p\pi^+$ / $W^- n \to n\pi^-$	1	-1	0
	$W^+ n \to n\pi^+$ / $W^- p \to p\pi^-$	1	1	0
	$W^+ n \to p\pi^0$ / $W^- p \to n\pi^0$	0	$\sqrt{2}$	0

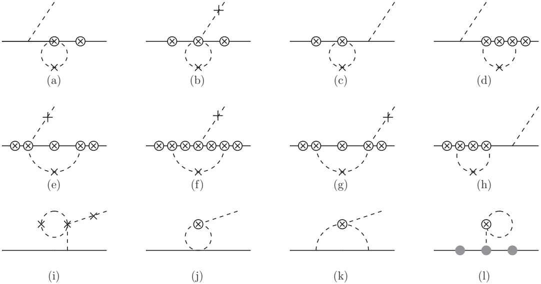
- The CC and NC amplitudes are related to each other
  - For CC, $H^\pm_\mu = \sqrt{2}\cos\theta_C (V^\pm_\mu - A^\pm_\mu)$, $\quad H^0_\mu = 0.$
  - For NC, $H^\pm_\mu = (1 - 2\sin^2\theta_W) V^\pm_\mu - A^\pm_\mu$, $\quad H^0_\mu = (-2\sin^2\theta_W) V^0_\mu$

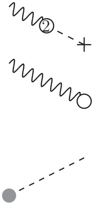
The hadronic amplitude

❑ Tree diagrams up through $O(p^3)$:



❑ All possible loop diagrams at $O(p^3)$:





89 diagrams & wave function renormalization & EOMS

Necessity of the Δ resonance

- Δ is strongly coupled to the final πN system
  - BR($\Delta \to \pi N) \simeq 99.4\%$
  - Close to πN threshold: $\Delta = m_\Delta - m_N \sim 300$ MeV
- Strategy: the δ-counting [Pascalutsa and Phillips, Phys. Rev. C67 (2012)]
  - hierachy of scales: $M_\pi \sim p \ll \Delta \ll \Lambda \sim 4\pi F_\pi$
  - expanding parameter: $\delta = \frac{\Delta}{\Lambda} \sim \frac{M_\pi}{\Delta} \sim \frac{p}{\Delta} \longrightarrow \frac{1}{\not{p} - m_\Delta} = \frac{1}{\not{p} - m_N - \Delta} \sim p^{-\frac{1}{2}}$
- Counting rule:

$$\text{chiral order } D = 4L + \sum_k k V^{(k)} - 2I_\pi - I_N - \frac{1}{2} I_\Delta$$

  - only trees of $O(p^{3/2})$ and $O(p^{5/2})$
  - No loop diagrams with explicit Δ up through $O(p^3)$
- The width effect

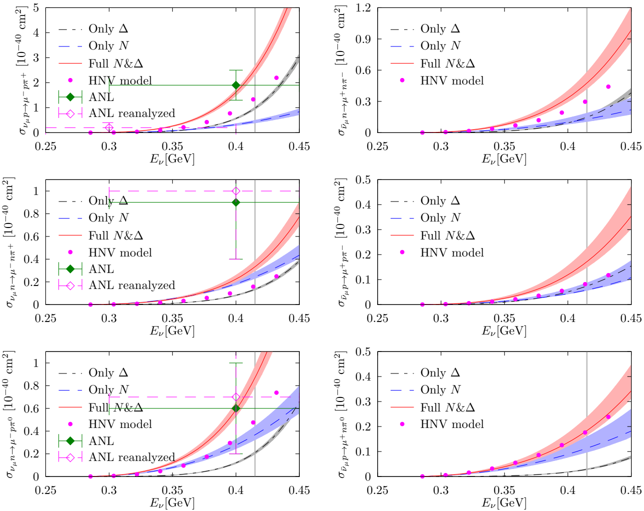
$$\frac{1}{m_\Delta^2 - s_\Delta} \to \frac{1}{m_\Delta^2 - i m_\Delta \Gamma_\Delta(s_\Delta) - s_\Delta}$$

Energy dependent width $\Gamma_\Delta(s_\Delta)$ calculated in the same scheme

[Gegelia et al, Phys. Lett. B(2016)]

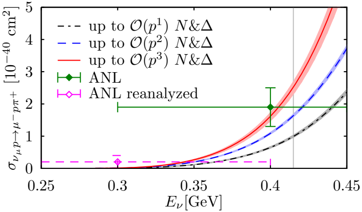
Cross sections for CC1π

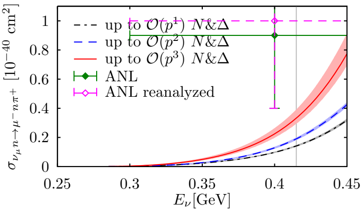
- Fairly good agreement with the ANL data for most of the channels except for $\nu_\mu n \to \mu^- n\pi^+$

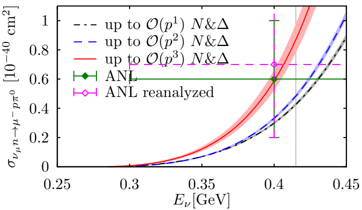


Cross sections for CC1π

- Order by order
  - Quite significant contribution when stepping from $O(p^2)$ and $O(p^3)$
  - Next-order effects could still be relevant
    (especially loops that πN can be put on-shell)

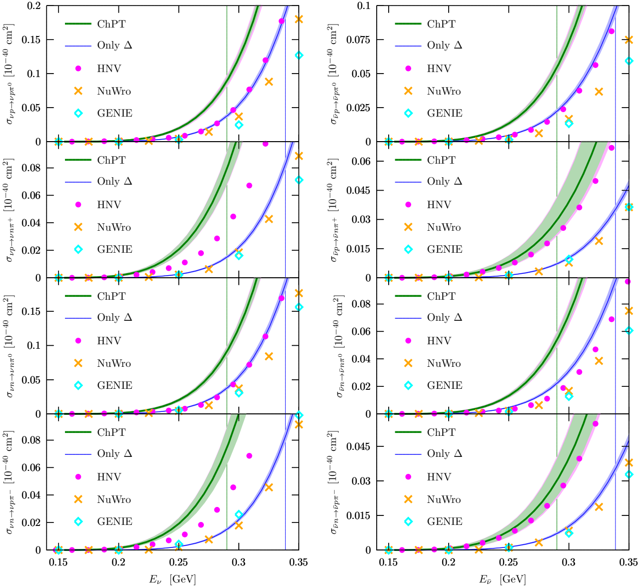






Cross sections for NC1π

- The $O(p^3)$ ChPT calculation produces considerably larger cross sections with respect to the HNV model in all reaction channels.
- Nuwro and GIENE results agree with the ChPT ones with Δ contribution.
- Non-resonant contribution is sizeable, not accounted by Nuwro and GIENE.



IV. Summary and outlook

Summary and outlook

- Systematically study the **NCE scattering** and **weak single pion production** off the nucleon for the first time within covariant BChPT up to $O(p^3)$.
  - **NCE**: The X sections are useful for a precise determination of the strangeness axial vector form factor in future
  - **CC**1π **& NC**1π:The Δ contributes significantly to all production channels
  - **NC**1π: Non-resonant contribution is sizeable which is not implemented in events generators like NuWro and GIENE

Provide a well-founded low energy benchmark for phenomenological models aimed at the description of weak pion production in the broad kinematic range of interest for current and future neutrino-oscillation experiments.

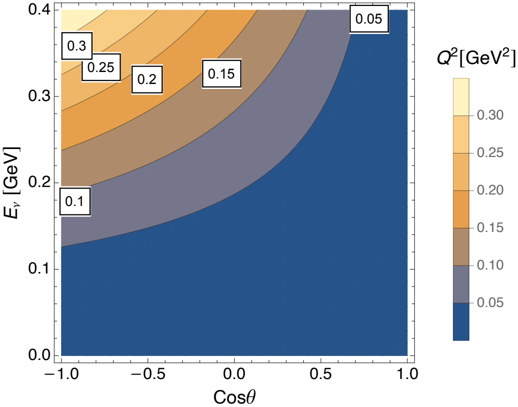
- Future application and perspective
  - Applied to study various low-energy theorems
  - Neutrino-nucleus scattering
  - Implement ChPT results in events generator?

Thank you very much for your attention!

Backup

Valid energy region of BChPT

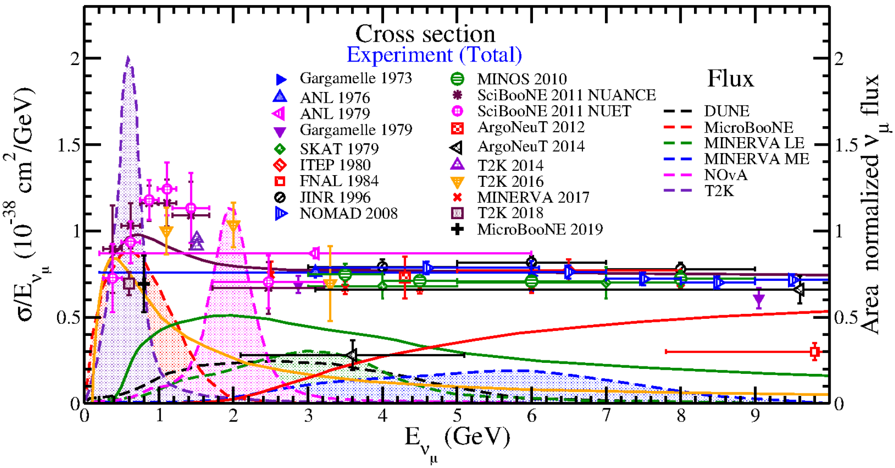
❑ Valid energy region of BChPT



☞ Square of mom. transfer $Q^2 \leq 0.2$ GeV^2 $\longrightarrow$ neutrino energy $E_\nu \leq 0.28$ GeV

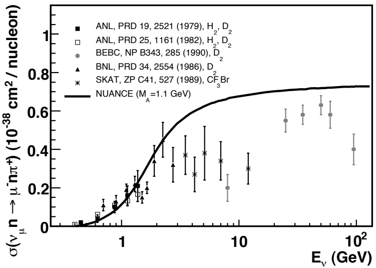
$$\boxed{\sigma = \int_{-1}^{+1} \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}\frac{\mathrm{d}Q^2}{\mathrm{d}x}\mathrm{d}x} \;, \quad Q^2 = \frac{2m_N E_\nu^2}{2E_\nu + m_N}(1-x), \quad x = \cos\theta \;, \quad \theta \in [0,\pi]$$

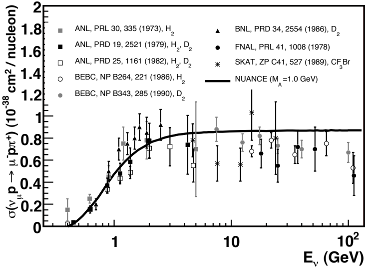
Flux X-section

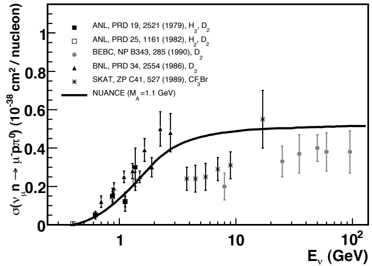


Experimental data

- CC1π





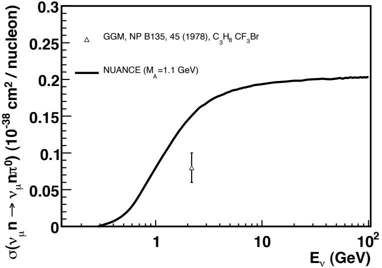


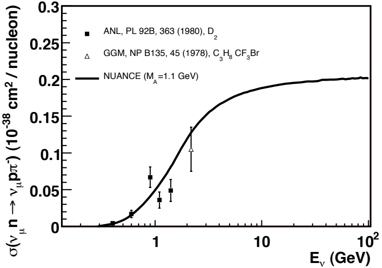
[Formaggio, Zeller, Rev. Mod. Phys. (2012)]

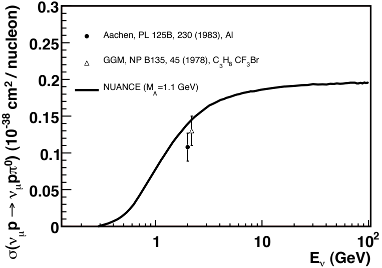
Experimental data

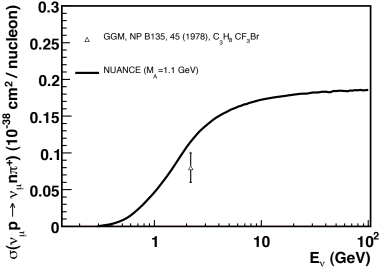
- NC1π: Very rare data below 1 GeV

[Formaggio, Zeller, Rev. Mod. Phys. (2012)]









Electroweak interaction in BChPT

- **Covariant BChPT** in SU(2) case.
  - Nucleonic Lagrangian [Fettes et al Ann. Phys. (2000)]

$$\mathcal{L}_N = \bar{N}\left[i\not{D} - m + \frac{g}{2}\not{u}\gamma_5\right]N + \bar{N}\left[c_j\mathcal{O}_j^{(2)} + d_k\mathcal{O}_k^{(3)}\right]N + \cdots$$

  - Purely mesonic Lagrangian [Gasser and Leutwyler, Ann. Phys. (1984)] [Gasser et al., Nucl. Phys. B307 (1988)]

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}[\Delta_\mu U(\Delta^\mu U)^\dagger + \chi U^\dagger + U\chi^\dagger] + \sum_{j=3,4,6}\ell_j\mathcal{O}_j^{(4)}$$

- **Electro-weak interactions** enter through external fields

[c.f. Scherer and Schindler, 2011, Springer]

  - Charged weak bosons $W^\pm$:

$$r_\mu = 0, \quad l_\mu = -\frac{g}{\sqrt{2}}(V_{ud}\,W_\mu^+\tau_+ + h.c.)$$

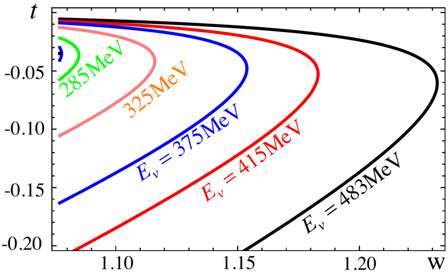
  - Neural weak boson Z^0:

$$r_\mu = e\tan(\theta_W)Z_\mu^0\frac{\tau_3}{2}, \quad l_\mu = -\frac{g}{\cos(\theta_W)}Z_\mu^0\frac{\tau^3}{2} + e\tan(\theta_W)Z_\mu\frac{\tau_3}{2},$$

$$v_\mu^{(s)} = \frac{e\tan(\theta_W)}{2}Z_\mu^0$$

Numerical settings

- Energies considered for $E_\nu \in [E_{\nu,th}, E_{\nu,\max} \equiv E_{\nu,th} + M_\pi]$
  - E.g., $E_{\nu,\max} = 415$ MeV for CC; $E_{\nu,\max} = 289$ MeV for NC
  - Well below the Δ peak $\to$ δ-counting is valid



W: CM energy of the final πN system (CC for example)

- Data for neutrino-induced single pion production off nucleons are very rare
- Values of the leading order constants

F_π	M_π	m_N	m_Δ	g_A	h_A
92.21	138.04	938.9	1232 MeV	1.27	1.43 ± 0.02

Low energy constants beyond LO

- Most of the LECs (16 out of 23) are previously determined from other processes or observables

	LEC	Value	Source
$\mathcal{L}_{\pi\pi}^{(4)}$	$\bar{\ell}_6$	16.5 ± 1.1	$\langle r^2 \rangle_\pi$ [Gasser, Leutwyler 1984]
$\mathcal{L}_{\pi N}^{(2)}$	$\tilde{c}_1$	-1.00 ± 0.04	πN scattering [Alarcon et al. 2013 & Chen et al. 2013]
	$\tilde{c}_2$	1.01 ± 0.04	
	$\tilde{c}_3$	-3.04 ± 0.02	
	$\tilde{c}_4$	2.02 ± 0.01	
	$\tilde{c}_6$	1.35 ± 0.04	μ_p and μ_n [Bauer et al. 2012 & PDG2016]
	$\tilde{c}_7$	-2.68 ± 0.08	
$\mathcal{L}_{\pi N}^{(3)}$	d_{1+2}^r	0.15 ± 0.20	πN scattering [Alarcon et al. 2013 & Chen et al. 2013]
	d_3^r	-0.23 ± 0.27	
	d_5^r	0.47 ± 0.07	
	d_{14-15}^r	-0.50 ± 0.50	
	d_{18}^r	-0.20 ± 0.80	
	d_6^r	-0.70	$\langle r_E^2 \rangle_N$ [Fuchs et al. 2014]
	d_7^r	-0.47	
	d_{22}^r	0.96 ± 0.03	$\langle r_A^2 \rangle_N$ [Yao et al. 2017]
$\mathcal{L}_{\pi N\Delta}^{(2)}$	b_1	$(4.98 \pm 0.27)/m_N$	$\Gamma_\Delta^{\rm em}$ [Bernard et al 2012]

- The remaining unknown LECs $\rightarrow$ set to natural size

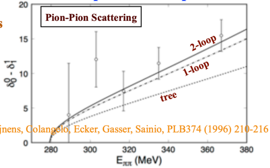
$$d_j^r = 0.0 \pm 1.0 \ \text{GeV}^{-2} \ , \quad j \in \{1, 8, 9, 14, 20, 21, 23\}$$

BChPT & PCB issue

- Pure Goldstone bosons: ChPT has gained great achievements

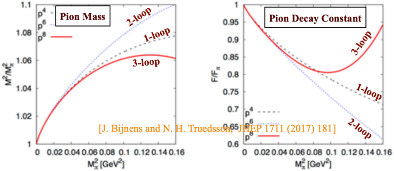
➡High-order calculations become standard
➡Fast convergence

up to Two-Loop



Bijnens, Colangelo, Ecker, Gasser, Sainio, PLB374 (1996) 210-216

up to Three-Loop



[J. Bijnens and N. H. Truedsson, JHEP 1711 (2017) 181]

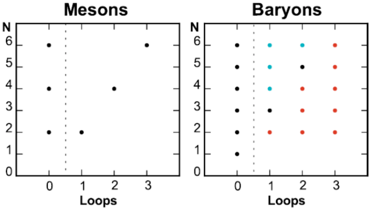
- Covariant ChPT including matter fields (Baryons, D/B mesons, Ξ_{cc})
  - ☞ Dimensional Regularization (DR) with standard $\overline{\mathrm{MS}}$-1 subtraction
  - ☞ **A systematic power counting rule is lost** due to the non-zero mass of matter fields in the chiral limit

Feynman Diagram

⇩

$\mathcal{O}(p^N)$

How important?

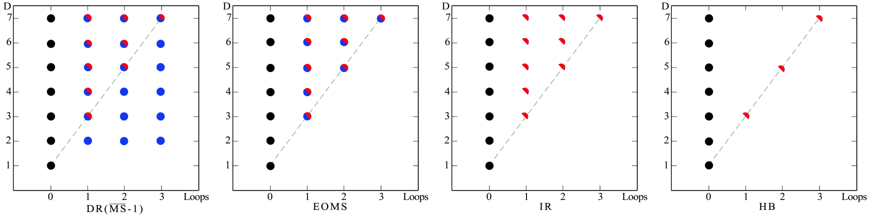


red dots denote possible PCB terms

Chiral order: $N = 4L - 2N_M - N_B + \Sigma_k k V_k$

Solution I: HB

Essence: The full integral can be separated into Infared Singular and Regular parts.



- Heavy baryon formalism (HB) [Jenkins and Manohar,PLB255' 91]

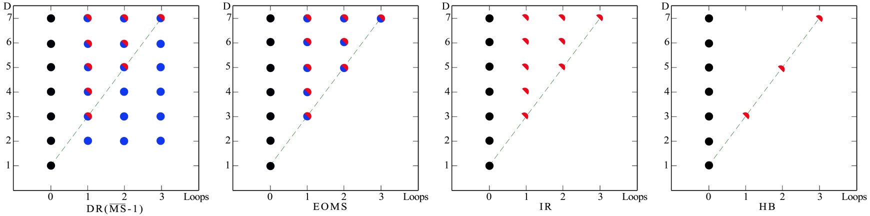
A simultaneous expansion in external momenta and $1/m_B$

- ☞ Non-covariant and slowly convergent in the threshold region. [N.Fettes,Ulf-G.Meissner and S.Steininger, NPA' 98, M.Mojzis,Eur.Phys.J.C2' 98]
- ☞ Even divergent in the sub-threshold region (e.g. scalar form factor). [V.Bernard,N.Kaiser and Ulf-G.Meissner,Int.J.Mod.Phys.E4' 95, T.Becher and H.Leutwyler,Eur.Phys.J.C9' 99]

- Infrared Regularization (IR)
- Extended-on-mass-shell scheme (EOMS)

Solution II: IR

Essence: The full integral can be separated into Infared Singular and Regular parts.



- Heavy baryon formalism (HB) [Jenkins and Manohar,PLB255' 91]
- Infrared Regularization (IR) [T.Becher and H.Leutwyler,Eur.Phys.J.C9' 99]

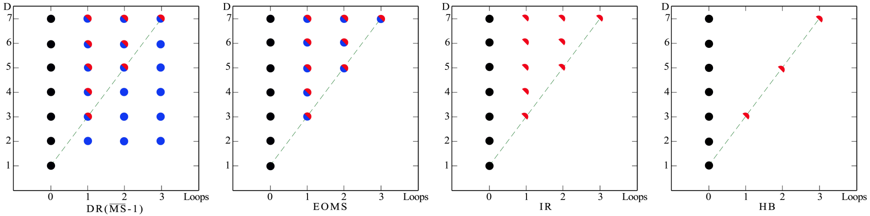
The whole series of the regular part in the full integral are dropped.

- ☞ Scale-dependence: amplitude and observables. [T.Becher and H.Leutwyler,JHEP0106' 01]
- ☞ Unphysical cuts(u=0). [J.M.Alarcon,J.Martin Camalich,J.A.Oller and L.Alvarez-Ruso,PRC83' 11]
- ☞ Bad predictions: e.g., huge Goldberger-Treiman relation violation (20-30%). [J.M.Alarcon,J.Martin Camalich,J.A.Oller and L.Alvarez-Ruso,PRC83' 11]

- Extended-on-mass-shell scheme (EOMS)

Solution III: EOMS

Essence: The full integral can be separated into Infared Singular and Regular parts.



- Heavy baryon formalism (HB) [Jenkins and Manohar,PLB255' 91]
- Infrared Regularization (IR) [T.Becher and H.Leutwyler,Eur.Phys.J.C9' 99]
- Extended-on-mass-shell scheme (EOMS) [T.Fuchs,J.Gegelia,G.Japaridze and S.Scherer,PRD68' 03]

step 1: **Ultraviolet (UV) divergence**
Dimensional regularization
MS-1 subtraction

→ Scale independent

finite ren. →

step 2: **PCB terms**
(polynomials)
removed by redefinition of LECs
in Effective Lagrangians

→ Correct power counting
(respectively faster convergence)

→ keep original analyticity
and all assumed symmetries

EOMS is a two-step renormalization scheme