Towards Inclusive Neutrino-Nucleon Interaction from Lattice QCD Calculation of Hadronic Tensor

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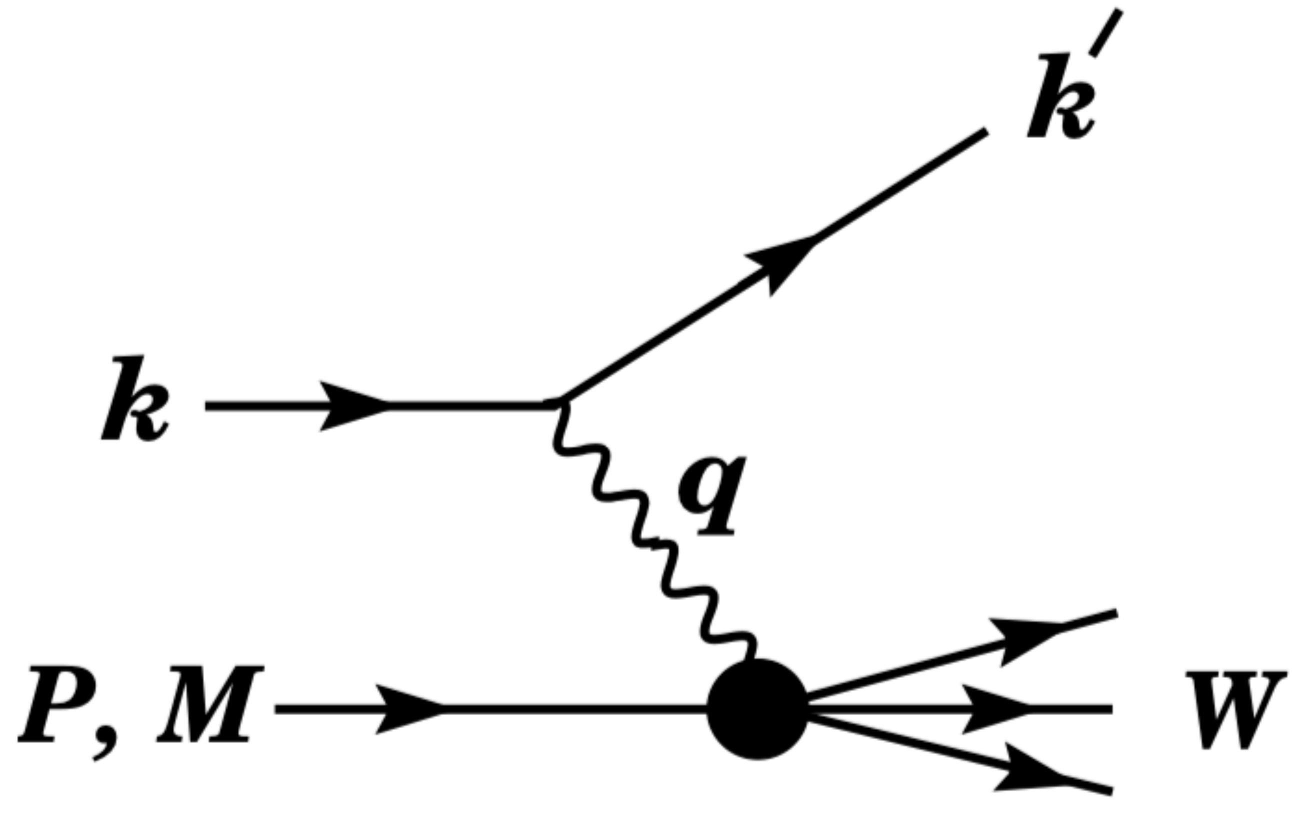
2311.04206

Hadronic tensor



for lepton-nucleon scatterings $\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4}\sum_j \eta_j L_j^{\mu\nu} \mathbf{W}^j_{\mu\nu}$

the hadronic tensor $W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s\left|\left[J^\dagger_\mu(z)\ J_\nu(0)\right]\right|p,s\right\rangle$ $W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}\left[T_{\mu\nu}\right]$

It can be further decomposed into structure functions, and encodes the nonperturbative nature of the nucleon.

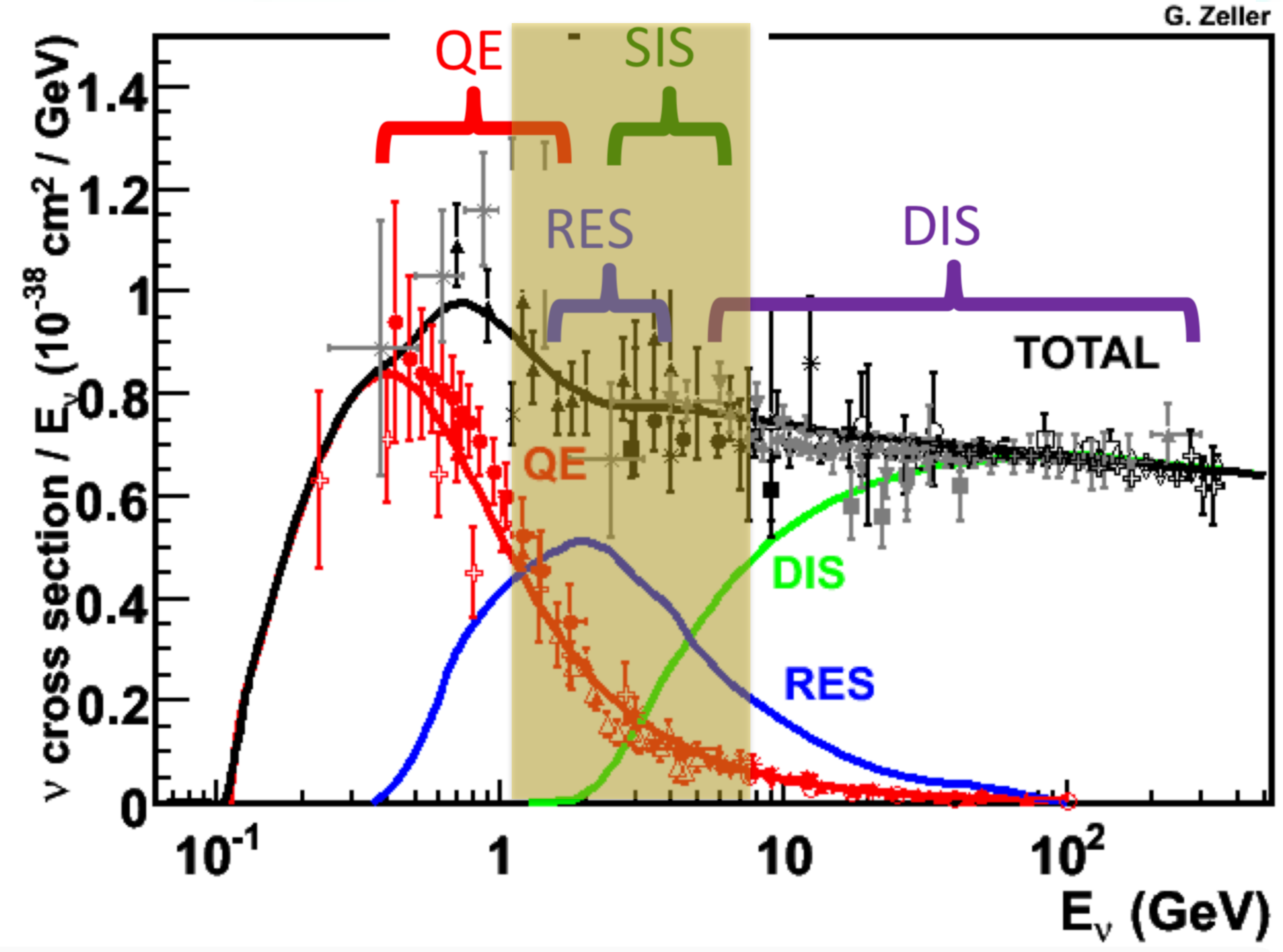
$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F_2(x,Q^2)$$

Hadronic tensor and neutrino-nucleus scattering

- ✦ Neutrino-nucleus scattering experiments to explore the properties of neutrinos.

- ✦ Besides nuclear effects and modeling, inputs of fundamental **neutrino-nucleon scattering** are needed.

- ✦ Challenge: at different neutrino energies, different contributions dominate the cross section.

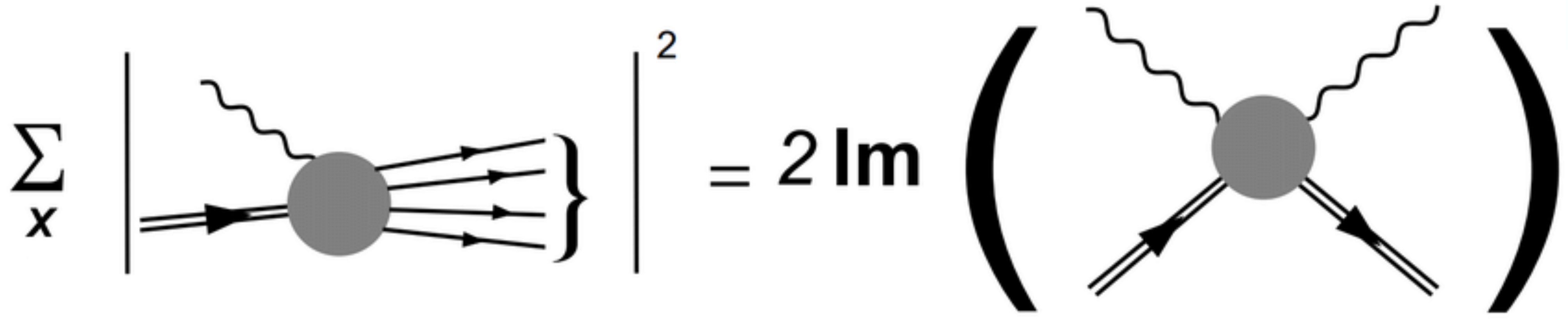


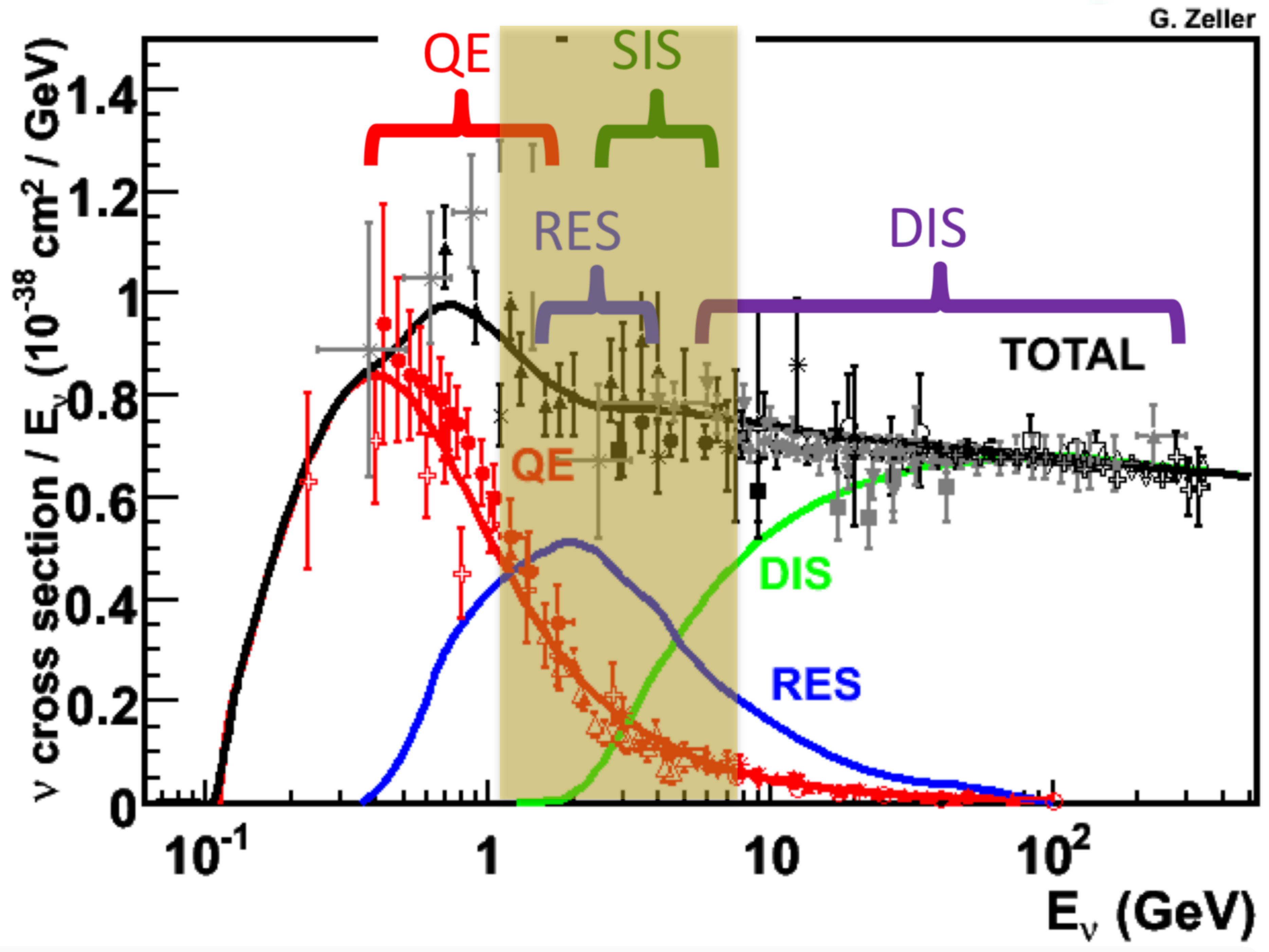
J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

Hadronic tensor and neutrino-nucleus scattering

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\left\langle p,s\left|\left[J^\dagger_\mu(z)J_\nu(0)\right]\right|s,s\right\rangle$$

$$= \frac{1}{2}\sum_n \int \prod_i^n \left[\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\right]\langle p,s\,|\,J^\dagger_\mu(0)\,|\,n\rangle\langle n\,|\,J_\nu(0)\,|\,p,s\rangle(2\pi)^3\boldsymbol{\delta}^4(\boldsymbol{q}-\boldsymbol{p}_n+\boldsymbol{p})$$





J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

USQCD white paper

the only way that lattice QCD could help in all energy regions

Calculating hadronic tensor on the lattice

Lattice QCD: Euclidean field theory using the path-integral formalism.

Time dependent matrix elements can be problematic.

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4 z e^{iq\cdot z}\left\langle p,s\left|\left[J_\mu^\dagger(z)J_\nu(0)\right]\right|s,s\right\rangle$$

$$W'_{\mu\nu} = \frac{1}{4\pi}\sum_n \int dt e^{\left(\nu-(E_n-E_p)\right)t}\int d^3\vec{z} e^{i\vec{q}\cdot\vec{z}}\langle p,s|J_\mu^\dagger(\vec{z})|n\rangle\langle n|J_\nu(0)|p,s\rangle$$

$$= \frac{1}{4\pi}\sum_n \frac{e^{\left(\nu-(E_n-E_p)\right)T}-1}{\nu-(E_n-E_p)}\int d^3\vec{z} e^{i\vec{q}\cdot\vec{z}}\langle p,s|J_\mu^\dagger(\vec{z})|n\rangle\langle n|J_\nu(0)|p,s\rangle$$

A simple change from Fourier transform to Laplace transform in the time direction leads to divergences when $\nu-(E_n-E_p)>0$.

Calculating hadronic tensor on the lattice

Define **Euclidean hadronic tensor**:

$$\tilde{W}_{\mu\nu}(\vec{p},\vec{q},\tau) = \frac{E_p}{m_N}\frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \to \sum_{\vec{x}_2 \vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s|J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)|p,s\rangle$$

$$= \sum_n \langle p,s|J_\mu^\dagger(\vec{q})|n\rangle\langle n|J_\nu(-\vec{q})|p,s\rangle e^{-(E_n-E_p)(t_2-t_1)} \equiv \sum_n A_n e^{-\nu_n\tau}$$

The energy transfer is determined by the energy of the intermediate states.

$$C_4 = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f}\sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\left\langle \chi_N(\vec{x}_f,t_f)J_\mu^\dagger(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\bar{\chi}_N(\vec{0},t_0)\right\rangle$$

$$C_2 = \sum_{\boldsymbol{x}_f} e^{-i\vec{p}\cdot\vec{x}_f}\left\langle \chi_N(\vec{x}_f,t_f)\bar{\chi}_N(\vec{0},t_0)\right\rangle$$

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD 62, 074501 (2000)

J. Liang et. al., EPJ Web Conf. 175, 14014 (2018)

J. Liang et. al., PRD 11, 114503 (2020)

Contractions

valence and connected-sea (CS) parton

CS anti-parton (Gottfried sum rule violation)

disconnected-sea parton and anti-parton

The CS anti-partons are supposed to be responsible for the Gottfried sum rule violation.

The latter three are **suppressed** when the momentum and energy transfers are large.

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD 62, 074501 (2000)

T.-J. Hou, M. Yan, J. Liang et. al., PRD106, 096008 (2022)

Back to Minkowski space

Formally, inverse Laplace transform $$W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau e^{\nu\tau}\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau)$$

Practically, need to solve the inverse problem $$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu)e^{-\nu\tau}$$

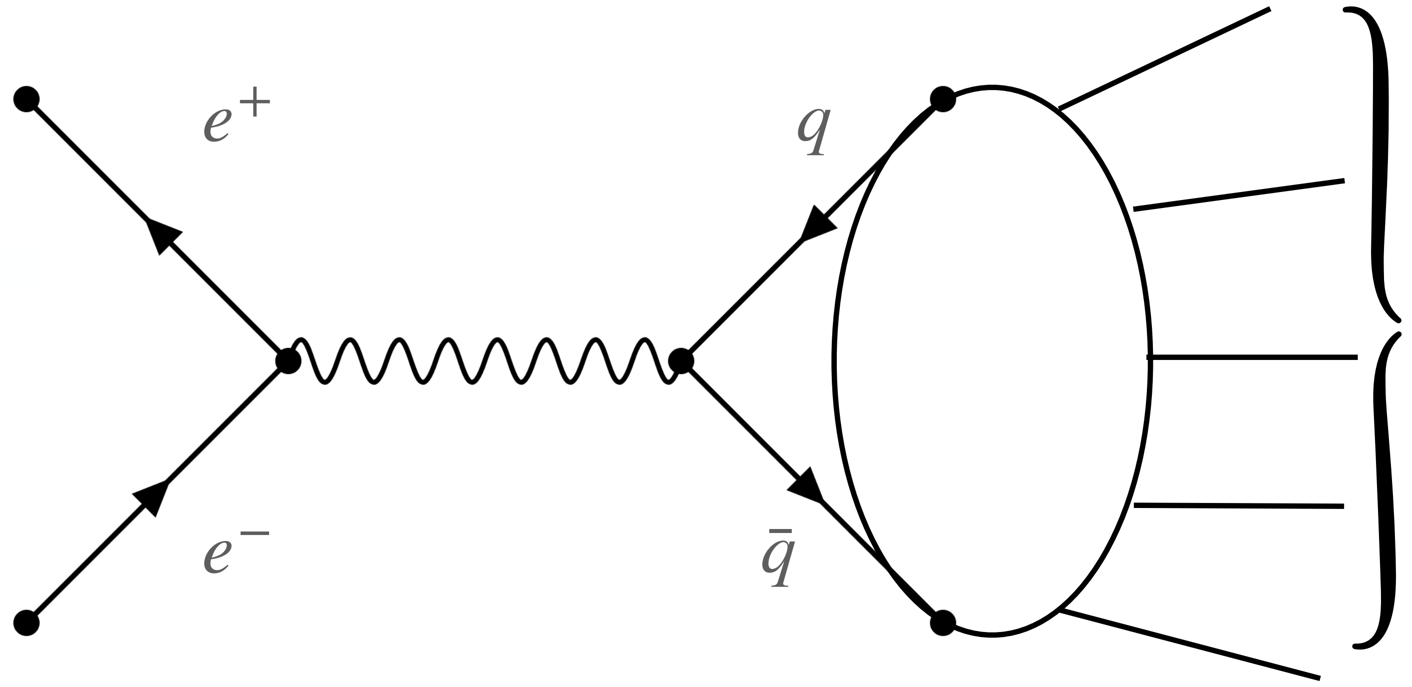
solving the inverse problem

- ✦ Backus-Gilbert (BG)
- ✦ Maximum Entropy (ME)
- ✦ **Bayesian Reconstruction (BR)** — best resolution, may have oscillations in the flat regions
- ✦ **Smoothed BR** — balanced resolution and smoothness
- ✦ …

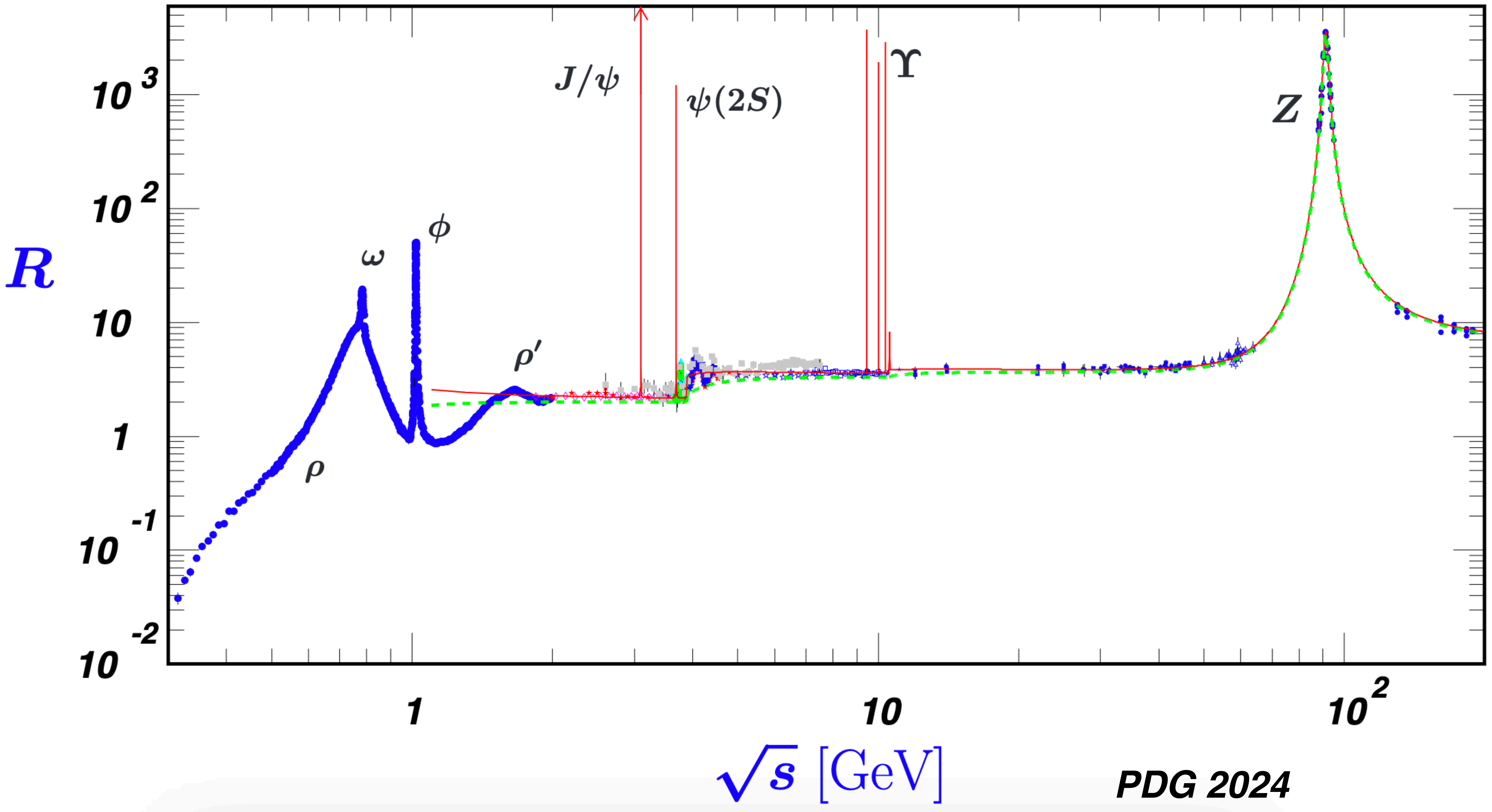
Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

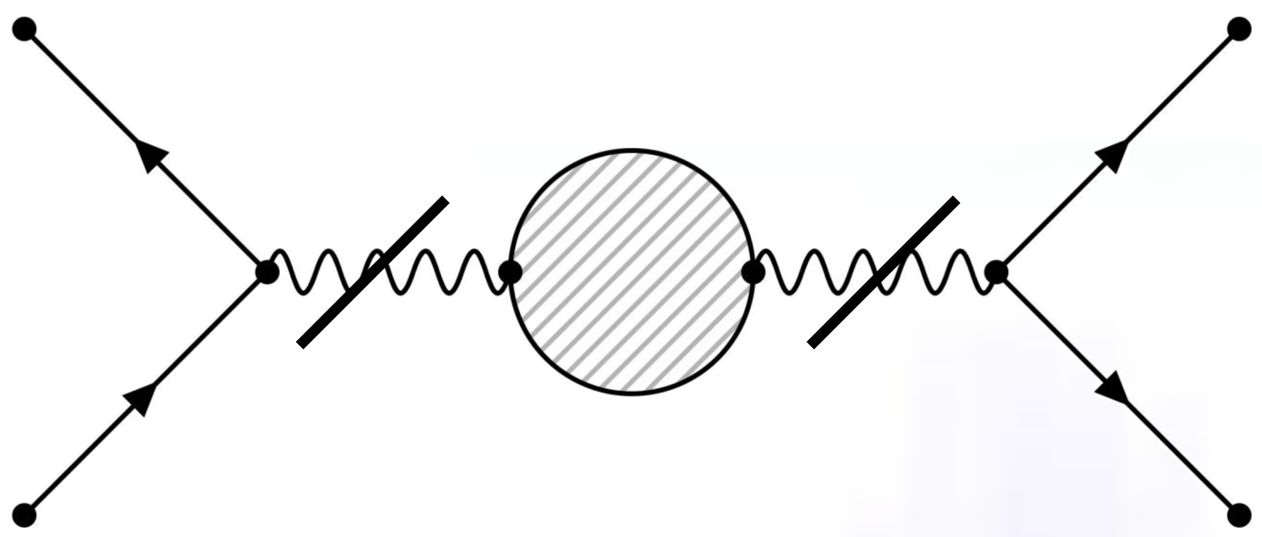
C. S. Fischer el. al., PRD98:014009 (2018)

A Benchmark test: R ratio from lattice QCD



$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



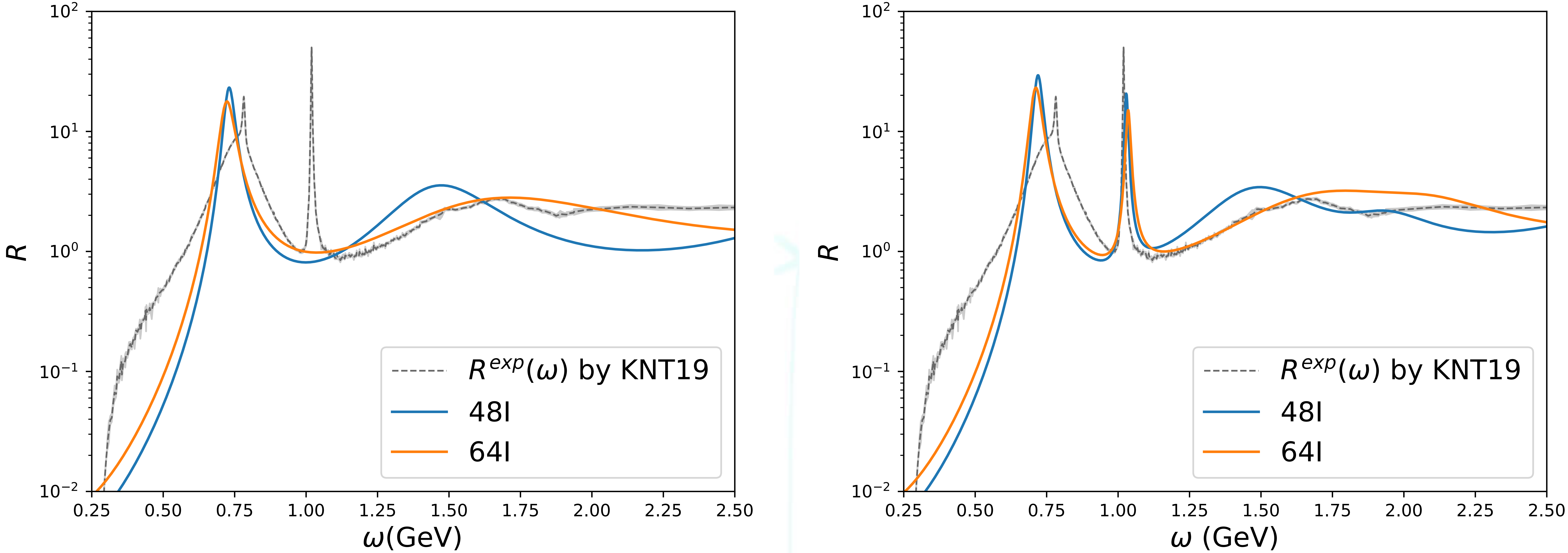


$$C_2(t) = \left\langle J_\mu^{\text{em}}(t)\ J_\mu^{\text{em}}(0) \right\rangle = \int d\omega \rho(\omega) e^{-\omega t}$$

$$R(\omega) = \frac{12\pi^2}{\omega^2}\rho(\omega)$$

$R(\omega)$ from solving the inverse problem

Results without smearing



Additional ϕ peek by applying separate BR on light and strange correlators

The R ratio from lattice QCD

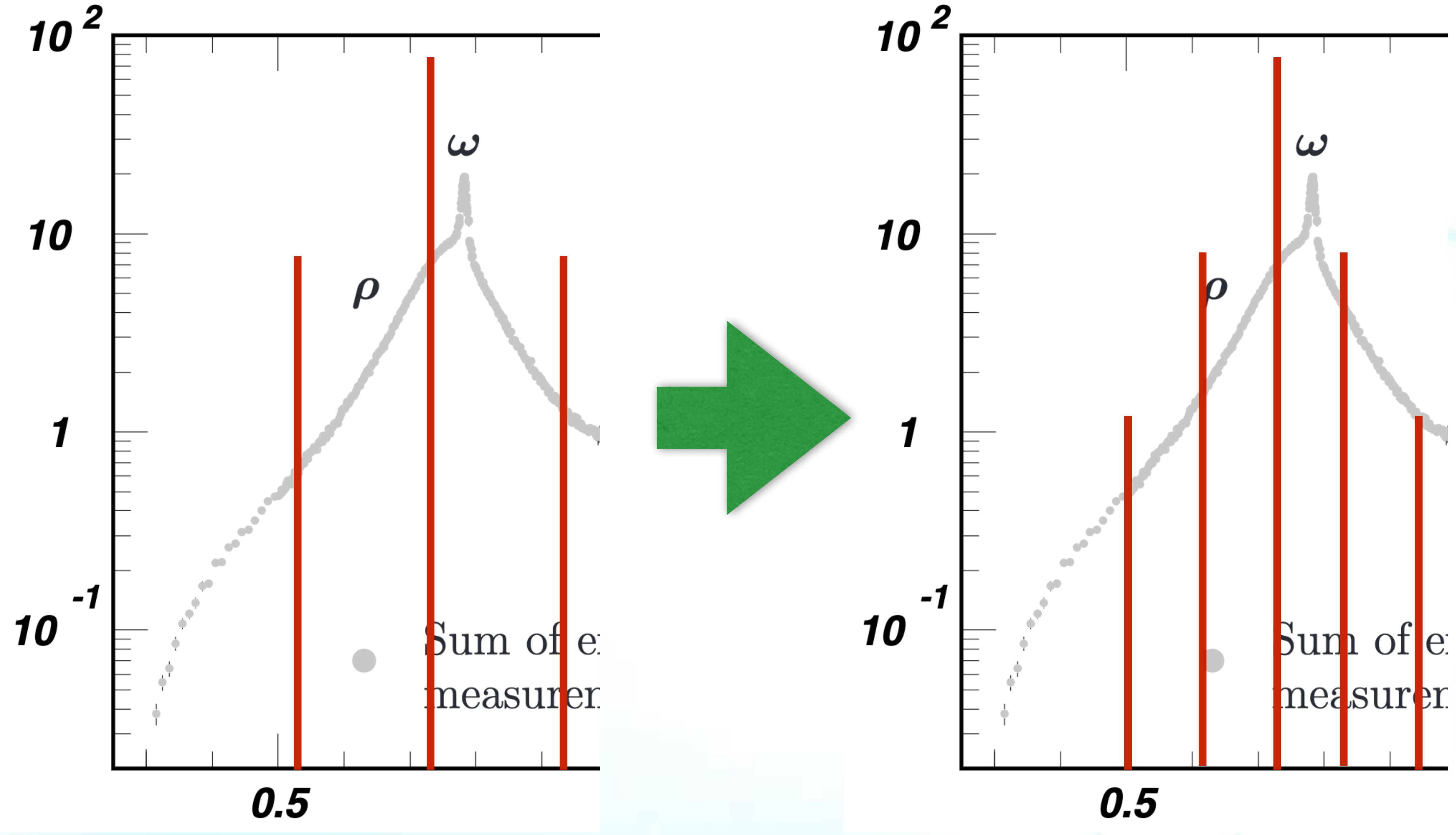
$$\rho(\omega) = \sum_n A_n \delta(\omega, \omega_n)$$

Lattice finite-volume discrete spectrum!

$$\rho^S(\omega, L, \Delta) = \int d\omega' \mathcal{S}(\omega, \omega') \rho(\omega', L)$$

$$\rho(\omega) = \lim_{\Delta \to 0} \lim_{L \to \infty} \rho^S(\omega, L, \Delta)$$

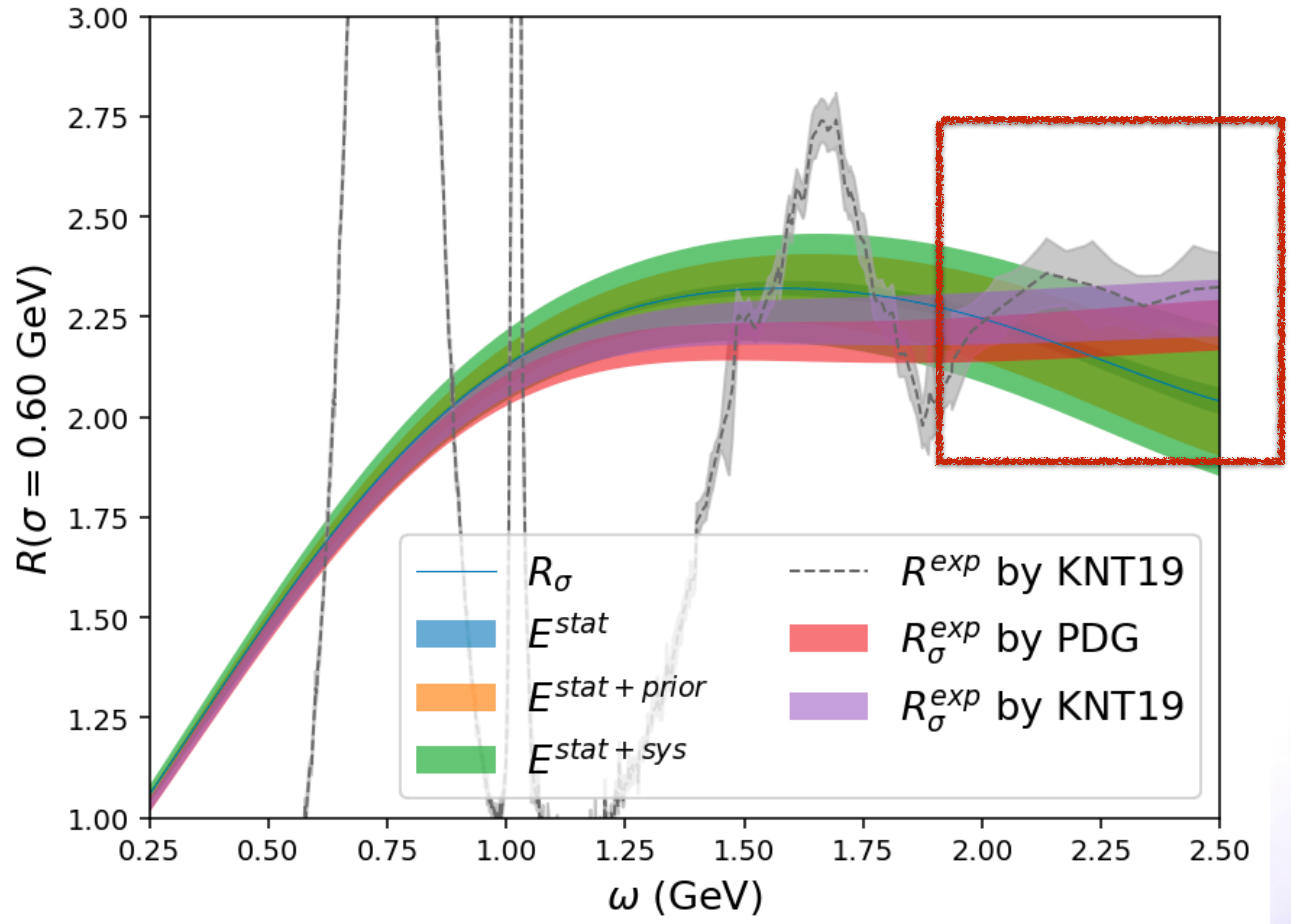
M. T. Hansen et al., Phys. Rev. D 96, 094513 (2017)



$$\mathcal{S}_\Delta(\omega, \omega') \sim \exp\left(-\frac{(\omega - \omega')^2}{2\Delta^2}\right)$$

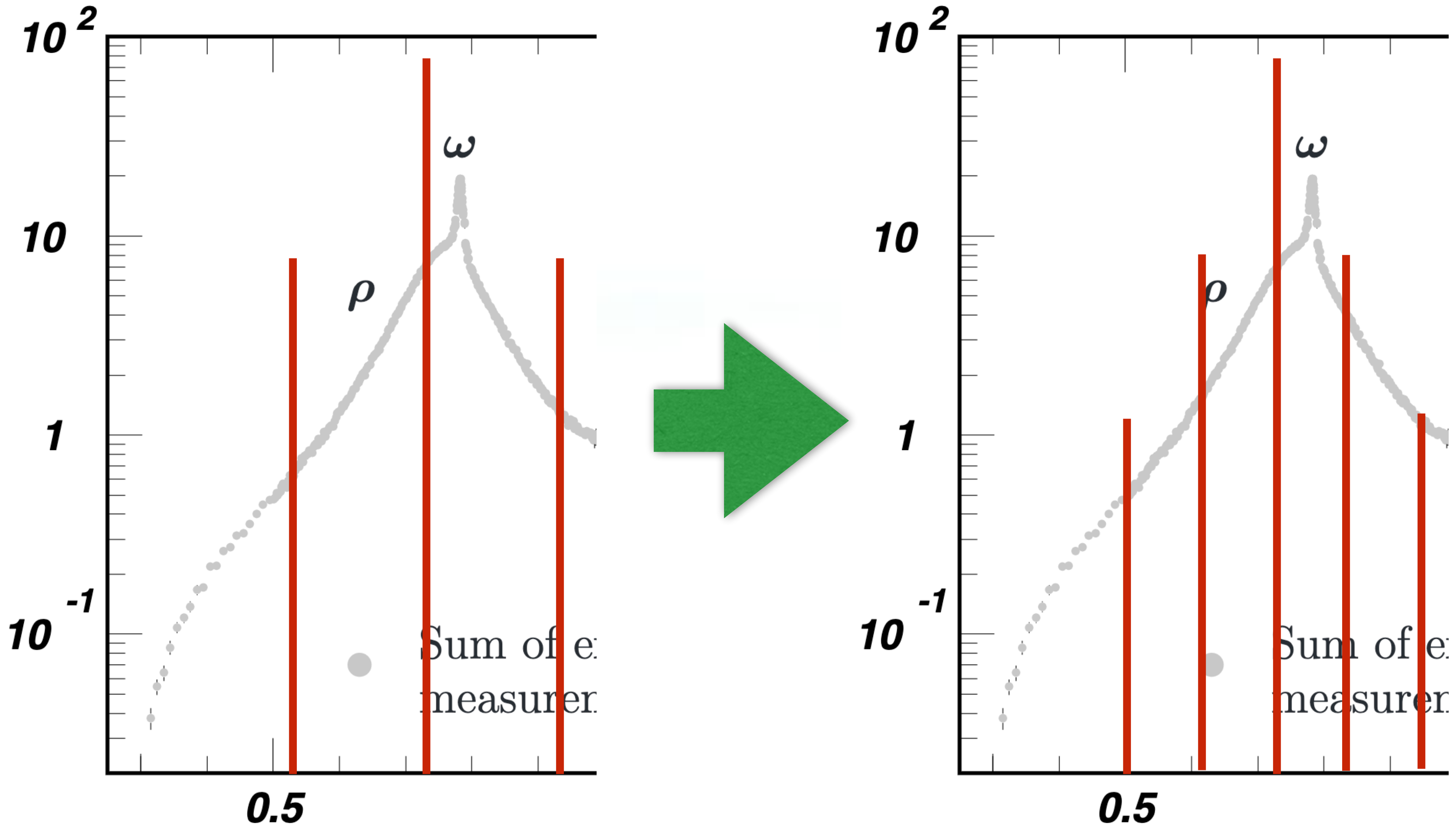
$$\rho^S(\omega, \Delta) = \lim_{L \to \infty} \rho^S(\omega, L, \Delta) \leftrightarrow \rho^S_P(\omega, \Delta)$$

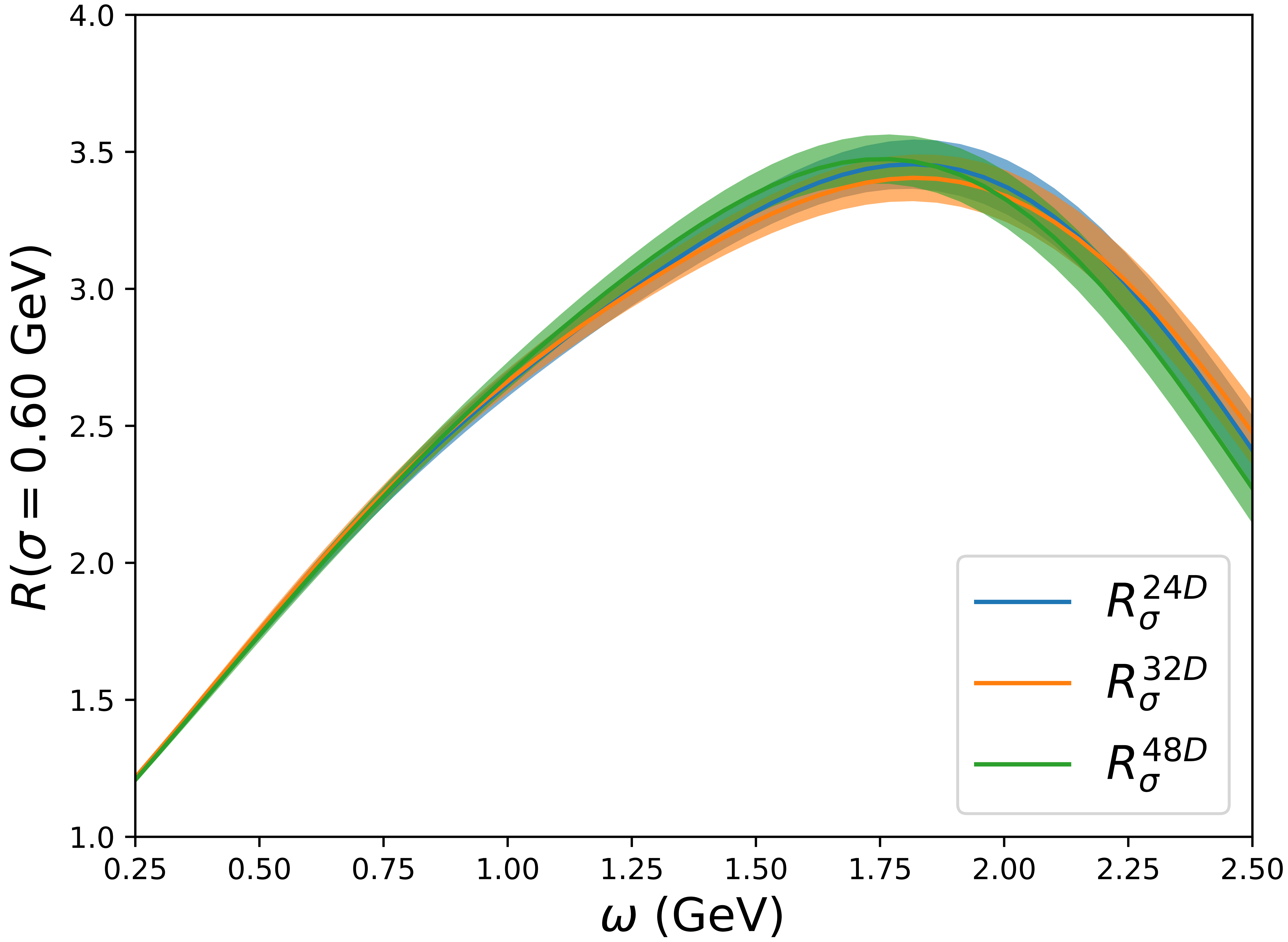
The R ratio from lattice QCD



In the smooth region, the smeared lattice results (inclusive contribution) are physical.

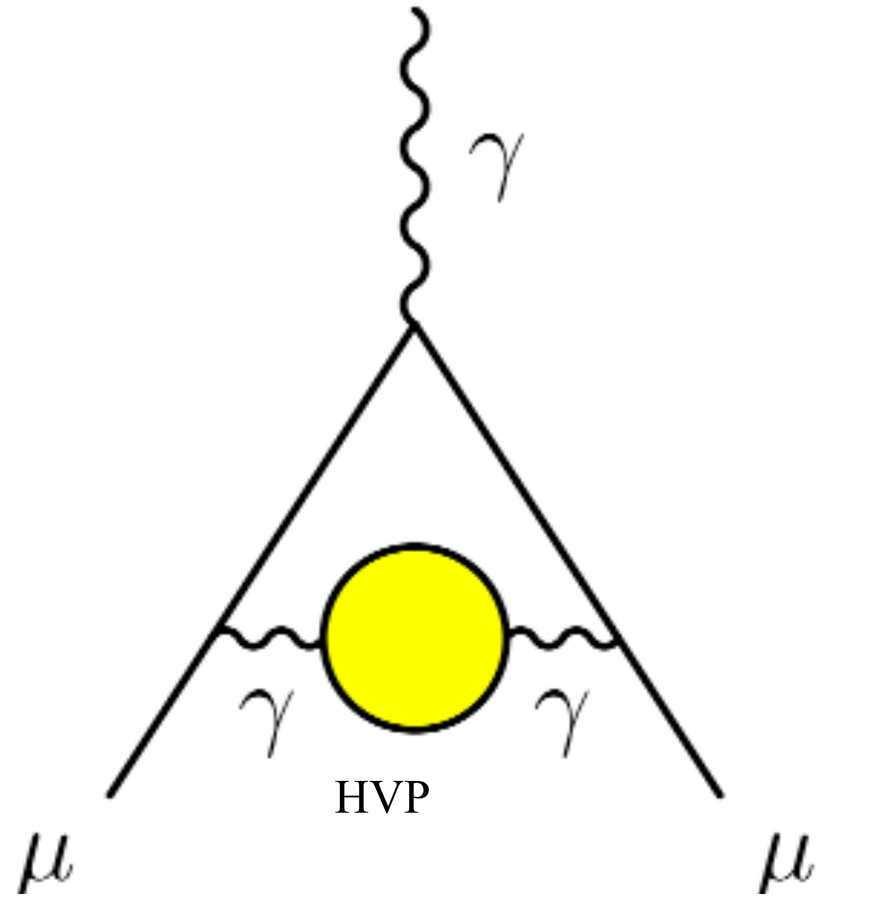
Volume dependence





After smearing, no significant volume dependence is found!

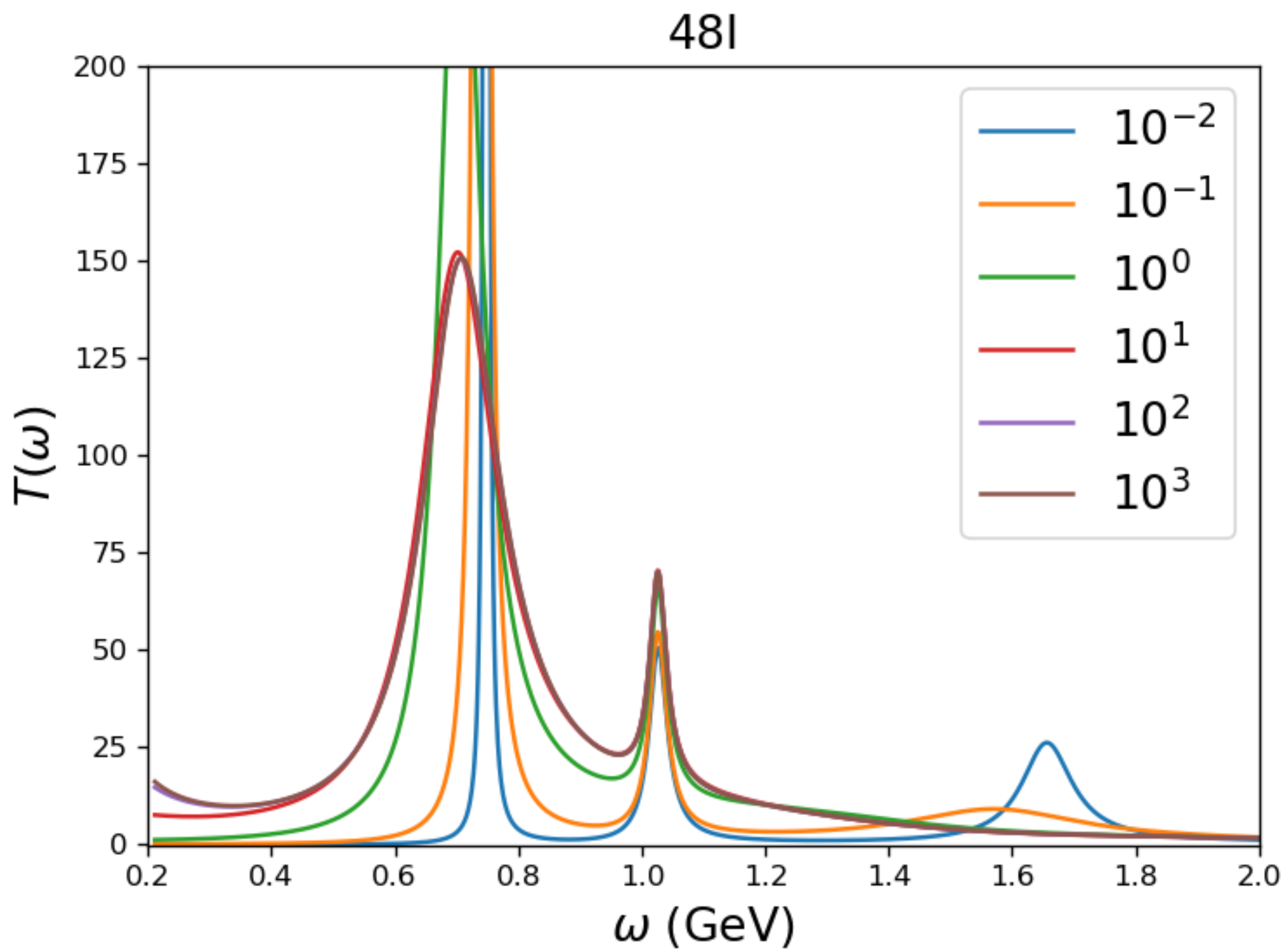
Preliminary results on muon anomaly



$$a_\mu^{\mathrm{HVP}}[\mathrm{LO}] = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{thr}}^{\infty} ds \frac{K(s)}{s^2} R(s) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{thr}}^{\infty} d\omega T(\omega)$$

$$C_2(t) = \int d\omega \frac{\rho(\omega)}{\omega^2} e^{-\omega t}\omega^2 = \int d\omega \frac{24\pi^2 K(\omega)\rho(\omega)}{\omega^5} \frac{\omega^5 e^{-\omega t}}{24\pi^2 K(\omega)}$$

$T(\omega)$



$$a_\mu^{a\to 0} = 6.822 \times 10^{-8}$$

The resonance effects are also properly extracted!

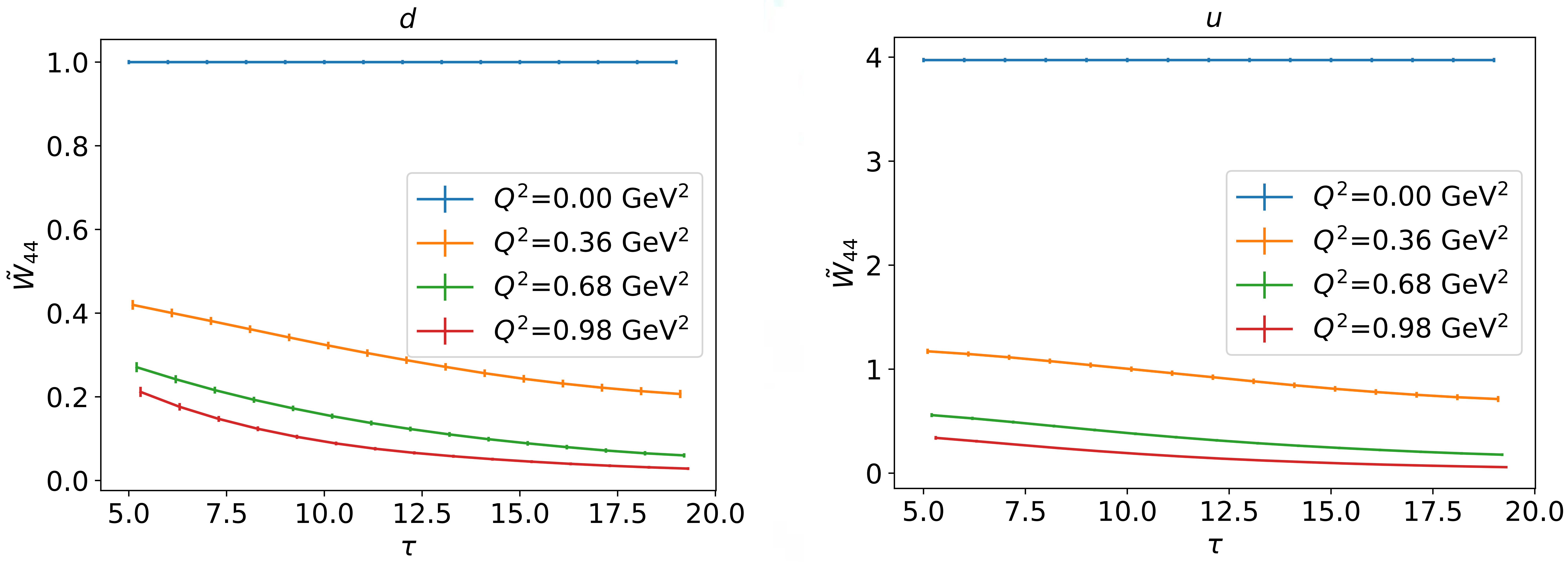
Nucleon hadronic tensor and form factors

$$\tilde{W}_{44}(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s|J_4(\vec{x}_2,t_2)J_4(\vec{x}_1,t_1)|p,s\rangle$$

$$= \sum_n \langle p,s|J_4^\dagger(\vec{q})|n\rangle\langle n|J_4(-\vec{q})|p,s\rangle e^{-(E_n-E_p)(t_2-t_1)} \equiv \sum_n A_n e^{-\nu_n\tau}$$

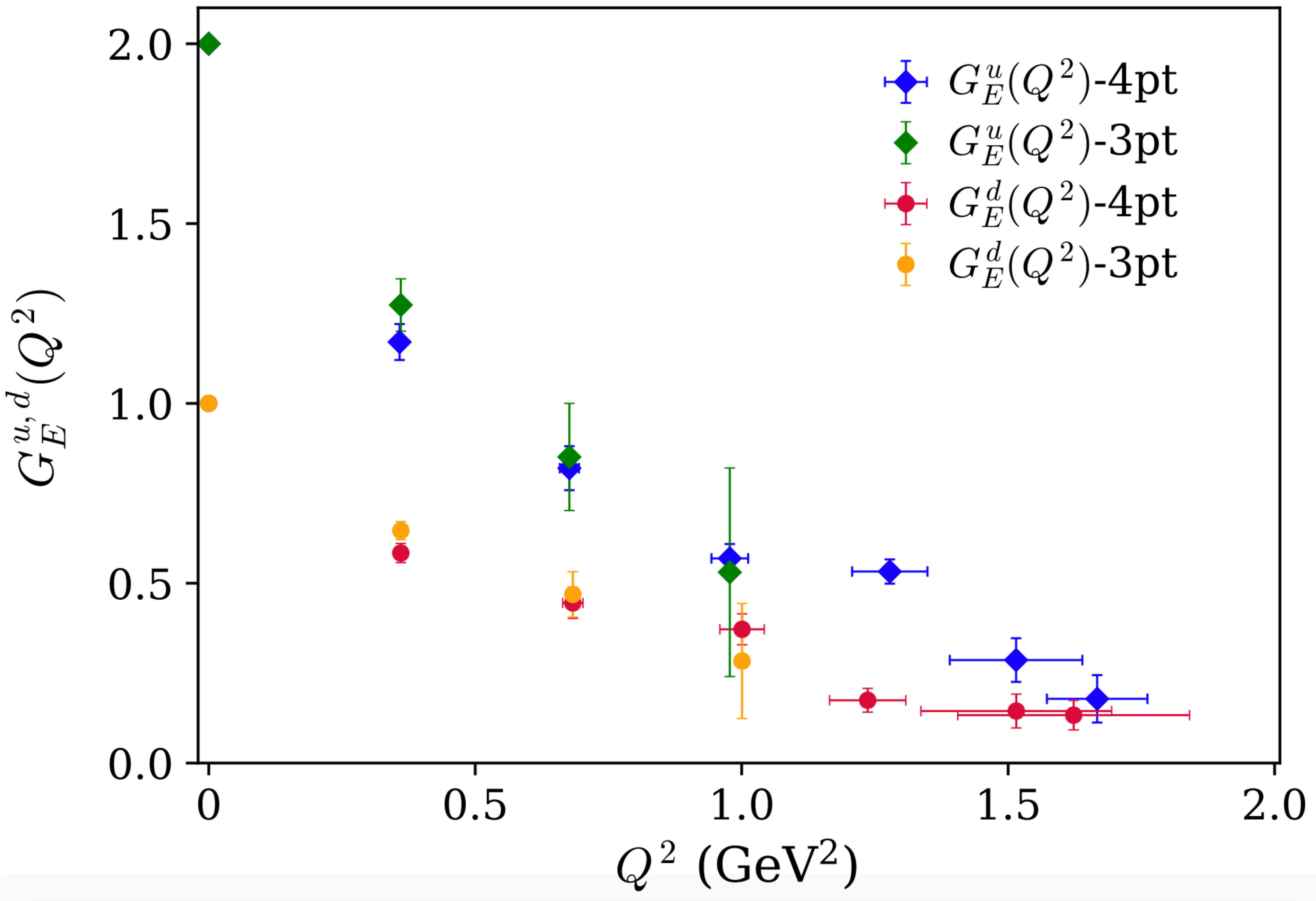
$$A_0 = \langle p,s|J_4(\vec{q})|n=0\rangle\langle n=0|J_4(-\vec{q})|p,s\rangle = G_E^2(Q^2)$$

$$A_1 = \langle p,s|J_4(\vec{q})|n=1\rangle\langle n=1|J_4(-\vec{q})|p,s\rangle = G_E^{*2}(Q^2)$$



Elastic form factor

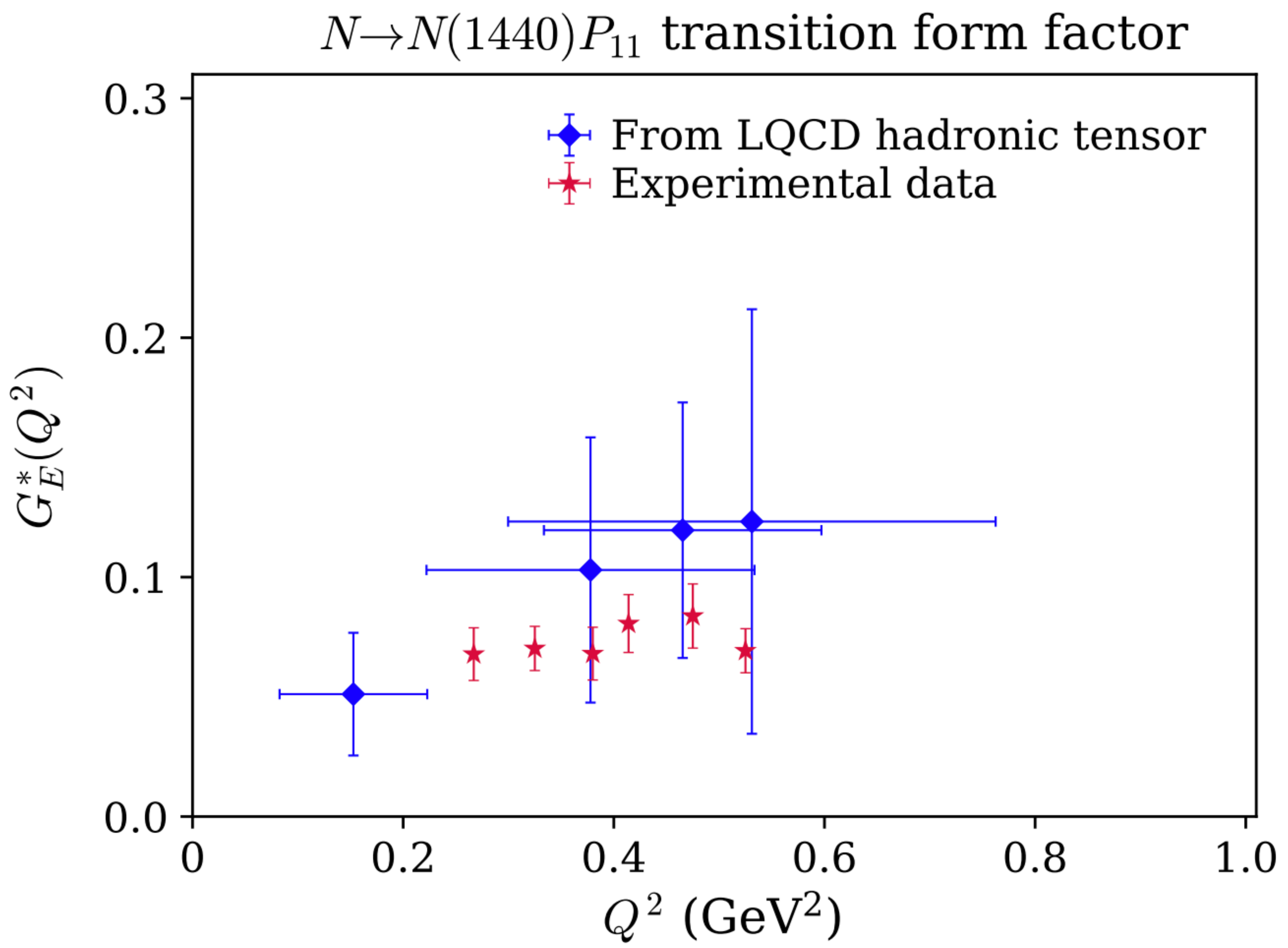
$$A_0 = \langle p, s | J_4(\vec{q}) | n = 0 \rangle \langle n = 0 | J_4(-\vec{q}) | p, s \rangle = G_E^2(Q^2)$$



J. Liang et. al., 2311.04206

The transition form factor

$$A_1 = \langle p, s \,|\, J_4(\vec{q}\,) \,|\, n = 1 \rangle \langle n = 1 \,|\, J_4(-\vec{q}\,) \,|\, p, s \rangle = G_E^{*2}(Q^2)$$



Not supposed to be a precise study

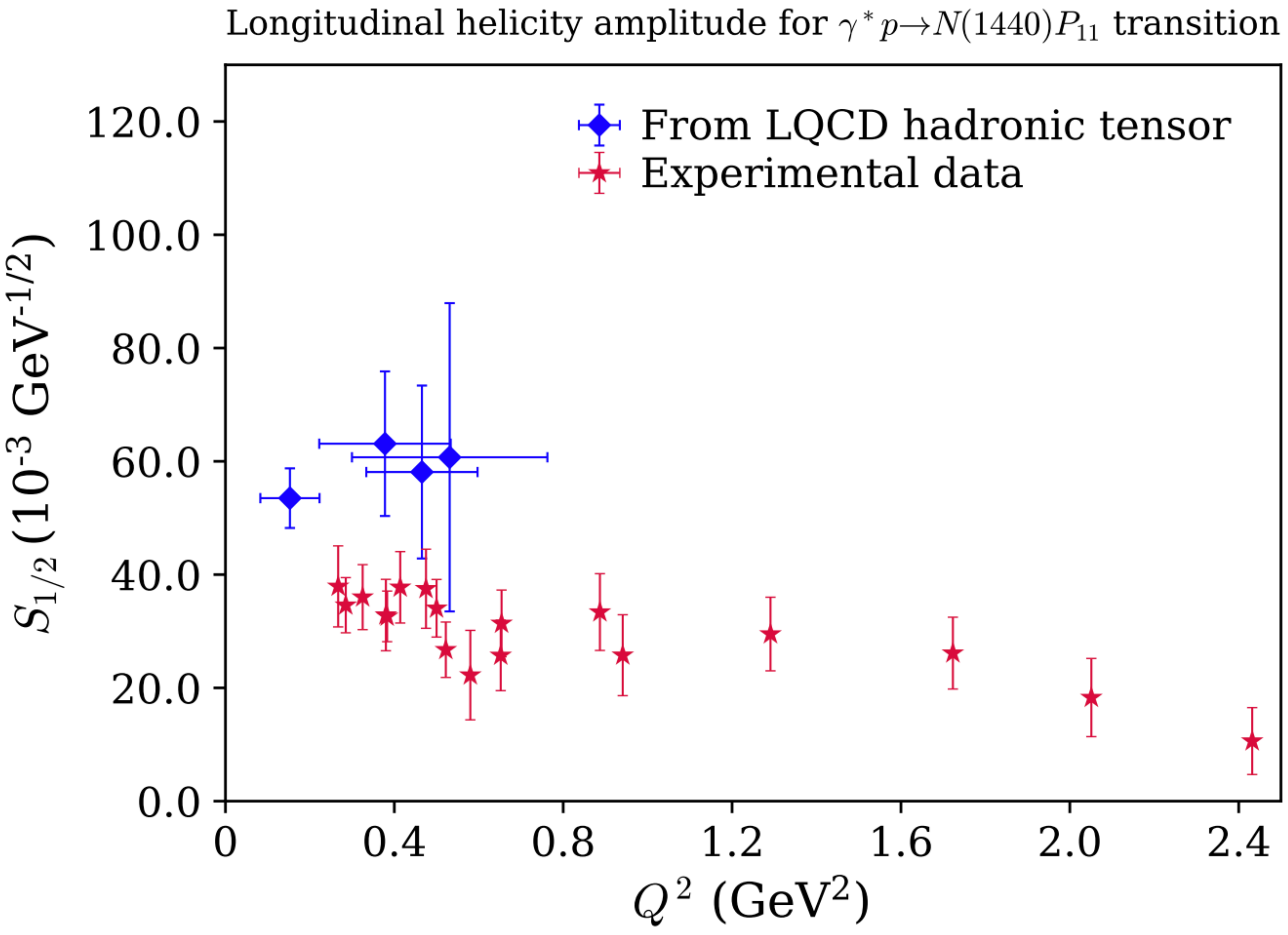
370 MeV pion mass at finite lattice spacing

a demonstration of the feasibility

J. Liang et. al., 2311.04206

The transition form factor

$$A_1 = \langle p, s \,|\, J_4(\vec{q}\,) \,|\, n=1 \rangle \langle n=1 \,|\, J_4(-\vec{q}\,) \,|\, p, s \rangle = G_E^{*2}(Q^2)$$



Not supposed to be a precise study

370 MeV pion mass at finite lattice spacing

a demonstration of the feasibility

amplitude involving resonances

J. Liang et. al., 2311.04206

Summary and outlook

- ✦ Calculating the hadronic tensor on the lattice helps study neutrino-nucleus scatterings from first principles.
- ✦ This is so far the only known lattice approach that gives inclusive results in both the RES and SIS regions.
- ✦ We now have obtained very nice results for the R-ratio and nucleon form factors as demonstrations of the method.
- ✦ New results of inclusive nu-N scatterings at higher energies are coming soon.

Thank you!