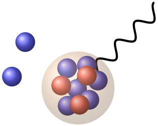
Two-neutron halos in EFT: neutron and E1 strength distributions

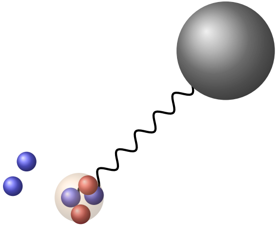
Matthias Göbel
Istituto Nazionale di Fisica Nucleare - Sezione di Pisa

in collaboration with
H.-W. Hammer and D. R. Phillips

September 26, 2024

Overview



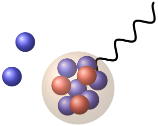


nn relative-energy distributions following core knockout

*E*1 strength distributions following Coulomb dissociation

Overview

halo effective field theory



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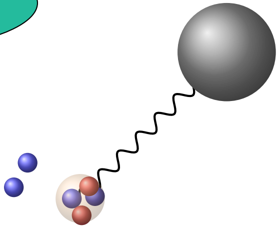
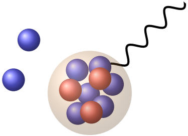


*E*1 strength distributions following Coulomb dissociation

universality of few-body systems

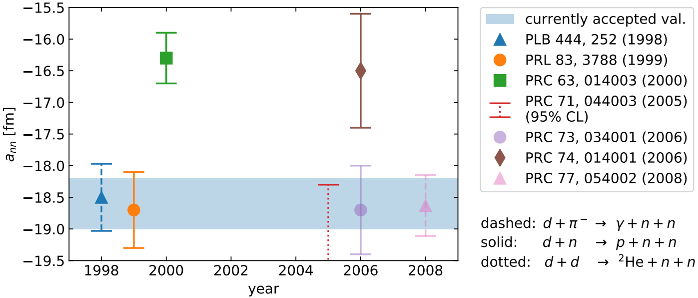
Part I



nn relative-energy distributions
of 2*n* halo nuclei

Application for the use of E_{nn} distribution: measuring the *nn* scattering length

- **motivation:** no high-precision value for the *nn* scattering length available



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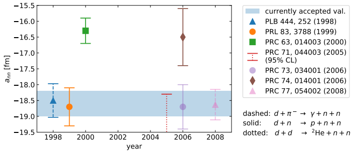

➔ use the reaction $^6\mathrm{He}(p, p'\alpha)nn$ to determine the scattering length from the final E_{nn} spectrum

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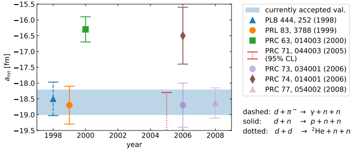

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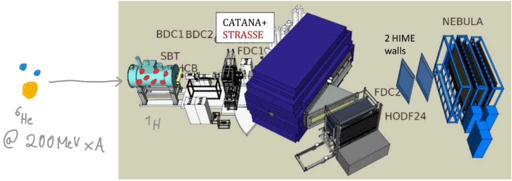

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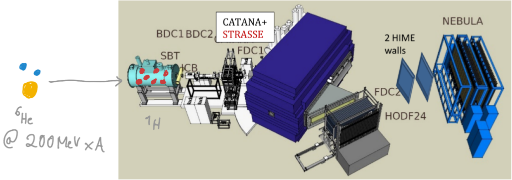

- advantages of this approach
  - different from the previous methods ➔ other systematics
  - final *nn* pair has high center-of-mass velocity in the lab system ➔ avoids problems with detection efficiency
- experiment proposal from Aumann & SAMURAI collaboration approved by RIKEN RIBF NP2012-SAMURAI55R1 (2020)

Obtaining the E_{nn} spectrum at the example of ^{6}He

- **approach:** ^{6}He in halo EFT
  1. calculate wave function $\Psi_c(p, q)$ (& do comparisons with model calc.)
  2. take final-state interaction (FSI) into account
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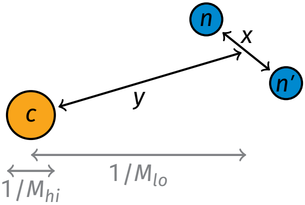
  - $\not\pi$EFT
  - core & valence nucleons as degrees of freedom
  - results are expanded in k/M_{hi}
    → systematic improvement possible



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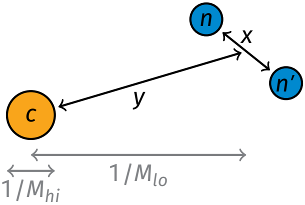
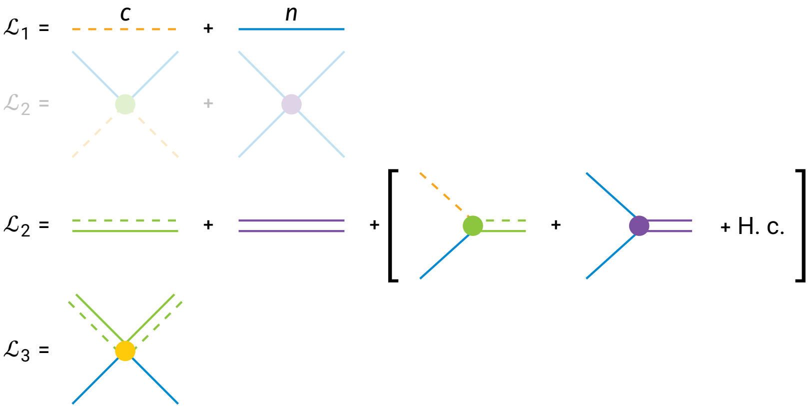


- **properties of ^{6}He**
  - Borromean 2n halo
  - separation of scales: $S_{2n} = 0.975\,\text{MeV} < E^*_\alpha \approx 20\,\text{MeV}$
  - quantum numbers: $J^\pi = 0^+$ (^{4}He: $J^\pi = 0^+$)
  - leading-order (LO) halo EFT interaction channels:
    - nn: $^1S_0$
    - nc: $^2P_{3/2}$ (not at LO: $^2P_{1/2}$, $^2S_{1/2}$)

halo EFT for ^{6}He formulated in Ji, Elster, Phillips, PRC 90 (2014)
review of halo EFT in Hammer, Ji, Phillips, JPG 44 (2017)

Lagrangian



Faddeev equations

use EFT in dimer formalism

1. step: obtain dressed dimer propagators

& renormalize using **input values**

a_1, r_1

a_0

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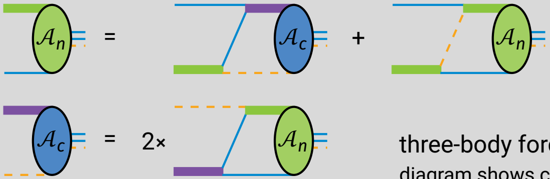
2. step: set up equations for Faddeev transition amplitudes



three-body force required for renormalization

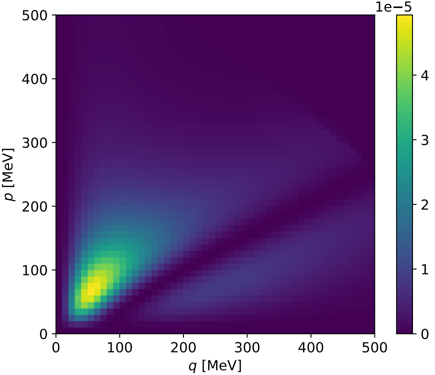
diagram shows case of vanishing three-body force

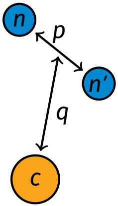
Results for the wave function

calculated ground-state wave functions and probability densities in halo EFT

Göbel, Hammer, Ji, Phillips, FBS 60 (2019)

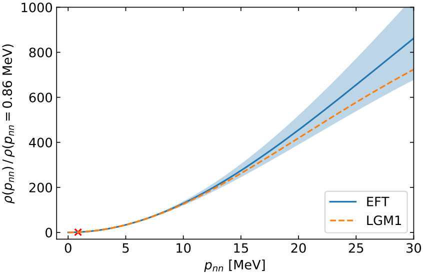
$\Psi_c^2(p,q)p^2q^2$





Compare ground-state distribution from EFT with model calculations

- use ground-state momentum distribution $\rho(p_{nn}) \approx \int \mathrm{d}q\, q^2 p_{nn}^2\, |\Psi_c(p_{nn}, q)|^2$
- model for comparison: local Gaussian model (LGM1), calc. done with FaCE Thompson, Nunes, Danilin, Comput. Phys. Commun. 161 (2004)



Göbel, Aumann, Bertulani, Frederico, Hammer, Phillips, PRC 104 (2021)

- model calc. within uncertainty band of EFT ✔

Handling the reaction $^6\text{He}(p, p'\alpha)nn$

- **initial state:** ^{6}He bound state $|\Psi\rangle$

$$\left(K_{nn} + K_{c(nn)} + V_{nn} + V_{nc} + V_{3B}\right)|\Psi\rangle = -B_3\,|\Psi\rangle$$

- **final state:** $|p, q\rangle_c$ all particles are free (state of definite momentum!)

$$\left(K_{nn} + K_{c(nn)}\right)|p, q\rangle_c = (-B_3 + E_{KO})\,|p, q\rangle_c$$

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final-state interactions (FSIs)

- definition: ints. which do not cause the transition but change the final state
- ∃ multiple FSIs in ^{6}He$(p, p'\alpha)nn$: V_{nn}, V_{nc}, V_{np}, V_{3B}, etc.
- kinematic suppression in ^{6}He$(p, p'\alpha)nn$ @ high beam energies ➔ only V_{nn} @ LO

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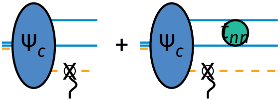
- FSI evaluated via nn Møller operator $\Omega^\dagger_{nn}$

$${}_c\langle p, q; \Omega_c|\,\Omega^\dagger_{nn} = {}_c\langle p, q; \Omega_c| + {}_c\langle p, q; \Omega_c|\,t_{nn}(E_p)G_0^{(nn)}(E_p)$$

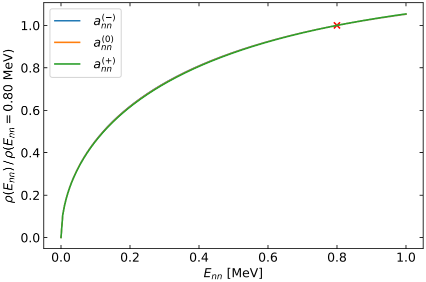
(eval. gives rise to singular integral)



E_{nn} spectrum before and after FSI

obtain distribution by using $\rho^{(t)}(p) = \int \mathrm{d}q\, p^2 q^2 \left|\Psi_c^{(\mathrm{wFSI})}(p,q)\right|^2$

variation of a_{nn}: $a_{nn}^{(-)} = -20.7$ fm, $a_{nn}^{(0)} = -18.7$ fm, $a_{nn}^{(+)} = -16.7$ fm



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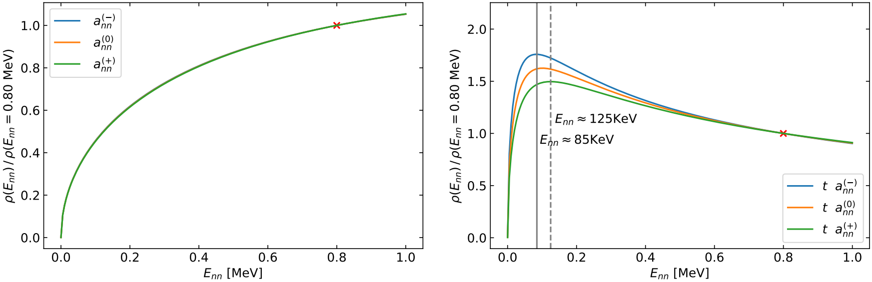

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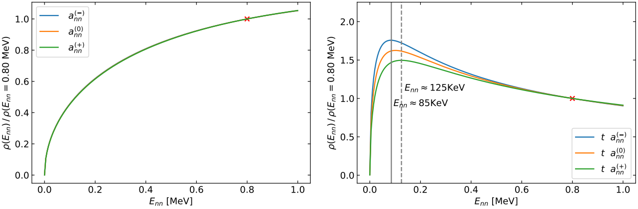


conclusions

- significant sensitivity on *nn* scattering length (≈ 10 % at peak position)
- sensitivity almost entirely caused by FSI ➔ ^{6}He is simply a suitable *n* source (nevertheless, ^{6}He wave function is an important ingredient)

E_{nn} spectrum of triton

T. Kirchner, M. Göbel, H.-W. Hammer

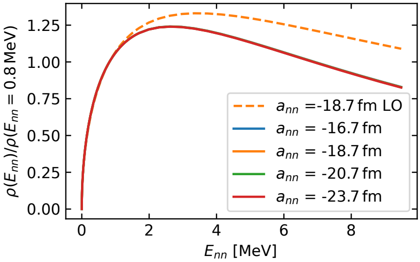
- experiment will also include $t(p, pp)nn$ reaction as a "cross check"
- same kinematics: suppression of non-nn FSIs
- overall procedure and inclusion of FSIs is the same
- as for ^{6}He for the ground state we solve three-body Faddeev eqs.,
  but this time in pionless EFT Bedaque, Hammer, van Kolck, NPA 676 (2000), Bedaque *et al.*, NPA 714 (2003)

interactions at LO and at NLO

- three two-body interaction channels: nn in $^1S_0$, np in $^1S_0$ and np in $^3S_1$
- two-body interactions specified in the form of t-matrices
  - for spin-singlet channels: $\tau_0 \propto \left(a_0^{-1} - r_0/2k^2 + ik\right)^{-1}$
  - for spin-triplet channel: pole expansion of the t-matrix: $\tau_d \propto \left(\gamma_d - \rho_d\left(\gamma_d^2 + p^2\right)/2 + ik\right)^{-1}$
- three-body interaction for renormalization

- NLO corrections are treated semi-perturbatively

E_{nn} spectrum of triton - before and after FSI
(preliminary)



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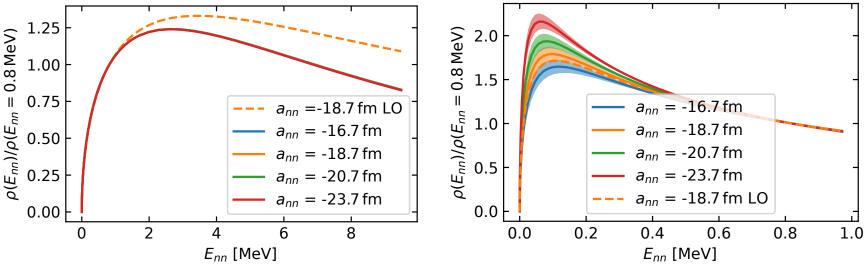

Kirchner, Göbel, Hammer, in preparation

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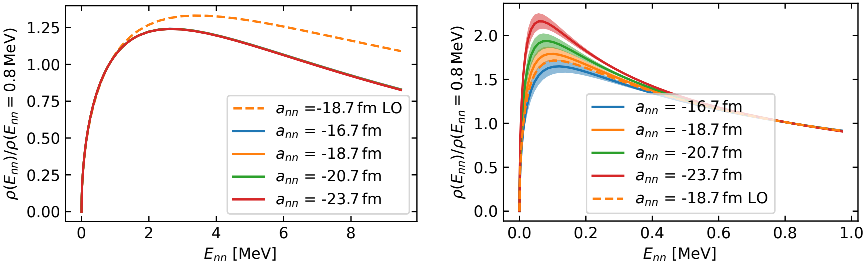

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→ situation analog to ^{6}He

Universality of *nn* distributions of 2*n* halos

- so far
  - *nn* distributions as means to study *nn* interaction, halos as neutron source
  - distributions have similar shapes
- now: distributions of different halos as means to study universality

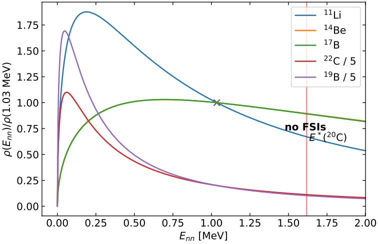
nucleus	core	$J\pi$	S_{2n} [keV]	E^* [keV]
Li-11		3/2 -	369.3 (6)	
	Li-9	3/2 -		2691 (5)
Be-14		0+	1270 (13)	
	Be-12	0+		2109 (1)
B-17		3/2 -	1380 (210)	
	B-15	?		
B-19		3/2 -	90 (560)	
	B-17	3/2 -		
C-22		0+	100	
	C-20	0+		1618 (11)

data from https://www.nndc.bnl.gov/nudat3/ [except S_{2n} of C-22]

- all can be described via *s*-wave interactions
- core spin = total spin
  ➔ core spin can be neglected
  (as long as V_{nc} is the same in $s_c - 1/2$ and $s_c + 1/2$)
- all have a separation of scales between S_{2n} and $E^*(c)$

Results: Different nuclei in comparison

normalization scheme: normalize to certain value @ some position
(experimentally useful)



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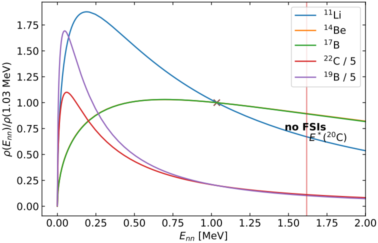

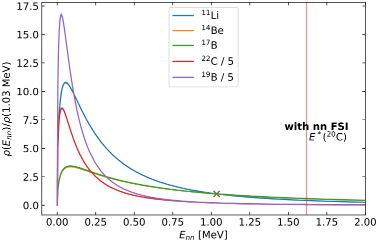

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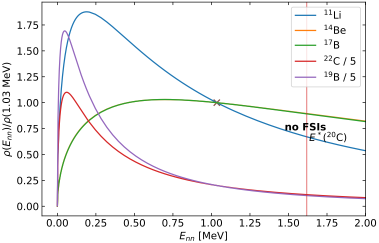
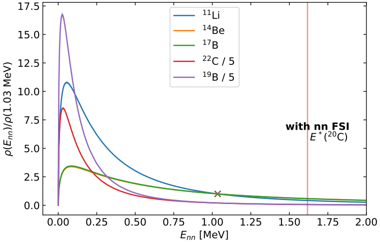
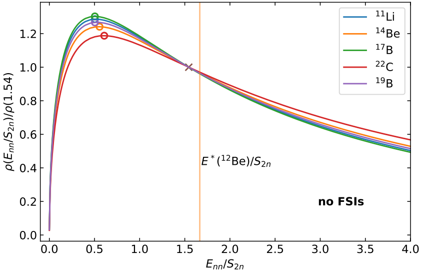




- hierarchy of S_{2n} becomes clearly visible:
  $S_{2n}(^{19}\mathrm{B}) < S_{2n}(^{22}\mathrm{C}) < S_{2n}(^{11}\mathrm{Li}) < S_{2n}(^{14}\mathrm{Be}) \approx S_{2n}(^{17}\mathrm{B})$
- significant influence of *nn* FSI

Going into the unitarity limit

use E_{nn}/S_{2n} instead of E_{nn} as variable



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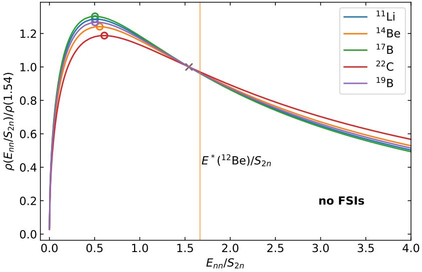

➔ curves almost on top of each other ➔ influence of a_{ij} and A on shape small

➔ test unitarity limit ($t_{ij} \propto (1/a_{ij} + ip)^{-1}$ -> $t_{ij} \propto (ip)^{-1}$)

Unitarity limit in nuclear physics discussed in König, Grießhammer, Hammer, van Kolck, PRL 118 (2017)

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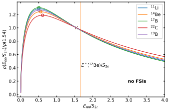

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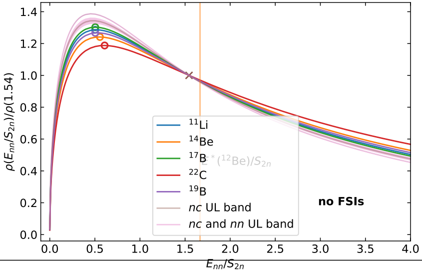

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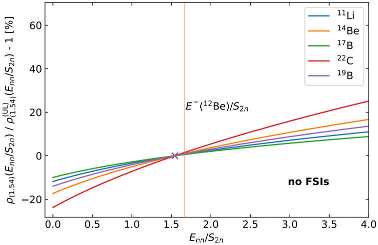
Going into the unitarity limit

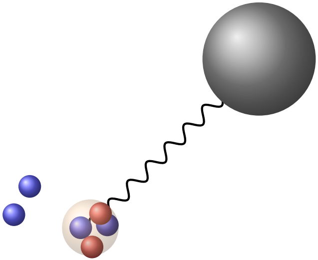
- universal description in terms of curve from double unitarity limit
- benchmark in terms of relative deviation from exact LO EFT result



Göbel, Hammer, Phillips, PRC 110 (2024)

- universality of the distribution
  $\tilde{\rho}(E_{nn}/S_{2n}; V_{nn}, V_{nc}, S_{2n}, A) \approx \tilde{\rho}(E_{nn}/S_{2n}; \bar{a}_{nn}, \bar{a}_{nc}, A) \approx \tilde{\rho}(E_{nn}/S_{2n}; \bar{a}_{nn}, A)$
  $\approx \tilde{\rho}(E_{nn}/S_{2n}; A) \approx \tilde{\rho}(E_{nn}/S_{2n})$
- started with LO EFT universalities, realized reduction-of-parameter universalities by going into the unitarity limit
- we extended universal description also to the final distribution

Part II



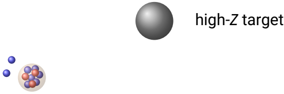
*E*1 strength distributions
following Coulomb dissociation
&
finite-range interactions

Remarks

$E1$ strength as an interesting observable

- parameterizes the Coulomb dissociation cross section: $\frac{\mathrm{d}\sigma}{\mathrm{d}E} \propto \frac{\mathrm{d}B(E1)}{\mathrm{d}E}$
- characteristic property of halo nuclei
- for $2n$ halos reltd. to a large core distance r_c

see, e.g., Forssén, Efros, Zhukov, NPA 697 (2002),
Acharya, Phillips, EPJ Web Conf. 113 (2016), Hagen, (2014)
review of low-energy dipole response in Aumann, EPJA 55 (2019)



Remarks

$E1$ strength as an interesting observable

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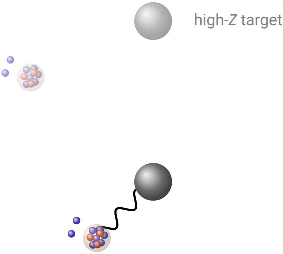

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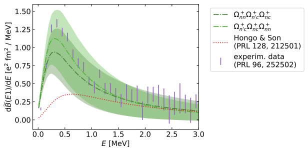

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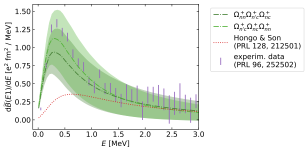

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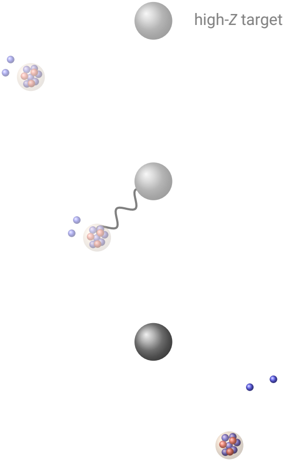

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$$\tilde{\mathcal{L}}_{nc}^{(2)} = d_m^\dagger \left(w_d \left(i\partial_0 + \frac{2}{2M} \right) + \Delta_d \right) d_m - \frac{g_d}{2} \left(d_m^\dagger \left[n \left(i\overleftrightarrow{\partial} \right) c \right]_m - \text{H. c.} \right)$$

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  - in QM this corresponds to energy-dependent potentials $H \to H(E)$
    solutions of H @ different E are in fact solutions of different H

    Formánek, Lombard, Mareš, CJP 54 (2004)

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  - ➔ corrections $\propto \partial_E V$ to expectation values and normalization are necessary
  - probability density of ^{6}He: $\forall\, p, q < M_{hi}$: corrections to the normalization have the greatest influence, others are small Göbel, Hammer, Ji, Phillips, FBS 60 (2019)

Finite-range EFT

- implication for $\frac{\mathrm{d}B(E1)}{\mathrm{d}E}$: shape can be calculated straightforwardly in zero-range halo EFT, calc. of the absolute values would be more intricinate
- avoid energy dependency by using rank-one separable int., which is finite-range

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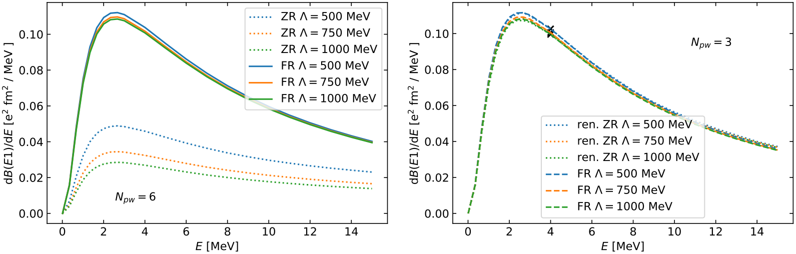
- EFT aim can still be realized: provide systematic improvability and in that way also uncertainty estimates (comparison of different-order results)
- finite-range interactions in use: Yamaguchi (YM) interactions Yamaguchi, PR 95 (1954)
  - work well in momentum-space Faddeev calculations
  - have already two parameters ➔ ideal for *p*-wave $n\alpha$ int.
  - extension of YM form factors ➔ more parameters ➔ reproducibility of more ERE terms
- Yamaguchi interaction is a rank-one separable interaction:
  $\langle p, l | V_{\bar{l}} | p', l' \rangle = \delta_{l,l'} \delta_{l,\bar{l}} g_l(p) \lambda_l g_l(p')$
  with YM form factors $g_l(p) := p^l \frac{\beta_l^4}{\left(p^2+\beta_l^2\right)^2}$

Leading-order results
(preliminary)



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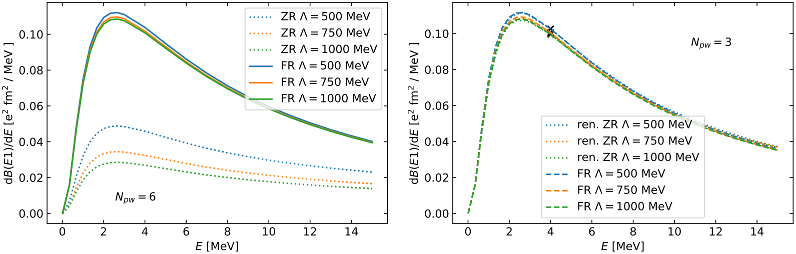

➔ zero-range (ZR) approach has convergence issue in Λ, related to $V(E)$

➔ finite-range (FR) approach is convergent

➔ results for the shape agree

Going to NLO
(preliminary)

- inclusion of the different NLO effects in the finite-range approach ($^2S_{1/2}$ nc int., r_0-term of $^1S_0$ nn int., (UT of $^2P_{3/2}$ nc int. in FR already LO))
- $\exists$ difficulties of NLO effects in the zero-range approach
  - r_0-term in the nn int. would create another $V(E)$
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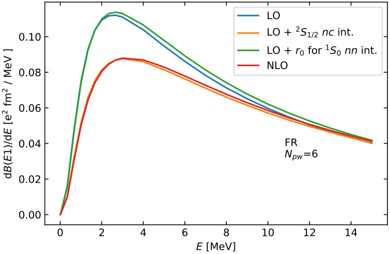

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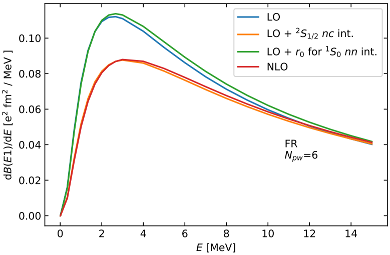


➔ NLO corrections have the expected size

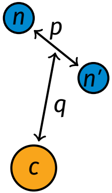
➔ NLO correction from $^2S_{1/2}$ *nc* int. much stronger than from *nn* r_0 term

Final-state interactions and partial waves

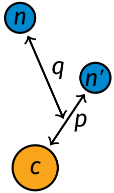
- *a priori* the matrix element of the t_i acting in the jk subsystem is known for i (= $\mathcal{S}_i$) as spectator
  ${}_i\langle p, q; \Omega | t_i(E_3) | p', q'; \Omega' \rangle_i \propto$
  $\delta_{\Omega,\Omega'} \delta_{(\Omega)_{jk},\Omega_i} \frac{\delta(q-q')}{q^2} \tau_{jk}\left(E_3 - q^2 / \left(2\mu_{i(jk)}\right)\right)$

➔ recoupling between states of different spectators and different partial waves in some cases necessary

spectator: *c*



spectator: *n*



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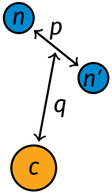
→ recoupling between states of different spectators and different partial waves in some cases necessary

- strategy: make use of relation for
  ${}_{\mathcal{S},\mathcal{S}'}\mathcal{T}^{p,q|p',q'}_{\Omega,\Omega'} f(p',q') \coloneqq$
  $\int \mathrm{d}p'\, p'^2 \int \mathrm{d}q'\, q'^2\, {}_{\mathcal{S}}\langle p,q;\Omega|p',q';\Omega'\rangle_{\mathcal{S}'}\, f(p',q')$
  - expression consists of sums over Clesch-Gordan coefficients, Wigner-3nj symbols, and partial-wave projections of f evaluated at shifted momenta (angle-dependent)

  → lower number of num. integrals compared to naive approach

spectator: c



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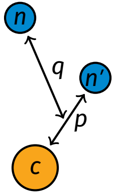
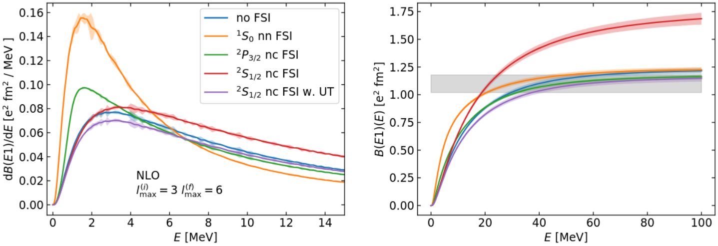


NLO results with FSIs
(preliminary)

- all FSIs (also ${}^2S_{1/2}$ nc FSI) in comparison on the basis of the NLO ground state
- overall $E1$ strength obtained from $\langle r_c^2 \rangle$ via sum rule

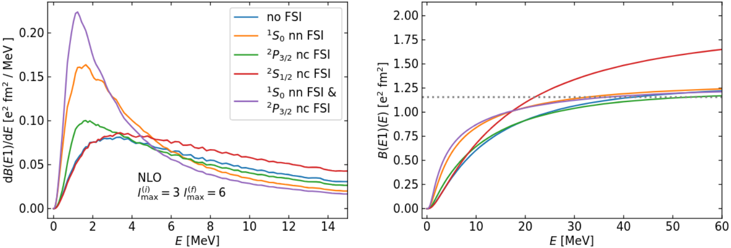


→ sum rule fulfilled except for one FSI, which is missing unitarity term (UT) according to power counting

→ nn FSI is more important than ${}^2P_{3/2}$ nc and ${}^2S_{1/2}$ nc FSIs

NLO results with FSIs
(preliminary)

- also results of second order in FSIs obtainable
- here shown: result based on $\Omega_{nn}^{\dagger}\Omega_{nc}^{\dagger}$



Göbel, Hammer, Phillips, in preparation

Conclusion & Outlook

Part I

Conclusions

- *nn* distributions of ^{6}He and triton have significant sensitivity on a_{nn}
- found universality of ground-state and final distributions
  - *nn* and *nc* interactions can be put in the unitarity limit

Outlook

- interesting possibilities for comparisons
- go to NLO to asses the accuracy of the univ. results better
- repeat the calcs. for different kinematics (additional FSIs)

Part II

Conclusions

- finite-range Halo EFT is an useful complement in the EFT toolbox
- NLO results for $E1$ distribution of ^{6}He
- found dominance of *nn* FSI

Outlook

- finalize calculations
- investigate universality of $E1$ distributions

Backup slides on $^{6}He(p, p'\alpha)nn$

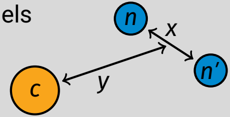
Compare ground-state distribution from EFT with model calculations

- compare ground-state distributions $\rho(p_{nn}) \approx \int \mathrm{d}q\, q^2 p_{nn}^2 \, |\Psi_c(p_{nn}, q)|^2$
- $\nexists$ published results for $\rho(p_{nn})$

→ use FaCE Thompson, Nunes, Danilin, Comput.Phys.Commun. 161 (2004)

computer code: Faddeev with Core Excitations (FaCE)

- solves the Schrödinger equation of three-body cluster models
- *input:*
  - local l-dependent two-body potentials (central or spin-orbit)
  - phenomenological three-body force
- *output:* hyperspherical wave function components $\chi^S_{K,l}(\rho)$ with $\rho^2 = x^2 + y^2$



Compare ground-state distribution from EFT with model calculations: defining the model

the two-body potentials in use

use local, l-dependent Gaussian potentials

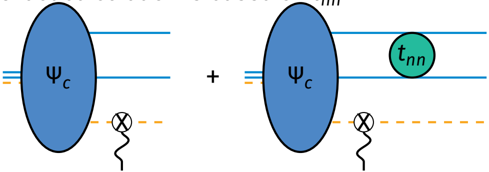
- central pot.: $\langle r; l, s | V_c^{(\tilde{l})} | r'; l', s' \rangle \coloneqq \delta_{l,l'} \delta_{l,\tilde{l}} \frac{\delta(r'-r)}{r'^2} \bar{V}_c^{(l)} \exp\left(-r^2 / \left(a_{c;l}^2\right)\right)$
- spin-orbit pot.

"standard setting": local Gaussian model 1 (LGM1)

- nn interaction: $V_c^{(0)}$
- nc interaction: $V_c^{(0)}, V_c^{(1)}, V_{SO}^{(1)}, V_c^{(2)}, V_{SO}^{(2)}$
- phenomenological three-body force

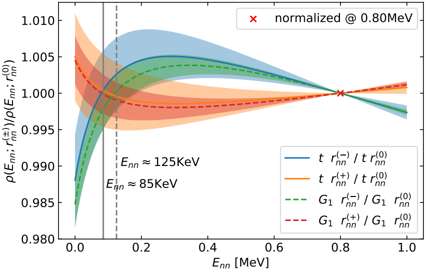
Calculating the wave function after FSI

- treatments of FSI are based on two-potential scattering theory
  Goldberger, Watson, "Collision Theory" (1964)
  - evaluate $T_{\beta\alpha} = \langle\beta|T^{(+)}_{U+V}|\alpha\rangle$
    with production potential V, FSI potential U and Hamiltonian H_0 of $|\alpha\rangle$ & $|\beta\rangle$
  - result of two-pot. scattering theory: dissection of $T^{(+)}_{U+V}$ for this matrix element
  - additional adjustments for the case that $U \in H_0^{(\alpha)} \neq H_0^{(\beta)}$
- two approaches on this basis:
  - approximation by using FSI enhancement factors
  - exact calculation
- exact calculation is based on t_{nn}



Sensitivity of the E_{nn} spectrum on effective range

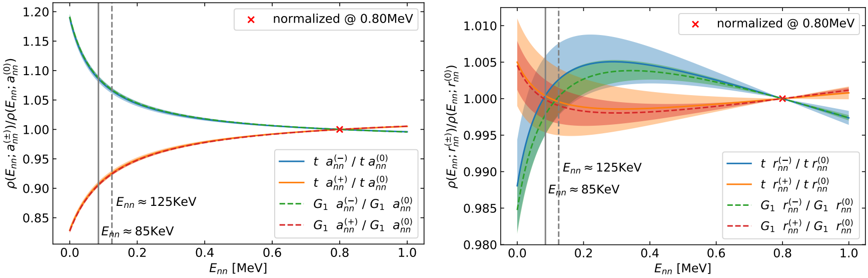
variation of r_{nn}: $r_{nn}^{(-)}$ = 2.0 fm, $r_{nn}^{(0)}$ = 2.73 fm, $r_{nn}^{(+)}$ = 3.0 fm
(simulation of an NLO effect)



conclusions

- significant sensitivity on *nn* scattering length
- almost no sensitivity on *nn* effective range

E_{nn} spectrum after FSI:
Ratio plots



conclusions

- influence of the *nn* scattering length at peak position ≈ 10 %
- influence of the *nn* effective range small

E_{nn} spectrum: uncertainty estimates

- estimate uncertainty of $\rho(E_{nn})$ based on $\Delta\rho(p_{nn}) \approx \rho(p_{nn})\frac{p_{nn}}{M_{hi}}$
- $M_{hi} \approx \sqrt{2\mu_{nn}E_{hi}}$ with $E_{hi} = E_\alpha^* \approx 20$ MeV
- results for the uncertainty estimates for $E_{nn} < 1$ MeV:
  - @LO: $\Delta\rho(E_{nn}) \lessapprox 20$ %
  - @NLO: $\Delta\rho(E_{nn}) \lessapprox 5$ %
  - @N^{2}LO: $\Delta\rho(E_{nn}) \lessapprox 1$ %

Backup slides
Universality of *nn* distributions

Some more formal thoughts about universality

- starting point: observable $\mathcal{O}$ being a function of
  - variable x (also measured)
  - system-specific parameters θ ➔$\mathcal{O}(x;\theta)$

Classifying the universality

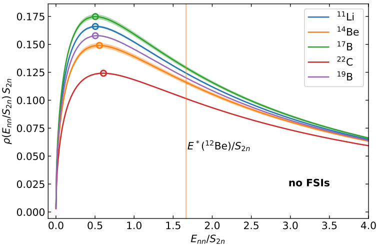
- different curves describing different systems are (nearly) on top of each other
- some attempts to classification
  - rescaling of the observable: $\mathcal{O}(x;\theta) = \tilde{\mathcal{O}}(x) f(\theta)$
    ➔$\tilde{\mathcal{O}}(x) = \mathcal{O}(x;\theta)/f(\theta)$ universal
  - + rescaling of the variable: $\mathcal{O}(x;\theta) = \tilde{\mathcal{O}}(x/g(\theta)) f(\theta)$
    ➔$\tilde{\mathcal{O}}(\tilde{x}) = \mathcal{O}(\tilde{x}g(\theta);\theta)/f(\theta)$ universal
  - reduction of parameters: $\mathcal{O}(x;\theta) = \mathcal{O}(x;\theta_{\mathbf{2}})$ with "$\theta_2 \subset \theta$"
- combinations of these universalities can appear

- leading-order Halo EFT description provides reduction-of-parameter universality
- question: Display E_{nn} distributions universality beyond that?

Analysis in terms of dimensionless variables

- analysis in terms of dimensionless variables can reduce the number of parameters
- starting point: LO EFT universality
  $\rho(E_{nn}; S_{2n}, V_{nn}, V_{nc}, V_3, \{m_i\}) = \rho\left(E_{nn}; S_{2n}, a_{nn}, a_{nc}, V_3^{(\mathrm{LO})}, A\right)$
- way of analysis: step by step
  Faddeev equations ➔ overall wave function ➔ E_{nn} distribution
- use $\sqrt{2\mu S_{2n}}$ as momentum scale
  & work with $\tilde{q} = q/\sqrt{2\mu S_{2n}}$ and $\bar{a}_{ij} = a_{ij}\sqrt{2\mu S_{2n}}$

➔ result: $\rho\left(E_{nn}; S_{2n}, a_{nn}, a_{nc}, V_3^{(\mathrm{LO})}, A\right) \propto S_{2n}^{-1}\tilde{\tilde{\rho}}(E_{nn}/S_{2n}; \bar{a}_{nn}, \bar{a}_{nc}, A)$



Understanding why the unitarity limit works so well

Dimensionless picture

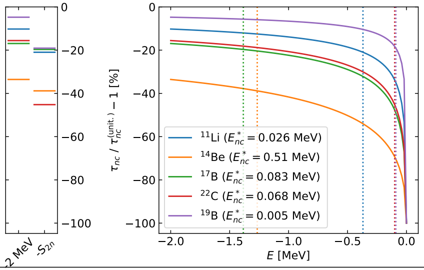
influence of $\bar{a}_{ij} = \sqrt{2\mu S_{2n}} a_{ij}$ depends on

- size of $\bar{a}_{ij}$: the larger $\bar{a}_{ij}$, the smaller the influence
- interplay of the different forces (where the t-matrix is probed)

Dimensionfull picture

influence of a_{ij} depends on

- size of a_{ij}: the larger a_{ij}, the smaller the influence
- size of S_{2n}: the larger S_{2n}, the smaller the influence of a_{ij}
- interplay of the different forces

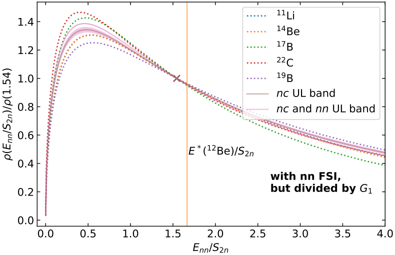


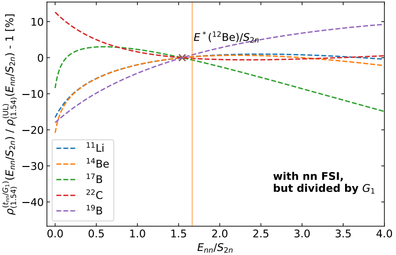
Universality of the final distributions

- universal curve: build on the ground-state findings
  ➔ use approx. technique of FSI enhancement factors
- benchmark: final distributions from LO EFT with FSI based on t-matrix

- ground state and *nn* FSI have unaligned universalities
  univ. driven by S_{2n} vs. "trivial univ." given by a_{nn}

➔ $\tilde{\rho}/G$ should be an universal function of E_{nn}/S_{2n}
$\tilde{\rho}^{(\mathrm{wFSI})}(E_{nn}/S_{2n}; V_{nn}, V_{nc}, S_{2n}, A) \approx \tilde{\rho}(E_{nn}/S_{2n}; A)\, G\big(a_{nn}\sqrt{2\mu E_{nn}}, r_{nn}\sqrt{2\mu E_{nn}}\big)$





Göbel, Hammer, Phillips, PRC 110 (2024)

Backup slides
$E1$ strength distribution of ^{11}Li

Different FSI approximation techniques

FSIs

- Møller operators

$$\Omega = \mathbb{1} + \int d\xi \, \frac{1}{E_\xi - H_0 - V} V \, |\xi\rangle \langle\xi|$$

$$= \mathbb{1} + \int d\xi \, G_0(E_\xi) t(E_\xi) \, |\xi\rangle \langle\xi|$$

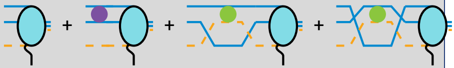
- matrix element of interest: ${}_c\langle p, q; \Xi_f | \Omega^\dagger \mathcal{M}(E1;\mu) | \Psi\rangle$ whereby $V = V_{nn} + V_{nc} + V_{n'c}$

→ approximations as an interesting alternative
  - insights in the role of specific interactions (e.g., *nn* or *nc*)
  - in certain kinematics favor specific interactions

Approximation strategies

- include only one FSI: use Ω_{nn} or Ω_{nc}

- use series in $G_0 t_{ij}$ up to certain order, e.g., first order:
  $\Omega \approx \mathbb{1} + G_0 t_{nn} + G_0 t_{nc} + G_0 t_{n'c}$

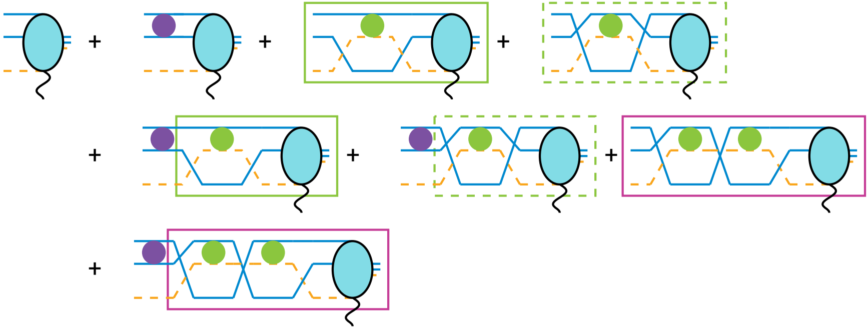


⚠ not necessarily unitary

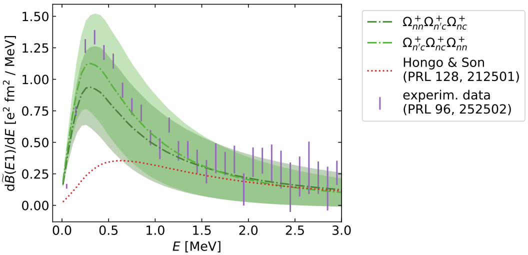
- use products of Møller operators, e.g.: $\Omega_{nc}\Omega_{nn}$ or $\Omega_{nc}\Omega_{n'c}\Omega_{nn}$
  ⚠ does not commute with $\mathcal{P}_{nn}$

$E1$ strength of two-neutron halo nuclei
Efficient organization of the calculations at the example of $\Omega^\dagger_{nn}\Omega^\dagger_{n'c}\Omega^\dagger_{nc}$

to simplify the calculation, certain subdiagrams can be reused



Results for ^{11}Li in comparison with experiment and other theory



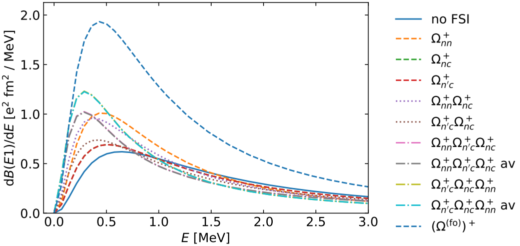
Göbel, Acharya, Hammer, Phillips, PRC 107 (2023)

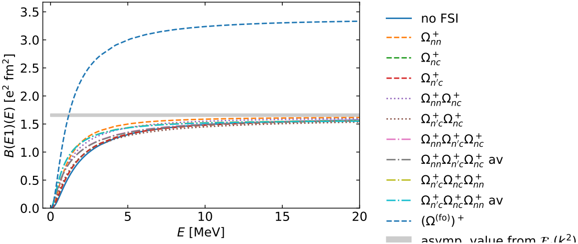
- EFT uncertainty bands given by $\sqrt{E/E_c^*}$
- uncertainty of FSI calculation from difference of the two third-order results

→ theory from Hongo & Son (Hongo, Son, PRL 128 (2022)) is less suitable for ^{11}Li

→ reasonable agreement with experimental data (Nakamura *et al.*, PRL 96 (2006))

Results for ^{11}Li - FSIs in detail





- convergence pattern in this approach is visible
- first-order approximation overshoots largely
- approximation scheme works, i.e., preserves probability

Backup slides
$E1$ strength distribution of ^{6}He

Final-state interactions and partial waves

- ^{11}Li
  - all FSIs are *s*-wave
  - one most important partial-wave component of the ground state
  - → evaluation of recoupling was necessary for combining different FSIs, but partial-wave structure was limited
- ^{6}He
  - FSIs are in different partial waves (*s* and *p* wave)
  - one most important partial-wave component of the ground state (*s* wave)
  - but: Could a *p*-wave FSI enhance a *p*-wave component of the ground state?
  - → similar approach to FSI via products of Møller operators useful, but be more general regarding partial waves
- plan for ^{6}He
  - evaluation of the $E1$ operator between arbitrary partial-wave states

$$
{}_c\langle x', y'; \Omega' | r_c Y_{1,\mu}(r_c) | x, y; \Omega \rangle_c = \delta_{s,s'} \delta_{\sigma,\sigma'} \delta_{l,l'} \delta_{j,j'}
$$

$$
\times \sqrt{\frac{3}{4\pi}} f_c y \sqrt{\hat{j} \hat{I} \hat{I}'} \sqrt{\hat{\lambda} \hat{J}} C^{\lambda',0}_{\lambda,0,1,0} C^{J',M'}_{J,M,1,\mu} (-1)^{2s+\sigma+\lambda'+j'} (-1)^{J+2J'} \begin{Bmatrix} 1 & I' & I \\ j' & J & J' \end{Bmatrix} \begin{Bmatrix} 1 & I' & I \\ \sigma & \lambda & \lambda' \end{Bmatrix}
$$

  with $\Omega = (l, [s_1, s_2]\, s)\, j\, (\lambda, \sigma)\, I; J, M$
  - evaluation of FSI between arbitrary many parital-wave states

Final-state interactions and partial-waves II
Equations for *nc* FSI

$$ {}_{\mathcal{S},\mathcal{S}'}\mathcal{T}^{p,q|p',q'}_{\Omega,\Omega'} f(p',q') := \int \mathrm{d}p'\, p'^2 \int \mathrm{d}q'\, q'^2\, {}_{\mathcal{S}}\langle p,q;\Omega|p',q';\Omega'\rangle_{\mathcal{S}'}\, f(p',q') $$

$$ {}_c\langle p,q;\Omega|(\Omega_{nc}-\mathbb{1})^\dagger \mathcal{M}_{E1,\mu}|\Psi\rangle = \sum_{\Omega_i,\Omega_m} f^{(\Omega_m,\Omega_i)}_{E1,\mu} \sum_{\substack{\Omega' \text{ with} \\ (\Omega')_{nc}=\omega_{nc}}} {}_{cn}\mathcal{T}^{p,q|p',q'}_{\Omega,\Omega'}\, g_{l_n}(p')\tau_{nc}\left(E_{p'}\right) $$

$$ \times \int \mathrm{d}\tilde{p}\, \tilde{p}^2 g_{l_n}(\tilde{p}) \frac{1}{E_{p'} - \tilde{p}^2/(2\mu_{nc}) + \mathrm{i}\epsilon}\, {}_{nc}\mathcal{T}^{\tilde{p},q'|p'',q''}_{\Omega',\Omega_m} D^{(\Omega_m,\Omega_i)}_{q''} \Psi_{c,\Omega_i}(p'',q'') $$

Example for the recoupling

```
part 1 of 26
I | S | Omegas
- | 3 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 |
1 | 1 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (1,1)1; 2,0 | (1,[1,0]1)3 (1,1)3; 2,0 | (1,[1,0]1)3 (2,1)3; 2,0 | (1,[1,0]1)3 (2,1)5; 2,0 |
- | 3 | (0,[1,1]0)0 (1,0)2; 2,0 | (0,[1,1]0)0 (1,0)2; 2,0 | (0,[1,1]0)0 (1,0)2; 2,0 | (0,[1,1]0)0 (1,0)2; 2,0 | (0,[1,1]0)0 (1,0)2; 2,0 |

part 2 of 26
I | S | Omegas
- | 3 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 |
1 | 1 | (1,[1,0]1)3 (3,1)5; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 |
- | 3 | (0,[1,1]0)0 (1,0)2; 2,0 | (0,[1,1]2)2 (1,0)2; 2,0 | (1,[1,1]0)2 (0,0)0; 2,0 | (1,[1,1]2)2 (0,0)0; 2,0 | (1,[1,1]0)2 (2,0)4; 2,0 |

part 3 of 26
I | S | Omegas
- | 3 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 |
1 | 1 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 |
- | 3 | (1,[1,1]2)2 (2,0)4; 2,0 | (1,[1,1]2)4 (2,0)4; 2,0 | (2,[1,1]0)4 (1,0)2; 2,0 | (2,[1,1]2)2 (1,0)2; 2,0 | (2,[1,1]2)4 (1,0)2; 2,0 |

part 4 of 26
I | S | Omegas
- | 3 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 |
1 | 1 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 |
- | 3 | (2,[1,1]0)4 (3,0)6; 2,0 | (2,[1,1]2)4 (3,0)6; 2,0 | (2,[1,1]2)6 (3,0)6; 2,0 | (3,[1,1]0)6 (2,0)4; 2,0 | (3,[1,1]2)4 (2,0)4; 2,0 |

part 5 of 26
I | S | Omegas
- | 3 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 |
1 | 1 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 | (1,[1,0]1)3 (0,1)1; 2,0 |
- | 3 | (3,[1,1]2)6 (2,0)4; 2,0 | (3,[1,1]0)6 (4,0)8; 2,0 | (3,[1,1]2)6 (4,0)8; 2,0 | (3,[1,1]2)8 (4,0)8; 2,0 | (4,[1,1]0)8 (3,0)6; 2,0 |
```

scheme: $(l, [2s_1, 2s_2]2s)\,2j\ \ (\lambda, 2\sigma)\,2I;\ 2J, 2M$

Backup slides
General slides

Calculation of wave functions & probability densities: Details

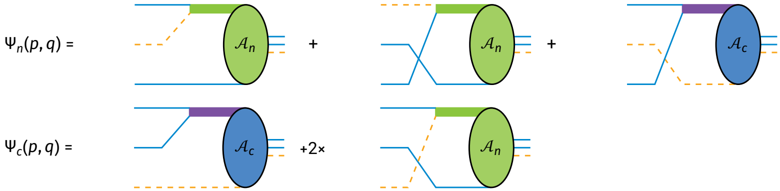
wave functions = overlaps of state $|\Psi\rangle$ with reference states

notation: $_i\langle p, q; (l, s)\, j, (\lambda, \sigma)\, \mathcal{J}; J, M | \Psi \rangle$

examples:

$$\Psi_n(p,q) \coloneqq {}_n\langle p, q; \left(1, \tfrac{1}{2}\right) \tfrac{3}{2}, \left(1, \tfrac{1}{2}\right) \tfrac{3}{2}; 0, 0 \,|\Psi\rangle$$

$$\Psi_c(p,q) \coloneqq {}_c\langle p, q; (0, 0)\, 0, (0, 0)\, 0; 0, 0 \,|\Psi\rangle$$



Calculation of wave functions & probability densities: Details

wave functions = overlaps of state $|\Psi\rangle$ with reference states

notation: ${}_i\left\langle p, q; \left(l, s\right) j, \left(\lambda, \sigma\right) \mathcal{J}; J, M \middle| \Psi \right\rangle$

examples:

$$\Psi_n(p,q) \coloneqq {}_n\left\langle p, q; \left(1, \tfrac{1}{2}\right) \tfrac{3}{2}, \left(1, \tfrac{1}{2}\right) \tfrac{3}{2}; 0, 0 \middle| \Psi \right\rangle$$

$$\Psi_c(p,q) \coloneqq {}_c\left\langle p, q; \left(0, 0\right) 0, \left(0, 0\right) 0; 0, 0 \middle| \Psi \right\rangle$$

complete description of ground state of ^{6}He possible in terms of

$$\Psi_c^{(l,s=0)}(p,q) \coloneqq {}_c\left\langle p, q; \left(l, 0\right) l, \left(l, 0\right) l; 0, 0 \middle| \Psi \right\rangle \quad \text{with } l \text{ even}$$

$$\Psi_c^{(l,s=1)}(p,q) \coloneqq {}_c\left\langle p, q; \left(l, 1\right) l, \left(l, 0\right) l; 0, 0 \middle| \Psi \right\rangle \quad \text{with } l \text{ odd}$$

($J^\pi = 0^+$ and antisymmetrization in *nn* subsystem restrict possible states)

General results from two-potential scattering theory

as discussed in Goldberger, Watson, "Collision Theory" (1964)

quantity of interest: $T_{\beta\alpha} = \langle\beta|T^{(+)}_{U+V}|\alpha\rangle$

- Lippmann-Schwinger equation for t-matrix: $T^{(+)}_{U+V} = (U + V) + (U + V)\, G_0^{(+)} T^{(+)}_{U+V}$
  - production potential V
  - FSI potential U
- with the asympt. states $|\alpha\rangle$ & $|\beta\rangle$
  - fulfilling $H_0 \left|i\right\rangle = E_i \left|i\right\rangle \quad (i \in \{\alpha, \beta\})$
  - $E_\alpha = E_\beta =: E$

method & results

- use Møller operators: $\Omega_V^{(\pm)} = \mathbb{1} + (E - H_0 - V \pm i\epsilon)^{-1}\, V$

→ dissection of $T^{(+)}_{U+V}$ possible: $T^{(+)}_{U+V} = \left(\Omega_U^{(-)}\right)^\dagger V \Omega^{(+)}_{U+V} + \left(\Omega_U^{(-)}\right)^\dagger U$

→ if $(H_0 + U)\,|\alpha\rangle = E_\alpha\,|\alpha\rangle$ holds instead of $H_0\,|\alpha\rangle = E_\alpha\,|\alpha\rangle$:

$$T_{\beta\alpha} = \langle\beta|\left(\Omega_U^{(-)}\right)^\dagger V\left(\mathbb{1} + (E - K - U - V + i\epsilon)^{-1}\, V\right)|\alpha\rangle$$

Faddeev equations: the quantum-mechanical picture

- equivalent to Schrödinger eq.: $H\,|\Psi\rangle = E\,|\Psi\rangle$

- introduce $|\psi_i\rangle \coloneqq G_0 V_i\,|\Psi\rangle$

➔ system of eqs. for $|\psi_i\rangle$: $|\psi_i\rangle = G_0 t_i \sum_{j\neq i} \left|\psi_j\right\rangle$

➔ $|\Psi\rangle = \sum_i |\psi_i\rangle$

- advantage: interactions can be specified in terms of t-matrices

- introduce $|F_i\rangle \coloneqq (G_0 t_i)^{-1}\,|\psi_i\rangle$

➔ system of eqs. for $|F_i\rangle$: $|F_i\rangle = \sum_{j\neq i} G_0 t_j \left|F_j\right\rangle$

➔ $|\Psi\rangle = \sum_i G_0 t_i\,|F_i\rangle$

- advantage: has nicer representations (function of only one variable can be formed in case of separable interactions)