# Weak decays of D mesons into three mesons

E Oset, Raquel Molina, Wei Hong Liang, Li Sheng Geng, Ju Jun Xie, Lian Rong Dai

Basic things about weak decays

The $D_s^+ \to \pi^+ \pi^0 \eta$

The $D_s^+ \to K^+ \pi^+ \pi^-$

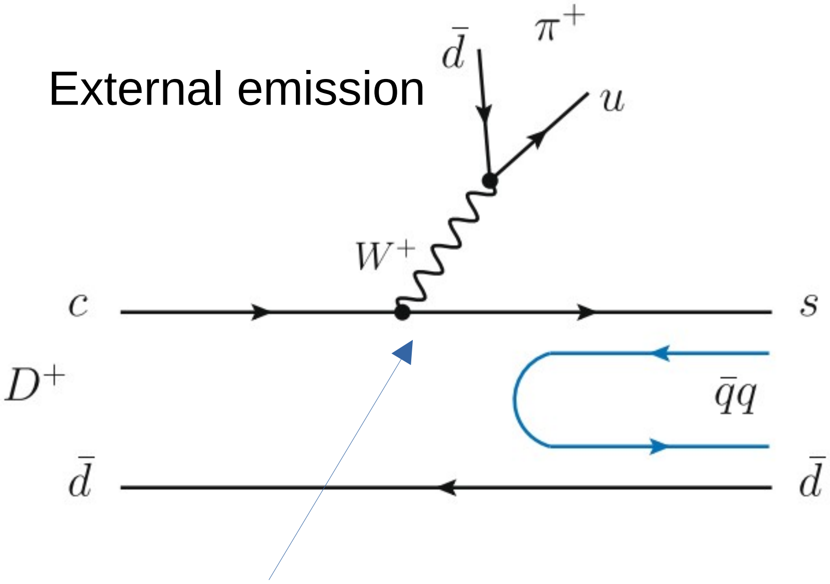
# ABC of weak interactions

| Charge | | | |
|---|---|---|---|
| 2/3 | u | c | t |
| -1/3 | d | s | b |

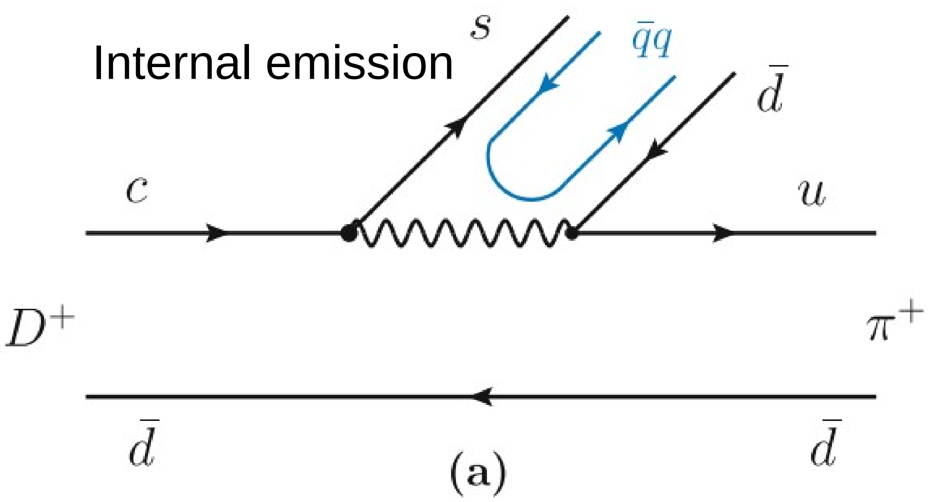
Cabibbo favored

Cabibbo suppressed

External emission




Internal emission




Color favored

$\gamma^\mu(1-\gamma_5)$

Color suppressed

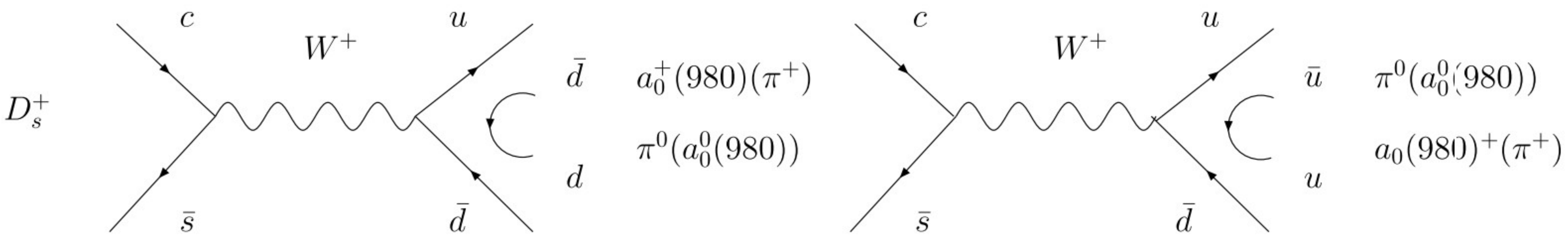



**Fig. 1.** Annihilation mechanisms assumed in Ref. [1] for the $D_s^+ \to \pi^0 a_0^+(980)$, $\pi^+ a_0^0(980)$.

This mechanism is much weaker than the others

# Theoretical interpretation of the $D_s^+ \to \pi^+\pi^0\eta$ decay and the nature of $a_0(980)$

Phys.Lett.B 803

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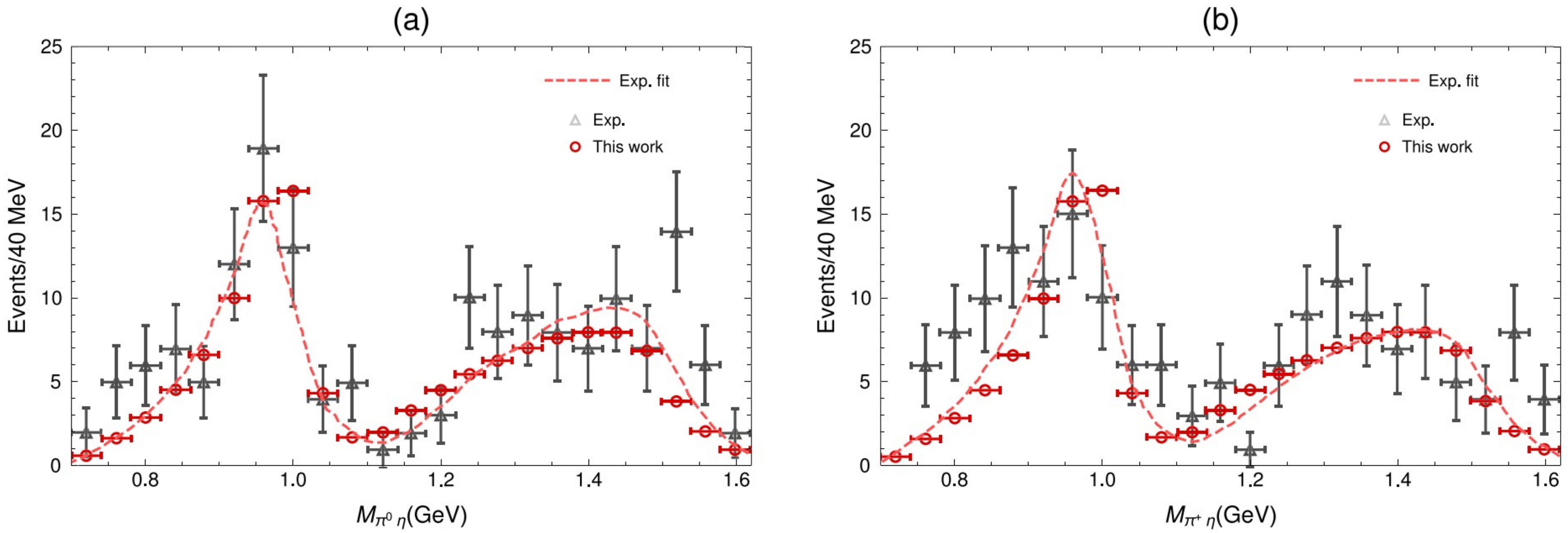



**Fig. 5.** Event distribution in 40 MeV bins of $d\Gamma/dM_{\pi\eta}$ compared with experiment with $M_{\pi^+\pi^0} > 1$ GeV. (a) for $\pi^0\eta$ distribution; (b) for $\pi^+\eta$ distribution. The dashed lines are taken from [1] after the non $\pi a_0$ events are removed.

[1] M. Ablikim, et al., BESIII Collaboration, Phys. Rev. Lett. 123 (2019) 112001

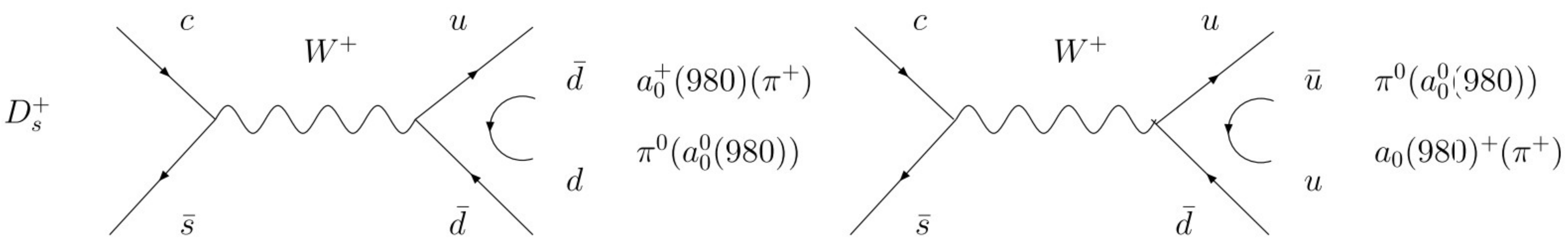
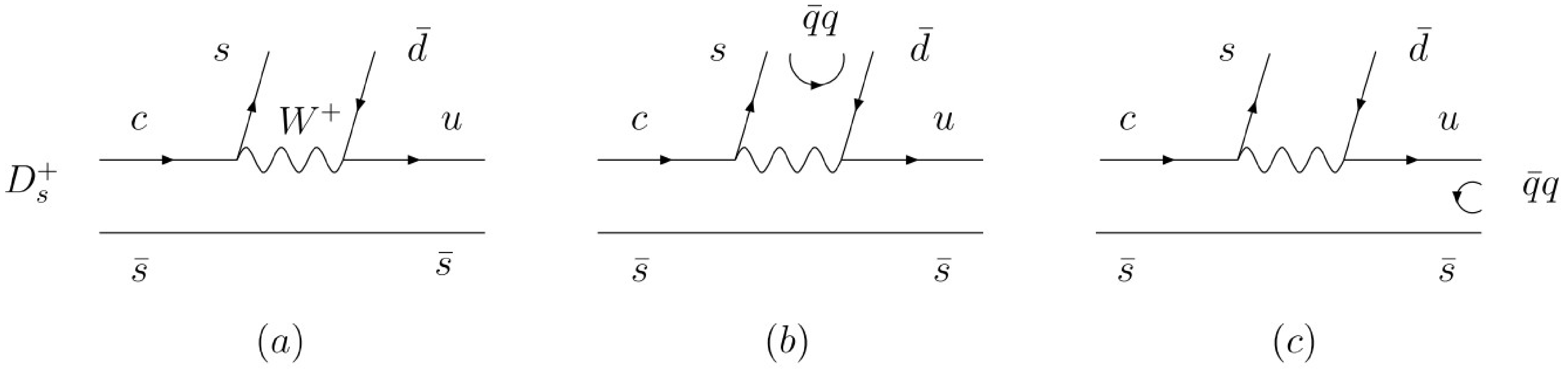



**Fig. 1.** Annihilation mechanisms assumed in Ref. [1] for the $D_s^+ \to \pi^0 a_0^+(980)$, $\pi^+ a_0^0(980)$.

and the decay is branded as a clean example of W -annihilation
with a rate which is one order of magnitude bigger than the typical W -annihilation rates

The theoretical interpretation is different. One can produce a given final state even if not produced in a first step through final state interaction




$$\sum_i s\bar{q}_i q_i \bar{d} = \sum_i M_{3i} M_{i2} = (M^2)_{32},$$

$$\sum_i u\bar{q}_i q_i \bar{s} = \sum_i M_{1i} M_{i3} = (M^2)_{13},$$

$M =$ q qbar

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \frac{2}{\sqrt{6}}\eta' \end{pmatrix}$$

$$(M^2)_{32} = \pi^+ K^- - \frac{1}{\sqrt{2}}\pi^0 \bar{K}^0,$$

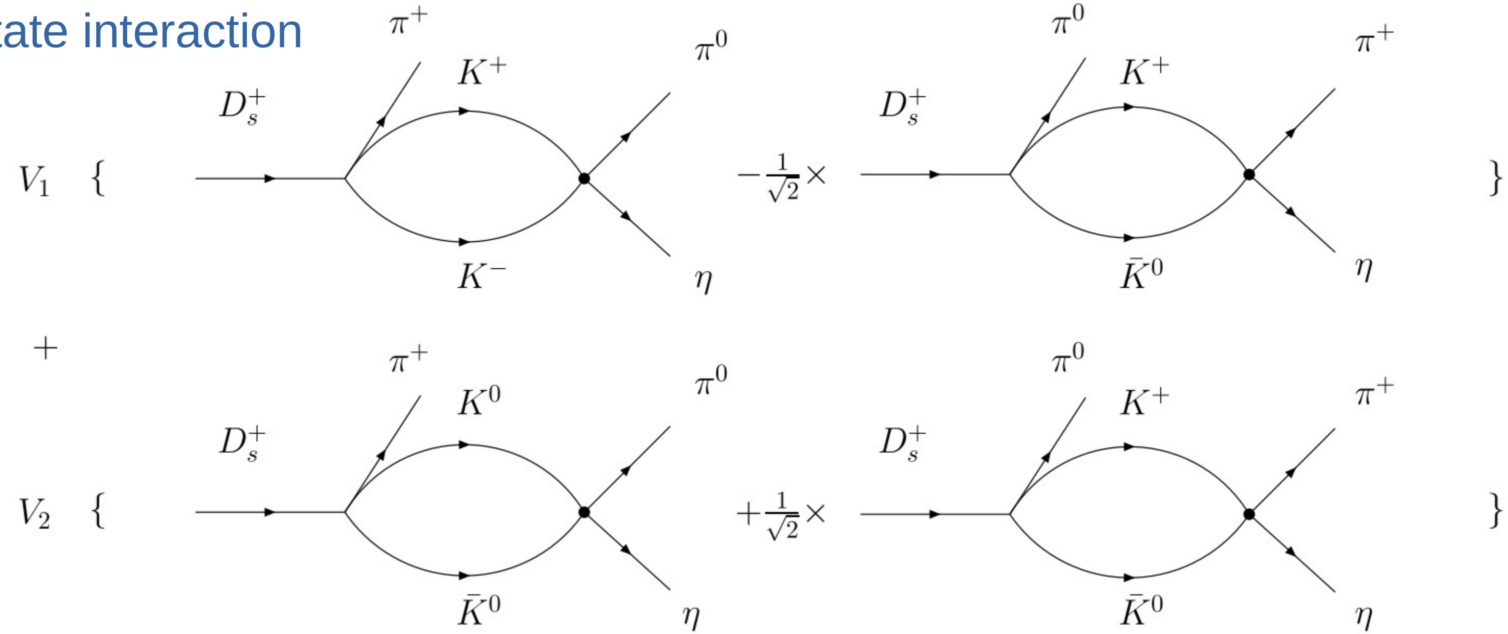
$$(M^2)_{13} = \frac{1}{\sqrt{2}}\pi^0 K^+ + \pi^+ K^0,$$

$$H_1 = (\pi^+ K^- - \frac{1}{\sqrt{2}}\pi^0 \bar{K}^0) K^+,$$

$$H_2 = (\frac{1}{\sqrt{2}}\pi^0 K^+ + \pi^+ K^0) \bar{K}^0.$$

NOTE: NONE OF THE STATES IS THE DESIRED ONE $\pi^+ \pi^0 \eta$

Final state interaction



$$t = V_1[G_{K\bar{K}}(M_{\pi^0\eta})t_{K^+K^-\to\pi^0\eta}(M_{\pi^0\eta}) - \frac{1}{\sqrt{2}}G_{K\bar{K}}(M_{\pi^+\eta})t_{K^+\bar{K}^0\to\pi^+\eta}(M_{\pi^+\eta})] + V_2[G_{K\bar{K}}(M_{\pi^0\eta})t_{K^0\bar{K}^0\to\pi^0\eta}(M_{\pi^0\eta}) + \frac{1}{\sqrt{2}}G_{K\bar{K}}(M_{\pi^+\eta})t_{K^+\bar{K}^0\to\pi^+\eta}(M_{\pi^+\eta})] \,,$$

$$t_{K^+K^-\to\pi^0\eta} = -\frac{1}{\sqrt{2}}t^{I=1}_{K\bar{K}\to\pi\eta},$$
$$t_{K^0\bar{K}^0\to\pi^0\eta} = \frac{1}{\sqrt{2}}t^{I=1}_{K\bar{K}\to\pi\eta},$$
$$t_{K^+\bar{K}^0\to\pi^+\eta} = -t^{I=1}_{K\bar{K}\to\pi\eta},$$

$$t = \bar{V}\left[G_{K\bar{K}}(M_{\pi^0\eta})t^{I=1}_{K\bar{K}\to\pi\eta}(M_{\pi^0\eta}) - G_{K\bar{K}}(M_{\pi^+\eta})t^{I=1}_{K\bar{K}\to\pi\eta}(M_{\pi^+\eta})\right] \,,$$

with $\bar{V} = (V_2 - V_1)/\sqrt{2}$.

$$\frac{d^2\Gamma}{dM_{\pi^0\eta}dM_{\pi^+\eta}} = \frac{1}{(2\pi)^3}\frac{M_{\pi^0\eta}M_{\pi^+\eta}}{8M^2_{D_s^+}}|t|^2$$

Up to a global normalization the decay amplitude is PARAMETER FREE

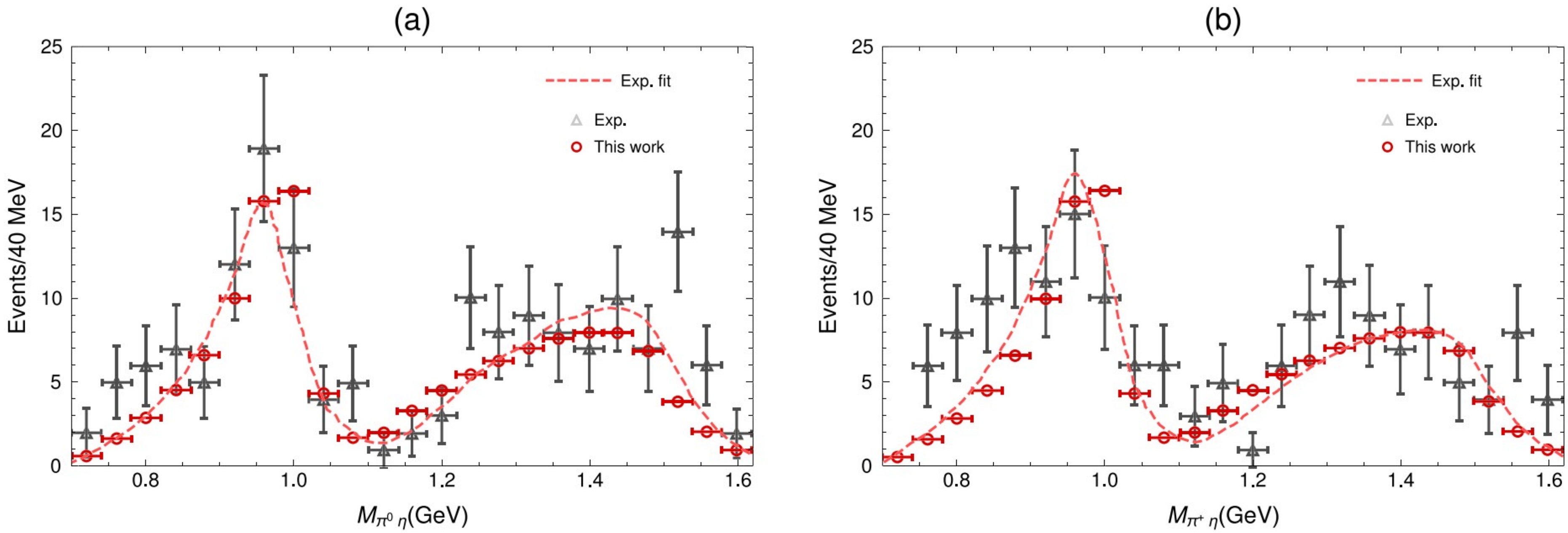
# The chiral unitary approach

$$t = [1 - VG]^{-1} V$$

$$G(s) = \int_{|\mathbf{q}|<q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1 \omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

$$V_{11} = -\frac{s}{2f^2}, \quad V_{12} = -\frac{s - m_\pi^2}{\sqrt{2} f^2},$$
$$V_{13} = -\frac{s}{4f^2}, \quad V_{14} = -\frac{s}{4f^2},$$
$$V_{15} = -\frac{\sqrt{2} m_\pi^2}{3f^2}, \quad V_{16} = 0,$$
$$V_{22} = -\frac{m_\pi^2}{2f^2}, \quad V_{23} = -\frac{s}{4\sqrt{2} f^2},$$
$$V_{24} = -\frac{s}{4\sqrt{2} f^2}, \quad V_{25} = -\frac{m_\pi^2}{3f^2},$$
$$V_{26} = 0, \quad V_{33} = -\frac{s}{2f^2},$$

$$V_{34} = -\frac{s}{4f^2}, \quad V_{35} = -\frac{\sqrt{2}(3s - m_K^2 - 2m_\eta^2)}{9f^2},$$
$$V_{36} = -\frac{3s - 2m_K^2 - m_\eta^2}{3\sqrt{6} f^2}, \quad V_{44} = -\frac{s}{2f^2},$$
$$V_{45} = -\frac{\sqrt{2}(3s - m_K^2 - 2m_\eta^2)}{9f^2}, \quad V_{46} = \frac{3s - 2m_K^2 - m_\eta^2}{3\sqrt{6} f^2},$$
$$V_{55} = -\frac{m_\pi^2 + 2m_K^2}{9f^2}, \quad V_{56} = 0,$$
$$V_{66} = -\frac{2m_\pi^2}{3f^2}. \tag{A4}$$

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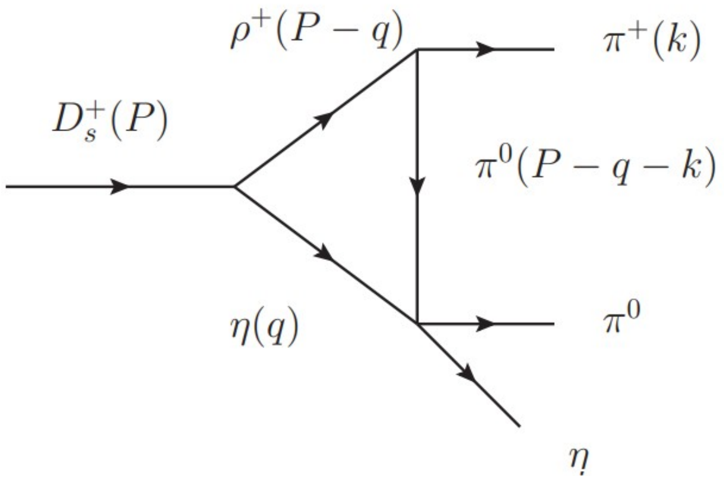
The cut is done in the experiment and in the theory to eliminate the contribution of the ρ

## Suggestion of alternative modes

Y.-K. Hsiao, Y. Yu, and B.-C. Ke, Eur. Phys. J. C 80, 895 (2020), 1909.07327.

X.-Z. Ling, M.-Z. Liu, J.-X. Lu, L.-S. Geng, and J.-J. Xie, Phys. Rev. D 103, 116016 (2021), 2102.05349.



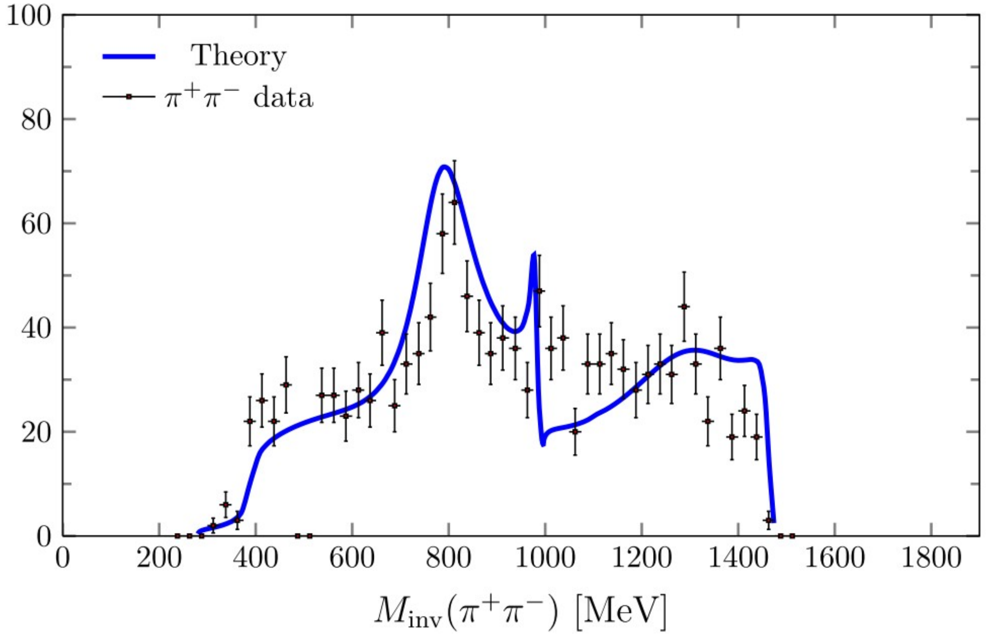

**Subtleties in triangle loops for $D_s^+ \rightarrow \rho^+\eta \rightarrow \pi^+\pi^0\eta$ in $a_0(980)$ production**

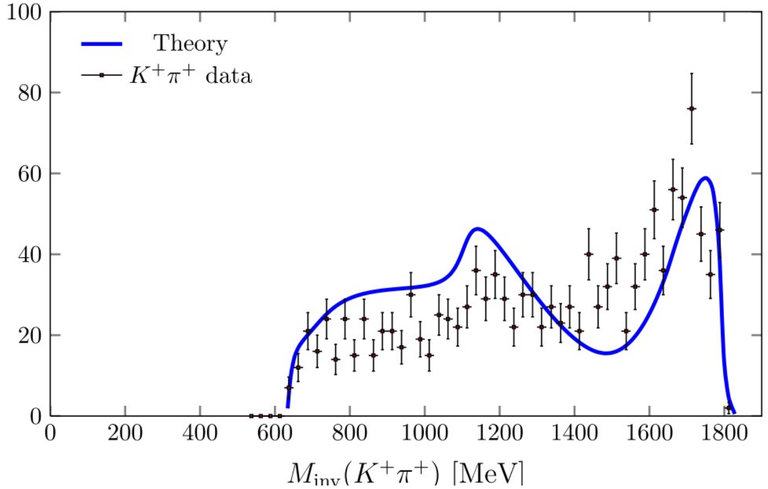
M.~Bayar, R.~Molina, E.~Oset, M.~Z.~Liu and L.~S.~Geng Phys.Rev.D 109 (2024) 7, 076027

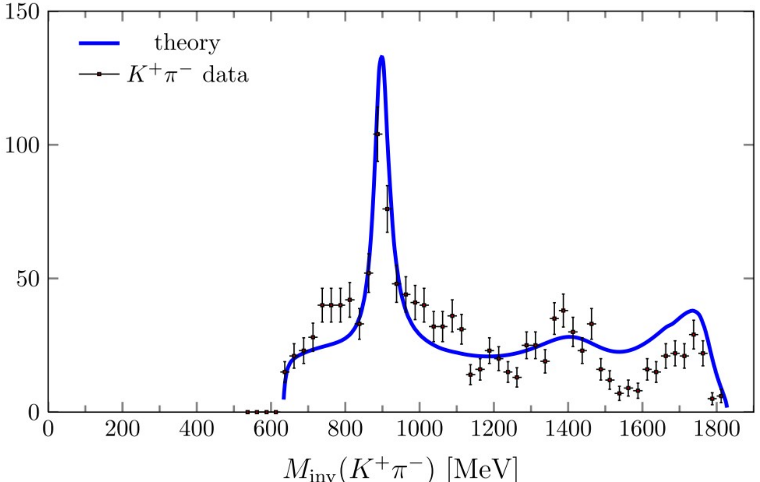
# Dynamical generation of the scalar $f_0(500)$, $f_0(980)$, and $K_0^*(700)$ resonances in the $D_s^+ \to K^+\pi^+\pi^-$ reaction

L. R Dai and E. Oset

PHYSICAL REVIEW D 109, 054008 (2024)




M. Ablikim et al. (BESIII Collaboration),
J. High Energy Phys. 08 (2022) 196.

The reaction is Cabibbo suppressed

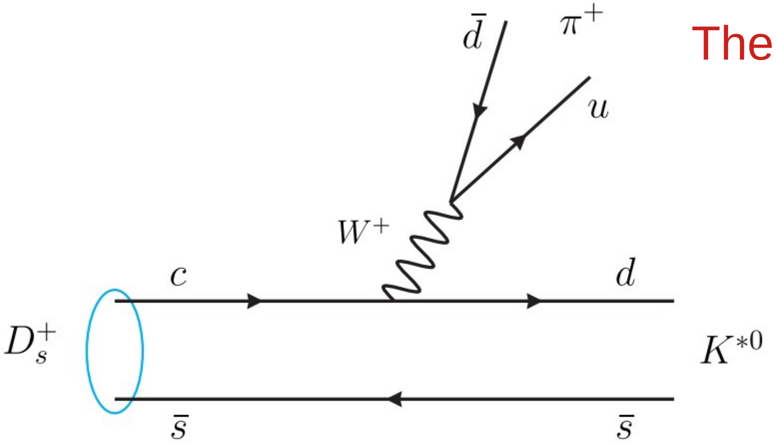



FIG. 1. Mechanism for production of $\pi^+ K^{*0}$ in $D_s^+$ decay with external emission.

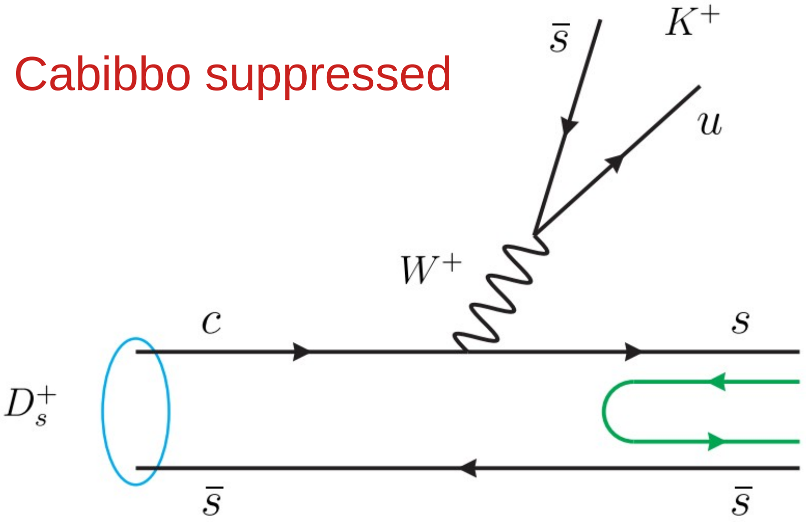



FIG. 2. $D_s^+ \to K^+ s\bar{s}$ with external emission and $s\bar{s}$ hadronization.

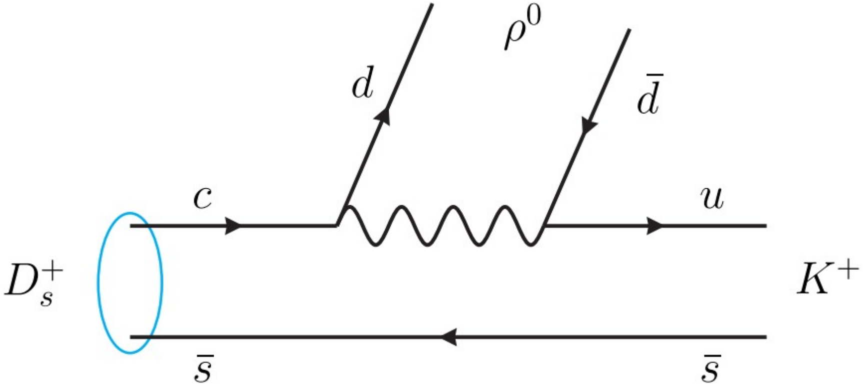



FIG. 3. Mechanism for $D_s^+ \to \rho K^+$ with internal emission.

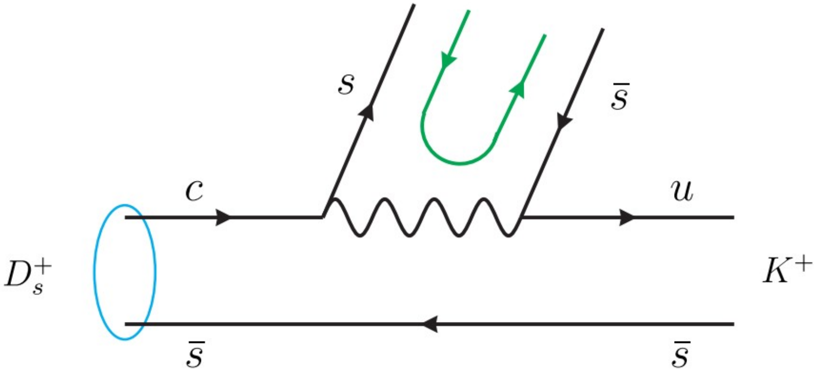



FIG. 4. $D_s^+ \to K^+ s\bar{s}$ with internal emission followed by $s\bar{s}$ hadronization.

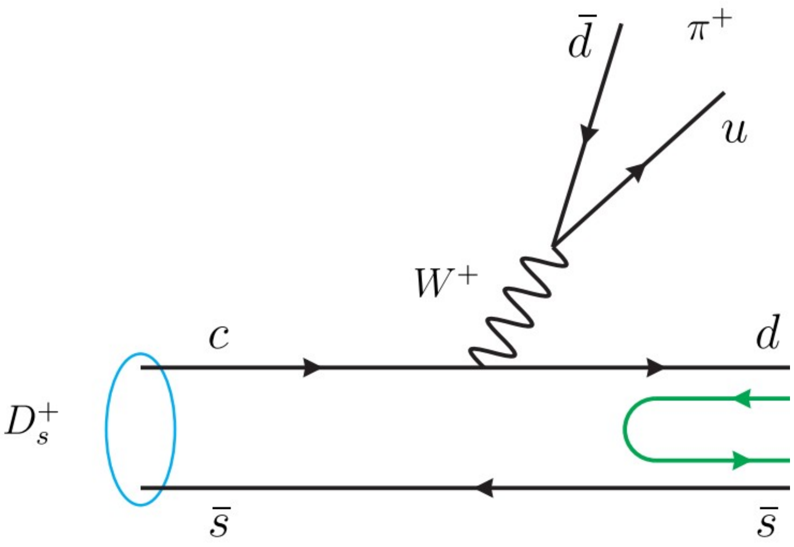



FIG. 5. $D_s^+ \to \pi^+ d\bar{s}$ with external emission and $d\bar{s}$ hadronization.

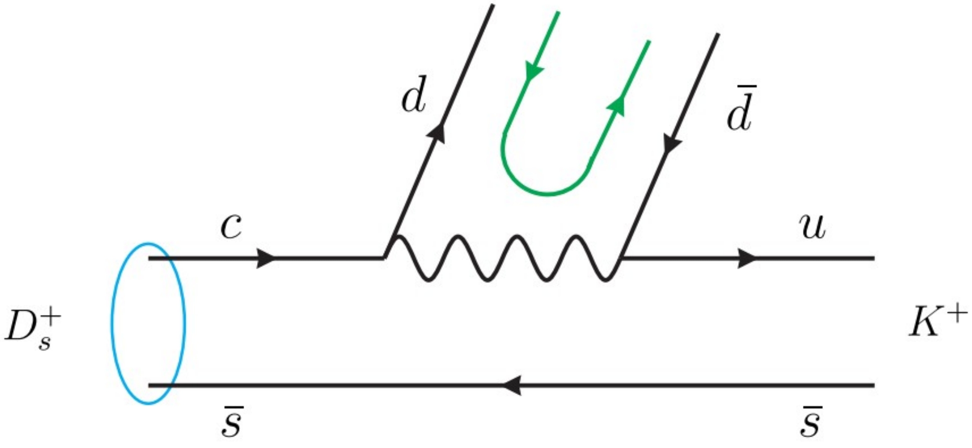



FIG. 6. $D_s^+ \to K^+ d\bar{d}$ with internal emission and $d\bar{d}$ hadronization.

(1) weight $\alpha$ for $K^{*0}$ production,

(2) weight $\alpha h$, the $h$ factor accounting for the mechanism of hadronization,

(3) weight $\gamma$ for $\rho^0$ production,

(4) weight $\gamma h$ since it involves an extra hadronization as in the case of (2),

(5) weight $\alpha h$ since it has the same topology as the case of (2), and

(6) weight $\gamma h$ since it involves an extra hadronization with respect to case of (3).

$$q\bar{q} \to P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} \end{pmatrix}$$

$$s\bar{s} \to \sum_i s\bar{q}_i q_i \bar{s} = \sum_i P_{3i} P_{i3} = (P^2)_{33}$$

$$= K^- K^+ + \bar{K}^0 K^0 + \frac{1}{3}\eta\eta$$

Goes with $K^+$

$$d\bar{s} \to \sum_i d\bar{q}_i q_i \bar{s} = \sum_i P_{2i} P_{i3} = (P^2)_{23}$$

$$= \pi^- K^+ - \frac{1}{\sqrt{2}}\pi^0 K^0.$$

Goes with $\pi^+$

$$d\bar{d} \to \sum_i d\bar{q}_i q_i \bar{d} = (P^2)_{22}$$

$$= \pi^- \pi^+ + \frac{\pi^0 \pi^0}{\sqrt{2}} + \frac{\eta\eta}{3} - \frac{2}{\sqrt{6}}\pi^0 \eta$$

$$+ K^0 \overline{K}^0.$$

Goes with $K^+$

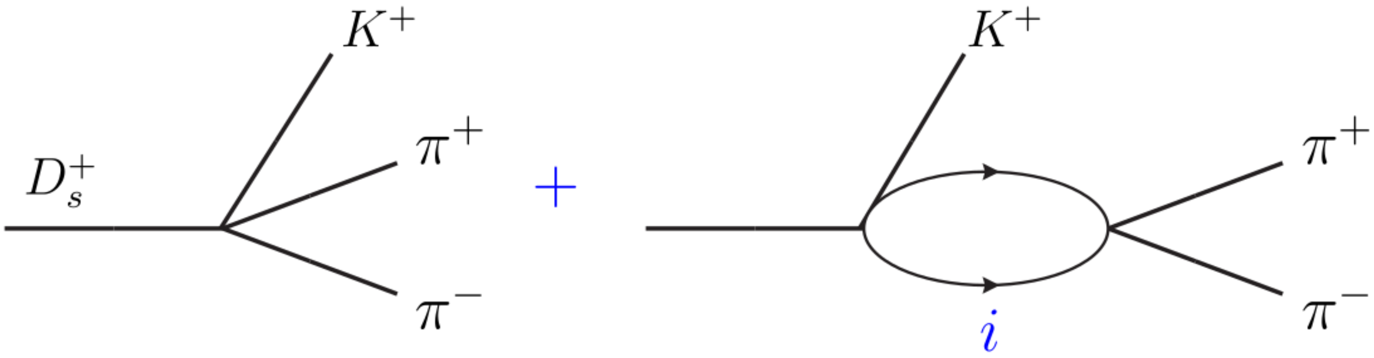
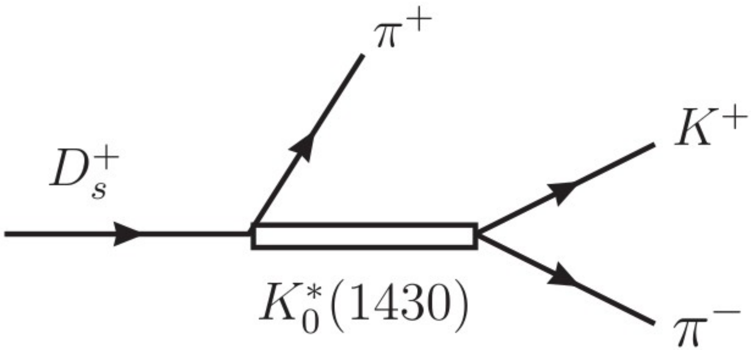
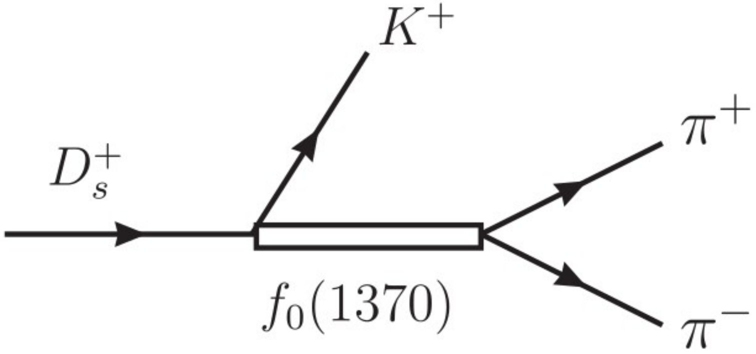



FIG. 7. Direct $K^+\pi^-\pi^+$ production (tree level) and production through intermediate states, $i = \pi^+\pi^-$, $\pi^0\pi^0$, $\eta\eta$, $\pi^0\eta$, $K^+K^-$, $K^0\bar{K}^0$ in general.

$$t^{(4+6)} = \gamma h \left\{ 1 + \sum_i W_i' G_i(M_{\rm inv}, \pi\pi) t_{i,\pi^+\pi^-}(M_{\rm inv}, \pi\pi) \right\}$$

For the G functions and the transition scattering matrices we use the chiral unitary approach



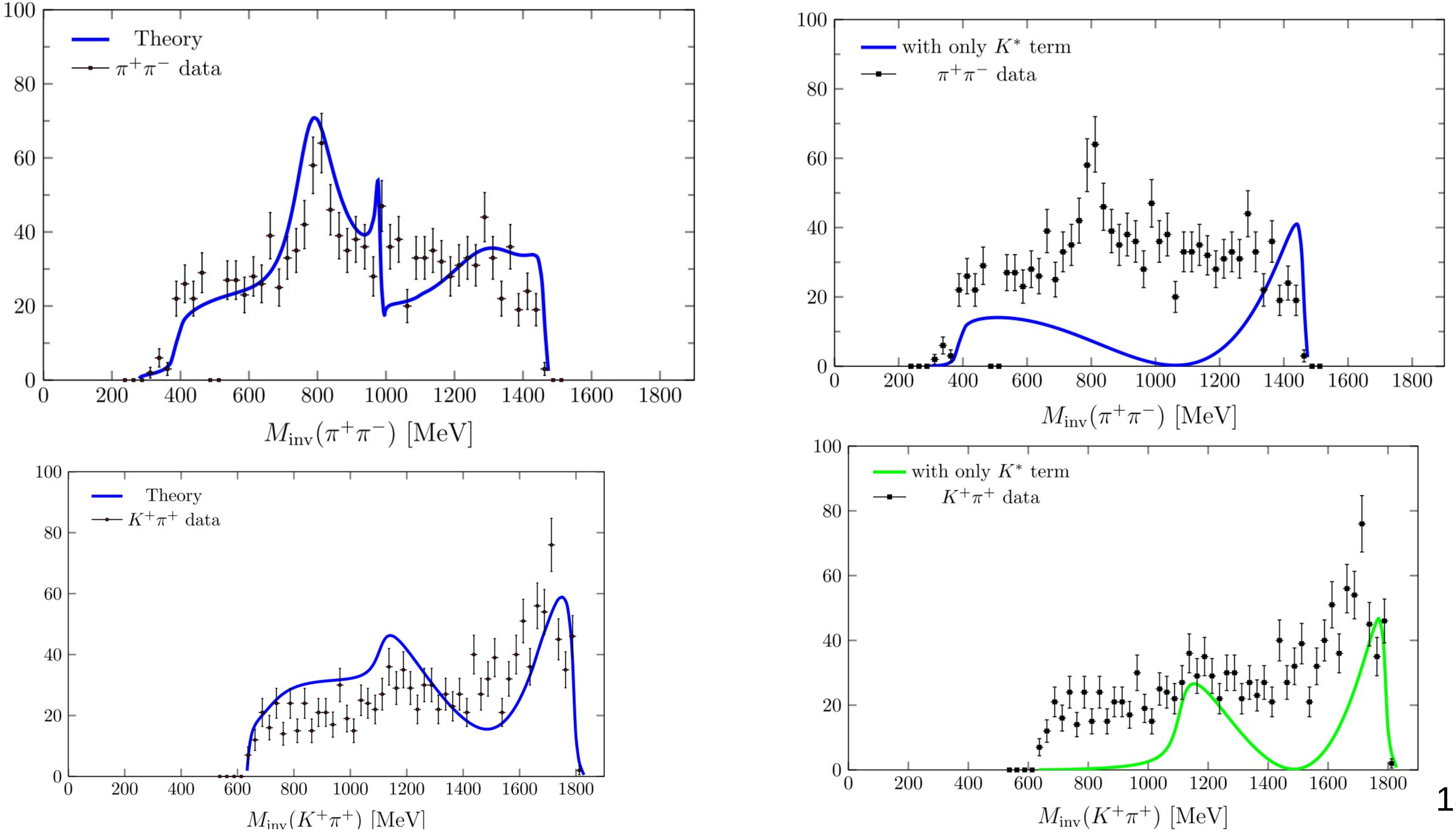

As in the experimental analysis, these are introduced by hand, with a weight to be fitted to the experiment

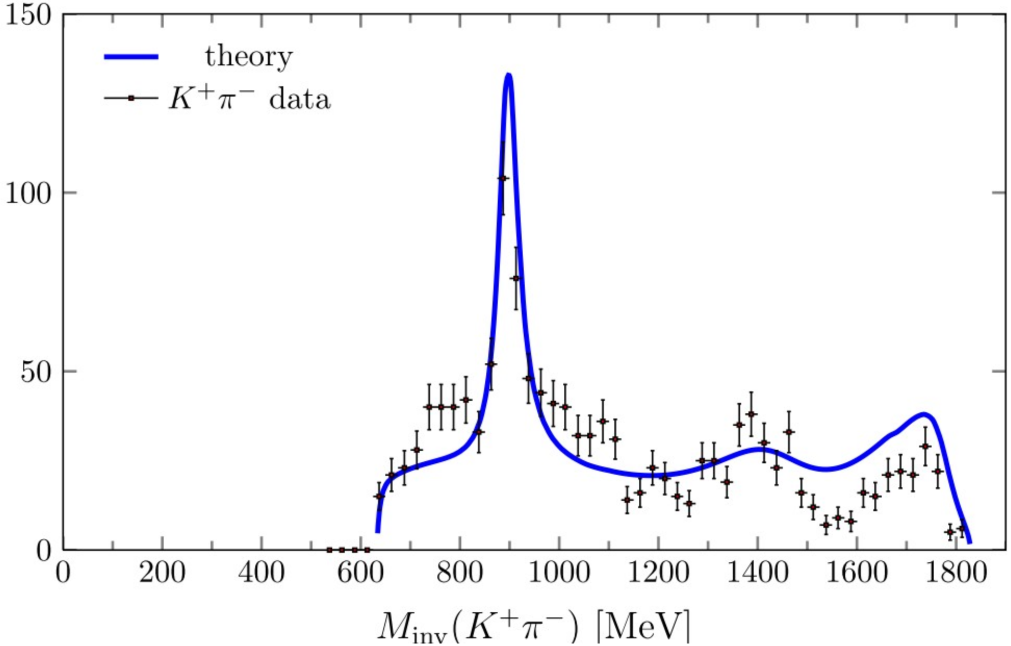
Now we have 6 parameters to fit the three distributions, 5 discounting the normalization

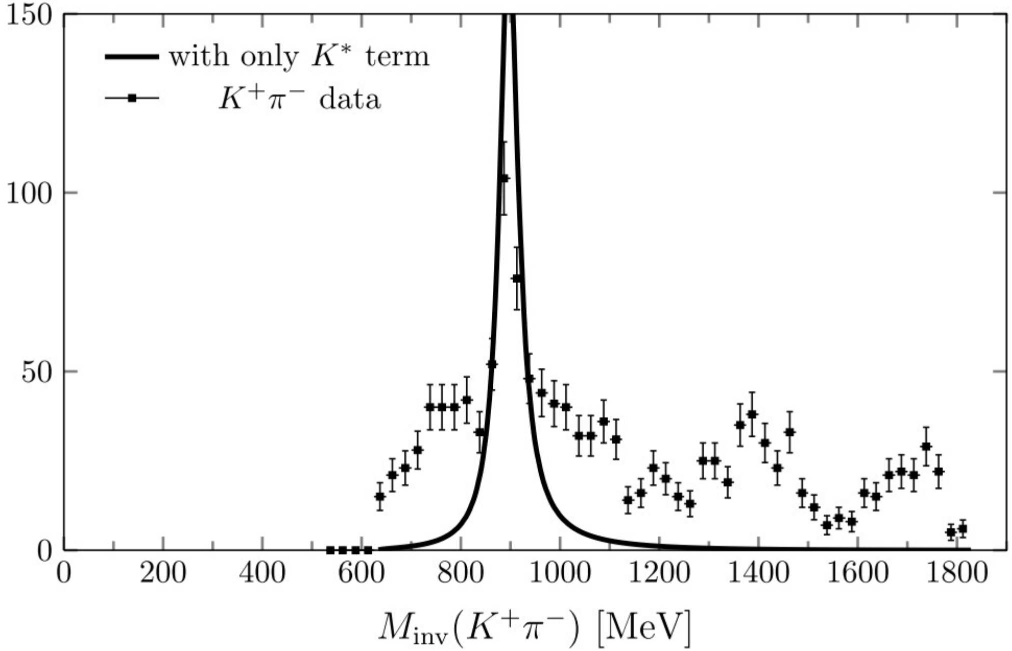
A fit to the data is done. Yet, the π π and K π terms are related by the given weights

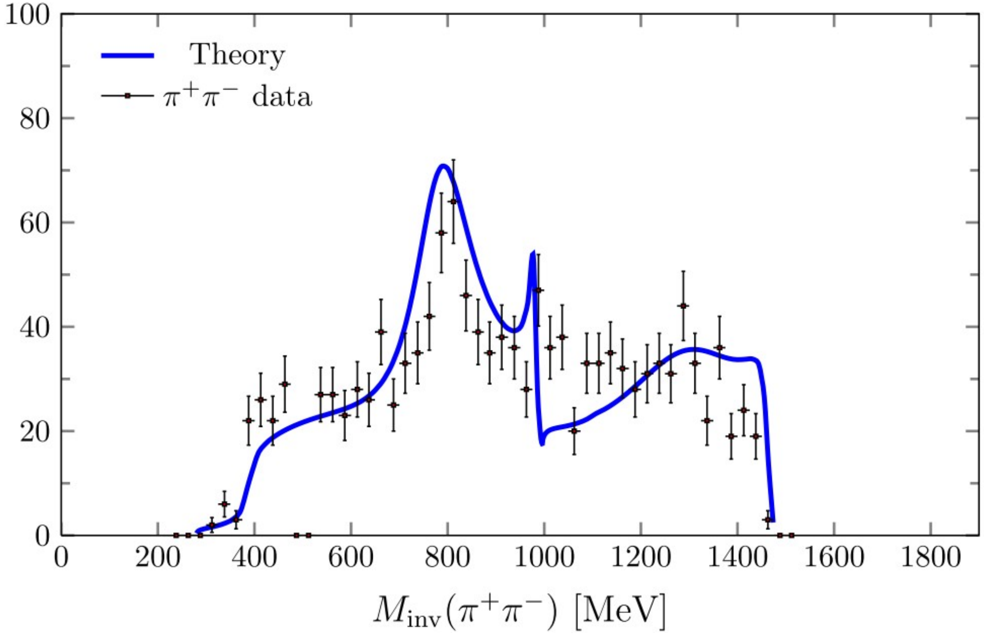
The interaction with coupled channels produce the $f_0(500)$, $f_0(980)$, $K^*_0(700)$ and

their relative strength is a prediction of the theory.


Theory
$\pi^+\pi^-$ data
$M_{\rm inv}(\pi^+\pi^-)$ [MeV]
with only $K^*$ term
$\pi^+\pi^-$ data
$M_{\rm inv}(\pi^+\pi^-)$ [MeV]
Theory
$K^+\pi^+$ data
$M_{\rm inv}(K^+\pi^+)$ [MeV]
with only $K^*$ term
$K^+\pi^+$ data
$M_{\rm inv}(K^+\pi^+)$ [MeV]


theory
$K^+\pi^-$ data
150
100
50
0
0 200 400 600 800 1000 1200 1400 1600 1800
$M_{\rm inv}(K^+\pi^-)$ [MeV]


with only $K^*$ term
$K^+\pi^-$ data
150
100
50
0
0 200 400 600 800 1000 1200 1400 1600 1800
$M_{\rm inv}(K^+\pi^-)$ [MeV]


Theory
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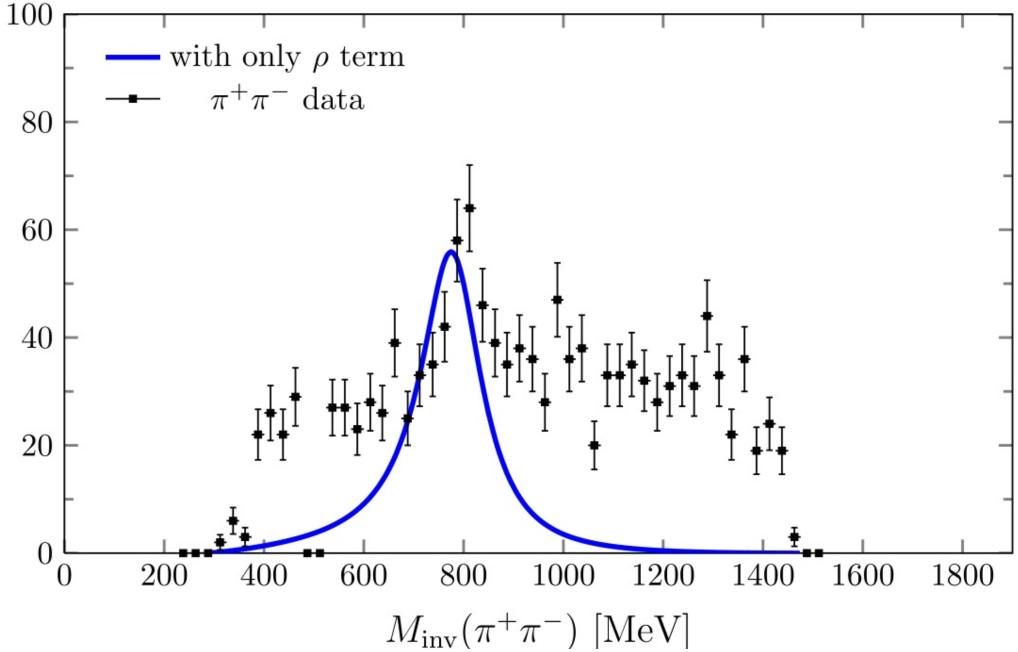
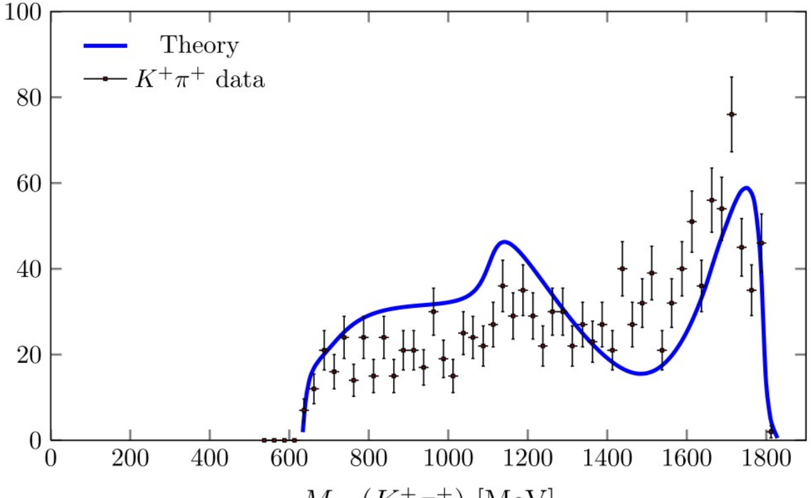
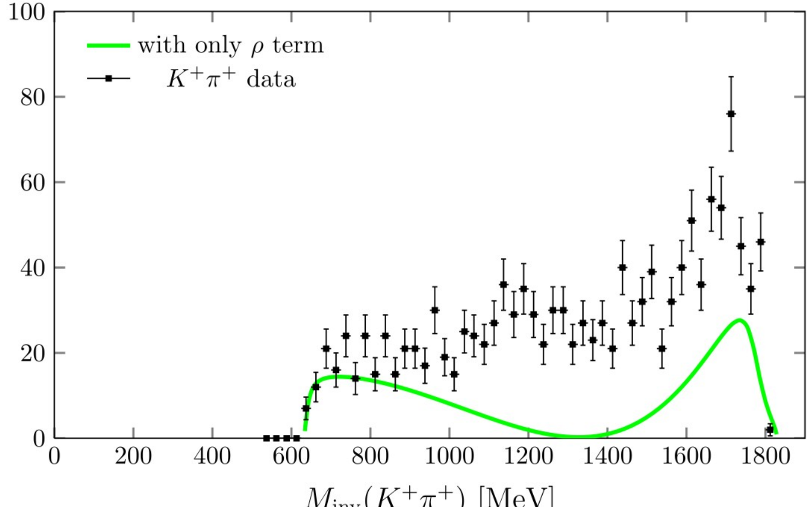
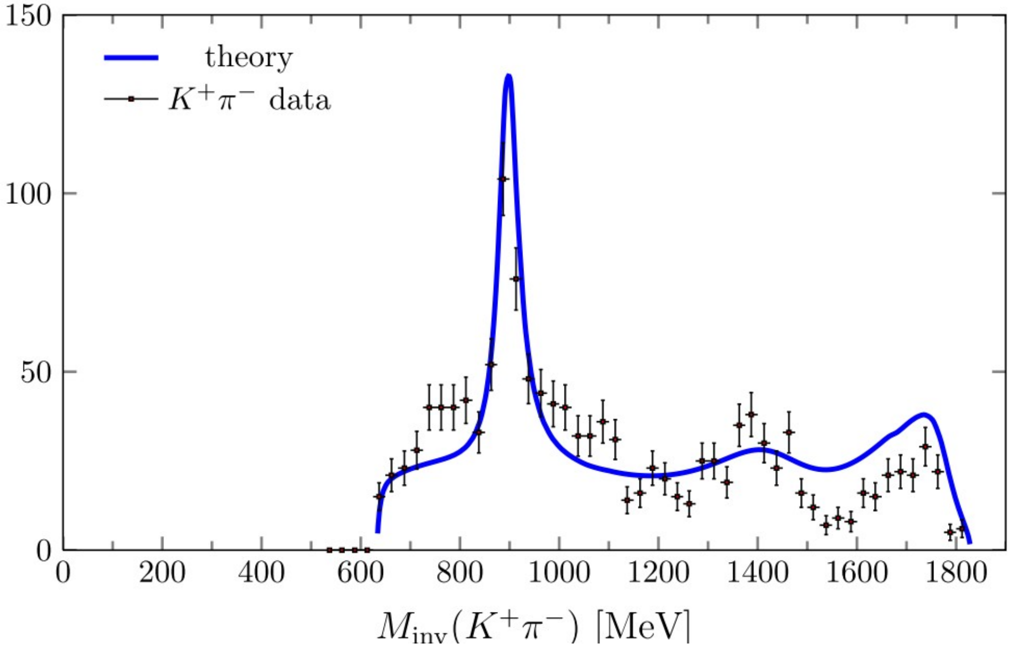
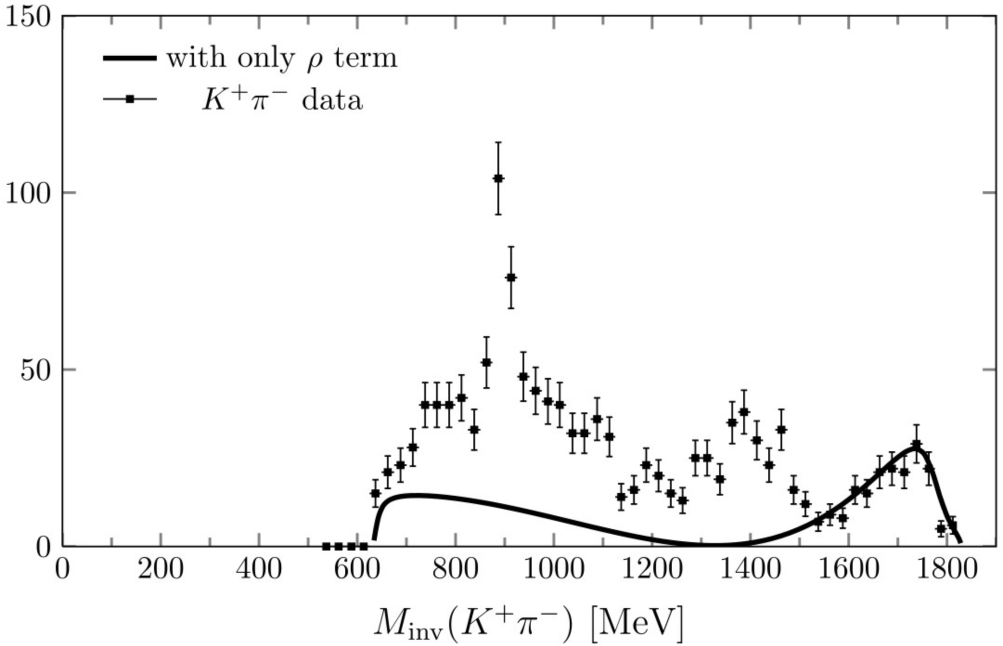

with only $\rho$ term
$\pi^+\pi^-$ data
$M_{\rm inv}(\pi^+\pi^-)$ [MeV]


Theory
$K^+\pi^+$ data


with only $\rho$ term
$K^+\pi^+$ data
$M_{\rm inv}(K^+\pi^+)$ [MeV]



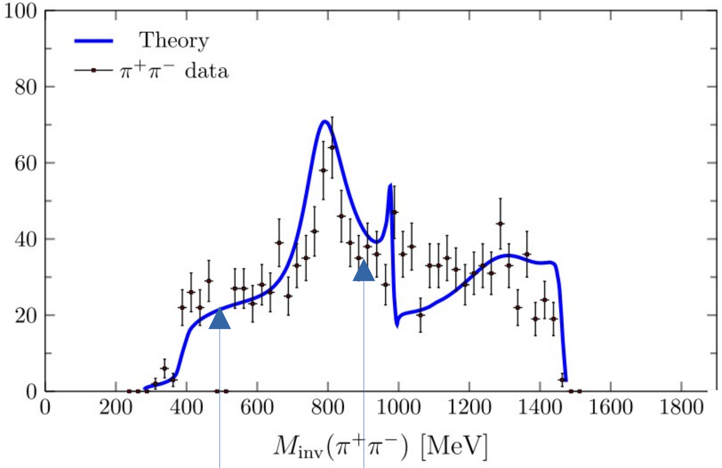
theory
$K^+\pi^-$ data
$M_{\mathrm{inv}}(K^+\pi^-)$ [MeV]

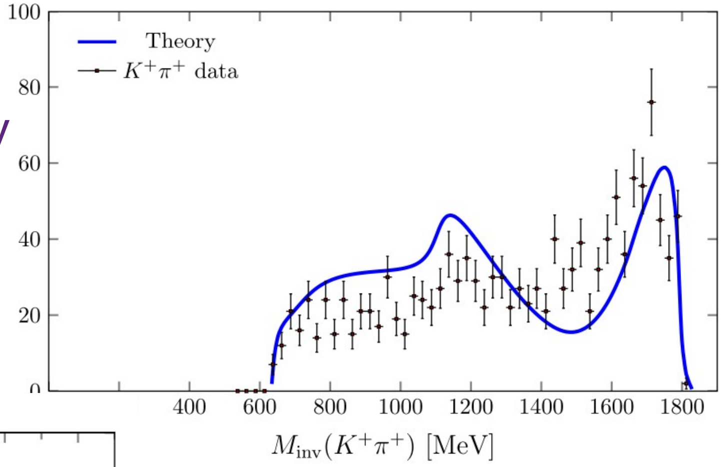
with only $\rho$ term
$K^+\pi^-$ data
$M_{\mathrm{inv}}(K^+\pi^-)$ [MeV]

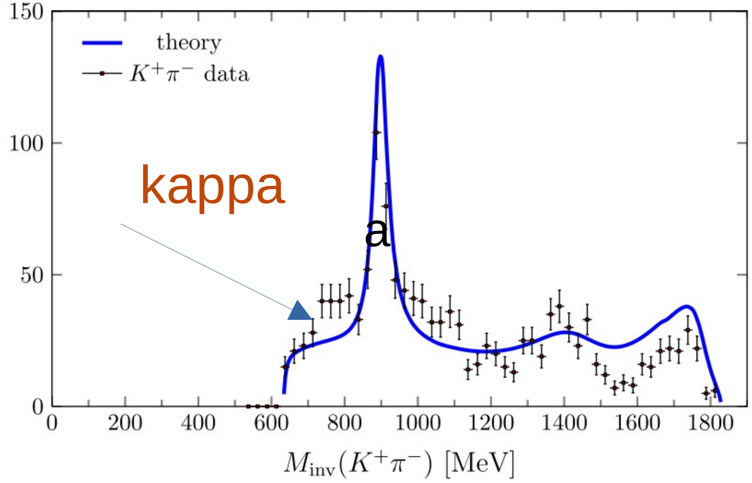
# Dynamical generation of the scalar $f_0(500)$, $f_0(980)$, and $K_0^*(700)$ resonances in the $D_s^+ \to K^+\pi^+\pi^-$ reaction

L. R. Dai and E. Oset, PHYSICAL REVIEW D 109, 054008 (2024)




M. Ablikim et al. (BESIII Collaboration), J. High Energy Phys. 08 (2022) 196.




$f_0(500)$ $f_0(980)$

kappa

# Conclusions

We start from the D decay at the quark level.  Look for all possible topologies

Hadronize q qbar pairs into pairs of mesons

Allow final state interaction of pairs of mesons.

Interpretation of the data can be done with a minimum of parameters

Some of the predictions are tied to the strong interaction of mesons and are parameter free.

D decays are very useful to investigate the strong interaction of mesons and the nature of some resonances.