中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

FB23

MSW Matter Potential at the One-loop Level in the Standard Model

Jihong Huang 黄吉鸿

Institute of High Energy Physics (IHEP)

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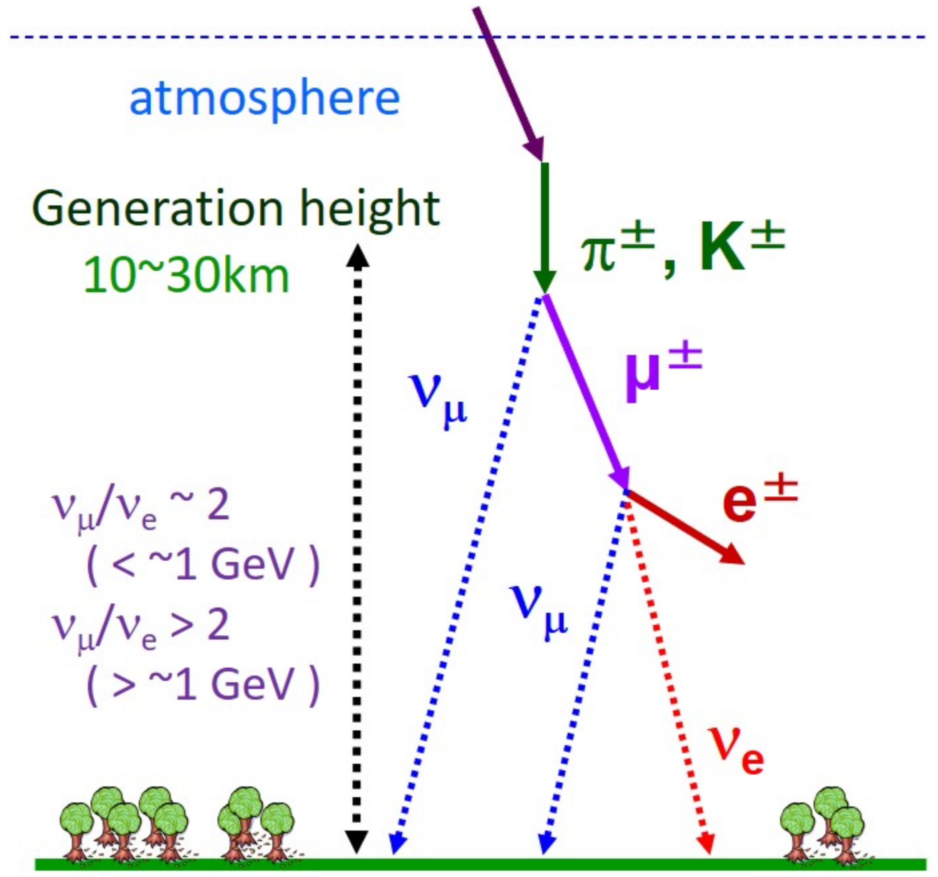
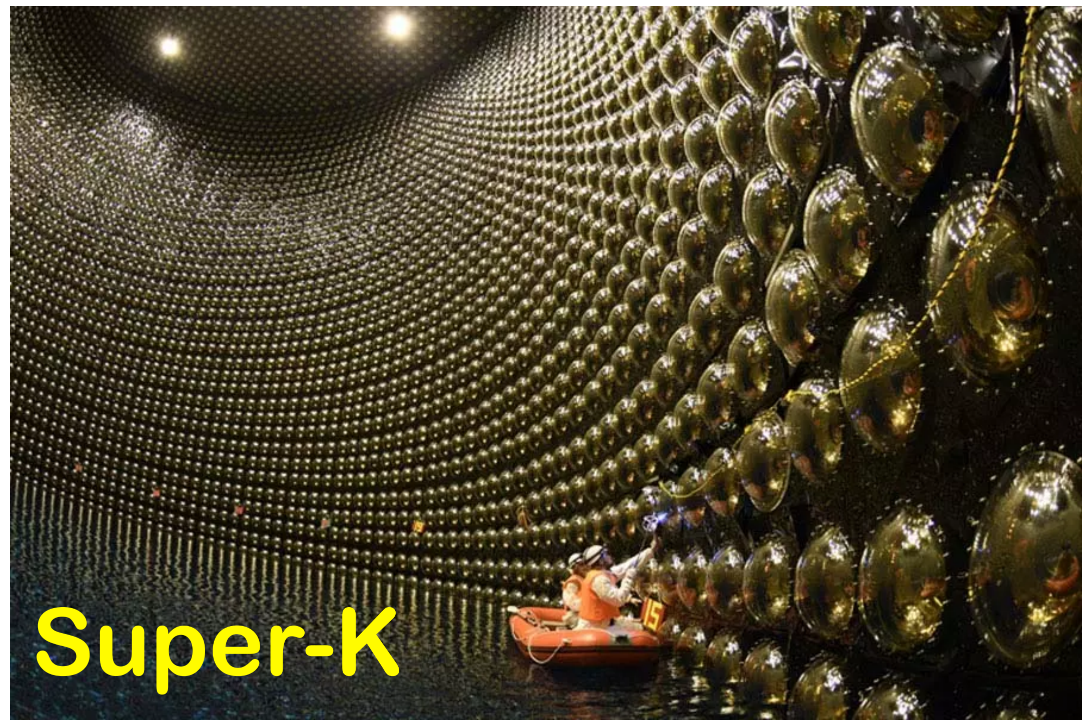
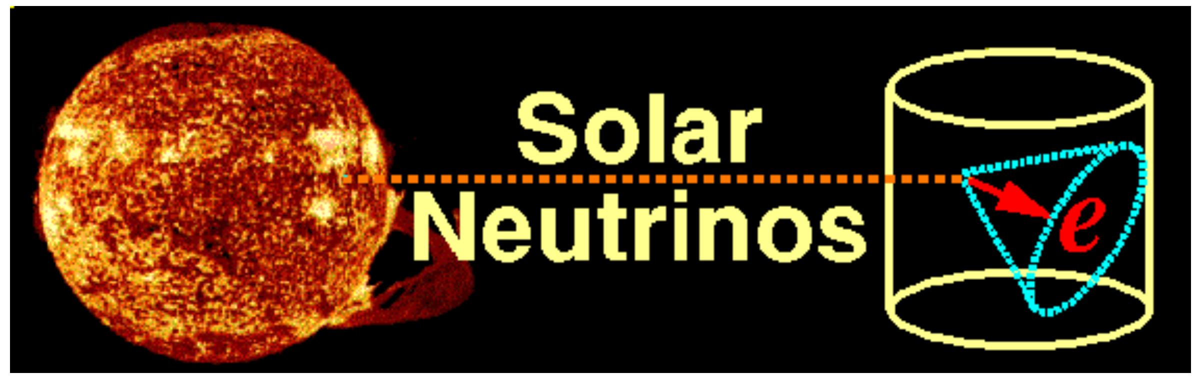
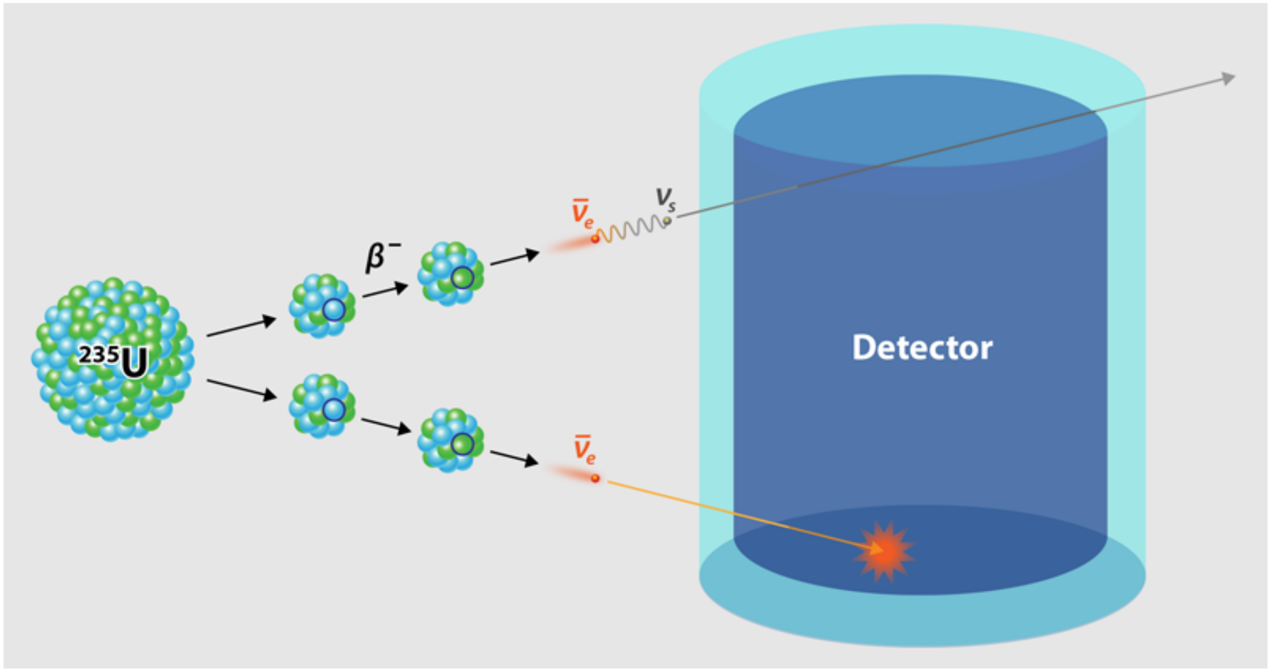
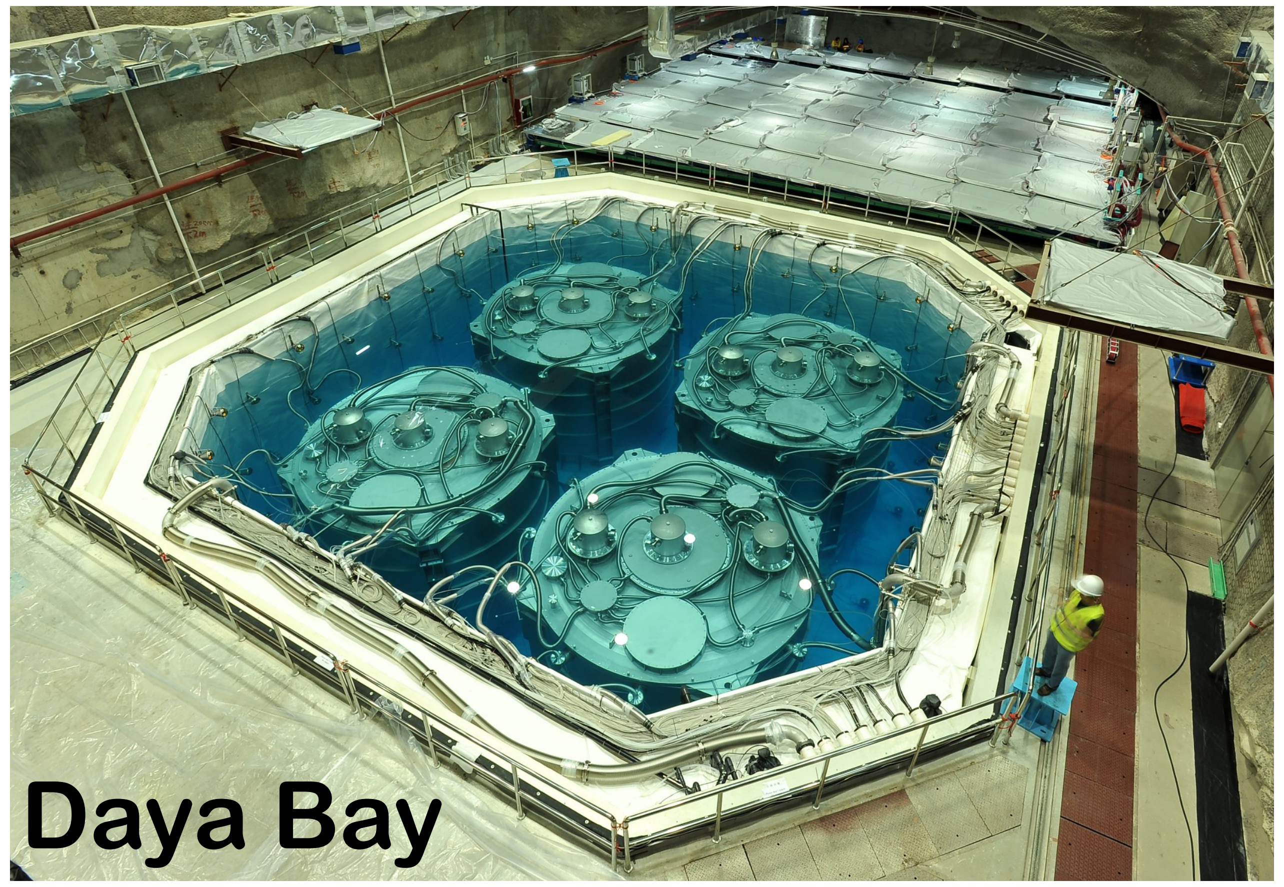
Neutrino Oscillations

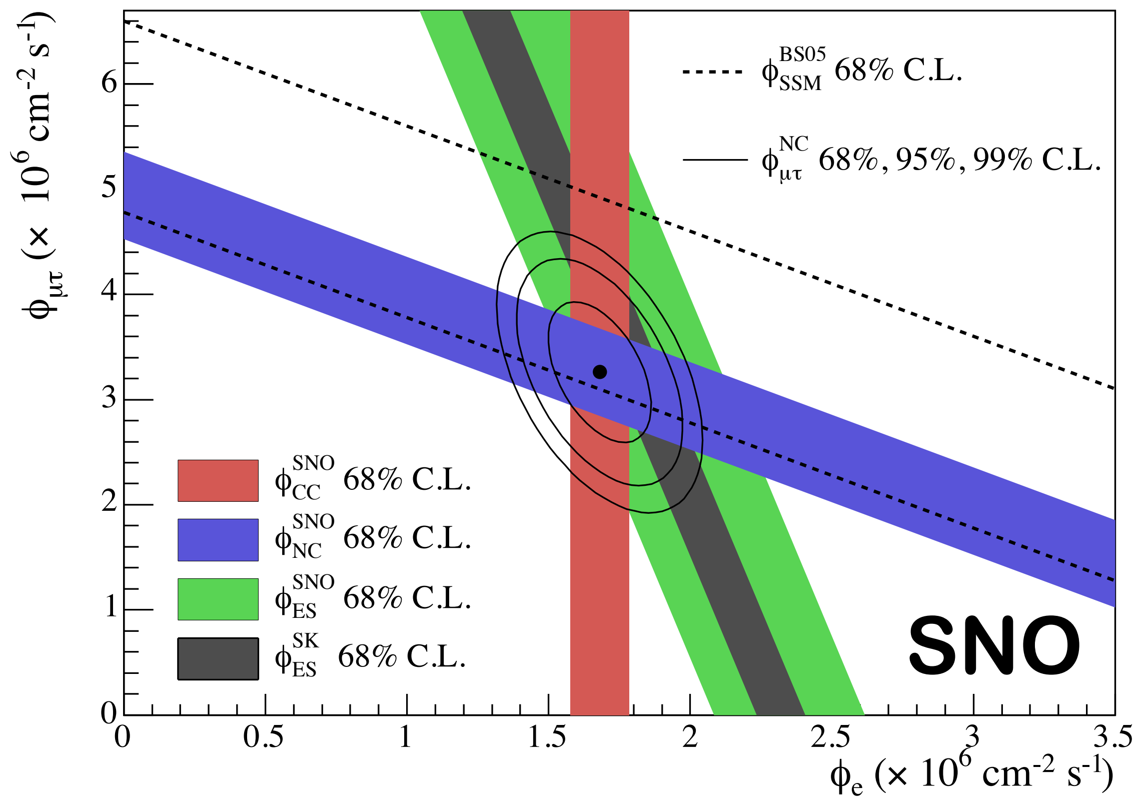


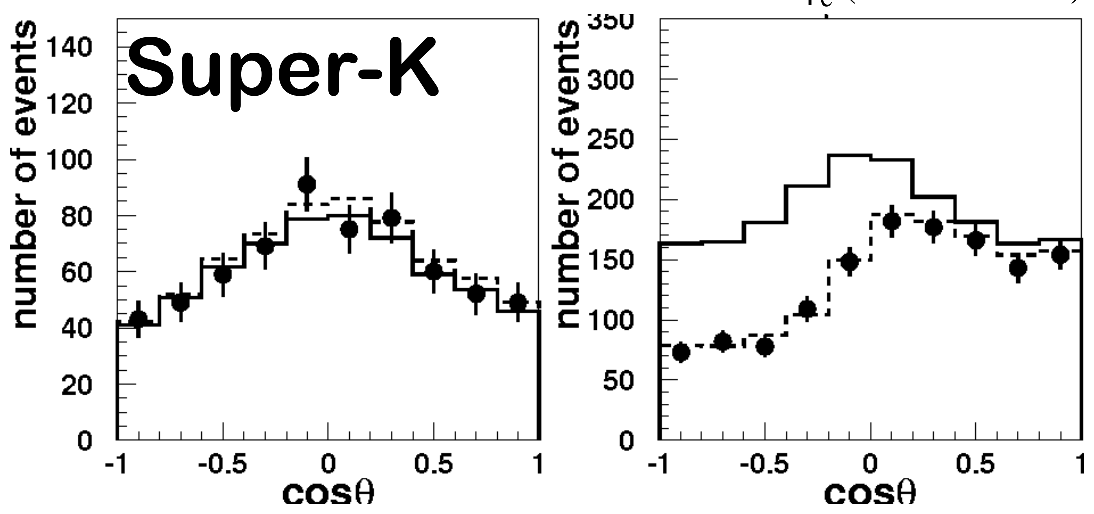


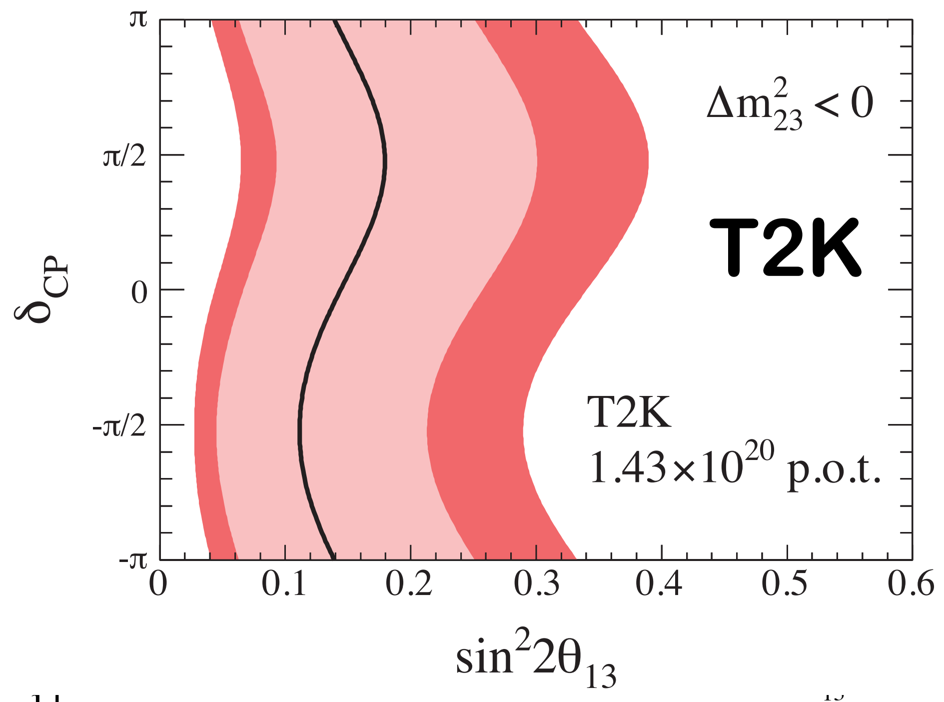


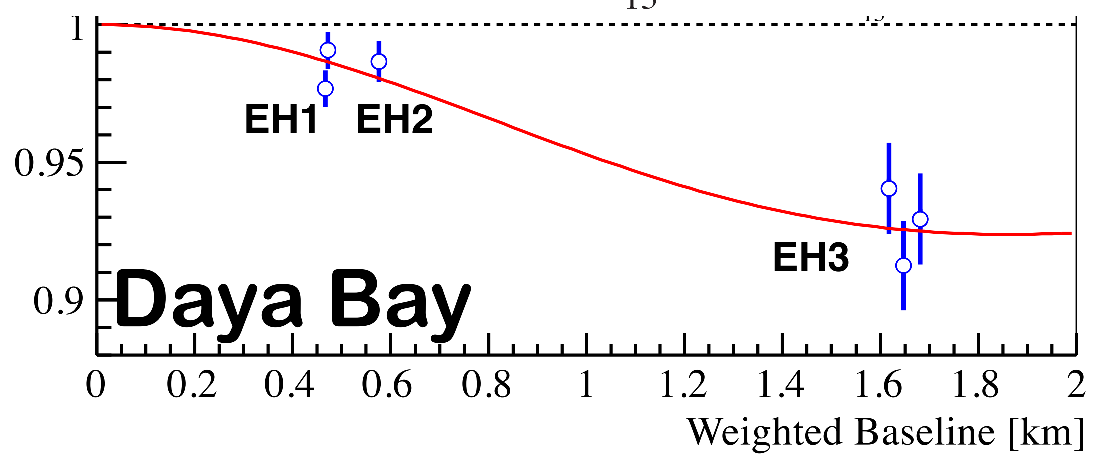


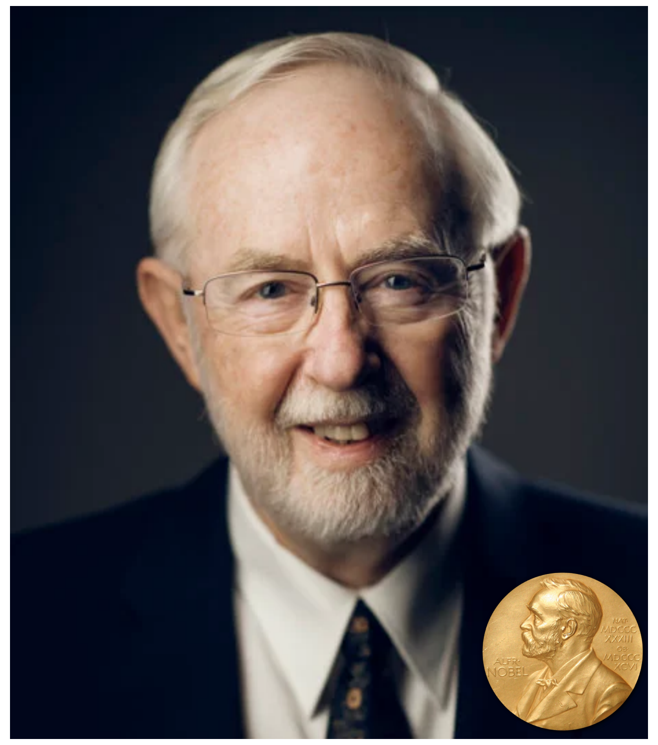












Three-flavor Oscillation

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} + 2\sum_{i<j}^{3}\mathrm{Im}\left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right)\sin\frac{\Delta m^2_{ji}L}{2E}$$

- **The PMNS matrix** $c_{ij} \equiv \cos\theta_{ij}\,, \quad s_{ij} = \sin\theta_{ij}$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

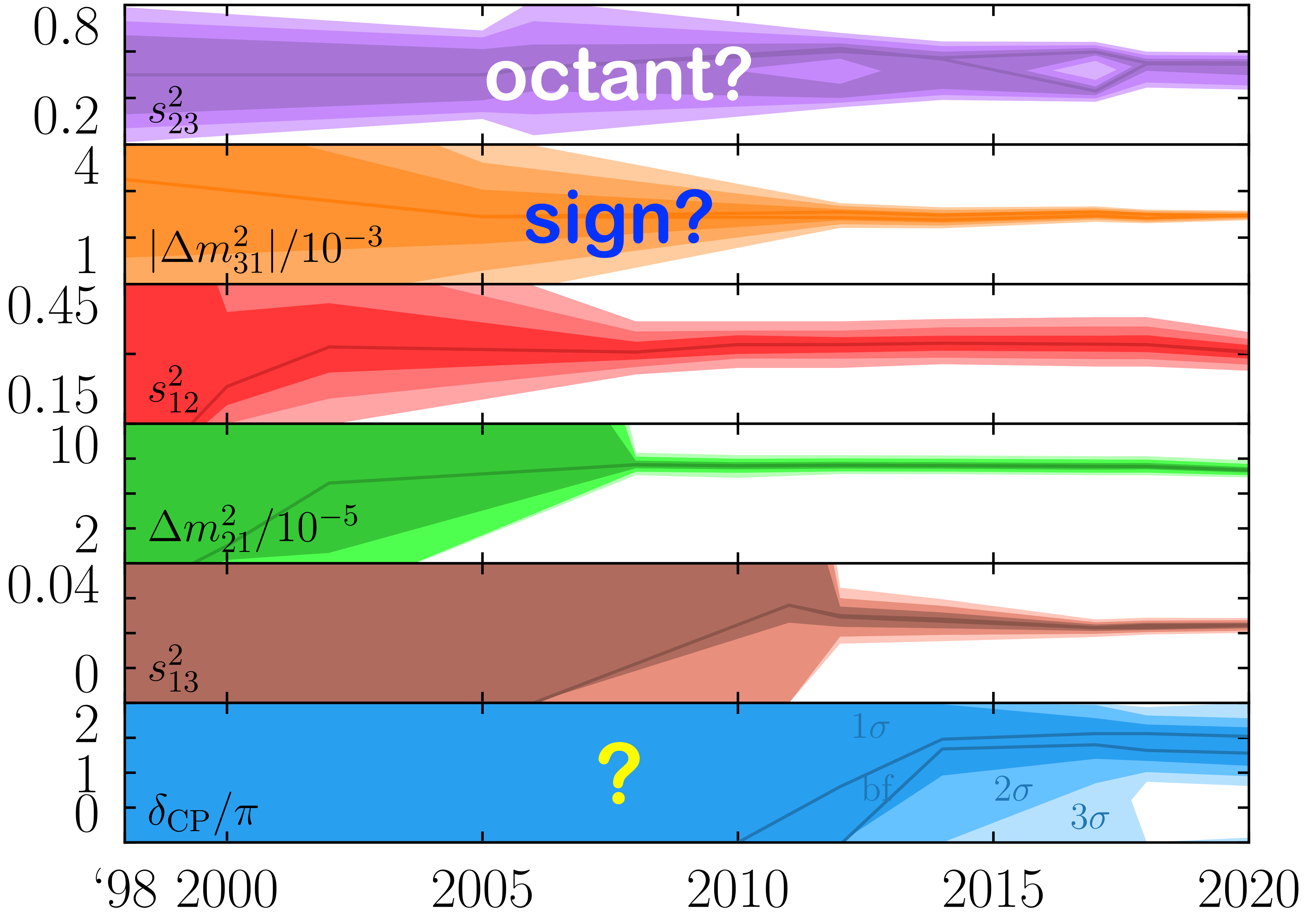
***three** mixing angles and **one** CP-violating phase*

- **Mass-squared differences** $\Delta m^2_{ji} \equiv m^2_j - m^2_i$

***two** mass-squared differences*

✓ ***Neutrinos are massive!***
✓ ***Leptonic flavor mixing is significant!***

2212.00809



Neutrino MSW Matter Effect

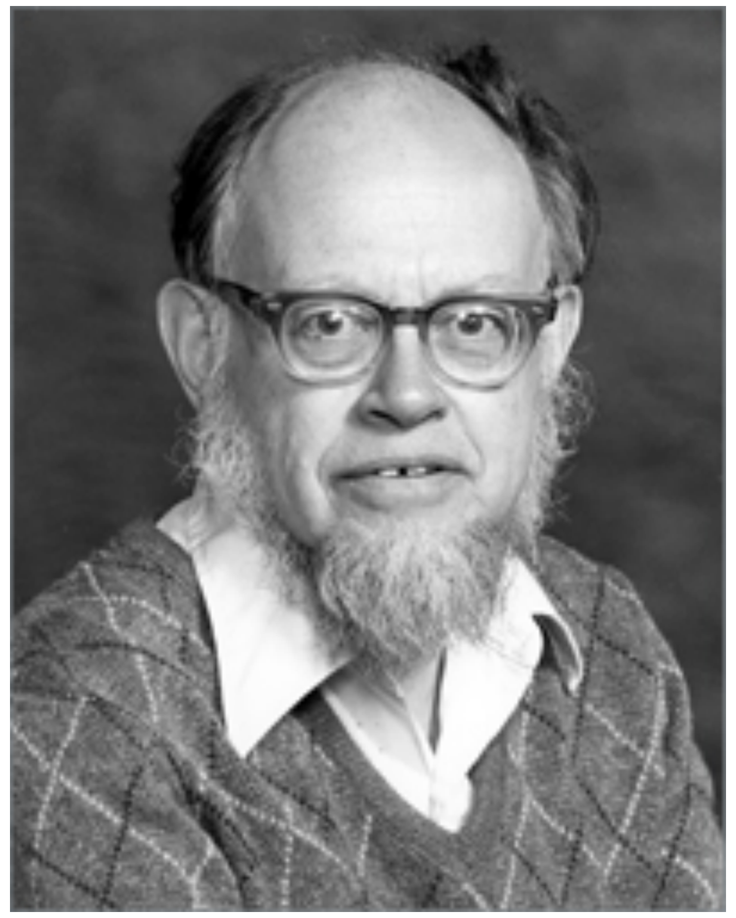
PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

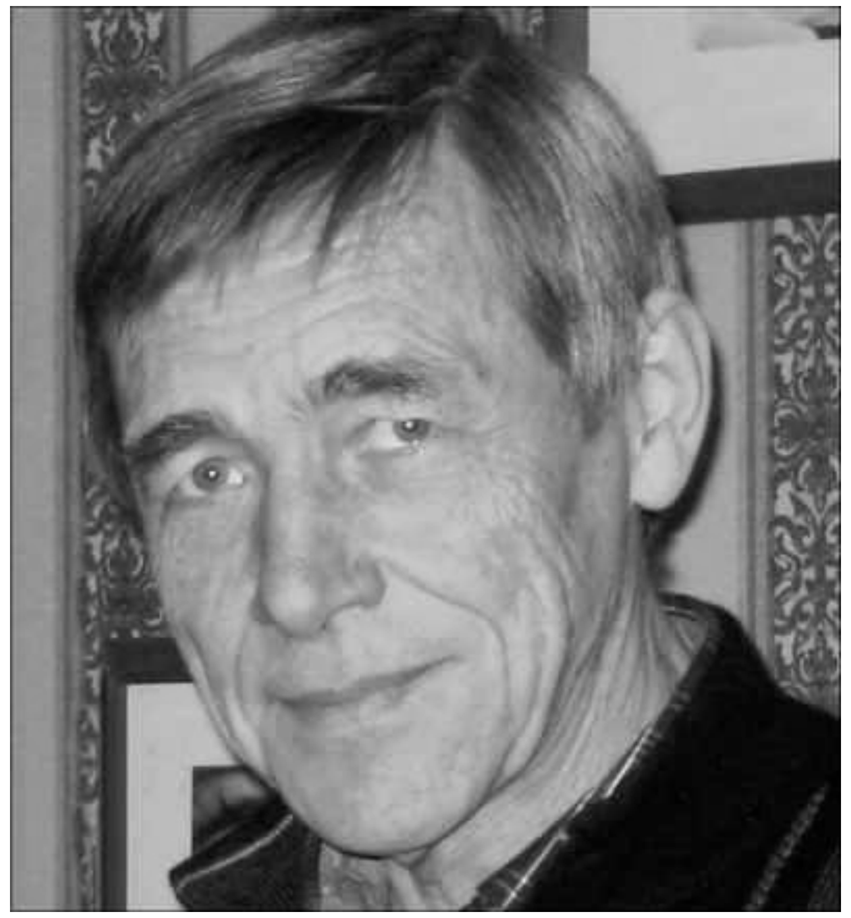
IL NUOVO CIMENTO VOL. 9 C, N. 1 Gennaio-Febbraio 1986

Resonant Amplification of ν Oscillations in Matter and Solar-Neutrino Spectroscopy.

S. P. MIKHEYEV and A. YU. SMIRNOV
Institute for Nuclear Research of Academy of Sciences
60th October Anniversary prosp. 7a, Moscow 117 342, *USSR*

(ricevuto il 3 Maggio 1985)

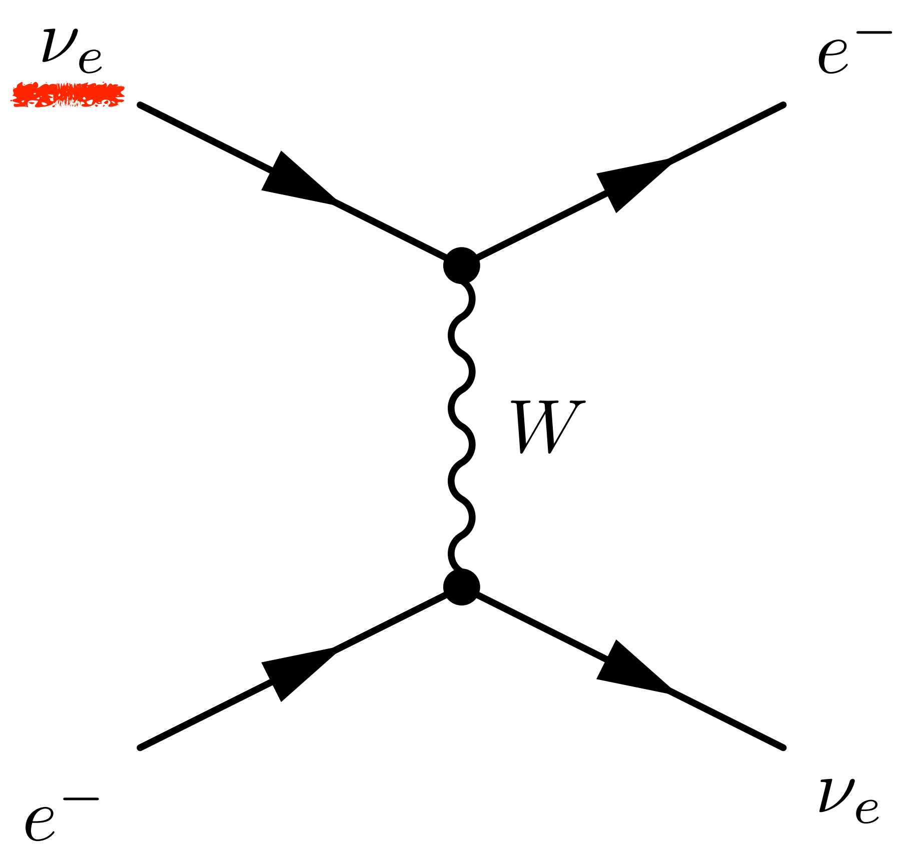
Summary. — For small mixing angles θ the amplification of ν oscillations in matter has the resonance form (resonance in neutrino energy or matter density). In the Sun resonance effect results in nontrivial changing (suppression) of ν-flux for a wide range of neutrino parameters $\Delta m^2 = (3 \cdot 10^{-4} \div 10^{-8})\ (\text{eV})^2$, $\sin^2 2\theta > 10^{-4}$.



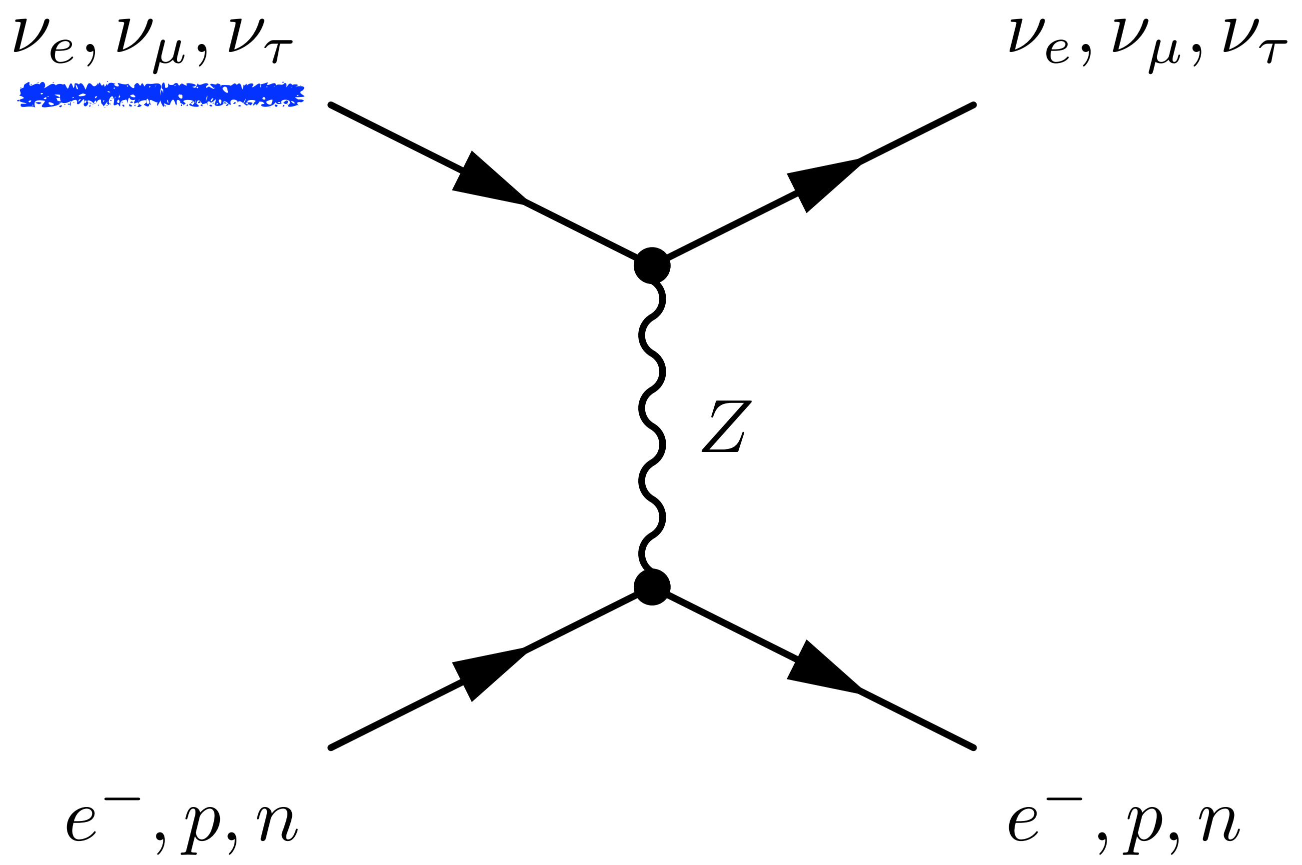
Stanislav Mikheyev

Alexei Smirnov

MSW Matter Potential @ Tree



charged current (CC)



neutral current (NC)

$$\mathscr{V}_e = \mathscr{V}_{\rm CC} + \mathscr{V}_{\rm NC}\,, \quad \mathscr{V}_\mu = \mathscr{V}_\tau = \mathscr{V}_{\rm NC}$$

- **NC couplings in the SM**

	$f = u$	$f = d$	$f = e$
$c^f_{\rm V,NC}$	$\frac{1}{2} - \frac{4}{3}s^2$	$-\frac{1}{2} + \frac{2}{3}s^2$	$-\frac{1}{2} + 2s^2$
$c^f_{\rm A,NC}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

- **CC couplings in the SM** $\quad c^e_{\rm V,CC} = c^e_{\rm A,CC} = 1$

- **Low-energy effective Hamiltonian**

$$\mathscr{H}^{\rm NC}_{\rm eff}(x) = \frac{G_\mu}{\sqrt{2}}\left[\overline{\nu_\alpha(x)}\gamma^\mu\left(1-\gamma^5\right)\nu_\alpha(x)\right]\left[\overline{f(x)}\gamma_\mu\left(c^f_{\rm V,NC} - c^f_{\rm A,NC}\gamma^5\right)f(x)\right]$$

$$\mathscr{H}^{\rm CC}_{\rm eff}(x) = \frac{G_\mu}{\sqrt{2}}\left[\overline{\nu_e(x)}\gamma^\mu\left(1-\gamma^5\right)\nu_e(x)\right]\left[\overline{e(x)}\gamma_\mu\left(c^e_{\rm V,CC} - c^e_{\rm A,CC}\gamma^5\right)e(x)\right]$$

homogeneous and **isotropic** background fermions

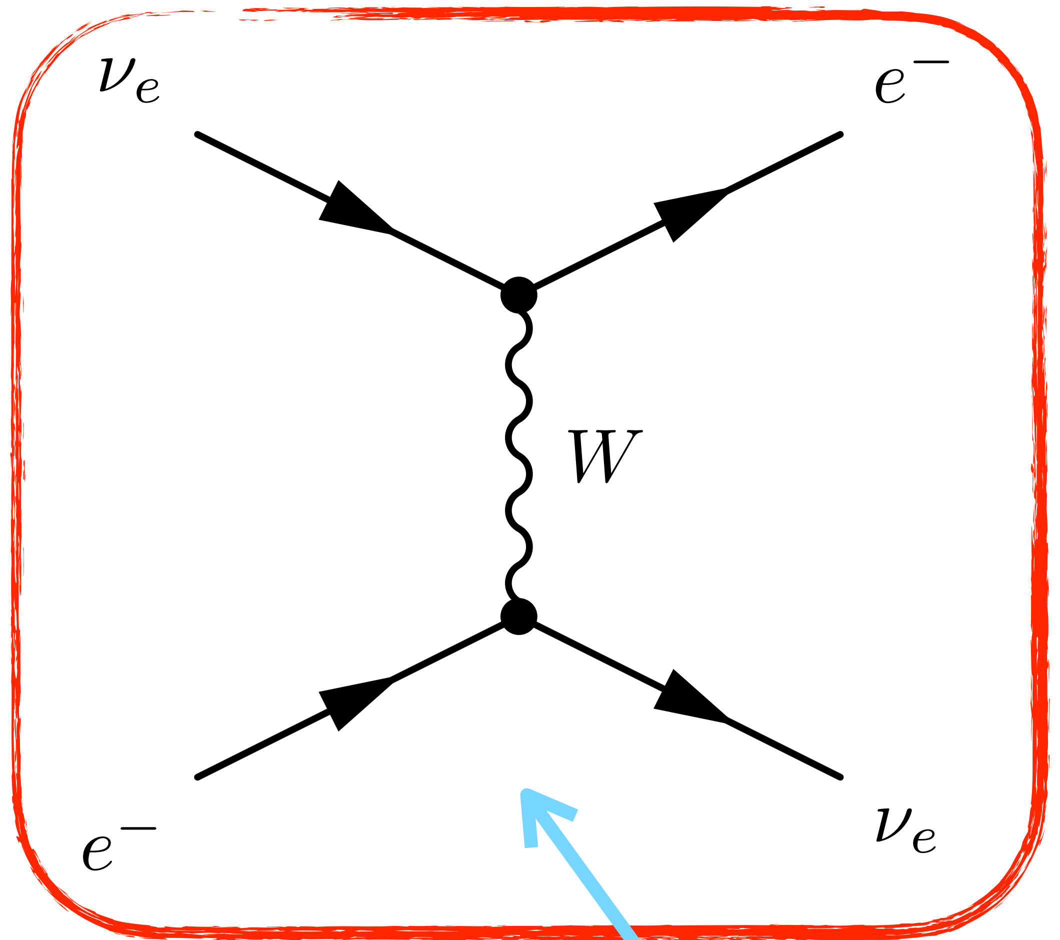
↓ average over all possible states of background fermions

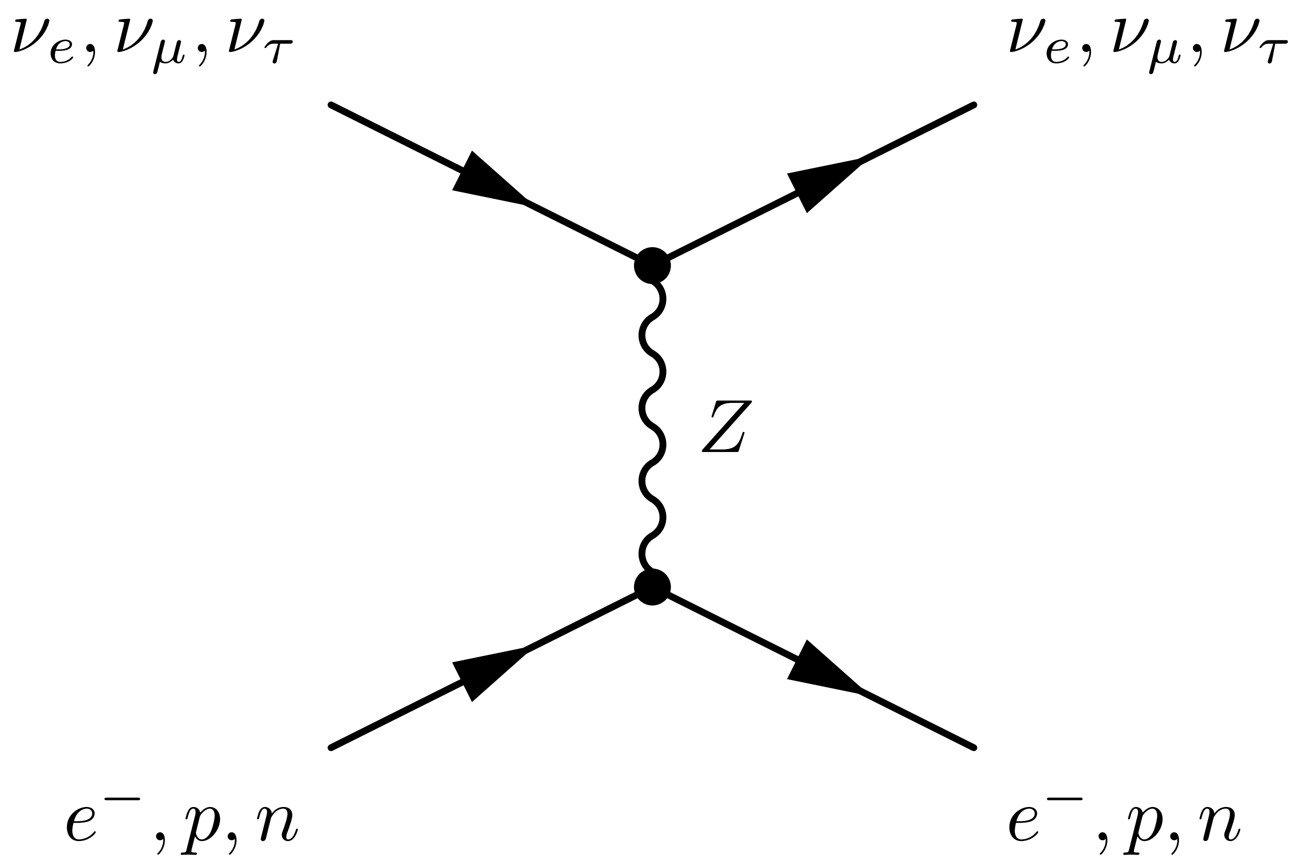
$$\mathscr{V}_{\rm CC} = \sqrt{2}G_\mu N_e c^e_{\rm V,CC} \quad \textit{only for } \nu_e$$

$$\mathscr{V}_{\rm NC} = \sqrt{2}G_\mu N_f c^f_{\rm V,NC} \quad \textit{universal for three flavors}$$

For antineutrinos, the matter potentials change accordingly to opposite signs.

MSW Matter Potential @ Tree





Only **CC potential** for ν_e is relevant for neutrino oscillation in matter.

- **Effective Hamiltonian in vacuum**

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = UH_0U^\dagger\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \quad H_0 = \frac{1}{2E}\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}$$

- **Effective Hamiltonian in matter**

$$H_\mathrm{m} = UH_0U^\dagger + \begin{pmatrix}\mathscr{V}_\mathrm{CC} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} + \begin{pmatrix}\mathscr{V}_\mathrm{NC} & 0 & 0\\ 0 & \mathscr{V}_\mathrm{NC} & 0\\ 0 & 0 & \mathscr{V}_\mathrm{NC}\end{pmatrix}$$

$$= \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}a & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]$$

$\mathscr{V}_\mathrm{CC} = \sqrt{2}G_\mu N_e$ → $a \equiv 2\sqrt{2}G_\mu N_e E$

◉ The energy of solar ^{8}B neutrinos is $E = 10$ MeV, take $N_e = 100\ N_A\ \mathrm{cm}^{-3}$ for $\rho = 150\ \mathrm{g\ cm}^{-3}$ in the solar center.

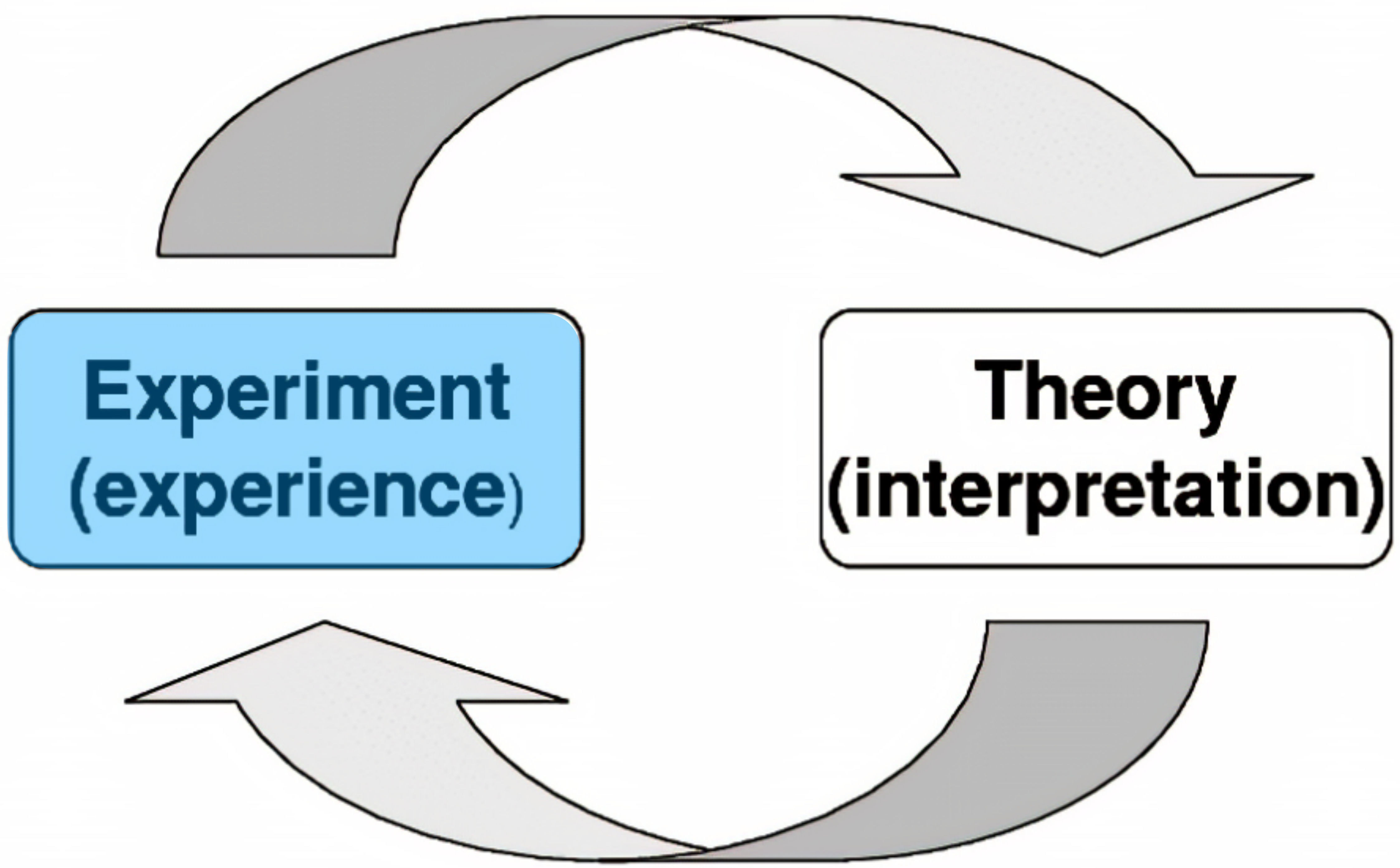
matter parameter $a = 2\sqrt{2}G_\mu N_e E \approx 1.53 \times 10^{-4}\ \mathrm{eV}^2$

mass-squared difference $\Delta m_{21}^2 \approx 7.41 \times 10^{-5}\ \mathrm{eV}^2$

MSW Matter Potential @ One-loop

WHY *matter potential @* ***1-loop*** *order?*

Is it ***necessary*** *to discuss* ***1-loop*** *effects for* ***neutrinos****?*



Electroweak Precision Tests

- ✓ *Yang-Mills gauge theory (1954)*
- ✓ *Brout-Englert-Higgs mechanism (1964)*
- ✓ *Glashow-Salam-Weinberg model (1960s)*
- ✓ ***Renormalizability*** *(1970s)*

predict →

- ✓ *Weak-neutral current (1973)*
- ✓ *W/Z gauge boson (1983)*

input parameters

- $\alpha \approx 1/137.036$ from Thomson scattering
- $G_\mu \approx 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ from μ lifetime
- $\sin^2\theta_\mathrm{w} \approx 0.231$ from neutrino-quark scattering

tree-level relation

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2\theta_\mathrm{w}}\ ,\ m_Z^2 = \frac{m_W^2}{1-\sin^2\theta_\mathrm{w}}$$

$$m_W^{\mathrm{ex}} \approx 80.9\ \mathrm{GeV}\ ,\ m_Z^{\mathrm{ex}} \approx 91.9\ \mathrm{GeV}$$

VS

$$m_W^{\mathrm{th}} \approx 77.6\ \mathrm{GeV}\ ,\ m_Z^{\mathrm{th}} \approx 88.5\ \mathrm{GeV}$$

loop corrections

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2\theta_\mathrm{w}(1-\Delta r)}$$

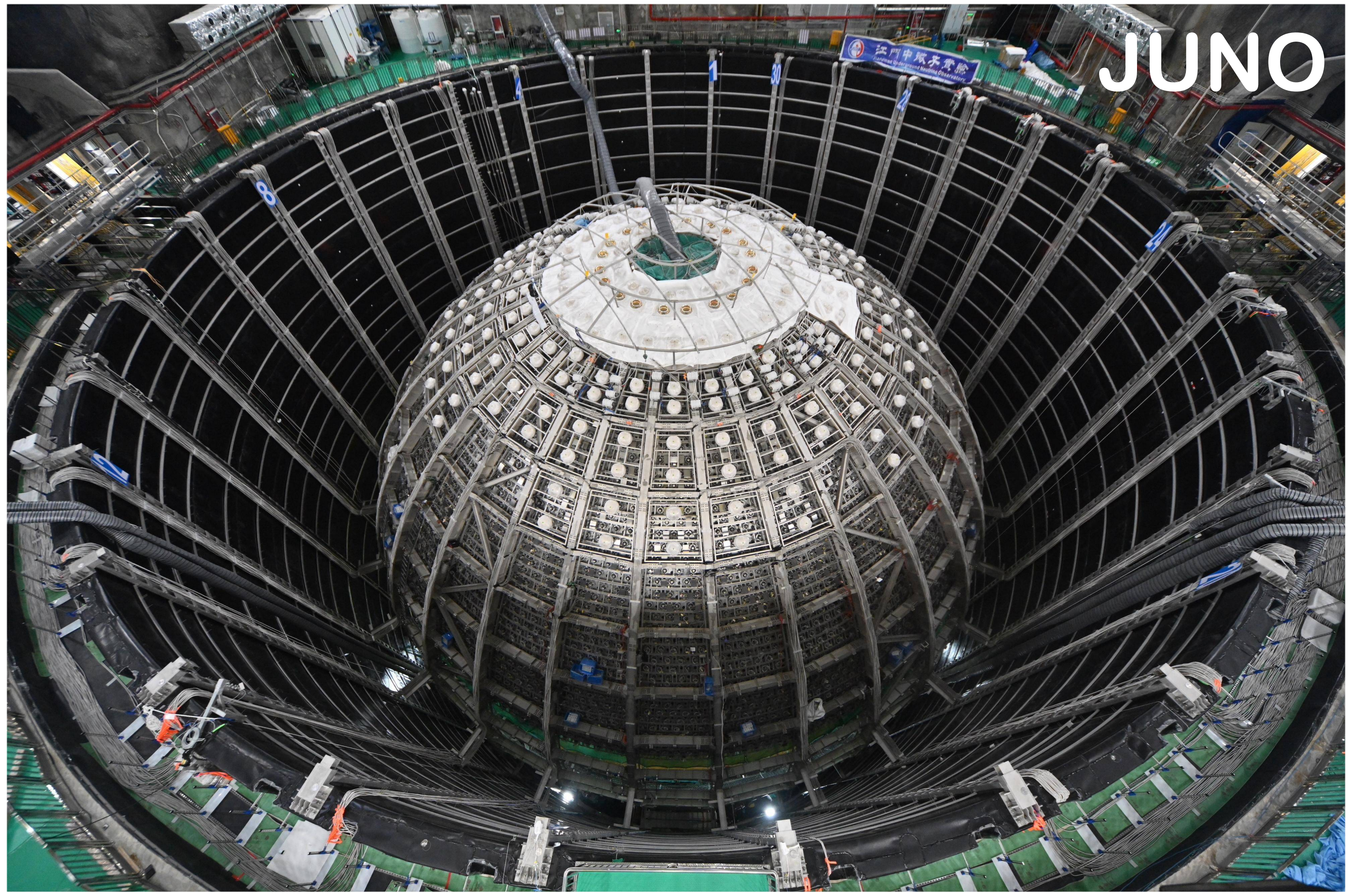
Testing a theory at its quantum level becomes possible if the following conditions are satisfied:

(i) existence of a theory that makes precise predictions beyond the lowest order,
(ii) availability of experiments which are sensitive to such small effects.

Both conditions have been fulfilled in case of QED.

W. Hollik, 1990

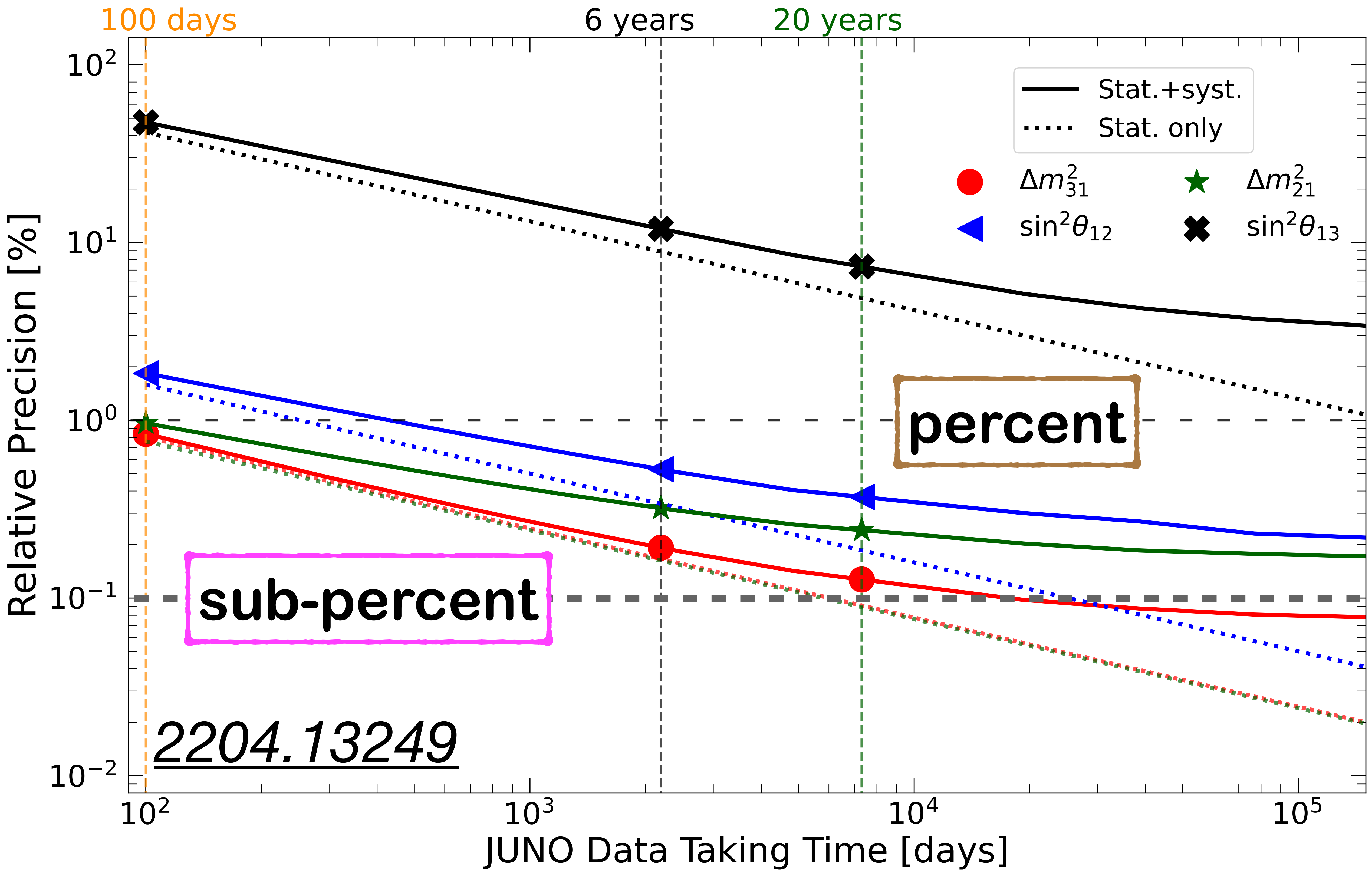
Precision Measurement on JUNO



- ✳ *700 m deep, 35.4 m diameter glass sphere*
- ✳ *20 kilotons liquid scintillator*
- ✳ *an energy resolution of 2.95% at 1 MeV [2405.17860]*

*after **6 years** of data taking*

✓ ***3-4σ*** *CL of mass-ordering &* ***sub-percent*** *precision*



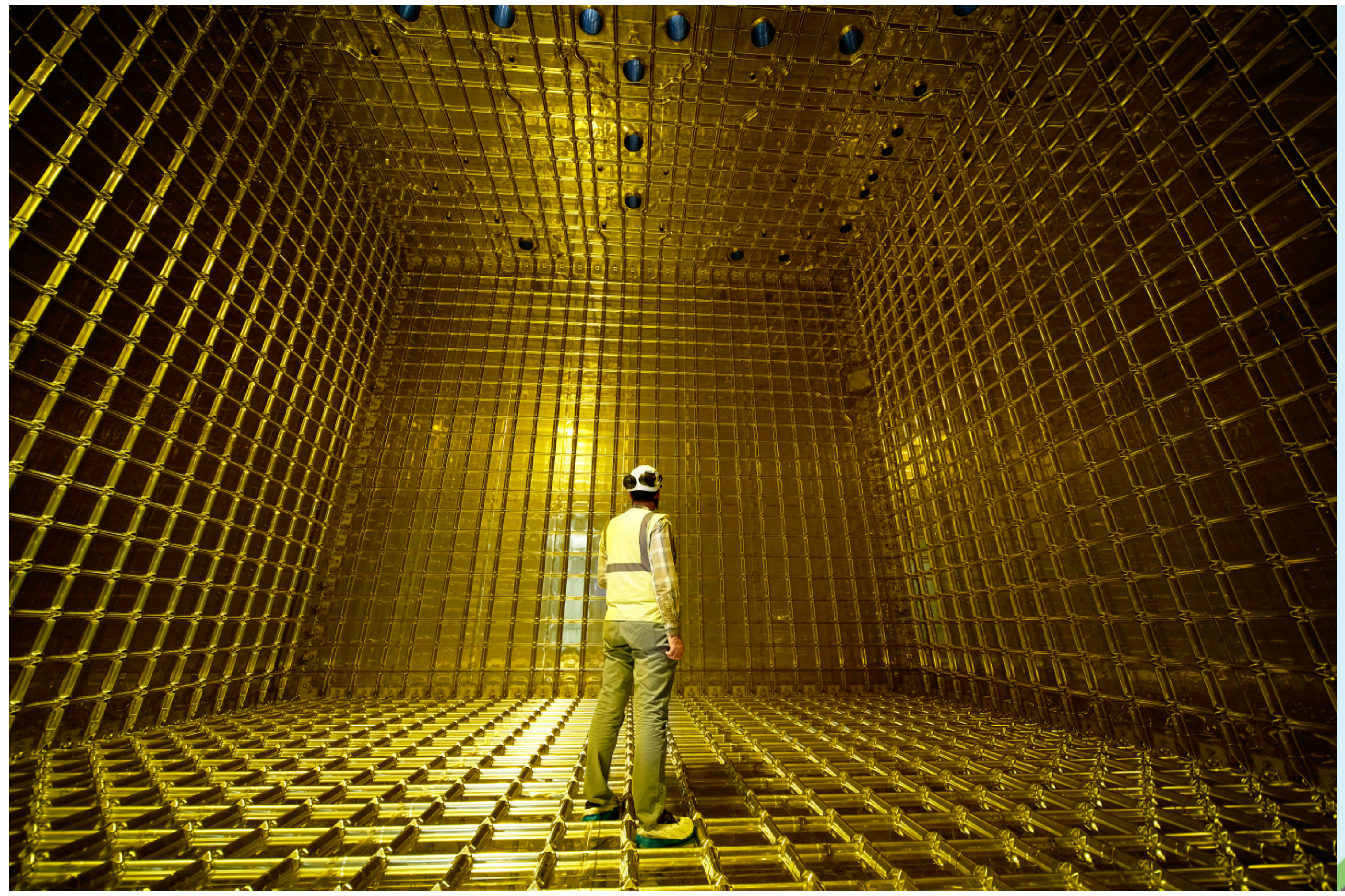
★ **Neutrino mass-ordering**

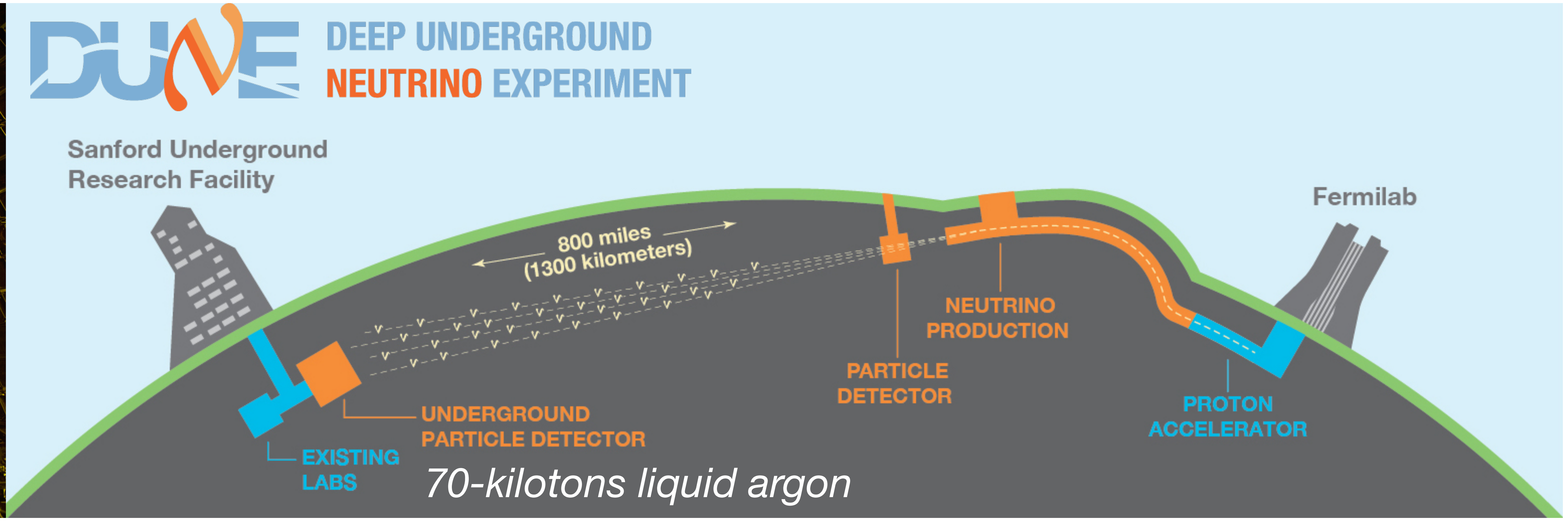
✦ **Precision measurements of oscillation parameters**

- Supernova Burst Neutrinos & DSNB
- Solar/atmospheric/terrestrial neutrinos
- Nucleon Decays (GUT)
- New physics searches

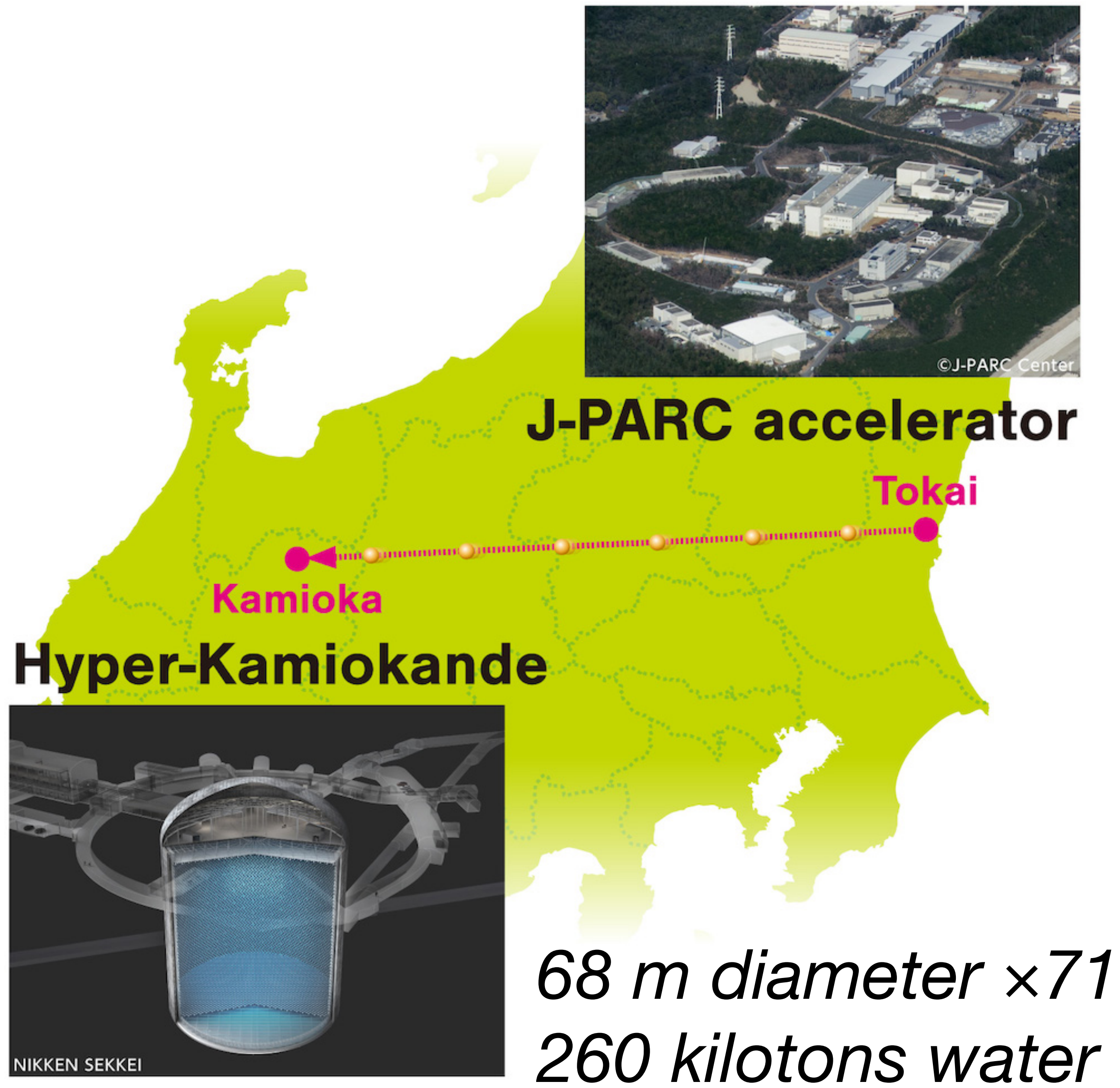
1507.05613

Sensitivity on DUNE & T2HK



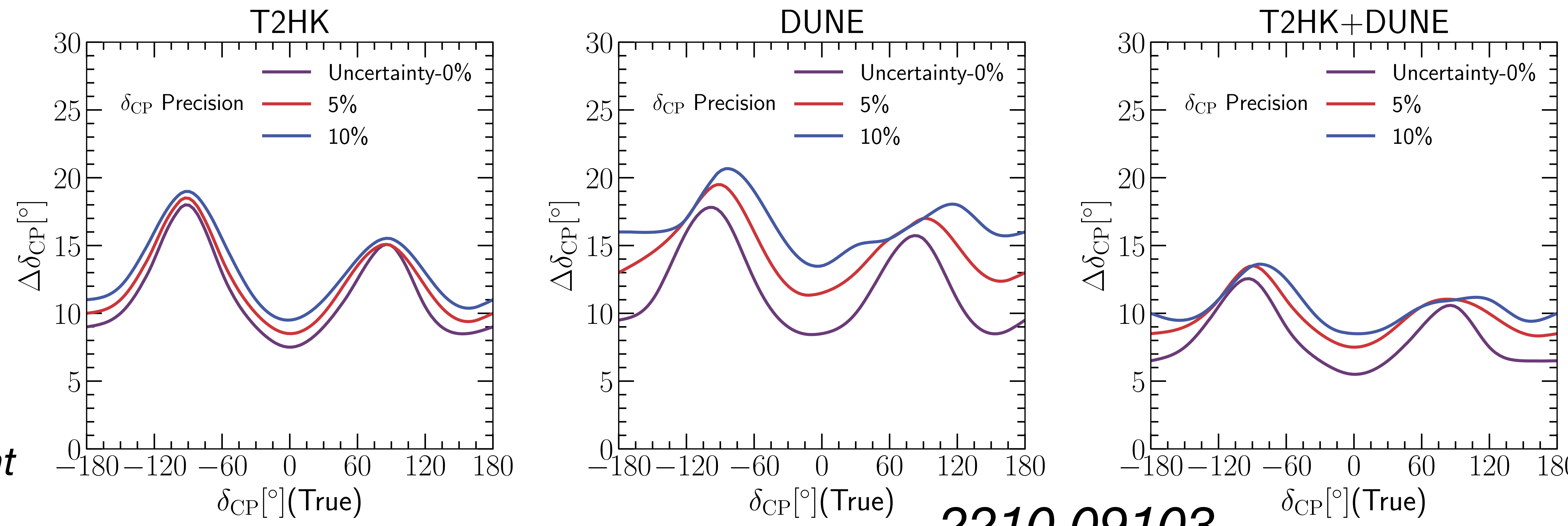


70-kilotons liquid argon



68 m diameter ×71 m hight
260 kilotons water

measure δ_{CP}, determine mass-ordering and θ_{23} octant



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Impact on Precision Measurement

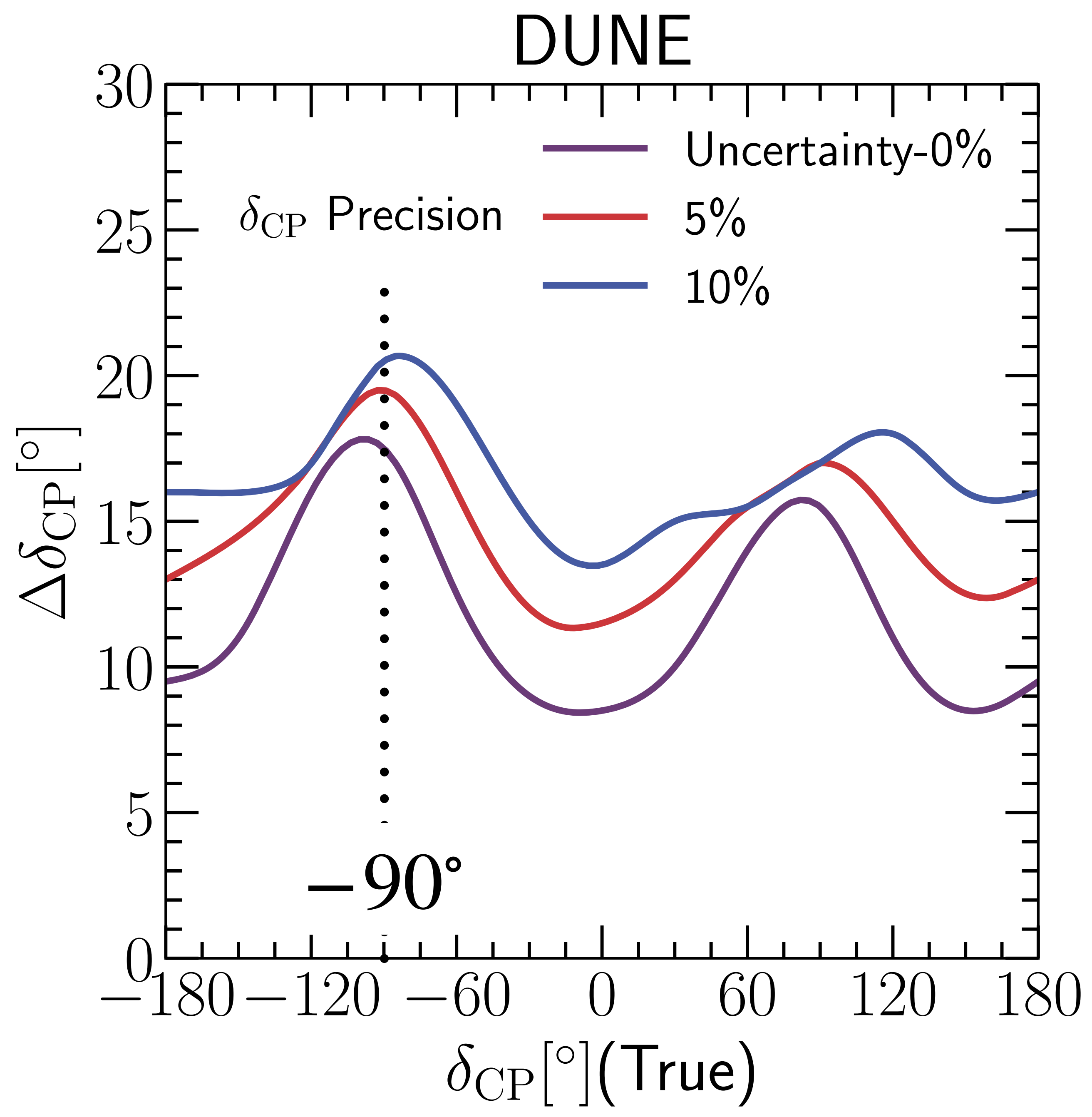
$$P\left(\nu_\mu \to \nu_e\right) \approx \sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2\left(\Delta_{31}-aL\right)}{\left(\Delta_{31}-aL\right)^2}\Delta_{31}^2$$
$$+\sin 2\theta_{23}\sin 2\theta_{13}\sin 2\theta_{12}\frac{\sin\left(\Delta_{31}-aL\right)}{\left(\Delta_{31}-aL\right)}\Delta_{31}\frac{\sin(aL)}{(aL)}\Delta_{21}\cos\left(\Delta_{31}+\delta_{CP}\right)$$
$$+\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(aL)}{(aL)^2}\Delta_{21}^2\,,$$

Matter effects are important in long-baseline accelerator experiments.

*Directly affect **mass-ordering** determination and **δ_{CP}** measurement.*

2210.09103

Sensitivity	Max $\lvert\Delta\rho/\rho\rvert$	T2HK	DUNE	T2HK+DUNE
Mass ordering	0%	1.03	7.8	11.7
significance	5%	1.00	7.5	11.4
$[\sigma]$	10%	0.96	7.2	11.1
Octant	0%	42.343° – 48.674°	42.21° – 49.03°	42.96° – 48.00°
at	5%	42.340° – 48.676°	42.16° – 49.11°	42.93° – 48.03°
5σ	10%	42.338° – 48.678°	42.12° – 49.18°	42.93° – 48.06°
CP violation	0%	21.4	37.4	54.9
fraction	5%	20.8	34.2	52.7
[%]	10%	19.8	32.0	50.2
CP Precision	0%	18°	16°	12°
at $\Delta\delta_{CP}$	5%	19°	19°	13°
1σ	10%	20°	21°	13°



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predict →

- ✓ *Weak-neutral current (1973)*
- ✓ *W/Z gauge boson (1983)*

Neutrino interactions

Perturbative calculations are possible for EW theory.

Neutrino precision measurement era

Experimental precision is comparable to the quantum corrections (percent-level).

Testing a theory at its quantum level becomes possible if the following conditions are satisfied:

(i) existence of a theory that makes precise predictions beyond the lowest order,
(ii) availability of experiments which are sensitive to such small effects.

Both conditions have been fulfilled in case of QED.

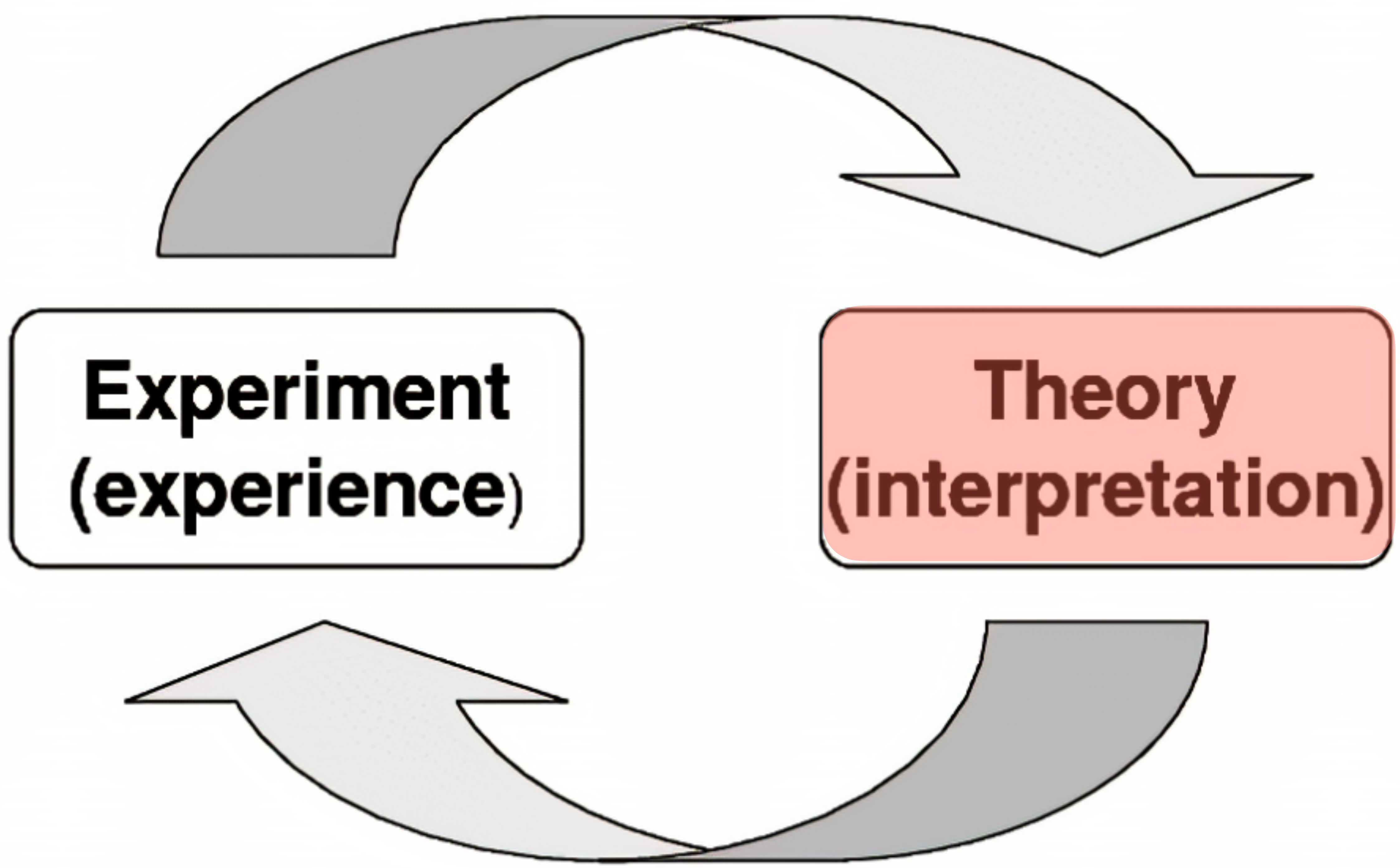
W. Hollik, 1990

Loop effects of neutrinos need to be considered!

MSW Matter Potential @ One-loop

WHY *matter potential @* ***1-loop*** *order?*

Is it ***necessary*** *to discuss* ***1-loop*** *effects for* ***neutrinos****?*



MSW Matter Potential @ One-loop

PHYSICAL REVIEW D VOLUME 35, NUMBER 3 1 FEBRUARY 1987

Radiative corrections to neutrino indices of refraction

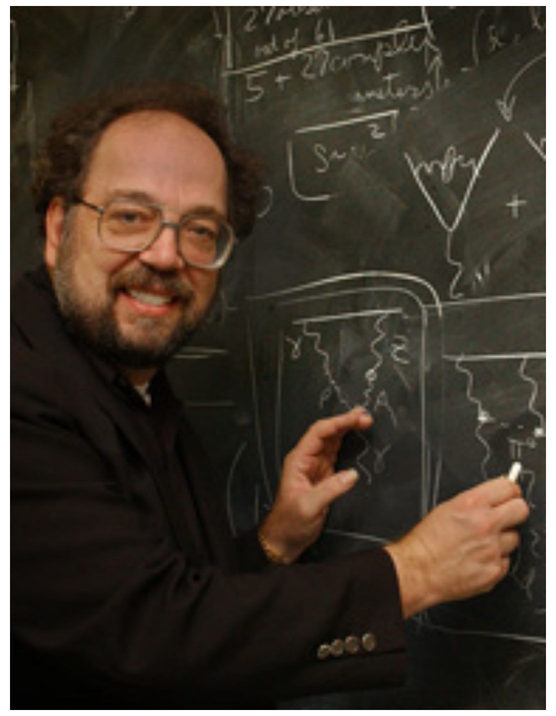
F. J. Botella,* C. -S. Lim, and W. J. Marciano
Physics Department, Brookhaven National Laboratory, Upton, New York 11973

(Received 10 September 1986) one year after M & S!

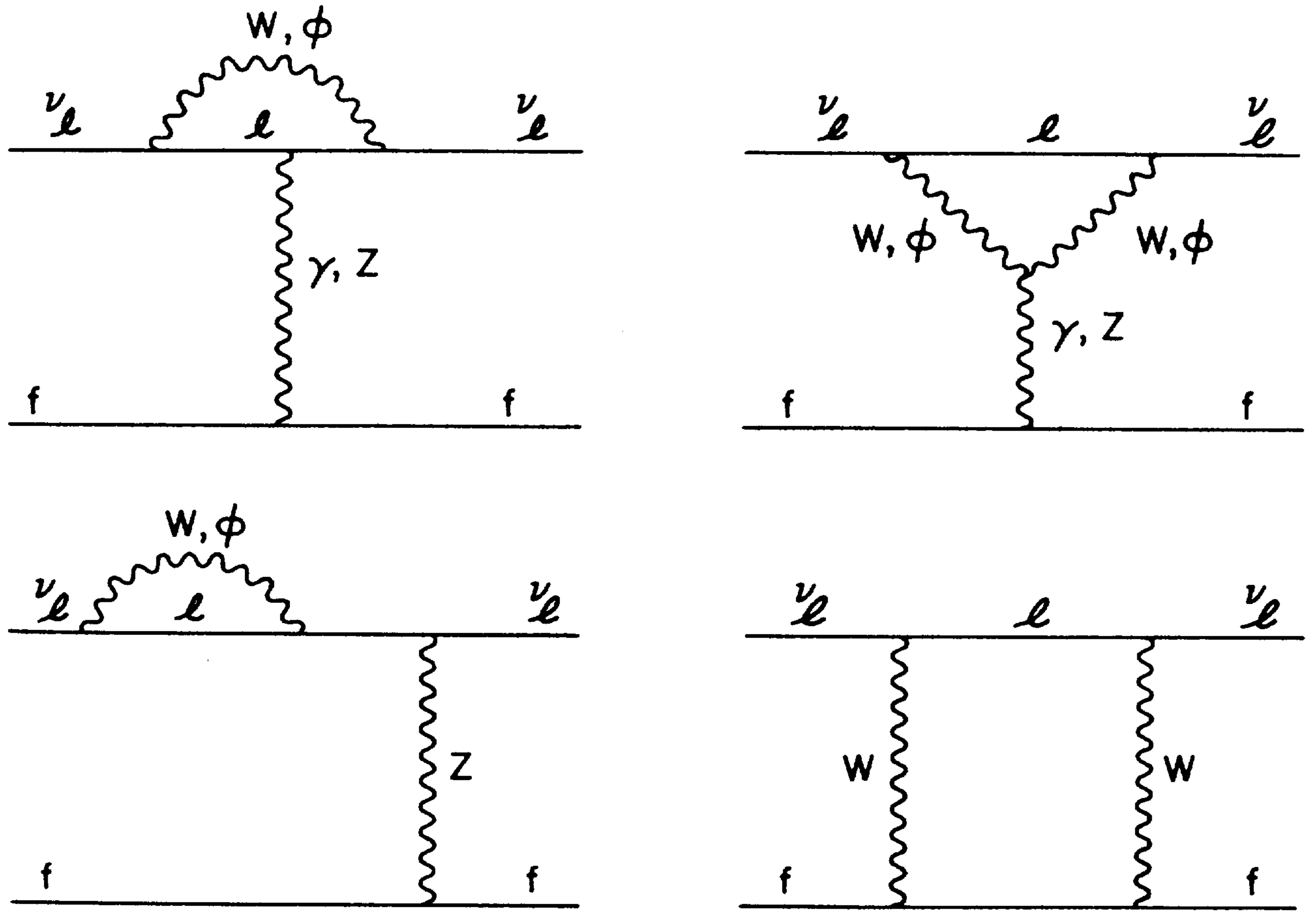
Quantum loop corrections to coherent forward neutrino scattering and indices of refraction n_{ν_l}, $l=e,\mu,\tau$ are examined in the standard $SU(2)_L\times U(1)$ model. For a neutral unpolarized medium with particle densities $N_e=N_p,N_n$ we find $p_\nu(n_{\nu_e}-n_{\nu_\mu})=-\sqrt{2}G_\mu N_e[1+O(\alpha m_\mu^2/m_W^2)]$ and

$$p_\nu(n_{\nu_\tau}-n_{\nu_\mu})=\frac{G_\mu}{\sqrt{2}}\frac{3\alpha}{2\pi\sin^2\theta_W}\frac{m_\tau^2}{m_W^2}[(N_p+N_n)\ln(m_\tau^2/m_W^2)+(N_p+\tfrac{2}{3}N_n)] .$$

Implications of our results for neutrino matter oscillations and elastic scattering are briefly discussed.



The effect of coherent forward scattering on neutrino oscillations in matter was investigated a number of years ago by Wolfenstein.[1] More recently, Mikheyev and Smirnov[2] employed that analysis to show how for a realistic range of neutrino masses and mixing parameters, neutrino matter oscillations between ν_e and ν_μ or ν_τ in the Sun's interior could be significantly enhanced and thus modify the spectrum of solar ν_e neutrinos. Such a scenario (henceforth referred to as the MSW effect) provides a natural solution to the solar neutrino puzzle,[3] i.e., why only about $\frac{1}{3}$ of the ν_e flux predicted by the standard solar model is experimentally observed.



in ordinary matter

$$m_e \ll m_\mu \ll m_\tau \; \& \; N_e = N_p = N_n$$

$$\epsilon_{\mu\tau} \equiv \frac{\widehat{\mathscr{V}}^{\tau}_{\rm NC} - \widehat{\mathscr{V}}^{\mu}_{\rm NC}}{\mathscr{V}_{\rm CC}}$$

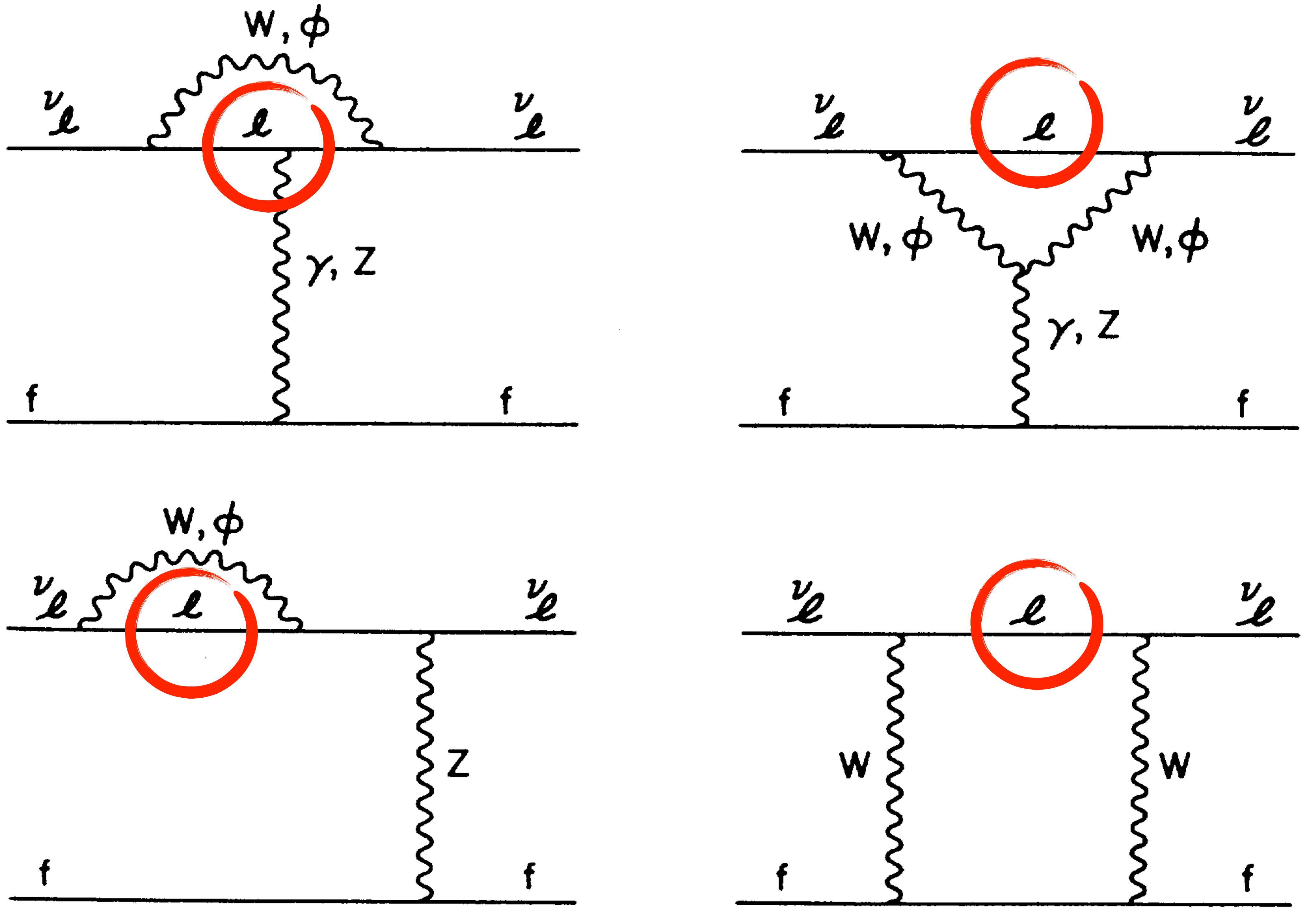
$$\approx -\frac{3\alpha}{2\pi\sin^2\theta_{\rm W}}\frac{m_\tau^2}{m_W^2}\left[\ln\left(\frac{m_\tau^2}{m_W^2}\right)+\frac{5}{6}\right] \approx 5.19\times 10^{-5}$$

Extremely small !

But greatly affect the flavor conversions of SN neutrinos with ***$\rho \sim 10^6$ g cm^{-3}.***

flavor-dependent corrections

MSW Matter Potential @ One-loop



$$H_{\rm m} = U H_0 U^\dagger + \begin{pmatrix} \mathscr{V}_{\rm CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \mathscr{V}_{\rm NC} & 0 & 0 \\ 0 & \mathscr{V}_{\rm NC} & 0 \\ 0 & 0 & \mathscr{V}_{\rm NC} \end{pmatrix}$$

$$\epsilon_{\mu\tau} \equiv \frac{\widehat{\mathscr{V}}^{\tau}_{\rm NC} - \widehat{\mathscr{V}}^{\mu}_{\rm NC}}{\mathscr{V}_{\rm CC}}$$

flavor-dependent parts

$$\approx -\frac{3\alpha}{2\pi \sin^2\theta_{\rm w}} \frac{m_\tau^2}{m_W^2} \left[\ln\left(\frac{m_\tau^2}{m_W^2}\right) + \frac{5}{6} \right] \approx 5.19 \times 10^{-5}$$

A complete one-loop calculation of the MSW potentials in the SM is needed.

- **NC potential**

  Flavor-dependent (1986) + Flavor-independent (**?**)

$$\begin{pmatrix} \mathscr{V}^{\rm FI}_{\rm NC} + \mathscr{V}^{e}_{\rm NC} & 0 & 0 \\ 0 & \mathscr{V}^{\rm FI}_{\rm NC} + \mathscr{V}^{\mu}_{\rm NC} & 0 \\ 0 & 0 & \mathscr{V}^{\rm FI}_{\rm NC} + \mathscr{V}^{\tau}_{\rm NC} \end{pmatrix}$$

  Important in active-sterile neutrino oscillations.

- **CC potential** (only for ν_e, **?**)

  Have not been studied thus far!

  Achieve high-precision measurements.

Corrections to Vector-type Couplings

WHY corrections to the vector-type couplings?

$$\mathscr{H}^{\rm NC}_{\rm eff}(x) = \frac{G_\mu}{\sqrt{2}} \left[\overline{\nu_\alpha(x)}\gamma^\mu\left(1-\gamma^5\right)\nu_\alpha(x)\right]\left[\overline{f(x)}\gamma_\mu\left(c^f_{\rm V,NC} - c^f_{\rm A,NC}\gamma^5\right)f(x)\right]$$

$$\mathscr{H}^{\rm CC}_{\rm eff}(x) = \frac{G_\mu}{\sqrt{2}} \left[\overline{\nu_e(x)}\gamma^\mu\left(1-\gamma^5\right)\nu_e(x)\right]\left[\overline{e(x)}\gamma_\mu\left(c^e_{\rm V,CC} - c^e_{\rm A,CC}\gamma^5\right)e(x)\right]$$

→

$$\mathscr{V}_{\rm CC} = \sqrt{2}G_\mu N_e c^e_{\rm V,CC}$$

$$\mathscr{V}_{\rm NC} = \sqrt{2}G_\mu N_f c^f_{\rm V,NC}$$

✓ Extract the **one-loop corrected vector-type coefficients** from **renormalized** scattering amplitudes

$$\Delta c^f_{\rm V,NC} \equiv \hat{c}^f_{\rm V,NC} - c^f_{\rm V,NC}$$

$$\Delta c^e_{\rm V,CC} \equiv \hat{c}^e_{\rm V,CC} - c^e_{\rm V,CC}$$

→

$$\Delta c^f_{\rm V,NC}/c^f_{\rm V,NC}$$

$$\Delta c^e_{\rm V,CC}/c^e_{\rm V,CC}$$

↑ *1-loop level* ↑ *relative corrections*

- **NC couplings in the SM**

	$f=u$	$f=d$	$f=e$
$c^f_{\rm V,NC}$	$\frac{1}{2}-\frac{4}{3}s^2$	$-\frac{1}{2}+\frac{2}{3}s^2$	$-\frac{1}{2}+2s^2$
$c^f_{\rm A,NC}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

- **CC couplings in the SM** $\quad c^e_{\rm V,CC} = c^e_{\rm A,CC} = 1$

On-shell Renormalization

1 **Perform the one-loop renormalization of the SM in the on-shell scheme.**

Input parameters: $\alpha\,,\ m_W\,,\ m_Z\,,\ m_h\,,\ m_f$

@ 't Hooft-Feynman gauge

- **Dimensional regularization**

  't Hooft & Veltman, 1972; Bollini & Giambiagi, 1972

- **On-shell renormalization scheme**

  Ross & Taylor, 1973; Sirlin, 1980; Aoki et al., 1982; Bohm, Spiesberger, Hollik, 1986; Hollik, 1990; Jegerlehner 1990; Denner, 1993; …

- *Renormalization constants (counterterms)*

$$e_0 = Z_e e = (1+\delta Z_e)e,$$
$$m_{W,0}^2 = m_W^2 + \delta m_W^2,$$
$$m_{Z,0}^2 = m_Z^2 + \delta m_Z^2,$$
$$m_{h,0}^2 = m_h^2 + \delta m_h^2,$$
$$m_{f,0}^2 = m_f^2 + \delta m_f^2,$$

$$W_{0\mu}^{\pm} = \sqrt{Z_W} W_\mu^{\pm} = \left(1+\frac{1}{2}\delta Z_W\right) W_\mu^{\pm}\,,$$
$$\begin{pmatrix} Z_{0\mu} \\ A_{0\mu} \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{ZZ} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} & 1+\frac{1}{2}\delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}\,,$$
$$h_0 = \sqrt{Z_h} h = \left(1+\frac{1}{2}\delta Z_h\right) h\,,$$
$$f_{i,0}^{\mathrm{L}} = \sqrt{Z_{ij}^{f,\mathrm{L}}} f_j^{\mathrm{L}} = \left(1+\frac{1}{2}\delta Z_{ij}^{f,\mathrm{L}}\right) f_j^{\mathrm{L}}\,,$$
$$f_{i,0}^{\mathrm{R}} = \sqrt{Z_{ij}^{f,\mathrm{R}}} f_j^{\mathrm{R}} = \left(1+\frac{1}{2}\delta Z_{ij}^{f,\mathrm{R}}\right) f_j^{\mathrm{R}}\,.$$

- *On-shell conditions*

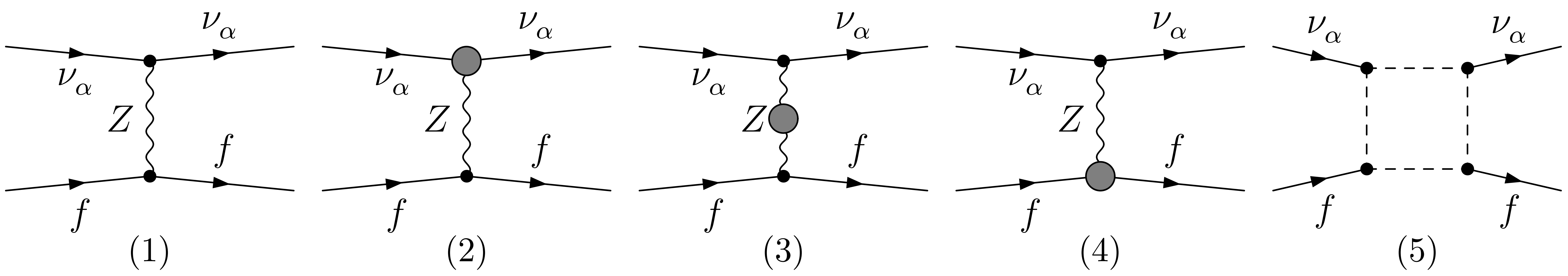
$$\delta m_W^2 = -\mathrm{Re}\Sigma_{\mathrm{T}}^W(m_W^2), \qquad \delta Z_W = \mathrm{Re}\frac{\partial \Sigma_{\mathrm{T}}^W(p^2)}{\partial p^2}\bigg|_{p^2=m_W^2},$$
$$\delta m_Z^2 = -\mathrm{Re}\Sigma_{\mathrm{T}}^Z(m_Z^2), \qquad \delta Z_Z = \mathrm{Re}\frac{\partial \Sigma_{\mathrm{T}}^Z(p^2)}{\partial p^2}\bigg|_{p^2=m_Z^2},$$
$$\delta m_h^2 = +\mathrm{Re}\Sigma^h(m_h^2), \qquad \delta Z_h = -\mathrm{Re}\frac{\partial \Sigma^h(p^2)}{\partial p^2}\bigg|_{p^2=m_h^2}.$$

$$\delta Z_{AA} = \frac{\partial \Sigma_{\mathrm{T}}^{AA}(p^2)}{\partial p^2}\bigg|_{p^2=0}, \qquad \delta Z_{AZ} = 2\mathrm{Re}\frac{\Sigma_{\mathrm{T}}^{AZ}(m_Z^2)}{m_Z^2}, \qquad \delta Z_{ZA} = -2\frac{\Sigma_{\mathrm{T}}^{AZ}(0)}{m_Z^2}.$$

One-loop Scattering Amplitudes

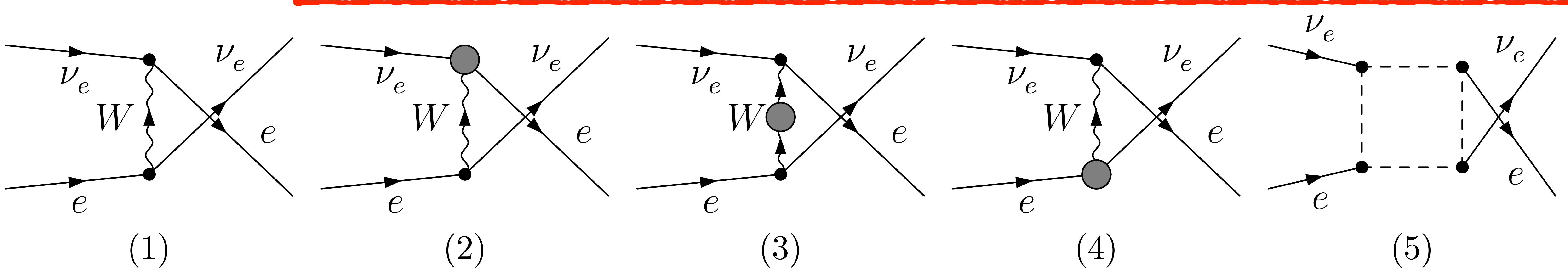
2 **Compute the one-loop neutrino scattering amplitudes in ordinary matter.**

JH & Shun Zhou, PRD (2023)



Finite corrections to the ***NC*** *coupling*

$$\Delta c^f_{\rm V,NC} = \left(-\frac{\Sigma^{\rm r}_Z}{m^2_Z} + s_{\rm 2w}\Gamma^{\rm r}_{\nu_\alpha\nu_\alpha Z}\right) c^f_{\rm V,NC} + s_{\rm 2w}\Gamma^{\rm r}_{ffZ} - \frac{4m^2_W}{g^2}\mathscr{M}^f_{\rm NC}$$



Finite corrections to the ***CC*** *coupling*

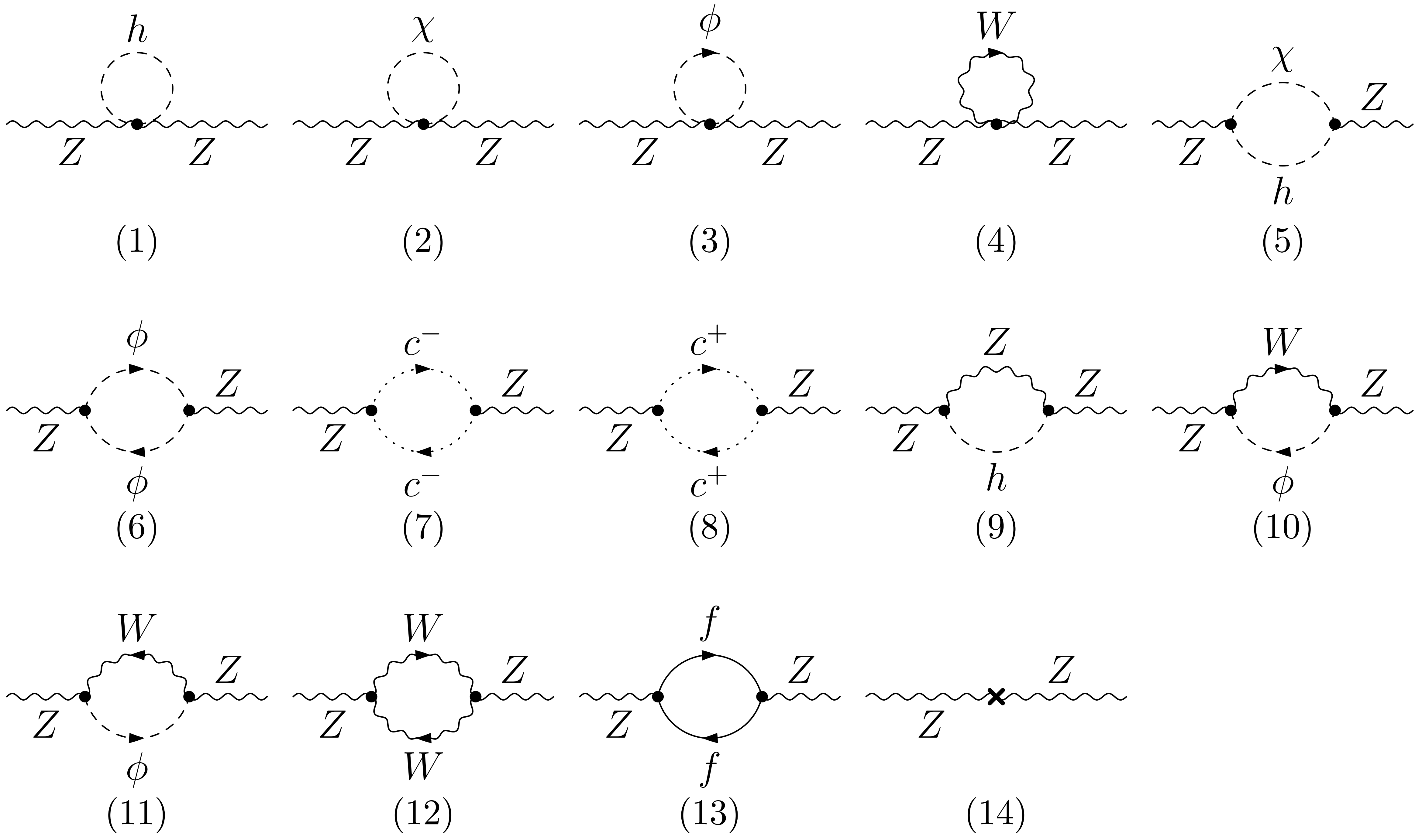
$$\Delta c^e_{\rm V,CC} = \left(-\frac{\Sigma^{\rm r}_W}{m^2_W} + 2\times\sqrt{2}s\Gamma^{\rm r}_{\nu_e e W}\right) c^e_{\rm V,CC} - \frac{4m^2_W}{g^2}\mathscr{M}_{\rm CC}$$

One-loop Scattering Amplitudes

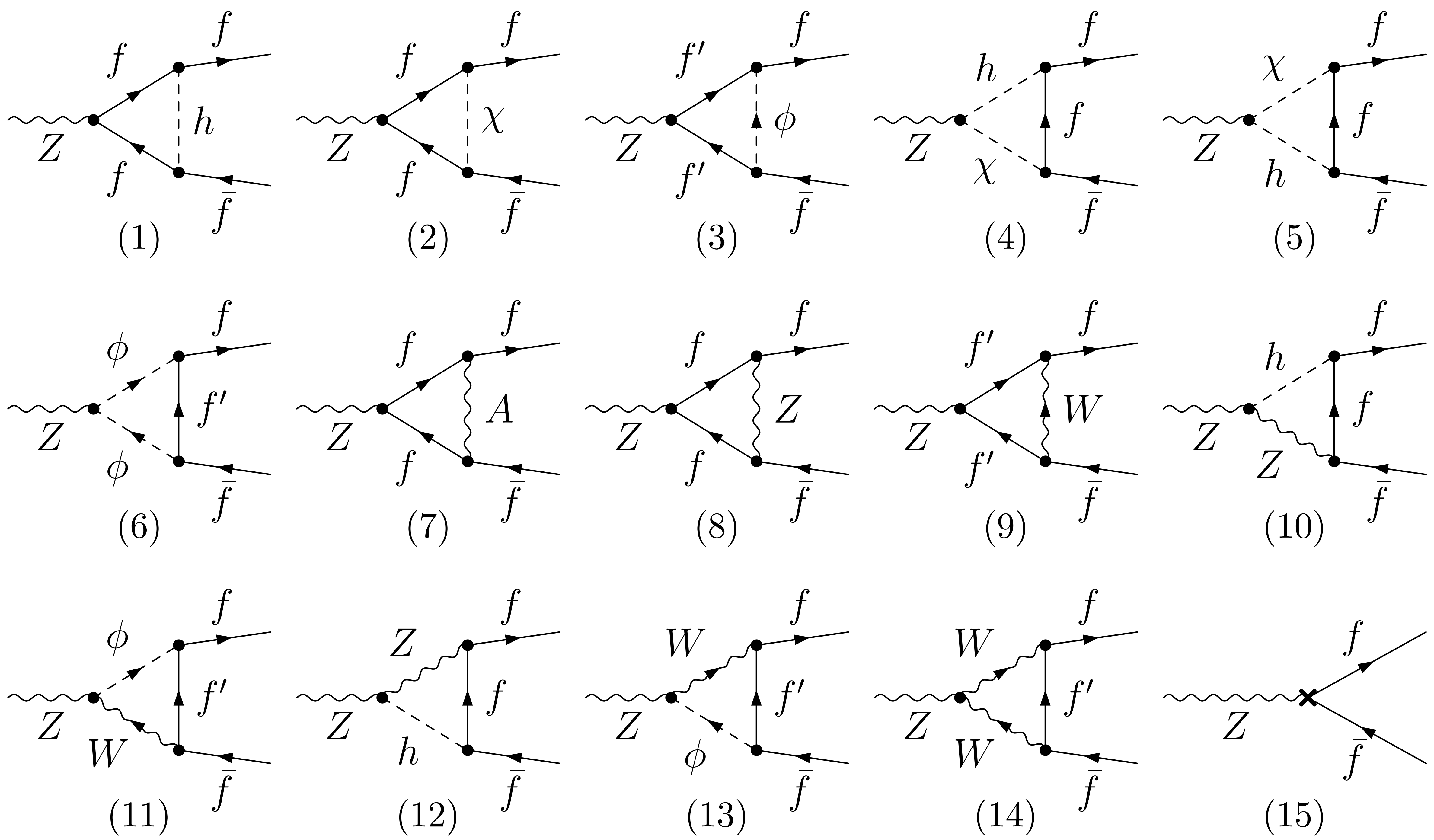
2 **Compute the one-loop neutrino scattering amplitudes in ordinary matter.**

JH & Shun Zhou, PRD (2023)

Z-boson self-energy @ 1-loop



Z-f-f vertex @ 1-loop



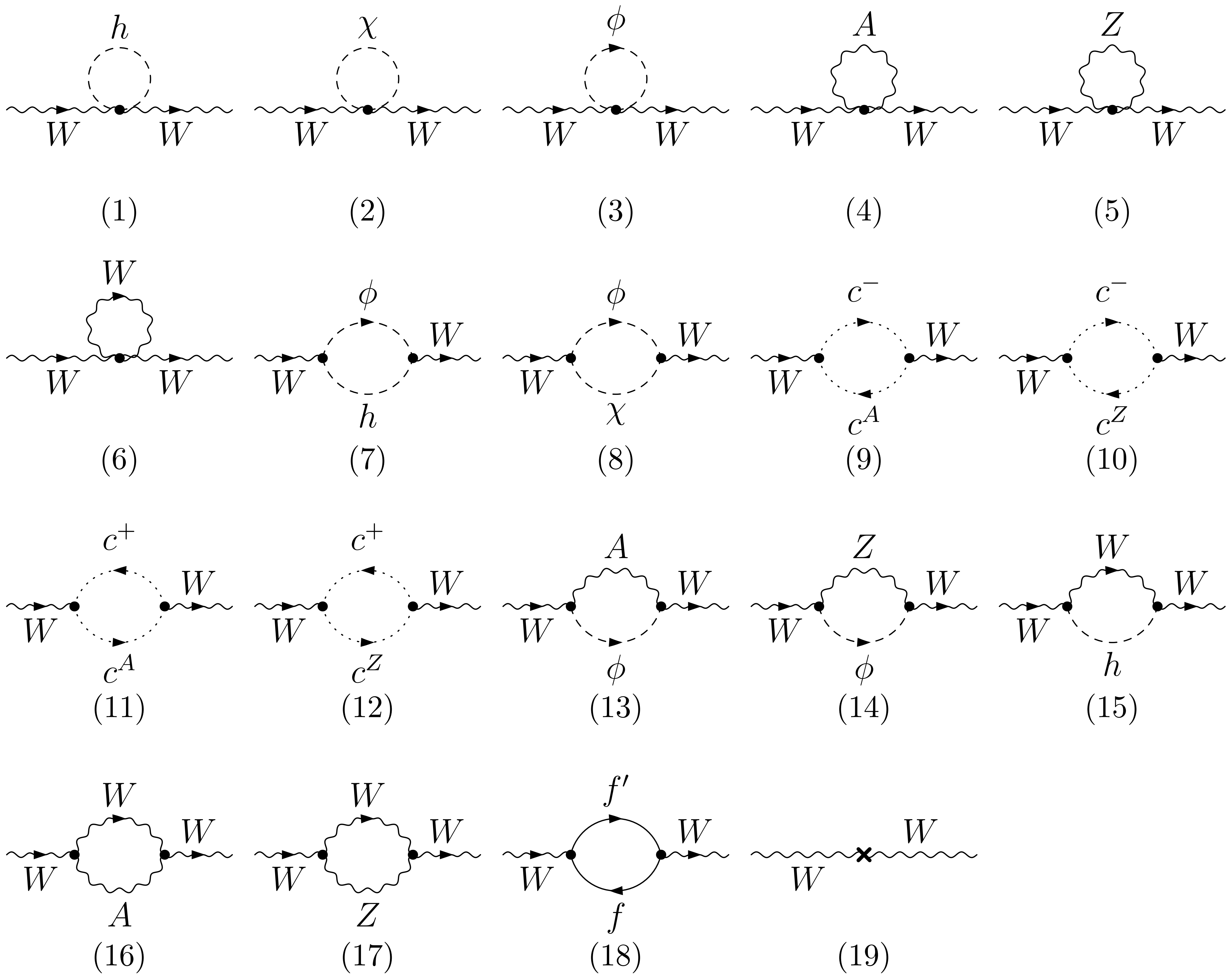
Including Goldstone bosons ϕ, χ for flavor-dependent terms.

One-loop Scattering Amplitudes

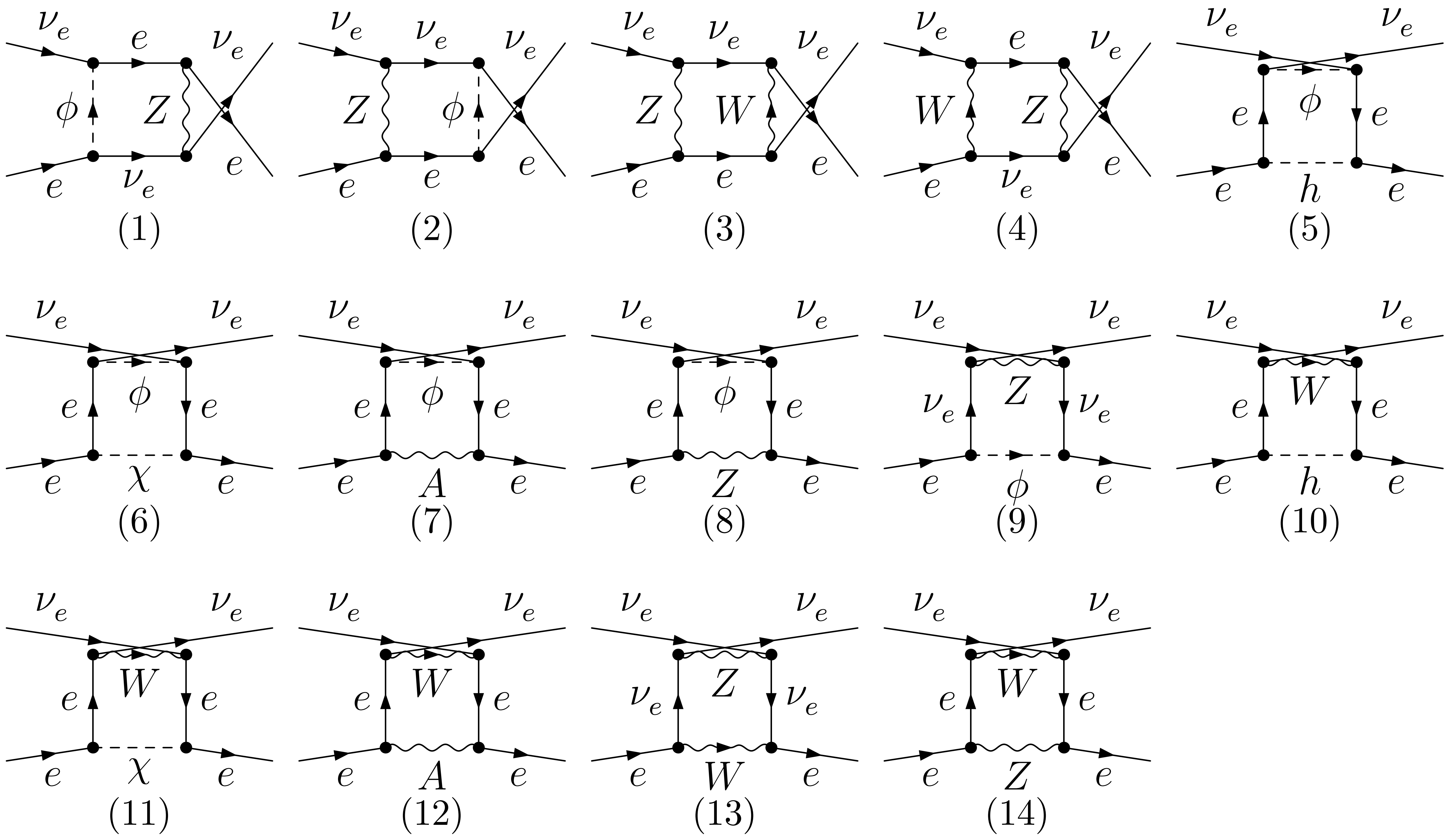
2 Compute the one-loop neutrino scattering amplitudes in ordinary matter.

JH & Shun Zhou, PRD (2023)

W-boson self-energy @ 1-loop



CC box diagrams @ 1-loop



Corrections to Vector-type Couplings

3 **Extract corrections to the vector-type couplings of CC and NC interactions.**

JH & Shun Zhou, PRD (2023)

*Finite corrections to the **NC** coupling*

$$\Delta c^f_{\rm V,NC} = \left(-\frac{\Sigma^{\rm r}_Z}{m^2_Z} + s_{2\rm w}\Gamma^{\rm r}_{\nu_\alpha\nu_\alpha Z} \right) c^f_{\rm V,NC} + s_{2\rm w}\Gamma^{\rm r}_{ffZ} - \frac{4m^2_W}{g^2}\mathscr{M}^f_{\rm NC}$$

Z-boson self-energy @ 1-loop

bosonic

$$\begin{aligned}(4\pi)^2\Sigma^{\rm r}_{Z-\rm b} = & \frac{g^2 m^2_Z}{8c^2(1-y_h)}\left(y^4_h - 6y^3_h + 17y^2_h - 22y_h + 4\right)\ln y_h \\ & -\frac{3}{2}g^2 m^2_Z\left(4c^4 + 4c^2 - 1\right){\rm DiscB}\left(m^2_Z, m_W, m_W\right) \\ & +\frac{g^2 m^2_Z}{4c^2(y_h - 4)}\left(y^3_h - 7y^2_h + 20y_h - 28\right){\rm DiscB}\left(m^2_Z, m_h, m_Z\right) \\ & +\frac{g^2 m^2_Z}{24c^2}\left(6y^2_h - 21y_h - 288c^6 - 264c^4 + 112c^2 + 49\right) ,\end{aligned}$$

fermionic

$$\begin{aligned}(4\pi)^2\Sigma^{\rm r}_{Z-\rm f} = \sum_f \frac{4e^2 m^2_Z}{12y_f - 3} & \left\{6y_f\left[a^2_f(1-4y_f) + 2v^2_f y_f\right]{\rm DiscB}\left(m^2_Z, m_f, m_f\right)\right. \\ & \left. +(4y_f - 1)\left[a^2_f(1-12y_f) + v^2_f(6y_f+1)\right]\right\} ,\end{aligned}$$

Flavor-independent !

NC box diagram contributions @ 1-loop

$$(4\pi)^2\mathcal{M}^u_{\rm NC} = -\frac{g^4}{8m^2_W}\left[\frac{5-4c^2}{4c^2} + x_\alpha\left(\ln x_\alpha + 1\right)\right] ,$$

$$(4\pi)^2\mathcal{M}^d_{\rm NC} = +\frac{g^4}{2m^2_W}\left[\frac{20c^2-1}{16c^2} + x_\alpha\left(\ln x_\alpha + 1\right)\right] ,$$

$$(4\pi)^2\mathcal{M}^e_{\rm NC} = +\frac{g^4}{2m^2_W}\left[\frac{28c^2-9}{16c^2} + x_\alpha\left(\ln x_\alpha + 1\right)\right] .$$

Flavor-dependent !

Consistent with previous results

Corrections to Vector-type Couplings

3 **Extract corrections to the vector-type couplings of CC and NC interactions.**

JH & Shun Zhou, PRD (2023)

*Finite corrections to the **CC** coupling*

$$\Delta c^e_{\rm V,CC} = \left(-\frac{\Sigma^{\rm r}_W}{m_W^2} + 2\times\sqrt{2}s\Gamma^{\rm r}_{\nu_e e W} \right) c^e_{\rm V,CC} - \frac{4m_W^2}{g^2}\mathscr{M}_{\rm CC}$$

$$\begin{aligned}(4\pi)^2\Gamma^{\rm r}_{\nu_e eW} = {} & \frac{g^2}{c^2}\left[\frac{\mathcal{F}_Z(m_Z^2)}{m_Z^2} - \frac{\mathcal{F}_W(m_W^2)}{4s^2m_W^2}\right] - e^2\left[\mathcal{F}'_A(0) - \frac{\mathcal{F}'_W(m_W^2)}{4s^2}\right] + \frac{g^2}{24s^2(4-x_h)}[(c^2-2)x_h^3 - (5c^2-13)x_h^2 \\ & + 4(c^2-8)x_h + 12(c^2+3)]{\rm DiscB}(m_W^2, m_h, m_W) - \frac{g^2}{24c^4s^2}(60c^8 - 8c^6 + 71c^4 - 22c^2 - 2){\rm DiscB}(m_W^2, m_W, m_Z) \\ & - \frac{g^2}{24s^2}(y_h^2 - 4y_h + 12){\rm DiscB}(m_Z^2, m_h, m_Z) + \frac{g^2}{24s^2}(48c^6 + 68c^4 - 16c^2 - 1){\rm DiscB}(m_Z^2, m_W, m_W) \\ & + \frac{g^2}{48s^2}(y_h^3 - 6y_h^2 + 18y_h - 20c^2)\ln y_h - \frac{g^2}{48}[(c^4+c^2+2)x_h^3 - (6c^2+9)x_h^2 + 18x_h + 168c^2 - 8]\ln x_h \\ & + \frac{g^2}{48c^6s^2}(c^6y_h^3 - 6c^6y_h^2 + 18c^6y_h - 48c^{10} - 36c^8 + 166c^6 - 119c^4 + 18c^2 + 2)\ln\left(\frac{m_W^2}{m_Z^2}\right) \\ & + \frac{g^2}{24c^4}[(c^2+2)y_h^2 - 6c^2y_h - 96c^8 - 224c^6 + 32c^4 + 23c^2 + 2].\end{aligned}$$

ν_e-e-W ***vertex @ 1-loop***

Numerical Results

4 **Evaluate the one-loop corrections to the MSW matter potentials.**

JH & Shun Zhou, PRD (2023)

- Input parameters
  - ***The fine structure constant***: $\alpha \equiv e^2/(4\pi) = 1/137.035999084$
  - ***The gauge-boson and Higgs-boson masses***:

$$m_W = 80.377\ \text{GeV},\ m_Z = 91.1876\ \text{GeV},\ m_h = 125.25\ \text{GeV}$$

  - ***The quark masses***:

$$m_u = 2.16\ \text{MeV},\ m_c = 1.67\ \text{GeV},\ m_t = 172.5\ \text{GeV}$$

$$m_d = 4.67\ \text{MeV},\ m_s = 93.4\ \text{MeV},\ m_b = 4.78\ \text{GeV}$$

  - ***The charged-lepton masses***:

$$m_e = 0.511\ \text{MeV},\ m_\mu = 105.658\ \text{MeV},\ m_\tau = 1.777\ \text{GeV}$$

} On-shell Masses

Numerical Results

4 Evaluate the one-loop corrections to the MSW matter potentials.

JH & Shun Zhou, PRD (2023)

Corrections to NC potential

	Self-energy	ν_α-ν_α-Z	f-f-Z	Box diagrams	$\Delta c^f_{\rm V,NC}$
$f = u$	-2.1×10^{-3}	5.1×10^{-3} 1.5×10^{-6} (fd)	-6.0×10^{-3}	7.9×10^{-4} -4.2×10^{-6} (fd)	-2.2×10^{-3}
$f = d$	3.7×10^{-3}	-8.8×10^{-3} -2.6×10^{-6} (fd)	-3.3×10^{-3}	-6.1×10^{-3} 1.7×10^{-5} (fd)	-1.5×10^{-2}
$f = e$	5.6×10^{-4}	-1.4×10^{-3} -3.9×10^{-7} (fd)	15.3×10^{-3}	-5.3×10^{-3} 1.7×10^{-5} (fd)	9.2×10^{-3}

from quarks to nucleons

$$\frac{\Delta c_{\rm V,NC}}{c_{\rm V,NC}} = \frac{N_p\left(2\Delta c^u_{\rm V,NC} + \Delta c^d_{\rm V,NC} + \Delta c^e_{\rm V,NC}\right) + N_n\left(\Delta c^u_{\rm V,NC} + 2\Delta c^d_{\rm V,NC}\right)}{N_n\left(c^u_{\rm V,NC} + 2c^d_{\rm V,NC}\right)} \approx 0.062 + 0.02\frac{N_p}{N_n}$$

8.2% for NC

Corrections to CC potential

Self-energy	ν_e-e-W	Box diagrams	$\Delta c^e_{\rm V,CC}$
-6.4×10^{-3}	4.5×10^{-2}	1.9×10^{-2}	5.8×10^{-2}

These corrections are at the same level as the experimental precisions.

5.8% for CC

Summary

- When neutrinos propagating through matter, the MSW matter effect caused by the **forward coherent scattering** can alter the flavor conversion.
- A **complete one-loop** calculation of the MSW matter potential is presented. The relative size of the correction to CC potential of electron-neutrinos is **5.8%**, while that to NC potential of all-flavor neutrinos can be as large as **8.2%**.
- Such corrections could affect the neutrino oscillations and be examined in the next-generation experiments. In the **neutrino precision measurement era**, higher-level calculations are necessary (MSW matter potential, neutrino-matter interactions…).

Weinberg's 2nd Laws of Progress in Theoretical Physics (1983):
"*Do not trust arguments based on the lowest order of perturbation theory.*"

Thanks for your attention!

BACKUP

MSW & Solar Neutrino Problem

Mixing angles in matter: $\tan 2\tilde{\theta} = \dfrac{\Delta m_{21}^2 s_{2\theta}}{\Delta m_{21}^2 c_{2\theta} - a}$

MSW resonance: $a = 2\sqrt{2} G_\mu N_e E = \Delta m_{21}^2 c_{2\theta}$

For high-energy ⁸B neutrinos:

- Solar-**core** production ($\tilde{\theta} \to \pi/2$)

$$\begin{pmatrix} \tilde{\nu}_1(0) \\ \tilde{\nu}_2(0) \end{pmatrix} = \begin{pmatrix} c_{\tilde{\theta}} & -s_{\tilde{\theta}} \\ s_{\tilde{\theta}} & c_{\tilde{\theta}} \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_\mu(0) \end{pmatrix}$$

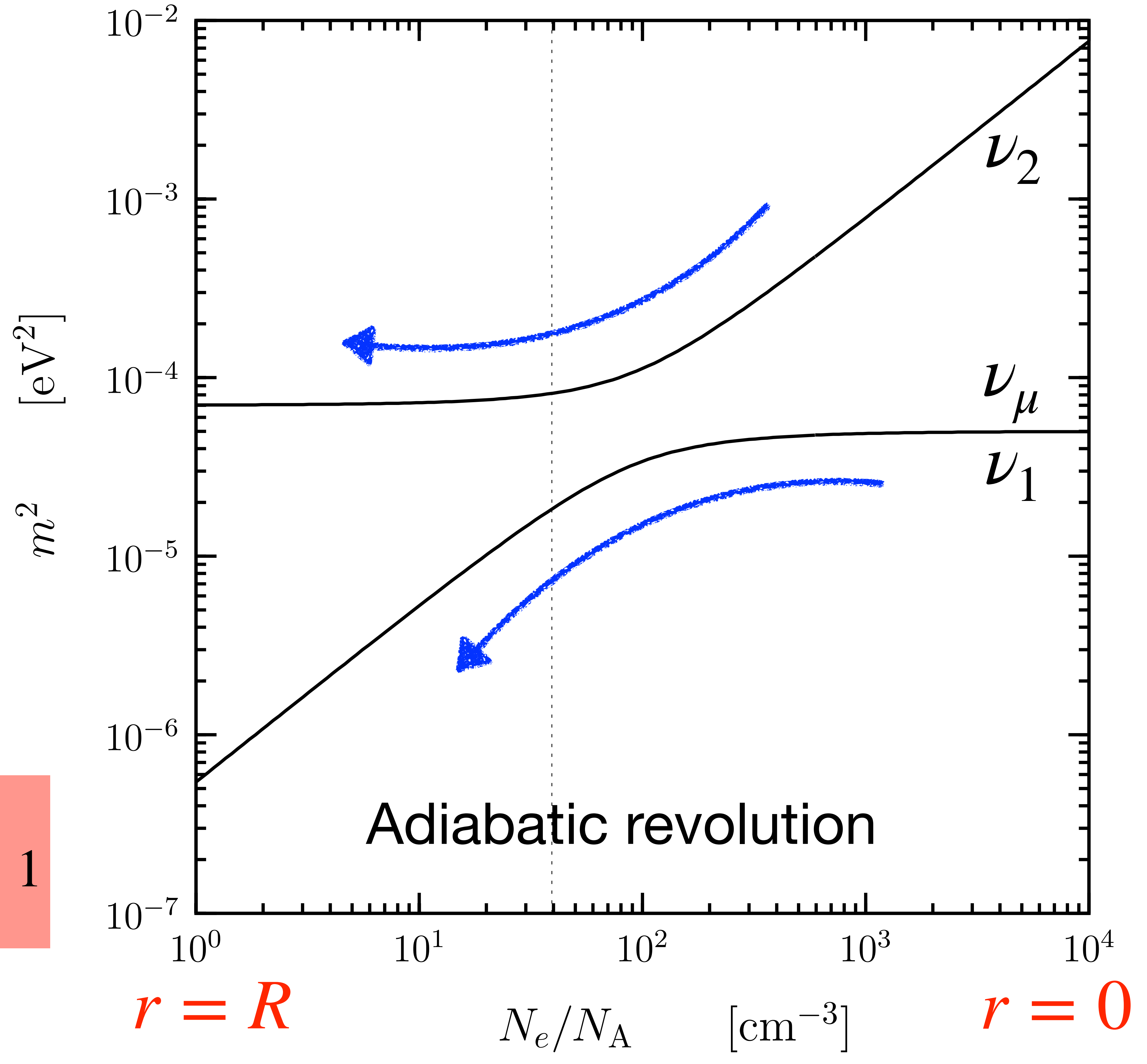
- Adiabatic evolution

$$\begin{pmatrix} \tilde{\nu}_1(r) \\ \tilde{\nu}_2(r) \end{pmatrix} = \begin{pmatrix} \tilde{\nu}_1(0) \\ \tilde{\nu}_2(0) \end{pmatrix}$$

Adiabatic condition: $\gamma \equiv \dfrac{\left(\Delta \tilde{m}_{21}^2\right)^2}{2E \sin 2\tilde{\theta} \, |\mathrm{d}A/\mathrm{d}r|} \gg 1$

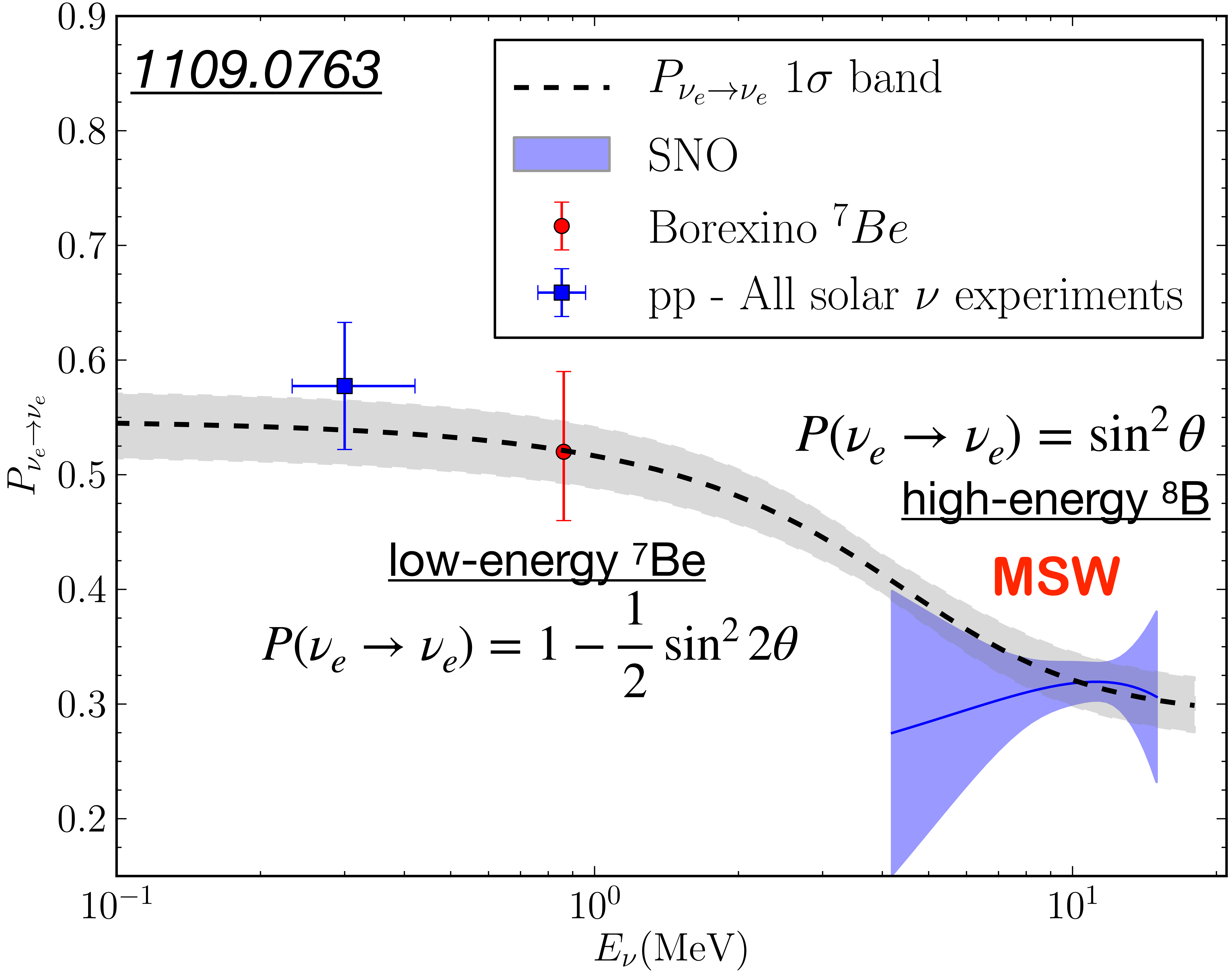
- Solar **surface** ($\tilde{\theta} \to \theta$)

$$\begin{pmatrix} \nu_e(R) \\ \nu_\mu(R) \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1(R) \\ \tilde{\nu}_2(R) \end{pmatrix}$$



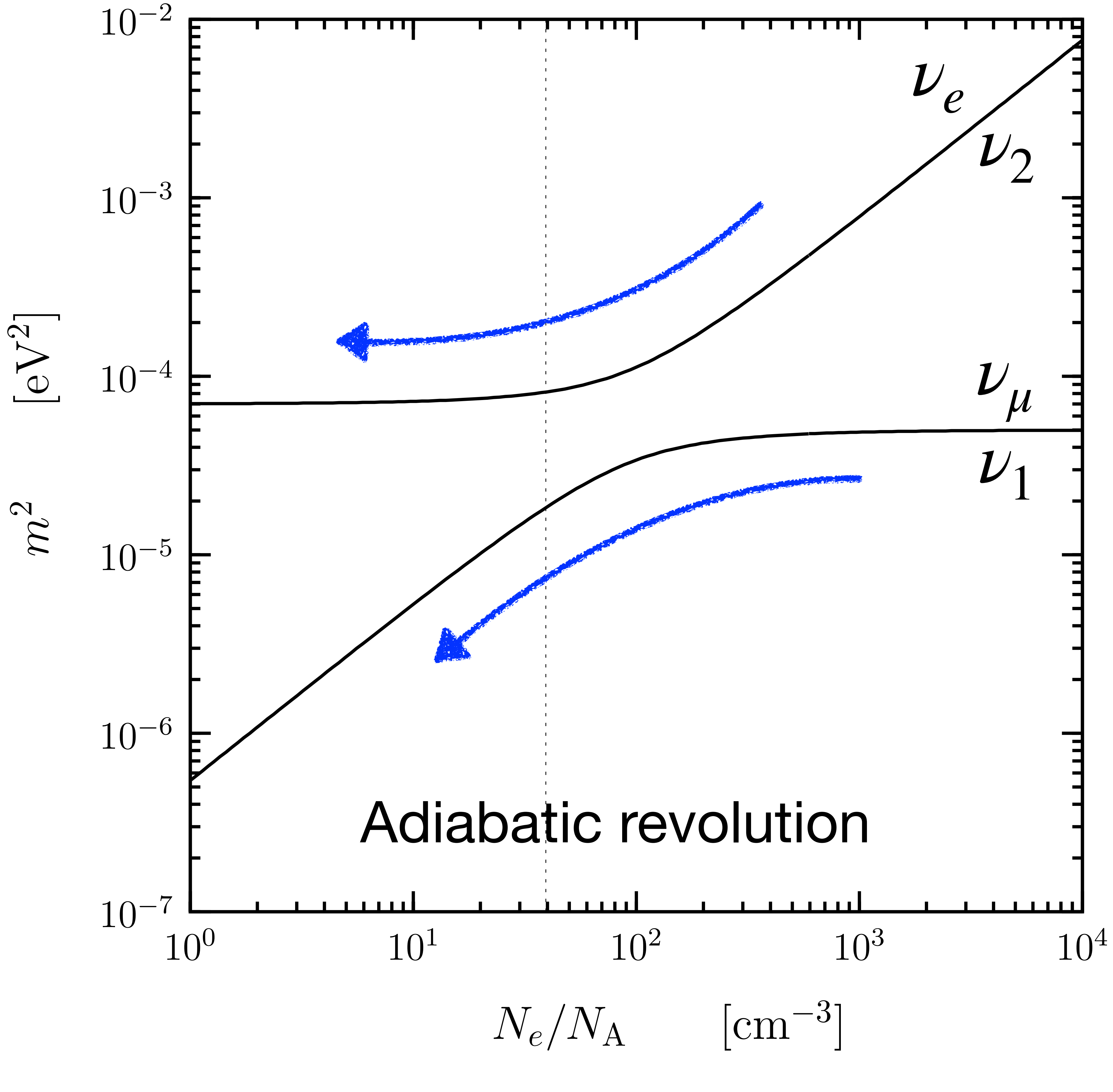
Survival probability: $P(\nu_e \to \nu_e) = \sin^2\theta$

MSW & Solar Neutrino Problem



MSW resonance:

$$a = 2\sqrt{2}G_\mu N_e E = \Delta m^2_{21} c_{2\theta}$$



Loop Functions

$$\mathrm{B}_0\left(p^2;m_0,m_1\right)=\Delta+\ln\left(\frac{\mu^2}{m_1^2}\right)+2+\mathrm{DiscB}(p^2,m_0,m_1)-\frac{m_0^2-m_1^2+p^2}{2p^2}\ln\left(\frac{m_0^2}{m_1^2}\right)$$

$$\mathrm{DiscB}\left(p^2,m_0,m_1\right)=\frac{\sqrt{\lambda\left(m_0^2,m_1^2,p^2\right)}}{p^2}\ln\left[\frac{m_0^2+m_1^2-p^2+\sqrt{\lambda\left(m_0^2,m_1^2,p^2\right)}}{2m_0m_1}\right]$$

$$\lambda\left(x,y,z\right)\equiv x^2+y^2+z^2-2xy-2yz-2zx$$

$$\mathcal{F}_Z(p^2)=\sum_f\{[4a_f^2m_f^2-p^2(a_f^2+v_f^2)]\mathrm{B}_0(p^2;m_f,m_f)-4(a_f^2+v_f^2)\mathrm{B}_{00}(p^2;m_f,m_f)+2(a_f^2+v_f^2)\mathrm{A}_0(m_f)\},\qquad(3.9)$$

$$\mathcal{F}_W(p^2)=\sum_{\{f,f'\}}[(m_f^2+m_{f'}^2)\mathrm{B}_0(p^2;m_f,m_{f'})-4\mathrm{B}_{00}(p^2;m_f,m_{f'})-p^2\mathrm{B}_0(p^2;m_f,m_{f'})+\mathrm{A}_0(m_f)+\mathrm{A}_0(m_{f'})],\qquad(3.10)$$

$$\mathcal{F}_A(p^2)=\sum_f Q_f^2[-4\mathrm{B}_{00}(p^2;m_f,m_f)-p^2\mathrm{B}_0(p^2;m_f,m_f)+2\mathrm{A}_0(m_f)],\qquad(3.11)$$

$$\mathcal{F}_{AZ}(p^2)=\sum_f Q_fv_f[-4\mathrm{B}_{00}(p^2;m_f,m_f)-p^2\mathrm{B}_0(m_Z^2;m_f,m_f)+2\mathrm{A}_0(m_f)],\qquad(3.12)$$

$x_i\equiv m_i^2/m_W^2\,,\ y_i\equiv m_i^2/m_Z^2$

$v_f\equiv c^f_{\mathrm{V,NC}}/s_{2\mathrm{w}}\,,\ a_f\equiv c^f_{\mathrm{A,NC}}/s_{2\mathrm{w}}$

$\overline{\text{MS}}$ vs On-shell Scheme

Generally speaking, the OS scheme is advantageous in the sense that the OS parameters can be directly extracted from experimental measurements.

The experimental determination of different $\overline{\text{MS}}$ parameters is usually carried out at different energy scales associated with relevant physical processes, so the RGEs should be implemented to obtain the complete set of $\overline{\text{MS}}$ parameters at a common scale.

However, the $\overline{\text{MS}}$ scheme together with the approach of EFT is practically more convenient to deal with higher-order corrections beyond one-loop and any theories with different mass scales.

In any case, our calculations of the one-loop matter potentials for neutrinos in the OS scheme are complementary to those in the $\overline{\text{MS}}$ scheme.

JUNO MSW

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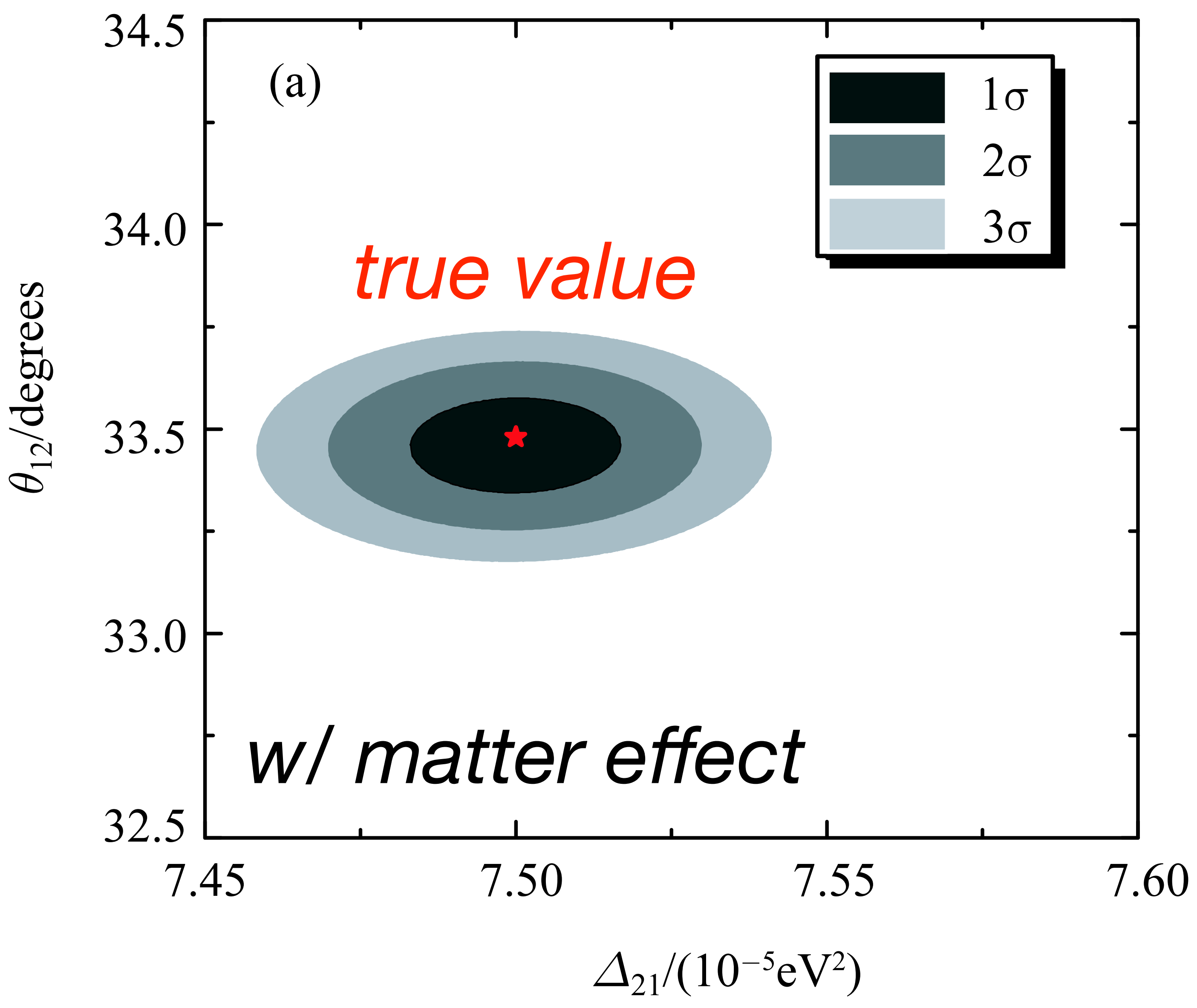
Terrestrial matter effects on reactor antineutrino oscillations at JUNO or RENO-50: how small is small? *

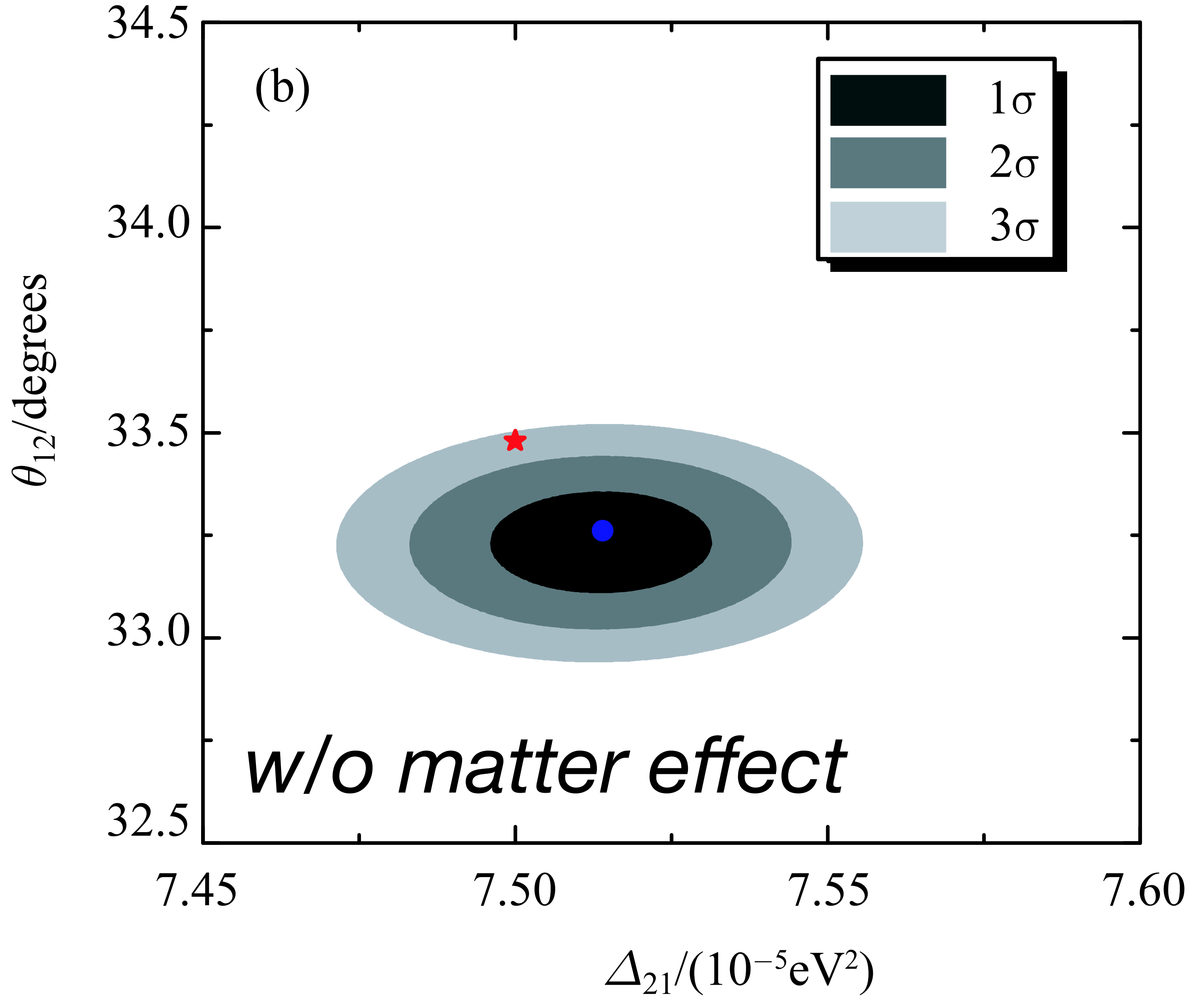
Yu-feng Li(李玉峰)[1)] Yi-fang Wang(王贻芳)[2)] Zhi-zhong Xing(邢志忠)[3)]

[1] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[2] School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

Abstract: We have carefully examined, in both analytical and numerical ways, how small the terrestrial matter effects can be in a given medium-baseline reactor antineutrino oscillation experiment like JUNO or RENO-50. Taking the forthcoming JUNO experiment as an example, we show that the inclusion of terrestrial matter effects may reduce the sensitivity of the neutrino mass ordering measurement by $\Delta\chi^2_{\rm MO} \simeq 0.6$, and a neglect of such effects may shift the best-fit values of the flavor mixing angle θ_{12} and the neutrino mass-squared difference Δ_{21} by about 1σ to 2σ in the future data analysis. In addition, a preliminary estimate indicates that a 2σ sensitivity of establishing the terrestrial matter effects can be achieved for about 10 years of data taking at JUNO with the help of a suitable near detector implementation.

$$E \sim 4 \text{ MeV}, \rho \sim 2.6 \text{ g cm}^{-3} \Longrightarrow A/\Delta_{21} \sim 10^{-2}$$





Weinberg's Laws

Weinberg's Laws of Progress in Theoretical Physics

- ***First Law (The conservation of Information)***: "You will get nowhere by churning equations."
- ***Second Law***: "Do not trust arguments based on the lowest order of perturbation theory."
- ***Third Law***: "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry."

From ***Why the Renormalization Group Is a Good Thing*** by Steven Weinberg in ***Asymptotic Realms of Physics: Essays in Honor of Francis E. Low*** (**1983**)