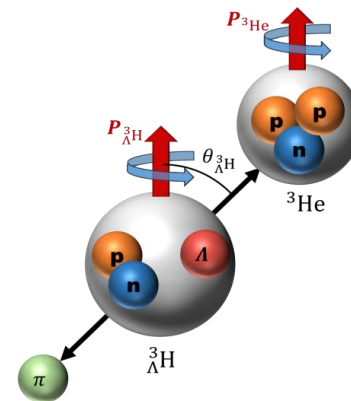
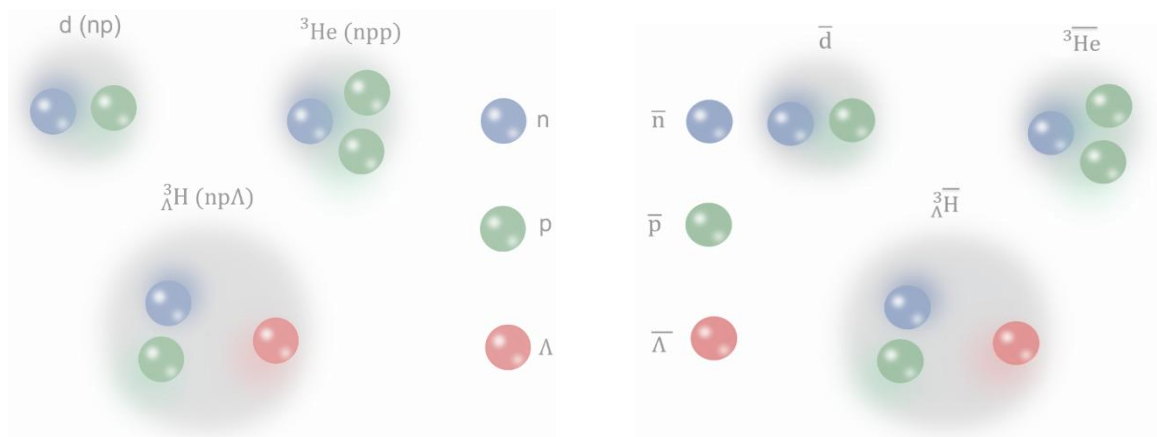
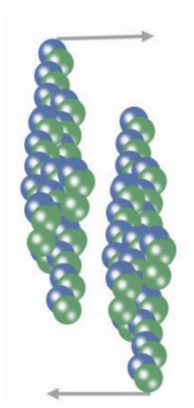


# The 23rd International Conference on Few-Body Problems in Physics (FB23)

## Production and Polarization of Hypernuclei in Heavy-ion Collisions

KJ Sun, Dai-Neng Liu, Yun-Peng Zhen, Jin-Hui Chen, Che Ming Ko, Yu-Gang Ma [arXiv:2405.12015](https://arxiv.org/abs/2405.12015)



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復旦大學  
FUDAN UNIVERSITY

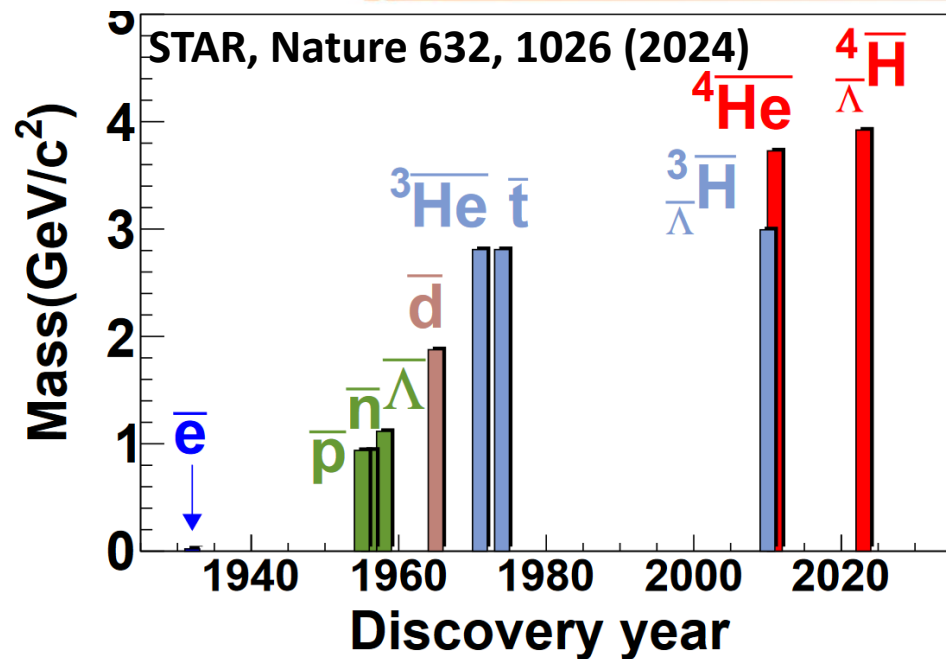
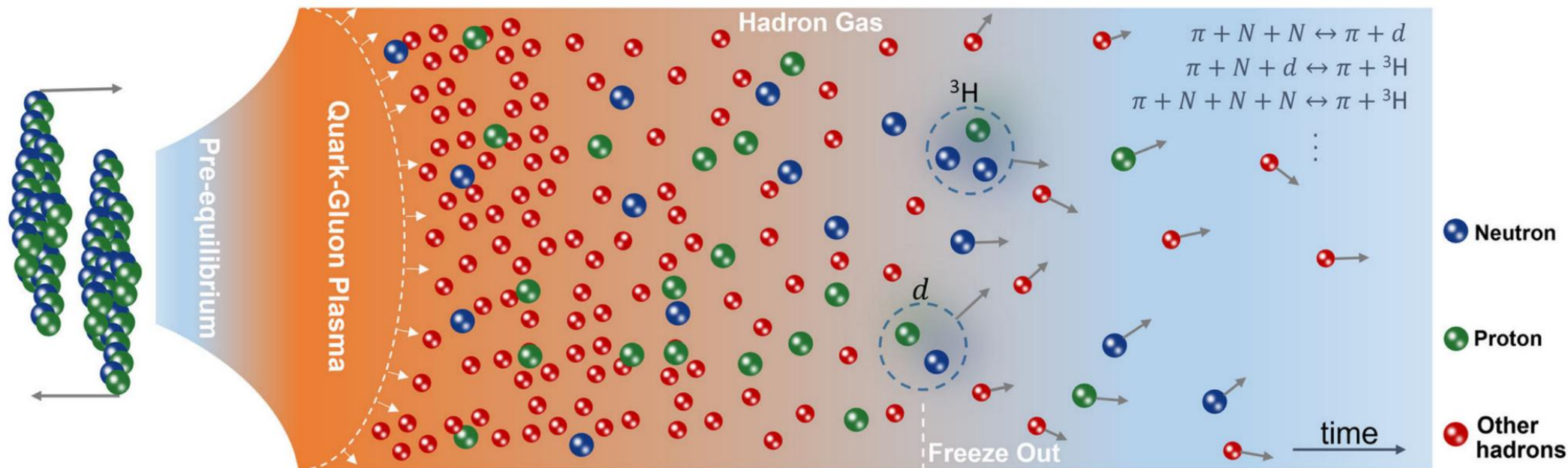
# Outline

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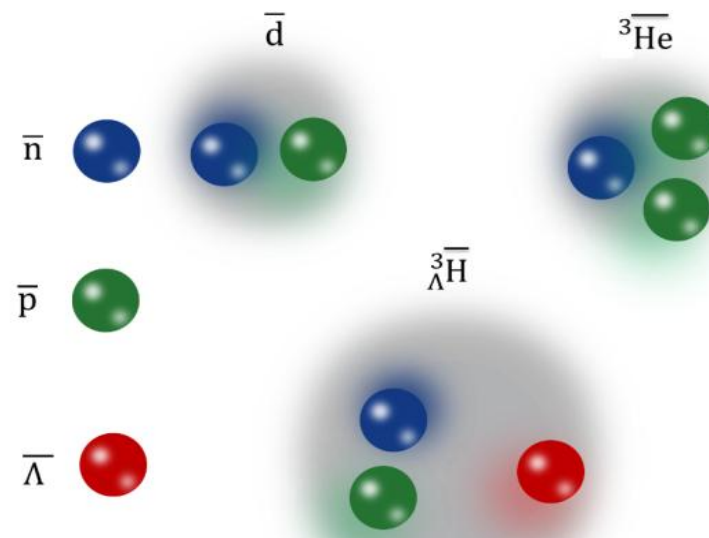
1. Property and production of (anti-)hypertriton in heavy-ion collisions
2. Polarization in heavy-ion collisions
3. **Results: (Anti-)Hypertriton polarization and its spin structure**
4. **Results: Effects of baryon spin correlations**
5. Summary and outlook

# 1. Polarization of light (anti-)(hyper-)nuclei

(1)

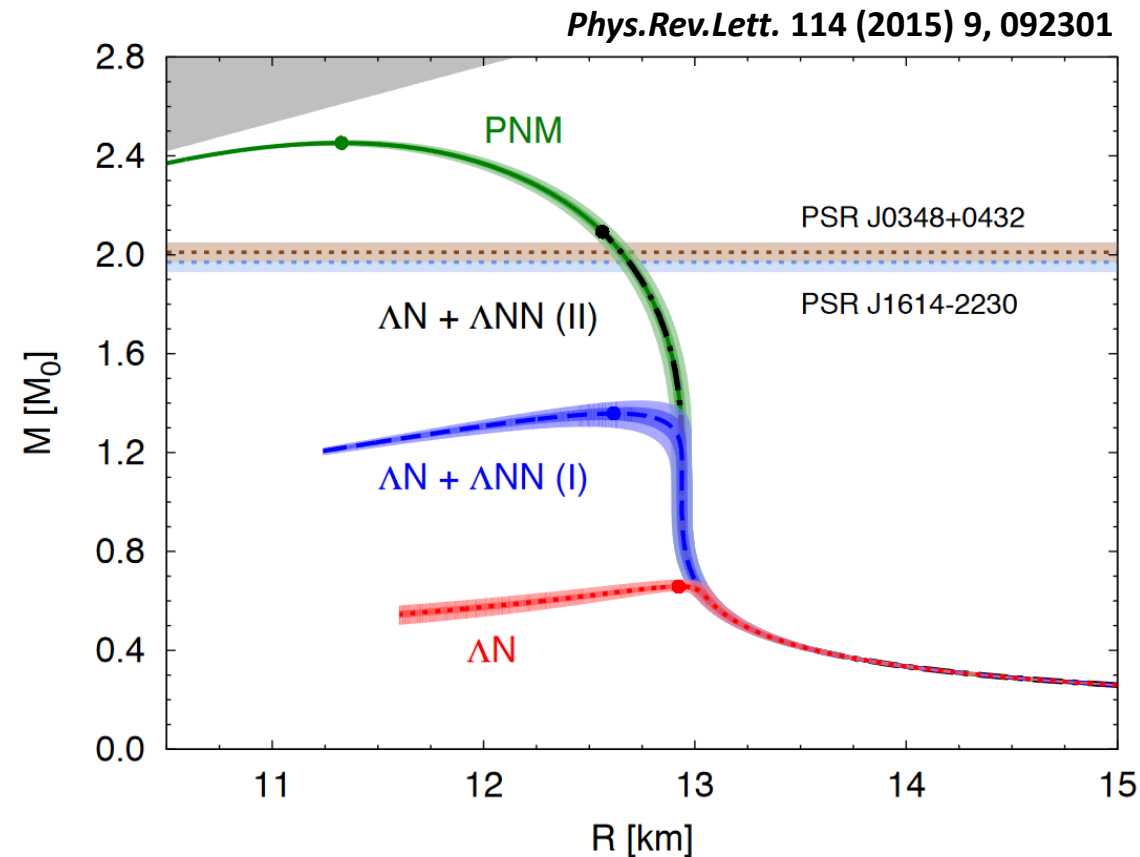
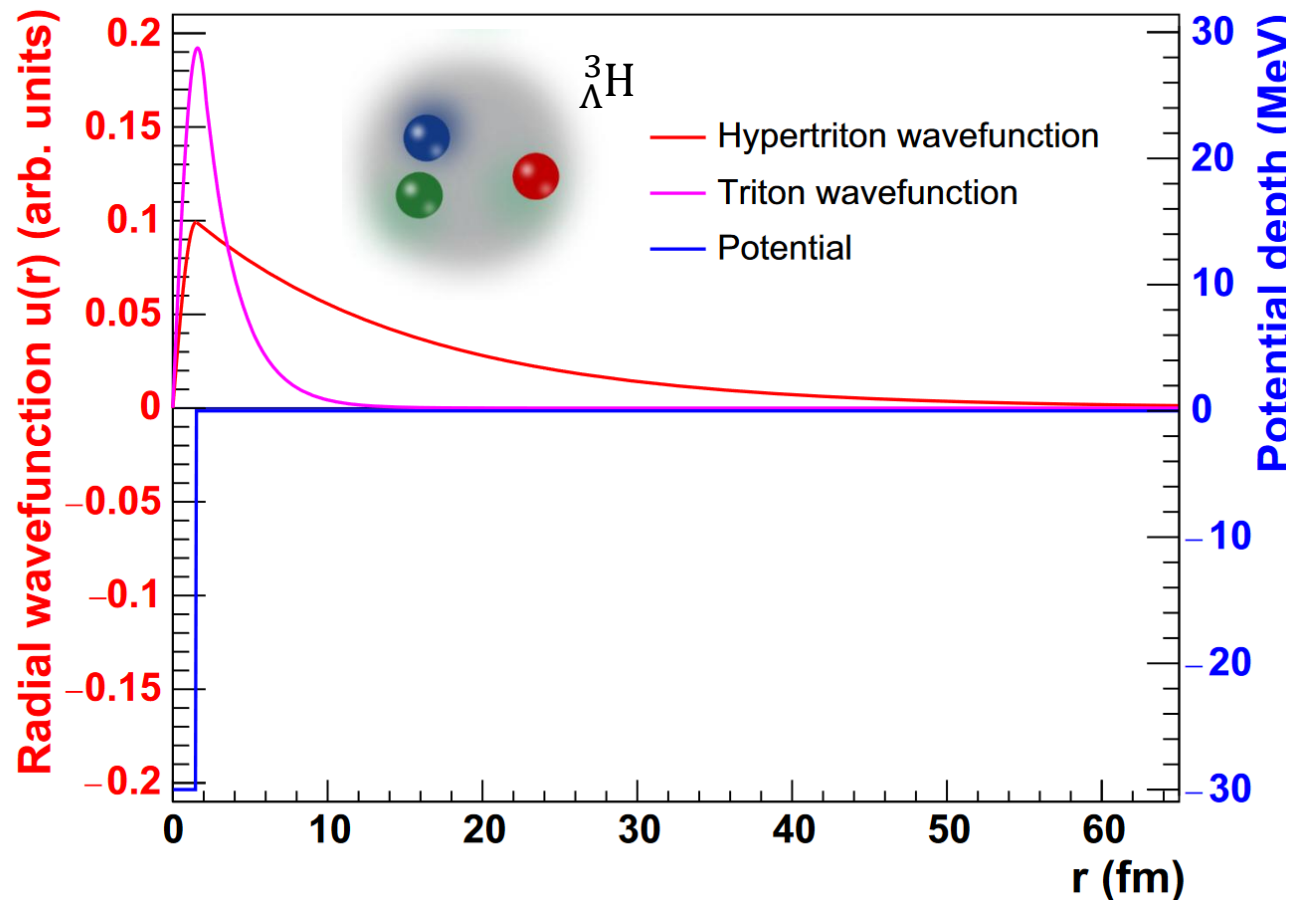


K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, C. Shen, Nature Commun. 15, 1074 (2024)



# 1. The halo-like nucleus: (anti-)hypertriton

(2)

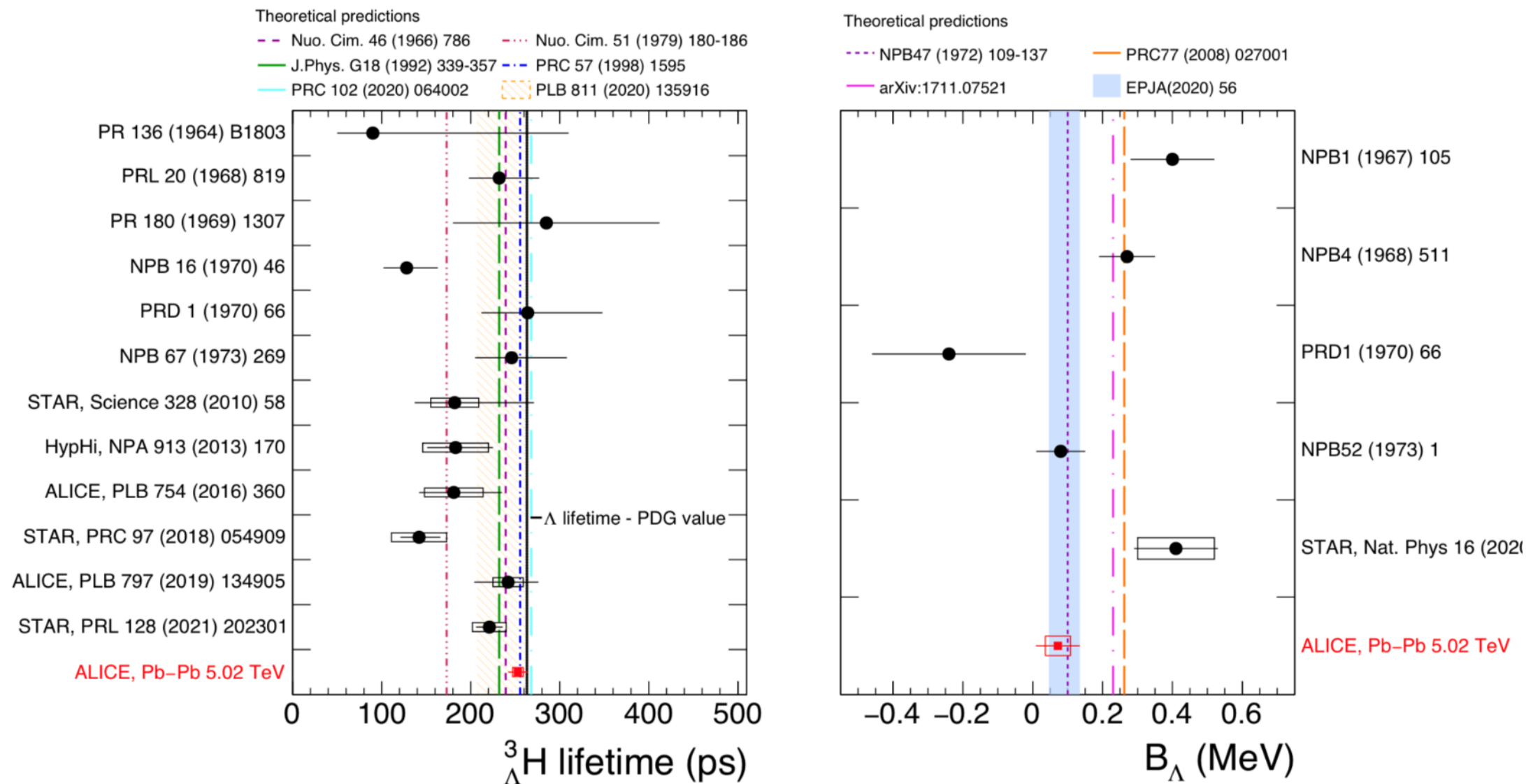


# 1. Binding energy and lifetime

(3)

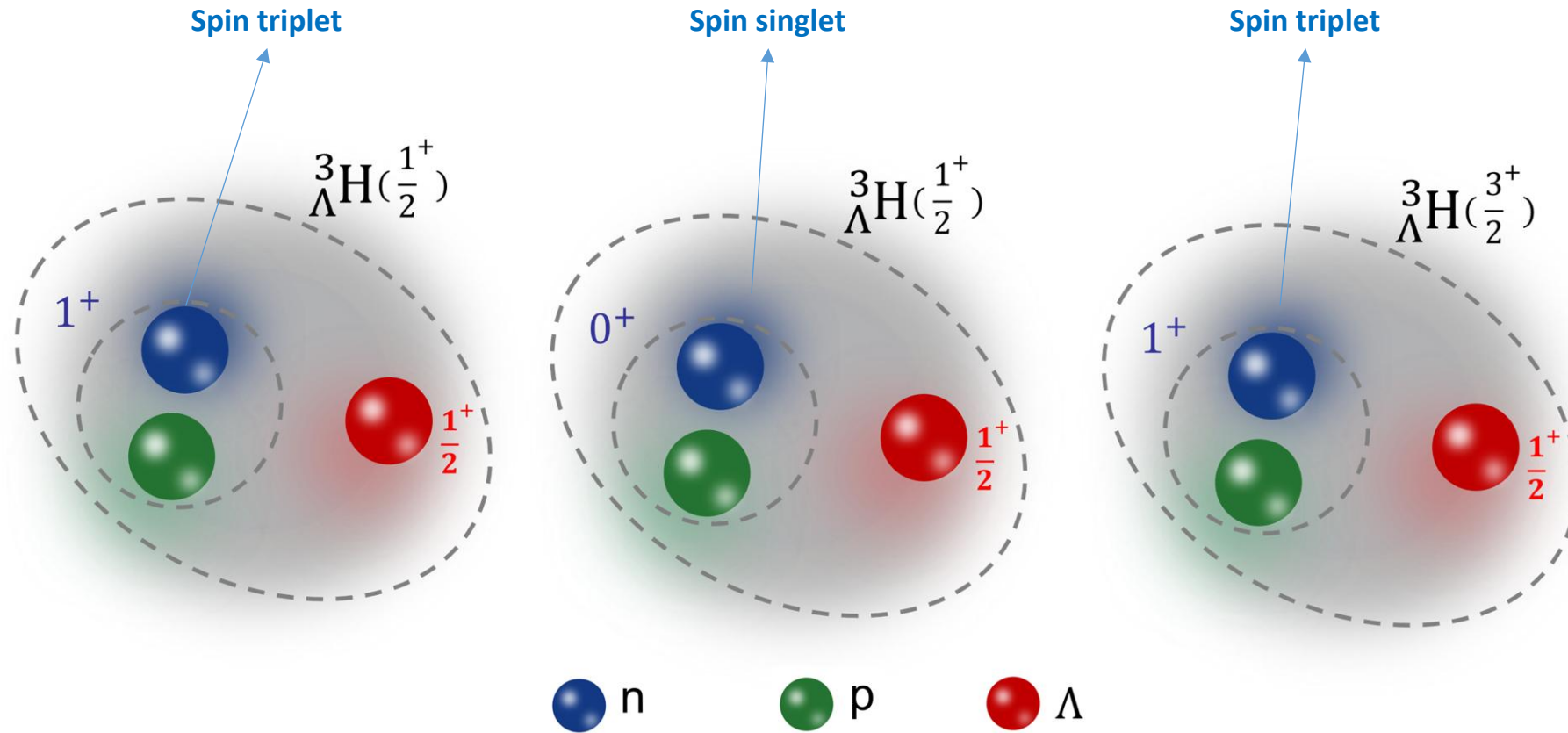
ALICE, PRL 131, 102302 (2023)

Y. G. Ma, Nucl. Sci. Tech. 3497 (2023)



# 1. Spin of (anti-)hypertriton ?

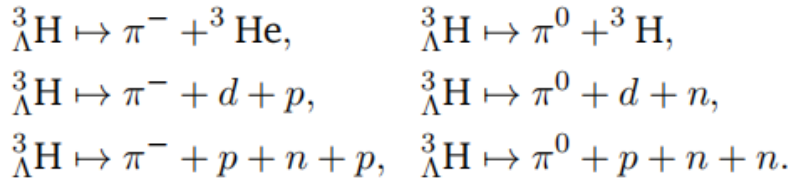
(4)





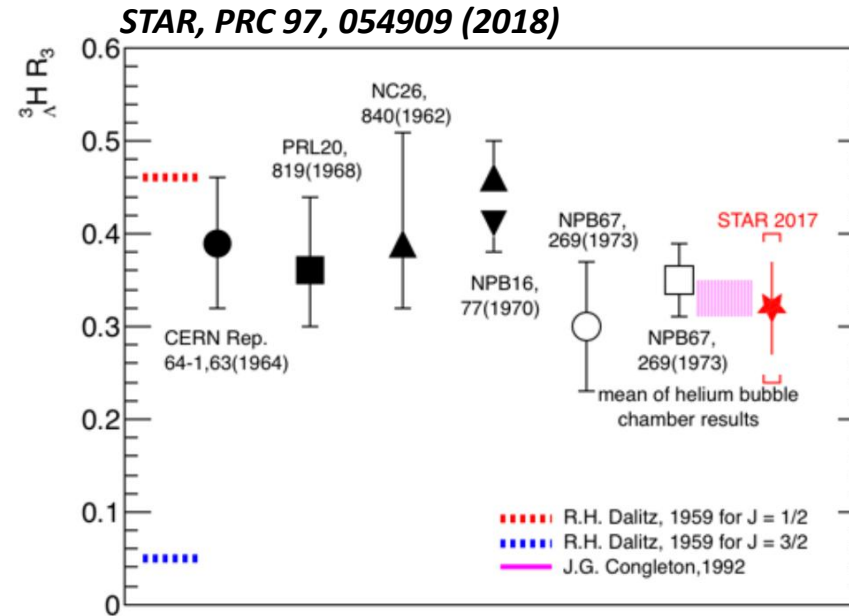
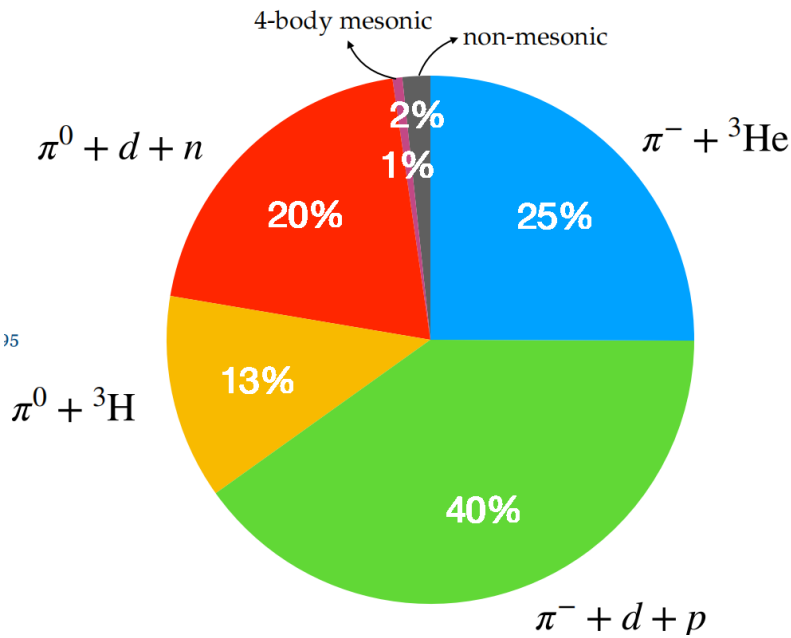
# 1. Spin of (anti-)hypertriton ?

(5)



Relative branching ratio:

$$R_3 = \frac{\text{B.R.}({}^3_{\Lambda}H \rightarrow {}^3\text{He}\pi^-)}{\text{B.R.}({}^3_{\Lambda}H \rightarrow {}^3\text{He}\pi^-) + \text{B.R.}({}^3_{\Lambda}H \rightarrow dp\pi^-)}$$



Favors spin 1/2

PHYSICAL REVIEW D **87**, 034506 (2013)  
**Light nuclei and hypernuclei from quantum chromodynamics in the limit of SU(3) flavor symmetry**  
 S. R. Beane,<sup>1</sup> E. Chang,<sup>2</sup> S. D. Cohen,<sup>3</sup> W. Detmold,<sup>4,5</sup> H. W. Lin,<sup>3</sup> T. C. Luu,<sup>6</sup> K. Orginos,<sup>4,5</sup>  
 A. Parreño,<sup>2</sup> M. J. Savage,<sup>3</sup> and A. Walker-Loud<sup>7,8</sup>

Label	A	s	I	$J^\pi$	Local SU(3) irreps	This work
$N$	1	0	1/2	1/2 <sup>+</sup>	8	8
$\Lambda$	1	-1	0	1/2 <sup>+</sup>	8	8
$\Sigma$	1	-1	1	1/2 <sup>+</sup>	8	8
$\Xi$	1	-2	1/2	1/2 <sup>+</sup>	8	8
$d$	2	0	0	1 <sup>+</sup>	$\overline{10}$	$\overline{10}$
$nn$	2	0	1	0 <sup>+</sup>	27	27
$n\Lambda$	2	-1	1/2	0 <sup>+</sup>	27	27
$n\Lambda$	2	-1	1/2	1 <sup>+</sup>	$8_A, \overline{10}$	-
$n\Sigma$	2	-1	3/2	0 <sup>+</sup>	27	27
$n\Sigma$	2	-1	3/2	1 <sup>+</sup>	10	10
$n\Xi$	2	-2	0	1 <sup>+</sup>	$8_A$	$8_A$
$n\Xi$	2	-2	1	1 <sup>+</sup>	$8_A, 10, \overline{10}$	-
$H$	2	-2	0	0 <sup>+</sup>	1, 27	1, 27
${}^3\text{H}, {}^3\text{H}$	3	0	1/2	1/2 <sup>+</sup>	$\overline{35}$	$\overline{35}$
${}^3\text{H}(1/2^+)$	3	-1	0	1/2 <sup>+</sup>	$\overline{35}$	-
${}^3_{\Lambda}H(3/2^+)$	3	-1	0	3/2 <sup>+</sup>	$\overline{10}$	$\overline{10}$
${}^3\text{He}, {}^3_{\Lambda}H, nn\Lambda$	3	-1	1	1/2 <sup>+</sup>	27, 35	27, 35
${}^3_{\Sigma}\text{He}$	3	-1	1	3/2 <sup>+</sup>	27	27
${}^4\text{He}$	4	0	0	0 <sup>+</sup>	$\overline{28}$	$\overline{28}$
${}^4_{\Lambda}\text{He}, {}^4_{\Lambda}\text{H}$	4	-1	1/2	0 <sup>+</sup>	$\overline{28}$	-
${}^4_{\Lambda\Lambda}\text{He}$	4	-2	1	0 <sup>+</sup>	27, $\overline{28}$	27, $\overline{28}$
$\Lambda\Xi^0 pnn$	5	-3	0	3/2 <sup>+</sup>	$\overline{10} + \dots$	$\overline{10}$

Favors spin 3/2

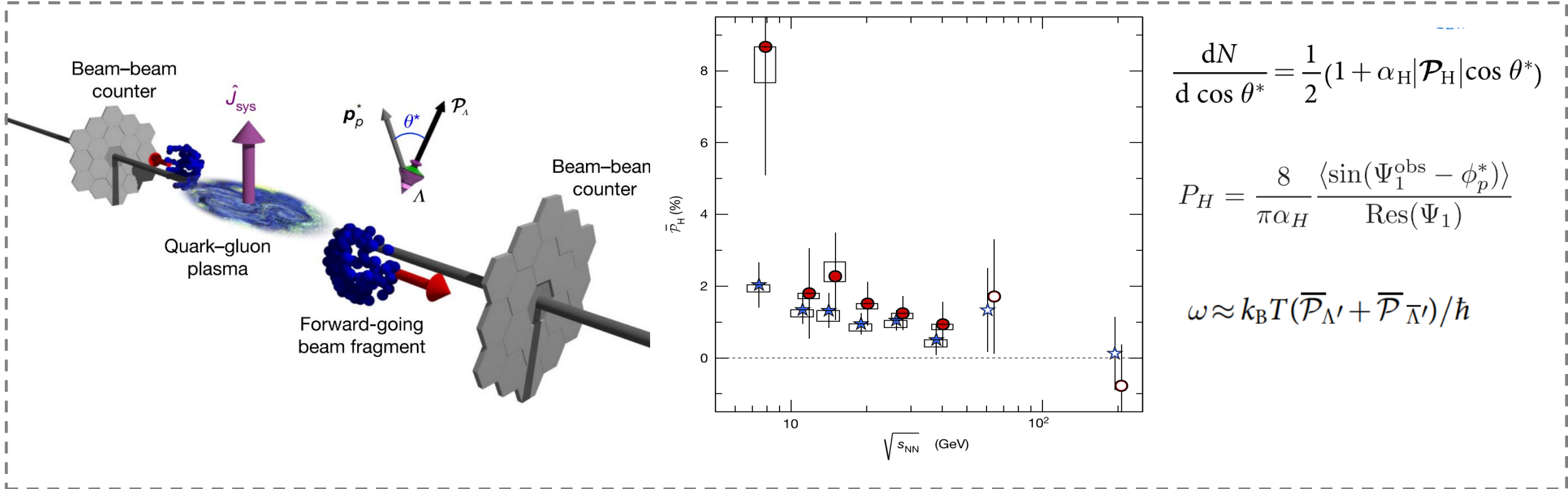
# 2. Polarization of hadrons in relativistic heavy-ion collisions

(6)

STAR, Nature 548, 62 (2017)

Z. T. Liang and X. N. Wang PRL 94, 102301 (2005)

F. Becattini, F. Piccinini, and J. Rizzo, PRC 77, 024906 (2008)



Spin polarization of Lambda hyperon



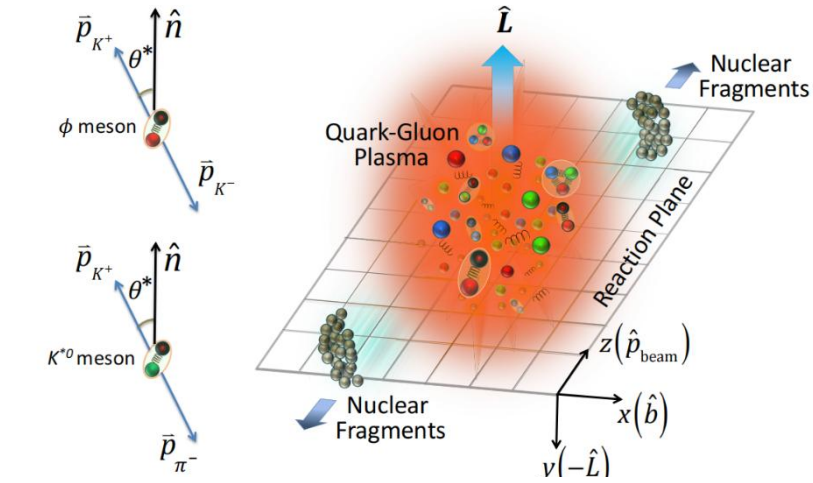
Vorticity of QGP



# 2. Polarization of hadrons in relativistic heavy-ion collisions

(7)

STAR, Nature 614, 7947 (2023)



## Spin alignment of mesons

$$\frac{dN}{d(\cos\theta^*)} \propto (1 - \rho_{00}) + (3\rho_{00} - 1)\cos^2\theta^*$$



## Fluctuation/correlation of strong force field

X. L. Sheng et al., PRL 131, 042304 (2023)

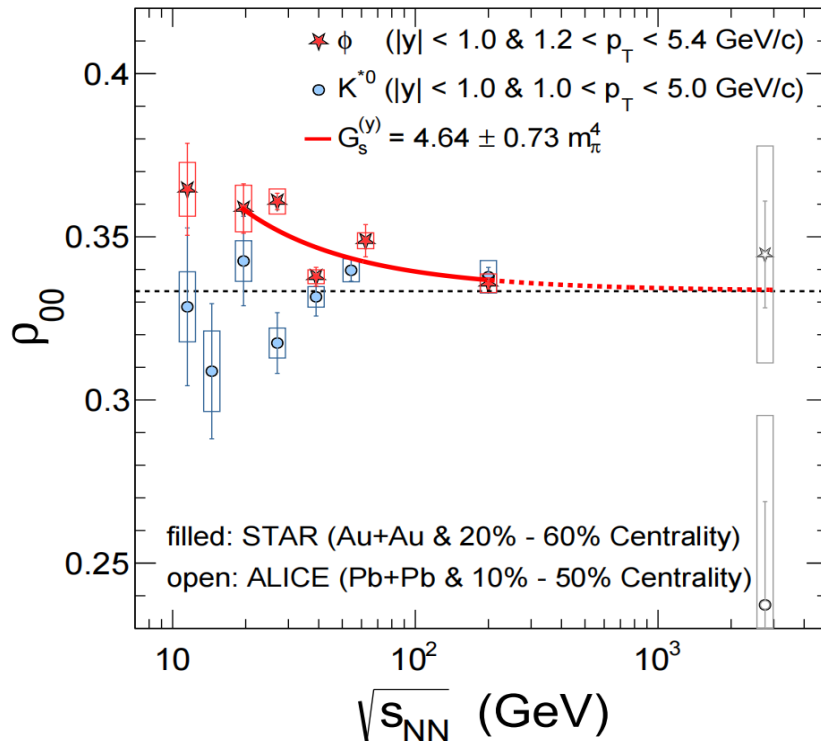
$$G_s^{(y)} \equiv g_\phi^2 \left[ 3\langle B_{\phi,y}^2 \rangle + \frac{\langle \mathbf{p}^2 \rangle_\phi}{m_s^2} \langle E_{\phi,y}^2 \rangle - \frac{3}{2} \langle B_{\phi,x}^2 + B_{\phi,z}^2 \rangle - \frac{\langle \mathbf{p}^2 \rangle_\phi}{2m_s^2} \langle E_{\phi,x}^2 + E_{\phi,z}^2 \rangle \right]$$

## Quark-antiquark spin correlation

J. P. Lv et al., arXiv:2402.13721

## Meson spectral property

F. Li and S. Liu, arXiv:2206.11890



# 2. Polarization of light (anti-)(hyper-)nuclei

(8)

## Unstable hadrons

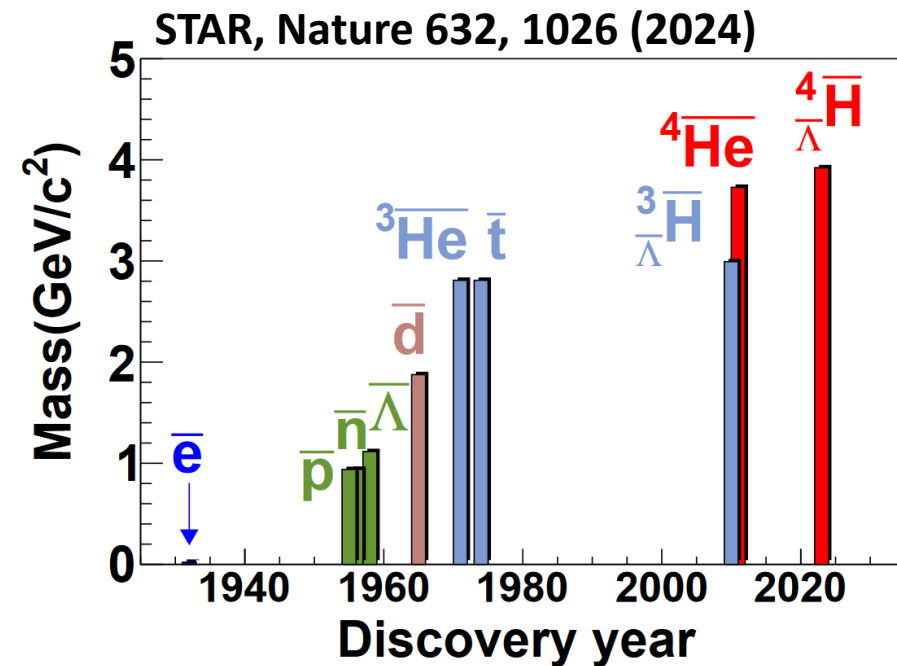
$\Lambda(uds)$   $\Xi(uss)$   $\Omega(sss)$   
 $\phi(s\bar{s})$   $K^{*0}(d\bar{s})$   $\rho^+(u\bar{d})$   
 $J/\psi(c\bar{c})$  ...

## Unstable (anti-)(hyper-)nuclei

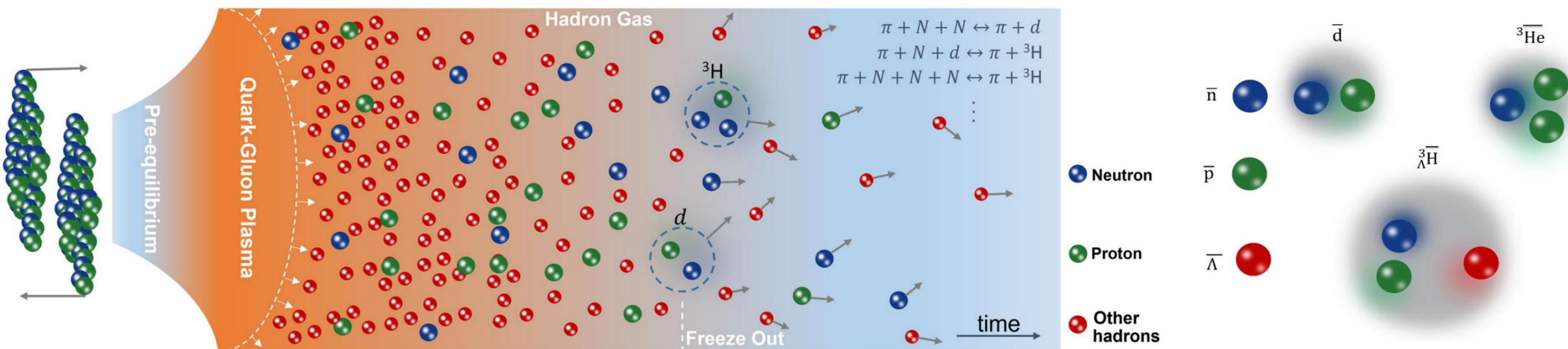
${}^3_{\Lambda}\text{H}(np\Lambda)$   ${}^4\text{Li}(nppp)$   
 ${}^3_{\Lambda}\bar{\text{H}}(\bar{n}\bar{p}\bar{\Lambda})$   ${}^4\bar{\text{Li}}(\bar{n}\bar{p}\bar{p}\bar{p})$   
 ...

## Stable (anti-)nuclei

$d(np)$   ${}^3\text{He}(npp)$   
 $\bar{d}(\bar{n}\bar{p})$   ${}^3\bar{\text{He}}(\bar{n}\bar{p}\bar{p})$   
 ...



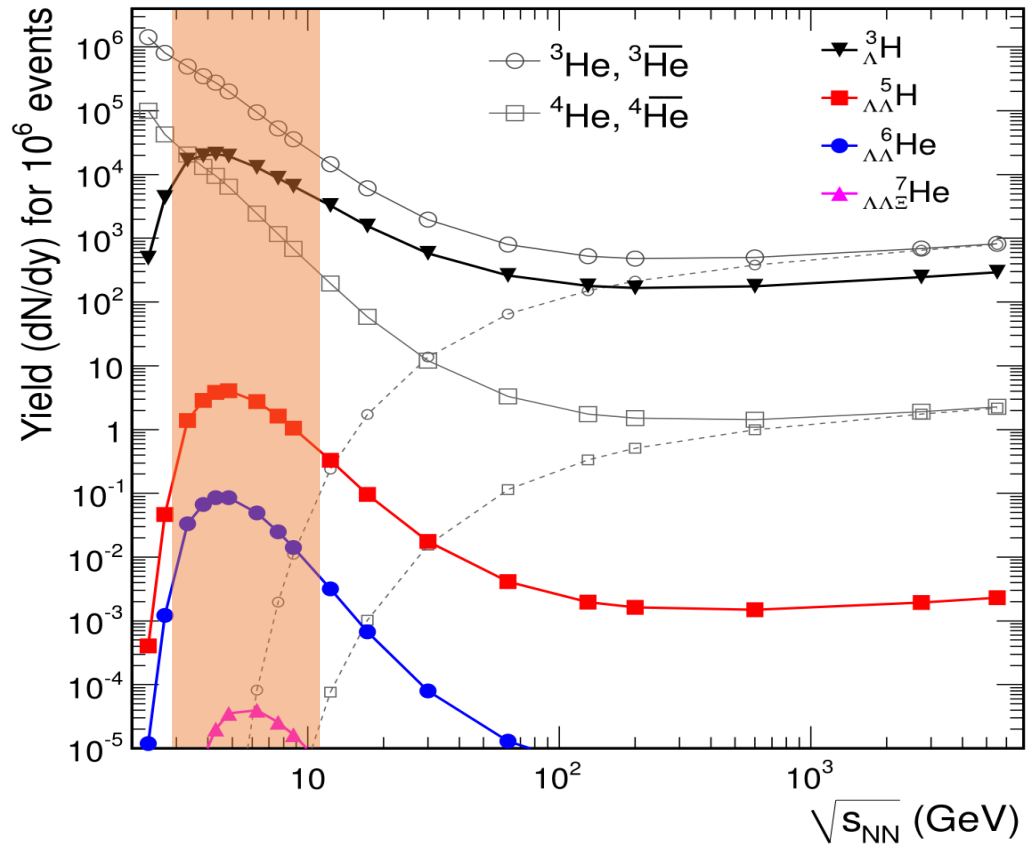
K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, C. Shen, Nature Commun. 15, 1074 (2024)



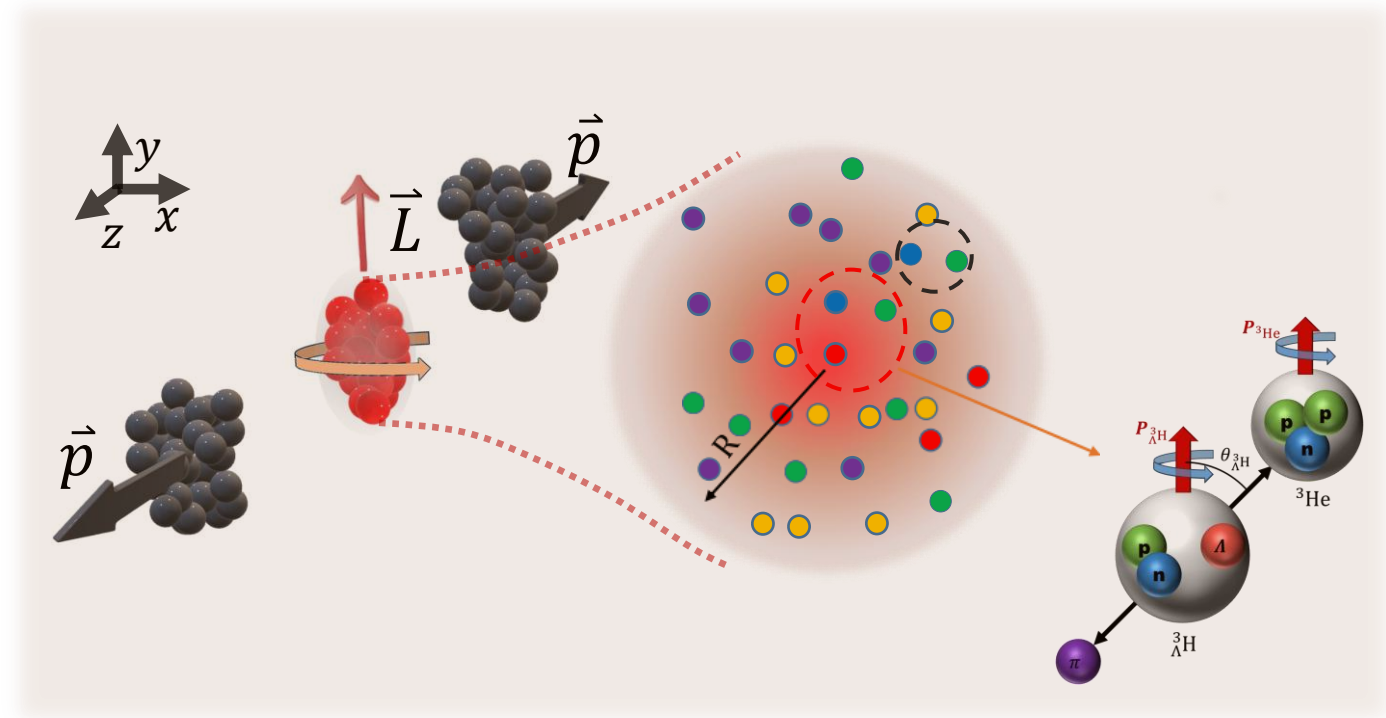
# 2. Polarization of light (anti-)(hyper-)nuclei

(9)

A. Andronic et al., Phys. Lett. B 697, 203-207 (2011)



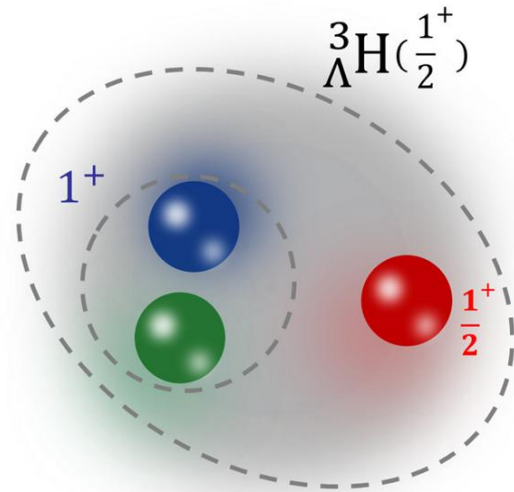
- FAIR/CBM (2.4-4.9 GeV)
- HIAF/CEE (2.1-4.5 GeV)
- NICA/MPD (4-11 GeV)



A novel tool to study the evolution of strongly-interacting matter at high-baryon density region

### 3. (Anti-)hypertriton polarization and its spin structure

(10)



$$\begin{aligned} |\frac{1}{2}, \uparrow\rangle_{\Lambda\text{H}} &= \frac{\sqrt{6}}{3} |\frac{1}{2}, \frac{1}{2}\rangle_n |\frac{1}{2}, \frac{1}{2}\rangle_p |\frac{1}{2}, -\frac{1}{2}\rangle_{\Lambda} \\ &- \frac{\sqrt{6}}{6} (|\frac{1}{2}, \frac{1}{2}\rangle_n |\frac{1}{2}, -\frac{1}{2}\rangle_p |\frac{1}{2}, \frac{1}{2}\rangle_{\Lambda} \\ &+ |\frac{1}{2}, -\frac{1}{2}\rangle_n |\frac{1}{2}, \frac{1}{2}\rangle_p |\frac{1}{2}, \frac{1}{2}\rangle_{\Lambda}), \end{aligned}$$

$$\begin{aligned} |\frac{1}{2}, \downarrow\rangle_{\Lambda\text{H}} &= -\frac{\sqrt{6}}{3} |\frac{1}{2}, -\frac{1}{2}\rangle_n |\frac{1}{2}, -\frac{1}{2}\rangle_p |\frac{1}{2}, \frac{1}{2}\rangle_{\Lambda} \\ &+ \frac{\sqrt{6}}{6} (|\frac{1}{2}, \frac{1}{2}\rangle_n |\frac{1}{2}, -\frac{1}{2}\rangle_p |\frac{1}{2}, -\frac{1}{2}\rangle_{\Lambda} \\ &+ |\frac{1}{2}, -\frac{1}{2}\rangle_n |\frac{1}{2}, \frac{1}{2}\rangle_p |\frac{1}{2}, -\frac{1}{2}\rangle_{\Lambda}). \end{aligned}$$

Coalescence model for hypertriton production (without baryon spin correlation)

$$E_i \frac{d^3 N_{i, \pm \frac{1}{2}}}{d\mathbf{p}_i^3} = \int_{\Sigma^\mu} d^3 \sigma_\mu p_i^\mu w_{i, \pm \frac{1}{2}}(\mathbf{x}_i, \mathbf{p}_i) \bar{f}_i(\mathbf{x}_i, \mathbf{p}_i)$$

$$w_{i, \pm \frac{1}{2}} = \frac{1}{2} [1 \pm \mathcal{P}_i(\mathbf{x}_i, \mathbf{p}_i)]$$

$$\hat{\rho}_i = \text{diag} \left( \frac{1 + \mathcal{P}_i}{2}, \frac{1 - \mathcal{P}_i}{2} \right)$$

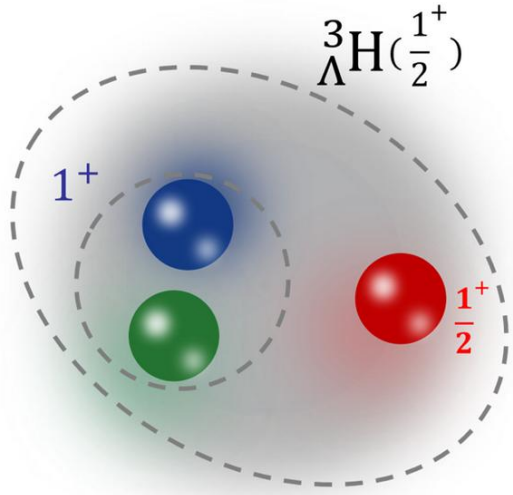
$$\bar{f}_i = \frac{g_i}{(2\pi)^3} [\exp(p_i^\mu u_\mu / T) / \xi_i + 1]^{-1}$$

$$\hat{\rho}_{np\Lambda} = \hat{\rho}_n \otimes \hat{\rho}_p \otimes \hat{\rho}_\Lambda$$

$$\begin{aligned} E \frac{d^3 N_{\Lambda\text{H}, \pm \frac{1}{2}}}{d\mathbf{P}^3} &= E \int \prod_{i=n,p,\Lambda} p_i^\mu d^3 \sigma_\mu \frac{d^3 p_i}{E_i} \bar{f}_i(\mathbf{x}_i, \mathbf{p}_i) \\ &\times \left( \frac{2}{3} w_{n, \pm \frac{1}{2}} w_{p, \pm \frac{1}{2}} w_{\Lambda, \mp \frac{1}{2}} + \frac{1}{6} w_{n, \pm \frac{1}{2}} w_{p, \mp \frac{1}{2}} w_{\Lambda, \pm \frac{1}{2}} \right. \\ &\quad \left. + \frac{1}{6} w_{n, \mp \frac{1}{2}} w_{p, \pm \frac{1}{2}} w_{\Lambda, \pm \frac{1}{2}} \right) \\ &\times W_{\Lambda\text{H}}(\mathbf{x}_n, \mathbf{x}_p, \mathbf{x}_\Lambda; \mathbf{p}_n, \mathbf{p}_p, \mathbf{p}_\Lambda) \delta(\mathbf{P} - \sum_i \mathbf{p}_i) \end{aligned}$$

### 3. (Anti-)hypertriton polarization and its spin structure

(11)



$$\begin{aligned}\mathcal{P}_{\Lambda\text{H}} &\approx \frac{\frac{2}{3}\mathcal{P}_n + \frac{2}{3}\mathcal{P}_p - \frac{1}{3}\mathcal{P}_{\Lambda} - \mathcal{P}_n\mathcal{P}_p\mathcal{P}_{\Lambda}}{1 - \frac{2}{3}(\mathcal{P}_n + \mathcal{P}_p)\mathcal{P}_{\Lambda} + \frac{1}{3}\mathcal{P}_n\mathcal{P}_p} \\ &\approx \frac{2}{3}\mathcal{P}_n + \frac{2}{3}\mathcal{P}_p - \frac{1}{3}\mathcal{P}_{\Lambda} \\ &\approx \mathcal{P}_{\Lambda}\end{aligned}$$

# 3. (Anti-)hypertriton polarization and its spin structure

(12)

K. J. Sun et al., arXiv:2405.12015(2024)

## Parity-violating weak decay

### $\Lambda$ hyperons

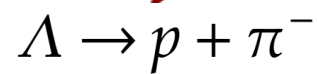
### Hypertriton

$\vec{S}_\Lambda^*$   
 $\vec{p}_p^*$   
 $\vec{p}_\pi^*$   
 $\theta^*$

$\mathcal{P}_H$ :  $\Lambda$  polarization  
 $\theta^*$ : angle between proton momentum and  $\Lambda$  spin  
 $\alpha_H$ :  $\Lambda$  decay parameter  
 $\alpha_\Lambda = 2\text{Re}(T_s^* T_p) = 0.732 \pm 0.01$   
 BESIII, Phys. Rev. Lett. 129, 131801 (2022)

$$\frac{dN}{d \cos \theta^*} = \text{Tr}[T^+ \hat{\rho} T]$$

$$\rho_\Lambda = \begin{pmatrix} \frac{1 + \mathcal{P}_\Lambda}{2} & \\ & \frac{1 - \mathcal{P}_\Lambda}{2} \end{pmatrix}$$



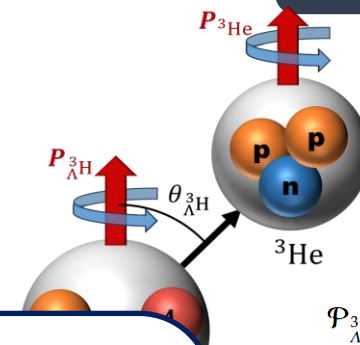
### The transition matrix

$$T(\Lambda \rightarrow \pi^- + p) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} T_s + T_p \cos \theta_p^* & T_p \sin \theta_p^* e^{-i\phi_p^*} \\ T_p \sin \theta_p^* e^{-i\phi_p^*} & T_s - T_p \cos \theta_p^* \end{pmatrix}$$

### The angular distribution

$$\frac{dN}{d \cos \theta^*} = \frac{1}{2} (1 + \alpha_H |\mathcal{P}_H| \cos \theta^*)$$

H denotes  $\Lambda$  and  $\bar{\Lambda}$



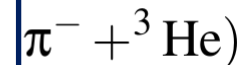
$$\rho_{\Lambda^3\text{H}} = \begin{pmatrix} \frac{1 + \mathcal{P}_{\Lambda^3\text{H}}}{2} & \\ & \frac{1 - \mathcal{P}_{\Lambda^3\text{H}}}{2} \end{pmatrix}$$

$\mathcal{P}_{\Lambda^3\text{H}}$ :  ${}^3\text{H}$  polarization  
 $\theta^*$ : Angle between  ${}^3\text{He}$  momentum and  ${}^3\text{H}$  rest frame  
 $\alpha_{\Lambda^3\text{H}}$ :  ${}^3\text{H}$  decay parameter

$$\alpha_{\Lambda^3\text{H}} \approx -\frac{1}{3T_s^2 + \frac{1}{3}T_p^2} \alpha_\Lambda$$

$$\approx -\frac{1}{2.58} \alpha_\Lambda$$

**Sign flip !**



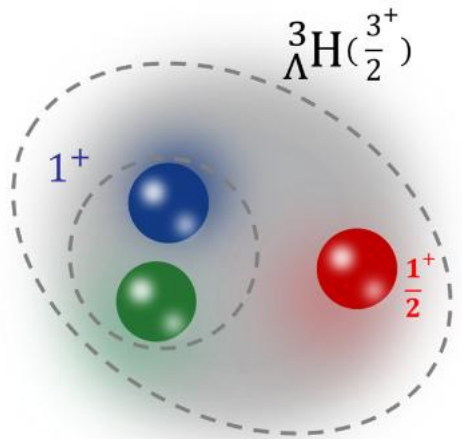
$$\begin{pmatrix} 3T_s - T_p \cos \theta^* & -T_p \sin \theta^* e^{i\phi^*} \\ -T_p \sin \theta^* e^{-i\phi^*} & 3T_s + T_p \cos \theta^* \end{pmatrix}$$

$$\frac{dN}{d \cos \theta^*} = \frac{1}{2} (1 + \alpha_{\Lambda^3\text{H}} \mathcal{P}_{\Lambda^3\text{H}} \cos \theta^*)$$



# 3. (Anti-)hypertriton polarization and its spin structure

(13)

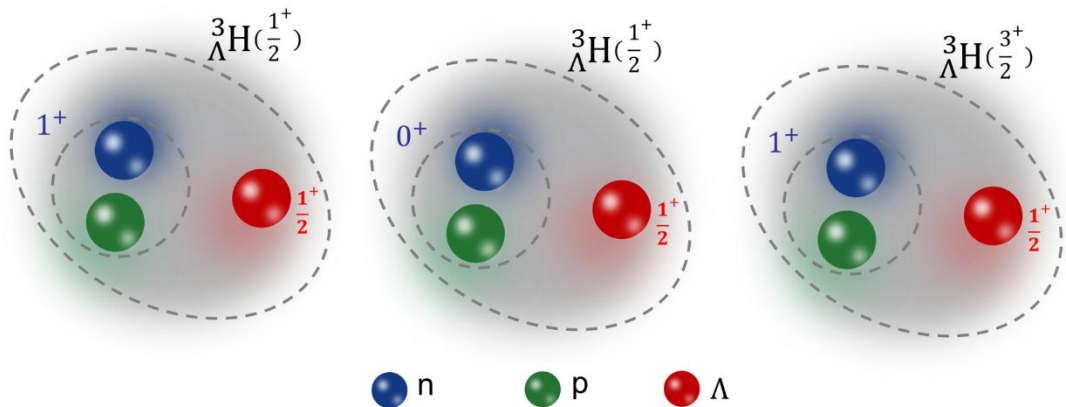


$$\hat{\rho}_{\Lambda}^{{}^3\text{H}} \approx \text{diag} \left[ \frac{(1 + \mathcal{P}_{\Lambda})^3}{4(1 + \mathcal{P}_{\Lambda}^2)}, \frac{(1 - \mathcal{P}_{\Lambda})(1 + \mathcal{P}_{\Lambda})^2}{4(1 + \mathcal{P}_{\Lambda}^2)}, \right. \\ \left. \frac{(1 - \mathcal{P}_{\Lambda})^2(1 + \mathcal{P}_{\Lambda})}{4(1 + \mathcal{P}_{\Lambda}^2)}, \frac{(1 - \mathcal{P}_{\Lambda})^3}{4(1 + \mathcal{P}_{\Lambda}^2)} \right]$$

$$T({}^3_{\Lambda}\text{H} \rightarrow \pi^- + {}^3\text{He}) = \frac{FT_p}{\sqrt{6\pi}} \begin{pmatrix} e^{i\phi^*} \sin\theta^* & 0 \\ -\frac{2}{\sqrt{3}} \cos\theta^* & \frac{e^{i\phi^*} \sin\theta^*}{\sqrt{3}} \\ -\frac{e^{-i\phi^*} \sin\theta^*}{\sqrt{3}} & -\frac{2}{\sqrt{3}} \cos\theta^* \\ 0 & -e^{-i\phi^*} \sin\theta^* \end{pmatrix}$$

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2} \left[ 1 + \left( \hat{\rho}_{\frac{1}{2}, \frac{1}{2}} + \hat{\rho}_{-\frac{1}{2}, -\frac{1}{2}} - \frac{1}{2} \right) (3\cos^2\theta^* - 1) \right]$$

$$\hat{\rho}_{\frac{1}{2}, \frac{1}{2}} + \hat{\rho}_{-\frac{1}{2}, -\frac{1}{2}} - \frac{1}{2} \approx -\frac{\mathcal{P}_{\Lambda}^2}{1 + \mathcal{P}_{\Lambda}^2} \approx -\mathcal{P}_{\Lambda}^2$$



$J^P$	structure	decay mode	$\frac{dN}{d\cos\theta^*}$
$\frac{1}{2}^+$	$\Lambda(\frac{1}{2}^+) - np(1^+)$	${}^3_{\Lambda}\text{H} \rightarrow \pi^- + {}^3\text{He}$	$\frac{1}{2} (1 - \frac{1}{2.58} \alpha_{\Lambda} \mathcal{P}_{\Lambda} \cos\theta^*)$
$\frac{1}{2}^+$	$\Lambda(\frac{1}{2}^+) - np(0^+)$	${}^3_{\Lambda}\text{H} \rightarrow \pi^- + {}^3\text{He}$	$\frac{1}{2} (1 + \alpha_{\Lambda} \mathcal{P}_{\Lambda} \cos\theta^*)$
$\frac{3}{2}^+$	$\Lambda(\frac{1}{2}^+) - np(1^+)$	${}^3_{\Lambda}\text{H} \rightarrow \pi^- + {}^3\text{He}$	$\frac{1}{2} (1 - \mathcal{P}_{\Lambda}^2 (3\cos^2\theta^* - 1))$
$\frac{1}{2}^-$	$\bar{\Lambda}(\frac{1}{2}^-) - \bar{n}\bar{p}(1^-)$	${}^3_{\Lambda}\bar{\text{H}} \rightarrow \pi^+ + {}^3\bar{\text{He}}$	$\frac{1}{2} (1 - \frac{1}{2.58} \alpha_{\bar{\Lambda}} \mathcal{P}_{\bar{\Lambda}} \cos\theta^*)$
$\frac{1}{2}^-$	$\bar{\Lambda}(\frac{1}{2}^-) - \bar{n}\bar{p}(0^-)$	${}^3_{\Lambda}\bar{\text{H}} \rightarrow \pi^+ + {}^3\bar{\text{He}}$	$\frac{1}{2} (1 + \alpha_{\bar{\Lambda}} \mathcal{P}_{\bar{\Lambda}} \cos\theta^*)$
$\frac{3}{2}^-$	$\bar{\Lambda}(\frac{1}{2}^-) - \bar{n}\bar{p}(1^-)$	${}^3_{\Lambda}\bar{\text{H}} \rightarrow \pi^+ + {}^3\bar{\text{He}}$	$\frac{1}{2} (1 - \mathcal{P}_{\bar{\Lambda}}^2 (3\cos^2\theta^* - 1))$

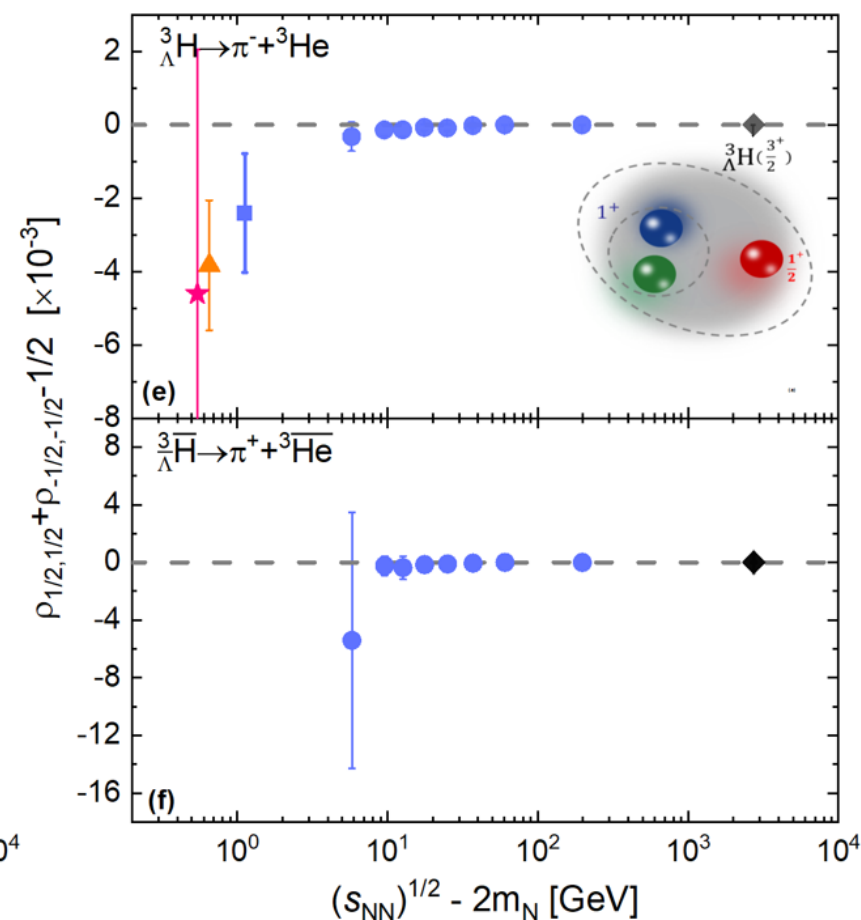
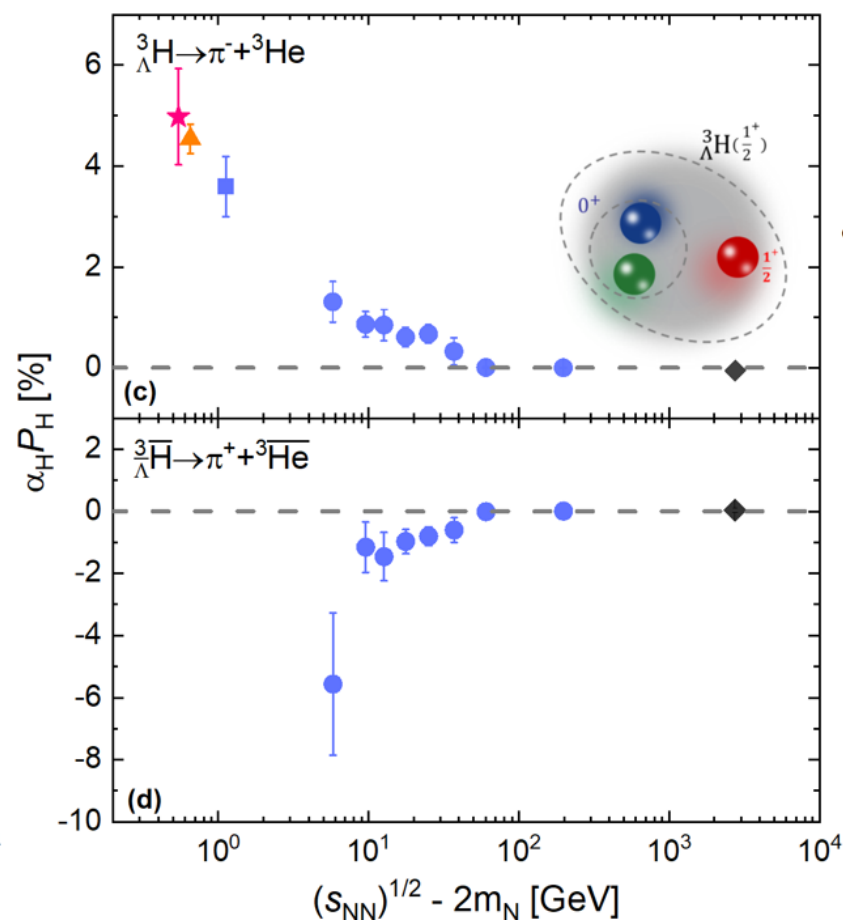
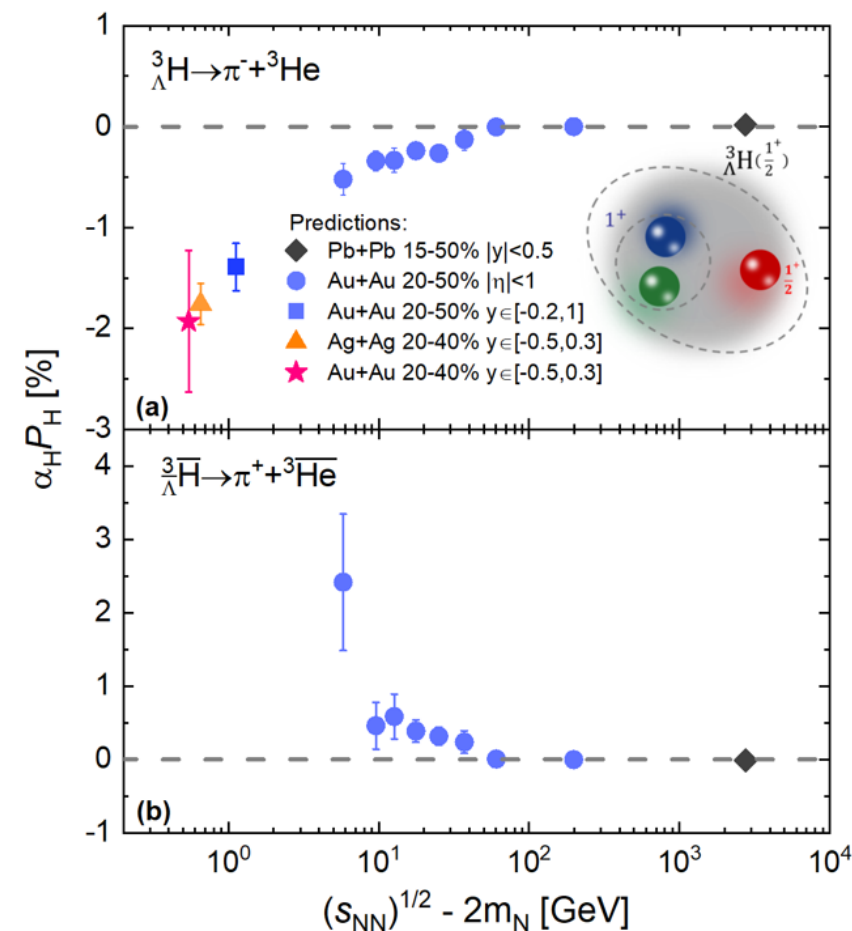
# 3. (Anti-)hypertriton polarization and its spin structure

(14)

The measurement of hypertriton polarization provides a novel method to uniquely determine its internal spin structure

$$\alpha_{\Lambda^3\text{H}} \approx -\frac{1}{2.58} \alpha_{\Lambda}$$

$$\alpha_{\Lambda^3\text{H}} \approx \alpha_{\Lambda}$$



# 4. Effects of baryon spin correlation

(15)

$$\begin{aligned}
 \hat{\rho}_{np\Lambda} &= \hat{\rho}_n \otimes \hat{\rho}_p \otimes \hat{\rho}_\Lambda + \frac{1}{2^2} (c_{np}^{\alpha\beta} \hat{\sigma}_{n,\alpha} \otimes \hat{\sigma}_{p,\beta} \otimes \hat{\rho}_\Lambda \\
 &\quad + c_{p\Lambda}^{\alpha\beta} \hat{\sigma}_{p,\alpha} \otimes \hat{\sigma}_{\Lambda,\beta} \otimes \hat{\rho}_n + c_{n\Lambda}^{\alpha\beta} \hat{\sigma}_{n,\alpha} \otimes \hat{\sigma}_{\Lambda,\beta} \otimes \hat{\rho}_p) \\
 &\quad + \frac{1}{2^3} c_{np\Lambda}^{\alpha\beta\gamma} \hat{\sigma}_{n,\alpha} \otimes \hat{\sigma}_{p,\beta} \otimes \hat{\sigma}_{\Lambda,\gamma}, \\
 \mathcal{P}_{\Lambda H}^3 &\approx \frac{\frac{2}{3}\langle\mathcal{P}_n\rangle + \frac{2}{3}\langle\mathcal{P}_p\rangle - \frac{1}{3}\langle\mathcal{P}_\Lambda\rangle - \langle\mathcal{P}_n\mathcal{P}_p\mathcal{P}_\Lambda\rangle + C_-}{1 - \frac{2}{3}(\langle(\mathcal{P}_n + \mathcal{P}_p)\mathcal{P}_\Lambda\rangle) + \frac{1}{3}\langle\mathcal{P}_n\mathcal{P}_p\rangle + C_+} \\
 C_- &= -\frac{1}{4}(\langle c_{np}^{zz}\mathcal{P}_\Lambda\rangle + \langle c_{p\Lambda}^{zz}\mathcal{P}_n\rangle + \langle c_{n\Lambda}^{zz}\mathcal{P}_p\rangle) - \frac{1}{4}\langle c_{np\Lambda}^{zzz}\rangle, \\
 C_+ &= \frac{1}{12}(\langle c_{np}^{zz}\rangle - 2\langle c_{p\Lambda}^{zz}\rangle - 2\langle c_{n\Lambda}^{zz}\rangle).
 \end{aligned}$$

**'genuine' correlation terms**

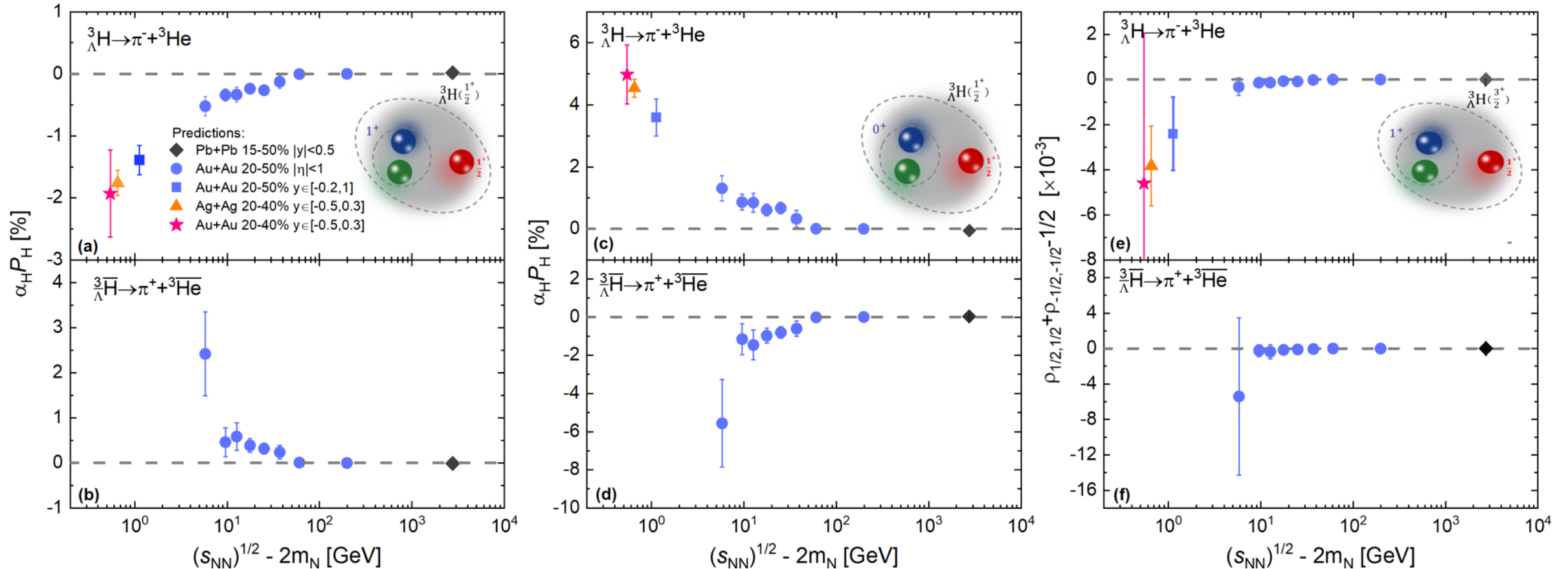
## Induced correlations

We can express the polarization of a particle as  $\mathcal{P} = \langle\mathcal{P}\rangle + \delta\mathcal{P}$  with  $\delta\mathcal{P}$  denoting its space and momentum dependent fluctuations, which leads to the relations  $\langle\mathcal{P}_n\mathcal{P}_p\rangle = \langle\mathcal{P}_n\rangle\langle\mathcal{P}_p\rangle + \langle\delta\mathcal{P}_n\delta\mathcal{P}_p\rangle$  and  $\langle\mathcal{P}_n\mathcal{P}_p\mathcal{P}_\Lambda\rangle = \langle\mathcal{P}_n\rangle\langle\mathcal{P}_p\rangle\langle\mathcal{P}_\Lambda\rangle + \langle\delta\mathcal{P}_n\delta\mathcal{P}_p\rangle\langle\mathcal{P}_\Lambda\rangle + \langle\delta\mathcal{P}_n\delta\mathcal{P}_\Lambda\rangle\langle\mathcal{P}_p\rangle + \langle\delta\mathcal{P}_p\delta\mathcal{P}_\Lambda\rangle\langle\mathcal{P}_n\rangle + \langle\delta\mathcal{P}_n\delta\mathcal{P}_p\delta\mathcal{P}_\Lambda\rangle$ . Assuming again  $\langle\mathcal{P}_n\rangle \approx \langle\mathcal{P}_p\rangle \approx \langle\mathcal{P}_\Lambda\rangle$  and neglecting the three-body correlation, we then have

$$\mathcal{P}_{\Lambda H}^3 \approx (1 - \langle\delta\mathcal{P}_n\delta\mathcal{P}_p\rangle - \langle\delta\mathcal{P}_p\delta\mathcal{P}_\Lambda\rangle - \langle\delta\mathcal{P}_n\delta\mathcal{P}_\Lambda\rangle)\langle\mathcal{P}_\Lambda\rangle.$$

This result suggests that it is possible to extract the information on the spin-spin correlations among nucleons and  $\Lambda$  hyperons from the measurement of hypertriton polarization in heavy-ion collisions, although it is non-trivial in practice.

- (Anti-)hypertriton is globally polarized in non-central heavy-ion collisions.
- (Anti-)hypertriton polarization and its decay pattern provide a novel method to uniquely determine the spin structure of its wavefunction.



**Backup**

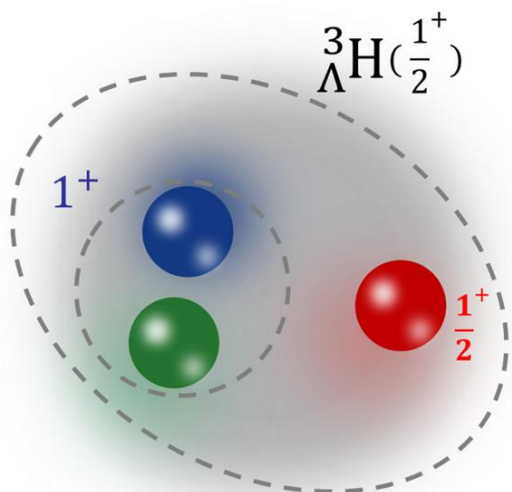
# 3. (Anti-)hypertriton polarization and its spin structure

Parity-violating weak decay:

$$T(\Lambda \rightarrow \pi^- + p) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} T_s + T_p \cos \theta_p^* & T_p \sin \theta_p^* e^{i\phi_p^*} \\ T_p \sin \theta_p^* e^{-i\phi_p^*} & T_s - T_p \cos \theta_p^* \end{pmatrix}$$

$$T({}^3_\Lambda\text{H} \rightarrow \pi^- + {}^3\text{He})$$

$$= \frac{F}{6\sqrt{\pi}} \begin{pmatrix} 3T_s - T_p \cos \theta^* & -T_p \sin \theta^* e^{i\phi^*} \\ -T_p \sin \theta^* e^{-i\phi^*} & 3T_s + T_p \cos \theta^* \end{pmatrix}$$



The normalized angular distribution of the  ${}^3\text{He}$  in the decay  ${}^3_\Lambda\text{H} \rightarrow \pi^- + {}^3\text{He}$  is given by

$$\frac{dN}{d \cos \theta^*} = \text{Tr}[T^+ \hat{\rho} T] = \frac{1}{2} (1 + \alpha_{{}^3_\Lambda\text{H}} \mathcal{P}_{{}^3_\Lambda\text{H}} \cos \theta^*), \quad (7)$$

in terms of the hypertriton decay parameter  $\alpha_{{}^3_\Lambda\text{H}} \approx -\frac{1}{3T_s^2 + \frac{1}{3}T_p^2} \alpha_\Lambda \approx -\frac{1}{2.58} \alpha_\Lambda$ . The angular distribution of  ${}^3\text{He}$  in the decay  ${}^3_\Lambda\text{H} \rightarrow \pi^- + {}^3\text{He}$  can thus be further expressed as

$$\frac{dN}{d \cos \theta^*} \approx \frac{1}{2} \left( 1 - \frac{1}{2.58} \alpha_\Lambda \mathcal{P}_\Lambda \cos \theta^* \right). \quad (8)$$

Compared to the angular distribution of the proton in the  $\Lambda$  decay, which has the form

**Sign flip !**

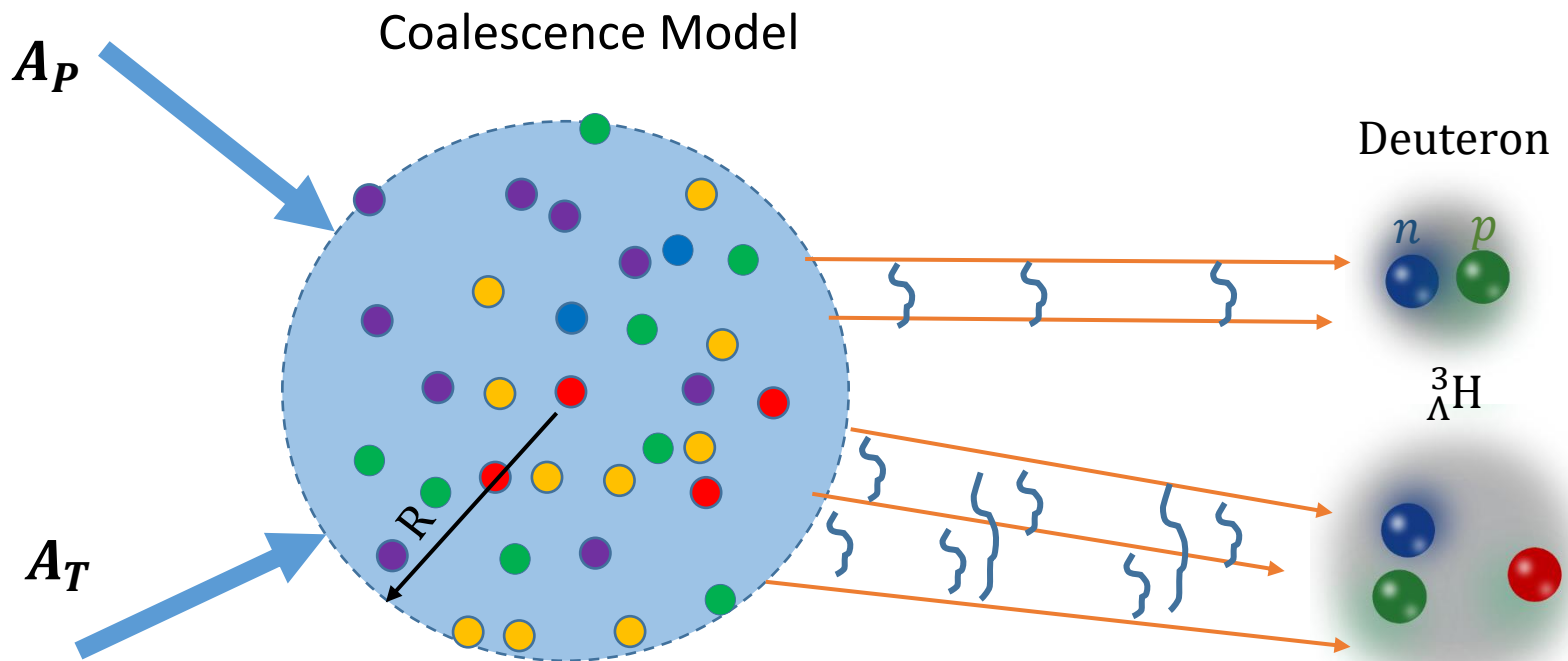
$$\frac{dN}{d \cos \theta_p^*} = \frac{1}{2} (1 + \alpha_\Lambda \mathcal{P}_\Lambda \cos \theta_p^*), \quad (9)$$

the  ${}^3\text{He}$  in  ${}^3_\Lambda\text{H}$  decay has an opposite sign in its angular dependence.



# 3. Final-state coalescence

(9)



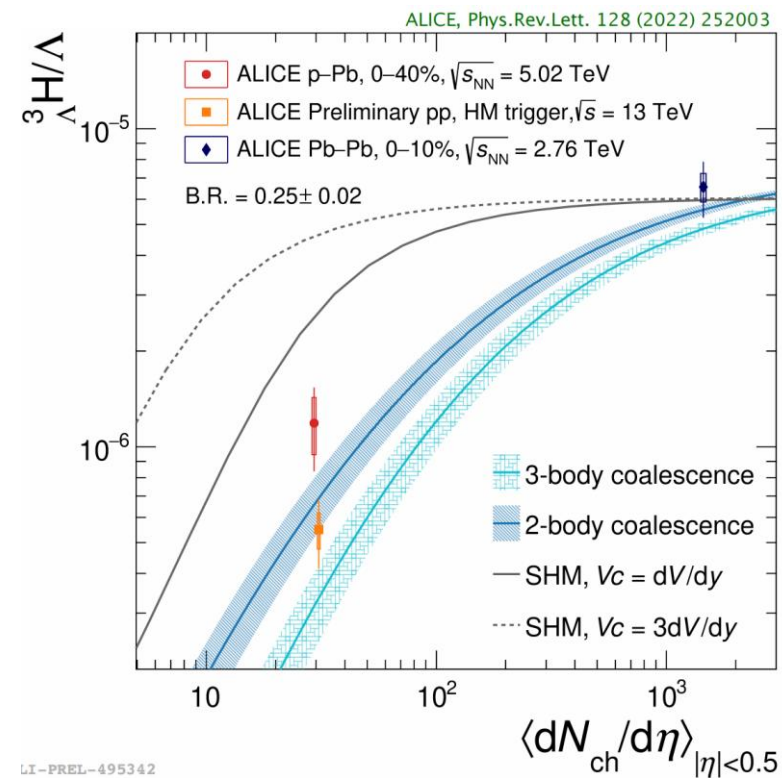
Density Matrix Formulation  
(sudden approximation)

$$N_A = Tr(\hat{\rho}_s \hat{\rho}_A)$$

$$= g_c \int d\Gamma \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

Wigner function of light cluster

Overlap between source distribution function and Wigner function of light nuclei

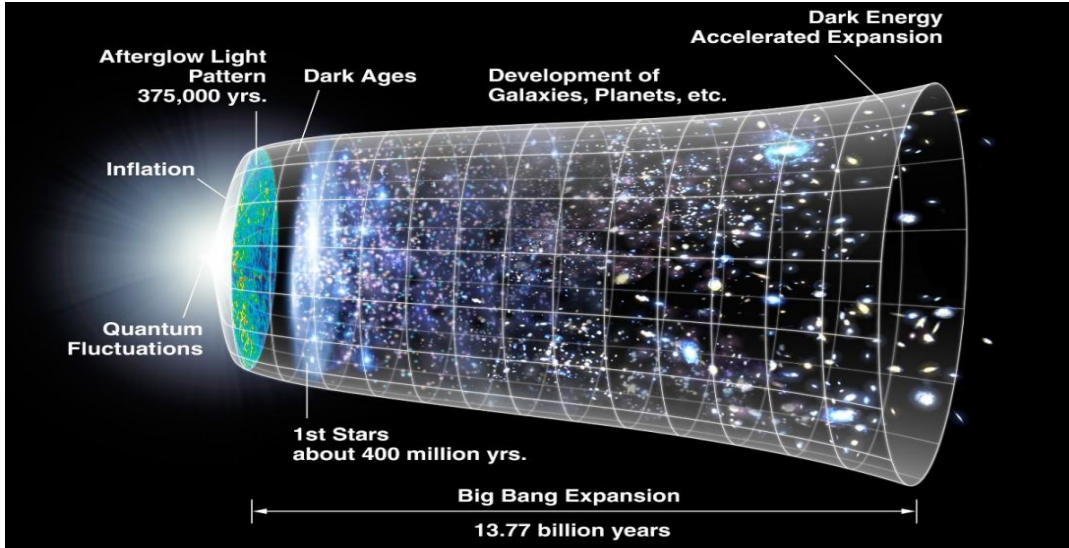


# 5. Little Bang Nucleosynthesis

Big-bang nucleosynthesis is responsible for the formation of light nuclei in our Universe.

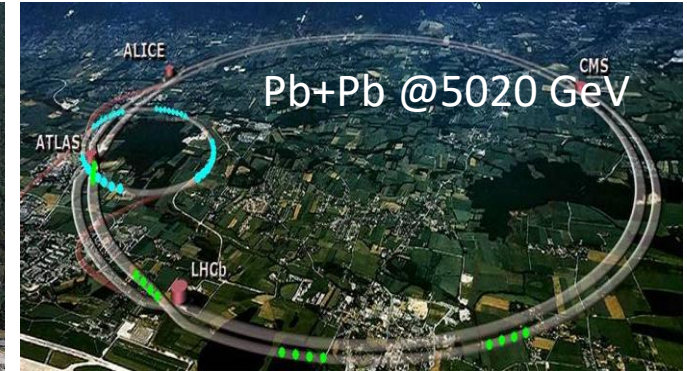
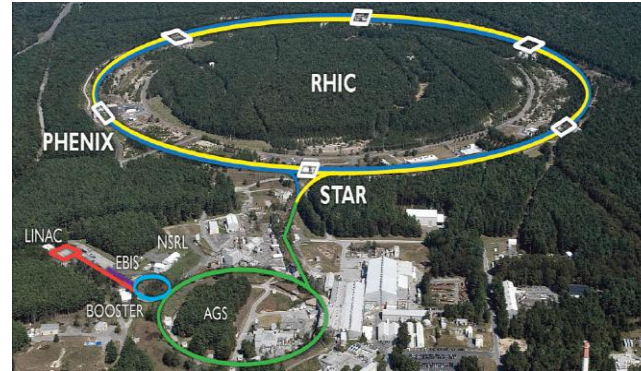
$$t \sim 100 \text{ s}, kT < 1 \text{ MeV}$$

K. A. Olive et al., Phys. Rept. 333, 389–407 (2000);  
 «The First Three Minutes» S. Weinberg

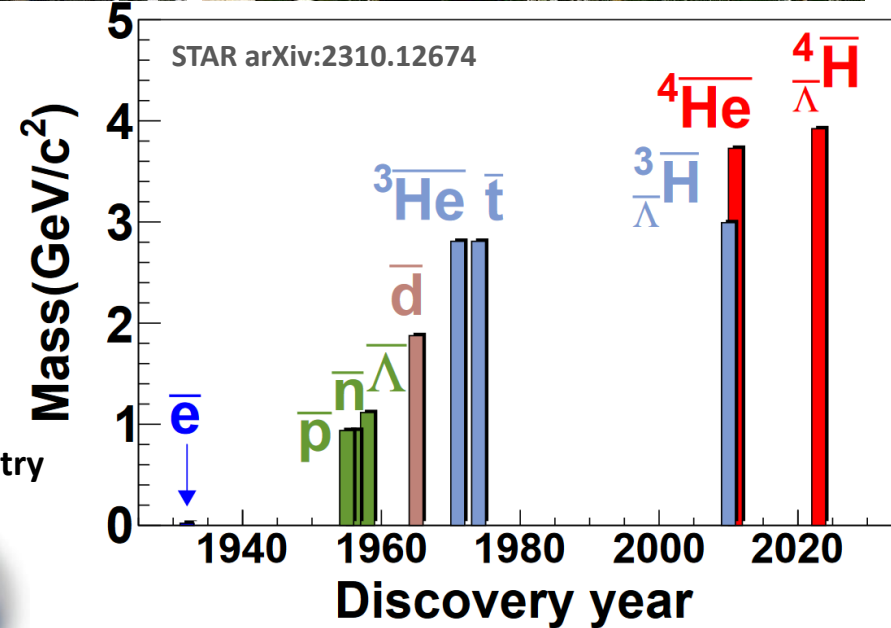


Synthesis of **antimatter** nuclei in little bangs of relativistic heavy-ion collisions

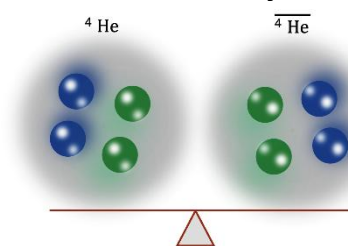
$$t \sim 10^{-22} \text{ s}, kT \sim 100 \text{ MeV}$$



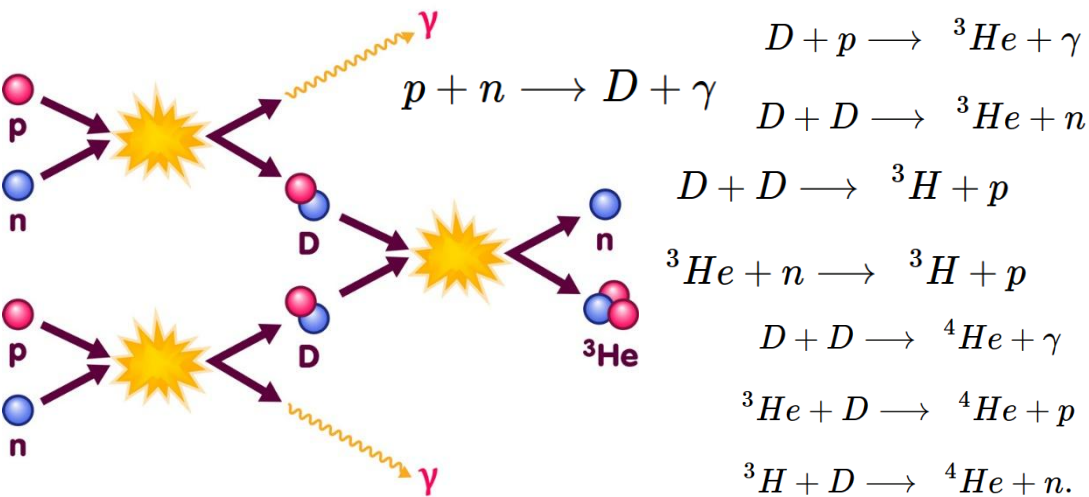
Antimatter factory



Matter-antimatter asymmetry

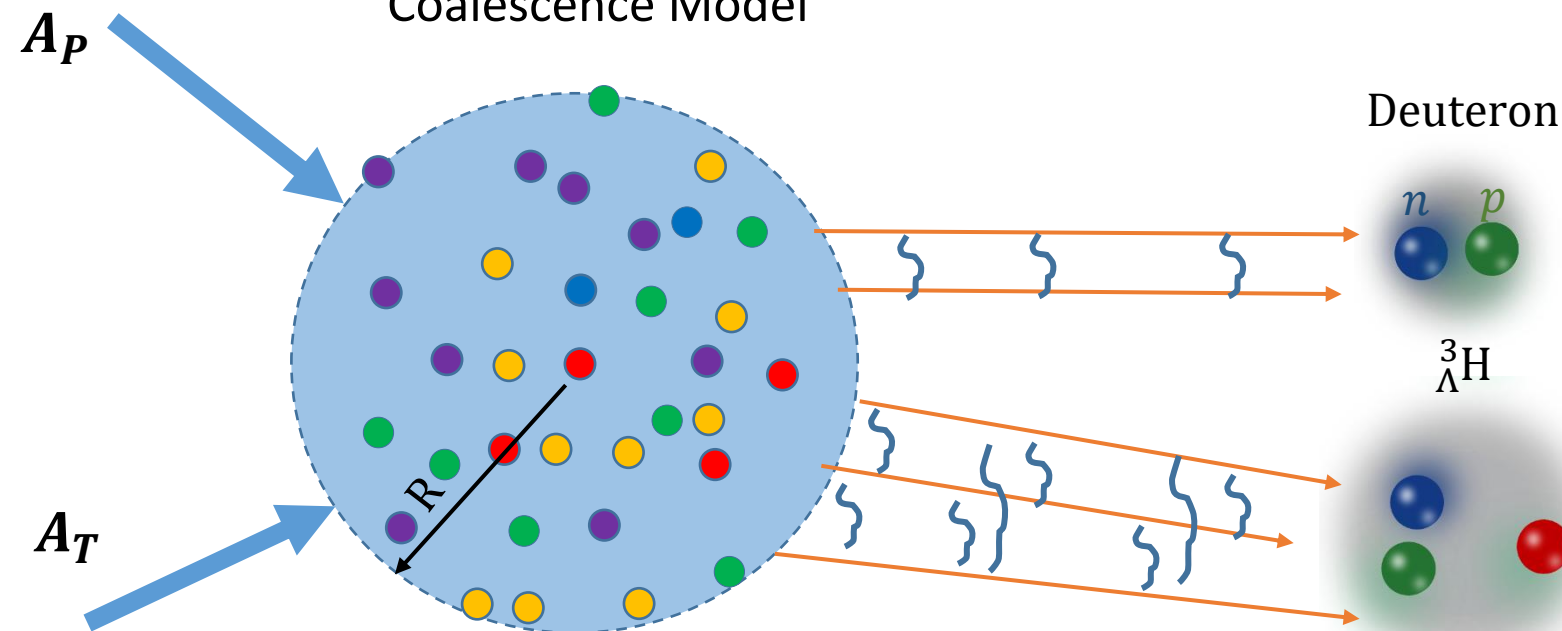


J. Chen et al., Phys. Rep. 760, 1 (2018);  
 P. Braun-Munzinger and B. Donigus NPA987, 144 (2019)



# 5. Final-state coalescence

## Coalescence Model



## Density Matrix Formulation (sudden approximation)

$$N_A = \text{Tr}(\hat{\rho}_s \hat{\rho}_A)$$

$$= g_c \int d\Gamma \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

Wigner function of light cluster

Overlap between source distribution function and Wigner function of light nuclei

Two-body coalescence  $a + b \rightarrow c$ :

$$N_c = \frac{2J_c + 1}{(2J_a + 1)(2J_b + 1)} \int \frac{dx_a dk_a}{(2\pi)^3} \frac{dx_b dk_b}{(2\pi)^3} f_a(x_a, k_a) f_b(x_b, k_b) W_c(x, k)$$

$$\approx \frac{2J_c + 1}{(2J_a + 1)(2J_b + 1)} \frac{N_a N_b}{\left(\frac{m_a m_b T}{m_a + m_b} (R_a^2 + R_b^2)\right)^{3/2}} \times \frac{1}{\left(1 + \frac{\sigma^2}{R_a^2 + R_b^2}\right)^{3/2}}$$

$$f_a = \frac{N_a}{(m_a T R_a^2)^{3/2}} e^{-\frac{k_a^2}{2m_a T} - \frac{x_a^2}{2R_a^2}}$$

$$W_c = 8e^{-x^2/\sigma^2 - \sigma^2 k^2}$$

$$N_a = \int \frac{dx_a dk_a}{(2\pi)^3} f_a(x_a, k_a) \quad 1 = \int \frac{dx dk}{(2\pi)^3} W_c(x, k)$$

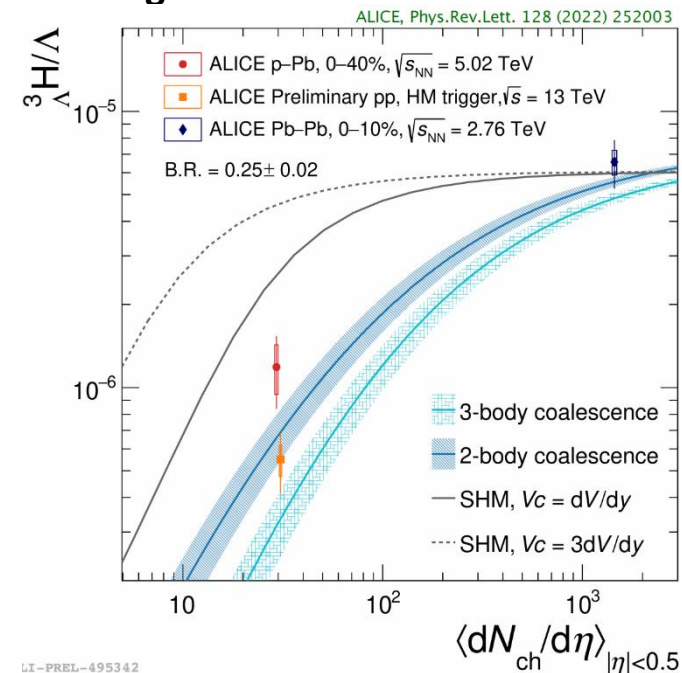
“Quantum mechanical correction”

$$N_d \propto \frac{1}{\left[1 + \left(\frac{2r_d^2}{3R^2}\right)\right]^{3/2}}$$

Production Structure

$$N_{\Lambda^3H} \propto \frac{1}{\left[1 + \left(\frac{r_{\Lambda^3H}^2}{2R^2}\right)\right]^3}$$

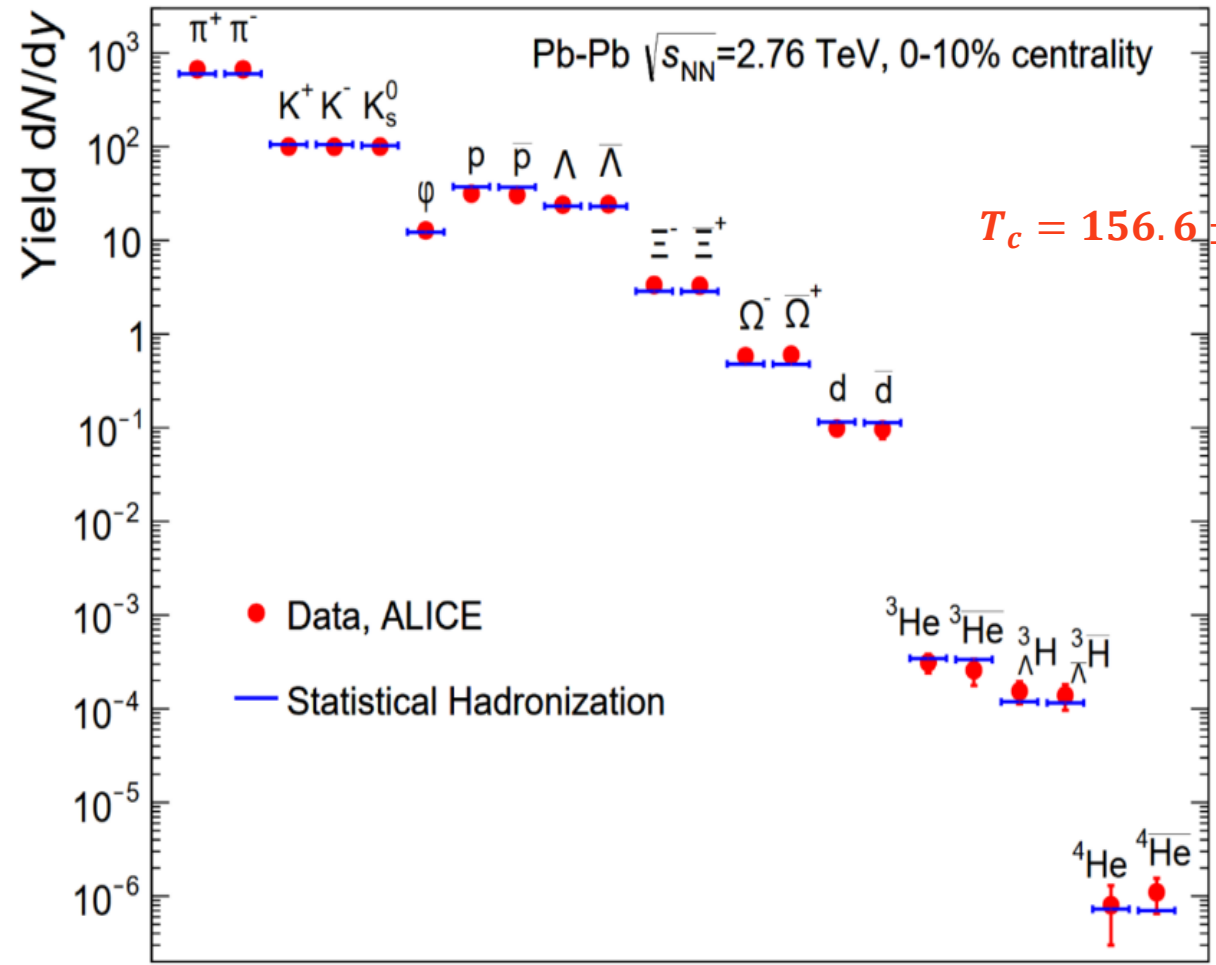
can be inferred from Femtoscopy





# 5. Statistical hadronization

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561, 321 (2018)

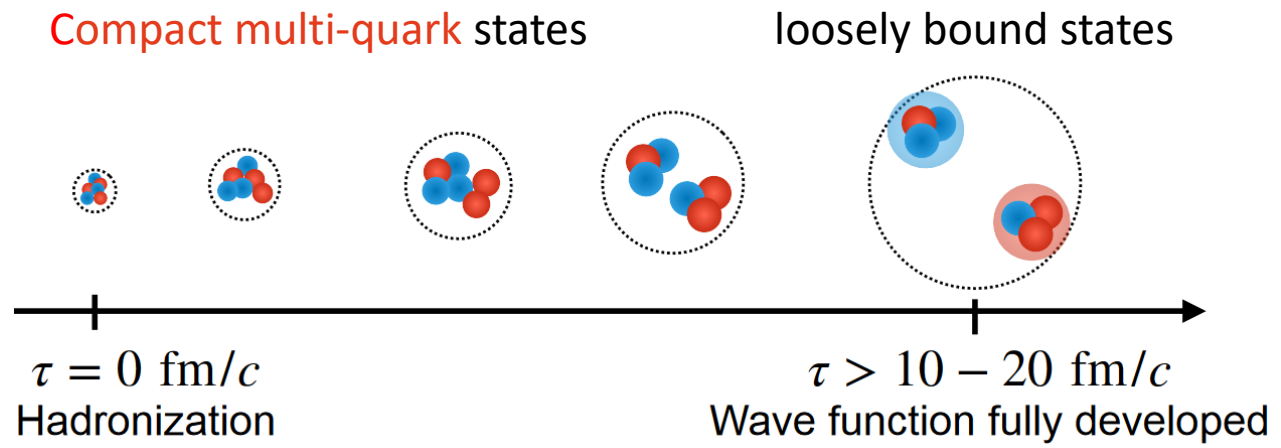


$$N_h \approx \frac{g_h V_C}{2\pi^2} m_h^2 T_C K_2\left(\frac{m_h}{T_C}\right)$$

$$\approx g_h V_C \left(\frac{m_h T_C}{2\pi}\right)^{3/2} e^{-m_h/T_C}$$

$T_C$ : Chemical freeze-out temperature, which is close to the chiral transition temperature (LQCD)

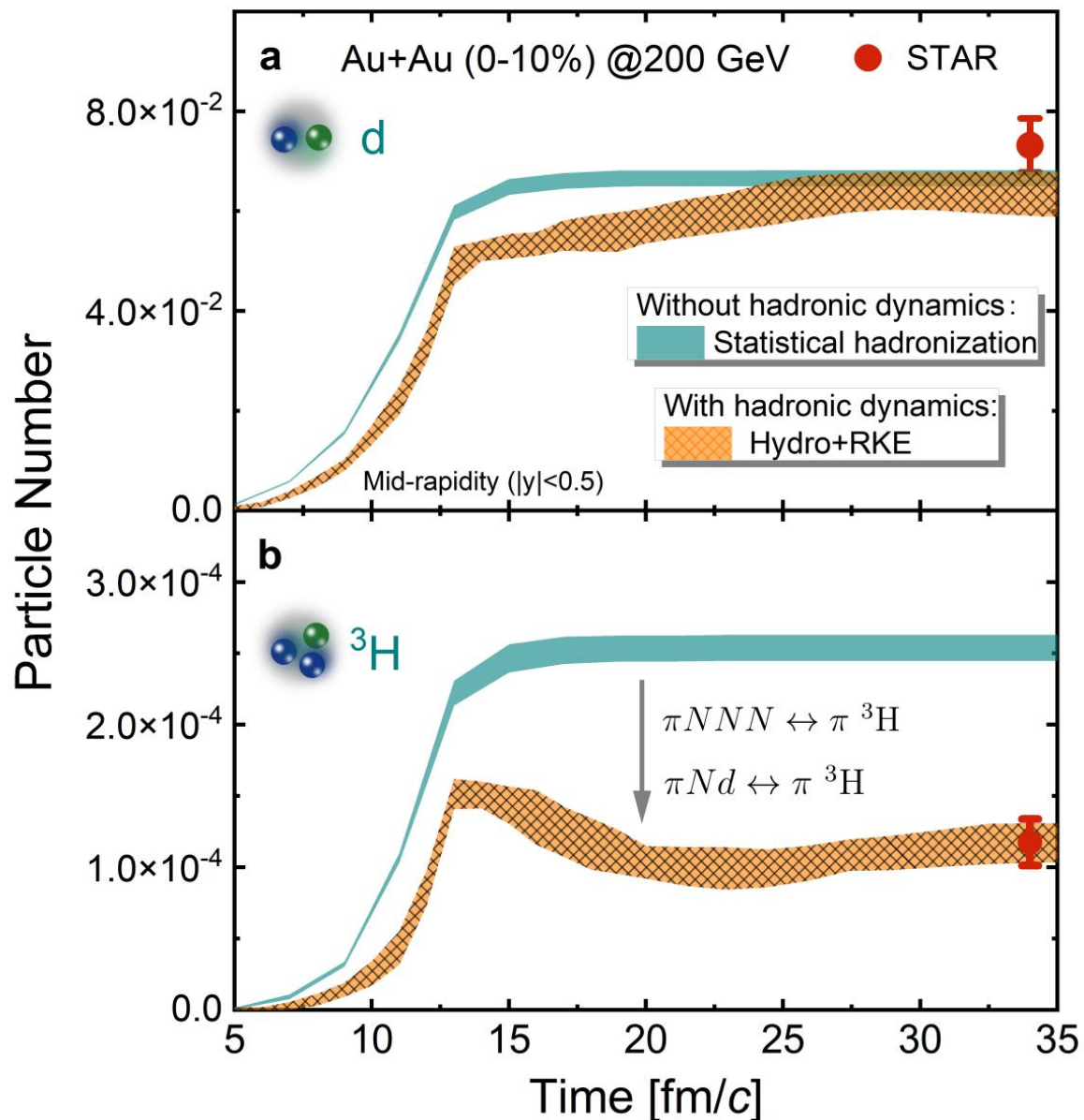
**All (stable) particles including light (hyper)nuclei are produced at the QCD phase boundary and share a common chemical freeze-out**



# 5. Relativistic kinetic equation

K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, C. Shen, *Nat. Commun.* 15, 1074 (2024)

Data from STAR, *PRL* 130, 202301 (2023)



Strong hadronic re-scattering effects

Relativistic kinetic equation for  $\pi NN \leftrightarrow \pi d$

$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = -\mathcal{K}^> f_d + \mathcal{K}^<(1 + f_d)$$

with collision integral:

$$\begin{aligned} \text{R.H.S.} = & \frac{1}{2g_d E_d} \int \prod_{i=1'}^{3'} \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d d^3 \mathbf{r}}{m_d} \\ & \times 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}) (|\mathcal{M}_{\pi+n \rightarrow \pi+n}|^2 + n \leftrightarrow p) \\ & \times \left[ - \left( \prod_{i=1'}^{3'} (1 \pm f_i) \right) g_\pi f_\pi g_d f_d + \frac{3}{4} \left( \prod_{i=1'}^{3'} g_i f_i \right) \right. \\ & \left. \times (1 + f_\pi)(1 + f_d) \right] \times (2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}}) \end{aligned}$$

Nonlocal collision integral to take into account the effects of **finite nuclei sizes**.  $W_d$  denotes deuteron Wigner function.

# 4. Effects of baryon spin correlation

(15)

Z. T. Liang, Chirality 2023

$$\left| \rho_{00}^V - \frac{1}{3} \right| \gg P_\Lambda^2 \sim P_q^2$$

$$\rho_{00}^V - \frac{1}{3} \sim \langle P_q P_{\bar{q}} \rangle$$

The STAR data show that:  $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$      $\langle P_q P_{\bar{q}} \rangle \gg \langle P_q \rangle \langle P_{\bar{q}} \rangle$

By studying  $P_H$ , we study the **average** of quark polarization  $P_q$ ;  
by studying  $\rho_{00}^V$ , we study the **correlation** between  $P_q$  and  $P_{\bar{q}}$ .

How to separate long range or local correlations

$$\rho_{10}^V = \frac{P_{qz}(1 + P_{\bar{q}y}) + (1 + P_{qy})P_{\bar{q}z} - iP_{qx}(1 + P_{\bar{q}y}) - i(1 + P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + \vec{P}_q \cdot \vec{P}_{\bar{q}})}$$

$$\rho_{0-1}^V = \frac{P_{qz}(1 - P_{\bar{q}y}) + (1 - P_{qy})P_{\bar{q}z} - iP_{qx}(1 - P_{\bar{q}y}) - i(1 - P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + \vec{P}_q \cdot \vec{P}_{\bar{q}})}$$

$$\rho_{1-1}^V = \frac{P_{qz}P_{\bar{q}z} - P_{qx}P_{\bar{q}x} + i(P_{qx}P_{\bar{q}y} + P_{qy}P_{\bar{q}x})}{3 + \vec{P}_q \cdot \vec{P}_{\bar{q}}}$$

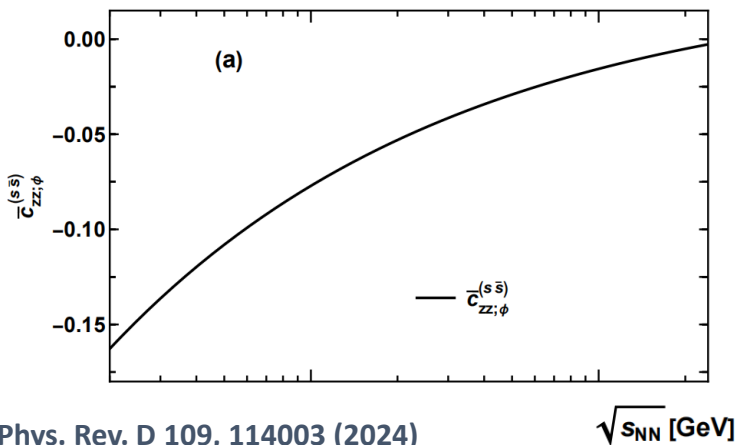
$$C_{NN}^{H_i \bar{H}_j} \equiv \frac{N_{H_i \bar{H}_j}^{\uparrow\uparrow} + N_{H_i \bar{H}_j}^{\downarrow\downarrow} - N_{H_i \bar{H}_j}^{\uparrow\downarrow} - N_{H_i \bar{H}_j}^{\downarrow\uparrow}}{N_{H_i \bar{H}_j}^{\uparrow\uparrow} + N_{H_i \bar{H}_j}^{\downarrow\downarrow} + N_{H_i \bar{H}_j}^{\uparrow\downarrow} + N_{H_i \bar{H}_j}^{\downarrow\uparrow}}$$

sensitive to the long range correlation

They should be sensitive to the local correlations.

## Global quark spin correlations in relativistic heavy ion collisions

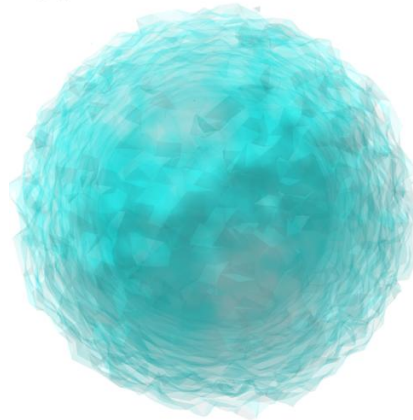
Ji-peng Lv,<sup>1,\*</sup> Zi-han Yu,<sup>1,†</sup> Zuo-tang Liang,<sup>1,‡</sup> Qun Wang,<sup>2,3,§</sup> and Xin-Nian Wang<sup>4,¶</sup>



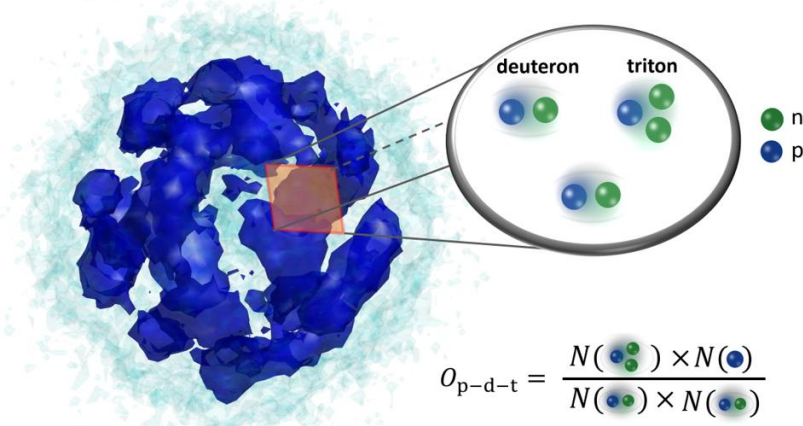
J. P. Lv et al., Phys. Rev. D 109, 114003 (2024)

C. M. Ko, NST 34, 80 (2023).

(a) Crossover



(b) First-order



$$O_{p-d-t} = \frac{N(\text{deuteron}) \times N(\text{triton})}{N(\text{neutron}) \times N(\text{proton})}$$

$$N_d \approx N_d^{(0)}(1 + C_{np}) + \frac{3}{2^{1/2}} \left( \frac{2\pi}{mT} \right)^{3/2} \times \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 C_2(\mathbf{x}_1, \mathbf{x}_2) \frac{e^{-\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{2\sigma_d^2}}}{(2\pi\sigma_d^2)^{3/2}}$$

$$N_t \approx \frac{3^{3/2}}{4} \left( \frac{2\pi}{mT} \right)^3 \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{x}_3 \rho_{nnp}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \times \frac{1}{3^{3/2}(\pi\sigma_t^2)^3} e^{-\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{2\sigma_t^2} - \frac{(\mathbf{x}_1 + \mathbf{x}_2 - 2\mathbf{x}_3)^2}{6\sigma_t^2}}$$

$$\rho_{nnp}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \approx \rho_n(\mathbf{x}_1)\rho_n(\mathbf{x}_2)\rho_p(\mathbf{x}_3) + C_2(\mathbf{x}_1, \mathbf{x}_2)\rho_p(\mathbf{x}_3) + C_2(\mathbf{x}_2, \mathbf{x}_3)\rho_n(\mathbf{x}_1) + C_2(\mathbf{x}_3, \mathbf{x}_1)\rho_n(\mathbf{x}_2) + C_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$

$$\rho_{np}(\mathbf{x}_1, \mathbf{x}_2) = \rho_n(\mathbf{x}_1)\rho_p(\mathbf{x}_2) + C_2(\mathbf{x}_1, \mathbf{x}_2)$$

$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[ 1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right) \right]$$

K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017);

K. J. Sun, C. M. Ko, and F. Li, PLB 816, 136258 (2021);