Exploring hadronic few-body interactions with femtoscopy

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on Few-Body Problems in Physics

Beijing, China
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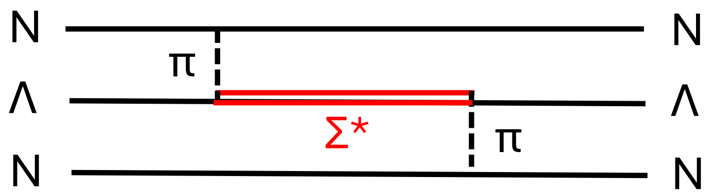
Three-body dynamics with hyperons

Dynamics of baryons involves formation of hadronic excitations

H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)



Short-range dynamics

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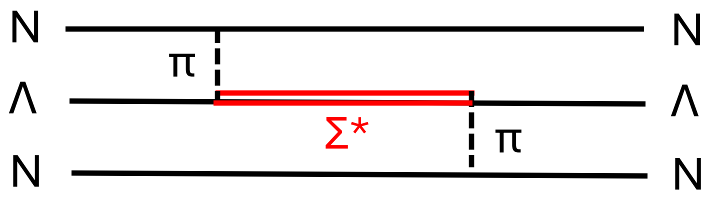
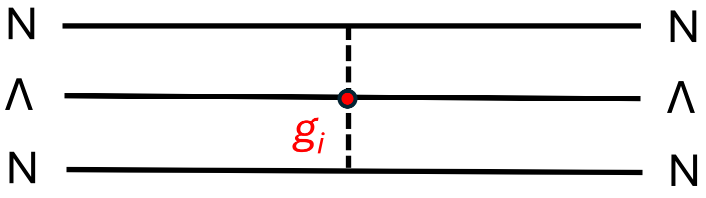


Short-range dynamics

Three-body forces:



g_i constants to be fixed by the experimental data

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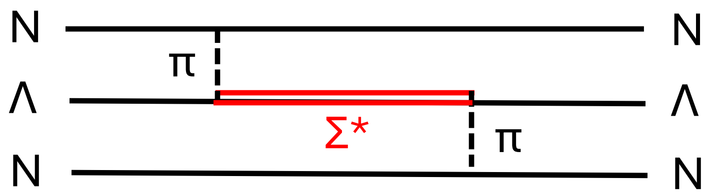

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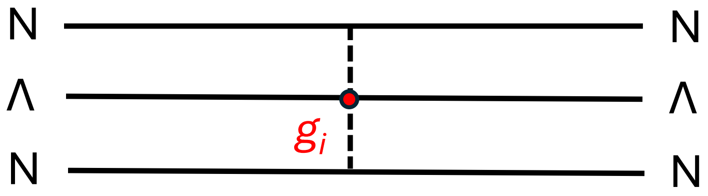

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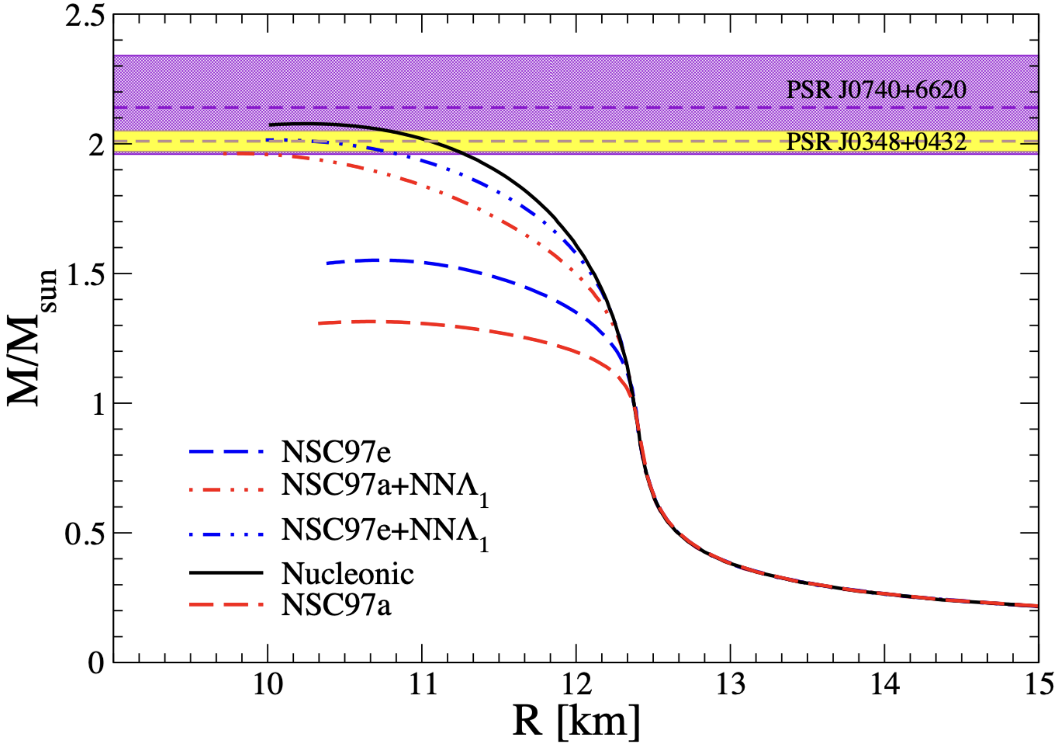
Experimental information from data on hypernuclei:

- Measurements of the binding energy for 35 hypernuclei
- Indication of a repulsive ΛNN interaction

A. Gal, E. V. Hungerford, and D. J. Millener, RMP 88, 035004 (2016)

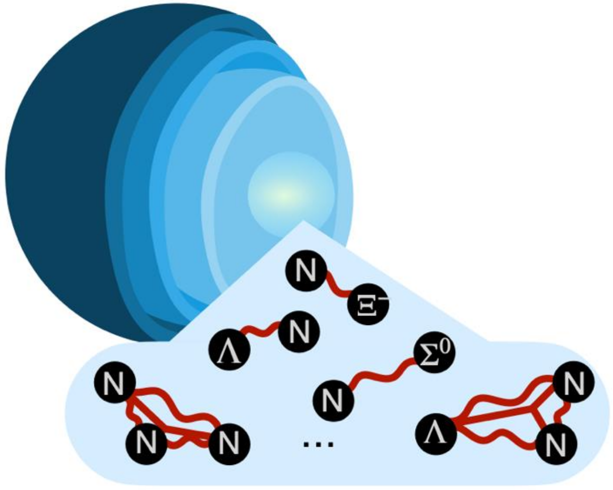
Three-body dynamics with hyperons

Equation of State → Mass-Radius relation



D. Logoteta et al., EPJA 55 (2019) 11, 207

- ○ Neutron stars (NS) are dense (3-5ρ_0) and compact (R = 10-14 km) objects
- ○ Equation of state (EOS) depends on constituents and interactions



- ○ Hyperon production might be energetically favorable, however, repulsive ΛNN is needed to reach 2M_0

Three-body dynamics with hyperons

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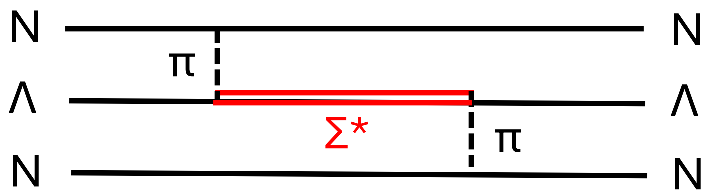

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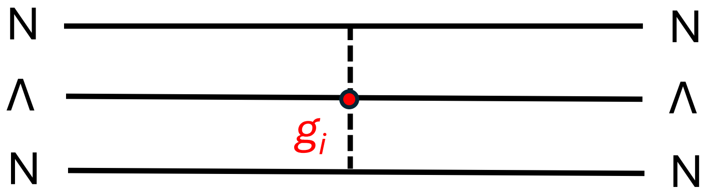

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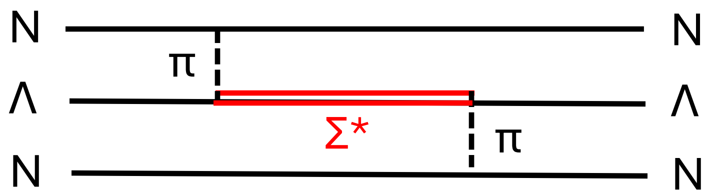

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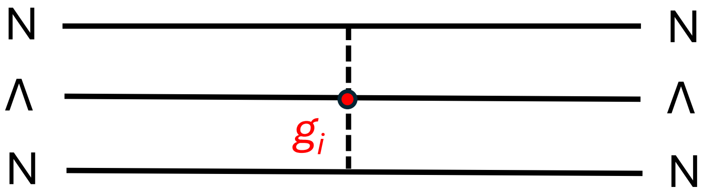

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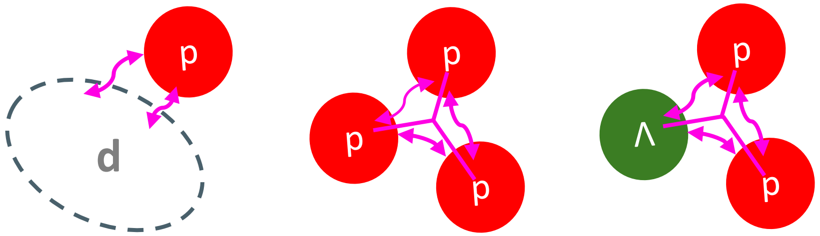
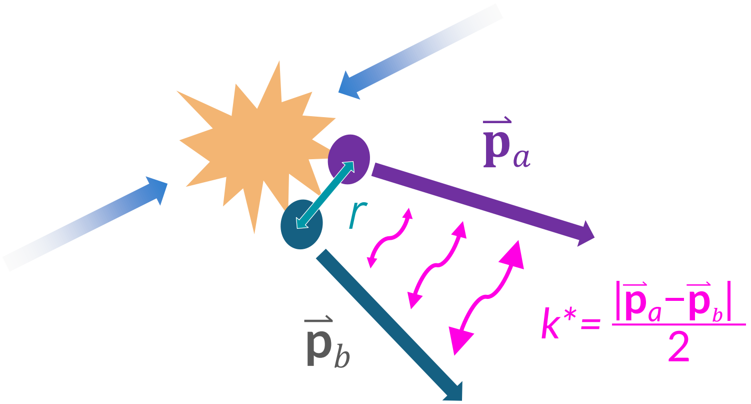
Can we exploit momentum correlations?



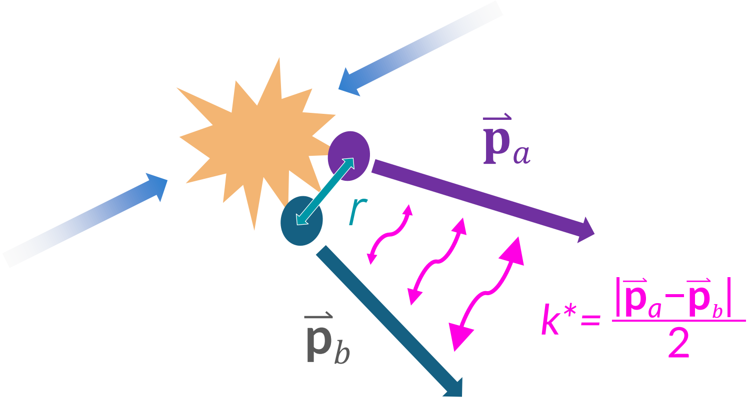
Correlation function



Emission source $S(\vec{r}\,)$

M. Lisa, S. Pratt et al., ARNPS 55 (2005), 357-402
L. Fabbietti et al., ARNPS 71 (2021), 377-402

Correlation function



Emission source $S(\vec{r}\,)$

$$C(k^*) = \mathcal{N}(k^*)\frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)} = \int S(\vec{r}^*)\left|\psi(\vec{k}^*,\vec{r}^*)\right|^2 d^3\vec{r}^*$$

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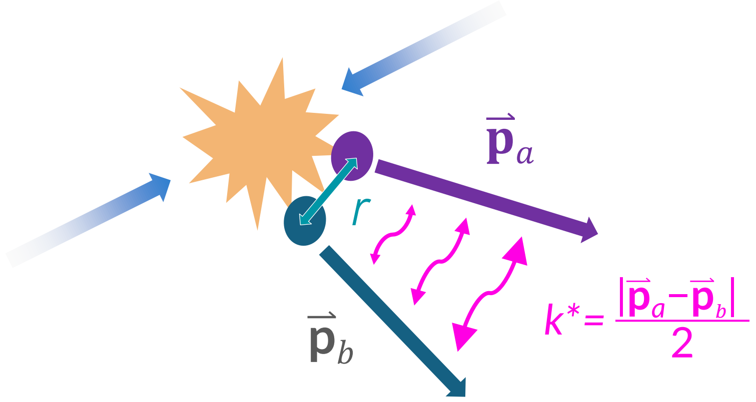
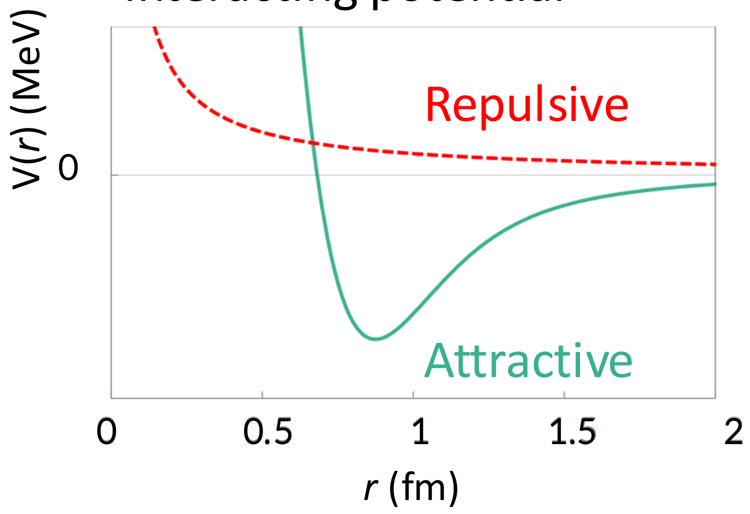


Emission source $S(\vec{r}\,)$

Interacting potential



Schrödinger equation

Two-particle wave function

D. Mihaylov et al., EPJC 78 (2018), 5, 394

$$C(k^*) = \mathcal{N}(k^*)\,\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)} = \int S(\vec{r}^{\,*})\left|\psi(\vec{k}^*,\vec{r}^{\,*})\right|^2 d^3\vec{r}^{\,*}$$

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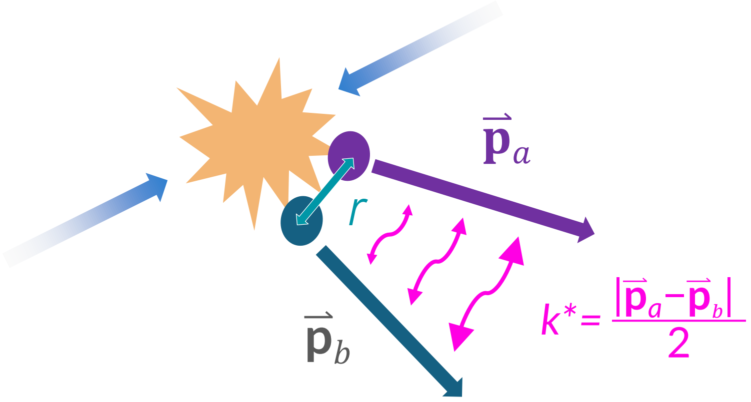

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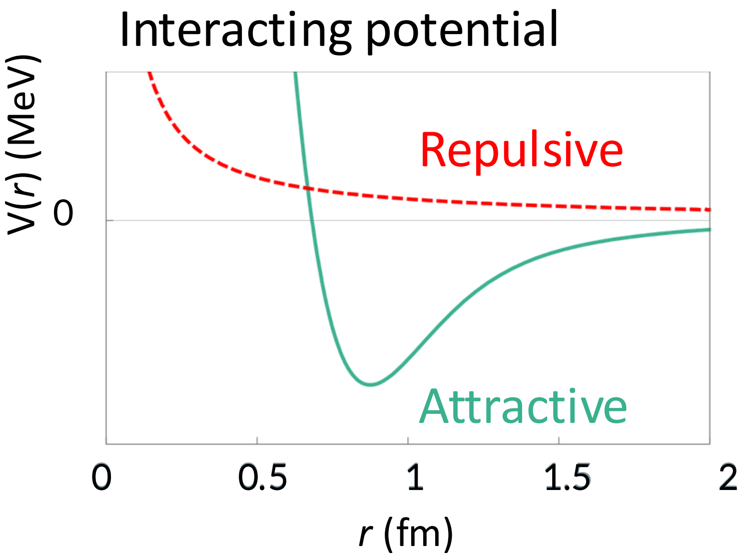

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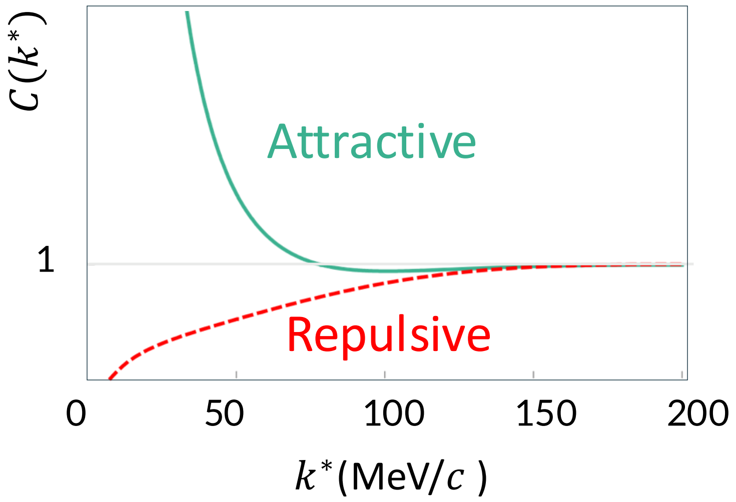

Correlation function $C(k^*)$

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Constraining the source

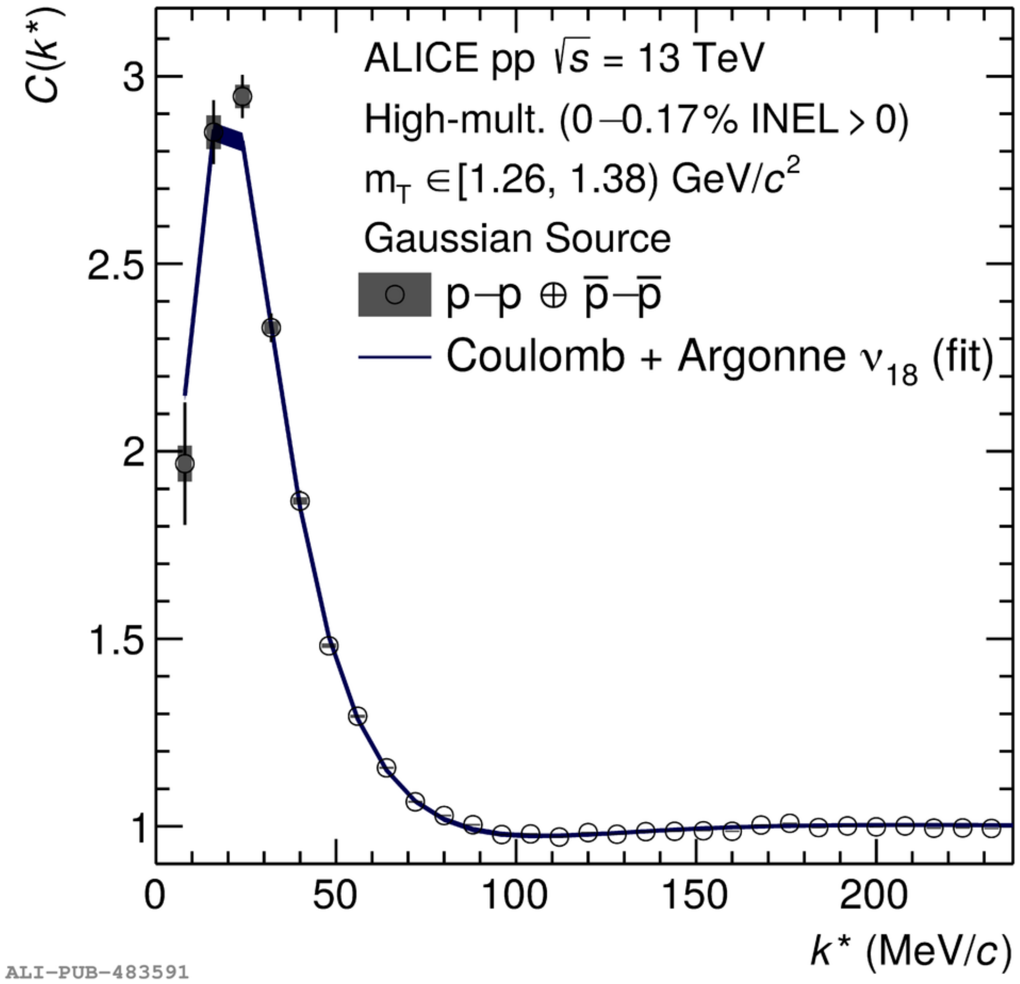
- Emitting source function anchored to p-p correlation function

$$C(k^*) = \int S(\vec{r}^*)\left|\psi(\vec{k}^*,\vec{r}^*)\right|^2 d^3\vec{r}^*$$

- Two-component model

$$S(r^*) = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^{*2}}{4r_{core}^2}\right) \times$$ Effect of short lived resonances (cτ ~ 1 fm)

ALICE Coll., PLB, 811 (2020), 135849



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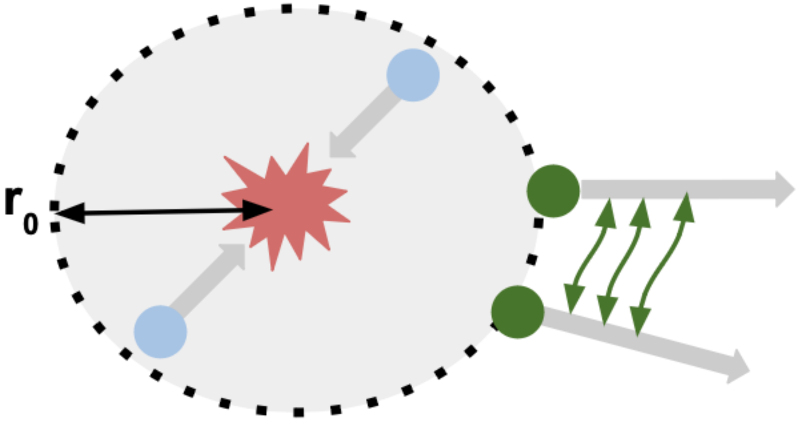

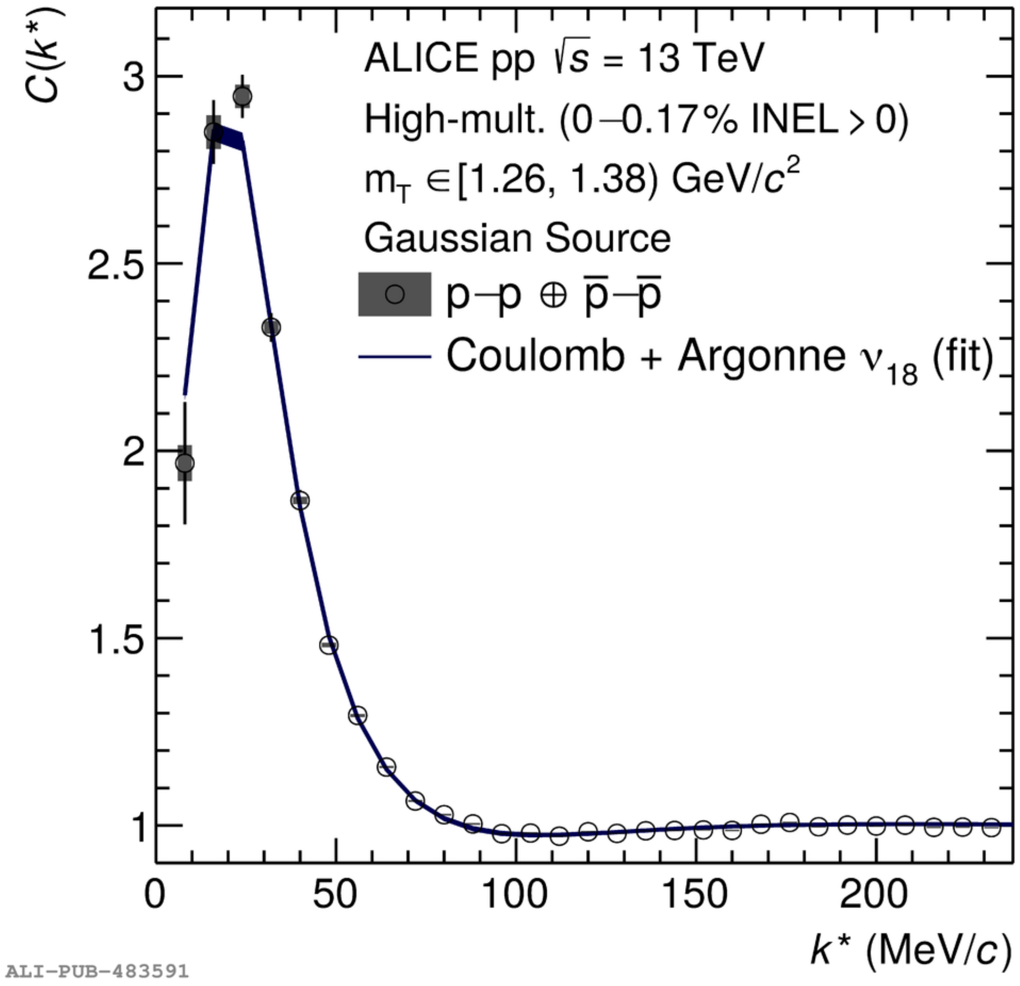

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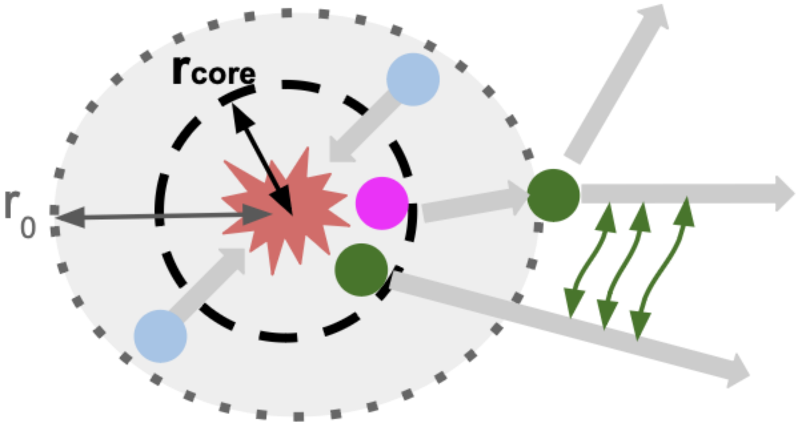

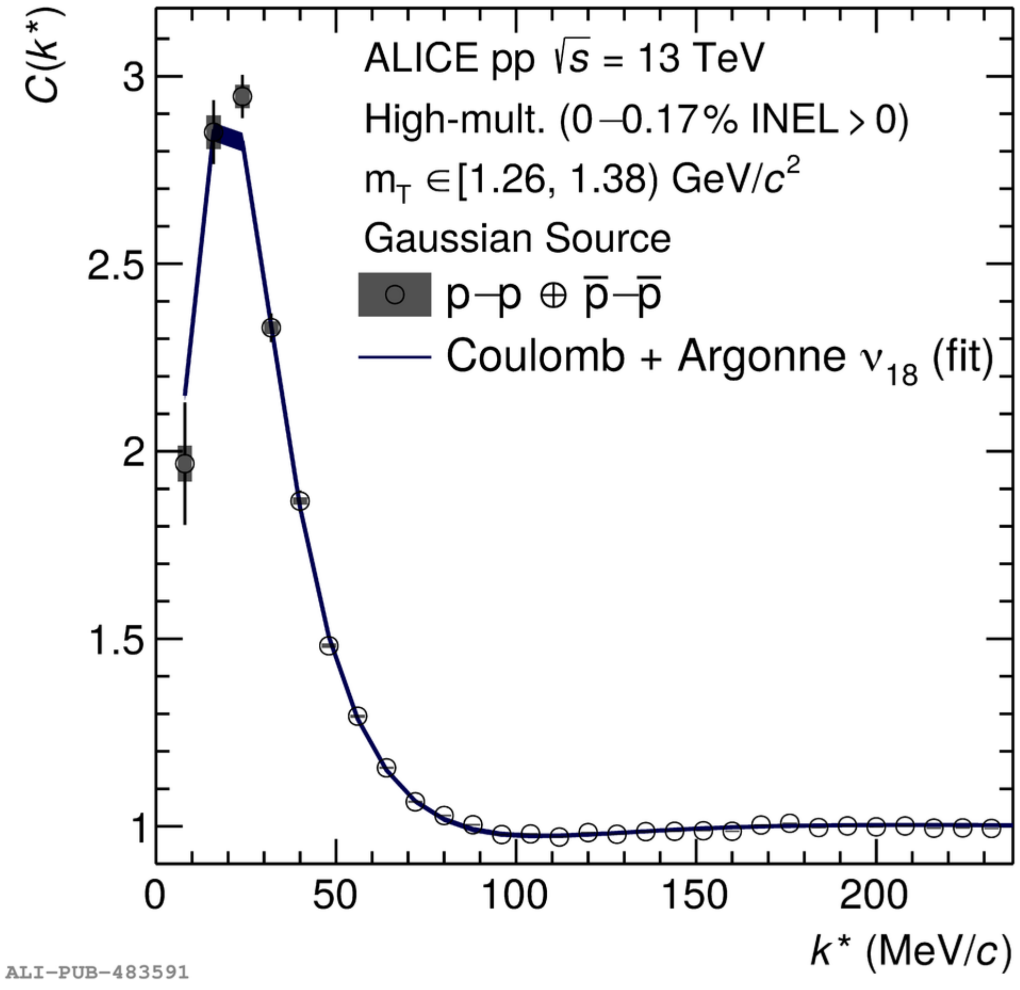

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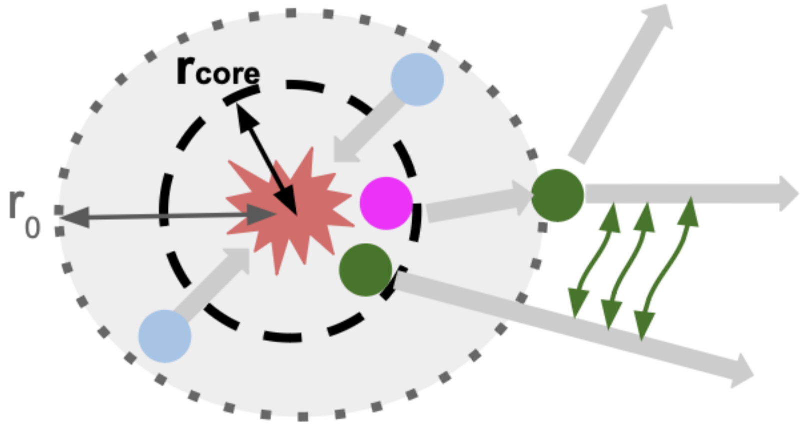
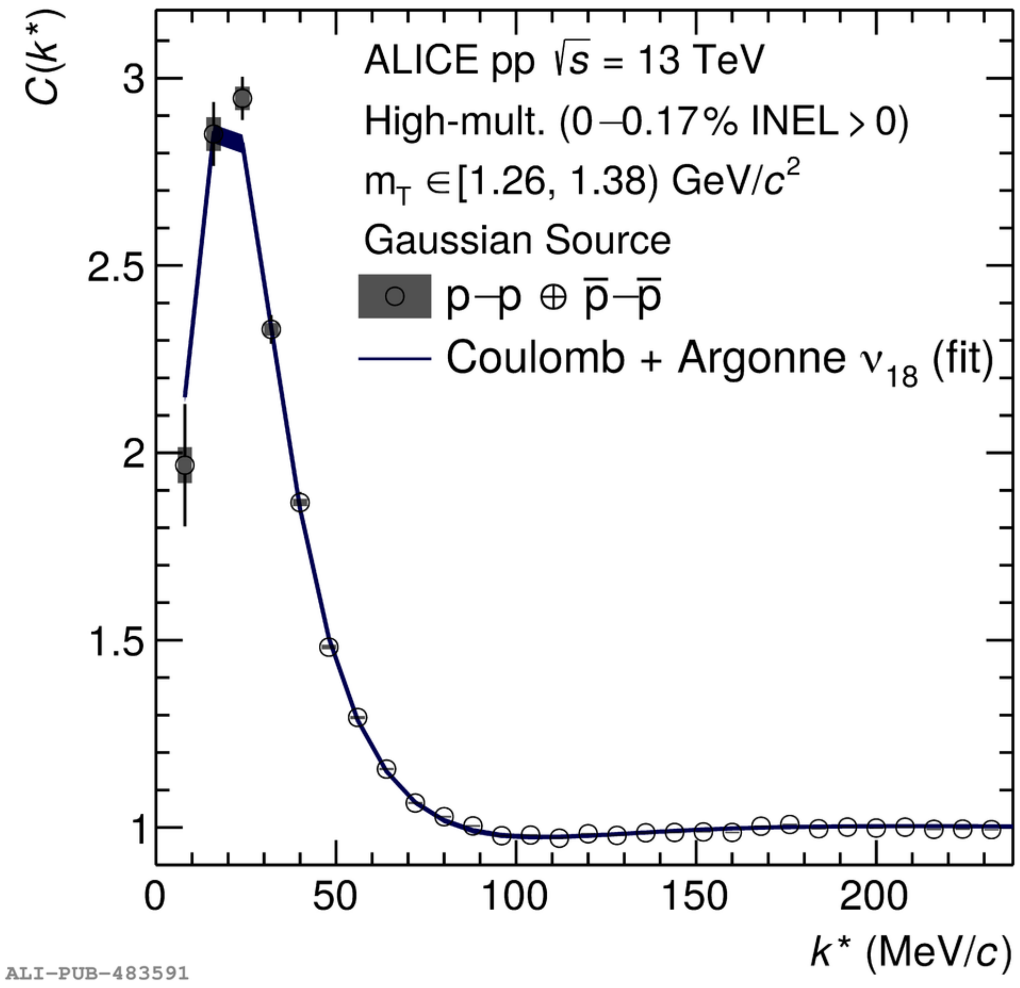


$$m_T = \sqrt{k_T^2 + m_{avrg}^2}$$

$$k_T = \frac{1}{2}\left|\vec{p}_{T,1} + \vec{p}_{T,2}\right|$$



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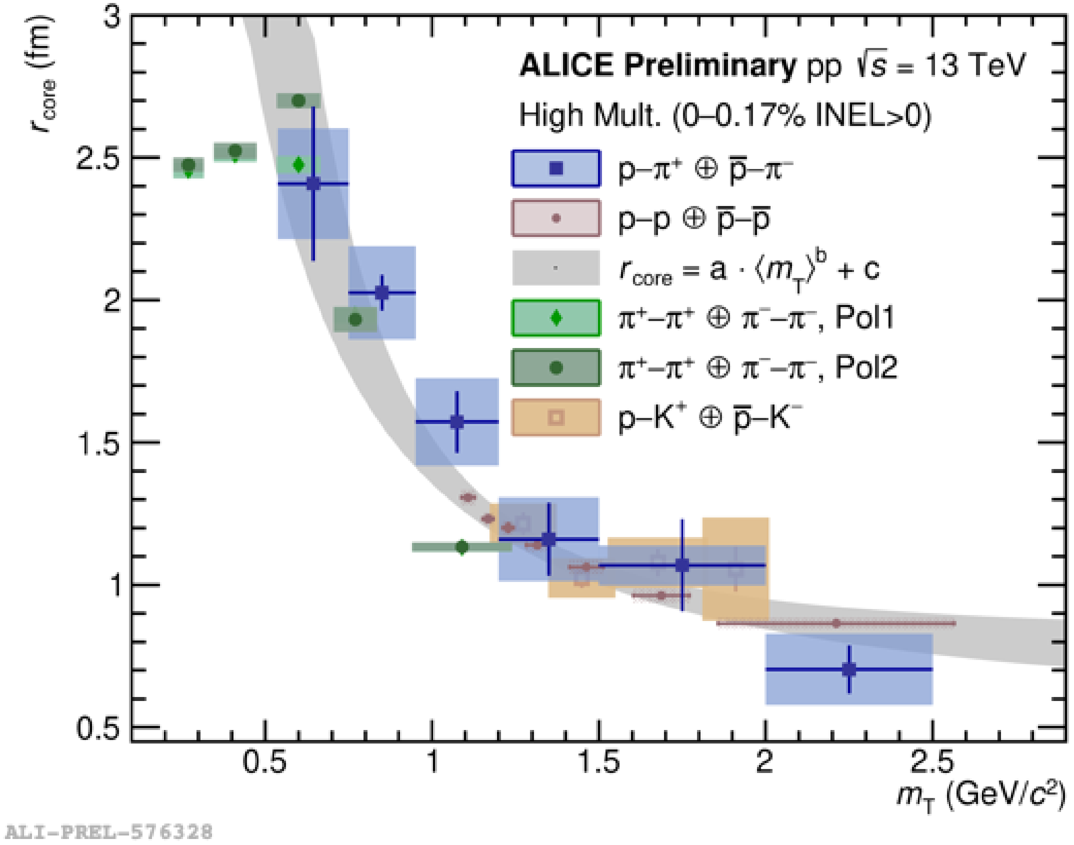
ALICE Coll., PLB, 811 (2020), 135849

- Universal source for all hadrons (cross-check for universality with K$^+$-p, π-π, p-π)
- **Small particle-emitting source created in pp collisions at the LHC**



ALICE Coll., arXiv:2311.14527 accepted by EPJC
ALICE Coll., PLB, 811 (2020), 135849
ALICE Coll., paper in preparation

NNN using proton-deuteron correlation

- Point-like particle models anchored to scattering experiments

  W. T. H. Van Oers et al., NPA 561 (1967);
  J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);
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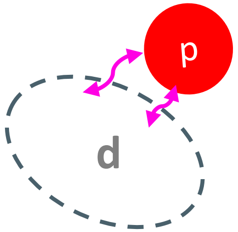
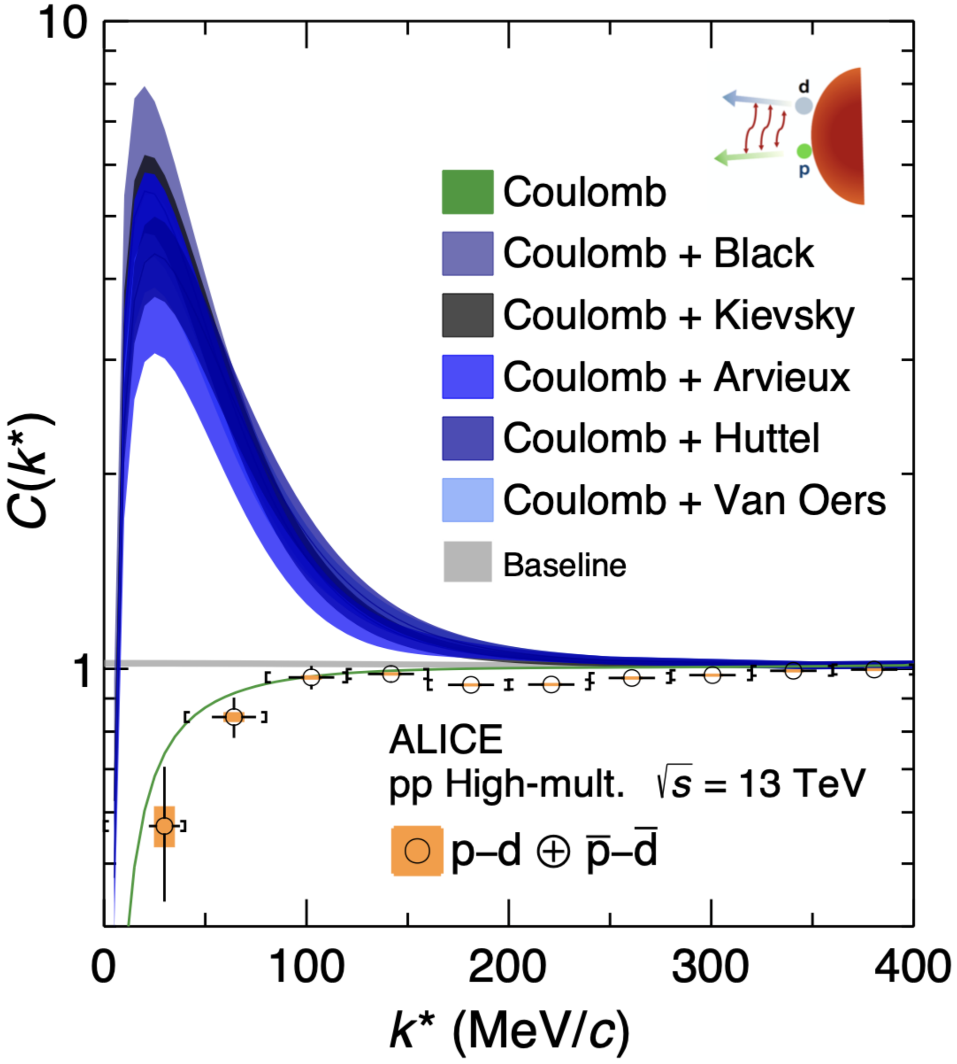
- Coulomb + strong interaction using Lednický model

  Lednický, R. Phys. Part. Nuclei 40, 307–352 (2009)
- Only s-wave interaction
- Source radius evaluated using the universal m_T scaling; cross-checked with K^+-d





ALICE Coll., PRX 14 (2024) 3, 031051

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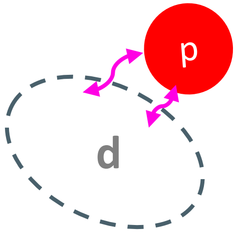
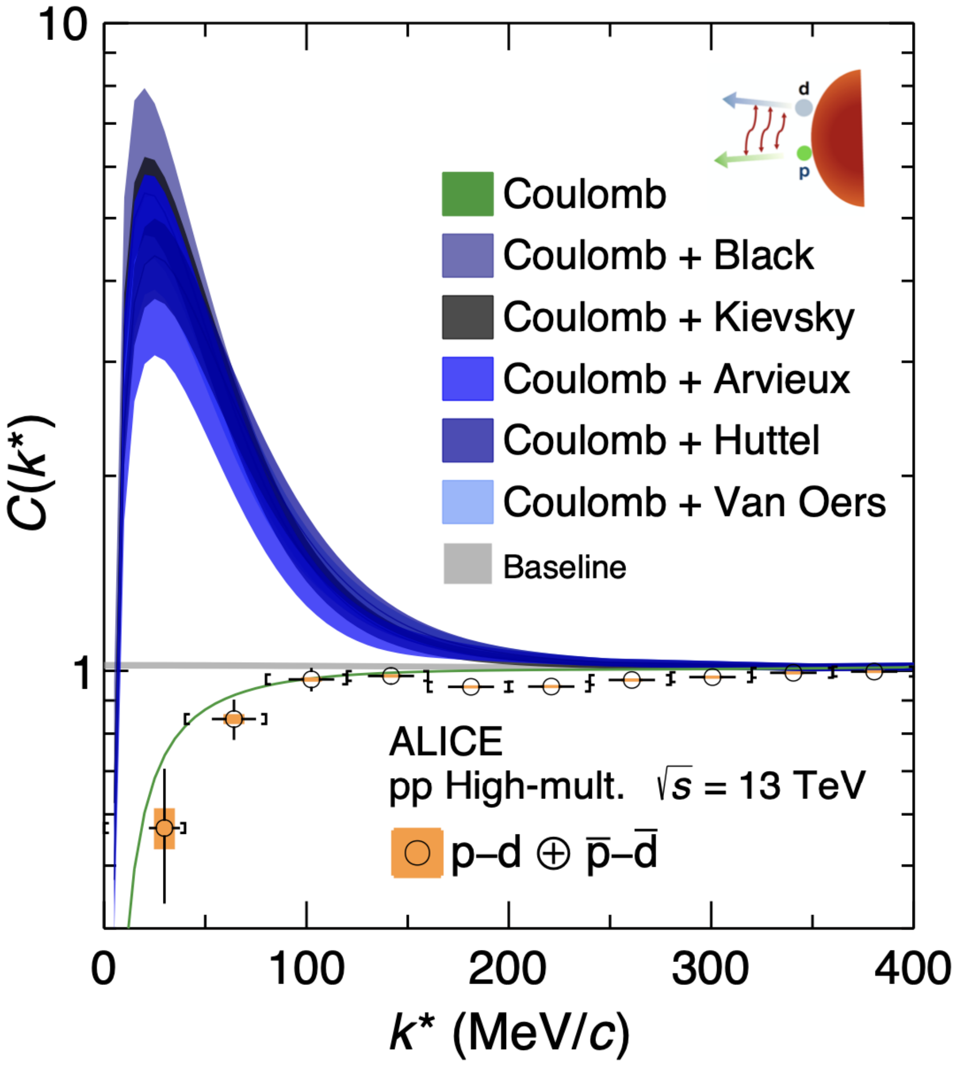
Point-like particle description fails for p-d in small systems





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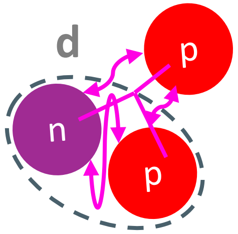
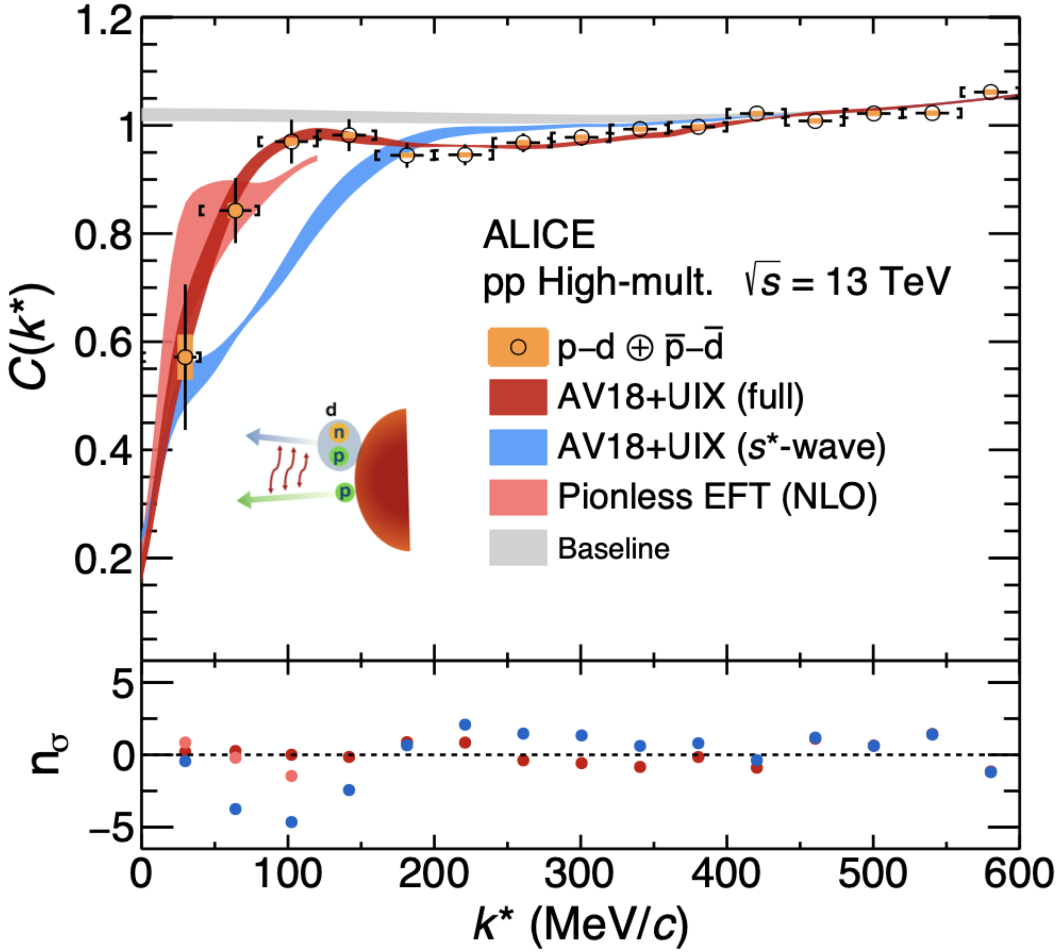
- Full three-body calculations are required: p–p–n forms p–d (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics





ALICE Coll., PRX 14 (2024) 3, 031051
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

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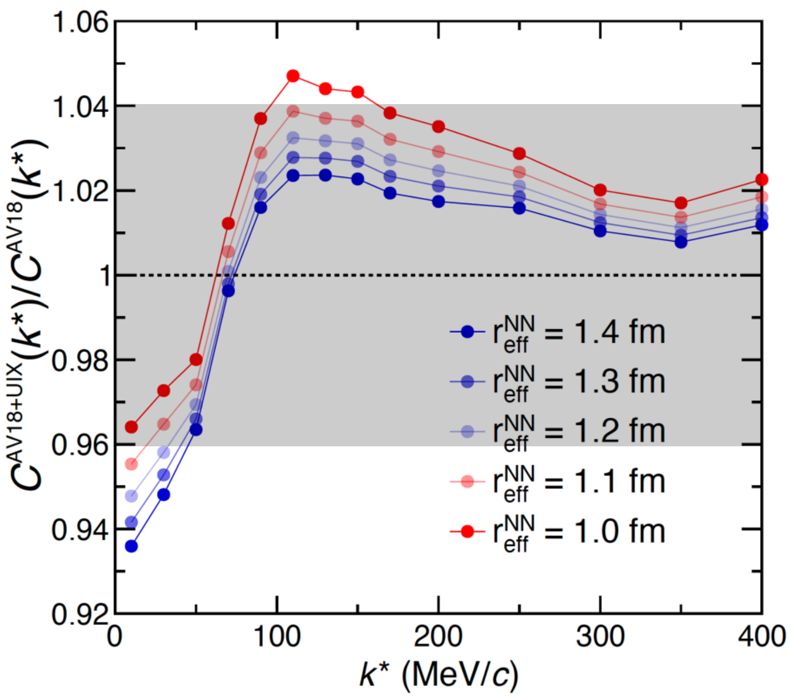
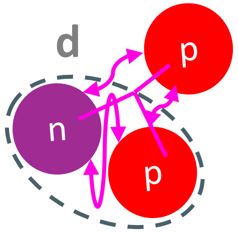
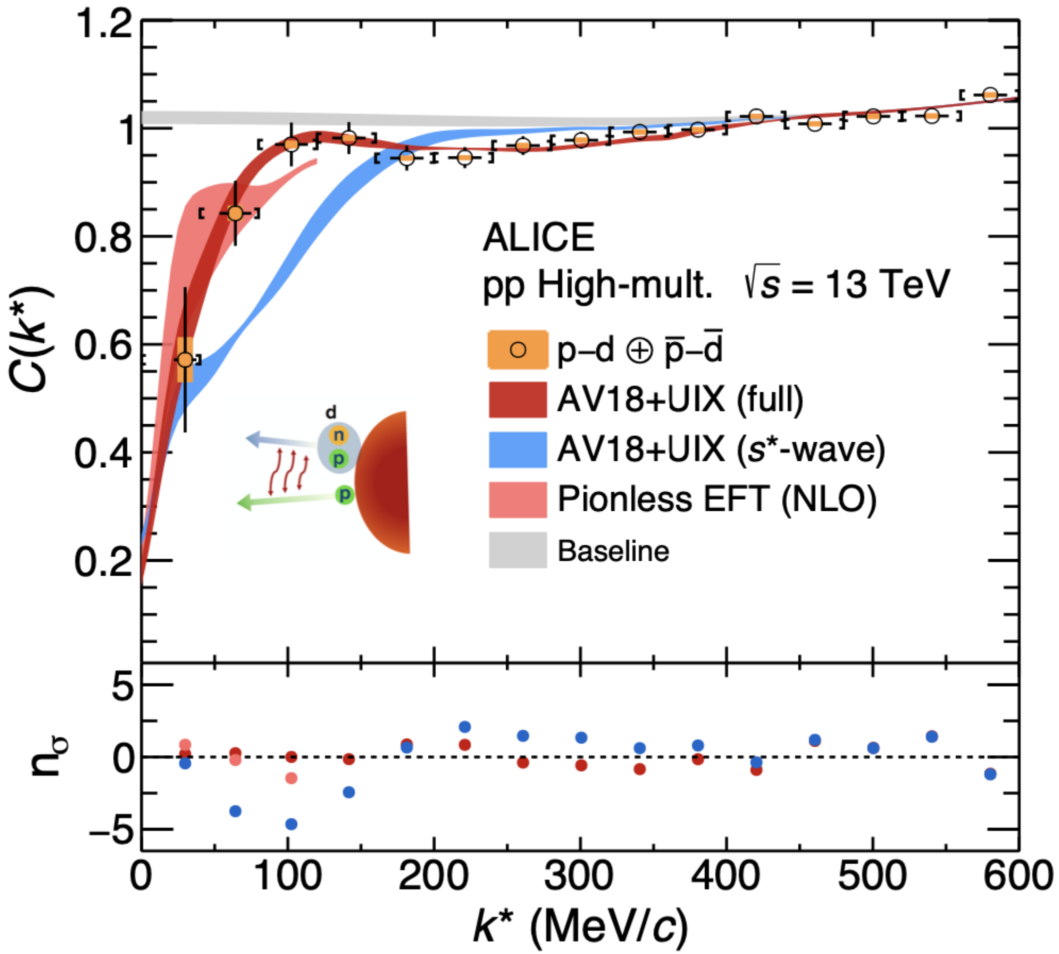
- Sensitivity to three-body forces up to 5%







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NNN using proton-deuteron correlation

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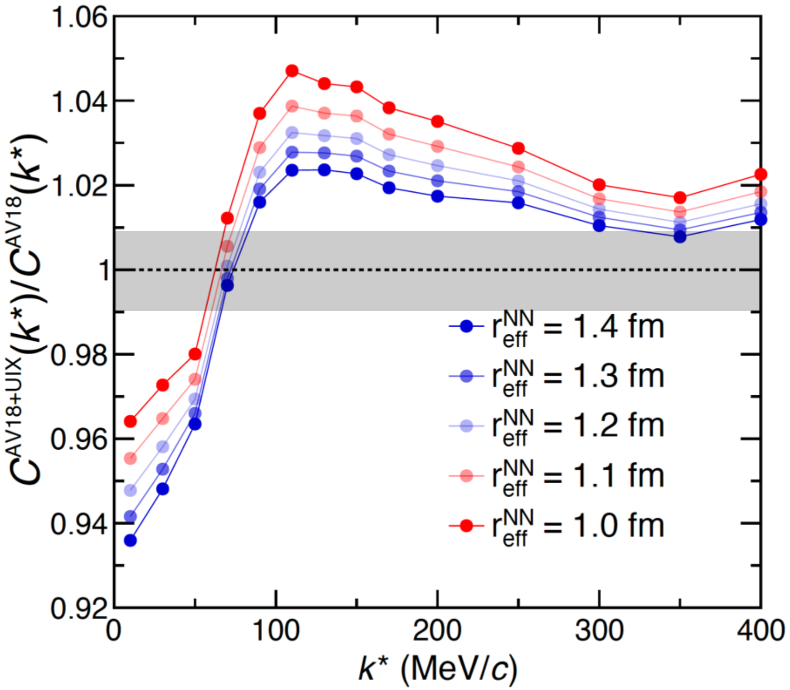
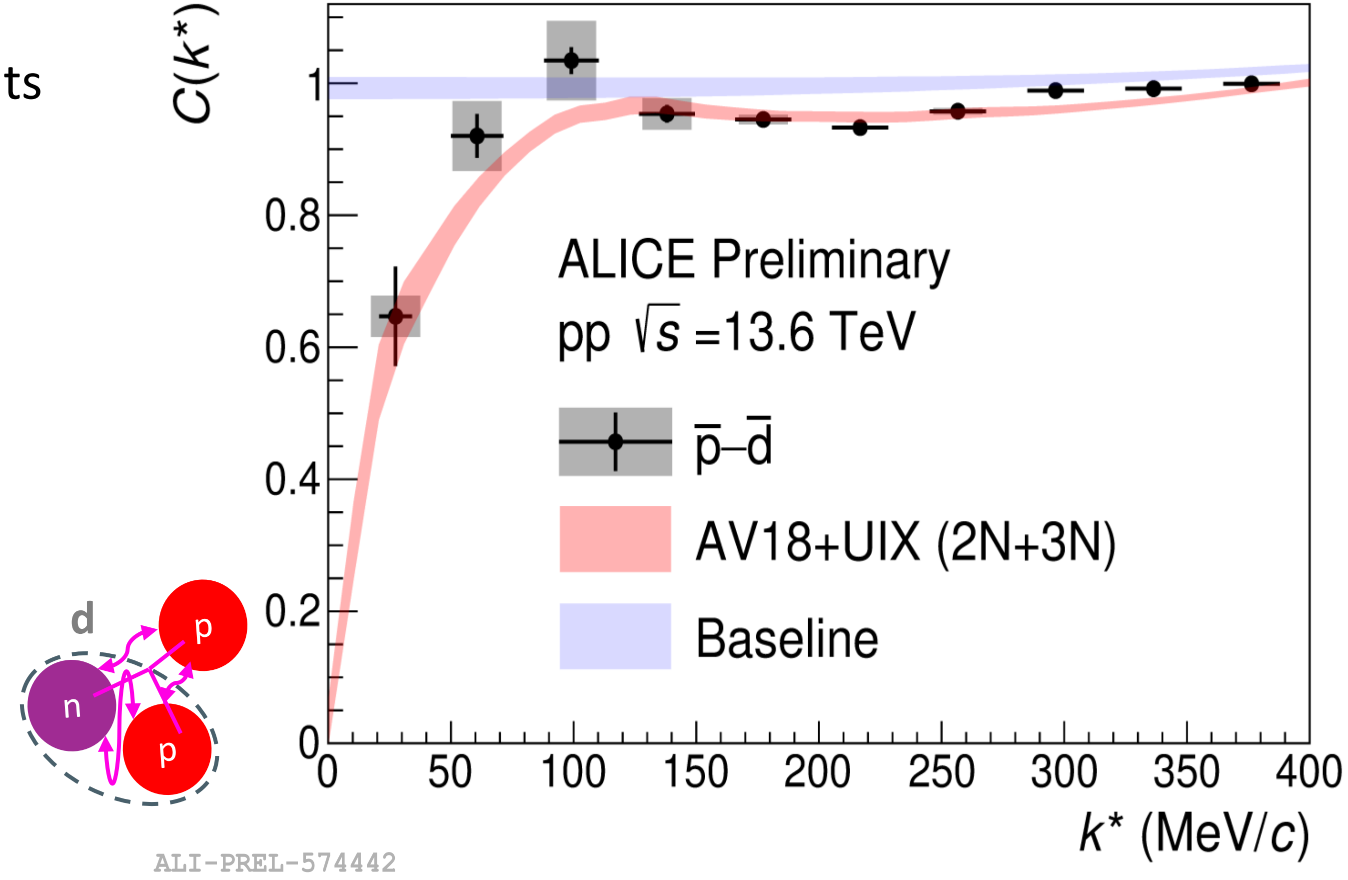
- Run 3 data from 2022 already analysed and results are promising!
- In Run 3 expected uncertainty of 1%





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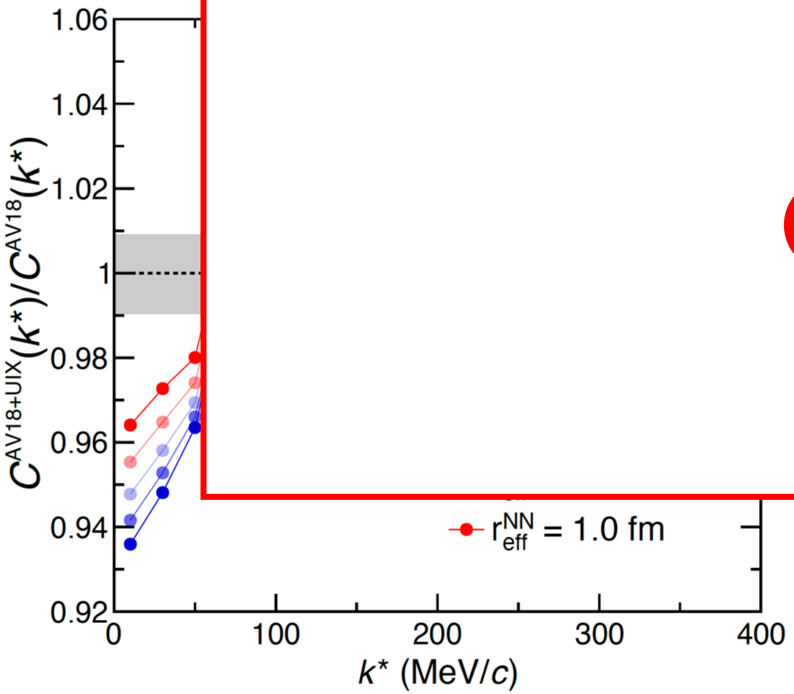
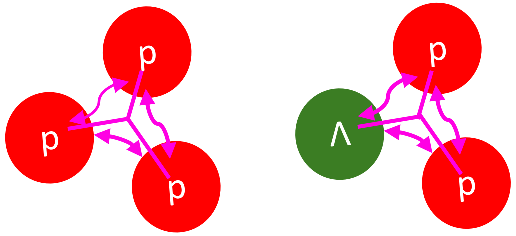
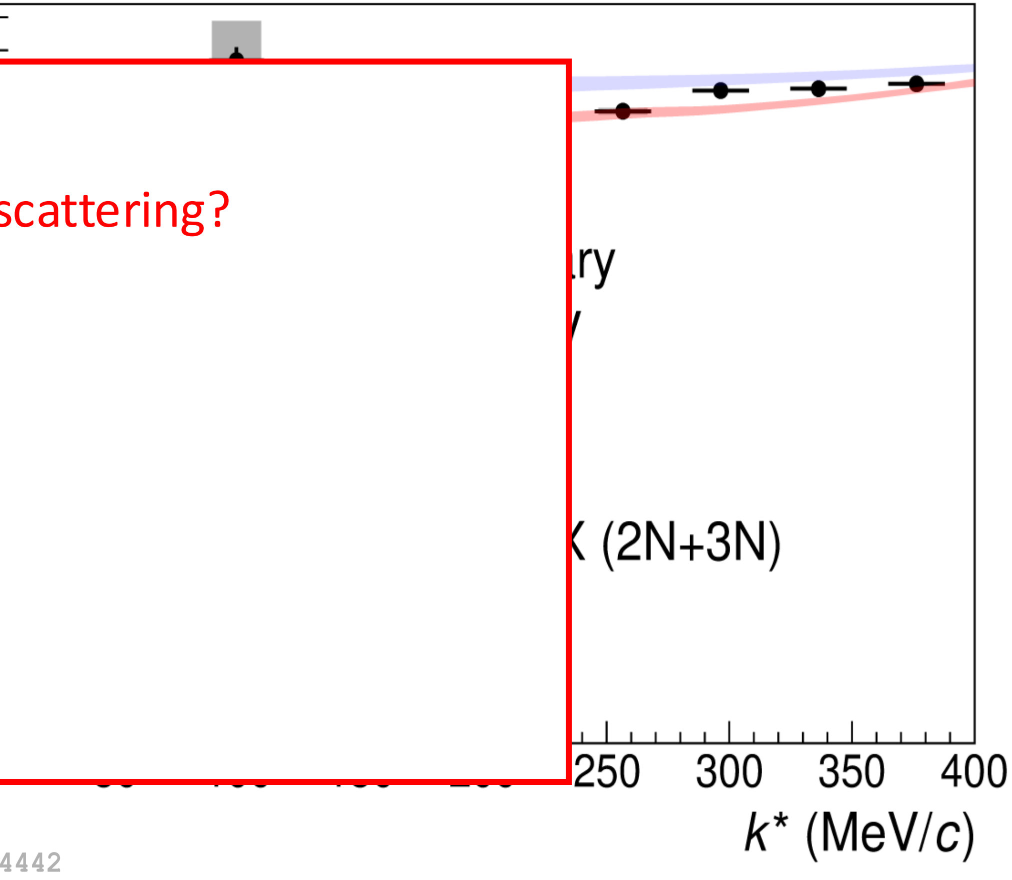


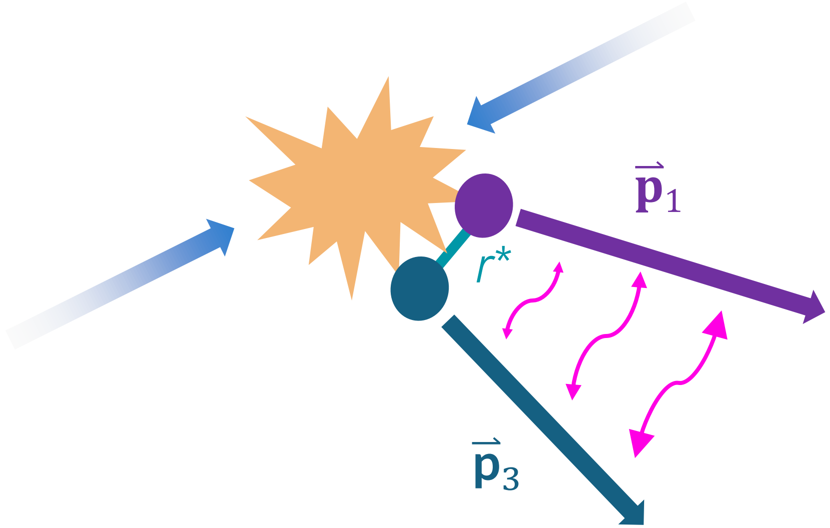
How to study elastic three hadron scattering?





ALI-PREL-574442

Two-body femtoscopy



Correlation function:

$$C(k^*) = \int S(\vec{r}^*)\left|\psi(\vec{k}^*,\vec{r}^*)\right|^2 d^3\vec{r}^*$$

Two-body scattering wave function

relative momentum:

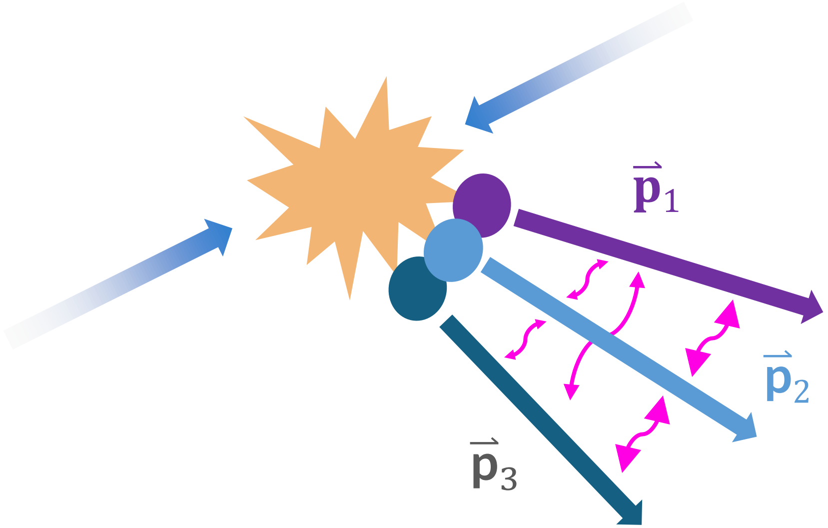
$$k^* = \frac{1}{2}\left|\vec{\mathbf{p}}_a + \vec{\mathbf{p}}_b\right|$$

relative distance:

$$r^*$$

M.Lisa, S. Pratt et al., ARNPS 55 (2005), 357-402
L. Fabbietti et al., ARNPS 71 (2021), 377-402

Three-body femtoscopy



Correlation function:

$$C(Q_3) = \int S(\rho)|\psi\,(Q_3, \rho\,)|^2\,\rho^5\,d\rho$$

Three-body scattering wave function

Hyper-momentum:

$$Q_3 = 2\sqrt{k_{12}^2 + k_{23}^2 + k_{31}^2}$$

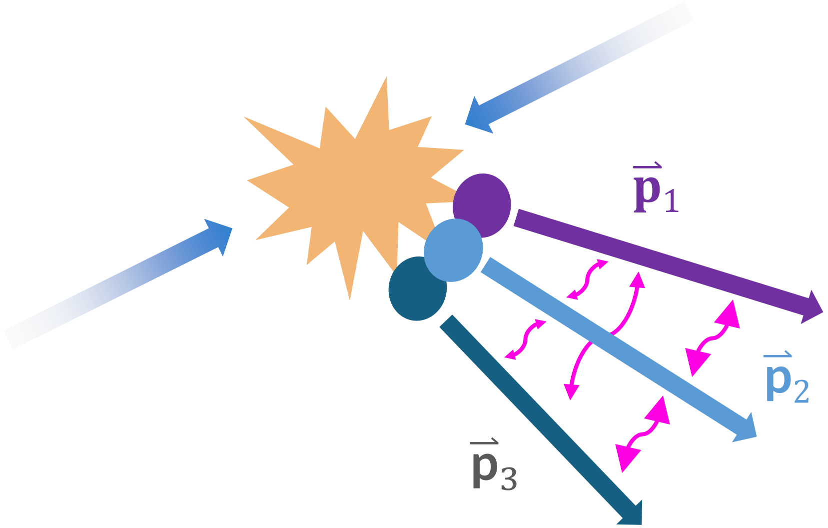
R. Del Grande et al. EPJC 82 (2022) 244
ALICE Coll., EPJ A 59, 145 (2023)

Hyper-radius:

$$\rho = 2\sqrt{r_{12}^2 + r_{23}^2 + r_{31}^2}$$

L. E. Marcucci et al., Front. in Phys. 8, 69 (2020).

Three-body femtoscopy



Correlation function:

$$C(Q_3) = \int S(\rho)|\psi\,(Q_3, \rho\,)|^2\,\rho^5\,d\rho$$

Three-body scattering wave function

Hyper-momentum:

$$Q_3 = 2\sqrt{k_{12}^2 + k_{23}^2 + k_{31}^2}$$

R. Del Grande et al. EPJC 82 (2022) 244
ALICE Coll., EPJ A 59, 145 (2023)

Hyper-radius:

$$\rho = 2\sqrt{r_{12}^2 + r_{23}^2 + r_{31}^2}$$

L. E. Marcucci et al., Front. in Phys. 8, 69 (2020).

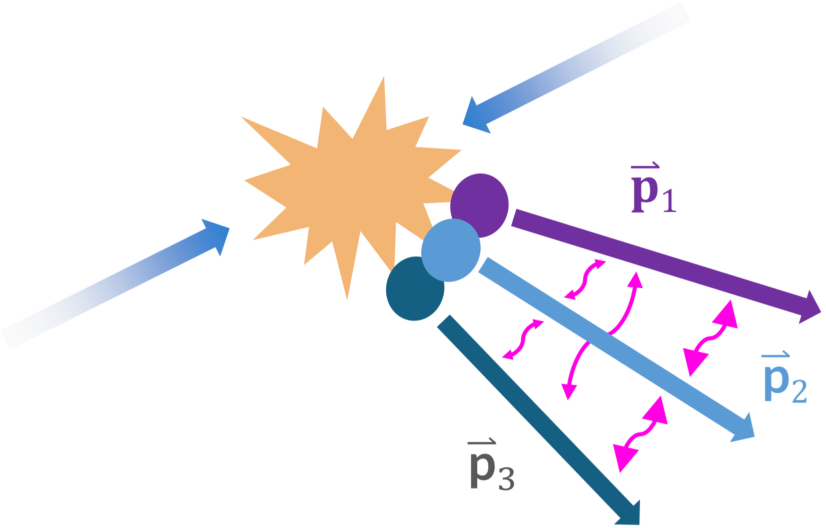
Exp:
ALICE Coll., EPJ A 59, 145 (2023)
ALICE Coll., EPJ A 59, 298 (2023)
ALICE Coll. arXiv:2308.16120 (2023), accepted by PRX
STAR Coll. arXiv:2208.05722 (2022), proceedings
HADES Coll. arXiv:2402.09280 (2024), proceedings

Theory (ALICE):
RDG et al. EPJC 82 (2022) 244
M. Viviani et al, PRC 108 (2023) 6, 064002
A. Kievsky, et al., PRC 109 (2024) 3, 034006
E. Garrido et al., arXiv: 2408.01750 (2024)

Three-body analyses in

- pp collisions at ALICE: p-p-p, p-p-Λ, p-p-K^{+-}, p-p-π^{+-}, K-d, p-d, Λ-d
- Au-Au collisions at STAR: p-d, d-d
- Ag-Ag collisions at HADES: p-d, p-t, p-^{3}He

Three-body femtoscopy



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- Ag-Ag collisions at HADES: p-d, p-t, p-^{3}He

Source function in pp collisions at the LHC

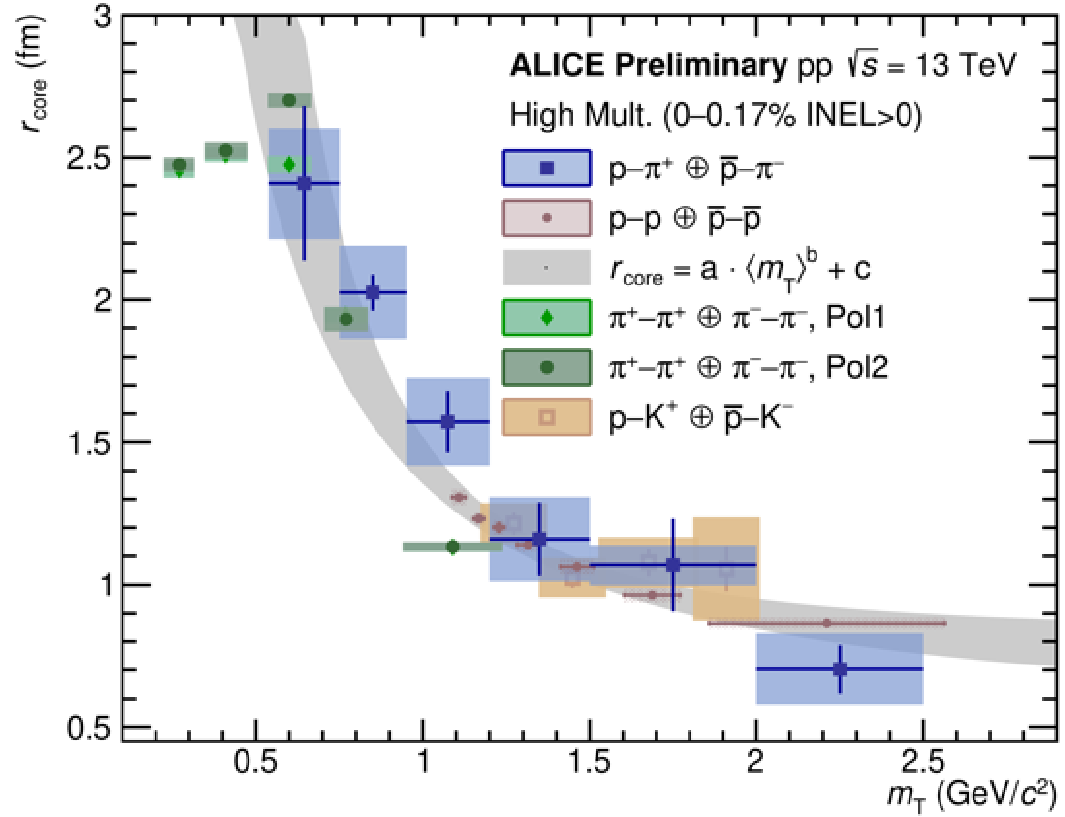
- Three-particle source as independent Gaussian emitter

$$S(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = S(\boldsymbol{x}_1)\,S(\boldsymbol{x}_2)\,S(\boldsymbol{x}_3)$$

r_{12}

r_{23}

- Pair radii r_{12}, r_{23}, r_{31} obtained from common source model using the m_T of the pairs in the triplets.



ALI-PREL-576328

ALICE Coll., arXiv:2311.14527 accepted by EPJC
ALICE Coll., PLB, 811 (2020), 135849
ALICE Coll., paper in preparation

Source function in pp collisions at the LHC

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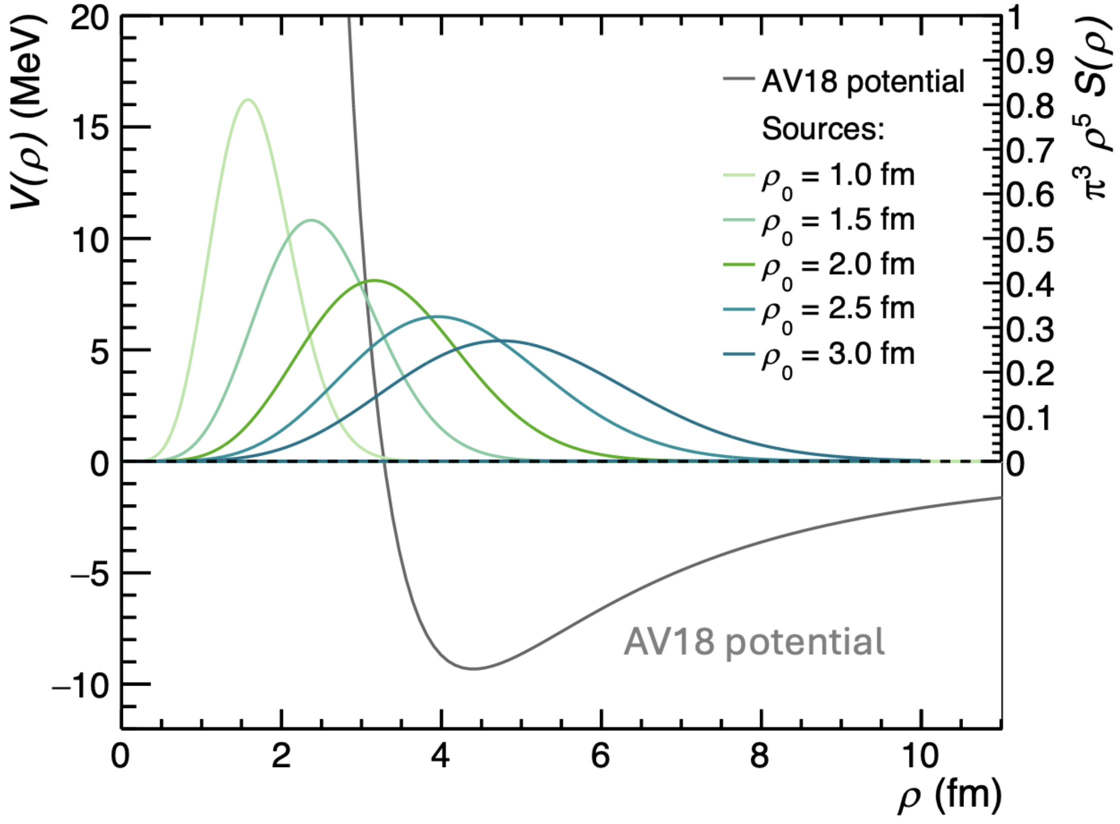
$$S(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \underbrace{S(\boldsymbol{x}_1)\;S(}_{r_{12}}\underbrace{\boldsymbol{x}_2)\;S(\boldsymbol{x}_3)}_{r_{23}}$$

- Pair radii r_{12}, r_{23}, r_{31} obtained from common source model using the m_{T} of the pairs in the triplets.
- In hyperspherical coordinates

$$S(\rho) = \frac{1}{\pi^3 \rho_0^6}\, e^{-\left(\frac{\rho}{\rho_0}\right)^2}$$

M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002
A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006
E. Garrido et al., arXiv: 2408.01750 (2024)

- The value of ρ_0 is determined the pair radii.
- **In pp collsions at $\sqrt{s}$ = 13 TeV the LHC source: $\rho_0 =$ 1-3 fm.**



A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

p-p-p correlation function in Run 2

- First-ever full three-body correlation functions calculations

three-proton wave function

$$C(Q_3) = \int \rho^5 d\rho\, S(\rho, \rho_0) |\Psi(\rho, Q_3)|^2$$

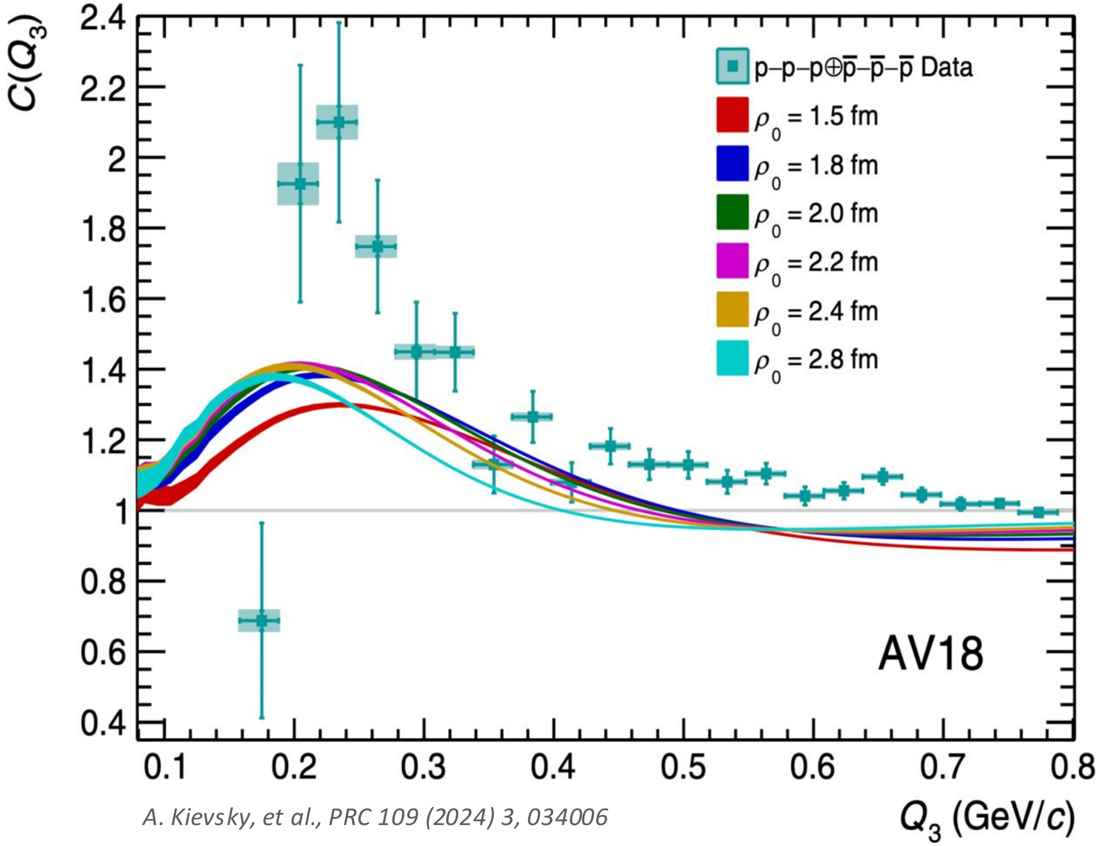
hyperradius

- Wave function via HH with AV18, Coulomb interaction and Quantum statistics

A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

- Negligible contribution from NNN (via UIX) found < 1%

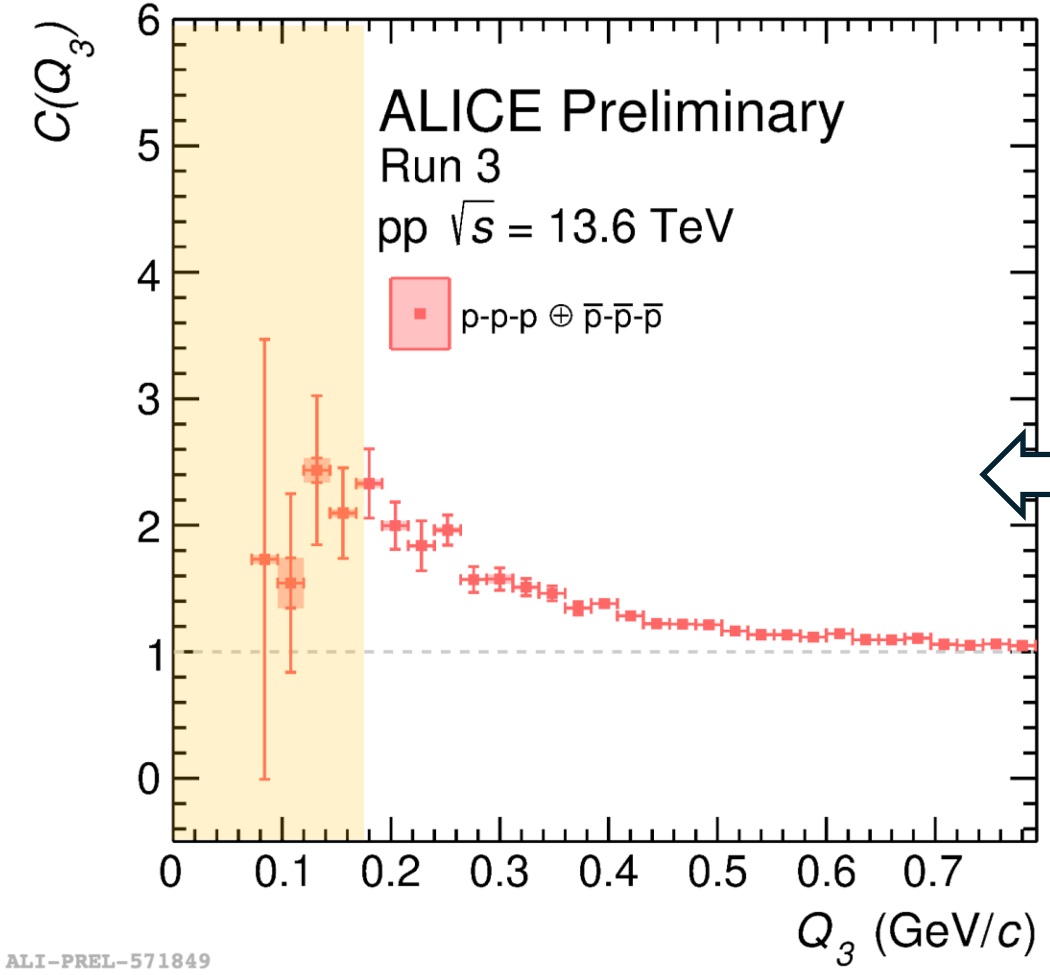
- Access the three-body source
- Need to study asymptotic behavior of wave function in detail (compare shape only since no normalization)



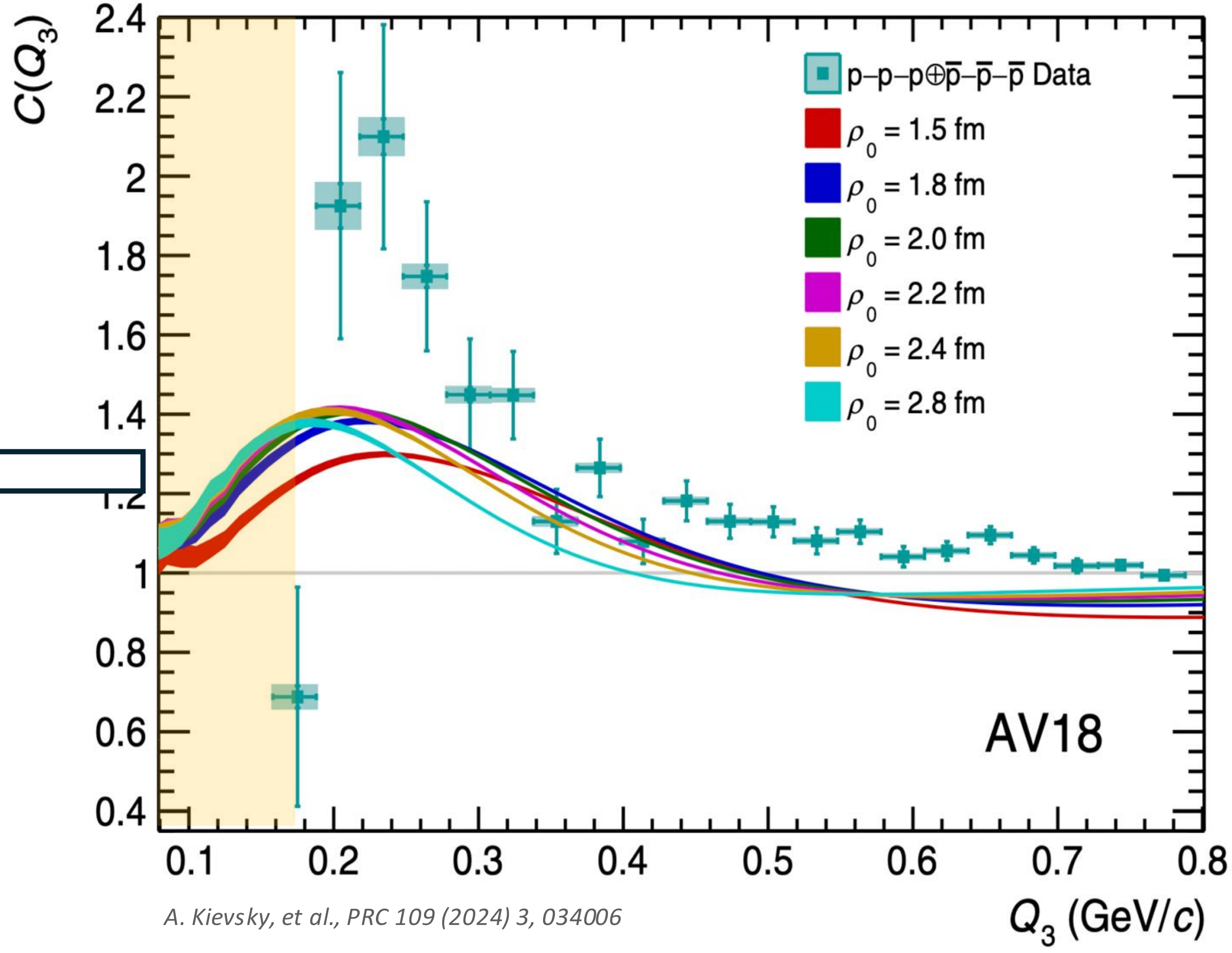
A. Kievsky, et al., PRC 109 (2024) 3, 034006

p-p-p correlation function in Run 3

- ○ ALICE Run 3 data from 2022 are promising!



- ○ ALICE Run 2 (2015-2018)



A. Kievsky, et al., PRC 109 (2024) 3, 034006

p-p-p correlation function in Run 3

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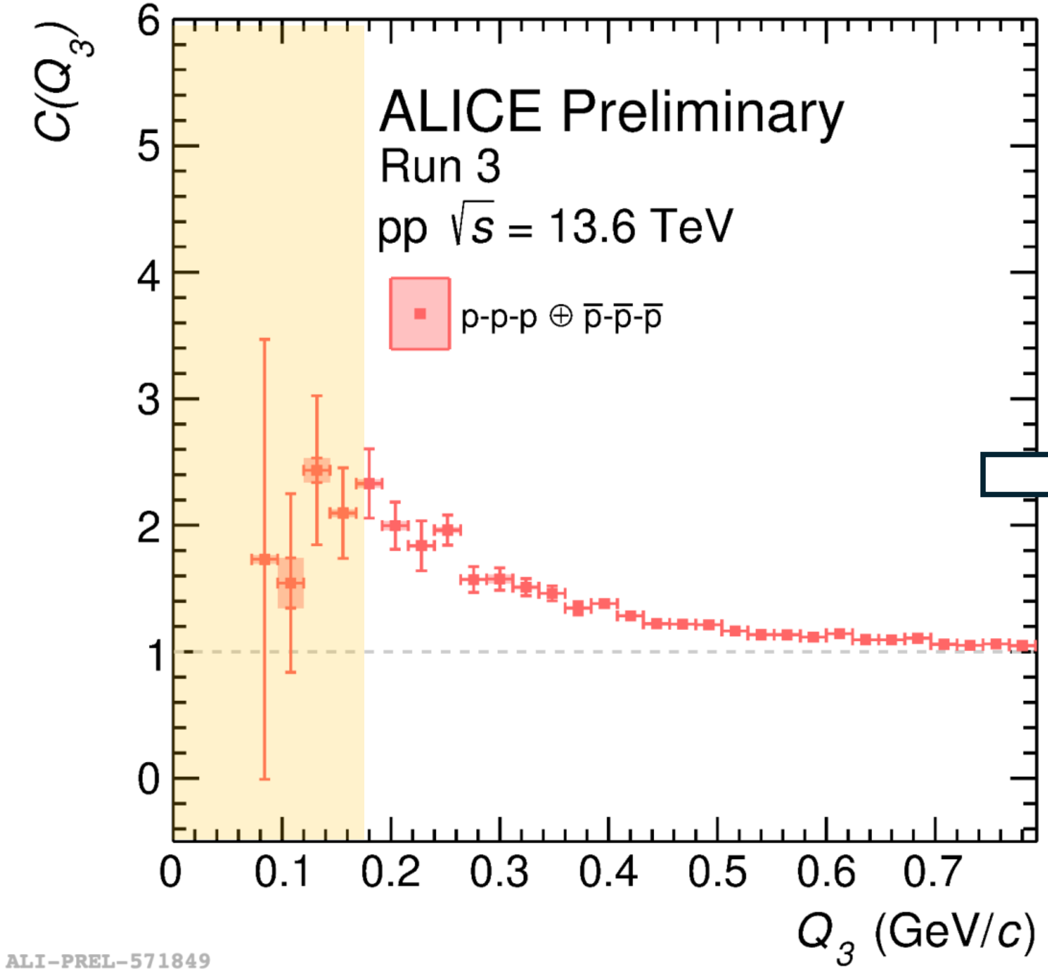
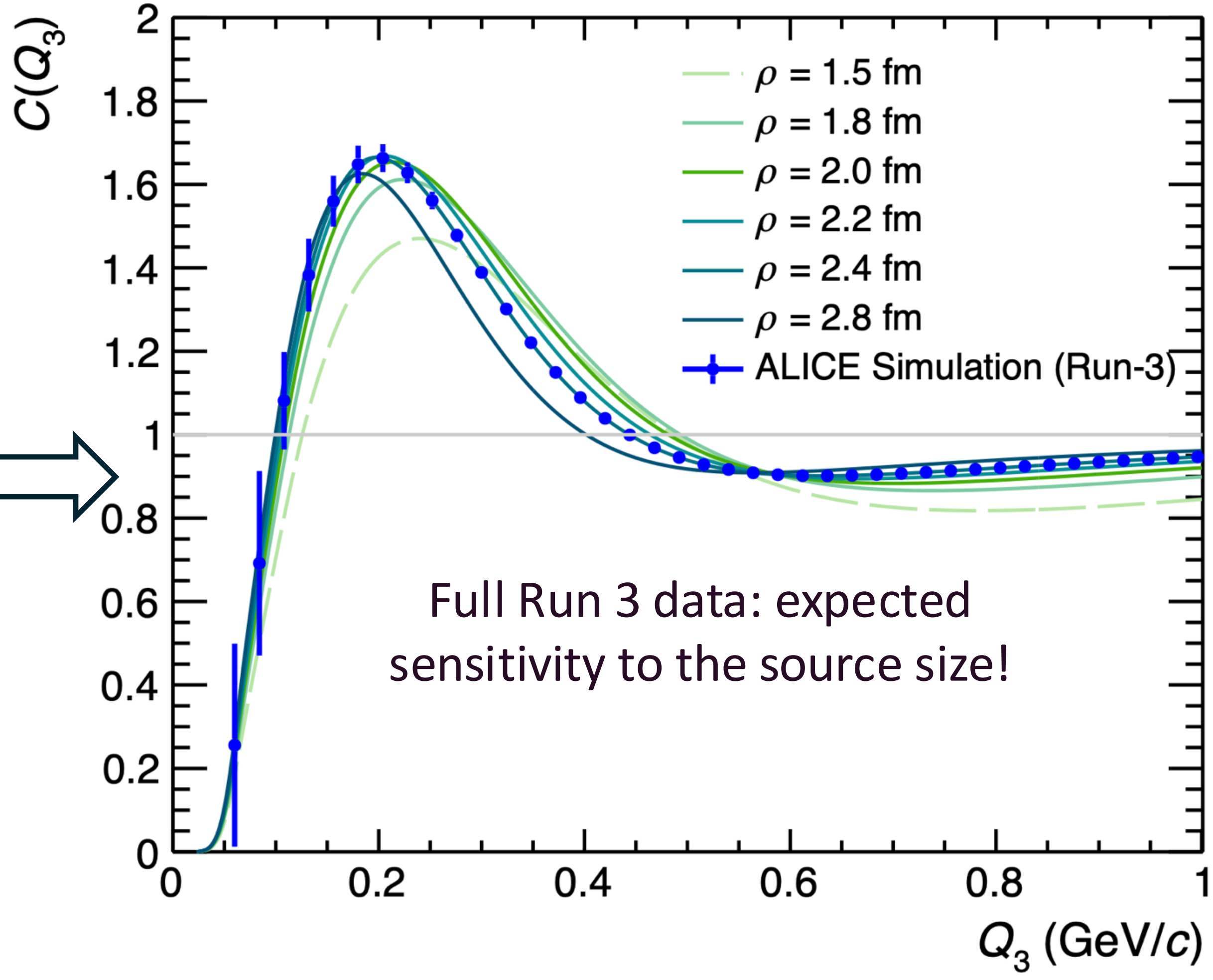


- Run 3: about 25 times larger statistical sample than 2022 alone



Full Run 3 data: expected sensitivity to the source size!

p-p-Λ correlation function in Run 2

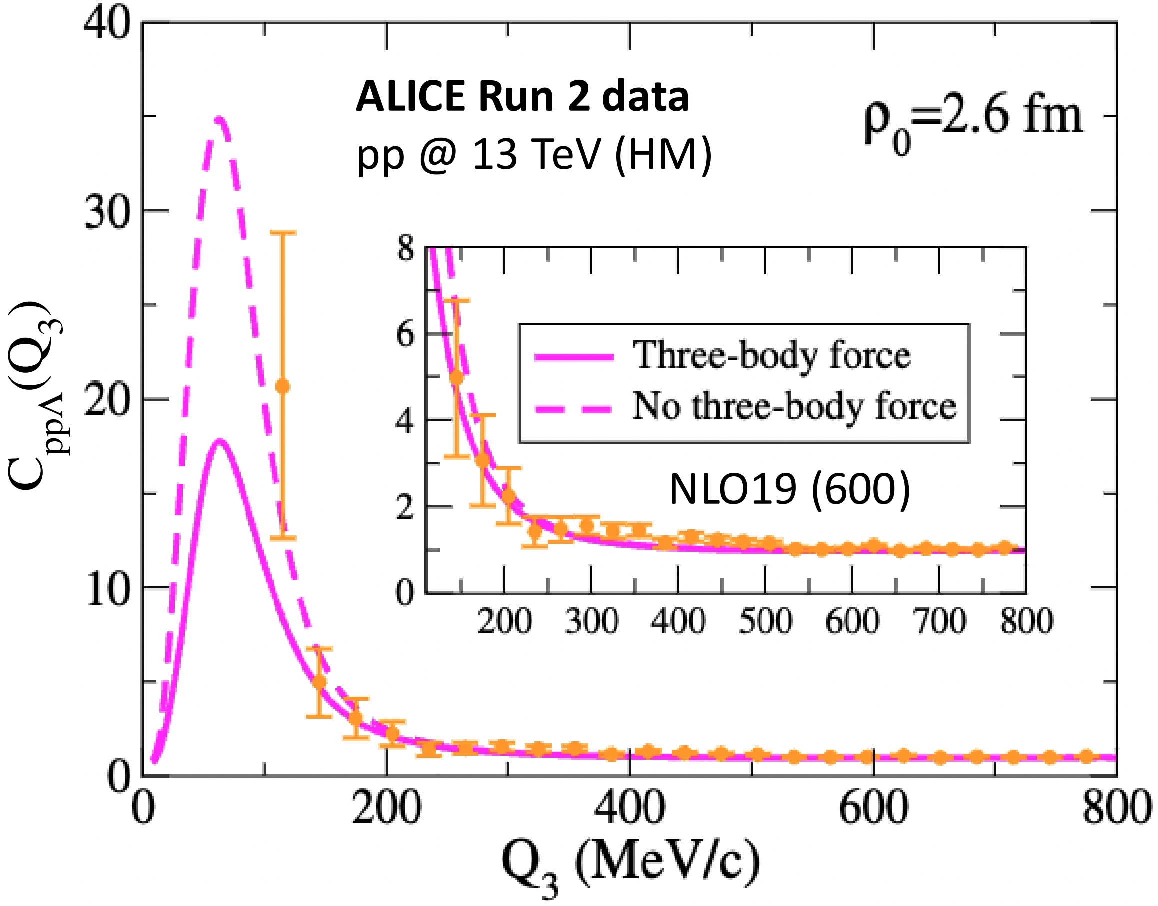
- Two-body NN and ΛN interactions provide an overbinding of the hypertriton

$\text{BE}({}^{3}_{\Lambda}\text{H}) = 2.904\ \text{MeV}$ exp: 2.39 MeV

E. Garrido et al., arXiv: 2408.01750 (2024)

Binding energy from: https://hypernuclei.kph.uni-mainz.de

→ three-body ΛNN interaction



E. Garrido et al., arXiv: 2408.01750 (2024)

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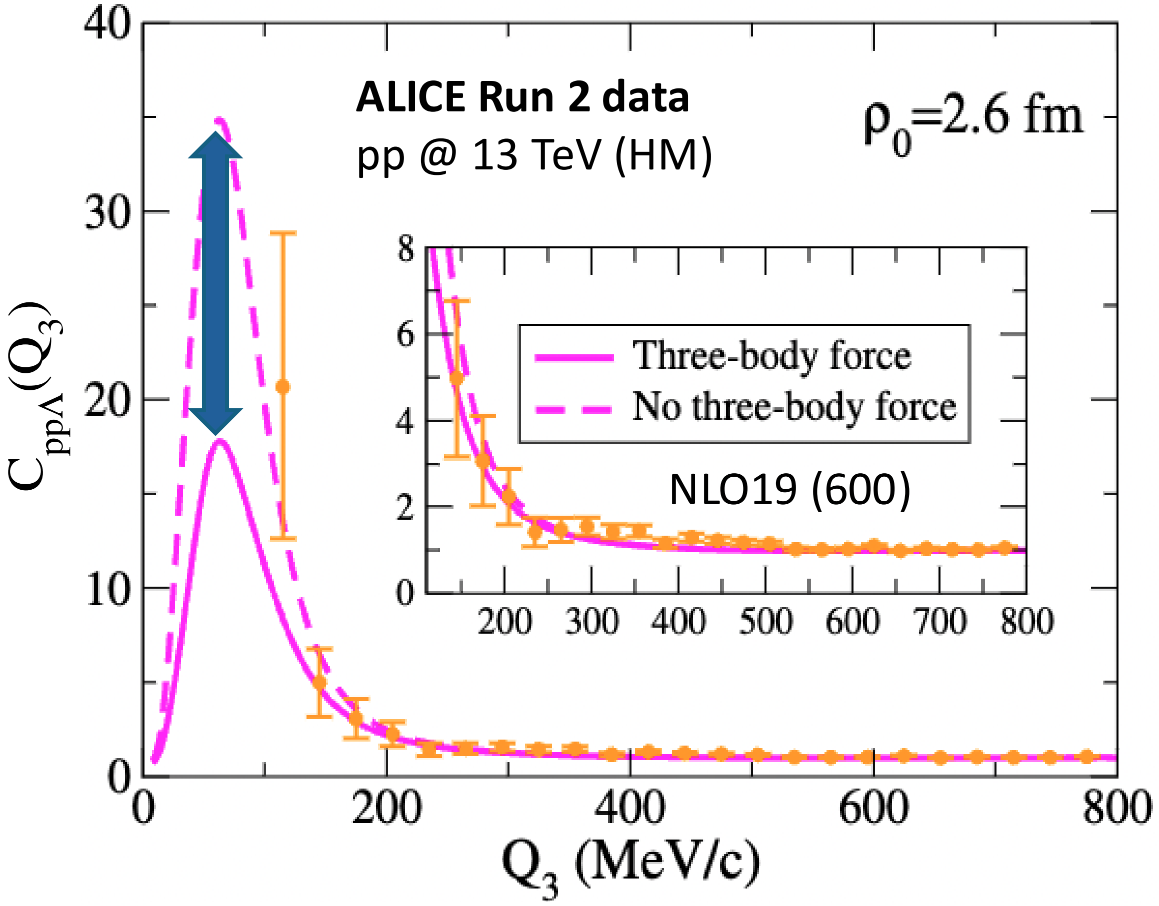
Binding energy from: https://hypernuclei.kph.uni-mainz.de

→ three-body ΛNN interaction

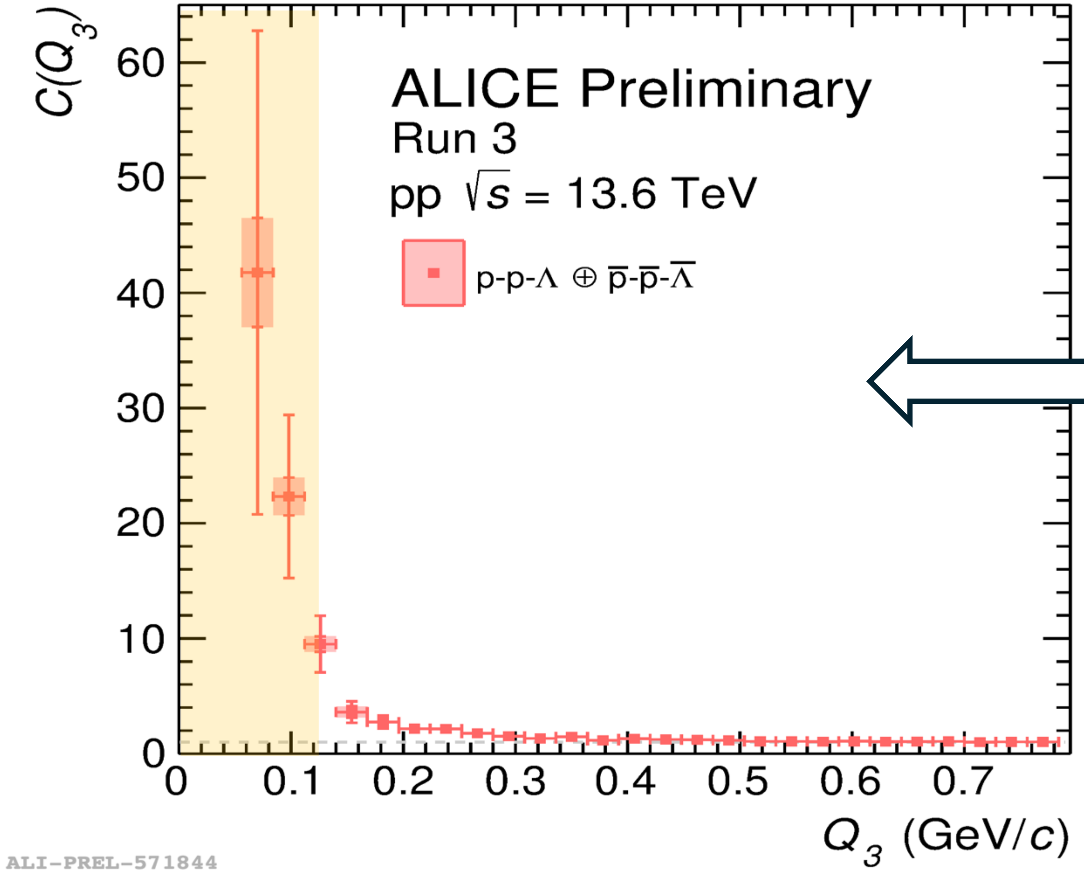
- **ΛNN interaction gives 40% effect**

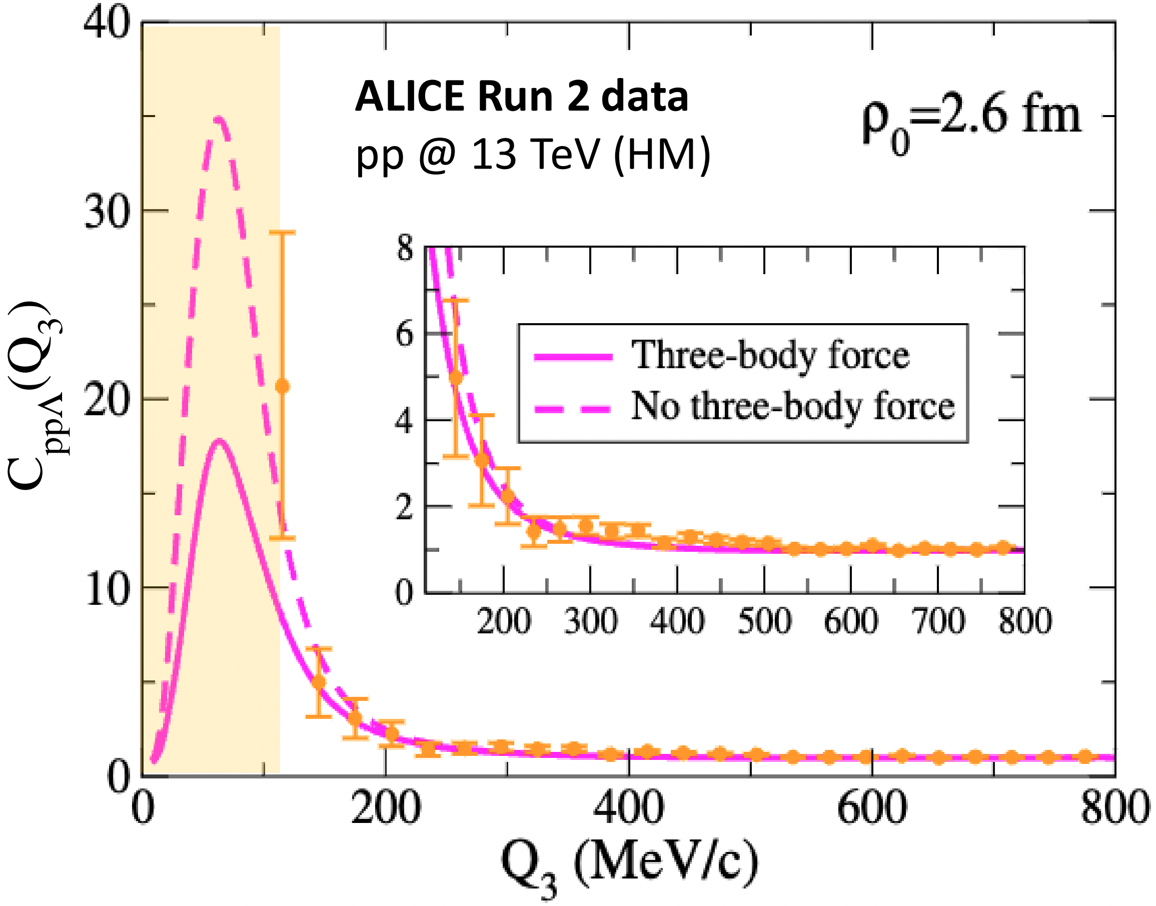


E. Garrido et al., arXiv: 2408.01750 (2024)

p-p-Λ correlation function in Run 3

- ALICE Run 3 data from 2022 are promising!
- By the end of Run 3 x100 improvement software trigger



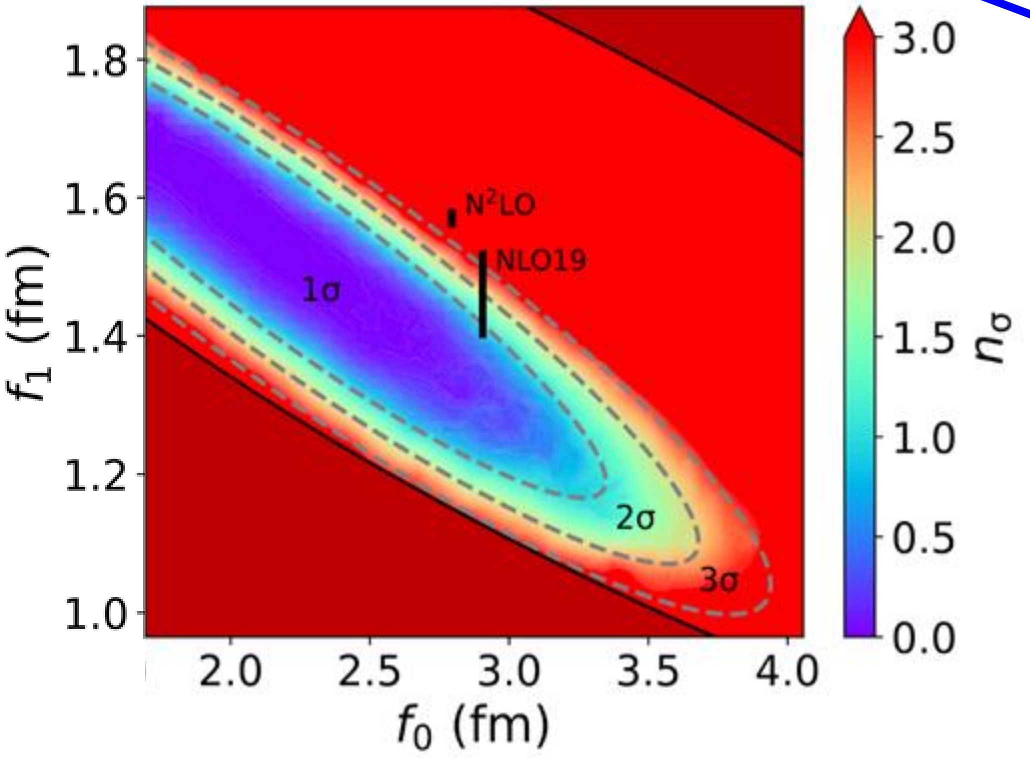


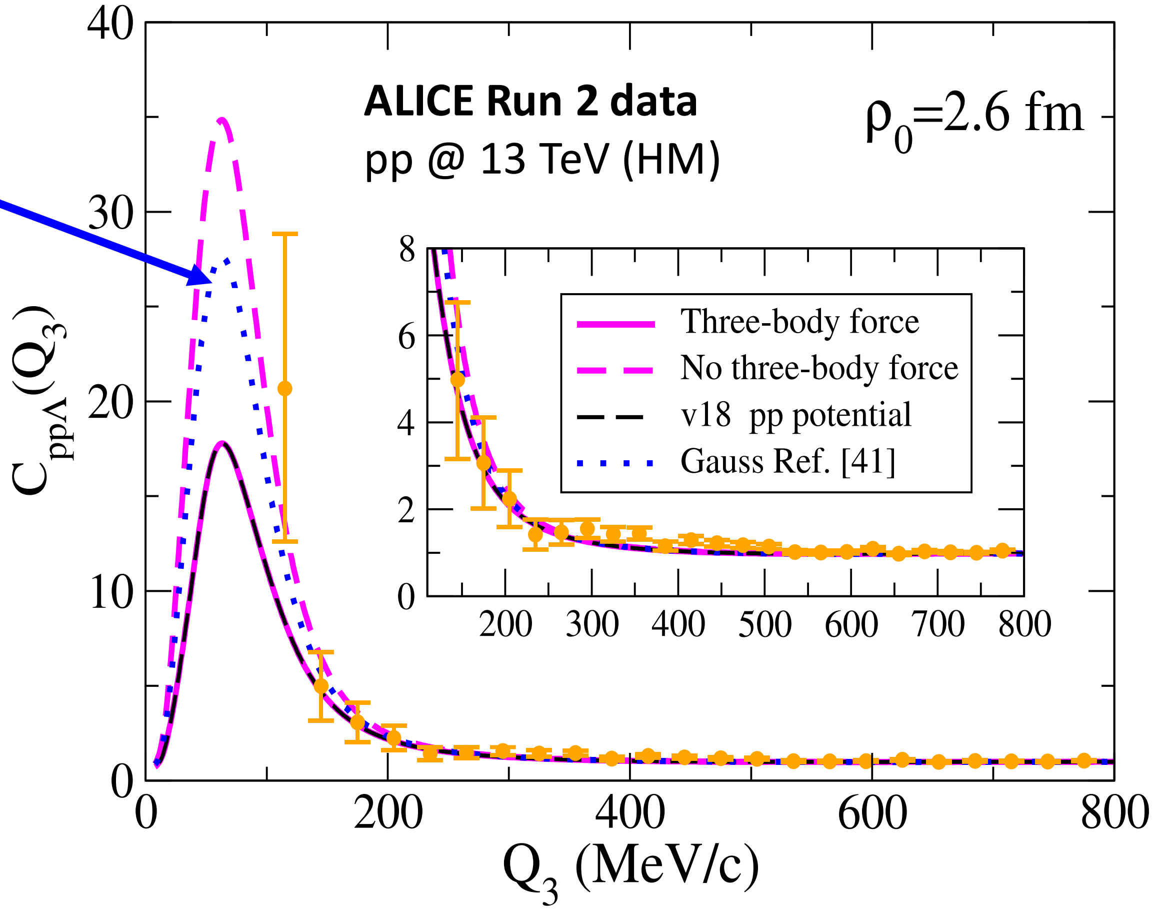
E. Garrido et al., arXiv: 2408.01750 (2024)

p-p-Λ correlation function

- **pΛ interaction**: scattering + femtoscopy

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550





Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

p-p-Λ correlation function

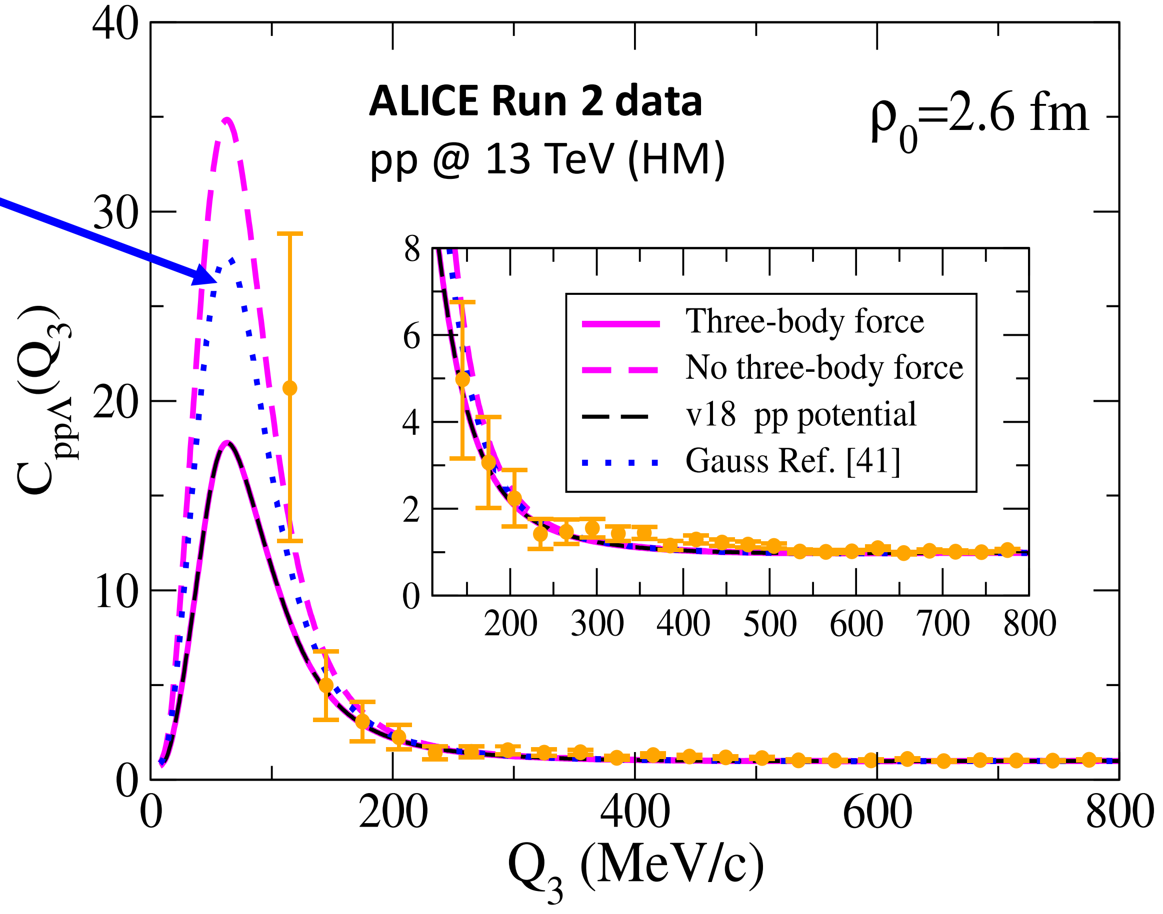
- **pΛ interaction**: scattering + femtoscopy

  D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550
- Hypertriton binding energy can be reproduced using ONLY ΛN interaction

  BE = 2.41 MeV **exp: 2.39 MeV**

  E. Garrido et al., arXiv: 2408.01750 (2024)
- Four-body systems:

  BE = 13.37 MeV* **exp: 10.651 MeV (${}^{4}_{\Lambda}\mathrm{H}$)**
  exp: 10.064 MeV (${}^{4}_{\Lambda}\mathrm{He}$)
- Future:
  Combined analysis of scattering data + hypernuclei + femtoscopy data



Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

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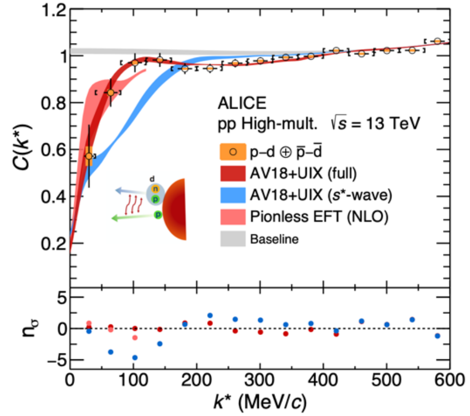
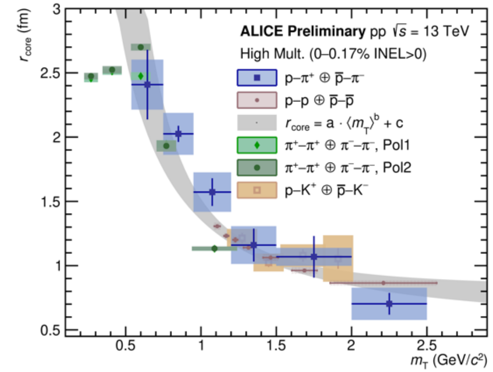
*Private comunication A. Kievsky and E. Garrido

Conclusions and Outlook

Exciting results from femtoscopy:

- p-d correlation function:
  - ✓ Many-body dynamics can be studied using hadron-deuteron correlations
  - ✓ Measured across several system sizes
- pp collisions at the LHC provide access to the interactions at short distances
- p-p-p correlation function:
  - ✓ negligible effect of three-body forces (< 1%)
- p-p-Λ correlations:
  - ✓ 40 % effect of three-body forces





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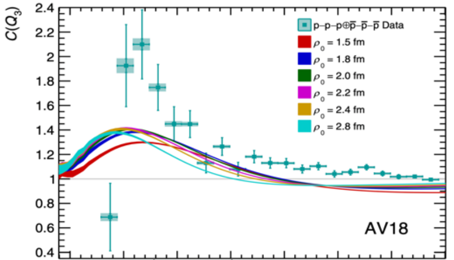
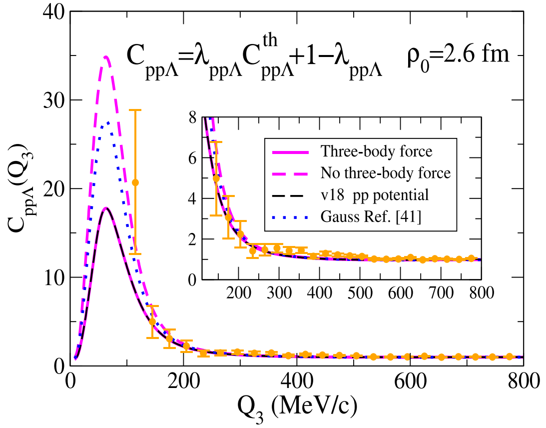


A. Kievsky, et al., PRC 109 (2024) 3, 034006



On-going

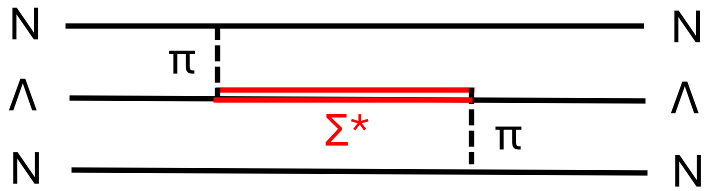
- Access to precise data on three-particle correlations in Run 3 and future Run 4
- New results expected from HADES and STAR for femtoscopy of light nucleii

Backup

Three-body dynamics with hyperons

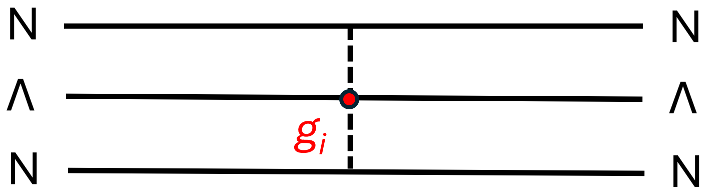
Dynamics of baryons involves formation of hadronic excitations

H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)



Short-range dynamics

Three-body forces:



g_i constants to be fixed by the experimental data

Experimental information from hypernuclear data:

- ○ **Strongly dependent on the ΛN interaction**
- ○ **Overbinding of hypernuclei** → repulsive ΛNN

Hypertriton:

	NLO19				SMS N2LO			Exp.
cutoff (MeV)	500	550	600	650	500	550	600	
$B(^3_\Lambda\mathrm{H})$ (MeV)	2.792	2.839	2.904	3.255	2.819	2.799	2.878	2.39

NLO19: J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91
N2LO: J.Haidenbauer, U. Meißner, A. Nogga, H. Le, EPJA 59 (2023), 3, 63
Exp: Mainz data base

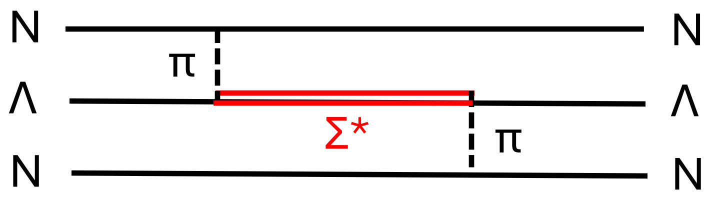
- ○ **Distances of about 2 fm** and in the hypertriton the Λ-d separation is 10 fm
- ○ In dense nuclear matter (neutron stars)
  Large densities (3-5ρ_0) → Small distances (0.8-1.0 fm)

Can we use three particle correlations?

Three-body dynamics with hyperons

Dynamics of baryons involves formation of hadronic excitations

H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)



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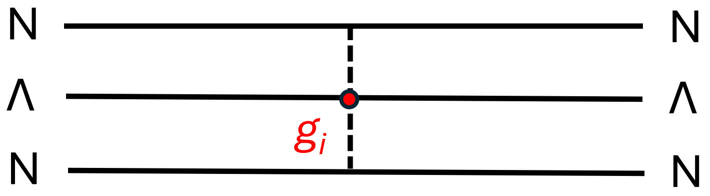
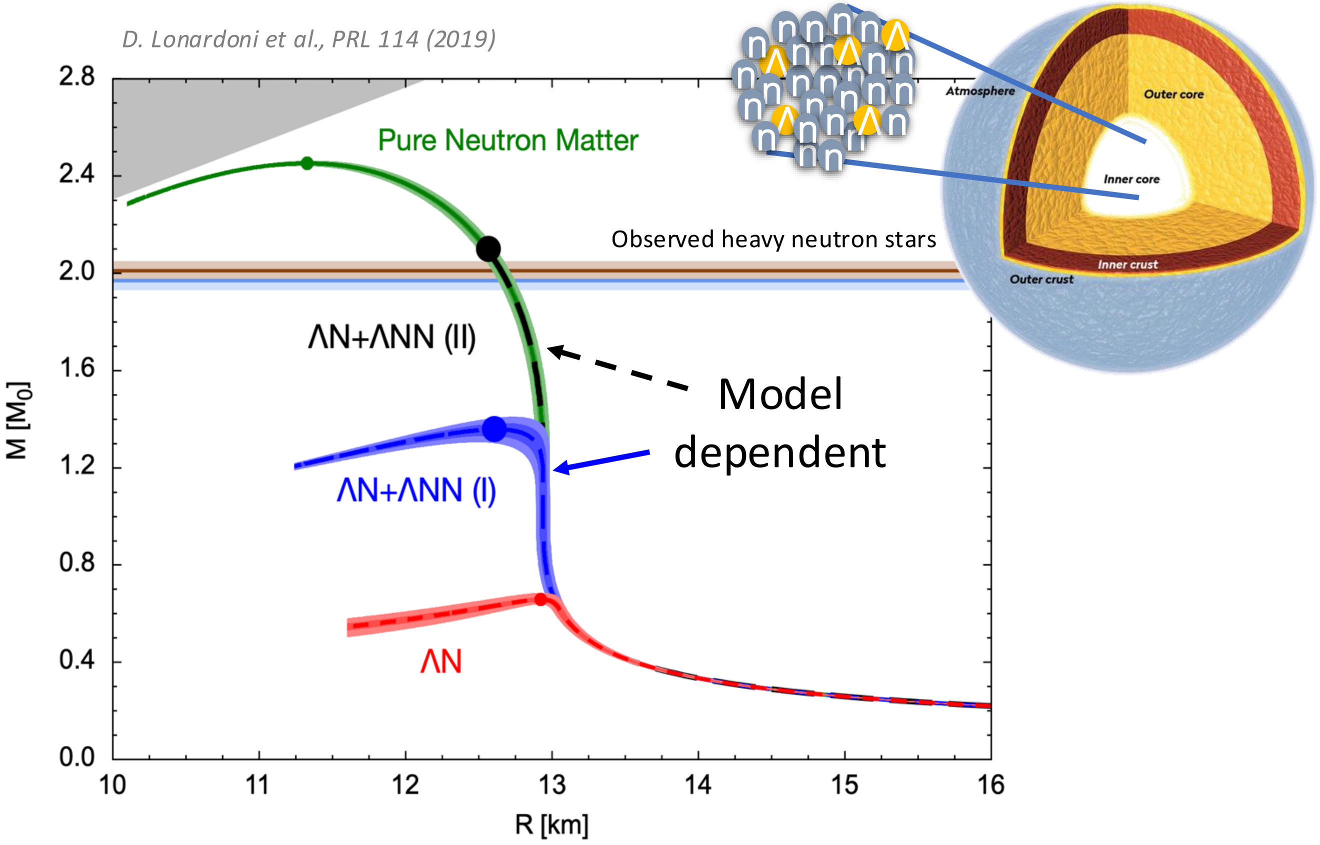


g_i constants to be fixed by the experimental data

Impact on dense nuclear matter (neutron stars)

Large densities (3-5ρ_0) → Small distances (0.8-1.0 fm)


D. Lonardoni et al., PRL 114 (2019)

Small particle distances can be accessed using femtoscopy!

Source function in pp collisions at the LHC

- Emitting source function anchored to p-p correlation function

$$\underbrace{C(k^*)}_{\text{measured}} = \int S(\vec{r}\,)\underbrace{\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2}_{\text{known interaction}} d^3\vec{r}$$

- Gaussian parametrization

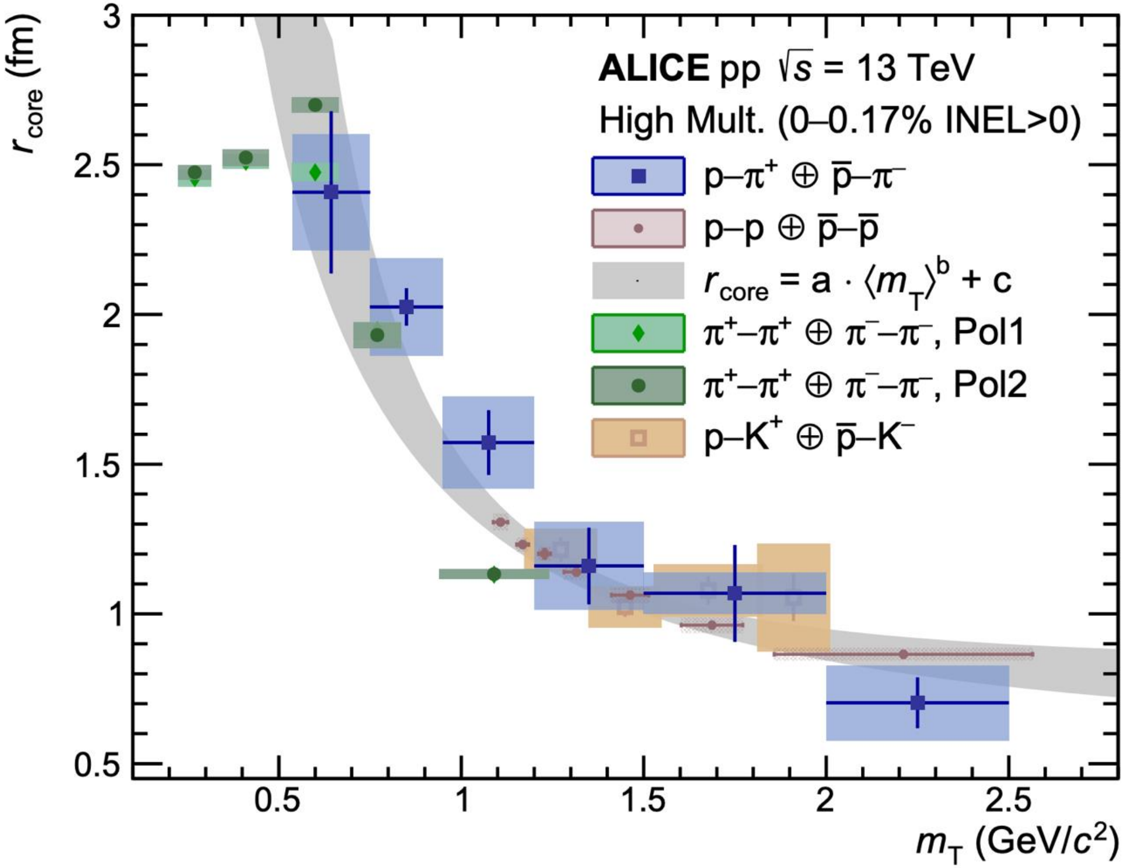
$$S(r) = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right) \times$$

Effect of short lived resonances (cτ ~ 1 fm)

ALICE Coll., PLB, 811 (2020), 135849

- One universal source for all hadrons (cross-check with K$^+$-p, π-π, p-Λ, p-π)
- **Small particle-emitting source created in pp collisions at the LHC**

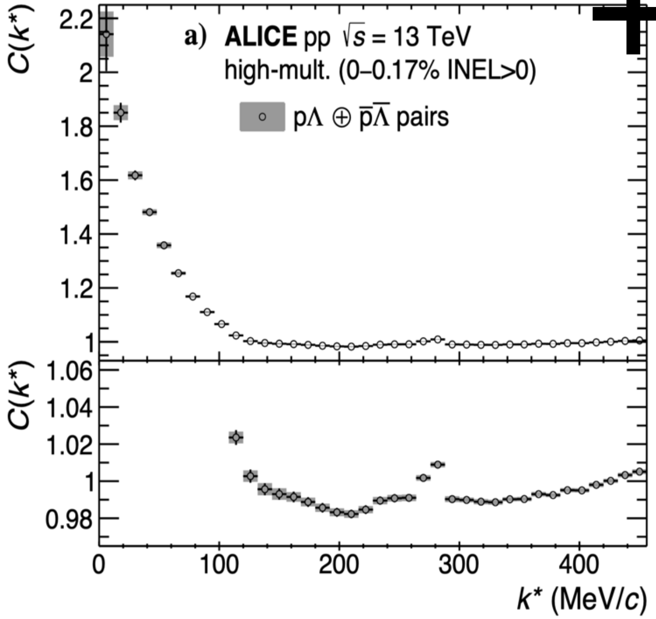
ALICE Coll., arXiv:2311.14527
ALICE Coll., PLB, 811 (2020), 135849
ALICE Coll, paper in preparation



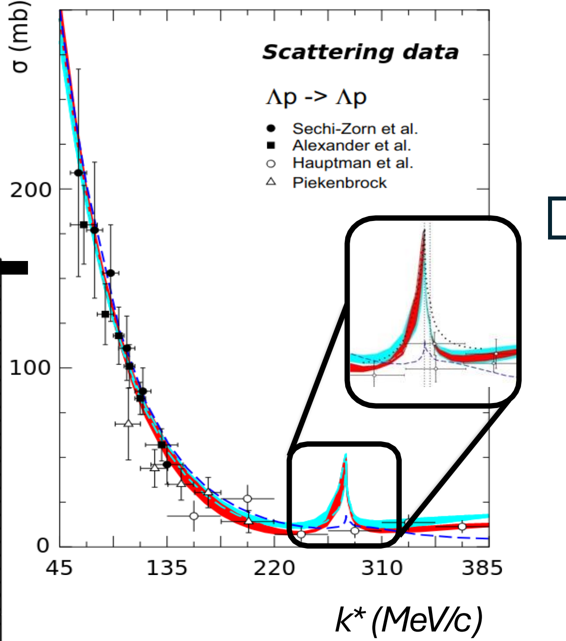
***Talk:** Anton Riedel 11 Sep, 09:00*

The pΛ interaction in the femtoscopy era

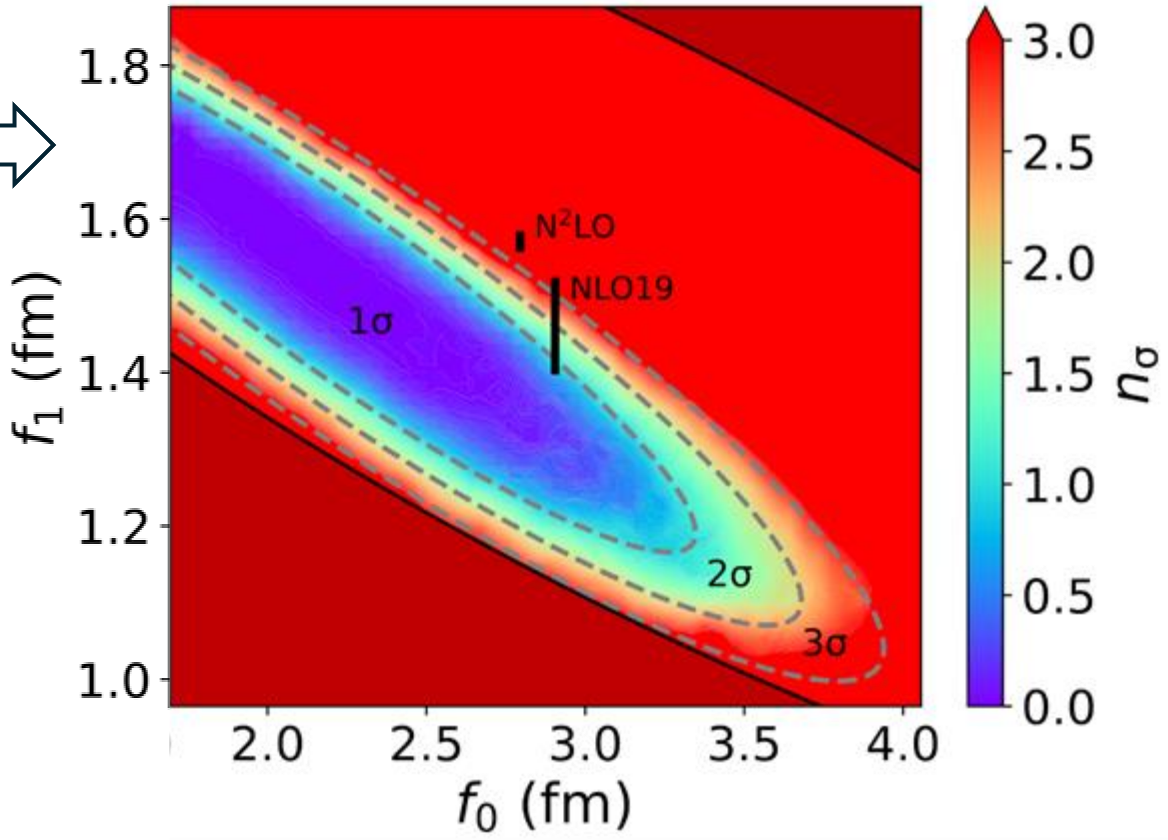
- **Improvement:** combined analysis of femtoscopic and scattering data



ALICE coll. PLB 833 (2022), 137272



J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91



D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

p-p-Λ correlation function

- Reference calculations:
  → two-body NN and ΛN interactions provide an overbinding of the hypertriton

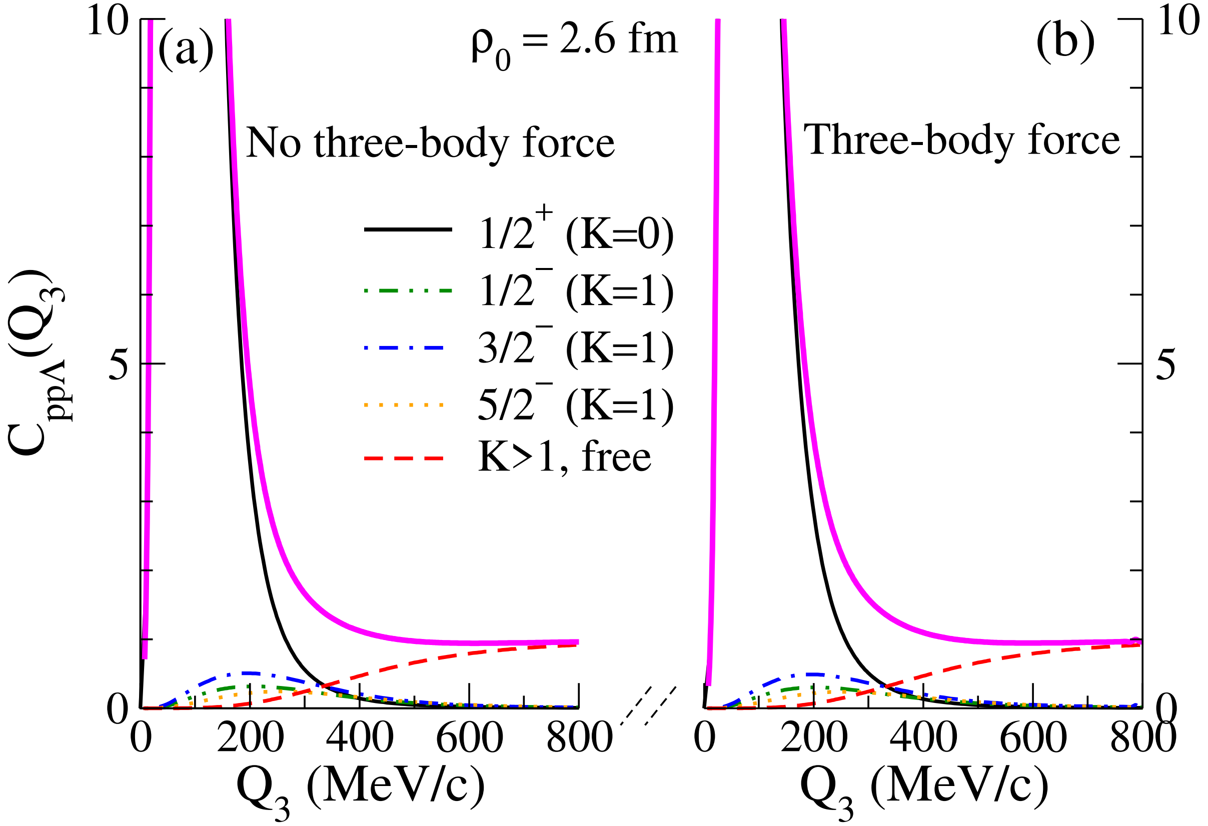
  BE($^{3}_{\Lambda}$H) = 2.904 MeV exp: 2.39 MeV

  E. Garrido et al., arXiv: 2408.01750 (2024)

  Binding energy from: https://hypernuclei.kph.uni-mainz.de

  → three-body ΛNN interaction

- ΛNN interaction gives 50% effect:
  only one partial wave (K=0) significantly contributes



Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

NNN using proton-deuteron correlations

- The p–d **correlation function**, assuming that p–p–n forms p–d

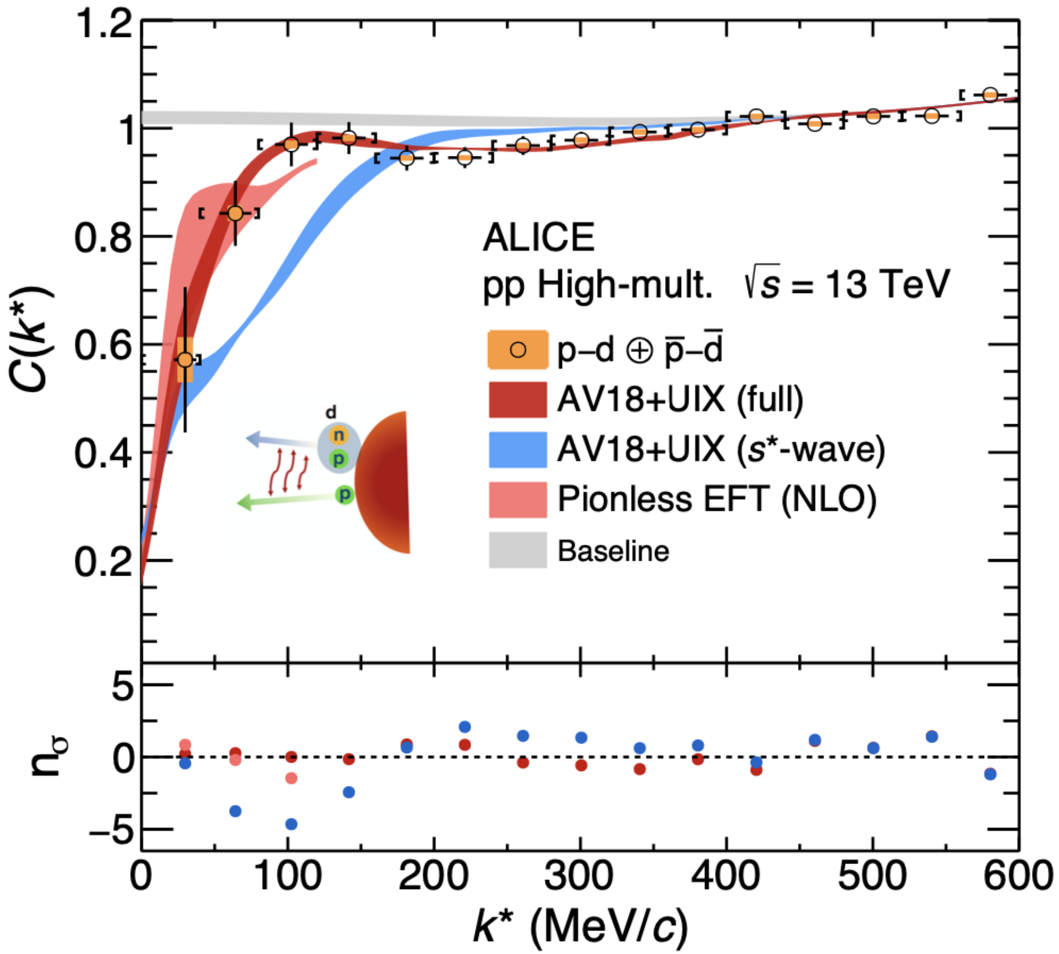
$$C_{pd}(k) = \frac{1}{A_d}\frac{1}{6}\sum_{m_1,m_2} \int d^3r_1 d^3r_2 d^3r_3 S_1(r_1) S_1(r_2) S_1(r_3) \left|\Psi_{m_1,m_2}\right|^2$$

where $S_1(r)$ is a single-particle Gaussian source and A_d is the formation probability of a deuteron

- The **three-body wavefunction** of the p–d System

$$\Psi_{m_2,m_1}(x,y) = \underbrace{\Psi^{free}_{m_2,m_1}}_{\text{Asymptotic solution}} + \underbrace{\sum_{LSJ}^{J\le\bar{J}} \sqrt{4\pi}\; i^L \sqrt{2L+1}\; e^{i\sigma_L} \left(1 m_2 \frac{1}{2} m_1 \middle| S J_z\right) (\mathrm{L0SJ_z}|\mathrm{JJ_z}) \tilde{\Psi}_{LSJJ_z}}_{\text{Three-body dynamics}}$$

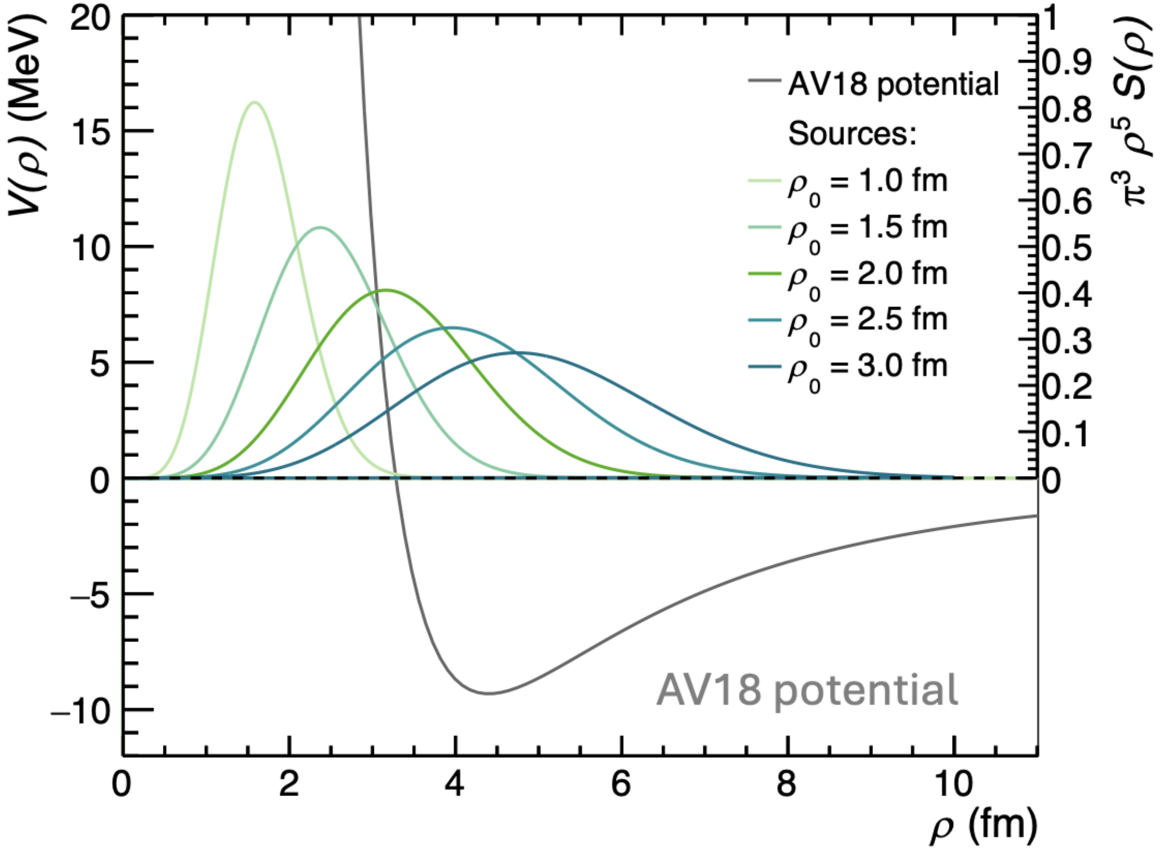
- **Hadron-nuclei correlations at the LHC can be used to study many-body dynamics**

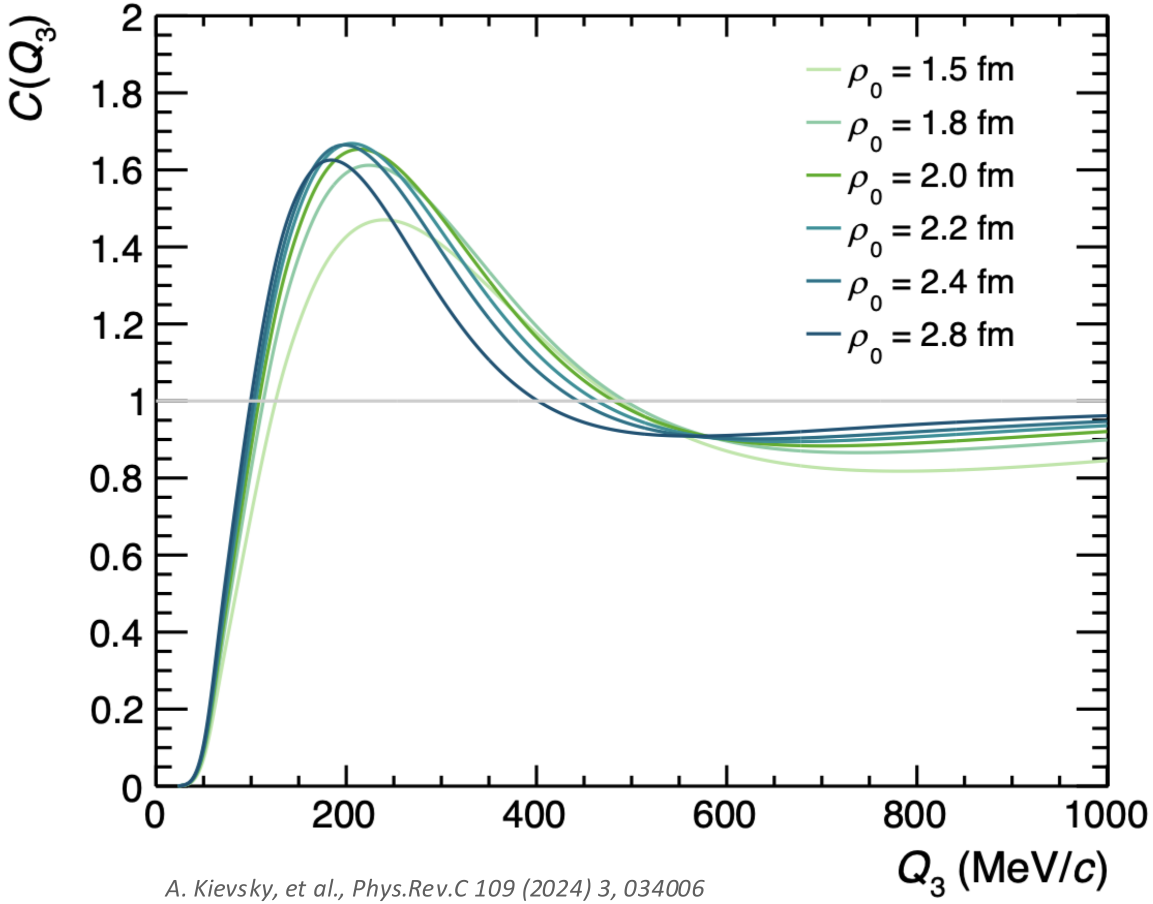


ALICE Coll. arXiv:2308.16120 (2023), accepted by PRX
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

Influence of the source size

- Using m_T (source size) differential studies we can probe the interaction with the distances



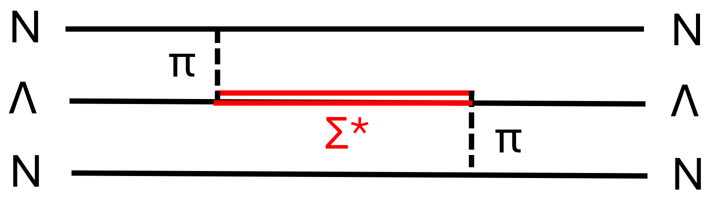


A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

Three-body dynamics with hyperons

Dynamics of baryons involves formation of hadronic excitations

H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)



Short-range dynamics

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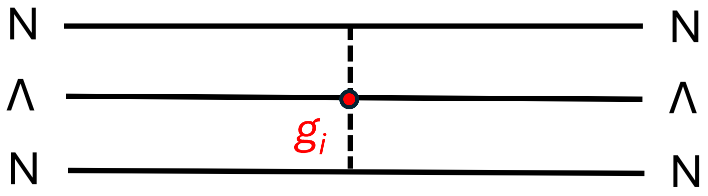


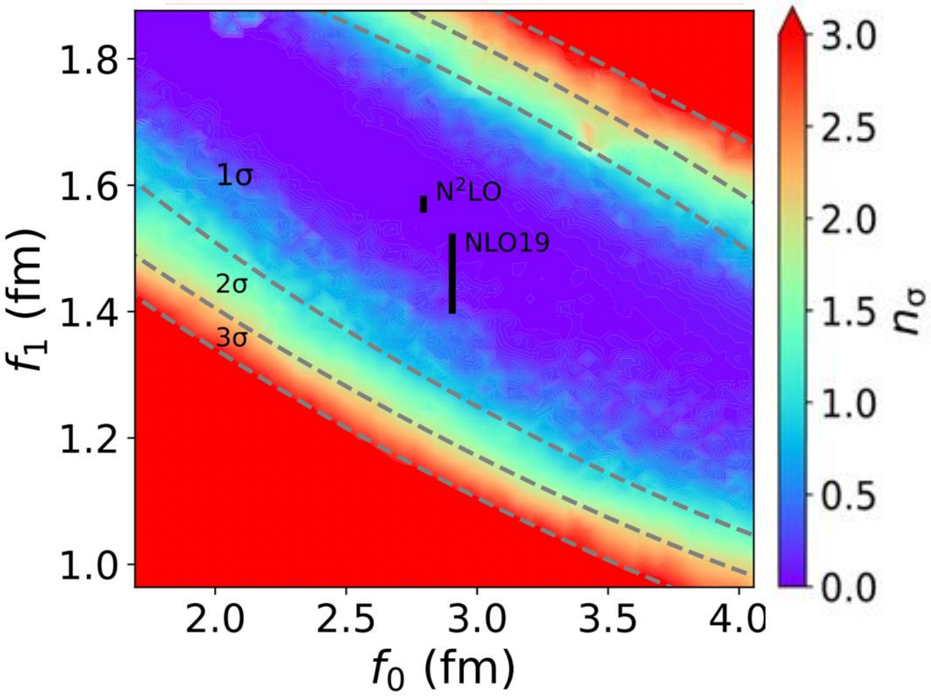
g_i constants to be fixed by the experimental data

Experimental information from hypernuclear data:

- **Distances of about 2 fm**
  - → in $^{3}_{\Lambda}\mathrm{H}$ the Λ is 10 fm from d

A. Gal, E. V. Hungerford, and D. J. Millener, RMP 88, 035004 (2016)

- **Strongly dependent on the ΛN interaction**



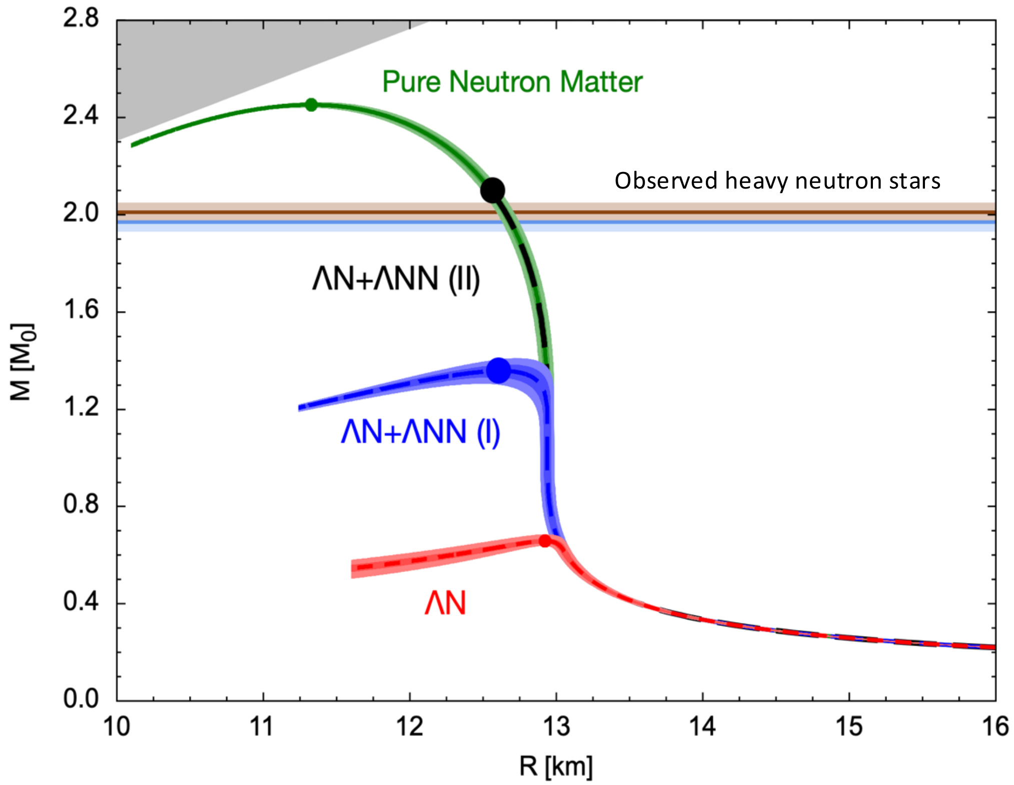
D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

The NΛ and NNΛ interactions in neutron stars

- High density in the core of neutron stars
  → Production of hyperons as Λ at $\rho = 2\text{-}3\rho_0$ and softening of the equation of state (EoS)
  → Incompatibility with astrophysical measurements of $M_{NS} \gtrsim 2\ M_\odot$

- Repulsive 3-body ΛNN interaction can stiffen the EoS but:
  → Effect on EoS largely model dependent
  D. Logoteta et al., EPJA 55 (2019); D. Lonardoni et al., PRL 114 (2019)
  → Is ΛNN interaction repulsive at finite densities?
  → How can we access the short range part of the interaction?

We can exploit femtoscopy measurements



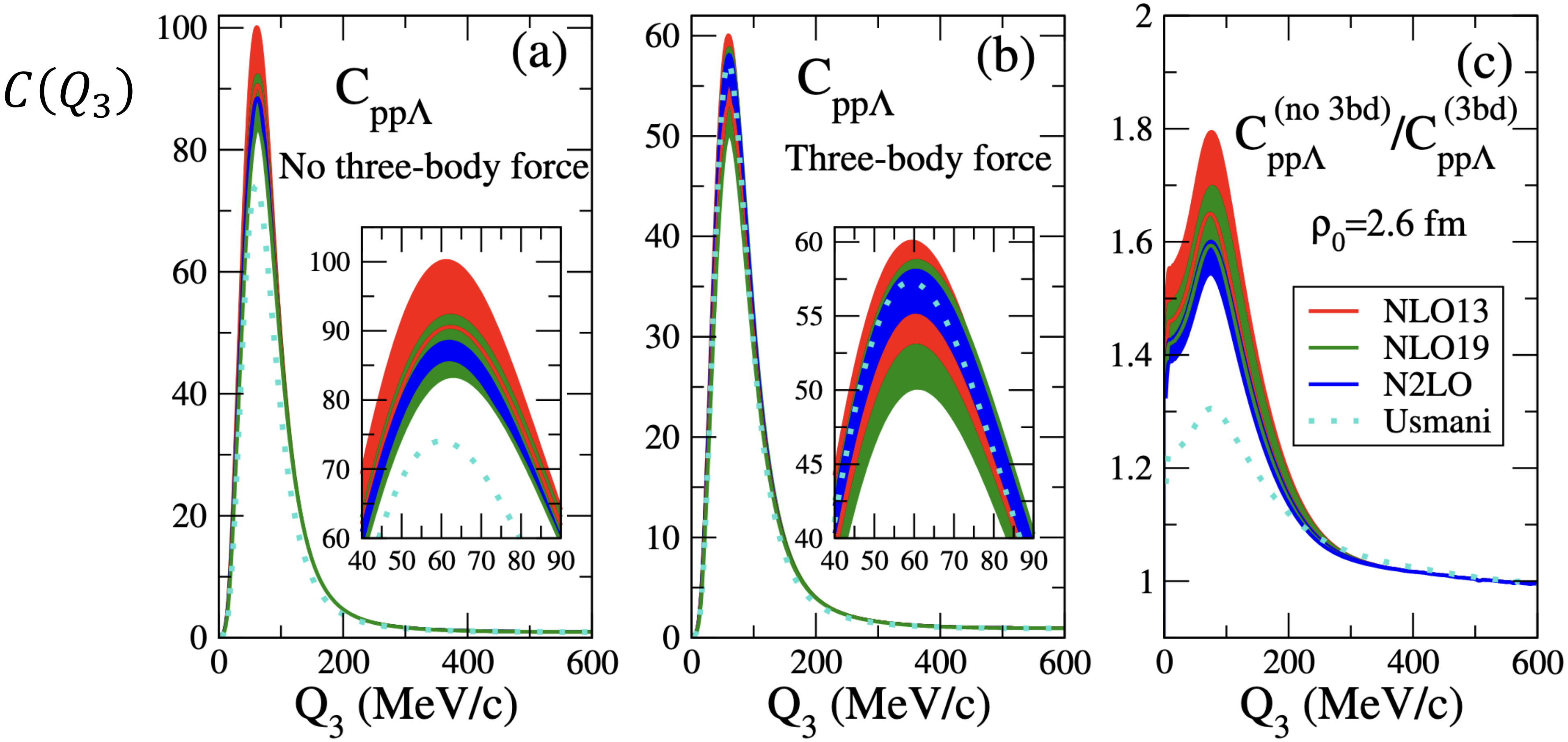
D. Lonardoni et al., PRL 114 (2019)

Talk by I. Vidana

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

Effect of the three-body interaction

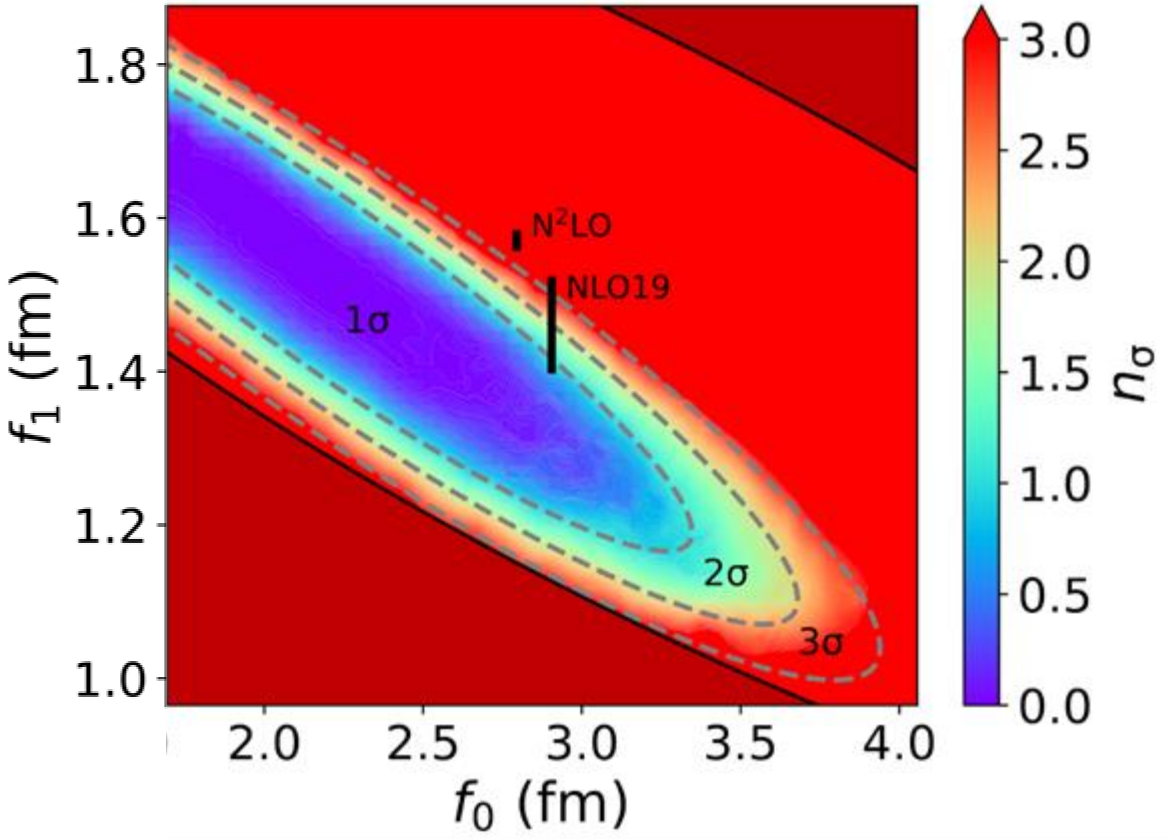
- Effect of the three-body interaction for different interaction models: 30% (Usmani) - 80% (NLO13)



The pΛ interaction in the femtoscopy era

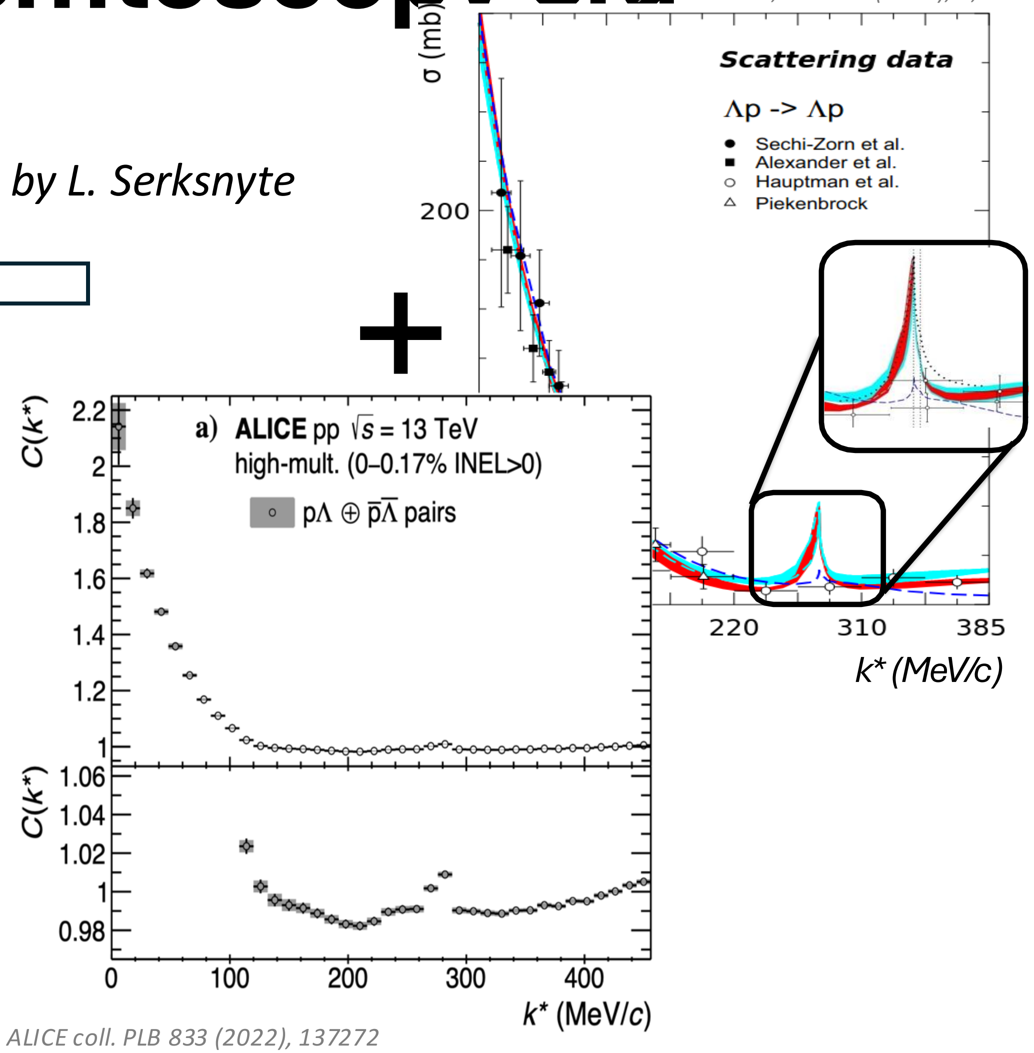
- **Improvement:** combined analysis of femtoscopic and scattering data

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550



Talk by L. Serksnyte

J. Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91



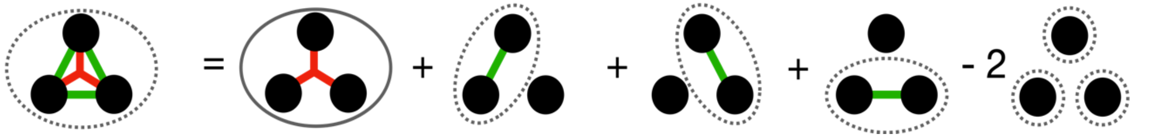
ALICE coll. PLB 833 (2022), 137272

p-p-p correlation function

- Cumulant method provides first hint of effects beyond two-body correlations

R. Kubo, J. Phys. Soc. Jpn. 17, 1100-1120 (1962)

R. Del Grande et al. EPJC 82 (2022) 244



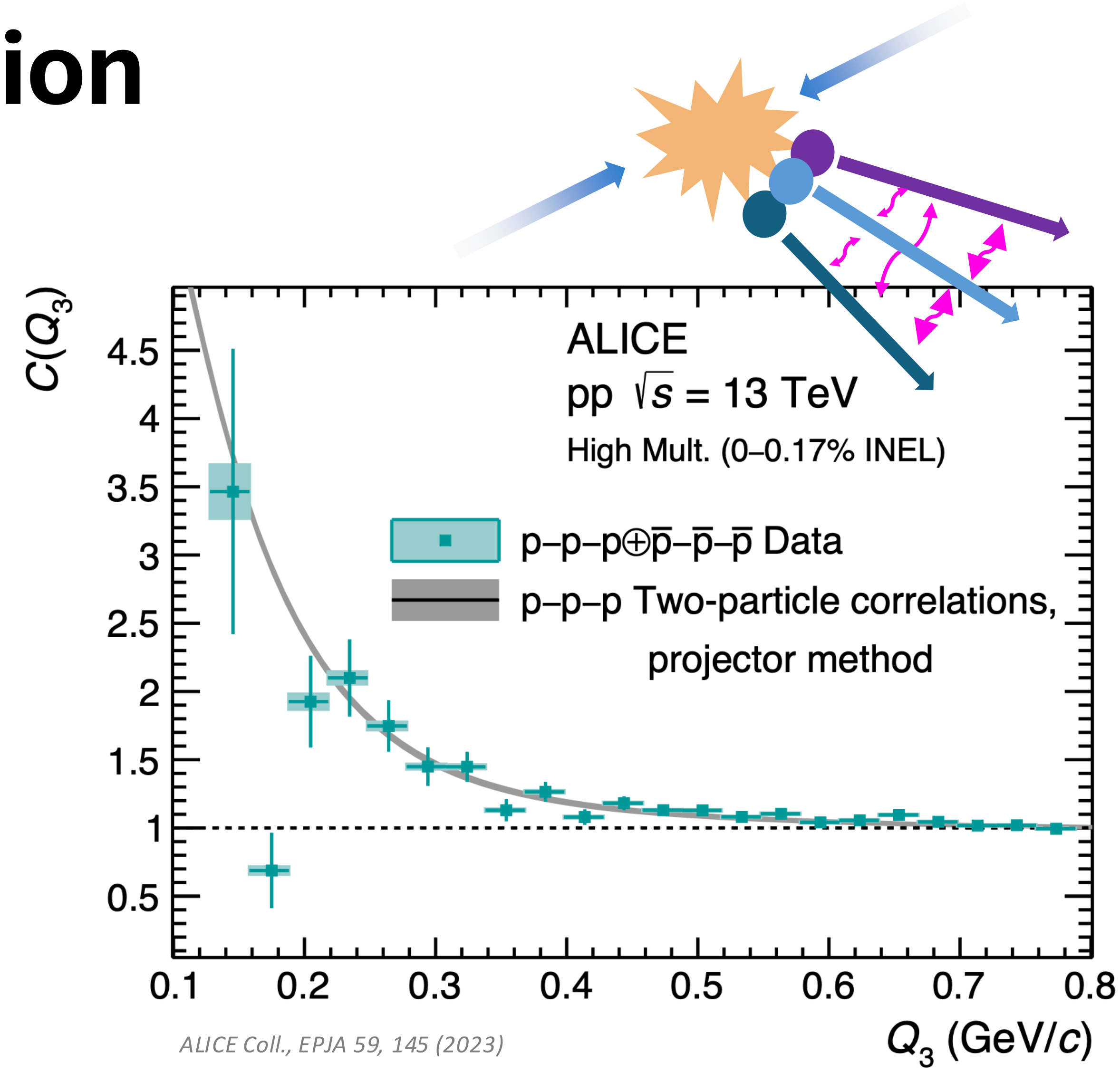
Measured three-body correlation

Genuine three-body correlations

Two-body correlations

Single particle contribution

- A deviation of nσ = 6.7 from lower-order contributions
- Theoretical predictions necessary to understand the origin of the deviation further



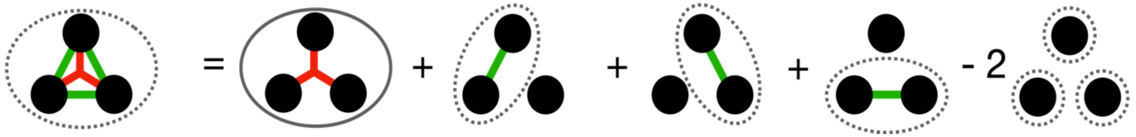
ALICE Coll., EPJA 59, 145 (2023)

p-p-Λ correlation function

- Cumulant method provides first hint of effects beyond two-body correlations

R. Kubo, J. Phys. Soc. Jpn. 17, 1100-1120 (1962)

R. Del Grande et al. EPJC 82 (2022) 244



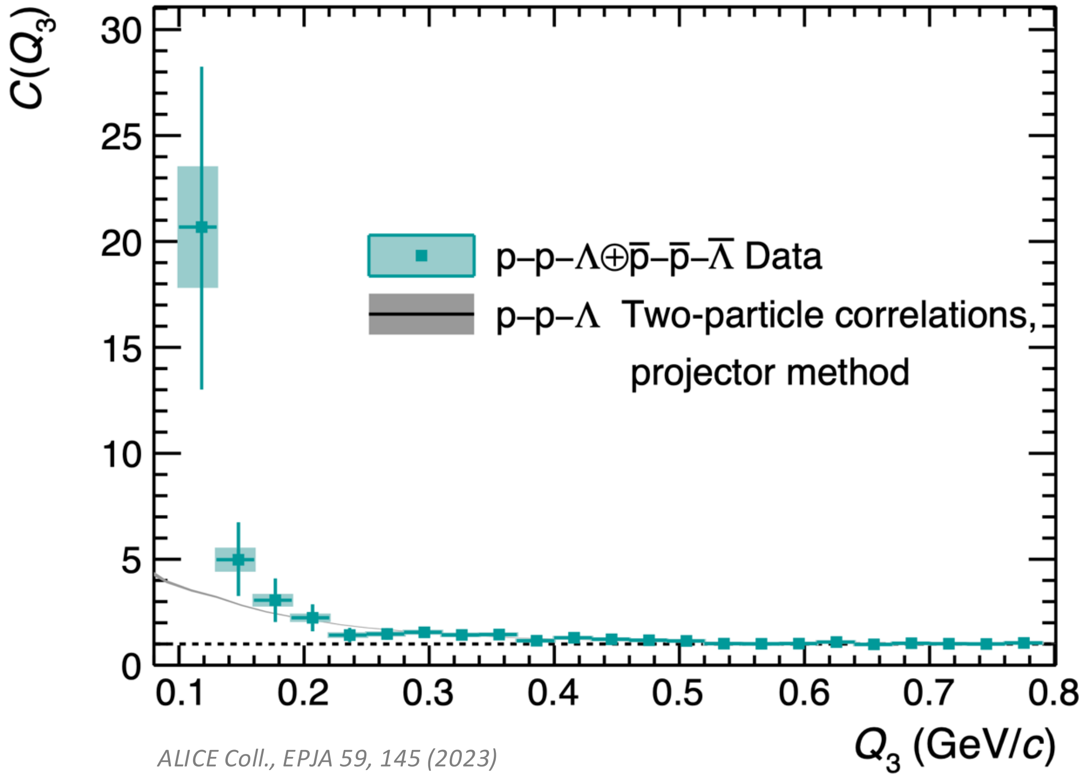
Measured three-body correlation

Genuine three-body correlations

Two-body correlations

Single particle contribution

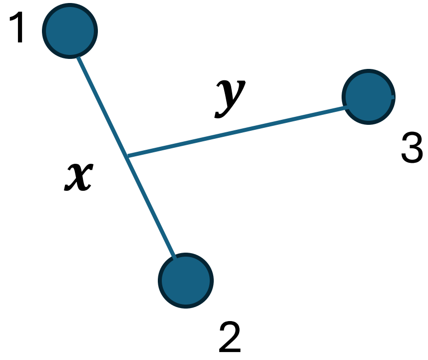
- Compatible with lower-order contributions (nσ = 0.8)



ALICE Coll., EPJA 59, 145 (2023)

Hyperspherical Harmonics formalism

- The Jacobi coordinates:



$$\begin{cases} \boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \\ \boldsymbol{y} = \sqrt{\frac{4}{(1+2m/M)}}\left(\boldsymbol{r}_3 - \frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right) \end{cases}$$

- We introduce the hyperradius and hyperangle:

$$\rho = \sqrt{x^2 + y^2} \qquad \phi = \arctan\left(\frac{y}{x}\right)$$

- Now the 6 variables are: $(\rho, \phi, \hat{\mathbf{x}}, \hat{\mathbf{y}})$ → 1 radius, 5 angles

Hyperspherical Harmonics formalism

Defining the wave function as:

$$\psi = \sum_{[K]} \rho^{-5/2} u_{[K]}(\rho) Y_{[K]}(\Omega)$$

Schroedinger equation with the interaction:

$$\left(\frac{\partial^2 u_{[K]}(\rho)}{\partial \rho^2} - \frac{(K+3/2)(K+5/2)}{\rho^2} u_{[K]}(\rho) \right) + \sum_{[K']} U_{[K][K']}(\rho)\, u_{[K']}(\rho) = Q^2\, u_{[K]}(\rho)$$

Where the hypercentral potential is obtained as

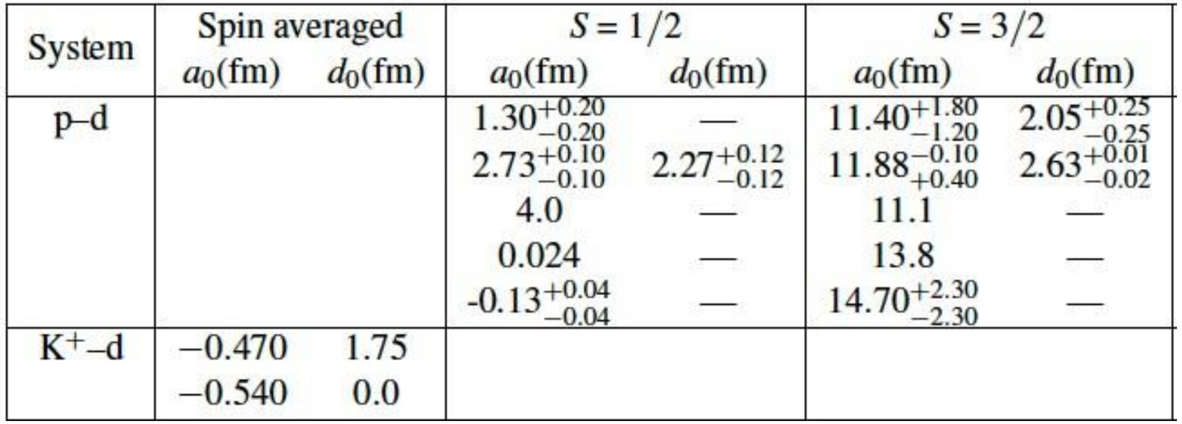
$$U_{[K][K']}(\rho) = \int d\Omega\, Y^*_{[K]}(\Omega)\left[V_{12} + V_{23} + V_{31} + V_{123}\right] Y_{[K']}(\Omega)$$

Kaon/Proton-deuteron correlation

- Effective two-body system
  - Coulomb + Strong interactions via Lednický model; only s-wave
  - Anchored to scattering experiments
  - Emission source: from m_T scaling

R. Lednický, Phys. Part. Nucl. 40, 307(2009)

W. T. H. Van Oers, & K. W. Brockman Jr, NPA 561 (1967); J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);
A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);



System	Spin averaged		$S=1/2$		$S=3/2$	
	a_0(fm)	d_0(fm)	a_0(fm)	d_0(fm)	a_0(fm)	d_0(fm)
p–d			$1.30^{+0.20}_{-0.20}$	—	$11.40^{+1.80}_{-1.20}$	$2.05^{+0.25}_{-0.25}$
			$2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$11.88^{-0.10}_{+0.40}$	$2.63^{+0.01}_{-0.02}$
			4.0	—	11.1	—
			0.024	—	13.8	—
			$-0.13^{+0.04}_{-0.04}$	—	$14.70^{+2.30}_{-2.30}$	—
K^+–d	−0.470	1.75				
	−0.540	0.0				

**R. Lednicky and V. L. Lyuboshits Sov. J. Nucl. Phys. 35 (1982)

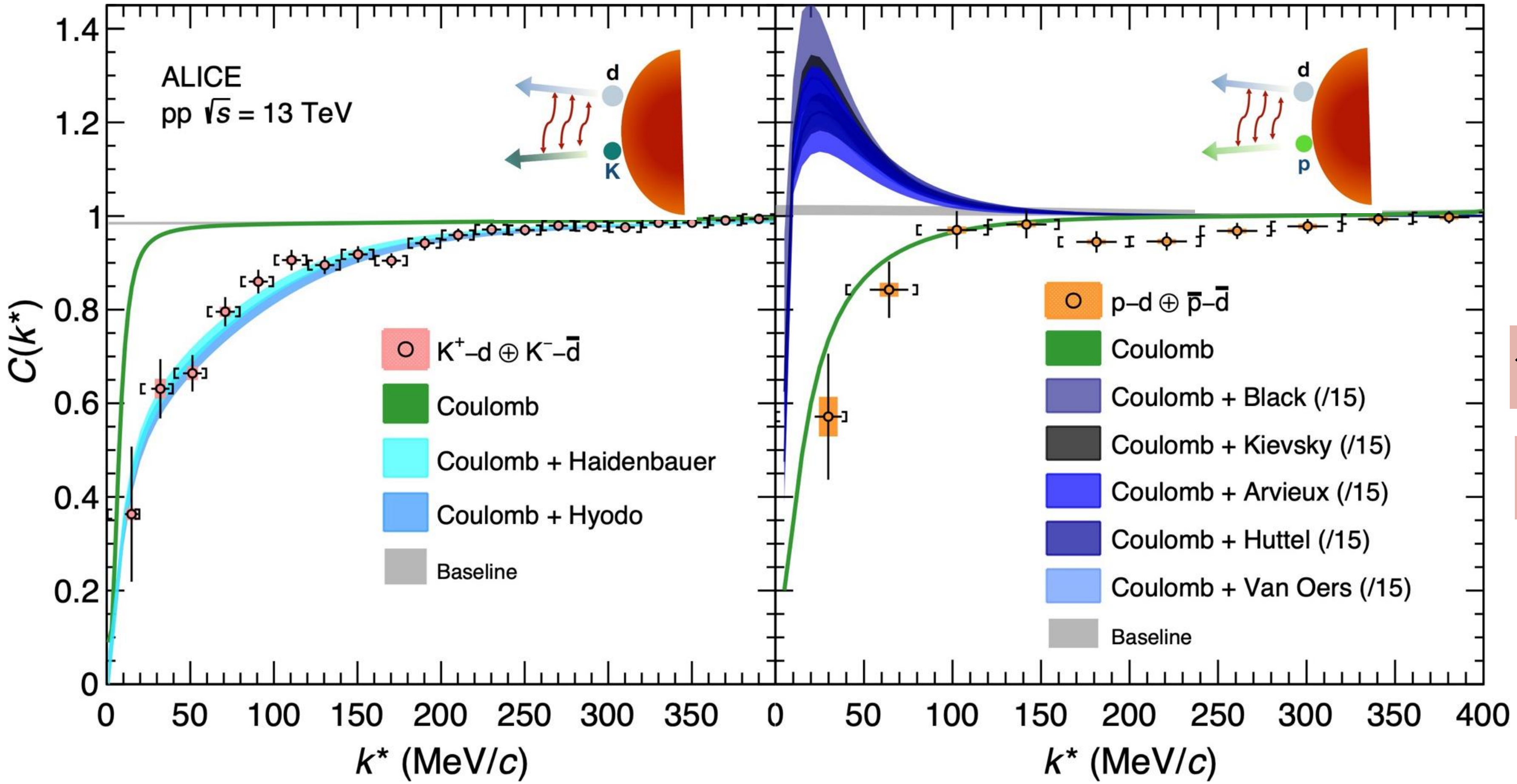
S = spin state
d_0^S = effective range
f_0^S = scattering length

$$f(k^*)^S = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^*\right)^{-1}$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2}\left|\frac{f(k^*)^S}{r_0}\right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0}\right) + \frac{2\Re f(k^*)^S}{\sqrt{\pi} r_0} F_1(2k^* r_0) - \frac{2\Im f(k^*)^S}{\sqrt{\pi} r_0} F_2(2k^* r_0)\right]$$

$$S(r) = (4\pi r_0^2)^{-3/2} \cdot exp\left(-\frac{r^2}{4r_0^2}\right)$$

Kaon/Proton-deuteron correlation



ALICE Coll., arXiv:2308.16120 (2023)

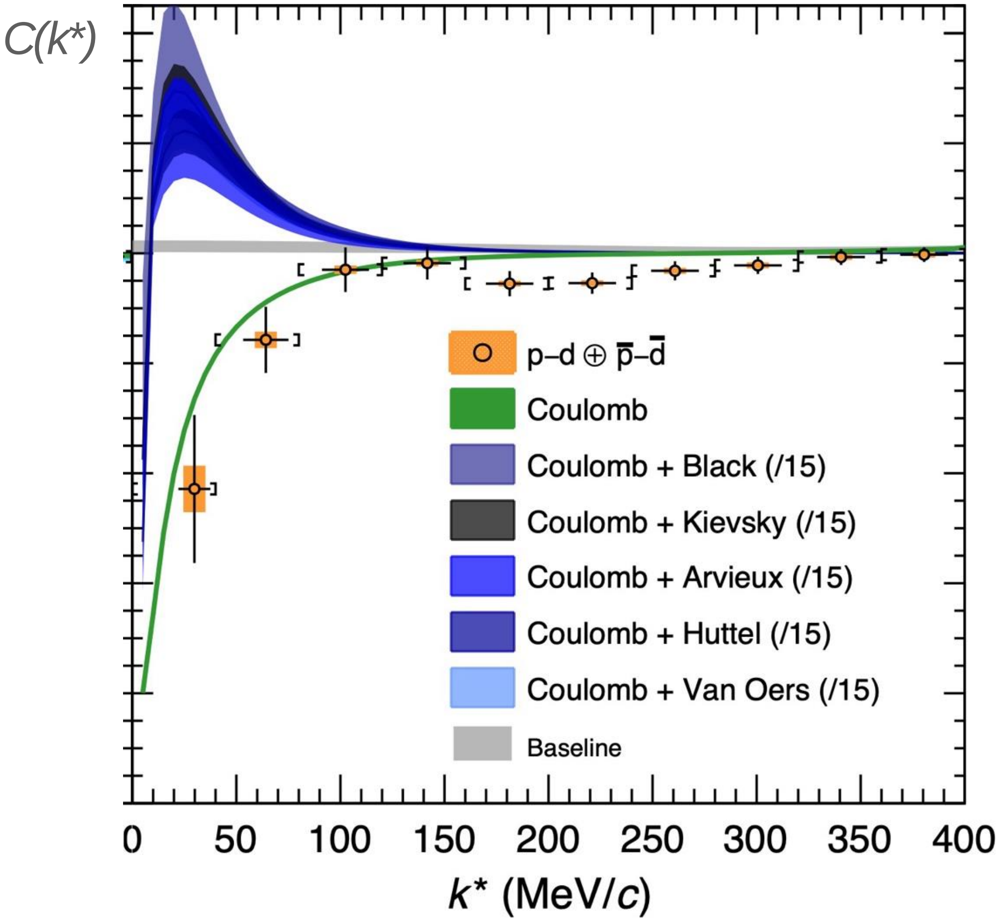
$$r_{\text{eff}}^{\text{Kd}} = 1.41^{+0.03}_{-0.06}\ \text{fm}$$

$$r_{\text{eff}}^{\text{pd}} = 1.059^{+0.04}_{-0.04}\ \text{fm}$$

It works very well for k-d since this interaction is only repulsive and there are no features of the interaction that appears only at short distances. The asymptotic description is sufficient

Proton-deuteron correlation

- The picture of two point-like particles does not work for p-d
  - the deuteron is a composite object
  - Pauli blocking at work for p-(pn) at short distances
  - The asymptotic interaction is different from the short distance one
  - One need a full-fledged three-body calculation



ALICE Coll., arXiv:2308.16120 (2023)

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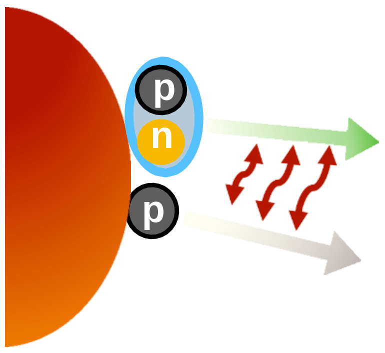
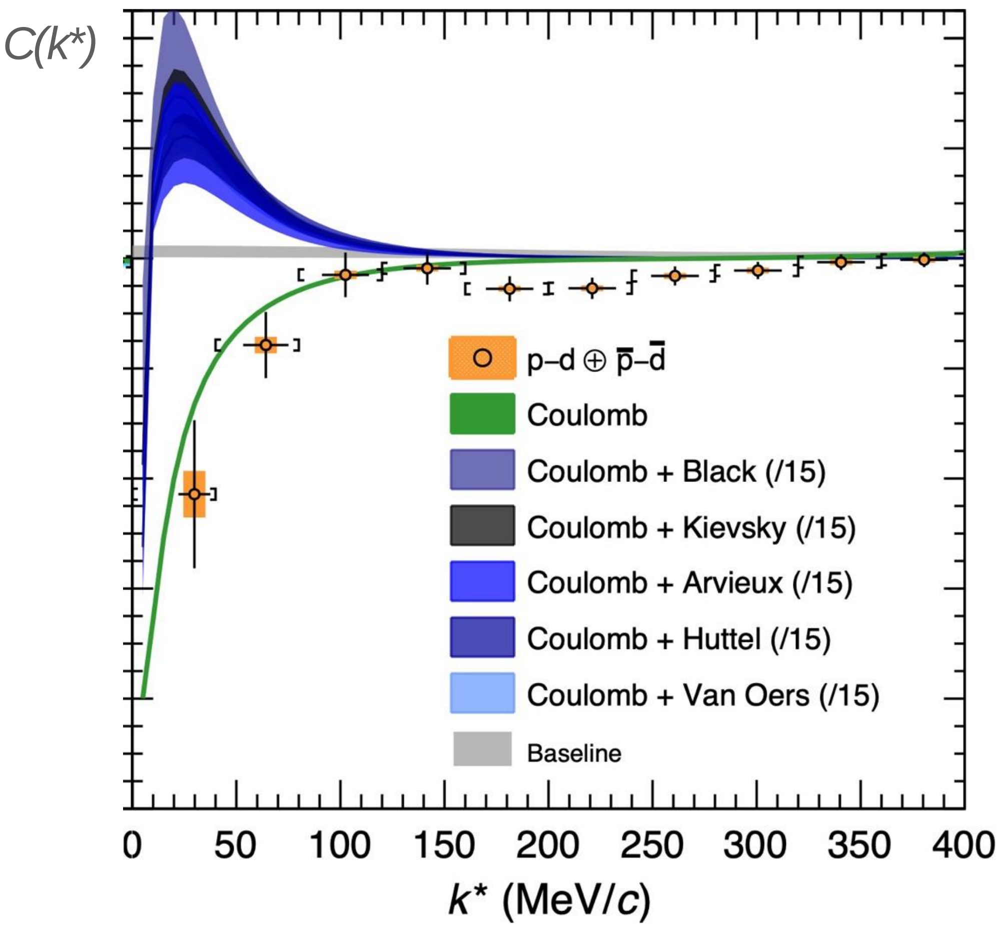


ALICE Coll., arXiv:2308.16120 (2023)



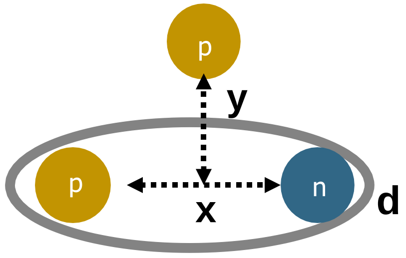
Pisa model: p-d as three-body system

- **Starting with the p-p-n state that goes into p-d state:**
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

$$\boxed{A_d} C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int d^3r_1 d^3r_2 d^3r_3 \boxed{S_1(r_1)S_1(r_2)S_1(r_3)} |\Psi_{m_2,m_1}|^2 ,$$

  - $\Psi_{m(,m)}(x, y)$ three-nucleon wave function asymptotically behaves as p–d state



Calculation done by PISA theory group: Michele Viviani, Alejandro Kievsky and Laura Marcucci

Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

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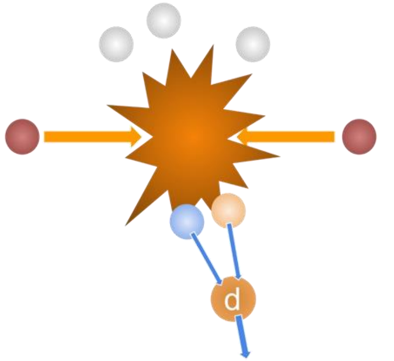
- $\Psi_{m(,m)}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state
- A_d is the deuteron formation probability using deuteron wavefunction



Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

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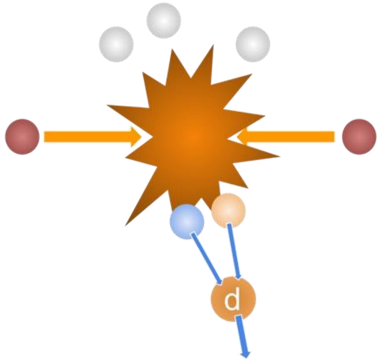
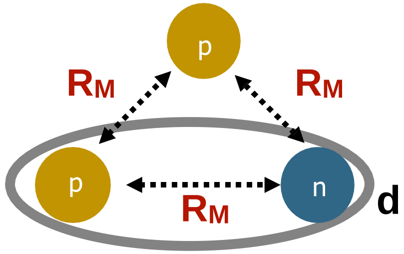


- $\Psi_{m_{(},m_{)}}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state
- A_d is the deuteron formation probability using deuteron wavefunction
- Final definition of the correlation with p-p source size R_M :

$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3}|\Psi_{m_2,m_1}|^2 .$$



Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

NNN using proton-deuteron correlations

- Point-like particle models anchored to scattering experiments

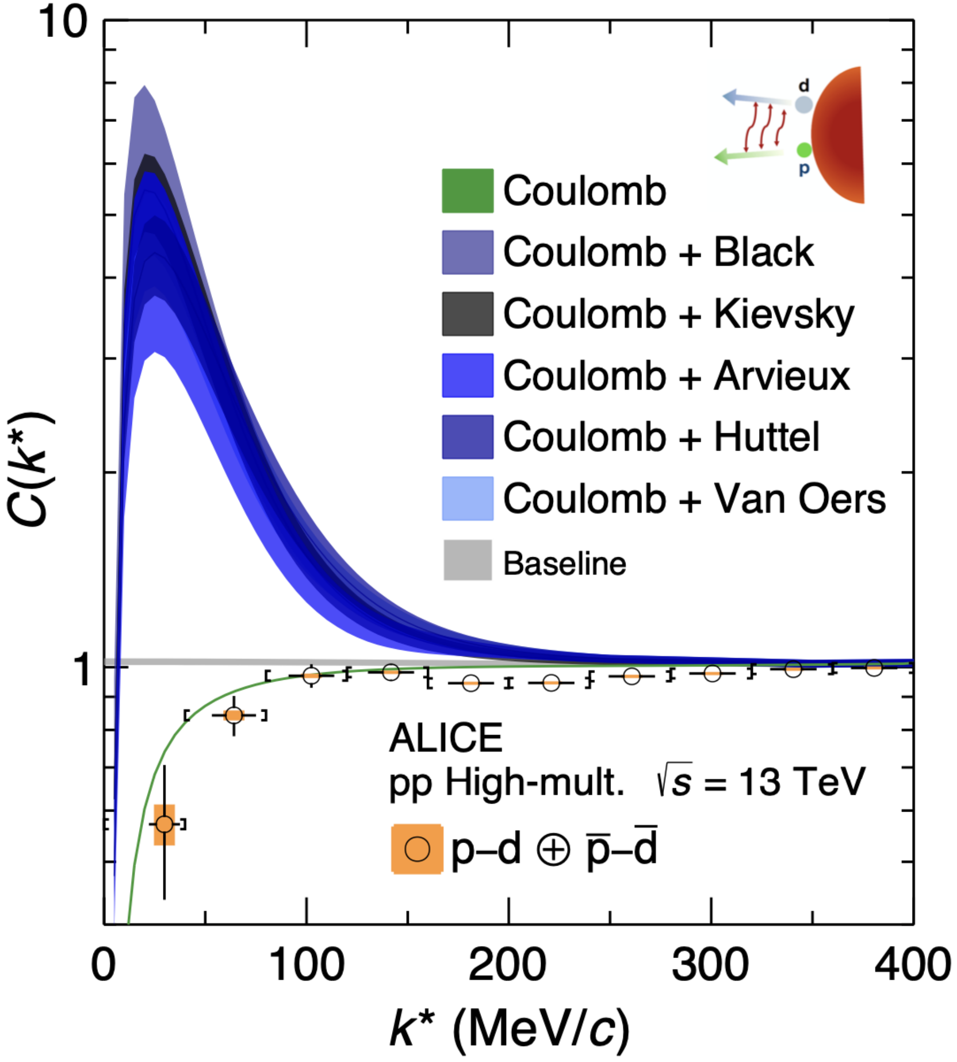
  W. T. H. Van Oers et al., NPA 561 (1967);
  J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);
  A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);

- Coulomb + strong interaction using Lednický model

  Lednický, R. Phys. Part. Nuclei 40, 307–352 (2009)

- Only s-wave interaction
- Source radius evaluated using the universal m_T scaling

Point-like particle description doesn't work for p-d



ALICE Coll. arXiv:2308.16120 (2023), accepted by PRX

p–d correlation function: d as composite object

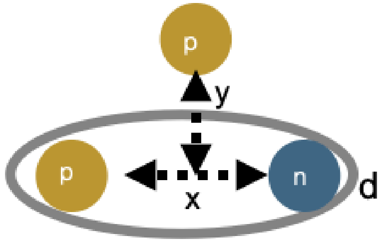
The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p–d state.

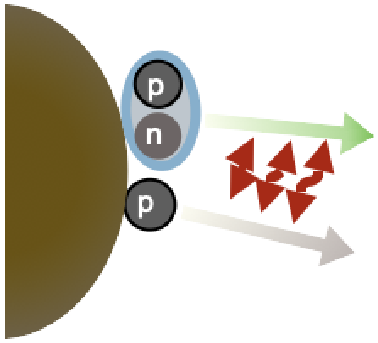
- **Three–body wavefunction for p–d:** $\Psi_{m_2,m_1}(x,y)$ describing three-body dynamics, anchored to p–d scattering observables.
  - x = distance of p-n system within the deuteron
  - y = p-d distance
  - m_2 and m_1 deuteron and proton spin
- $\Psi_{m_2,m_1}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state:

$$\boxed{\Psi_{m_2,m_1}(\boldsymbol{x},\boldsymbol{y}) = \underbrace{\Psi^{(\text{free})}_{m_2,m_1}}_{\text{Asymptotic form}} + \sum_{LSJ}^{J\leq\bar{J}} \sqrt{4\pi}\, i^L \sqrt{2L+1}\, e^{i\sigma_L} \underbrace{(1 m_2 \tfrac{1}{2} m_1|SJ_z)(L0SJ_z|JJ_z)\widetilde{\Psi}_{LSJJ_z}}_{\text{Strong three-body interaction}}\,.}$$

➡ $\tilde{\Psi}_{LSJJ_Z}$ describe the configurations where the three particles are close to each other

➡ $\Psi^{(\text{free})}_{m_1,m_2}$ an asymptotic form of p-d wave function





Kievsky et al, Phys. Rev. C 64 (2001) 024002
Kievsky et al, Phys. Rev. C 69 (2004) 014002
Deltuva et al, Phys. Rev. C71 (2005) 064003

p–d correlation function

- **Starting with the PPN state that goes into pd state**:
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

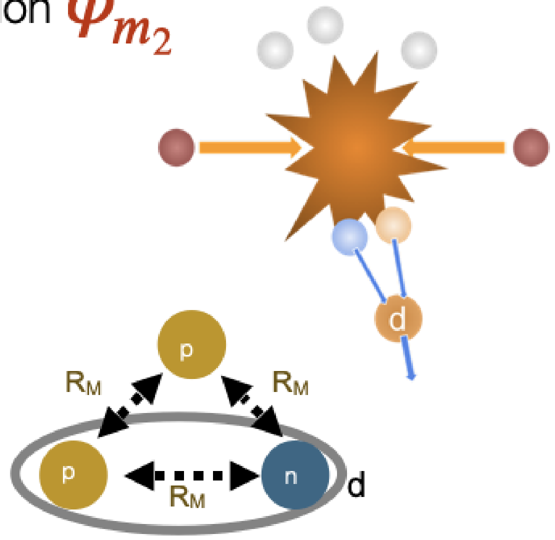
$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int d^3r_1 d^3r_2 d^3r_3\, S_1(r_1)S_1(r_2)S_1(r_3)|\Psi_{m_2,m_1}|^2 ,$$

  - Where A_d is the deuteron formation probability using deuteron wavefunction ϕ_{m_2}

$$A_d = \frac{1}{3}\sum_{m_2}\int d^3r_1 d^3r_2\, S_1(r_1)S_1(r_2)|\phi_{m_2}|^2 ,$$

  - Final definition of the correlation with p–p source size R_M :

$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int \rho^5 d\rho d\Omega\, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3}|\Psi_{m_2,m_1}|^2 .$$

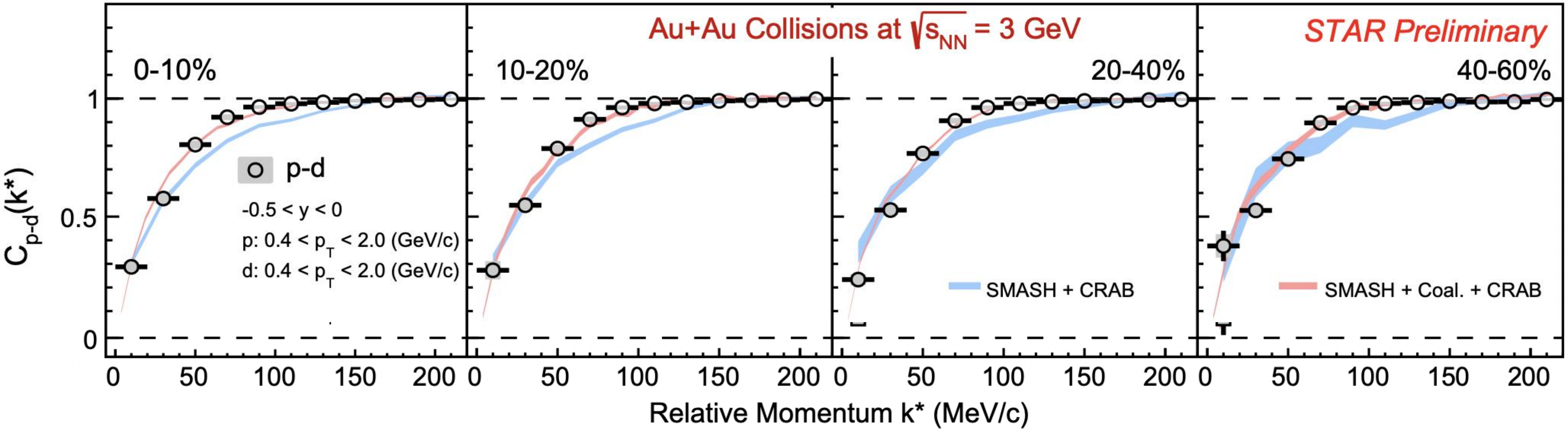


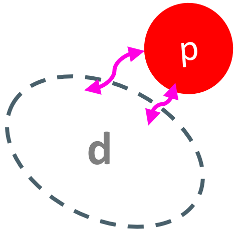
Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

NNN using proton-deuteron correlation

- In large systems (STAR and HADES) point-like description seems to work

STAR Coll. arXiv:2208.05722 (2022)
HADES Coll. arXiv:2402.09280 (2024)





- Analysis based on phase shifts[1] and models the source using a transport model[2] and correlation afterburner[3]
- Shows sensitivity to the production mechanism of the d

[1] B. K. Jennings et al. PRC 33 (1986),
[2] W. Zhao et al. PRC 98 5 (2018),
[3] S. Pratt https://web.pa.msu.edu/people/pratts/freecodes/crab/home.html.

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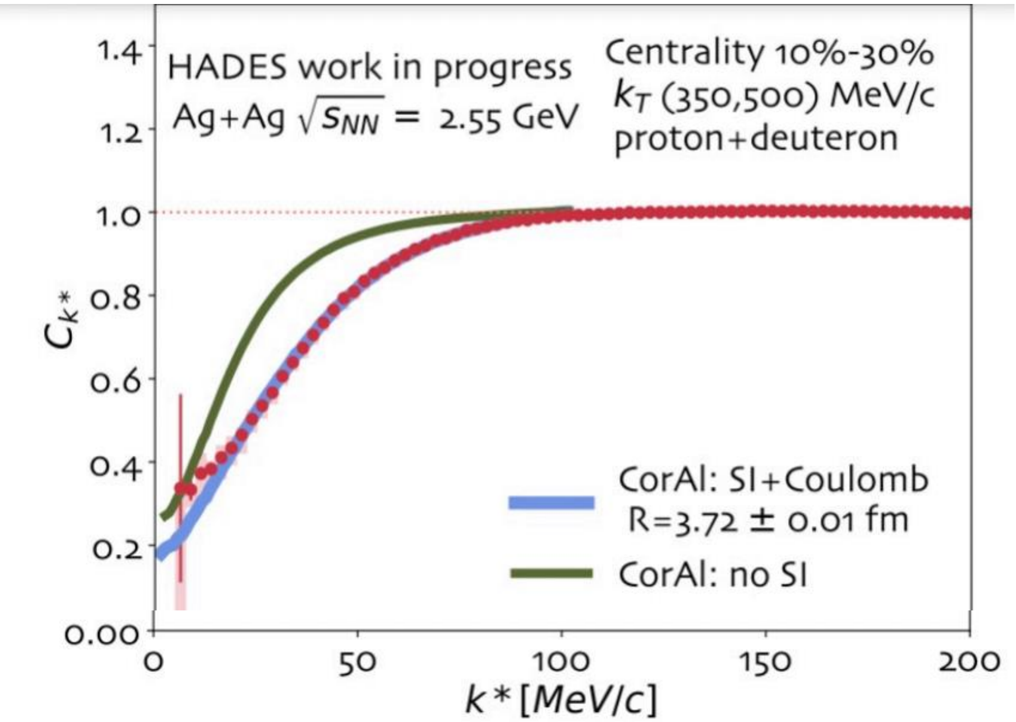

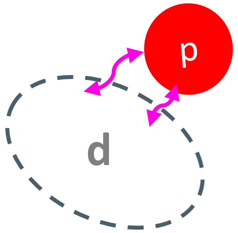

- Analysis uses phase shifts[1] source is fitted[2]

1: T.C. Black et al., PLB 471 (1999),
2: CorAl by Scott Pratt: https://github.com/scottedwardpratt/coral