

PROTON STRUCTURE IN A LIGHT-FRONT HAMILTONIAN APPROACH



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Co-host Chinese Physical Society (CPS) High energy Physics Branch of CPS

BLFQ Collaboration

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Introduction
○○○

BLFQ
○○○

$|qqq\rangle + |qqqg\rangle$
○○○○○○○○○○○○

$|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$
○○○○○○

Conclusions
○○○○○

Overview



Introduction

Basis Light-Front Quantization (BLFQ) to

Proton : $(|qqq\rangle + |qqqg\rangle)$

Proton : $(|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle)$

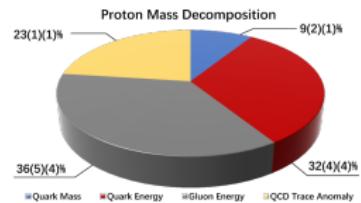
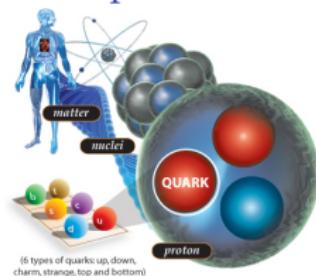
Conclusions

PRD 108 094002 (2023); PLB 847 138305 (2023); PLB 855 138829 (2024); PLB 855 138831 (2024); [2408.11298 \[hep-ph\]](#)

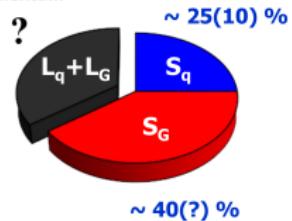
Fundamental Properties: Mass and Spin

- About 99% of the visible mass is contained within nuclei
- Nucleon: composite particles, built from nearly massless quarks ($\sim 1\%$ of the nucleon mass) and gluons
- Quantitative decomposition of *nucleon spin* in terms of quark and gluon degrees of freedom is not yet fully understood.
- To address these fundamental issues → nature of the subatomic force between quarks and gluons, and the internal landscape of nucleons.
- Ideal facilities : EIC & EicC

talked by H. Gao and S. Peng



Orbital angular momentum



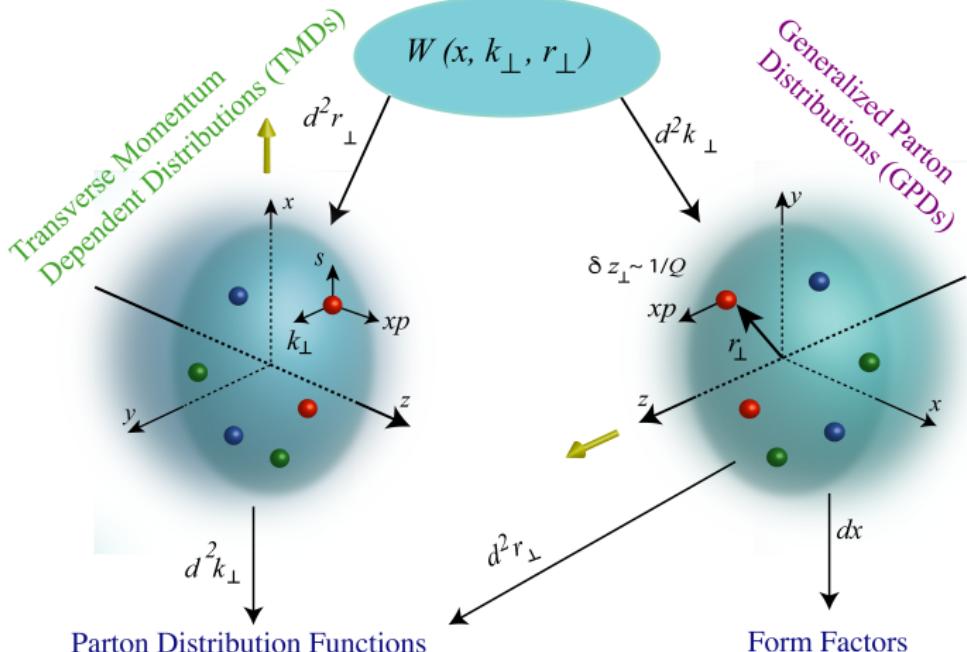
¹ Yang, et al., PRL 121, 212001 (2018)

² Bhattacharya, et al., PRL 128, 182002 (2022); PRL 133, 051901 (2024)

³ Pictures (top to bottom) adopted from A. Signori, S. Peng, C. Lorce

Hadron Tomography

Wigner Distributions



- $x \rightarrow$ longitudinal momentum fraction; $k_\perp \rightarrow$ parton transverse momentum; $r_\perp \rightarrow$ transverse distance from the center.

Nonperturbative Approaches



Lagrangian formalism

Euclidean space-time

correlators: $\langle \mathcal{O}(x_1, \dots, x_n) \rangle$

$$\langle \mathcal{O} \rangle = \int \mathcal{D}_\psi \mathcal{O} \exp \left(-S_E[\psi] \right)$$

—

Hamiltonian formalism

Minkowski space-time

wave functions: $|\psi_h\rangle$

$$H|\psi_h\rangle = E_h|\psi_h\rangle$$

$$i \frac{\partial}{\partial t} |\psi_h(t)\rangle = H|\psi_h(t)\rangle$$



Lattice QCD, Dyson-Schwinger, FRG

DLCQ, BLFQ, Tamm-Danoff, RGPEP

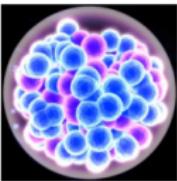
Schrödinger Equation $H|\psi\rangle = E|\psi\rangle$



nonrelativistic

few-body

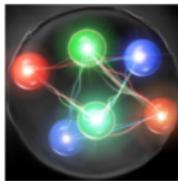
quant, AMO



nonrelativistic

many-body

nucl, quant chem



relativistic

many-body

hadron, QFT

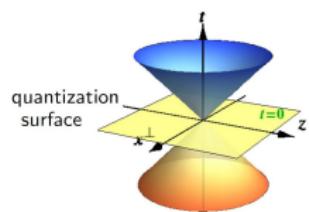
¹ Adopted from Yang Li

Basis Light-Front Quantization (BLFQ)

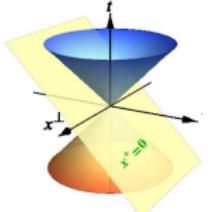
A computational framework for solving relativistic many-body bound state problems in quantum field theories



instant form
time variable $t = x^0$



front form
 $x^+ \triangleq x^0 + x^3$



$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- $P^\pm \equiv P^0 \pm P^3$: light-front momentum (Hamiltonian)
- First-principle / effective Hamiltonian as input
- Access to mass (M) and LFWFs

- Evaluate observables

$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

GOAL

Light-front wave-functions

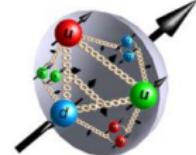
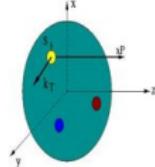
3D imaging

Proton spin

FFs

GPDs

TMDs...



¹Vary, Honkanen, Li, Maris, Brodsky, Harindranath, et. al., Phys. Rev. C 81, 035205 (2010).



- Fock expansion of baryonic bound states:

$$|\text{Proton}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots,$$

Solution proposed by BLFQ

Discrete basis and their direct product

2D HO $\phi_{nm}(p^\perp)$ in the transverse plane

Plane-wave in the longitudinal direction

Light-front helicity state for spin d.o.f.

Truncation

$$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$$

$$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$$

$$\sum_i (m_i + \lambda_i) = M_J$$

$$\alpha_i = (k_i, n_i, m_i, \lambda_i)$$

$$|\alpha\rangle = \otimes_i |\alpha_i\rangle$$

Fock sector truncation

Large N_{\max} and $K \rightarrow$ High UV cutoff & low IR cutoff

- Exact factorization between center-of-mass motion and intrinsic motion

¹Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Nucleon within BLFQ



- The LF eigenvalue equation: $H_{\text{eff}}|\Psi\rangle = M^2|\Psi\rangle$

$$\begin{aligned}
 H_{\text{eff}} = & \sum_a \frac{\vec{p}_{\perp a}^2 + m_a^2}{x_a} + \frac{1}{2} \sum_{a \neq b} \kappa^4 \left[x_a x_b (\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a} (x_a x_b \partial_{x_b})}{(m_a + m_b)^2} \right] \\
 & + \frac{1}{2} \sum_{a \neq b} \frac{C_F 4\pi \alpha_s}{Q_{ab}^2} \bar{u}_{s'_a}(k'_a) \gamma^\mu u_{s_a}(k_a) \bar{u}_{s'_b}(k'_b) \gamma^\nu u_{s_b}(k_b) g_{\mu\nu}
 \end{aligned}$$

Publications:

- Mondal et al., Phys. Rev. D 102, 016008 (2020) : Form Factors, PDFs, ...
- Xu et al., Phys. Rev. D 104, 094036 (2021) : Nucleon structure, ...
- Liu et al., Phys. Rev. D 105, 094018 (2022) : Angular Momentum, ...
- Hu et al., Phys. Lett. B 833, 137360 (2022) : TMDs, ...
- Kaur et al., Phys. Rev. D 109, 014015 (2024) : Chiral-odd GPDs, ...
- Zhang et al., Phys. Rev. D 109, 034031 (2024) : Twist-3 GPDs, ...
- Liu et al., Phys. Lett. B 855, 138809 (2024) : Skewed GPDs, ...
- Nair et al., Phys. Rev. D 110, 056027 (2024) : GFFs, ...
- Peng et al., coming soon : Double parton correlations, ...

Proton with One Dynamical Gluon

$$P^+ P^- |\Psi\rangle = M^2 |\Psi\rangle$$

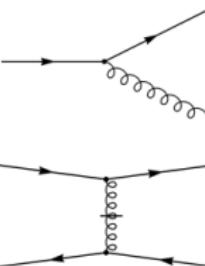
$$|\text{proton}\rangle = \psi_{uud} |uud\rangle + \psi_{uudg} |uudg\rangle$$



QCD Interaction:

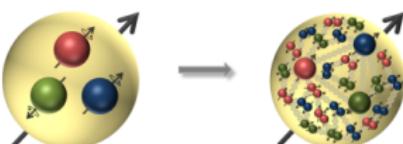
$$P^- = P_{\text{QCD}}^- + P_C^-$$

$$\begin{aligned} P_{\text{QCD}}^- = & \int dx^- d^2x^\perp \left\{ \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+} \psi \right. \\ & - \frac{1}{2} A_a^i [m_g^2 + (i\partial^\perp)^2] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi \\ & \left. + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \right\}, \end{aligned}$$



Confinement only in leading Fock:

$$P_C^- P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \left\{ \{ \vec{r}_{ij\perp}^2 - \frac{\partial_{x_i} (x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \} \right.$$



Parameters:

Truncation: Nmax=9, K=16.5

HO parameters: b=0.7GeV, $b_{\text{inst}}=3\text{GeV}$

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

¹ S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys. Rev. D 108 (2023) 094002.

² Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).

³ Li, Maris, Zhao and Vary, Phys. Lett. B (2016); M. Burkardt, Phys. Rev. D 58, 096015 (1998).

Proton with One Dynamical Gluon

Fock expansion:

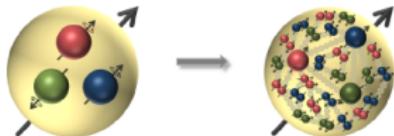
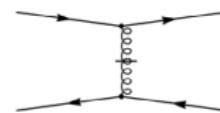
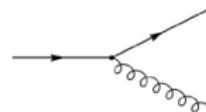
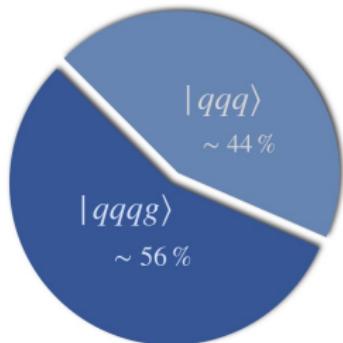
$$|\text{Proton}\rangle = a|uud\rangle + b|uudg\rangle + \dots$$

Light-front effective Hamiltonian :

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^2 + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$



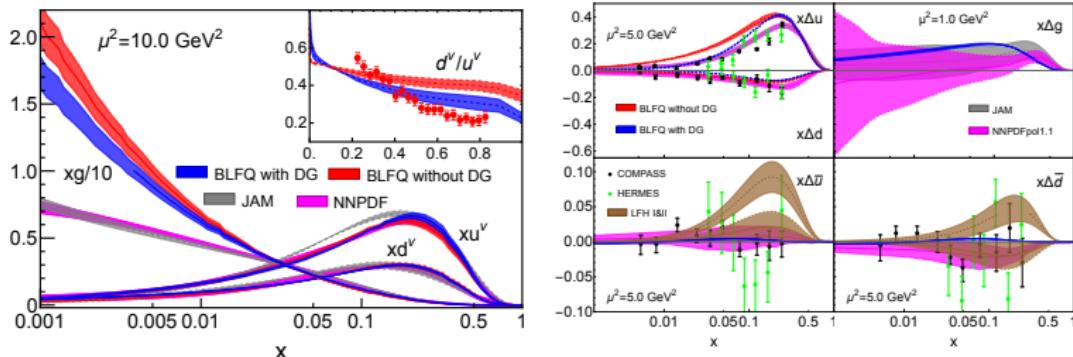
Fock Sector Decomposition



¹S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.

Unpolarized and Helicity PDFs

BLFQ: PRD 108 (2023) 094002

Diagonalizing H_{eff} \Rightarrow LF wavefunction \Rightarrow Initial PDFs \Rightarrow Scale evolution

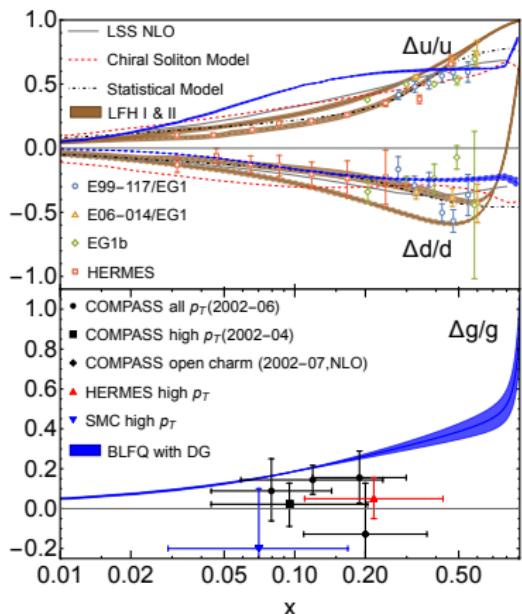
$$\boxed{\text{Model scale } \mu_0^2 = 0.24 \pm 0.01 \text{ GeV}^2}$$

- Quark momentum : $\langle x \rangle_u = 0.261 \pm 0.005$, $\langle x \rangle_d = 0.109 \pm 0.005$ at 10 GeV 2 .
- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004$, $\frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002$.
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, PHENIX: $\Delta G^{[0.02, 0.3]} = 0.2 \pm 0.1$.
- Sea quarks: solely generated from the QCD evolution.

¹LFH: PRL 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003.

Helicity Asymmetries

BLFQ: PRD 108 (2023) 094002



- Experimentally, the expected increase of $\Delta u/u$ is observed.
- For d quark: remains negative in the experimentally covered region.
- Global analyses favor negative values of $\Delta d/d$ at large- x .

BLFQ Predictions for Spin Decomposition

Fock expansion:

$$| \text{Proton} \rangle = a | uud \rangle + b | uudg \rangle + \dots$$



Quark and gluon helicities :

$$\Delta\Sigma_q = \int dx \Delta q(x)$$

$$\Delta\Sigma_g = \int dx \Delta G(x)$$

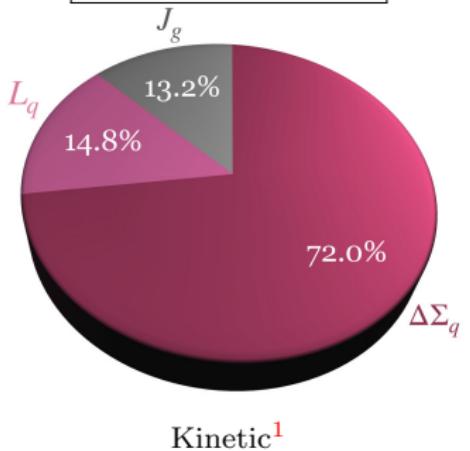
Total AM² :

$$J_i = \frac{1}{2} \int dx x [H_i(x, 0, 0) + E_i(x, 0, 0)]$$

Kinetic OAM² :

$$L_q = \frac{1}{2} \int dx [x \{H_q(x, 0, 0) + E_q(x, 0, 0)\} - \tilde{H}_q(x, 0, 0)]$$

$$\frac{1}{2}\Sigma_q + L_q + J_g = \frac{1}{2}$$



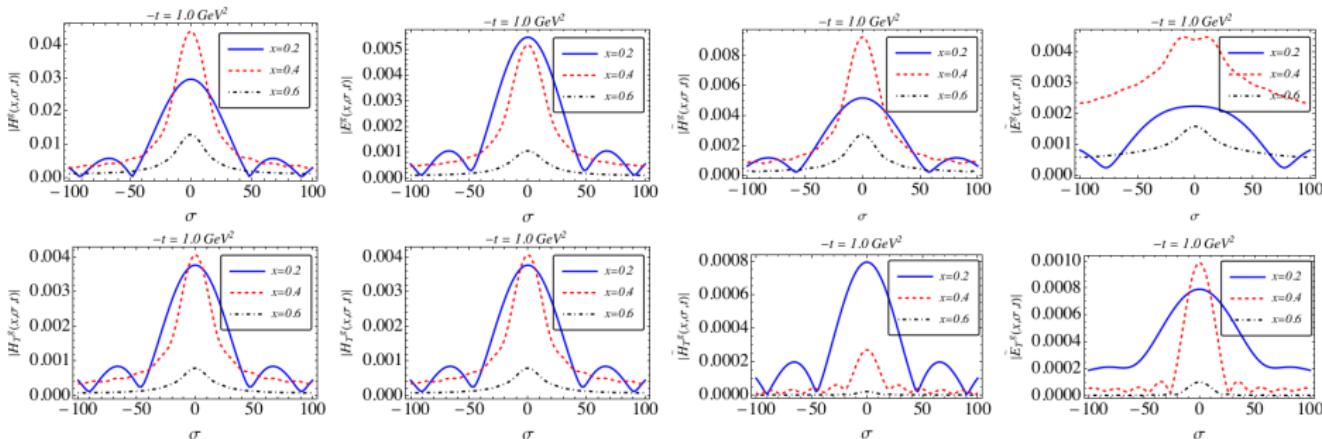
¹S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108, 094002 (2023).

²X. Ji, Phys.Rev.Lett. 78, 610 (1997).

Gluon GPDs in Longitudinal Position Space



$$f(x, \sigma, t) = \int_0^{\xi_f} \frac{d\xi}{2\pi} e^{i\xi P^+ b^- / 2} G(x, \xi, t) = \int_0^{\xi_f} \frac{d\xi}{2\pi} e^{i\xi \sigma} G(x, \xi, t),$$



- Boost-invariant longitudinal position space, $\sigma = \frac{1}{2} b^- P^+$.
- Pattern similar to that observed in single-slit optical experiments.
- Influenced by the functional behavior of the GPDs; differ from quark GPDs.
- Similar pattern in DVCS amplitude, Parton density, Wigner distributions....

¹Zhang, Liu, Xu, CM, et. al., coming soon.

²Liu, Xu, CM, et. al., PLB 855 (2024); Brodsky, et. al., PLB 641 (2006); CM, EPJC 77 (2017);

x-Dependent Squared Radius

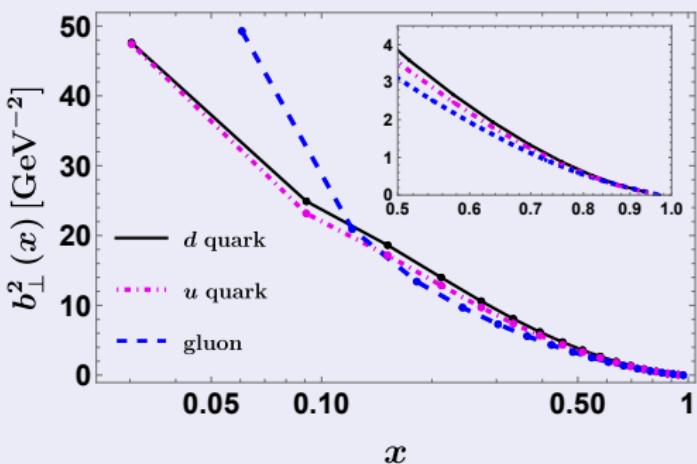


$$\langle b_\perp^2 \rangle^i(x) = \frac{\int d^2 \vec{b}_\perp b_\perp^2 \mathcal{H}^i(x, b_\perp)}{\int d^2 \vec{b}_\perp \mathcal{H}^i(x, b_\perp)},$$

- Transverse squared radius:

$$\langle b_\perp^2 \rangle = \sum_i e_q \int_0^1 dx f^i(x) \langle b_\perp^2 \rangle^i(x)$$

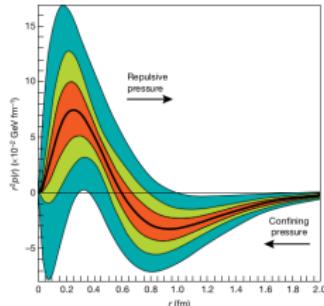
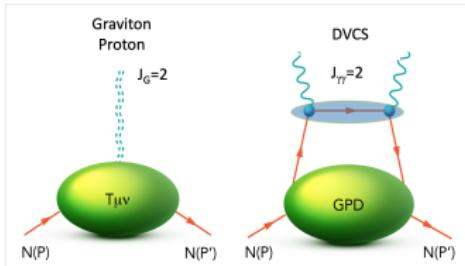
- BLFQ¹: $\langle b_\perp^2 \rangle = 0.47 \pm 0.04 \text{ fm}^2$
- Experimental data²:
 $\langle b_\perp^2 \rangle_{\text{exp}} = 0.43 \pm 0.01 \text{ fm}^2$



¹ B. Lin, S. Nair, S. Xu, CM, X. Zhao, J. P. Vary, PLB 847, 138305 (2023).

² R. Dupre, M. Guidal and M. Vanderhaeghen, PRD 95, 011501 (2017).

Proton Gravitational Form Factors



- Parametrization of matrix element in terms of GFFs

$$\langle P' | T_i^{\mu\nu}(0) | P \rangle = \bar{U}' \left[-B_i(q^2) \frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2) + B_i(q^2)) \frac{1}{2} (\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) + C_i(q^2) \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu} \right] U$$

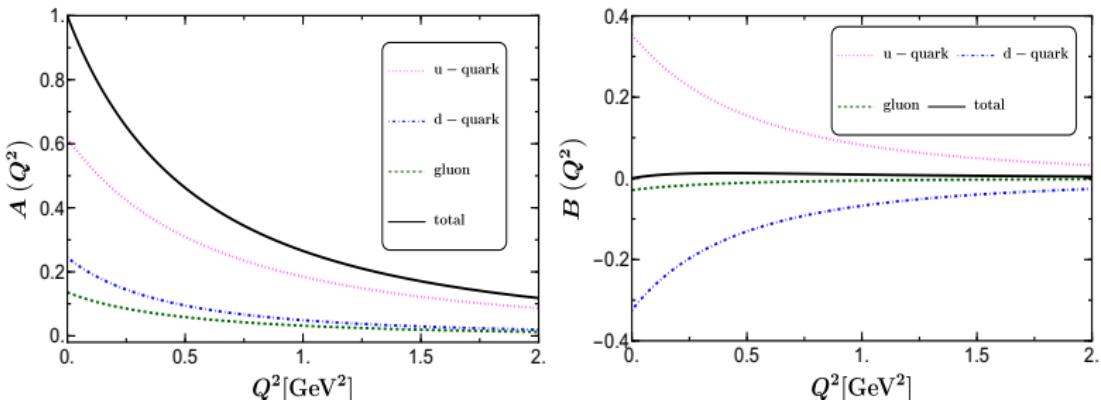
- Momentum sum rule : $\sum_i A^i(0) = 1$
- Gravitomagnetic moment sum rule : $\sum_i B^i(0) = 0$
- Spin sum rule: $J^i = \frac{1}{2} [A^i(0) + B^i(0)]$
- $4C(q^2) = D(q^2)$ provides shear forces and the pressure distributions

¹Burkert *et. al.*: Rev. Mod. Phys. 95, 041002 (2023); Ji, Phys. Rev. Lett. 78, 610 (1997)

$A(Q^2)$ and $B(Q^2)$



$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle$$



- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- Spin sum rule: $J^i = \frac{1}{2} (A^i(0) + B^i(0))$

$$\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0$$

- $D(Q^2) = 4C(Q^2)$: T^{ij} components

¹S. Nair, CM, et. al. coming soon...

Quark TMDs

Leading Twist TMDs



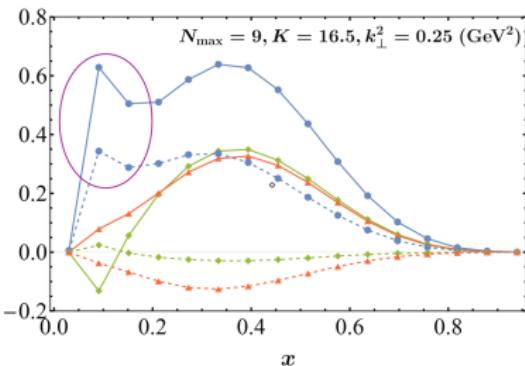
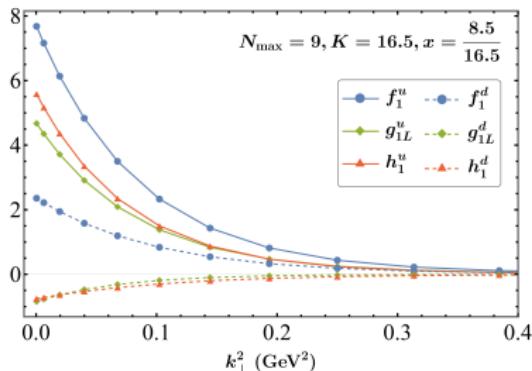
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \bullet$		$h_1^\perp = \bullet - \bullet$ Boer-Mulders
	L		$g_{1L}^\perp = \bullet \rightarrow - \bullet \rightarrow$ Helicity	$h_{1L}^\perp = \bullet \rightarrow - \bullet \rightarrow$
	T	$f_{1T}^\perp = \bullet \uparrow - \bullet \downarrow$ Sivers	$g_{1T}^\perp = \bullet \uparrow - \bullet \uparrow$	$h_{1T}^\perp = \bullet \uparrow - \bullet \uparrow$ Transversity

- Positivity bounds

$$f_1^g(x, \mathbf{k}_\perp^2) > 0, \quad f_1^g(x, \mathbf{k}_\perp^2) \geq |g_{1L}^g(x, \mathbf{k}_\perp^2)|,$$

$$f_1^g(x, \mathbf{k}_\perp^2) \geq \frac{|\mathbf{k}_\perp|}{M} |g_{1T}^g(x, \mathbf{k}_\perp^2)|,$$

$$f_1^g(x, \mathbf{k}_\perp^2) \geq \frac{|\mathbf{k}_\perp|^2}{2M^2} |h_1^{\perp g}(x, \mathbf{k}_\perp^2)|$$



¹Hongyao Yu, et. al. coming soon...

²A. Accardi et al., Eur.Phys.J.A 52 (2016) 9, 268.



Gaussian Ansatz Compatibility?

- To check compatibility of BLFQ results with the Gaussian ansatz :

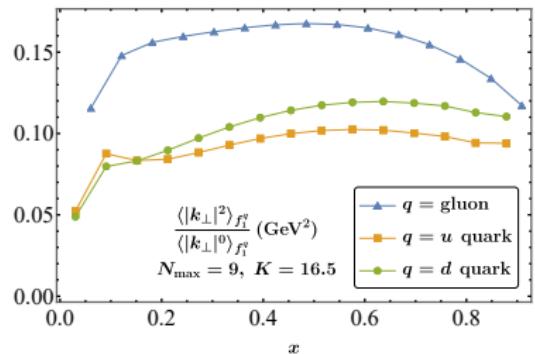
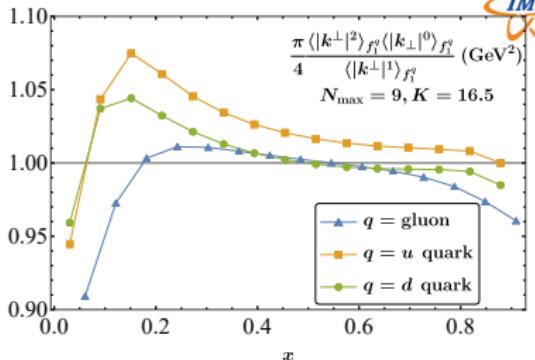
$$f_1^i(x, k_\perp^2) \approx a \frac{\exp\left(-\frac{|k_\perp|^2}{r}\right)}{\pi r}$$

where $a = \langle |k_\perp|^0 \rangle_{f_1^i}$ and
 $r = \langle |k_\perp|^2 \rangle_{f_1^i}$

- If the Gaussian ansatz holds :

$$\frac{\langle |k_\perp|^2 \rangle_{f_1^i} \times \langle |k_\perp|^0 \rangle_{f_1^i}}{(\langle |k_\perp|^1 \rangle_{f_1^i})^2} \times \frac{\pi}{4} = 1$$

BLFQ results do not support Gaussian ansatz



¹ Hongyao Yu, et. al. in preparation

² Hongyao Yu, et. al. Phys.Lett.B 855, 138831 (2024)



Effective Hamiltonian with Dynamical Gluon and Sea Quarks

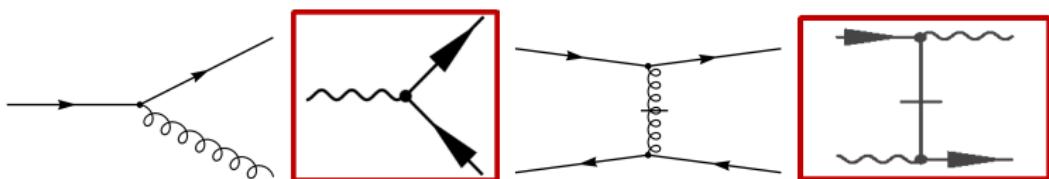
Fock expansion:

$$| \text{Proton} \rangle = a | uud \rangle + b | uudg \rangle + c_1 | uudu\bar{u} \rangle + c_2 | uudd\bar{d} \rangle + c_3 | uuds\bar{s} \rangle + \dots$$

Light-front QCD Hamiltonian :

$$H_{\text{LF}} = \sum_a \frac{\vec{p}_{\perp a}^2 + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$

$$\begin{aligned} H_{\text{vertex}} + H_{\text{inst}} &= g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \\ &\quad + \frac{1}{2} g_s^2 \bar{\psi} \gamma^\mu A_\mu \frac{\gamma^+}{(i\partial^+)} A_\nu \gamma^\nu \psi \end{aligned}$$



¹ Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

² Siqi Xu, Yiping Liu, CM, et. al., 2408.11298 [hep-ph]

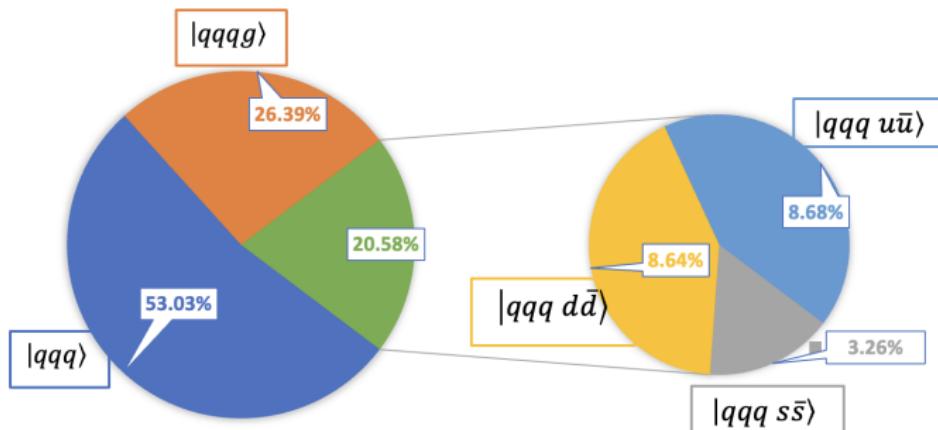
Fock Sector Decomposition

$$|P_{proton}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqq d\bar{d}\rangle + |qqq s\bar{s}\rangle$$

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 16$

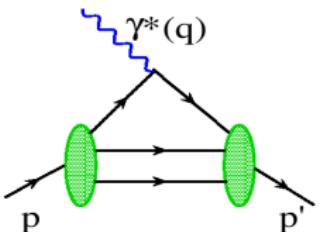
m_u	m_d	m_f	g	b	b_{inst}
0.99 GeV	0.94 GeV	5.9 GeV	3.0	0.6 GeV	2.7 GeV

In five quark Fock sector, we use current quark mass



¹ Siqi Xu, Yiping Liu, CM, et. al., 2408.11298 [hep-ph]

Proton EM Form Factors



Sach's form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2),$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2).$$

- EM current: $J^\mu = \bar{\psi} \gamma^\mu \psi$
- $\langle p'; \uparrow | J^+(0) | p; \uparrow (\downarrow) \rangle \sim F_1(2)(q^2)$
- Two FFs: $F_{1(2)}(q^2 = -Q^2)$

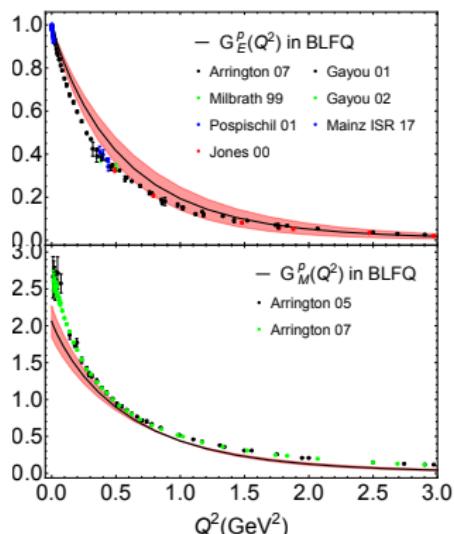
Proton Radii

$$\langle r_E^2 \rangle = -6 \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0},$$

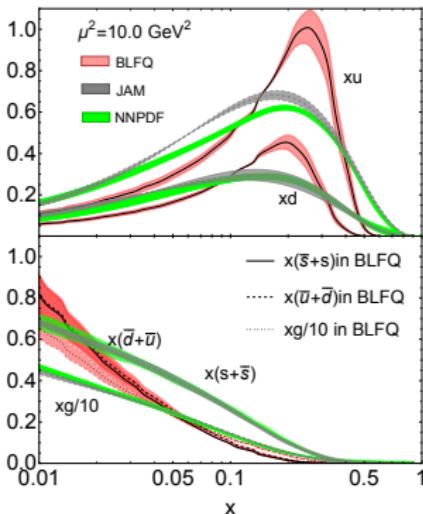
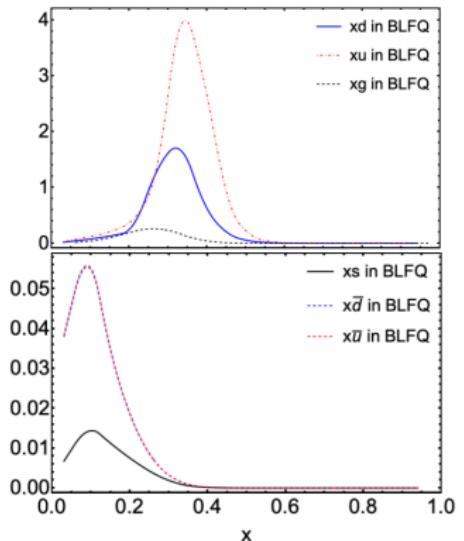
$$\langle r_M^2 \rangle = -\frac{6}{G_M(0)} \frac{dG_M(Q^2)}{dQ^2} \Big|_{Q^2=0}.$$

$$\sqrt{\langle r_E^2 \rangle} = 0.72 \pm 0.05 (0.840^{+0.003}_{-0.002}) \text{ fm}$$

$$\sqrt{\langle r_M^2 \rangle} = 0.73 \pm 0.02 (0.849^{+0.003}_{-0.003}) \text{ fm}$$



Unpolarized PDFs

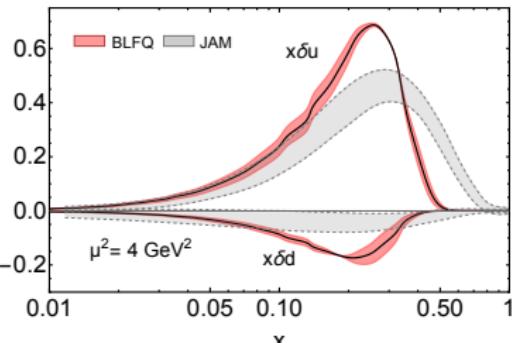
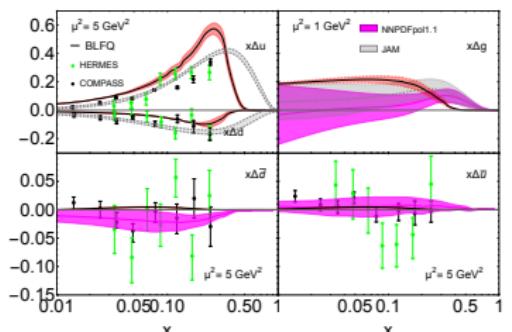


Diagonalizing H_{LFQCD} \Rightarrow LFWFs \Rightarrow Initial PDFs \Rightarrow Scale evolution

- Model scale $\mu_0^2 = 0.22 \pm 0.02 \text{ GeV}^2$, $\langle x \rangle_{u+d} = 0.37 \pm 0.01 \text{ at } 10 \text{ GeV}^2$.
- Longitudinal excitations challenging, in absence of confining potential.

¹ Siqi Xu, Yiping Liu, CM, et. al., 2408.11298 [hep-ph]

Helicity and Transversity PDFs



- Gluon spin $\Delta G = 0.29 \pm 0.03^{\textcolor{red}{1}}$ for $x_g \in [0.05, 0.2]$ at 10 GeV²
- NNPDF² analysis: $\Delta G = 0.23(6)$; lattice QCD³: $\Delta G = 0.251(47)(16)$
- Tensor Charges: $\delta u = 0.81 \pm 0.08$, $\delta d = -0.22 \pm 0.01^{\textcolor{red}{1}}$
- JAM⁴ analysis: $\delta u = 0.71(2)$, $\delta d = -0.200(6)$; lattice QCD⁵: $\delta u = 0.784(28)$, $\delta d = -0.204(11)$.

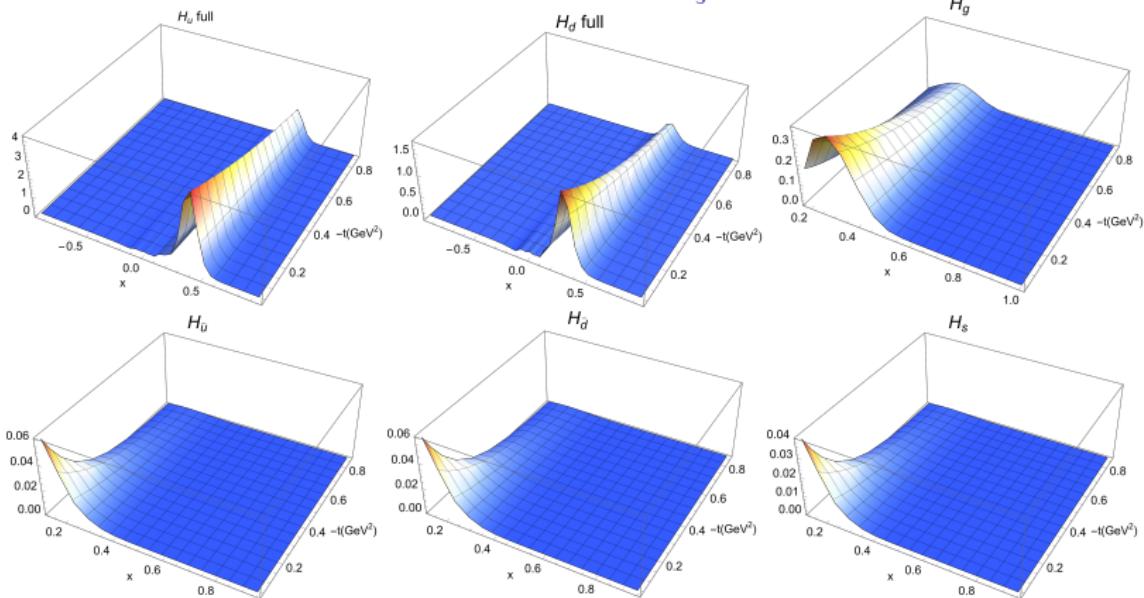
¹ Siqi Xu, Yiping Liu, CM, et. al., 2408.11298 [hep-ph]

² E. R. Nocera, et. al. (NNPDF), Nuclear Physics B 887, 276 (2014)

³ Y.-B. Yang, et. al. (Lattice) Phys. Rev. Lett. 118, 102001 (2017)

⁴ C. Cocuzza, et. al. (JAM), Phys. Rev. Lett. 132, 091901 (2024)

⁵ R. Gupta, et. al. (Lattice), Phys. Rev. D 98, 091501 (2018)

GPDs at Skewness $\xi = 0.1$ 

- u and d GPDs from $-1 < x < 1$; exhibit similar distributions
- At $\xi = 0.1$, DGLAP region dominates
- Discontinuity at $x = \pm\xi$
- Gluon and sea quarks GPDs in DGLAP region

¹Y. Liu, S. Xu, CM, et. al., coming soon...

Conclusions



- Basis Light-front Quantization : A non-perturbative approach based on light-front QCD Hamiltonian
- **LF Hamiltonian** \Rightarrow **Wavefunctions** \Rightarrow **Observables**.
- $|qqq\rangle + |qqqg\rangle$ ($P^- = P_{\text{QCD}}^- + P_{\text{C}}^-$) \Rightarrow Provides good description of data/global fits for various observables.
- $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$, ($P^- = P_{\text{QCD}}^-$) \Rightarrow Provides qualitative description of data/global fits for mass, spin, EMFFs, PDFs, axial and tensor charges

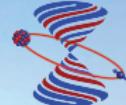
Outlook

- Expand Fock sectors
- Include three-gluon and four-gluon interactions in the Hamiltonian.

Enormous amount of possibilities with future EICs Thank You



LIGHT CONE 2024



Hadron Physics in the EIC era

The Institute of Modern Physics,
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- » Hadron/nuclear structure
- » Spin physics
- » Relativistic many-body physics
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- » Lattice field theories
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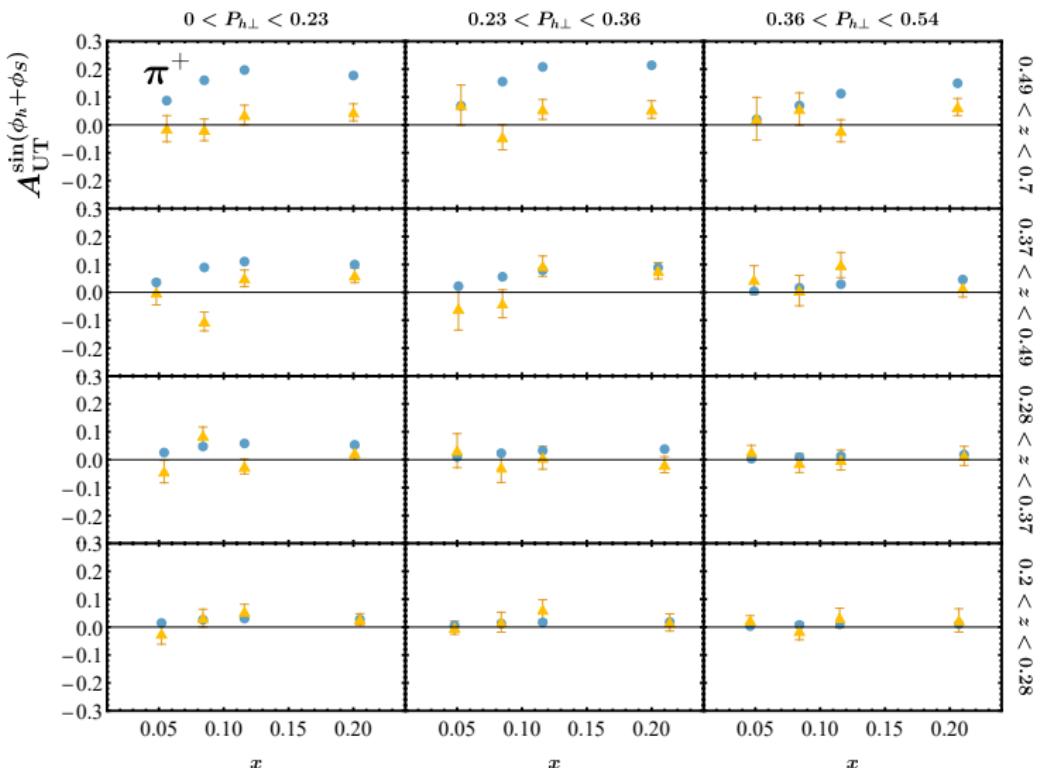
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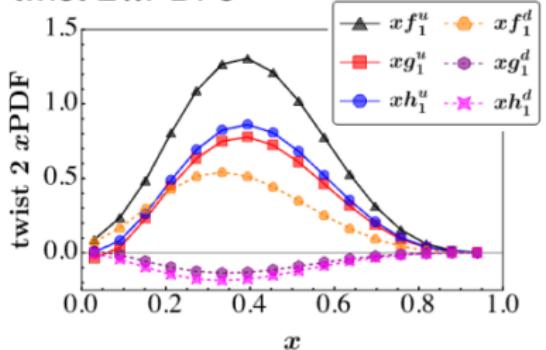
Spin Asymmetry in SIDIS

¹Honhyao, et. al. in preparation

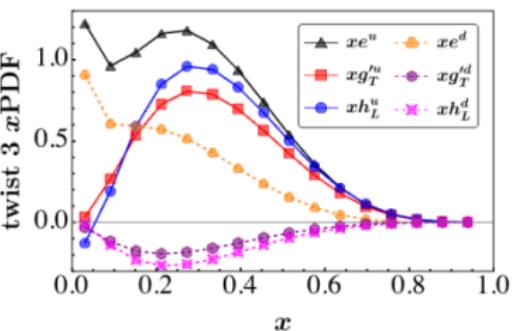
xPDFs: Twist-2 vs Twist-3



twist-2 xPDFs



twist-3 xPDFs

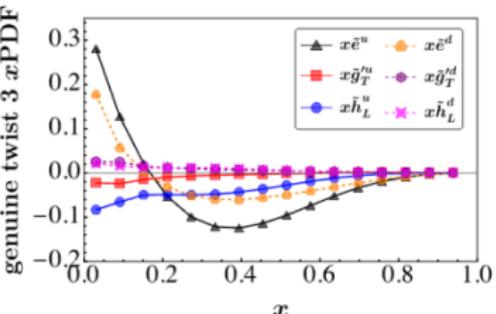


genuine twist-3 xPDFs

$$\int \frac{d^2 k_\perp}{(2\pi)^2} f(x, k_\perp) = f(x)$$

Twist-3 PDFs:

- more concentrating in small x
- similar magnitude to twist-2 PDFs



¹Zhimin Zhu, et. al. in preparation

Light-Front QCD with Light-Cone Gauge ($A^+ = 0$)

$$\begin{aligned}
 \hat{P}_{\text{LFQCD}}^- &= \frac{1}{2} \int dx^- d^2x^\perp \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi + A^{ia} (i\partial^\perp)^2 A^{ia} \\
 &+ g_s \int dx^- d^2x^\perp \bar{\psi} \gamma_\mu A^{\mu a} T^a \psi \\
 &+ \frac{g_s^2}{2} \int dx^- d^2x^\perp \bar{\psi} \gamma_\mu A^{\mu a} T^a \frac{\gamma^+}{i\partial^+} (\gamma_\nu A^{\nu b} T^b \psi) \\
 &+ \frac{g_s^2}{2} \int dx^- d^2x^\perp \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} (\bar{\psi} \gamma^+ T^a \psi) \\
 &- g_s^2 \int dx^- d^2x^\perp i f^{abc} \bar{\psi} \gamma^+ T^c \psi \frac{1}{(i\partial^+)^2} (i\partial^+ A^{\mu a} A_\mu^b) \\
 &+ g_s \int dx^- d^2x^\perp i f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c \\
 &+ \frac{g_s^2}{2} \int dx^- d^2x^\perp i f^{abc} i f^{ade} i\partial^+ A^{\mu b} A_\mu^c \frac{1}{(i\partial^+)^2} (i\partial^+ A^{\nu d} A_\nu^e) \\
 &- \frac{g_s^2}{4} \int dx^- d^2x^\perp i f^{abc} i f^{ade} A^{\mu b} A^{\nu c} A_\mu^d A_\nu^e.
 \end{aligned}$$

Diagrammatic representation of the terms:

- Top term: Quark loop with a quark-gluon vertex.
- Second term: Gluon loop with a quark-gluon vertex.
- Third term: Quark loop with a quark-gluon vertex.
- Fourth term: Gluon loop with a quark-gluon vertex.
- Fifth term: Quark loop with a quark-gluon vertex.
- Sixth term: Gluon loop with a quark-gluon vertex.
- Seventh term: Quark loop with a quark-gluon vertex.
- Eighth term: Gluon loop with a quark-gluon vertex.

¹ S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998).