The 23rd International Conference on
Few-Body Problems in Physics (FB23)

Study of exotic hadrons in a multiquark model

Yan-Rui Liu
(School of Physics, Shandong Universiy, Jinan, China)

Sep. 26, 2024, Beijing, China

Related papers:

Y.R. Liu, X. Liu, S.L. Zhu, Phys. Rev. D 93, 074023 (2016)
J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, Phys. Rev. D 94, 094031 (2016)
S.Q. Luo, K. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Eur. Phys. J. C 77, 709 (2017)
J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Phys. Rev. D 95, 034002 (2017)
J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, Phys. Rev. D 97, 094015 (2018)
J. Wu, X. Liu, Y.R. Liu, S.L. Zhu, Phys. Rev. D 99, 014037 (2019)
J.B. Cheng, Y.R. Liu, Phys. Rev. D 100, 054002 (2019)
J.B. Cheng, S.Y. Li, Y.R. Liu, Y.N. Liu, Z.G. Si, T. Yao, Phys. Phys. D 101, 114017 (2020)
J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, Chin. Phys. C 45, 043102 (2021)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, Phys. Rev. D 108, 056015 (2023)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, Chin. Phys. C 48, 063109 (2024)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, arXiv: 2401.00115 [hep-ph]

Content

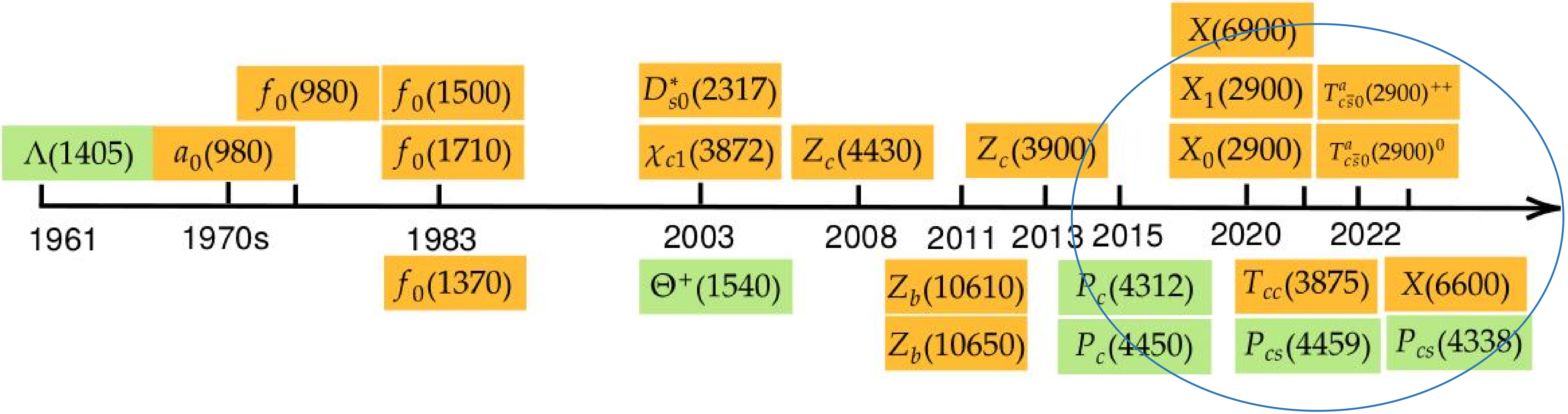
1. Introduction

2. Formalism

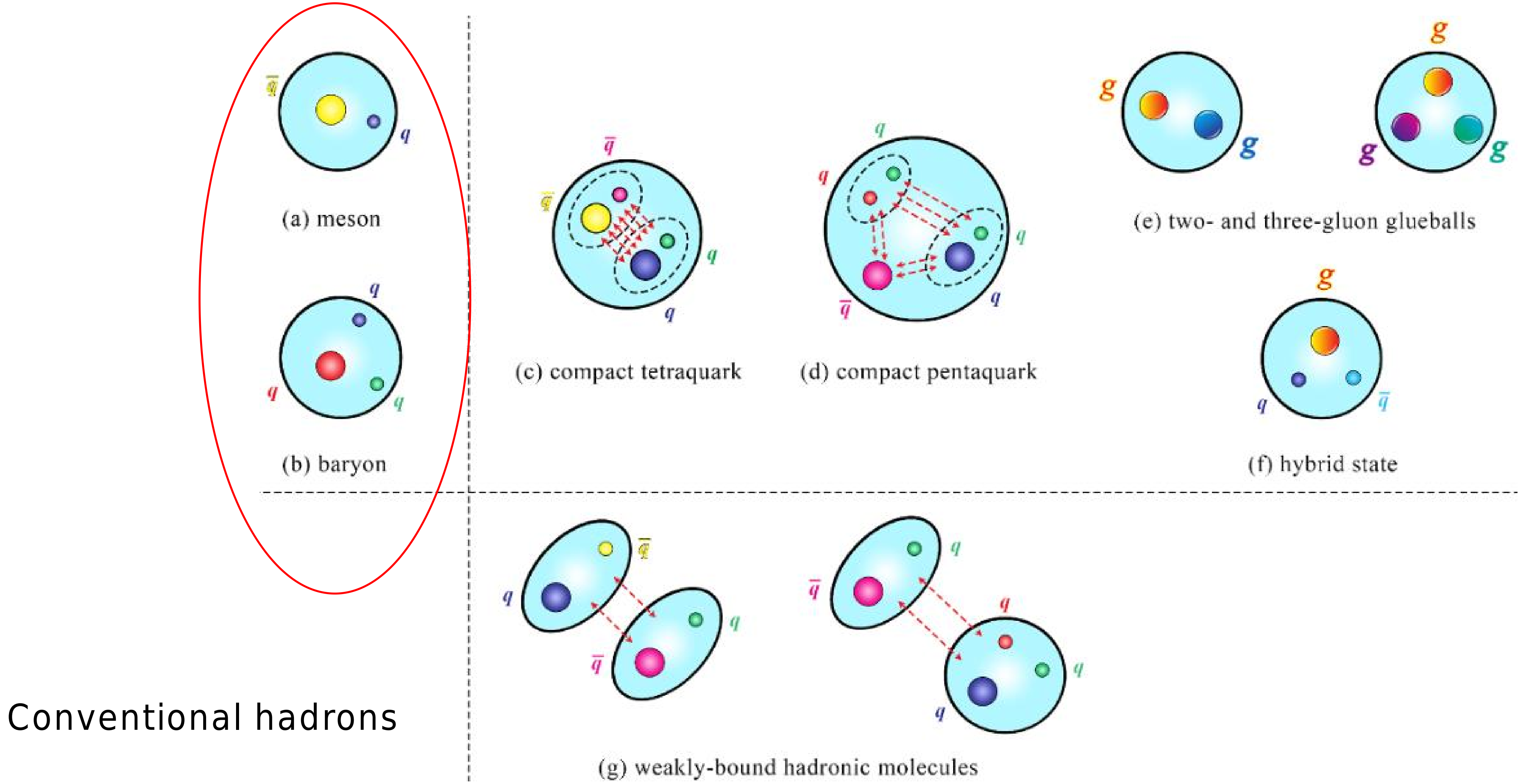
3. X(3960), $X_0(4140)$; T_{cc}; X(6600); $T_{cs,c\bar{s}}(2900)$

4. Summary

Some of the observed exotic states



Introduction

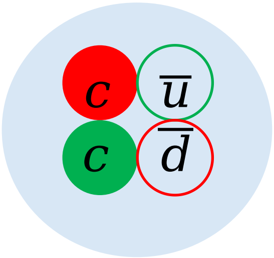


H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Rep. Prog. Phys. 86, 026201 (2023)

$(n = u, d)$

$cs\overline{cs}$

State	Mass(MeV)	Γ (MeV)	Observed channels
$\chi_{c1}(4140)$[CDF:2009jgo]	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$	$B^+ \to J/\psi \phi K^+$
$X(4350)$[Belle:2009rkh]	$4350^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	$\gamma\gamma \to J/\psi \phi$
$\chi_{c1}(4274)$[CDF:2011pep]	$4274^{+8.4}_{-6.7} \pm 1.9$	$32.3^{+21.9}_{-15.3} \pm 7.6$	$B^+ \to J/\psi \phi K^+$
$\chi_{c0}(4500)$[LHCb:2016axx]	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$B^+ \to J/\psi \phi K^+$
$\chi_{c0}(4700)$[LHCb:2016axx]	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$B^+ \to J/\psi \phi K^+$
$X(4630)$[LHCb:2021uow]	$4626 \pm 16^{+18}_{-110}$	$174 \pm 27^{+134}_{-73}$	$B^+ \to J/\psi \phi K^+$
$X(4685)$[LHCb:2021uow]	$4684 \pm 7^{+13}_{-16}$	$126 \pm 15^{+37}_{-41}$	$B^+ \to J/\psi \phi K^+$
$X(3960)$[LHCb:2022aki]	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	$B^+ \to D_s^+ D_s^- K^+$
$X_0(4140)$[LHCb:2022aki]	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	$B^+ \to D_s^+ D_s^- K^+$



T_{cc}

$cc\overline{cc}$

State	Mass (MeV)	Γ(MeV)	Observed channels
$X(6900)$[LHCb:2020bwg]	$6905 \pm 11 \pm 7$ MeV	$80 \pm 19 \pm 33$ MeV	di-J/ψ
$X(6600)$[CMS:2023owd]	$6552 \pm 10 \pm 12$ MeV	$124^{+32}_{-26} \pm 33$ MeV	diJ/ψ
$X(7200)$[CMS:2023owd]	$7287^{+20}_{-18} \pm 5$MeV	$95^{+59}_{-40} \pm 19$MeV	di-J/ψ
$X(6400)$[ATLAS:2023bft]	$6.41 \pm 0.08^{+0.08}_{-0.03}$ GeV	$0.59 \pm 0.35^{+0.12}_{-0.2}$GeV	di-J/ψ
$X(6600)$[ATLAS:2023bft]	$6.63 \pm 0.05^{+0.08}_{-0.01}$ GeV	$0.35 \pm 0.11^{+0.11}_{-0.04}$GeV	di-J/ψ
$X(7200)$[ATLAS:2023bft]	$7.22 \pm 0.03^{+0.01}_{-0.04}$GeV	$0.095 \pm 0.6^{+0.06}_{-0.05}$ GeV	$J/\psi + \psi(2S)$

$cs\overline{nn}$

$cn\overline{sn}$

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

Hidden-charm pentaquark-like baryons ($n = u, d$):

	State	Mass (MeV)	Γ (MeV)	observable channels
	$P_c(4380)^+$[LHCb:2015yax]	$4380 \pm 8 \pm 29$	$215 \pm 18 \pm 86$	$\Lambda_b^0 \to J/\psi p K^-$
$\overline{c}cuud$	$P_c(4312)^+$[LHCb:2019kea]	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Lambda_b^0 \to J/\psi p K^-$
	$P_c(4440)^+$[LHCb:2019kea]	$4440 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.2}$	$\Lambda_b^0 \to J/\psi p K^-$
	$P_c(4457)^+$[LHCb:2019kea]	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Lambda_b^0 \to J/\psi p K^-$
$\overline{c}cuds$	$P_{cs}(4459)^0$[LHCb:2020jpq]	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$\Xi_b^- \to J/\psi \Lambda K^-$
	$P_c(4337)^+$[LHCb:2021chn]	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$B_s^0 \to J/\psi p\bar{p}$
	$P_{cs}(4338)^0$ [LHCb:2022jad]	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$B^- \to J/\psi \Lambda\bar{p}$

Can we understand above exotic mesons and baryons in the compact multiquark picture?

S-wave states here

Formalism: color-magnetic interaction (CMI) model [symmetry analysis]

$$H = \sum_i m_i + H_{eff},$$
$$H_{eff} = -\sum_{i<j} C_{ij} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j.$$

$$M = \sum_i m_i + E_{\rm CMI}$$

- Although problems:
  - Dynamics (no);
  - Spatial information (no);
  - Effective quark masses (system to system);
  - Effective coupling constants (conventional → multiquark?);
  - Estimated masses (uncertainty).
- Simple for estimation of rough positions of multiquark states
- CMI model for mass splittings can catch basic features of spectra
- In the model:
  (1) Construct flavor-color-spin wave function bases;
  (2) Mixing between different color-spin structures
  → Base independent results

In original CMI:

$$M = \sum_i m_i + E_{\rm CMI}$$

In threshold scheme:

$$M = [M_{ref} - (E_{\rm CMI})_{ref}] + E_{\rm CMI}$$

where ref=hadron-hadron state and M_{ref} is its (measured) threshold.

In the scheme we will use:

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

where we assume X(4140) as the ground 1^{++} $cs\overline{cs}$ compact tetraquark and Δ_{ij} denotes mass gap between two different quarks.

Hadron	CMI	Hadron	CMI	Parameter(MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn} = 18.4$
Σ	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{ns}$	$C_{ns} = 12.4$
Ξ^0	$\frac{8}{3}(C_{ss} - 4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss} + C_{ns})$	
Ω	$8C_{ss}$			$C_{ss} = 6.5$
Λ	$-8C_{nn}$			
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}} = 6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}} = 6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}} = 2.1$
B_s	$-16C_{b\bar{s}}$	B^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}} = 2.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}} = 5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}} = 2.9$
Σ_c	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{cn}$	$C_{cn} = 4.0$
Ξ_c'	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{cn} - \frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{cn} + \frac{8}{3}C_{cs}$	$C_{cs} = 4.8$
Σ_b	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{bn}$	$C_{bn} = 1.3$
Ξ_b'	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{bn} - \frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{bn} + \frac{8}{3}C_{bs}$	$C_{bs} = 1.2$

Original CMI:

$$H = \sum_i m_i - \sum_{i<j} C_{ij} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j,$$

$$\begin{aligned} m_n &= 361.8 \text{ MeV}, \\ m_s &= 540.4 \text{ MeV}, \\ m_c &= 1724.8 \text{ MeV}, \\ m_b &= 5052.9 \text{ MeV}. \end{aligned}$$

$(n = u, d)$

TABLE III. Comparison for hadron masses between experimental data and theoretical estimation. All the values are in units of MeV.

Hadron	Theory	Experiment	Deviation	Hadron	Theory	Experiment	Deviation
D	1975.9	1864.8	111.1	D^*	2121.0	2007.0	114.0
D_s	2154.5	1968.3	186.2	D_s^*	2299.5	2112.1	187.4
η_c	3361.0	2983.6	377.4	J/ψ	3474.1	3096.9	377.2
Σ_c	2452.9	2454.0	1.1	Σ_c^*	2516.9	2518.4	−1.5
Ω_c	2796.2	2695.2	101.0	Ω_c^*	2845.3	2765.9	79.4
Ξ_c	2525.9	2471.0	54.9	Ξ_c'	2612.3	2577.9	34.4
Ξ_c^*	2680.6	2645.9	34.7				

Bad theoretical results! mainly due to *quark masses*

- Alternative schemes to study multiquark spectrum:

$$M = \sum_i m_i + E_{\mathrm{CMI}} \quad \Longrightarrow \quad \boxed{M = [M_{ref} - (E_{\mathrm{CMI}})_{ref}] + E_{\mathrm{CMI}}}$$

$\rightarrow M_{high}$

(1) Reference scale → hadron-hadron threshold

M= $[M_{ref=(meson-meson)} - (E_{CMI})_{ref}] + E_{CMI}$,

(same quark content for ref and multiquark)

more reasonable tetraquark masses than original CMI.

But, from studies for

$cs\bar{c}\bar{s}$, $QQ\bar{Q}\bar{Q}$, $qq\bar{Q}\bar{Q}$, $Qq\bar{Q}\bar{q}$, $QQ\bar{Q}\bar{q}$ $\rightarrow M_{low}$

[1605.01134, 1608.07900, 1609.06117, 1707.01180, 1810.06886, 2001.05287, 2008.00737]

(2) Reference scale → mass of X(4140) $\rightarrow M_{reasonable}$

Assumption: $X(4140)$ observed in $J/\psi\phi$ as the ground 1^{++} $cs\bar{c}\bar{s}$ tetraquark

Formalism: mass splitting model

Li et al., PRD 108, 056015 (2023)

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

where $\Delta_{ij} = m_i - m_j$ denotes the effective quark mass gap between quark i and quark j.

C_{ij}	n	s	c	b	$C_{i\bar{j}}$	$\bar{n}$	$\bar{s}$	$\bar{c}$	$\bar{b}$
n	18.3	12.1	4.0	1.3	n	29.8	18.7	6.6	2.1
s		6.5	4.3	1.3	s		9.8	6.7	2.3
c			3.5	2.0	c			5.3	3.3
b				1.9	b				2.9

Consistent with Buccella et al., EPJC 49, 743 (2007).

$$\frac{C_{cc}}{C_{c\bar{c}}} = \frac{C_{bb}}{C_{b\bar{b}}} = \frac{C_{bc}}{C_{b\bar{c}}} = \frac{C_{nn}}{C_{n\bar{n}}} \approx \frac{2}{3}$$

Godfrey-Isgur model: $m_{B_c^*} - m_{B_c} = 70$ MeV

Wu et al., PRD 99, 014037 (2019);
Cheng et al., PRD 101, 114017 (2020)

Δ_{bc}=3340.2MeV,

Δ_{cn}=1280.7MeV,

Δ_{sn}=90.6MeV,

Δ_{cs}=1180.6MeV,

Δ_{bs}=4520.2MeV.

Approximate relations:

$$\Delta_{cn} \approx \Delta_{cs} + \Delta_{sn},$$

$$\Delta_{bs} \approx \Delta_{bc} + \Delta_{cs}.$$

$(n = u, d)$

Why is X(4140) selected?

1. It is a $J/\psi\phi$ resonance confirmed by different experiments. $J^{PC}=1^{++}$ determined; suppressed mixing with $c\bar{c}$ states;

2. $J^{PC}=1^{++}$ partner states X(4274) and X(4140) can be consistently interpreted as compact $cs\overline{cs}$ tetraquark states;

[Stancu, J.Phys.G 37, 075017 (2010); Wu et. al., PRD 94, 094031 (2016)]

3. X(4140) as the reference state can provide more reasonable explanations for other observed $cs\overline{cs}$ states.

[Li et. al., Chin.Phys.C 48, 063109 (2024)]

Formalism: mass fomulas for compact tetraquark states

In original CMI (v1):

$$M = \sum_i m_i + E_{\rm CMI}$$

additional attraction needed

In threshold scheme (v2):

$$M = [M_{ref} - (E_{\rm CMI})_{ref}] + E_{\rm CMI}$$

superfluous attraction included

where ref=hadron-hadron state and mesured M_{ref} is its threshold.

In the scheme we will use (v3):

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

Usually, $M_{v2} < M_{v3} < M_{v1}$

(size: hadron-hadron state > compact tetraquark > conventional hadron)

Formalism: rearrangement decay

- Combine information from spectrum and decay to analyze multiquark properties
- A simple decay scheme:

1. decay Hamiltonian is a constant: $\boldsymbol{H_{decay}} = \mathcal{C}$

system-dependent $\mathcal{C}$

2. measured width ≈sum of two-body rearrangement decay widths: $\boldsymbol{\Gamma_{exp}} \approx \boldsymbol{\Gamma_{sum}}$

$$\mathcal{M} = \langle initial|H_{decay}|final\rangle = \mathcal{C}\sum_{ij} x_i y_j$$

$$\Psi_{initial} = \sum_i x_i(q_1 q_2 \bar{q}_3 \bar{q}_4),$$

$$\Psi_{final} = \sum_i y_i(q_1 q_2 \bar{q}_3 \bar{q}_4).$$

$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M^2_{initial}}$$

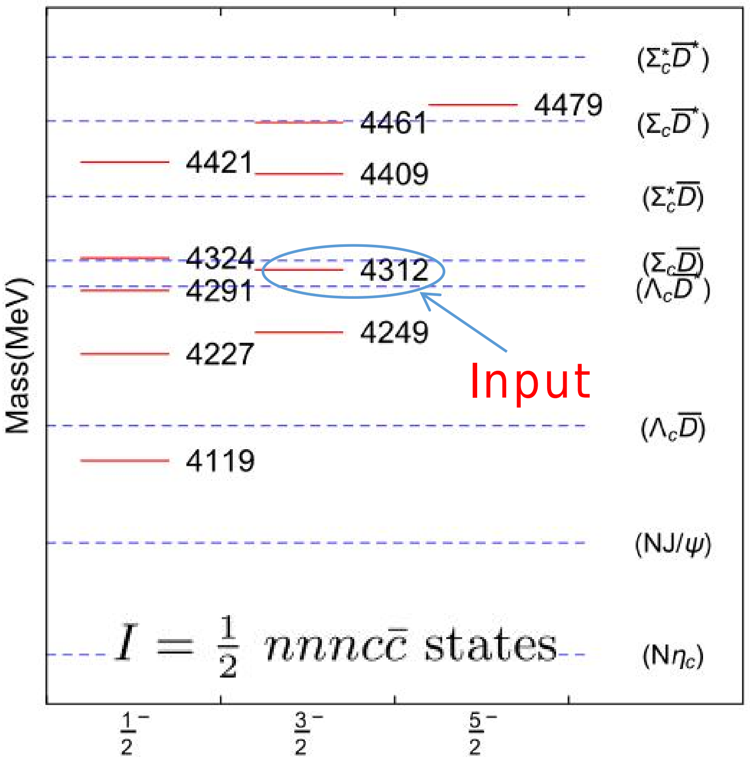
We analyzed masses and widths of the Pc states in:

PRD100, 054002(2019); PRD 108, 056015 (2023)

State	Mass(MeV)	Γ (MeV)	Observed channels
$P_\psi^N(4380)^+$[22]	$4380 \pm 8 \pm 29$	$215 \pm 18 \pm 86$	$\Lambda_b^0 \to J/\psi p K^-$
$P_\psi^N(4312)^+$[23]	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_\psi^N(4440)^+$[23]	$4440 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.2}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_\psi^N(4457)^+$[23]	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_{\psi s}^\Lambda 4459)^0$[54]	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$\Xi_b^- \to J/\psi \Lambda K^-$
$P_\psi^N(4337)^+$[53]	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$B_s^0 \to J/\psi p \bar{p}$
$P_{\psi s}^\Lambda(4338)^0$ [55]	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$B^- \to J/\psi \Lambda \bar{p}$

Example of formalism: Pc states $(n = u, d)$

$(nnn)_{8_c}(c\bar{c})_{8_c}$-$(nnn)_{1_c}(c\bar{c})_{1_c}$



Th.

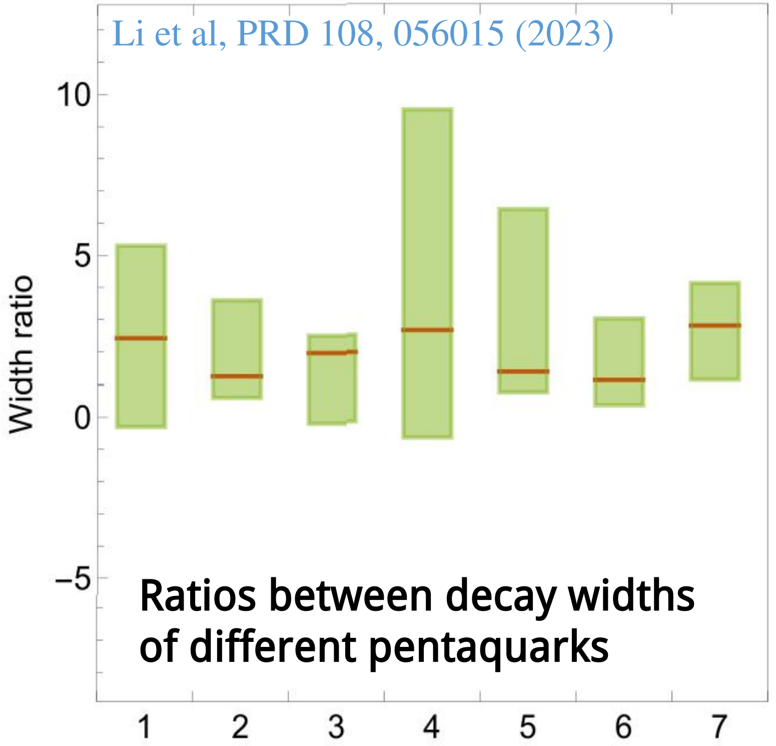
$$\Gamma(\tilde{P}_c(4421)^+) : \Gamma(\tilde{P}_c(4461)^+) = 2.42,$$
$$\Gamma(\tilde{P}_c(4421)^+) : \Gamma(\tilde{P}_c(4312)^+) = 1.24,$$
$$\Gamma(\tilde{P}_c(4312)^+) : \Gamma(\tilde{P}_c(4461)^+) = 1.96,$$
$$\Gamma(\tilde{P}_c(4324)^+) : \Gamma(\tilde{P}_c(4461)^+) = 2.64,$$
$$\Gamma(\tilde{P}_c(4324)^+) : \Gamma(\tilde{P}_c(4312)^+) = 1.35,$$
$$\Gamma(\tilde{P}_c(4324)^+) : \Gamma(\tilde{P}_c(4421)^+) = 1.09.$$

Exp.

$$\Gamma(P_c(4440)^+) : \Gamma(P_c(4457)^+) = 3.2^{+2.1}_{-3.5},$$
$$\Gamma(P_c(4440)^+) : \Gamma(P_c(4312)^+) = 2.1^{+1.5}_{-1.5},$$
$$\Gamma(P_c(4312)^+) : \Gamma(P_c(4457)^+) = 1.5^{+1.0}_{-1.7},$$
$$\Gamma(P_c(4337)^+) : \Gamma(P_c(4457)^+) = 4.5^{+5.0}_{-5.2},$$
$$\Gamma(P_c(4337)^+) : \Gamma(P_c(4312)^+) = 3.0^{+3.4}_{-2.3},$$
$$\Gamma(P_c(4337)^+) : \Gamma(P_c(4440)^+) = 1.4^{+1.6}_{-1.1}.$$

J.B. Cheng, Y.R. Liu, PRD100, 054002(2019);

Li et al, PRD 108, 056015 (2023)



$P_c(4457)^+$, $P_c(4440)^+$, $P_c(4337)^+$ can be regarded as the J=3/2, J=1/2, and J=1/2 pentaquark states, respectively.

For $P_c(4457)^+$ $\Gamma(\Sigma_c^*\bar{D}) : \Gamma(\Lambda_c\bar{D}) : \Gamma(NJ/\psi) = 2.3 : 4.0 : 1.0$

For $P_c(4440)^+$ $\Gamma(\Lambda_c\bar{D}^*) : \Gamma(\Sigma_c\bar{D}) : \Gamma(\Lambda_c\bar{D}) : \Gamma(NJ/\psi) : \Gamma(N\eta_c) = 45.5 : 3.0 : 3.0 : 7.5 : 1.0$

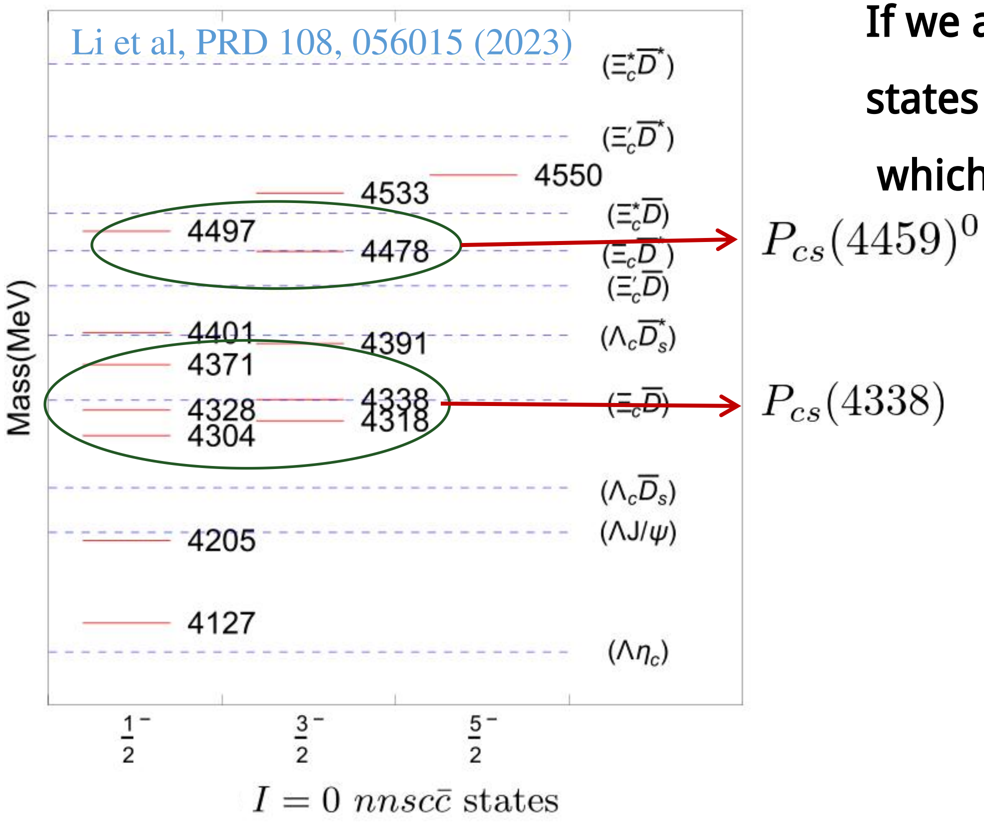
For $P_c(4312)^+$ $\Gamma(NJ/\psi) : \Gamma(\Lambda_c\bar{D}^*) = 1.1$

For $P_c(4337)^+$ $\Gamma(\Lambda_c\bar{D}) : \Gamma(NJ/\psi) = 1.3$

Predictions

Example of formalism: Pcs states $(n = u, d)$

$(nns)_{8_c}(c\bar{c})_{8_c}$-$(nns)_{1_c}(c\bar{c})_{1_c}$



If we assign the $P_{cs}(4459)^0$, $P_{cs}(4338)^0$ to be J=3/2 pentaquark states $\tilde{P}_{cs}(4478)$, $\tilde{P}_{cs}(4338)$, respectively, $\Gamma(\tilde{P}_{cs}(4478)) : \Gamma(\tilde{P}_{cs}(4338)) \sim 0.12$ which is contradicted with the experimantal value.

Other possible assignments:

$$\begin{aligned}
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4371)^0) &= 0.15,\\
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4328)^0) &= 0.56,\\
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4318)^0) &= 2.57,\\
\Gamma(\tilde{P}_{cs}(4478)^0) : \Gamma(\tilde{P}_{cs}(4304)^0) &= 0.17,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4371)^0) &= 0.72,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4338)^0) &= 0.61,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4328)^0) &= 2.78,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4318)^0) &= 12.71,\\
\Gamma(\tilde{P}_{cs}(4497)^0) : \Gamma(\tilde{P}_{cs}(4304)^0) &= 0.83.
\end{aligned}$$

Theoretical widths are much smaller than the measured results.

$J^P = \frac{1}{2}^-$

Both $P_{cs}(4459)^0$ and $P_{cs}(4338)^0$ can be regarded as $\frac{1}{2}^-$ pentaquark states.

For $P_{cs}(4459)^0$, $\Gamma(\Lambda_c \bar{D}_s^*) : \Gamma(\Xi_c \bar{D}^*) : \Gamma(\Lambda J/\Psi) = 2.3 : 1.1 : 1.0$

For $P_{cs}(4338)$, $\Gamma(\Lambda J/\Psi) : \Gamma(\Lambda_c \bar{D}_s) = 3.0$

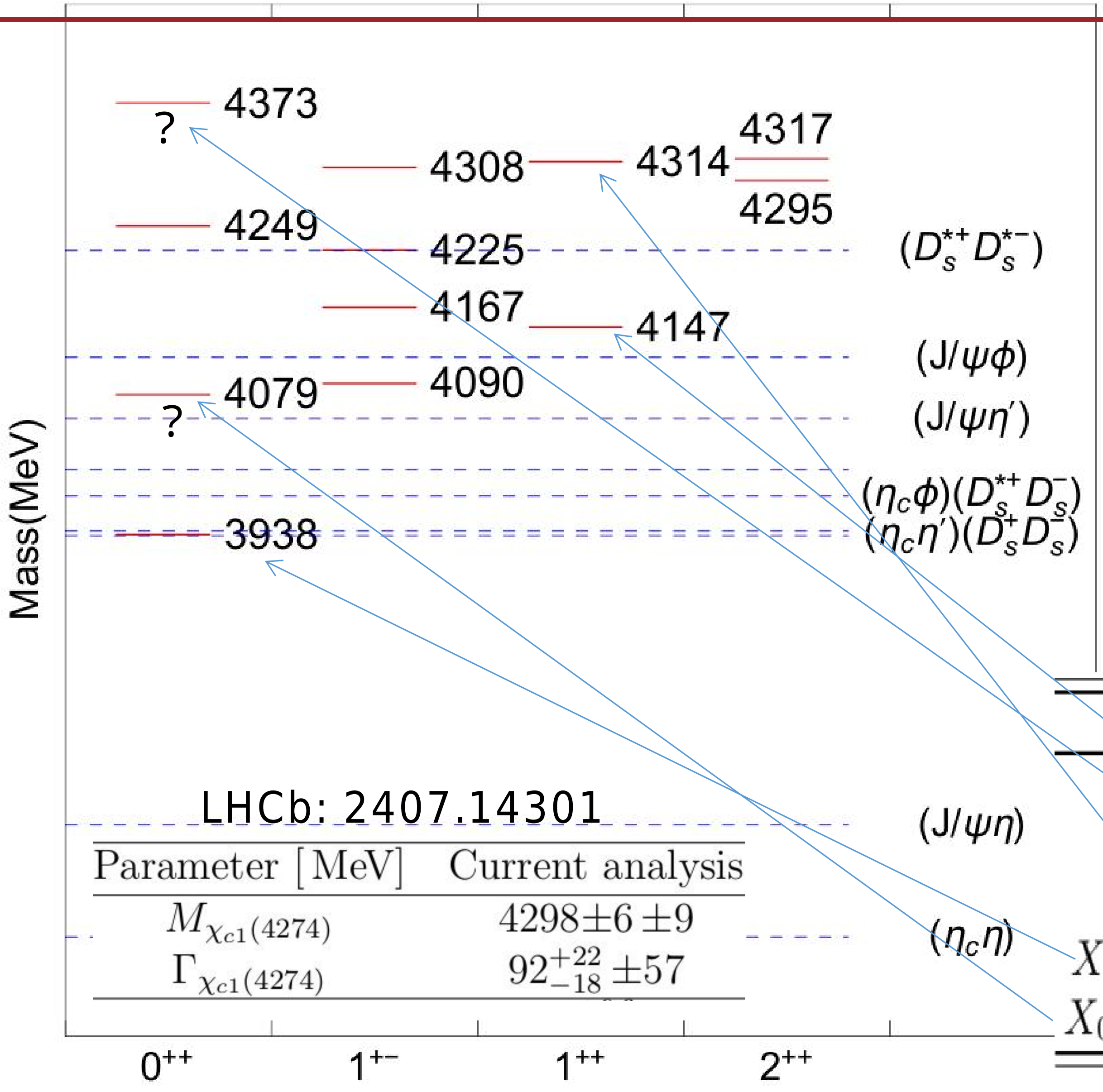
The J=5/2 state，the lighest J=3/2 state, and the lighest J=1/2 state are narrow.

Predictions

Exp: $\Gamma(P_{cs}(4459)^0) : \Gamma(P_{cs}(4338)^0) = 2.5^{+1.6}_{-1.4}$

(1) $cs\bar{c}\bar{s}$ states

Li et al., Chin.Phys.C 48, 063109 (2024)



LHCb: 2407.14301

Parameter [MeV]	Current analysis
$M_{\chi_{c1}(4274)}$	$4298 \pm 6 \pm 9$
$\Gamma_{\chi_{c1}(4274)}$	$92^{+22}_{-18} \pm 57$

$M_{X(4140)} = 4146.5$ MeV $\quad \mathcal{C} = 7282.15$ MeV

$\chi_{c1}(4140)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
19^{+7}_{-5} OUR AVERAGE					
$162 \pm 21 ^{+24}_{-49}$	24k	[1] AAIJ	21E	LHCB	$B^+ \to J/\psi\phi K^+$
$15.3^{+10.4}_{-6.1} \pm 2.5$	19	[2] AALTONEN	17	CDF	$B^+ \to J/\psi\phi K^+$
$16.3 \pm 5.6 \pm 11.4$	616	[3] ABAZOV	15M	D0	$p\bar{p} \to J/\psi\phi$ + anything
$20 \pm 13 ^{+3}_{-8}$	52	[4] ABAZOV	14A	D0	$B^+ \to J/\psi\phi K^+$
$28 ^{+15}_{-11} \pm 19$	0.3k	[5] CHATRCHYAN	14M	CMS	$B^+ \to J/\psi\phi K^+$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$83 \pm 21 ^{+21}_{-14}$	4289	[6,7] AAIJ	17C	LHCB	$B^+ \to J/\psi\phi K^+$
$11.7^{+8.3}_{-5.0} \pm 3.7$	14	[8,9] AALTONEN	09AH	CDF	$B^+ \to J/\psi\phi K^+$

State	$Mass_{PDG}$(MeV)	Γ_{PDG} (MeV)	Γ_{sum}
$\chi_{c1}(4140)$	4146.5 ± 3.0	19^{+7}_{-5}	83.0 MeV
$X(4350)$	$4350^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	74.0 MeV
$\chi_{c1}(4274)$	4286^{+8}_{-9}	51 ± 7	76.9 MeV
$X(3960)$[LHCb:2022aki]	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	31.8 MeV
$X_0(4140)$[LHCb:2022aki]	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	60.2 MeV

(1) $cs\bar{c}\bar{s}$ states

J^{PC}	Mass	Channels					Γ_{sum}
2^{++}		$J/\psi\phi$	$D_s^{*+}D_s^{*-}$				
	4316.9	(83.4, 53.8)	(47.5, 23.9)				77.7
	4294.6	(16.6, 10.2)	(52.5, 23.2)				33.4
1^{++}		$J/\psi\phi$	$(D_s^{*+}D_s^- - D_s^+D_s^{*-})/\sqrt{2}$				
	4313.6	(99.8, 63.9)	(8.2, 13.0)				76.9
	4146.5	(0.2, 0.1)	(91.8, 82.9)				83.0
0^{++}		$J/\psi\phi$	$\eta_c\eta'$	$\eta_c\eta$	$D_s^{*+}D_s^{*-}$	$D_s^+D_s^-$	
	4372.6	(57.1, 40.9)	(0.0, 0.0)	(0.0, 0.0)	(52.8, 32.9)	(0.1, 0.2)	74.0
	4249.0	(39.5, 21.2)	(0.8, 0.7)	(0.7, 0.8)	(42.7, 11.4)	(2.3, 2.2)	36.3
	4078.5	(3.1, –)	(18.0, 10.4)	(16.0, 16.6)	(3.8, –)	(49.2, 33.2)	60.2
	3938.2	(0.3, –)	(34.0, –)	(30.3, 28.2)	(0.8, –)	(48.4, 3.6)	31.8
1^{+-}		$J/\psi\eta'$	$J/\psi\eta$	$\eta_c\phi$	$D_s^{*+}D_s^{*-}$	$(D_s^{*+}D_s^- + D_s^+D_s^{*-})/\sqrt{2}$	
	4308.0	(0.9, 0.6)	(0.8, 0.8)	(8.5, 6.9)	(97.7, 46.9)	(0.2, 0.4)	55.5
	4225.1	(3.0, 1.8)	(2.7, 2.7)	(36.8, 26.0)	(1.6, 0.1)	(23.4, 30.3)	60.9
	4166.9	(2.2, 1.1)	(1.9, 1.9)	(54.5, 33.8)	(0.1, –)	(49.6, 50.8)	87.5
	4089.8	(46.9, 13.5)	(41.7, 38.2)	(0.2, 0.1)	(0.6, –)	(26.8, 9.3)	61.0

$$\Gamma(J/\psi\phi) : \Gamma(D_s^*\bar{D}_s) \simeq 4.9,$$

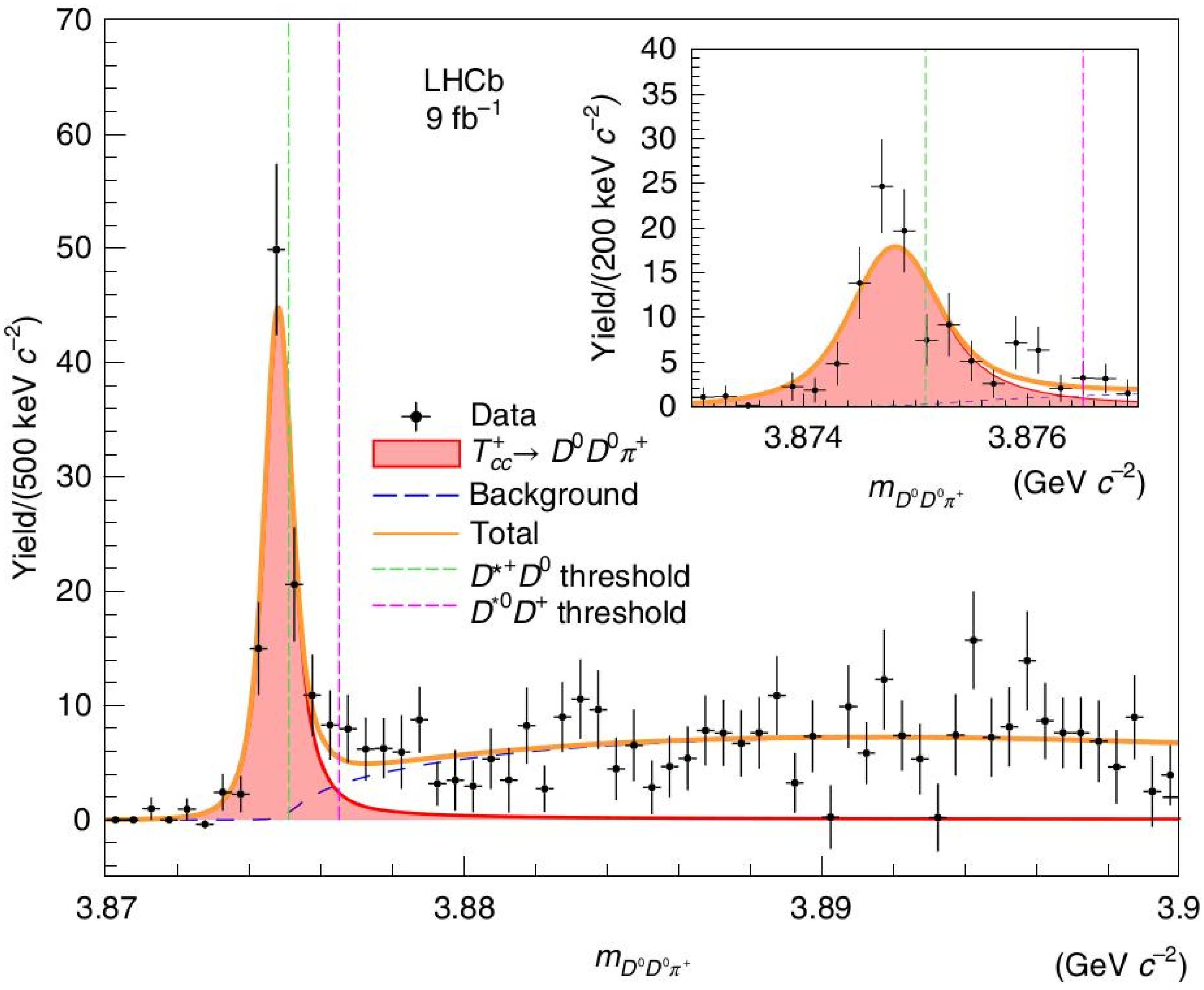
$$\Gamma(J/\psi\phi) : \Gamma(D_s^*\bar{D}_s) \simeq 0.0.$$

$$\Gamma(\eta_c\eta') : \Gamma(\eta_c\eta) : \Gamma(D_s^+D_s^-) \simeq 1 : 1.6 : 3.2,$$

$$\Gamma(\eta_c\eta) : \Gamma(D_s^+D_s^-) \simeq 7.8.$$

Li et al., Chin.Phys.C 48, 063109 (2024)

(2) $QQ\overline{qq}$ states



LHCb, Nature Phys. 18, 751 (2022):

$$m_{D^{*+}} + m_{D^0} = 3875.1 \text{ MeV}$$

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{BW} = 273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

LHCb, Nature Commun. 13,3351 (2022):

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

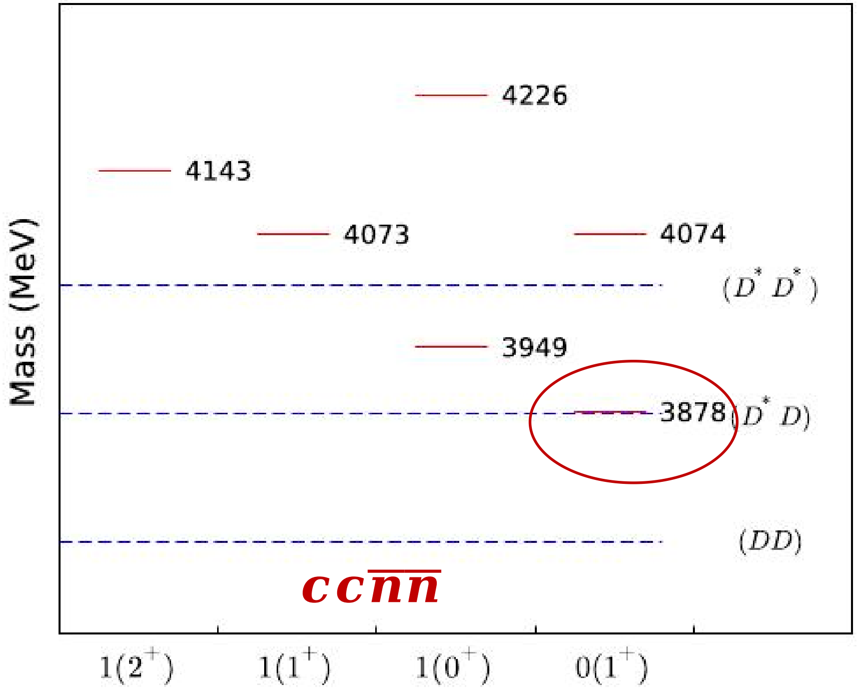
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

Minimal quark content: $cc\bar{u}\bar{d}$

(2) Lowest $I(J^P) = 0(1^+)\ cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV}$$



$\mathcal{C}$=7282.15 MeV from X(4140)

$I(J^P)$	Mass	Channels ($cc\bar{n}\bar{n}$)		Γ
$1(2^+)$	[4143.2]	D^*D^* [(33.3, 20.8)]		[20.8]
$1(1^+)$	[4072.8]	D^*D [(16.7, 53.0)]		[53.0]
$1(0^+)$	[4225.9 / 3948.8]	D^*D^* [(55.7, 43.2) / (2.6, –)]	DD [(0.3, 0.3) / (41.4, 35.9)]	[43.5 / 35.9]
$0(1^+)$	[4074.0 / 3878.2]	D^*D^* [(48.4, 20.9) / (1.6, –)]	D^*D [(6.2, 19.8) / (18.8, 7.2)]	[40.7 / 7.2]

Width sensitive to mass for near-threshold states.

If M → 3876 MeV, Γ=3.0 MeV;
If M → 3880 MeV, Γ=9.7 MeV.

With measured mass $M_{Tcc} = M_{D^{*+}} + M_D - 273$ keV, quasi-two-body decay width [Capstick, Roberts, PRD 49, 4570 (1994)]:

$$k_{max} = \frac{\sqrt{M^2_{T^+_{cc}} - (2M_{D^0} + M_\pi)^2}\sqrt{M^2_{T^+_{cc}} - M^2_\pi}}{2M_{T^+_{cc}}}$$

$$\Gamma = \int_0^{k_{max}} dk \frac{\Gamma_{D^{*+}\to D^0\pi^+}}{(M_{T^+_{cc}} - E_{D^{*+}}(k) - E_{D^0}(k))^2 + \frac{1}{4}\Gamma_{D^{*+}}} \frac{k^2|\mathcal{M}|^2}{(2\pi)^2 M_{T^+_{cc}} E_{D^{*+}}(k) E_{D^0}(k)}$$

~105 keV

(2) Lowest $I(J^P) = 0(1^+)\ cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$

PHYSICAL REVIEW D **104,** 114009 (2021)

Color and baryon number fluctuation of preconfinement system in production process and T_{cc} structure

Yi Jin,[1] Shi-Yuan Li,[2] Yan-Rui Liu,[2] Qin Qin,[3] Zong-Guo Si,[2] and Fu-Sheng Yu[4,5,6]

IV. CONCLUSION

The consistency between the theoretical analysis on the T_{cc} production by Qin, Shen and Yu [37] and the data [8,9] strongly favors that the newly discovered resonance T_{cc} is produced as a real four-quark state. We in this paper clarify

lation. The cross section $pp \to T_{DD^*} + X$ is around $3 \times 10^2\, pb$, which is one order lower than that of the production rate of the four-quark state [37].

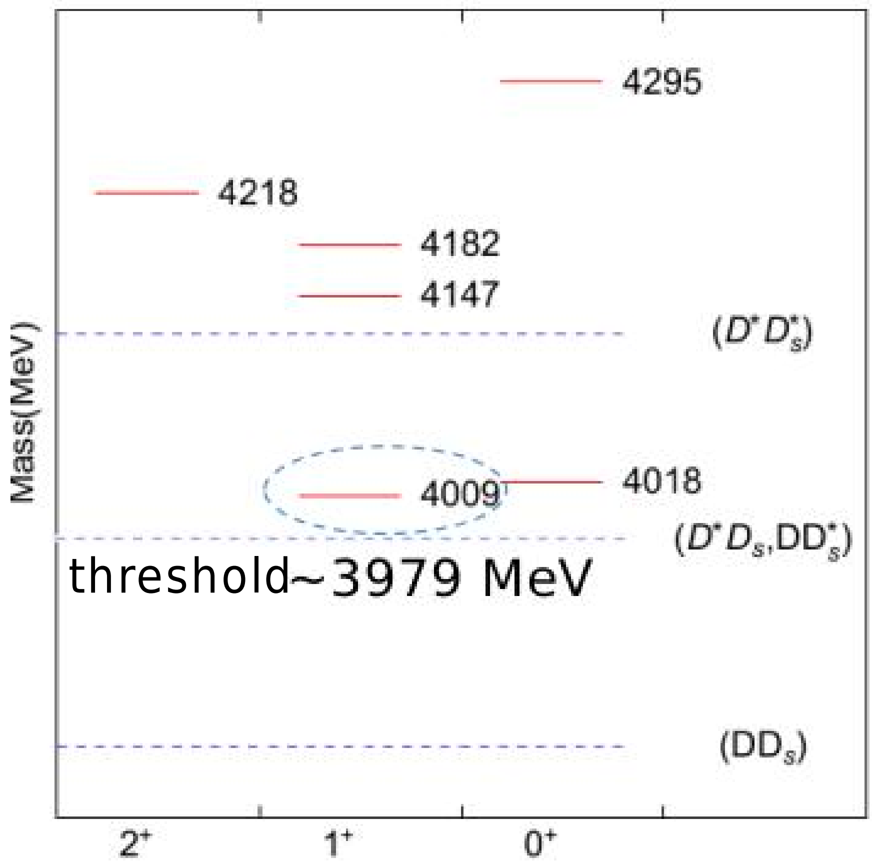
Chinese Physics C Vol. 45, No. 10 (2021) 103106

Discovery potentials of double-charm tetraquarks*

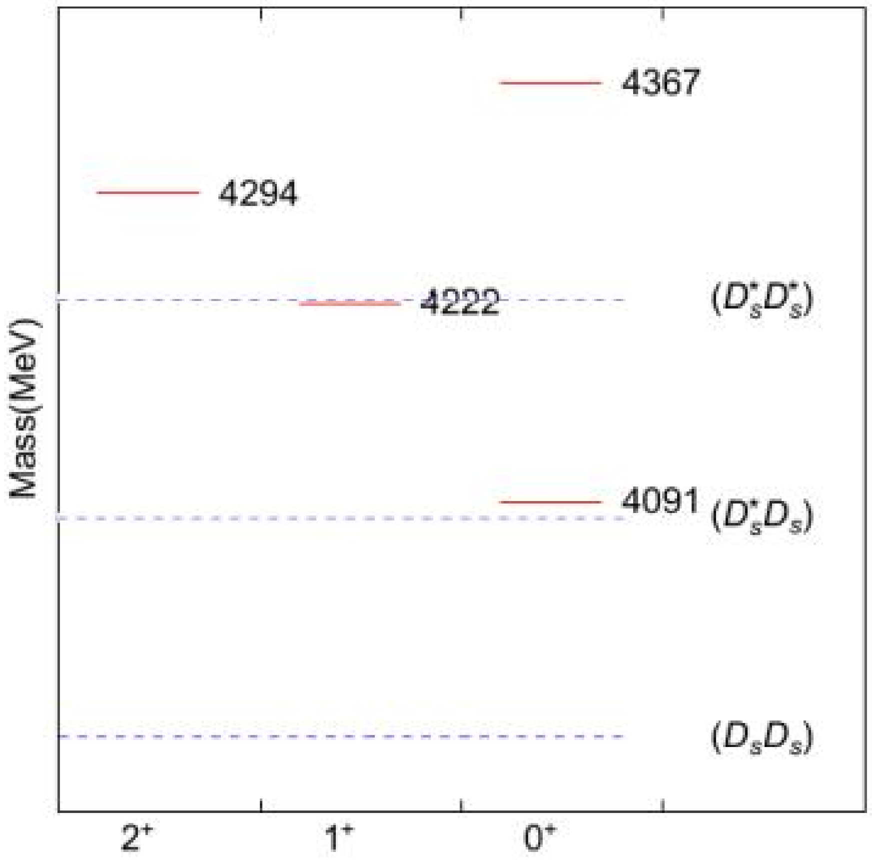
Qin Qin(秦溱)[1†] Yin-Fa Shen(沈胤发)[1] Fu-Sheng Yu(于福升)[2,3,4‡]

From mass, width, and production properties, it is possible to assign the LHCb T_{cc} as the lowest $I(J^P) = 0(1^+)\ cc\bar{u}\bar{d}$ tetraquark state.

(2) $c\,c\,\overline{n}\,\overline{s}$ and $c\,c\,\overline{s}\,\overline{s}$ states: spectrum



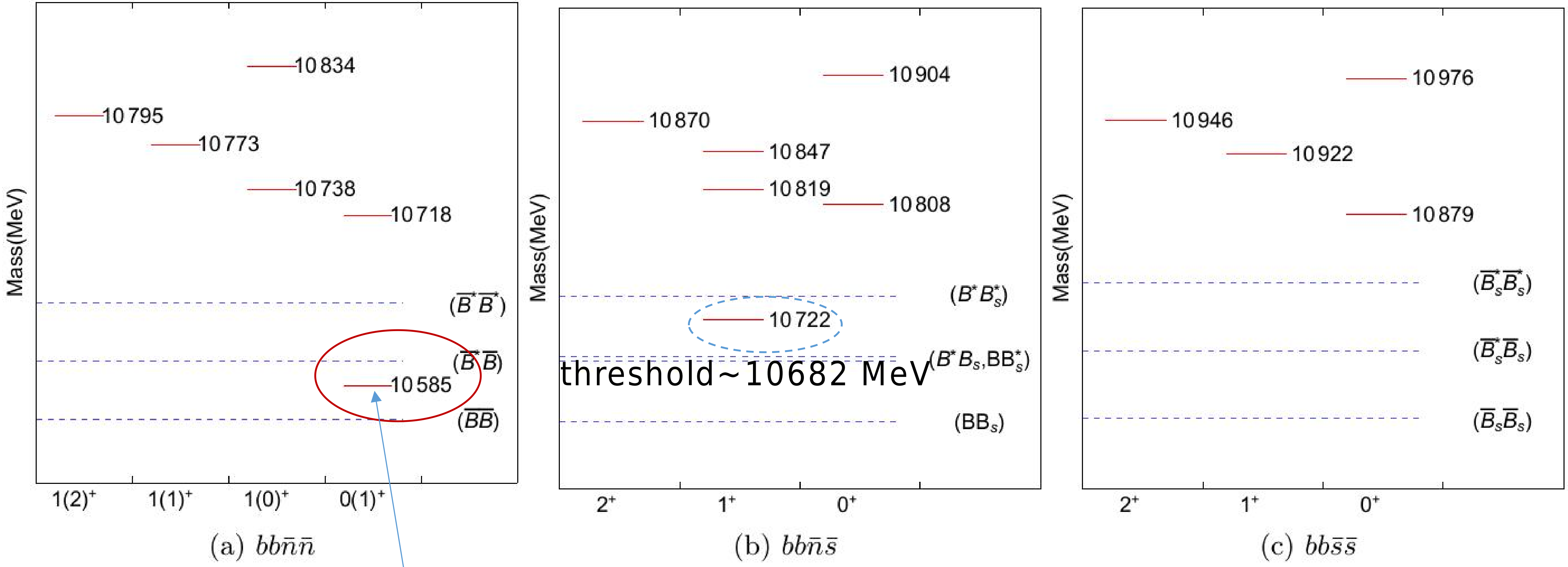
(b) $cc\bar{n}\bar{s}$



(c) $cc\bar{s}\bar{s}$

Belle: PRD 105, 032002 (2022): No $X_{cc\bar{s}\bar{s}}$ is observed

(2) $bb\bar{q}\bar{q}$ states: spectrum



Almost all theoretical studies support this bound $bb\bar{u}\bar{d}$.

Table 10. Stability of the double-heavy tetraquarks in various studies. The meanings of "S," "US," and "ND" are "stable," "unstable," and "not determined," respectively.

Reference	$(cc\bar{n}\bar{n})$	$(cc\bar{n}\bar{s})$	$(cc\bar{s}\bar{s})$	$(bb\bar{n}\bar{n})$	$(bb\bar{n}\bar{s})$	$(bb\bar{s}\bar{s})$	$(bc\bar{n}\bar{n})$	$(bc\bar{n}\bar{s})$	$(bc\bar{s}\bar{s})$
This work	US	US	US	S	S	US	ND	US	US
[8]	S	S		S	S		S	US	
[11]	S	S	US	S	S	US	S	S	US
[16]	S			S					
[18]	S			S			S		
[19]	US			S			S		
[20]	US			S	S		US	US	
[24]	S			S			S		
[28]	S	US	US	S	S	US	S	US	US
[29]	S			S			S		
[30]	US	US	US	S	US	US	US	US	US
[31]	US	US	US	S	US	US	US	US	US
[32]			US			US			US
[33]	US	US	US	S	S	S			
[34]								S	S
[39]	US			S					
[44, 45]	US	US		S	S		S	US	
[47]							S		
[48]				S	S		US	US	
[63]	US			S			ND		
[69]							ND	US	
[83]	US	US	US	S	S	US	US	US	US
[84]	US	US	US	S	S	US	US	US	US

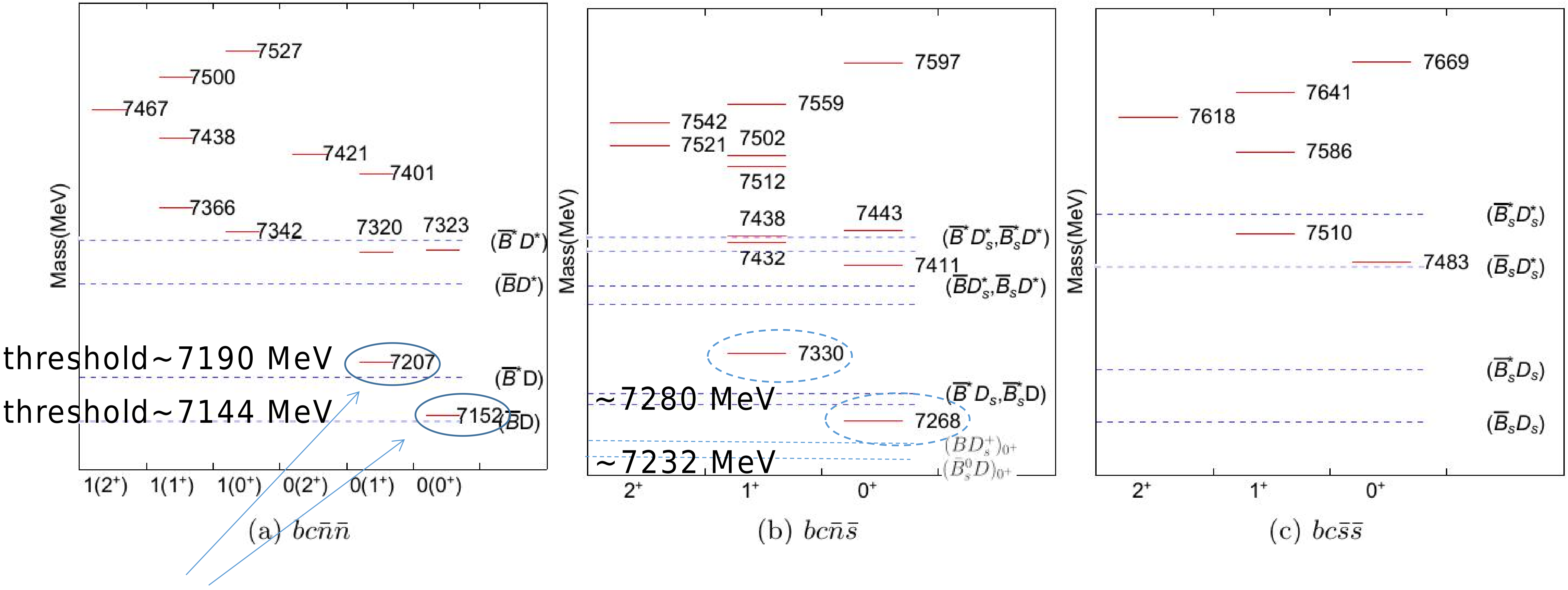
J.B. Cheng et al, CPC 45, 043102 (2021)

$$T_{cc} < 3965 \text{ MeV}$$

$$T_{bb} < 10627 \text{ MeV}$$

$$T_{bc} < 7199 \text{ MeV}$$

(2) $bc\bar{q}\bar{q}$ states: spectrum



(a) $bc\bar{n}\bar{n}$ (b) $bc\bar{n}\bar{s}$ (c) $bc\bar{s}\bar{s}$

J.B. Cheng et al, CPC 45, 043102 (2021): 7167 MeV & 7223 MeV;
Karliner, Rosner, PRL 119, 202001 (2017): 11 MeV below BD;
Alexandrou et al, PRL 132, 151902 (2024): shallow bound $bc\bar{u}\bar{d}$ with J=0 and 1.

(2) $QQ\bar{q}\bar{q}$ states: rearrangement decay

$cc\bar{n}\bar{n}$

$I(J^P)$	Mass	Channels		Γ
		D^*D^*		
$1(2^+)$	4143.2	(33.3, 20.8)		20.8
		D^*D		
$1(1^+)$	4072.8	(16.7, 53.0)		53.0
		D^*D^*	DD	
$1(0^+)$	4225.9	(55.7, 43.2)	(0.3, 0.3)	43.5
	3948.8	(2.6, −)	(41.4, 35.9)	35.9
		D^*D^*	D^*D	
$0(1^+)$	4074.0	(48.4, 20.9)	(6.2, 19.8)	40.7
	3878.2	(1.6, −)	(18.8, 7.2)	7.2

$bb\bar{n}\bar{n}$

$I(J^P)$	Mass	Channels		Γ
		B^*B^*		
$1(2^+)$	10795.3	(33.3, 5.3)		5.3
		$\bar{B}^*\bar{B}$		
$1(1^+)$	10772.9	(16.7, 11.5)		11.5
		$\bar{B}^*\bar{B}^*$	$\bar{B}\bar{B}$	
$1(0^+)$	10834.4	(57.4, 10.3)	(1.2, 0.3)	10.5
	10738.4	(0.9, 0.1)	(40.5, 7.2)	7.4
		$\bar{B}^*\bar{B}^*$	$\bar{B}^*\bar{B}$	
$0(1^+)$	10717.8	(41.2, 4.6)	(12.2, 7.0)	11.6
	10584.5	(8.8, −)	(12.8, −)	0

$cc\bar{n}\bar{s}$

J^P	Mass	Channels			Γ
		$D^*D_s^*$			
2^+	4217.5	(33.3, 35.5)			35.5
		$D^*D_s^*$	D^*D_s	DD_s^*	
1^+	4182.2	(49.6, 42.7)	(4.1, 6.4)	(4.5, 7.0)	56.1
	4146.6	(0.0, 0.0)	(16.7, 24.1)	(16.6, 23.7)	47.8
	4009.2	(0.4, −)	(20.9, 13.9)	(20.5, 13.1)	27.0
		$D^*D_s^*$	DD_s		
0^+	4295.1	(55.3, 76.7)	(0.2, 0.4)		77.1
	4018.8	(3.0, −)	(41.5, 65.0)		65.0

$bb\bar{n}\bar{s}$

J^P	Mass	Channels			Γ
		$B^*B_s^*$			
2^+	10869.9	(33.3, 10.0)			10.0
		$\bar{B}^*\bar{B}_s^*$	$\bar{B}^*\bar{B}_s$	$\bar{B}\bar{B}_s^*$	
1^+	10846.5	(0.1, 0.0)	(15.1, 4.9)	(18.2, 5.9)	10.9
	10819.1	(44.2, 10.4)	(11.3, 3.4)	(8.8, 2.6)	16.3
	10722.2	(5.7, −)	(15.3, 2.3)	(14.7, 2.1)	4.3
		$\bar{B}^*\bar{B}_s^*$	$\bar{B}\bar{B}_s$		
0^+	10903.8	(56.7, 18.9)	(0.7, 0.3)		19.2
	10807.8	(1.7, 0.4)	(41.0, 13.8)		14.2

$cc\bar{s}\bar{s}$

J^P	Mass	Channels		Γ
		$D_s^*D_s^*$		
2^+	4293.5	(33.3, 14.6)		14.6
		$D_s^*D_s$		
1^+	4222.0	(16.7, 42.7)		42.7
		$D_s^*D_s^*$	D_sD_s	
0^+	4366.6	(55.0, 33.6)	(0.1, 0.1)	33.8
	4090.7	(3.3, −)	(41.5, 29.1)	29.1

$bb\bar{s}\bar{s}$

J^P	Mass	Channels		Γ
		$B_s^*B_s^*$		
2^+	10946.1	(33.3, 4.7)		4.7
		$\bar{B}_s^*\bar{B}_s$		
1^+	10921.6	(16.7, 10.3)		10.3
		$\bar{B}_s^*\bar{B}_s^*$	$\bar{B}_s\bar{B}_s$	
0^+	10975.7	(55.8, 8.7)	(0.3, 0.1)	8.8
	10878.7	(2.5, 0.2)	(41.3, 6.5)	6.8

Ratios between partial widths as predictions for tetraquark states that have two or three rearrangement decay channels.

$bc\bar{s}\bar{s}$

J^P	Mass	Channels			Γ
		$\bar{B}_s^*D_s^*$			
2^+	7617.9	(33.3, 12.8)			12.8
		$\bar{B}_s^*D_s^*$	$\bar{B}_s^*D_s$	$\bar{B}_sD_s^*$	
1^+	7640.5	(46.2, 19.7)	(0.4, 0.2)	(8.1, 4.1)	24.1
	7586.4	(3.0, 0.9)	(1.4, 0.8)	(29.3, 12.3)	14.1
	7510.2	(0.8, −)	(39.8, 18.2)	(4.3, 1.0)	19.2
		$\bar{B}_s^*D_s^*$	$\bar{B}_sD_s$		
0^+	7668.6	(55.2, 26.1)	(0.2, 0.1)		26.3
	7482.6	(3.2, −)	(41.5, 20.6)		20.6

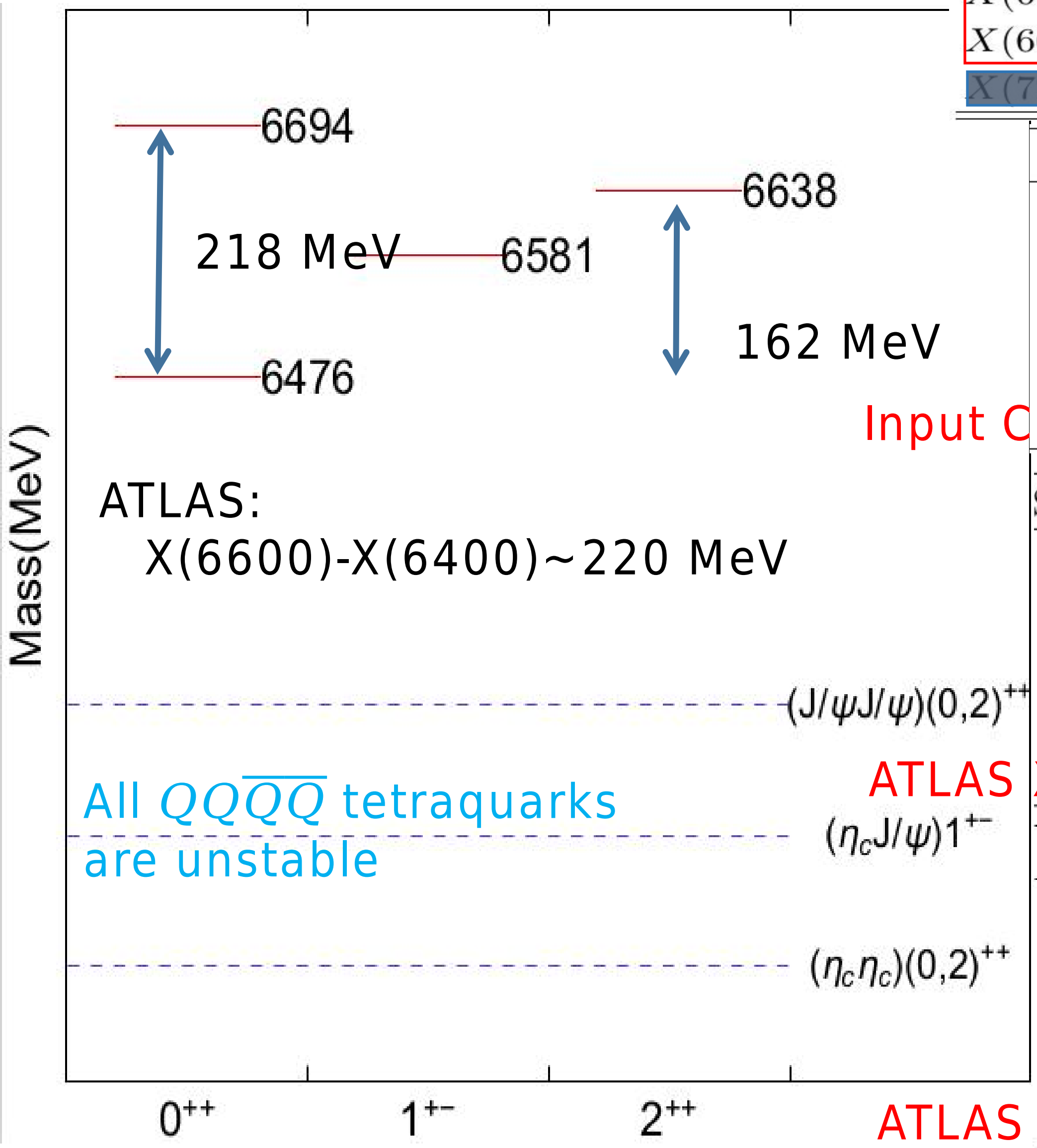
(3) $cc\bar{c}\bar{c}$ states

TABLE I: Masses (M) and widths (Γ) of the fully-heavy tetraquark states in units of MeV. Their observation channels are presented in the last column. Both CMS and ATLAS Collaborations used two models in determining the resonance parameters.

Collaboration	State	(M, Γ)		Observation Channel
LHCb [1]	X(6900)	$(6905 \pm 11 \pm 7,\ 80 \pm 19 \pm 33)$		$J/\psi J/\psi$
CMS[3]		Interference model	No-interference model	$J/\psi J/\psi$
	X(6600)	$(6638^{+43+16}_{-38-31},\ 440^{+230+110}_{-200-240})$	$(6552 \pm 10 \pm 12,\ 124^{+32}_{-26} \pm 33)$	
	X(6900)	$(6847^{+44+48}_{-28-20},\ 191^{+66+52}_{-49-17})$	$(6927 \pm 9 \pm 4,\ 122^{+24}_{-21} \pm 18)$	
	X(7200)	$(7134^{+48+41}_{-25-15},\ 97^{+40+29}_{-29-26})$	$(7287^{+20}_{-18} \pm 5,\ 95^{+59}_{-40} \pm 19)$	
ATLAS[4]		Model A	Model B	$J/\psi J/\psi$
	X(6400)	$(6410 \pm 80^{+80}_{-30},\ 590 \pm 350^{+120}_{-200})$	$(6650 \pm 20^{+30}_{-20},\ 440 \pm 50^{+60}_{-50})$	
	X(6600)	$(6630 \pm 50^{+80}_{-10},\ 350 \pm 110^{+110}_{-40})$		
	X(6900)	$(6860 \pm 30^{+10}_{-20},\ 110 \pm 50^{+20}_{-10})$	$(6910 \pm 10 \pm 10,\ 150 \pm 30 \pm 10)$	
		Model α	Model β	$J/\psi\psi(2S)$
	X(7200)	$(7200 \pm 30^{+10}_{-40},\ 90 \pm 60^{+60}_{-50})$	$(6960 \pm 50 \pm 30,\ 510 \pm 170^{+110}_{-100})$	

(3) $cc\overline{c}\overline{c}$ states

State	Mass (MeV)	Γ(MeV)	Observed channels
$X(6900)$[LHCb:2020bwg]	$6905 \pm 11 \pm 7$ MeV	$80 \pm 19 \pm 33$ MeV	di-J/ψ
$X(6600)$[CMS:2023owd]	$6552 \pm 10 \pm 12$ MeV	$124^{+32}_{-26} \pm 33$ MeV	diJ/ψ
$X(7200)$[CMS:2023owd]	$7287^{+20}_{-18} \pm 5$MeV	$95^{+59}_{-40} \pm 19$MeV	di-J/ψ
$X(6400)$[ATLAS:2023bft]	$6.41 \pm 0.08^{+0.08}_{-0.03}$ GeV	$0.59 \pm 0.35^{+0.12}_{-0.2}$GeV	di-J/ψ
$X(6600)$[ATLAS:2023bft]	$6.63 \pm 0.05^{+0.08}_{-0.01}$ GeV	$0.35 \pm 0.11^{+0.11}_{-0.04}$GeV	di-J/ψ
$X(7200)$[ATLAS:2023bft]	$7.22 \pm 0.03^{+0.01}_{-0.04}$GeV	$0.095 \pm 0.6^{+0.06}_{-0.05}$ GeV	$J/\psi + \psi(2S)$



Input CMS X(6600)→ (6476.4)

System	$J^{P(C)}$	Mass	Channels: $J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 80.2)			80.2
	1^{+-}	6581.0		(16.7, 172.1)		172.1
	0^{++}	6694.3	(54.9, 138.2)		(0.1, 0.3)	138.5
		6476.4	(3.5, 6.9)		(41.6, 110.7)	117.7

ATLAS X(6400)→ (6476.4); Input ATLAS X(6600) → (368.6)

System	$J^{P(C)}$	Mass	Channels: $J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 213.5)			213.5
	1^{+-}	6581.0		(16.7, 458.0)		458.0
	0^{++}	6694.1	(54.9, 367.7)		(0.1, 0.9)	368.6
		6476.4	(3.5, 18.5)		(41.6, 294.8)	313.2

ATLAS X(6400)→ (6476.4); Input ATLAS X(6600)→ (352.3)

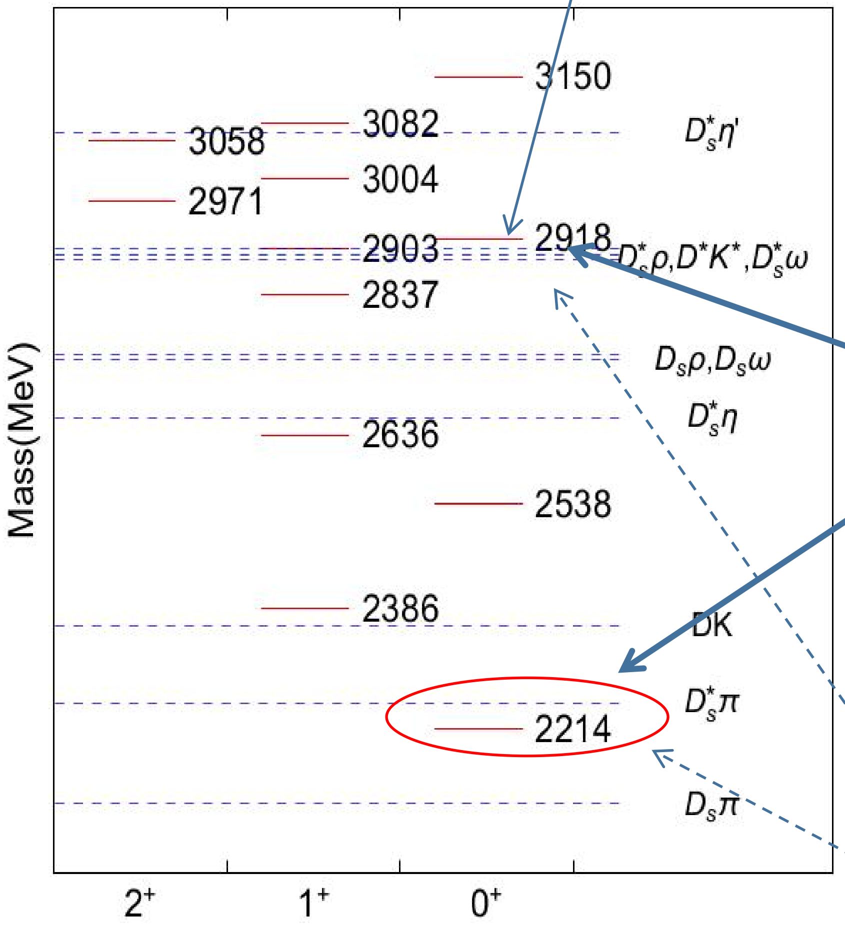
System	$J^{P(C)}$	Mass	Channels: $J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 352.3)			352.3
	1^{+-}	6581.0		(16.7, 755.8)		755.8
	0^{++}	6694.1	(54.9, 606.8)		(0.1, 1.4)	608.2
		6476.4	(3.5, 30.5)		(41.6, 486.4)	516.9

(4) $Qq\overline{q}\overline{q}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

P=- →

$\mathcal{C}$=13.577 GeV ←



$I(J^P)$	Mass	$D_s^*\rho$	D^*K^*	Γ
$1(2^+)$	3058.0	(95.8, 340.4)	(26.9, 94.8)	435.2
	2971.3	(4.2, 10.9)	(73.1, 179.3)	190.1

$I(J^P)$	Mass	$D_s^*\rho$	$D_s^*\pi$	$D_s\rho$	D^*K^*	D^*K	DK^*	Γ
$1(1^+)$	3082.1	(71.1, 266.6)	(0.0, 0.2)	(4.5, 22.7)	(22.5, 84.1)	(0.8, 4.6)	(9.3, 46.7)	424.9
	3004.4	(26.8, 80.9)	(0.3, 2.0)	(22.7, 103.6)	(61.8, 181.3)	(1.3, 7.5)	(1.3, 6.1)	381.4
	2902.7	(0.0, 0.0)	(0.7, 4.2)	(44.1, 166.0)	(12.6, 2.7)	(0.0, 0.2)	(12.4, 45.5)	218.5
	2837.3	(1.3, –)	(0.0, 0.0)	(26.8, 80.2)	(0.1, –)	(3.2, 16.5)	(70.9, 198.4)	295.1
	2636.4	(0.8, –)	(10.7, 51.5)	(1.8, –)	(1.5, –)	(72.7, 258.8)	(4.4, –)	310.3
	2385.9	(0.0, –)	(88.2, 253.0)	(0.0, –)	(1.4, –)	(22.1, –)	(1.7, –)	253.0

$I(J^P)$	Mass	$D_s^*\rho$	$D_s\pi$	D^*K^*	DK	Γ
$1(0^+)$	3149.7	(63.3, 266.6)	(0.0, 0.3)	(46.3, 196.1)	(0.2, 1.7)	464.6
	2917.9	(34.1, 54.9)	(0.7, 5.1)	(47.4, 56.9)	(3.1, 20.1)	137.0
	2537.5	(2.5, –)	(17.3, 96.4)	(4.1, –)	(67.0, 297.1)	393.5
	2214.3	(0.1, –)	(81.9, 235.6)	(2.1, –)	(29.6, –)	235.6

$I(J^P)$	Mass	$D_s^*\omega$	D^*K^*	Γ
$0(2^+)$	3058.0	(95.8, 333.6)	(26.9, 94.8)	428.4
	2971.3	(4.2, 10.4)	(73.1, 179.3)	189.7

$I(J^P)$	Mass	$D_s^*\omega$	$D_s^*\eta$	$D_s^*\eta'$	$D_s\omega$	D^*K^*	D^*K	DK^*	Γ
$0(1^+)$	3082.1	(71.1, 262.0)	(0.0, 0.1)	(0.0, 0.0)	(4.5, 22.5)	(22.5, 84.1)	(0.8, 4.6)	(9.3, 46.7)	420.0
	3004.4	(26.8, 78.5)	(0.2, 0.8)	(0.2, –)	(22.7, 102.3)	(61.8, 181.3)	(1.3, 7.5)	(1.3, 6.1)	376.4
	2902.7	(0.0, 0.0)	(0.3, 1.4)	(0.4, –)	(44.1, 162.4)	(12.6, 2.7)	(0.0, 0.2)	(12.4, 45.5)	212.1
	2837.3	(1.3, –)	(0.3, 0.0)	(0.0, –)	(26.8, 77.1)	(0.1, –)	(3.2, 16.5)	(70.9, 198.4)	292.0
	2636.4	(0.8, –)	(5.0, –)	(5.7, –)	(1.8, –)	(1.5, –)	(72.7, 258.8)	(4.4, –)	258.8
	2385.9	(0.0, –)	(41.5, –)	(46.7, –)	(0.0, –)	(1.4, –)	(22.1, –)	(1.7, –)	–

$I(J^P)$	Mass	$D_s^*\omega$	$D_s\eta$	$D_s\eta'$	D^*K^*	DK	Γ
$0(0^+)$	3149.7	(63.3, 263.3)	(0.0, 0.1)	(0.0, 0.1)	(46.3, 196.1)	(0.2, 1.7)	461.3
	2917.9	(34.1, 47.8)	(0.4, 2.0)	(0.4, 0.1)	(47.4, 56.9)	(3.1, 20.1)	126.9
	2537.5	(2.5, –)	(8.1, 12.6)	(9.1, –)	(4.1, –)	(67.0, 297.1)	309.7
	2214.3	(0.1, –)	(38.6, –)	(43.3, –)	(2.1, –)	(29.6, –)	–

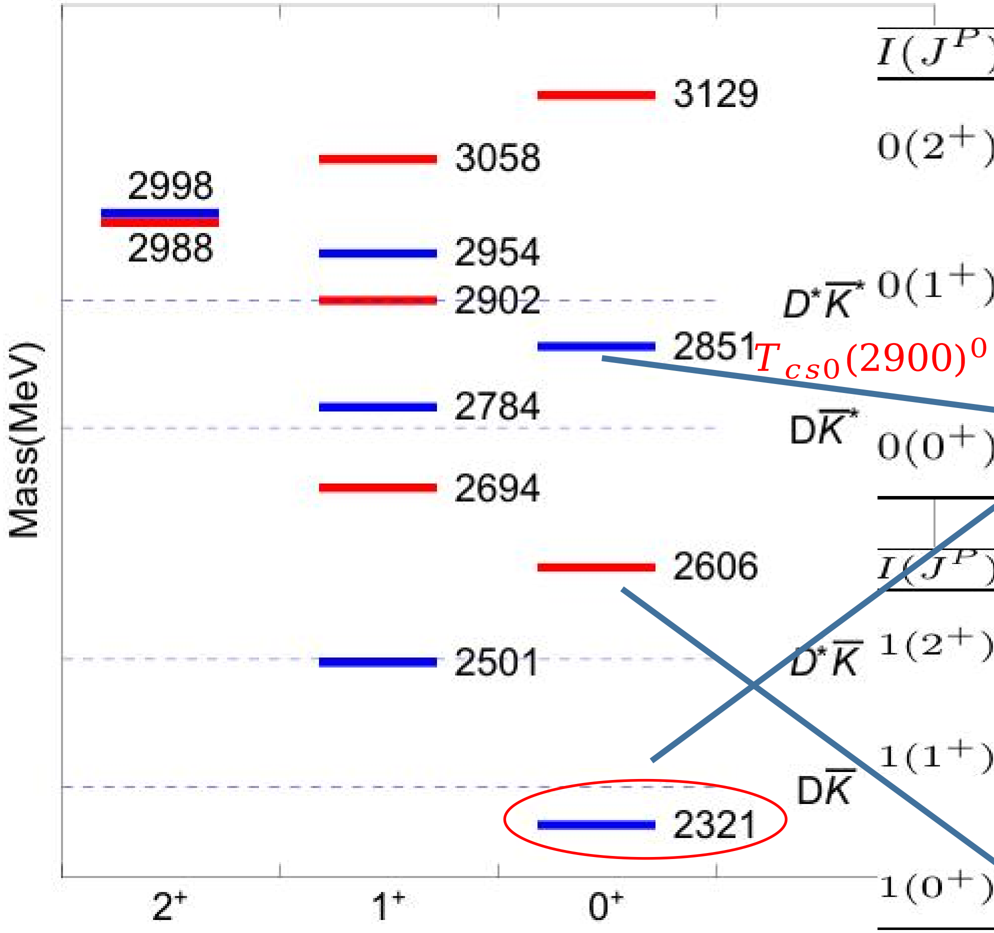
$cn\overline{sn}$: I=0 & I=1 degenerate

(4) $Qq\bar{q}\bar{q}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
[redacted]			
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

P=- →

$\mathcal{C}$=13.577 GeV ←



(e) $I = 0/1\ cs\bar{n}\bar{n}$ states

$I(J^P)$	Mass	Channels			Γ
$0(2^+)$	2998.3	D^*K^*: (66.7, 380.2)			380.2
$0(1^+)$	2953.5	$D^*\bar{K}^*$: (47.6, 203.0)	$D^*\bar{K}$: (1.3, 14.5)	$D\bar{K}^*$: (10.6, 88.5)	306.1
	2784.3	(2.4, −)	(1.8, 17.8)	(47.6, 151.9)	169.7
	2500.6	(0.1, −)	(55.2, −)	(0.1, −)	0
$0(0^+)$	2850.5	$D^*\bar{K}^*$: (41.4, −)	$D\bar{K}$: (2.7, 34.4)		34.4
	2320.7	(0.3, −)	(55.6, −)		0

$I(J^P)$	Mass	Channels			Γ
$1(2^+)$	2987.5	D^*K^*: (33.3, 180.1)			180.1
$1(1^+)$	3058.0	$D^*\bar{K}^*$: (46.9, 329.8)	$D^*\bar{K}$: (0.4, 4.5)	$D\bar{K}^*$: (7.1, 69.0)	403.3
	2902.3	(2.2, 0.3)	(1.1, 12.1)	(30.7, 225.3)	237.7
	2693.7	(0.9, −)	(40.2, 333.5)	(3.9, −)	333.5
$1(0^+)$	3128.9	$D^*\bar{K}^*$: (55.1, 451.1)	$D\bar{K}$: (0.1, 2.1)		453.2
	2606.4	(3.2, −)	(41.5, 419.3)		419.3

Summary

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

With one mass formulae and a simple decay scheme:

- X(3960) is a good candidate of the lowest $\mathbf{0^{++}}$ $\boldsymbol{cs\bar{c}\bar{s}}$ tetraquark state.

- The lowest $\mathbf{0(1^{+})}$ $\boldsymbol{cc\bar{u}\bar{d}}$ tetraquark state can be used to understand the LHCb $\boldsymbol{T_{cc}}$ state. [mass, width, production]

- X(6660)/X(6400) consistent with $\mathbf{0^{++}/0^{++}}$ $\boldsymbol{cc\bar{c}\bar{c}}$ tetraquark states [$\mathbf{2^{++}/0^{++}?}$].

- $\boldsymbol{T^{a}_{c\bar{s}0}(2900)}$ as the second highest I=1 $\boldsymbol{cn\bar{s}\bar{n}}$ tetraquark state; $\boldsymbol{T_{cs0}(2900)}$ as the higher I=0 $\boldsymbol{cs\bar{u}\bar{d}}$ tetraquark state.

Thank you for your attention!