天津大学
Tianjin University

23rd International Conference on Few-Body Problems in Physics
September 22-27, 2024 Beijing, China

Exploration for Hyperon Halo with Density Functional Theory

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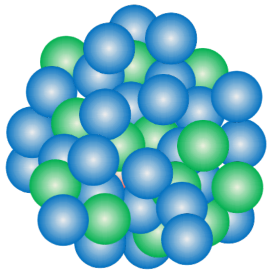
Outline

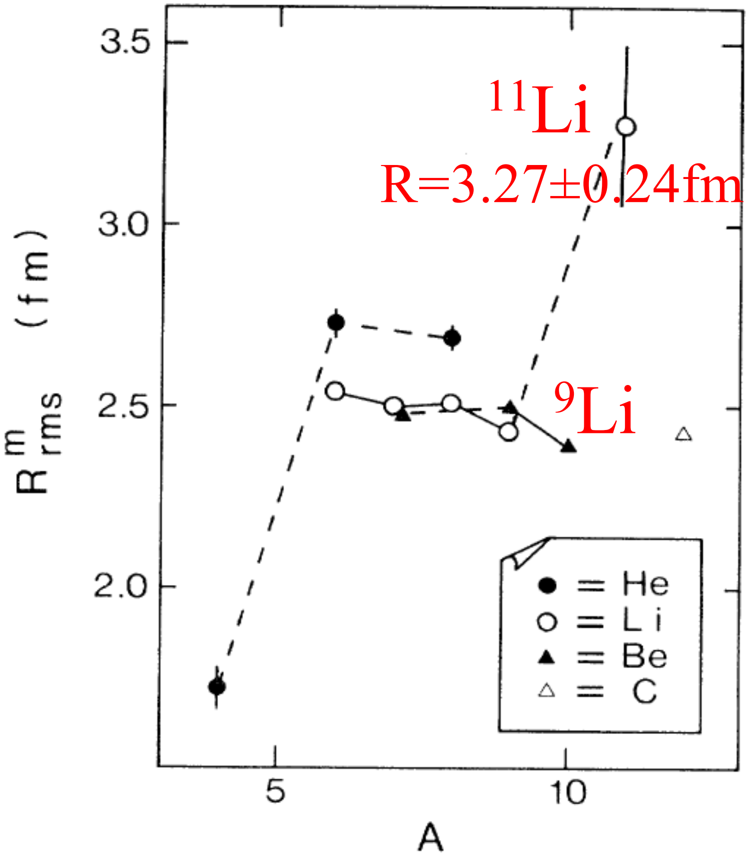
- Introduction
- Density Functional theory（DFT） for hypernucleus
- Hyperon halo orbits in C hypernuclei
- Hyperon halo orbits in Zr hypernuclei
- Possible hyperon halo in O hypernuclei
- Summary

Introduction

Stable nucleus

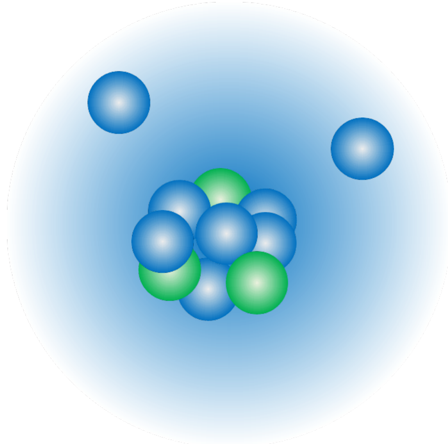


Neutron-rich nuclei

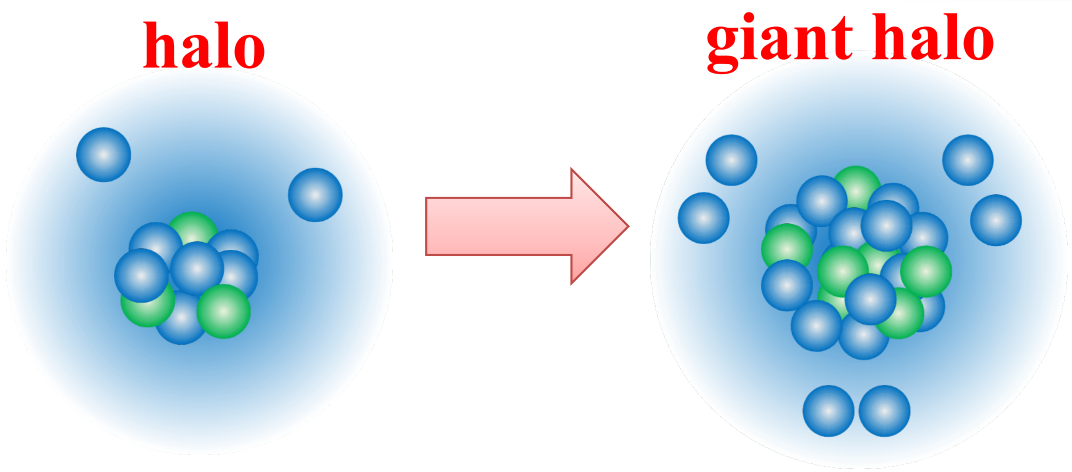


Tanihata, et al., PRL55, 2676 (1985)

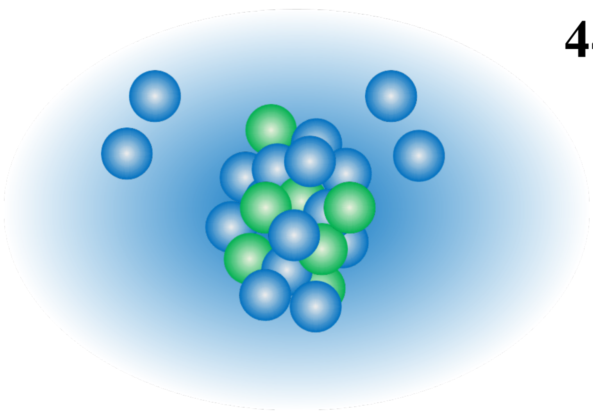
HALO

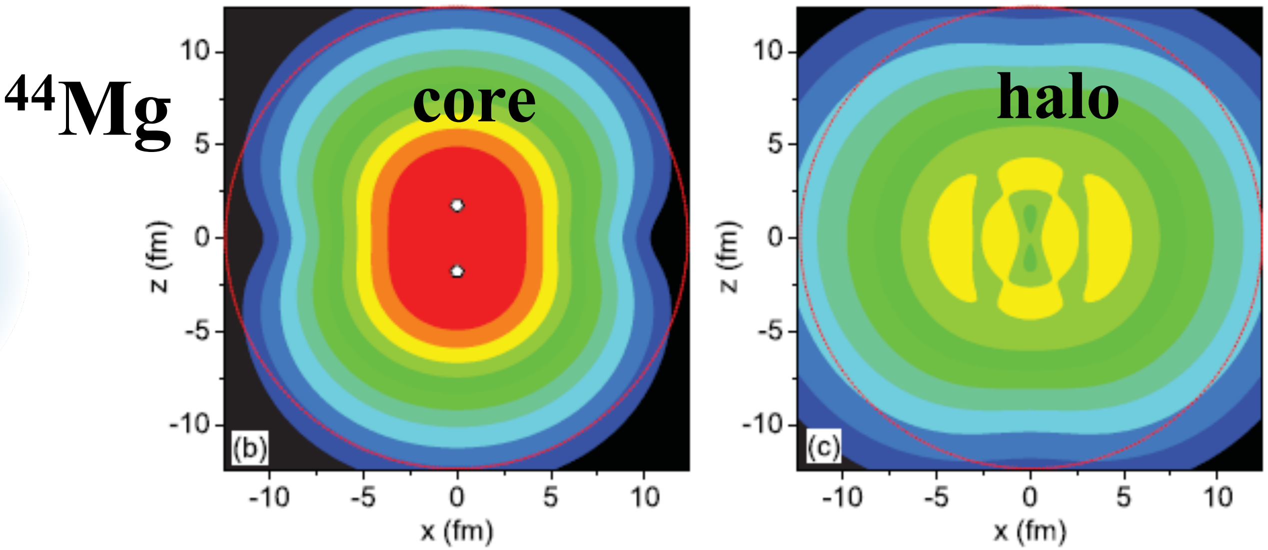


Introduction



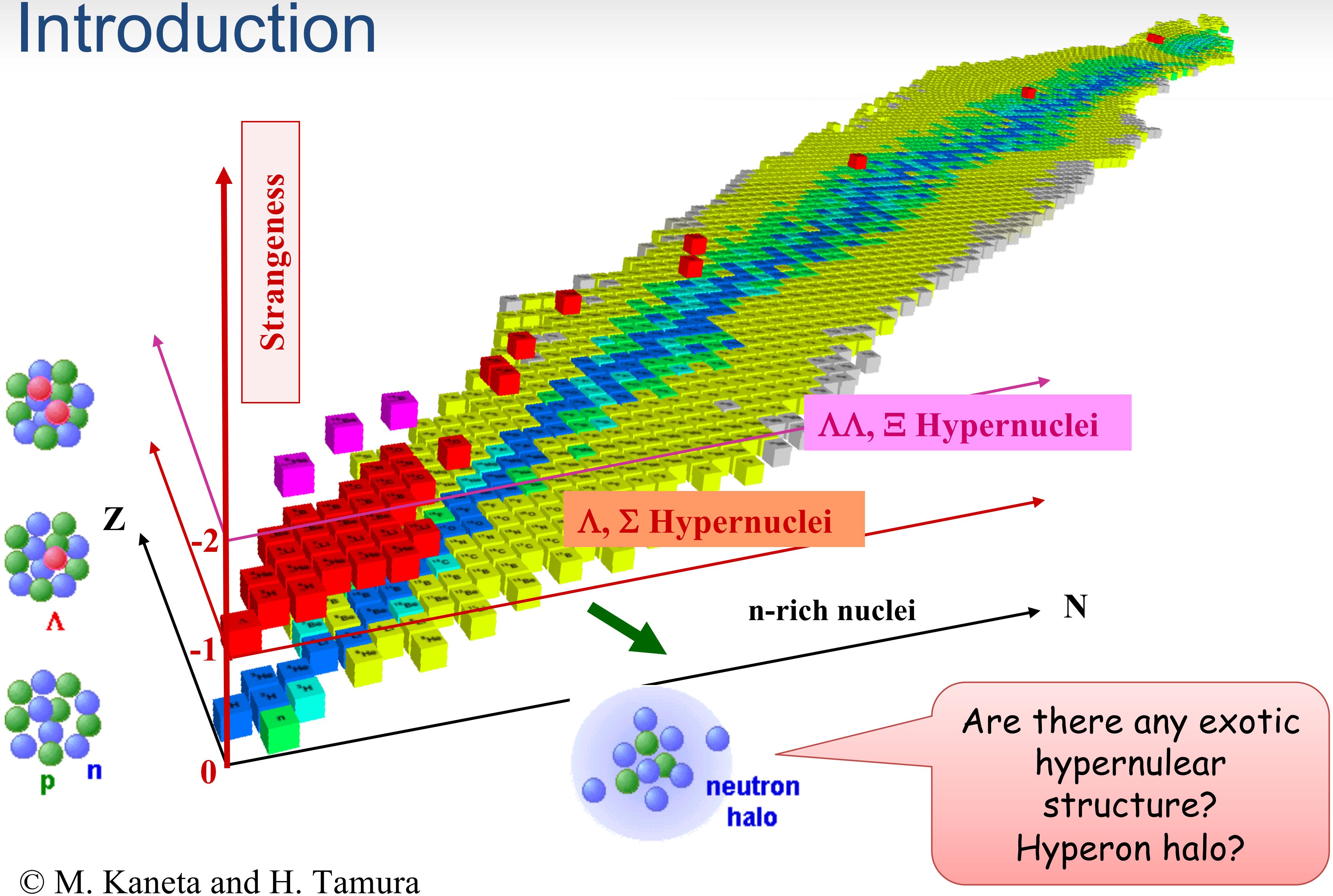
J. Meng, and P. Ring. PRL. 80:460 (1998).
J. Meng, H. Toki, J. Y. Zeng, S.Q. Zhang, S.G. Zhou. PRC, 65:041302(R) (2002).
J. Terasaki, S. Q. Zhang, S. G. Zhou, J. Meng, PRC 74, 054318 (2006)
M. Grasso, S. Yoshida, N. Sandulescu, N. Van Giai, PRC74, 064317 (2006)
J. Meng, and S. G. Zhou, J. Phys. 42, 093101(2015)





S.G. Zhou, S. G. Zhou, J. Meng, P. Ring, E. G. Zhao, PRC 82, 011301(R) (2010)
Y. Chen. L.L. Lu, H.Z. Liang, J. Meng, PRC.85.067301(2012)

Introduction



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http://lambda.phys.tohoku.ac.jp/strangeness/index-j.html

Introduction

- Hyperon effect on the neutron drip line
  - Few body model

    E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto, Phys. Rev. C 53, 2075 (1996).
  - Shell model

    R. Wirth, R. Roth, Phys. Lett. B 779 (2018) 336–341
  - Skyrme Hartree-Fock model

    X. R. Zhou, A. Polls, H. J. Schulze, and I. Vidana, Phys. Rev. C 78, 054306 (2008).
  - Relativistic Hartree-Bogoliubov

    D. Vretenar, W. Pöschl, G. A. Lalazissis, and P. Ring, Phys. Rev. C 57, R1060(R) (1998).

    H. F. Lv, J. Meng, S. Q. Zhang, and S. G. Zhou, Eur. Phys. J. A 17, 19 (2003).
- Hyperon drip line (multistrangeness)
  - Skyrme Hartree-Fock model

    J. Margueron, E. Khan, and F. Gulminelli, Phys. Rev. C 96, 054317 (2017).
  - Hartree-Fock-Bogoliubov

    H. Güven, K. Bozkurt, E. Khan, and J. Margueron, Phys. Rev. C 98, 014318 (2018).
  - Relativistic Hartree-Bogoliubov

    H. F. Lv, J. Meng, Chinese Phys. Lett. 19, No. 12, 1775 (2002)

    H. F. Lv, Chinese Phys. Lett. 25, 3613 (2008).

    Y. T. Rong, P. Zhao, and S. G. Zhou, Phys. Lett. B 807, 135533 (2020).

DFT for hypernuclei

★ Skyrme-like interaction

✓ NN interaction D. Vautherin and D.M. Brink, PRC 5, 626 (1972).

$$v_{NN}(\mathbf{r}_1-\mathbf{r}_2) = t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)+\frac{1}{2}t_1(1+x_1P_\sigma)\left[\boldsymbol{k}'^2\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_2)\boldsymbol{k}^2\right]$$
$$+t_2(1+x_2P_\sigma)\boldsymbol{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\boldsymbol{k}+iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}'\delta(\mathbf{r}_1-\mathbf{r}_2)\times\boldsymbol{k}, \quad (1)$$

$$v_{den-NN}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \frac{1}{6}t_3(1+x_3P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)\rho^\alpha\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right), \quad (2)$$

✓ NΛ interaction D. E. Lanskoy and Y. Yamamoto, PRC 55, 2330 (1997).

$$v_{\Lambda N}(\mathbf{r}_\Lambda-\mathbf{r}_N) = t_0^\Lambda(1+x_0^\Lambda P_\sigma)\delta(\mathbf{r}_\Lambda-\mathbf{r}_N)+\frac{1}{2}t_1^\Lambda\left[\boldsymbol{k}'^2\delta(\mathbf{r}_\Lambda-\mathbf{r}_N)+\delta(\mathbf{r}_\Lambda-\mathbf{r}_N)\boldsymbol{k}^2\right]$$
$$+\ t_2^\Lambda\boldsymbol{k}'\delta(\mathbf{r}_\Lambda-\mathbf{r}_N)\cdot\boldsymbol{k}+iW_0^\Lambda\boldsymbol{k}'\delta(\mathbf{r}_\Lambda-\mathbf{r}_N)\cdot(\boldsymbol{\sigma}_N+\boldsymbol{\sigma}_\Lambda)\times\boldsymbol{k} \quad (3)$$

$$v_{den-\Lambda N}(\mathbf{r}_\Lambda,\mathbf{r}_N,\rho) = \frac{3}{8}t_3^\Lambda(1+x_3^\Lambda P_\sigma)\delta(\mathbf{r}_\Lambda-\mathbf{r}_N)\rho^\gamma\left(\frac{\mathbf{r}_\Lambda+\mathbf{r}_N}{2}\right), \quad (4)$$

✓ **ΛΛ** interaction D. E. Lanskoy, PRC 58, 3351 (1998).

$$V_{\Lambda\Lambda} = \lambda_0\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\frac{1}{2}\lambda_1\left[\boldsymbol{k}'^2\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}^2\right]+\lambda_2\boldsymbol{k}'\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}+\lambda_3\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\rho_N^\alpha\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right)$$

Hyperon halo orbits in C hypernuclei

★Energy density functional

✓ NN interaction SIII, SLy4, SkM*

✓ NΛ interaction LY5 $W_0^\Lambda = 62$ MeV fm^5 D. E. Lanskoy and Y. Yamamoto, PRC 55, 2330 (1997)

to reproduce the spin-orbit splitting of 1p states (0.152 MeV) in $^{13}_{\Lambda}$C

H. Kohri et al., Phys. Rev.C 65, 034607 (2002)

LY5r $W_0^\Lambda = 4.7$ MeV fm^5

Table 3.6: The calculated binding energies of $^{13}_{\Lambda}$C obtained by SkM* and Set V in Ref. [15] with $W_0^\Lambda = 4.7$ MeV fm^5. The experiment data are taken from Ref. [24] [H. Kohri, et al., Phys. Rev. C 65, 9 (2002)]. All the values are in MeV.

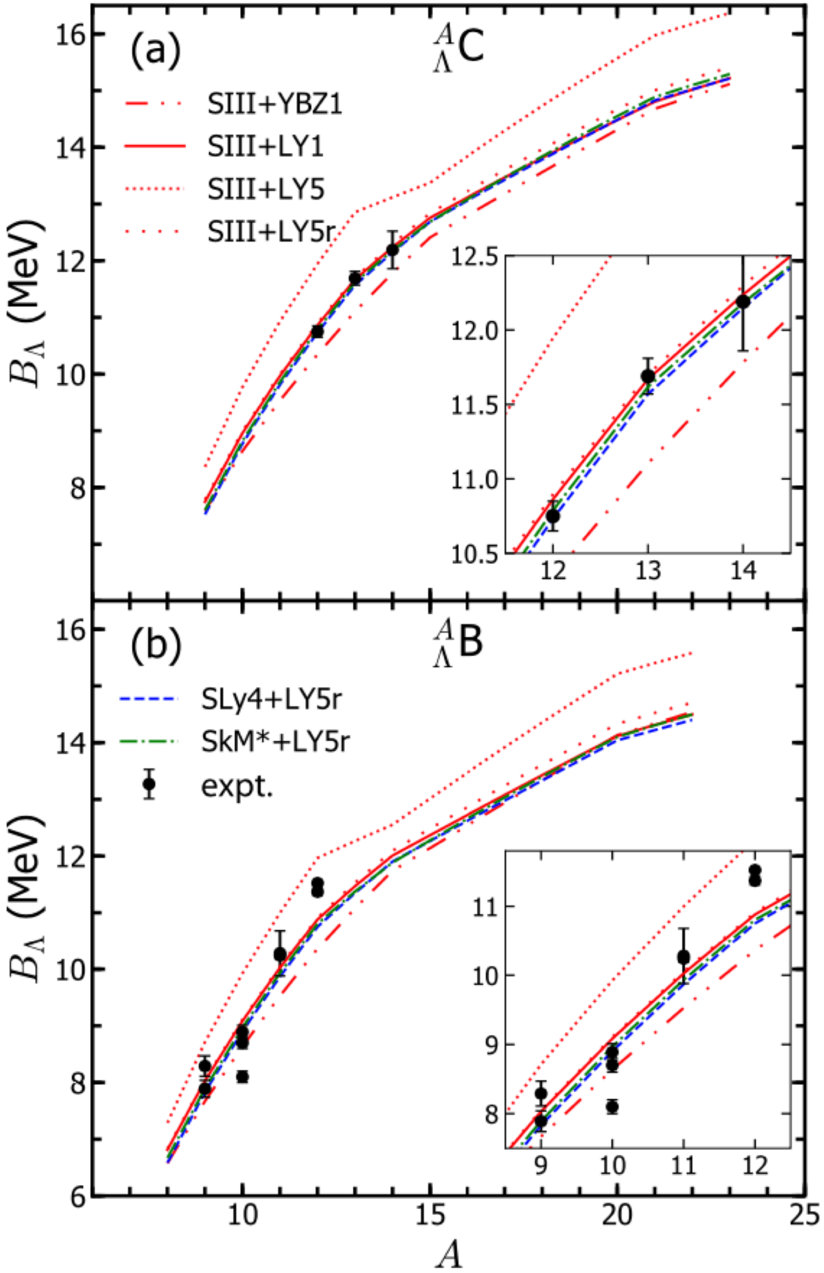
Λ orbit	e_Λ	B_Λ	B_Λ^{exp}	$B_\Lambda(s_{1/2}) - B_\Lambda$	$E_{\text{ex}}^{\text{exp}}$
$s_{1/2}$	−13.156	11.618	11.69 ± 0.12	−−	−−
$p_{1/2}$	−1.782	0.273		11.344	10.982 ± 0.031(stat) ± 0.056(syst)
$p_{3/2}$	−1.936	0.428		11.190	10.830 ± 0.031(stat) ± 0.056(syst)

0.155 Cal.

0.152 Exp.

$$B_\Lambda = B(^{A+1}_{\Lambda}Z) - B(^{A}Z)$$

Hyperon halo orbits in C hypernuclei

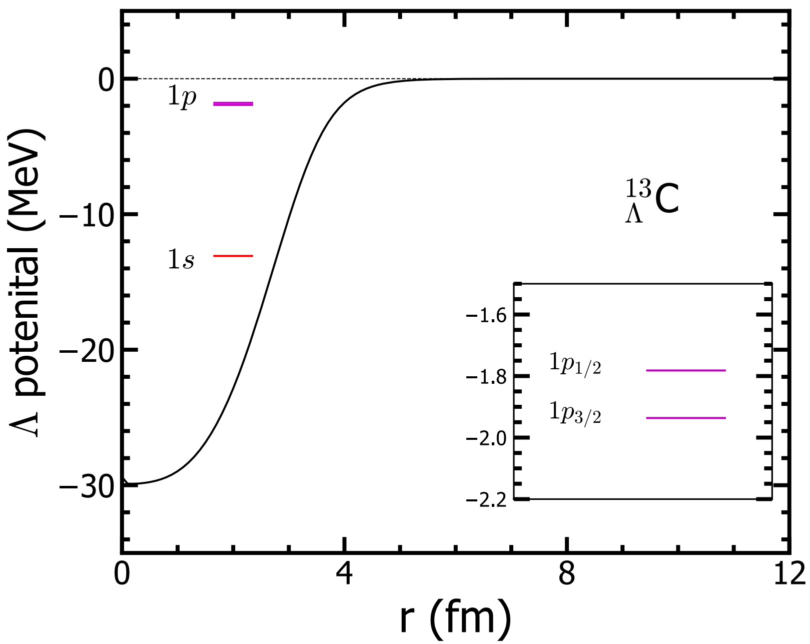


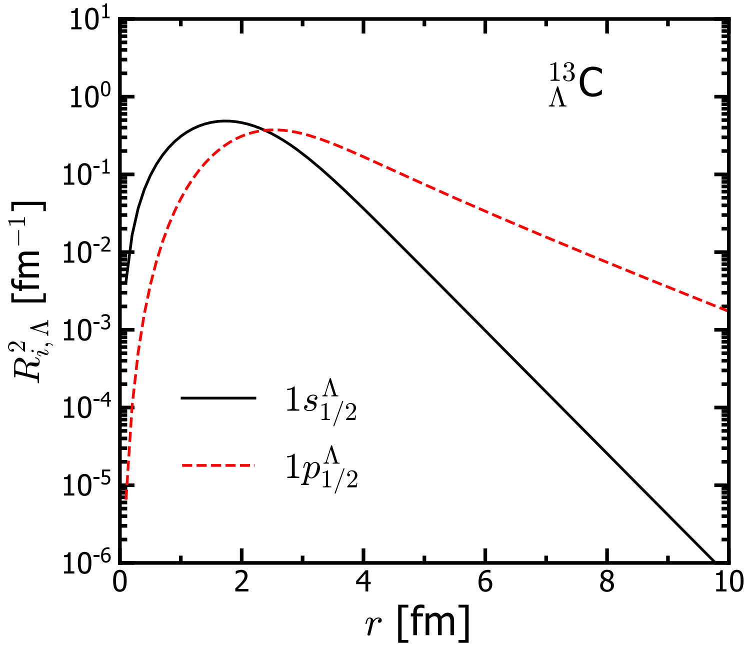
Good agreement with the present data!

Y. Z., H. Sagawa, and E. Hiyama, Phys. Rev. C **103**, 034321 (2021) .

Hyperon halo orbits in C hypernuclei

Y. Z., H. Sagawa, and E. Hiyama, Phys. Rev. C **103**, 034321 (2021)





Λ orbit	s.p. energy	rms radius
$1s_{1/2}$	-13.156	2.144
$1p_{1/2}$	-1.782	3.410

60% increase

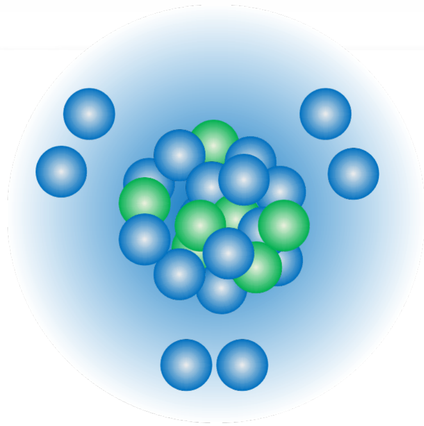
Halo orbits

Hyperon halo orbits in Zr hypernuclei

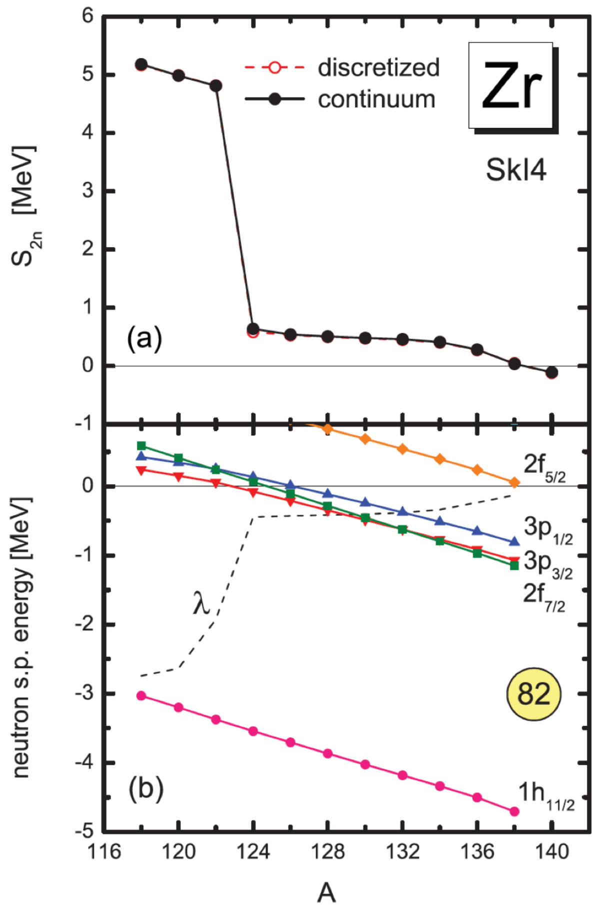
- Zr isotopes

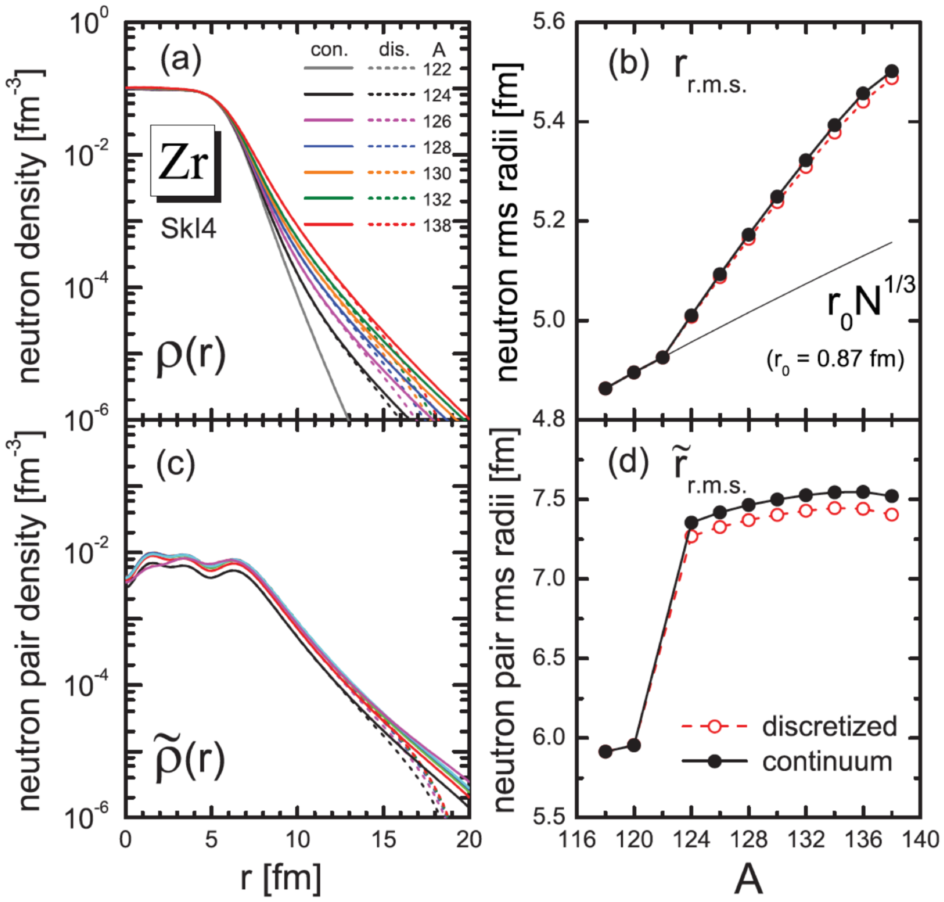
Giant halo Zr (A>124)

J. Meng and P. Ring, PRL 80, 460 (1998).



Y. Z., M. Matsuo, and J. Meng, PRC 86, 54318 (2012).





How about 1 or 2 Λ particles are added?

Hyperon halo orbits in Zr hypernuclei

- Hartree-Fock-Bogoliubov (HFB) model

✓ NN interaction: SkI4 P.-G. Reinhard and H. Flocard, NPA 584, 467 (1995).

✓ NN pairing: DDDI

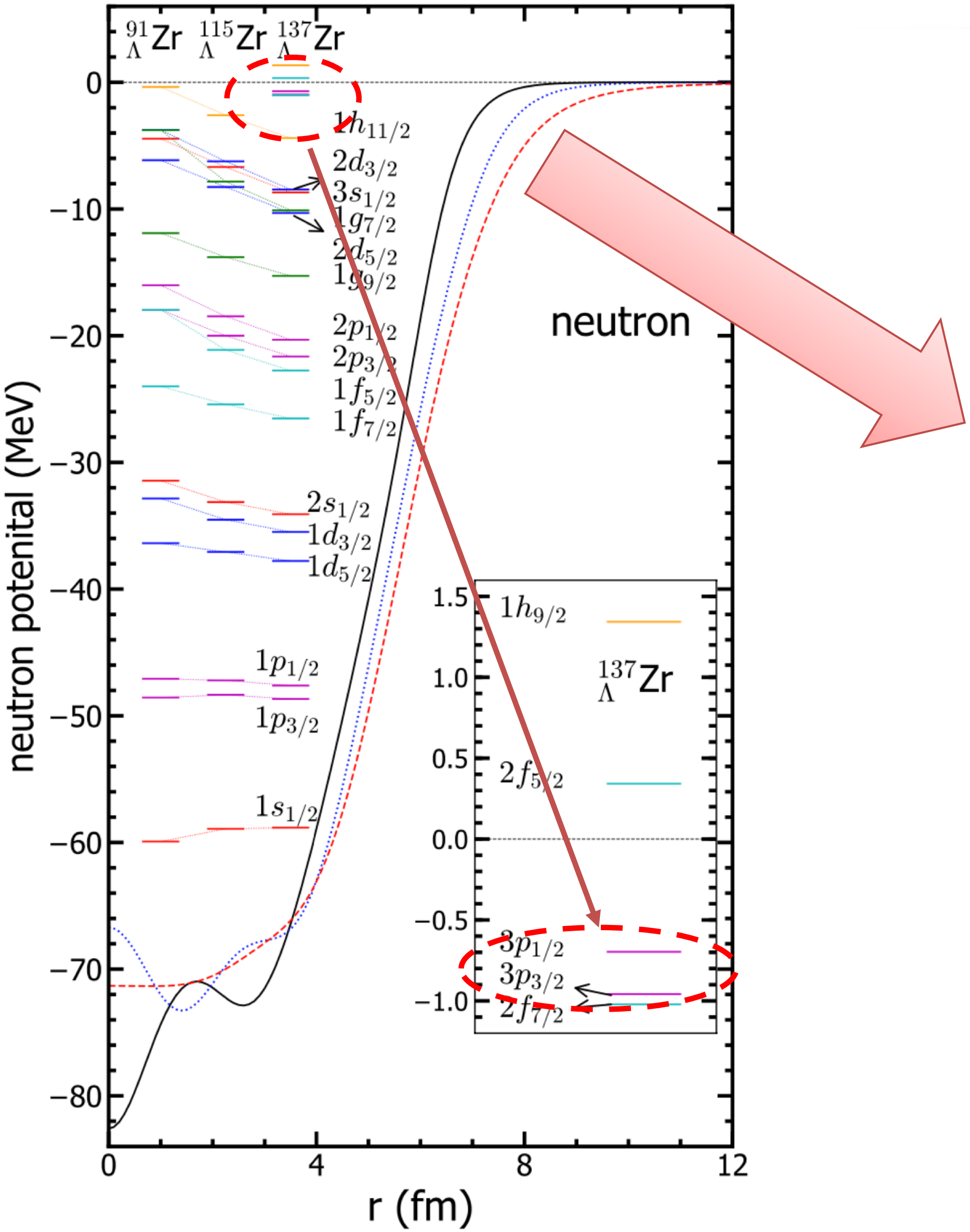
$$\Delta(\boldsymbol{r}) = \frac{1}{2} V_0 \left[1 - \eta \left(\frac{\rho_q(\boldsymbol{r})}{\rho_0} \right)^{\alpha} \right] \tilde{\rho}(\boldsymbol{r}), \quad q = n \text{ or } p.$$

V_0=-300 MeV fm^3, η=0.5, α=0.5, ρ_0=0.16 fm^{-3}, E_{cut}=70 MeV

M. Grasso et al. PRC 74(2006)64317.

✓ N**Λ** interaction: LY5r Y. Z., H. Sagawa, and E. Hiyama, Phys. Rev. C 103, 034321 (2021)

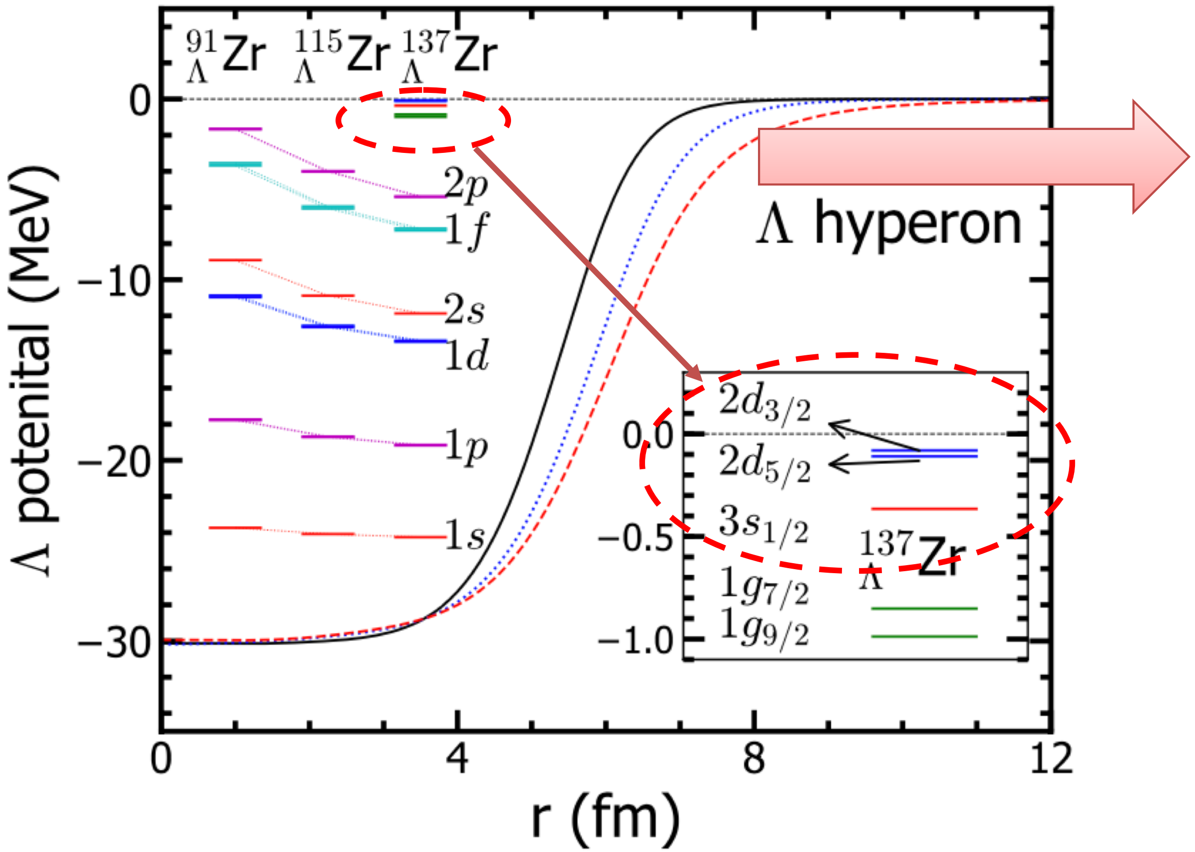
Hyperon halo orbits in Zr hypernuclei



In ^{136}Zr, the diffusive neutron potential can hold the weakly bound p orbits, which can hold up to 6 neutrons → giant neutron halo

Y. Z., H. Sagawa, and E. Hiyama, Prog. Theor. Exp. Phys. **2022** 023D01 (2022)

Hyperon halo orbits in Zr hypernuclei



In ^{136}Zr, the diffusive neutron density leads to a diffusive hyperon potential also.

	state	s.p.e.	v^2	r_{rms}
n	$3p_{1/2}$	−0.726	0.872	8.7611
	$3p_{3/2}$	−0.986	0.926	8.3981
	$2f_{5/2}$	0.292	0.155	7.2496
	$2f_{7/2}$	−1.047	0.886	6.9517
	$1h_{9/2}$	1.271	0.050	6.1079
	$1h_{11/2}$	−4.432	0.992	5.7730
total				5.4624
Λ	$3s_{1/2}$	−0.370	0.500	9.9822
	$2d_{3/2}$	−0.090	0.0	8.5794
	$2d_{5/2}$	−0.118	0.0	8.4148
	$1g_{7/2}$	−0.870	0.0	5.8904
	$1g_{9/2}$	−1.006	0.0	5.8721

5 almost degenerate weakly bound low-l orbits can hold up to 12 Λ hyperons

Possible hyperon halo in O hypernuclei

■ Hartree-Fock-Bogoliubov (HFB) model

✓ NN interaction: SIII

✓ NN pairing: DDDI $\Delta(\boldsymbol{r}) = \frac{1}{2} V_0 \left[1 - \eta \left(\frac{\rho_q(\boldsymbol{r})}{\rho_0} \right)^{\alpha} \right] \tilde{\rho}(\boldsymbol{r}), \quad q = n \text{ or } p.$

V_0= -458.4 MeV fm^3, η=0.83, α=0.51, ρ_0=0.08 fm^{-3}, E_{cut}= 60 MeV

neutron drip line: ^{24}O

✓ N**Λ** interaction: LY5r $W_0^{\Lambda} = 4.7$ MeV fm^5

YZ et al., PRC 103, 034321 (2021)

✓ **ΛΛ** pairing: DDDI volume-type

V_0= -139 MeV fm^3, η=0, α=0.51, E_{cut}= 60 MeV

H. Güven, et al., PRC 98, 014318 (2018)

✓ **ΛΛ** interaction:

$$V_{\Lambda\Lambda} = \lambda_0 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + \frac{1}{2}\lambda_1 \left[\boldsymbol{k}'^2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k}^2 \right] + \lambda_2 \boldsymbol{k}' \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k} + \lambda_3 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \rho_N^{\alpha} \left(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2} \right)$$

$\lambda_0 = -76.2$ MeV fm^3 $B_{\Lambda\Lambda}^{\rm exp} = 19.07 \pm 0.08 \pm 0.07$ MeV of $^{11}_{\Lambda\Lambda}$Be

$\lambda_1 = 57.5$ MeV fm^5

Ekawa et al., PTEP. 2019, 021D02 (2019)

λ_2= ?

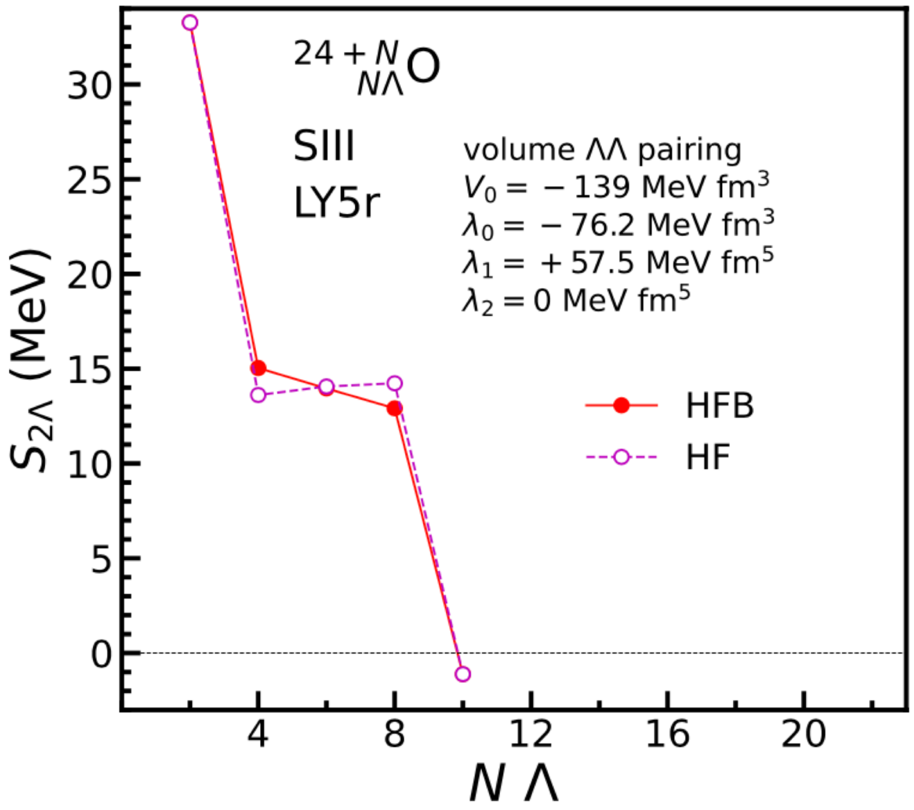
Try: λ_2= 0 or -100 MeV fm^5

Possible hyperon halo in O hypernuclei

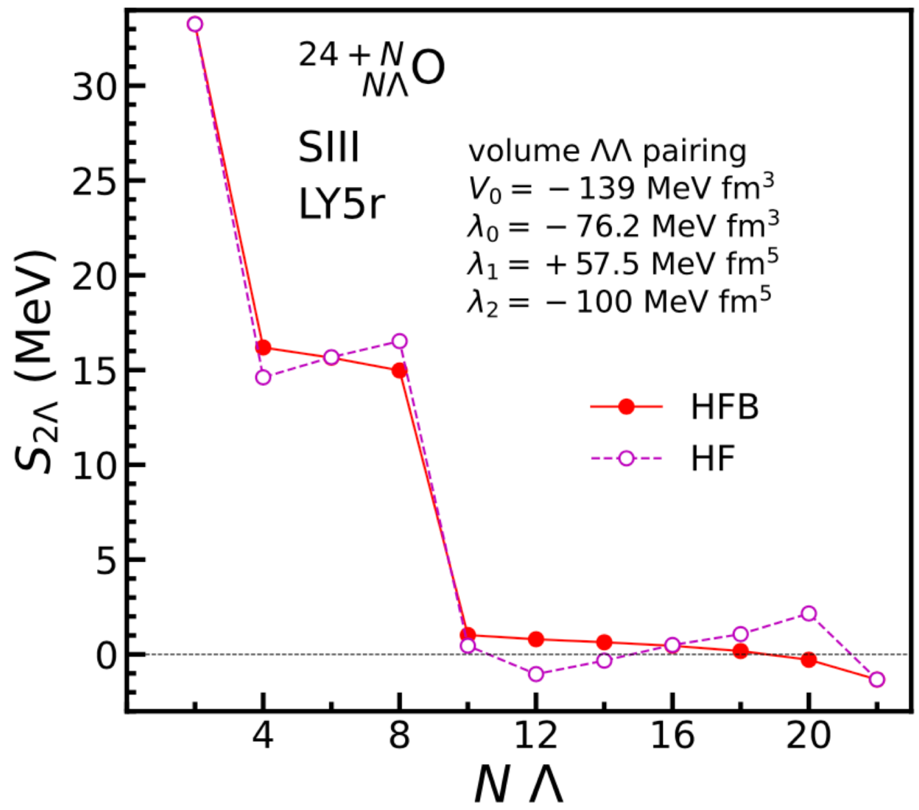
✓ **ΛΛ** interaction

$$V_{\Lambda\Lambda} = \lambda_0\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\frac{1}{2}\lambda_1\left[\boldsymbol{k}'^2\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}^2\right]+\lambda_2\boldsymbol{k}'\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}+\lambda_3\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\rho_N^{\alpha}\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right)$$

$\lambda_2 = 0$ MeV fm^5



$\lambda_2 = -100$ MeV fm^5



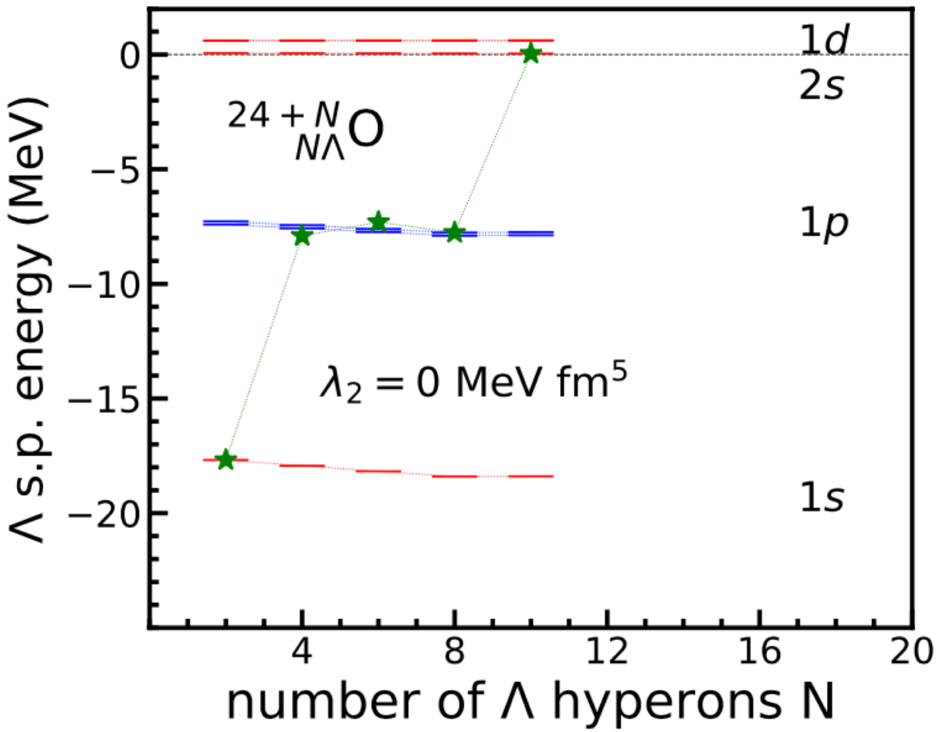
! With more attractive p-wave interaction, the hyperon drip line could be extended.

Possible hyperon halo in O hypernuclei

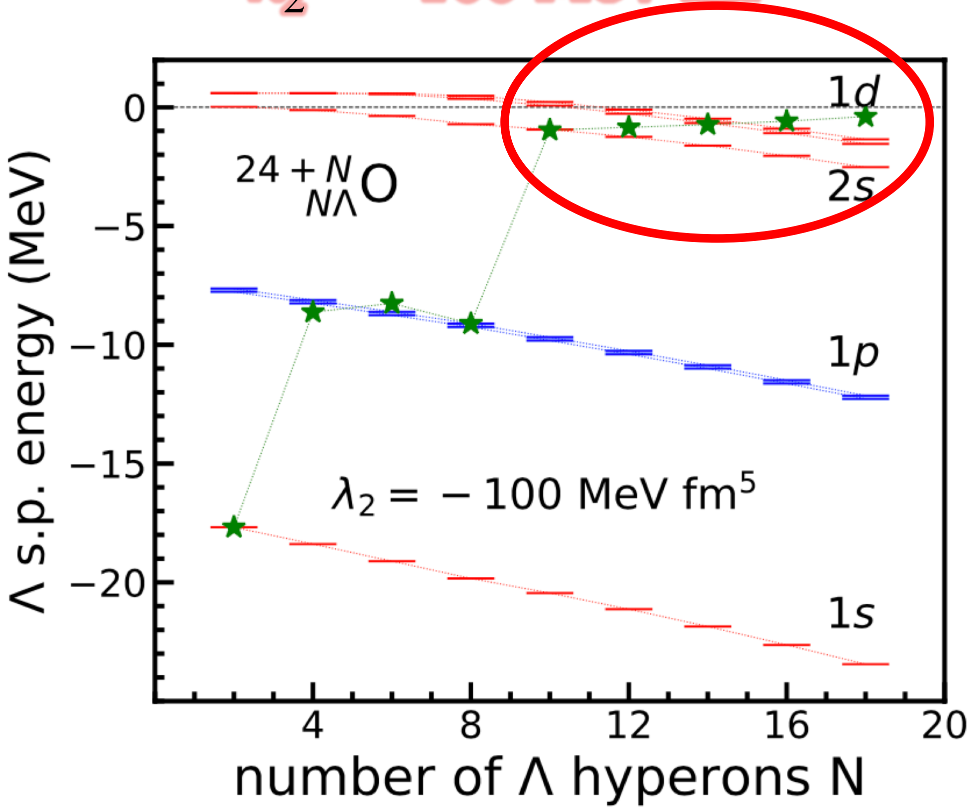
✓ **ΛΛ** interaction

$$V_{\Lambda\Lambda} = \lambda_0 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + \frac{1}{2}\lambda_1 \left[\boldsymbol{k}'^2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k}^2\right] + \lambda_2 \boldsymbol{k}' \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k} + \lambda_3 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\rho_N^{\alpha}\left(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}\right)$$

$\lambda_2 = 0$ MeV fm^5



$\lambda_2 = -100$ MeV fm^5



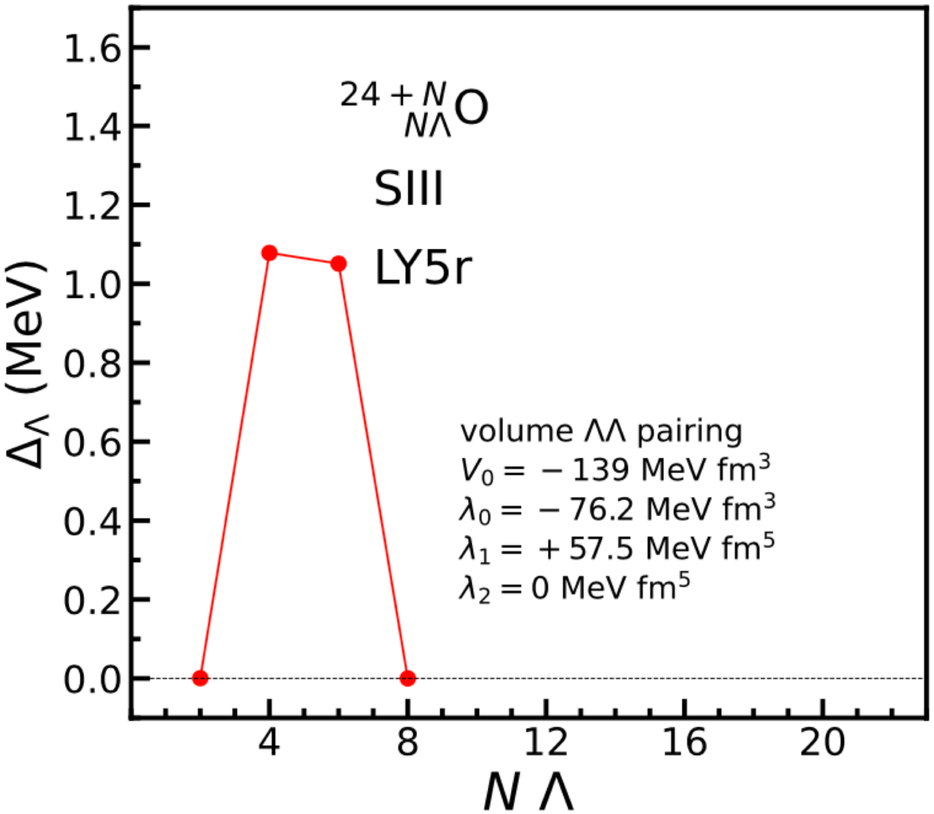
! 2s and 1d orbits are pulled down from the continuum to the weakly bound states as hyperon number increases.

Possible hyperon halo in O hypernuclei

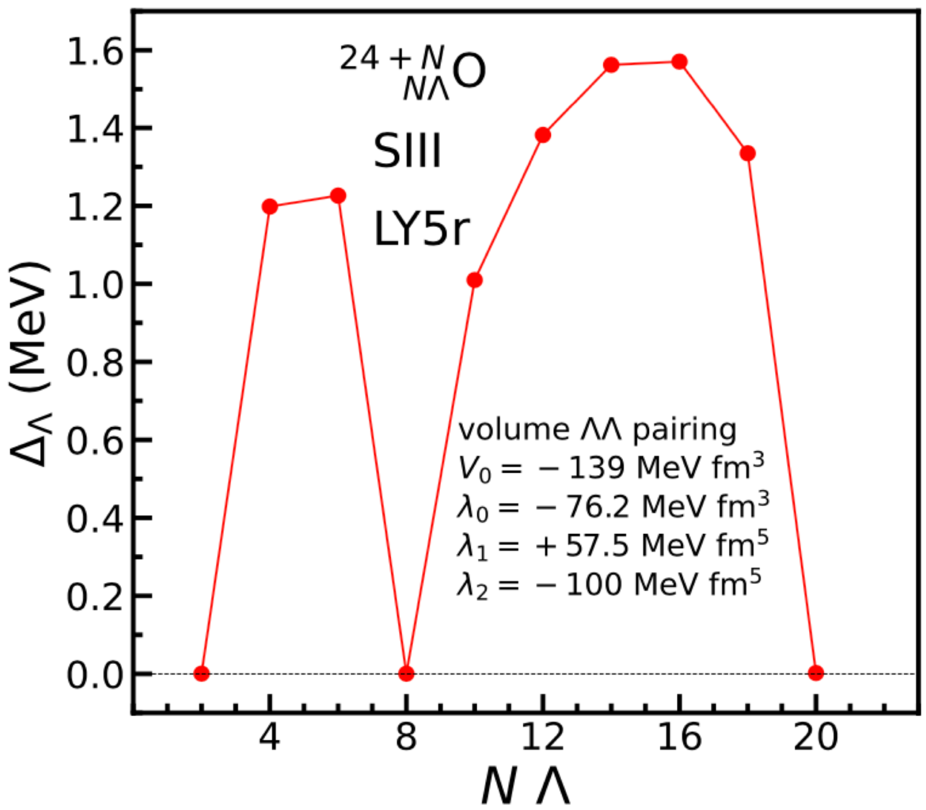
✓ **ΛΛ** interaction

$$V_{\Lambda\Lambda} = \lambda_0\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\frac{1}{2}\lambda_1\left[\boldsymbol{k}'^2\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}^2\right]+\lambda_2\boldsymbol{k}'\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}+\lambda_3\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\rho_N^{\alpha}\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right)$$

λ_2= 0 MeV fm^5



λ_2= -100 MeV fm^5

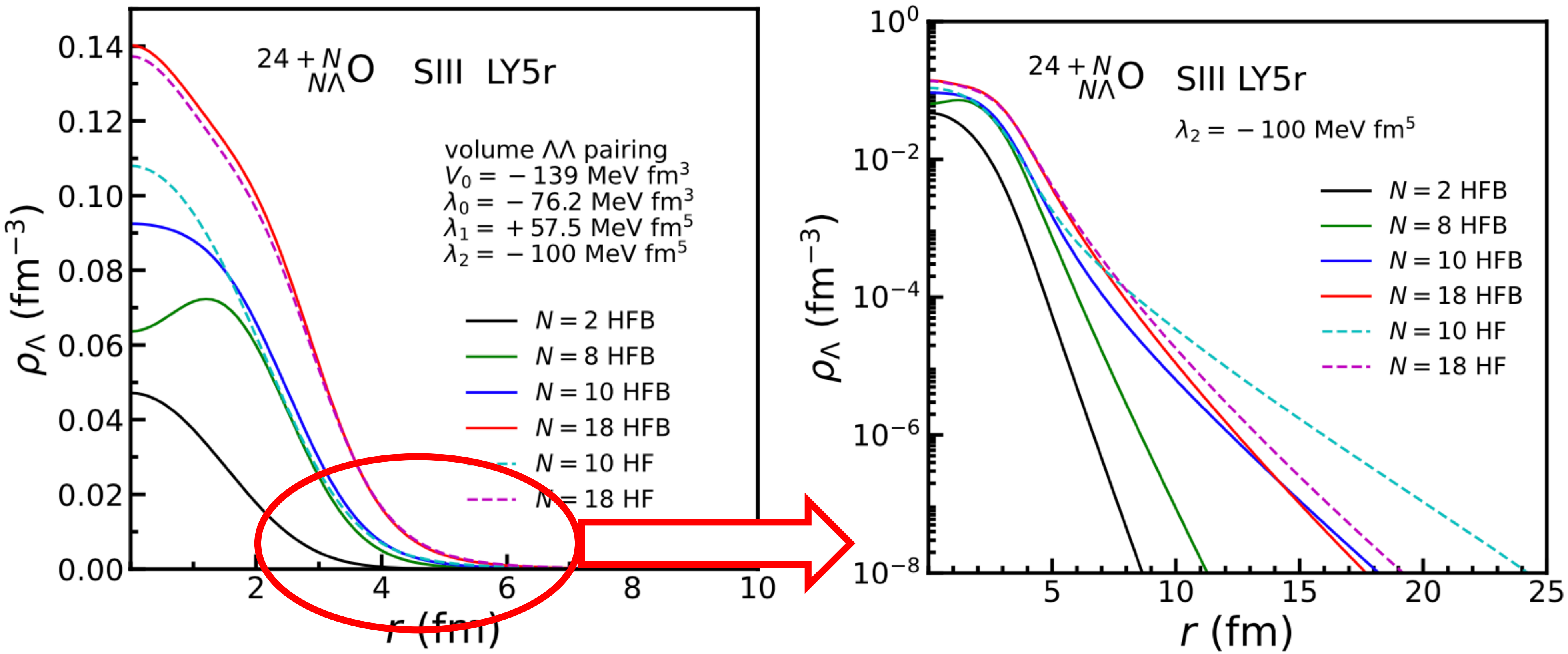


! Obvious hyperon shell closure at N=2, 8, 20

Possible hyperon halo in O hypernuclei

$\lambda_2 = -100$ MeV fm^5

- Hyperon density distribution



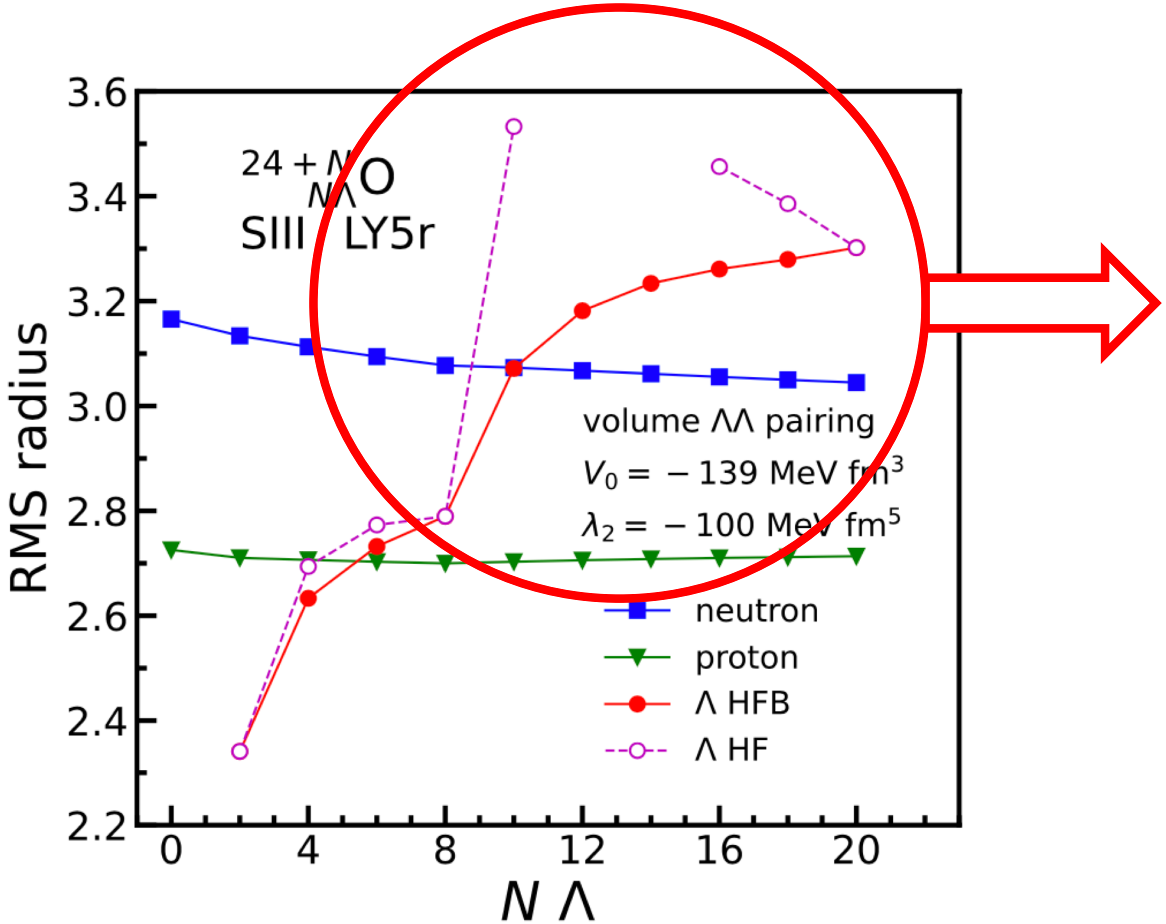
! Obvious increase of the hyperon density from N=8 to N=10

! "pairing anti-halo" effect

Possible hyperon halo in O hypernuclei

$\lambda_2 = -100$ MeV fm^5

- RMS radius of neutron, proton and hyperon

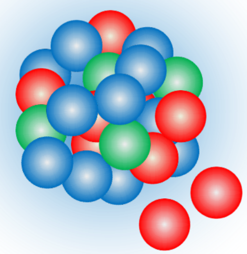


! quick increase of hyperon RMS radius, even larger than neutron after N=10

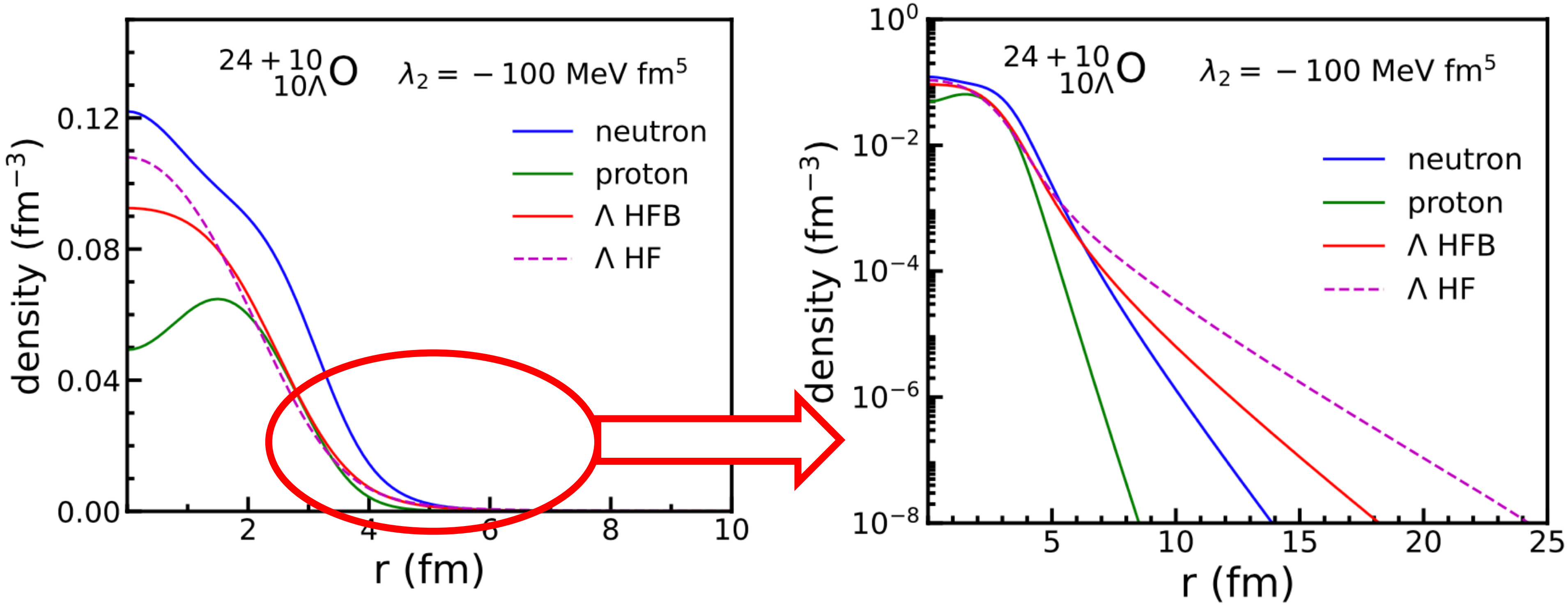
! "pairing anti-halo" effect

Possible hyperon halo in O hypernuclei

λ_2= -100 MeV fm^5



- Density of neutron, proton and hyperon in $^{24+10}_{10\Lambda}O$



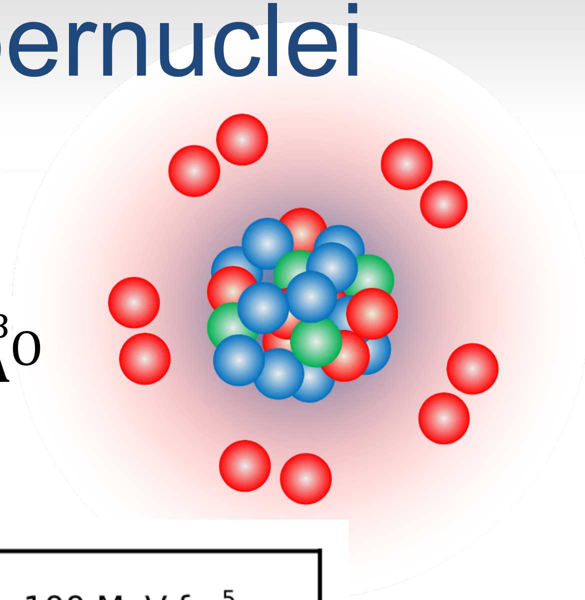
! Large tail of hyperon density in HF cal. → large hyperon RMS radius

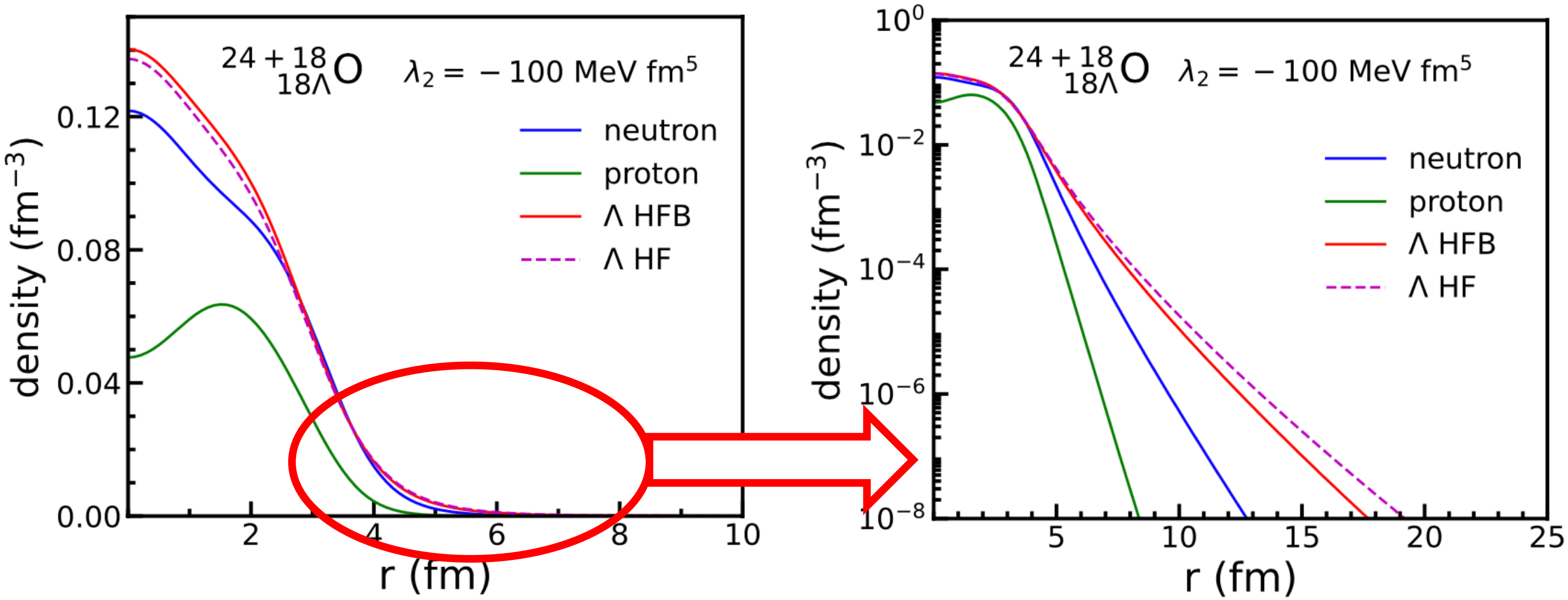
! "pairing anti-halo" effect

Possible hyperon halo in O hypernuclei

λ_2= -100 MeV fm^5

- Density of neutron, proton and hyperon in $^{24+18}_{18\Lambda}O$





! The weakly bound levels can hold up to N=10 hyperons, which may form the hyperon halo.

Summary

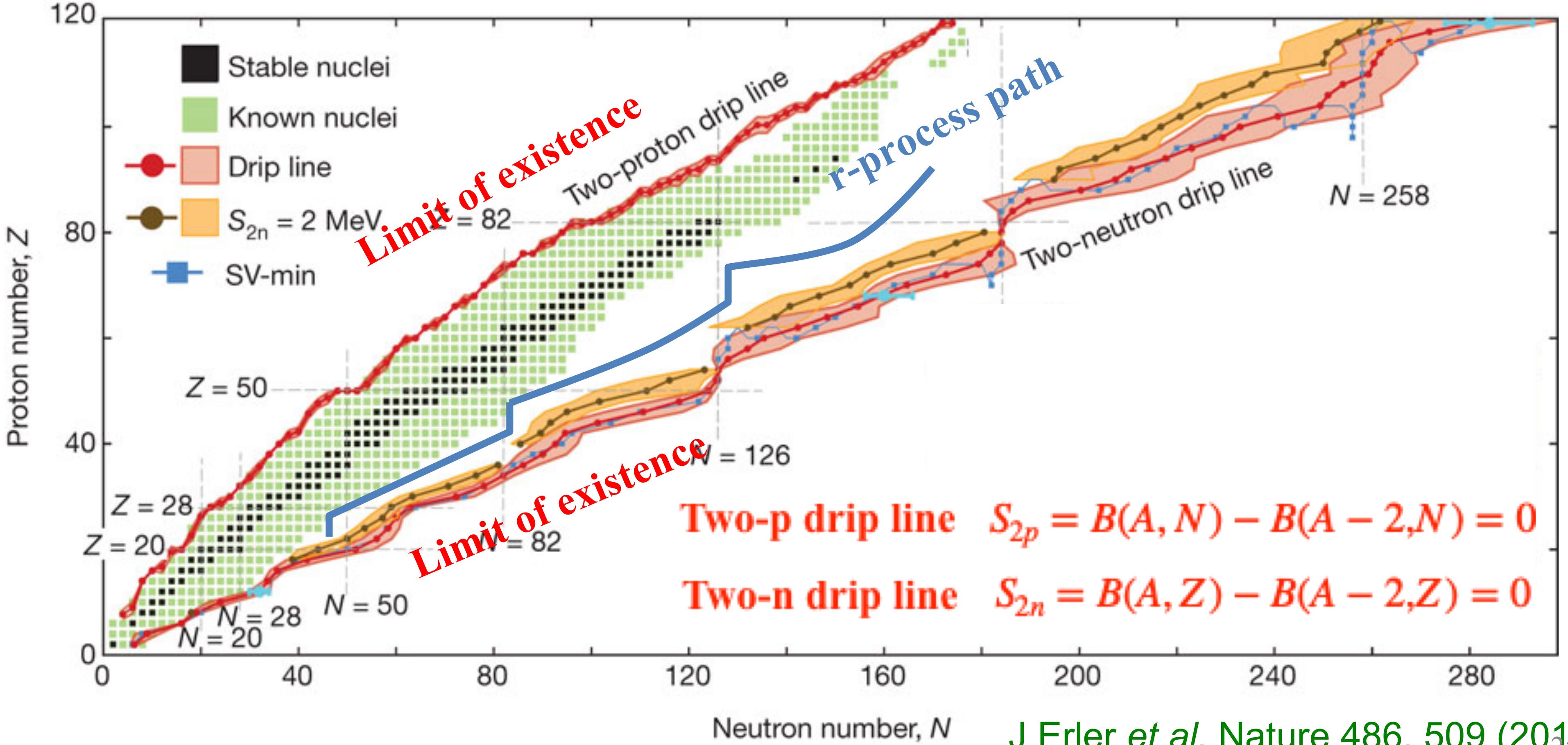
- Skyrme energy density functional theory for hypernucleus including **Λ**N interaction (with spin-orbit interaction) and **ΛΛ** interaction was developed. The spin-orbit splitting and the **ΛΛ** binding energy data was reproduced.
- We studied the possible halo orbits of **Λ** hyperon in C, Zr and O isotopes.
  - ✓ C, halo orbits: 1p, 2s (neutron-rich)
  - ✓ Zr, halo orbits: 3s, 2d, 1g, almost degenerate. They can hold even more hyperons than neutrons in the giant neutron halo.
  - ✓ ^{24}O + **Λ** hyperon: **ΛΛ** interaction and pairing are important to determine the **Λ** drip line and the **Λ** halo.

Thank you!

Appendix

Introduction *Nuclear Landscape*

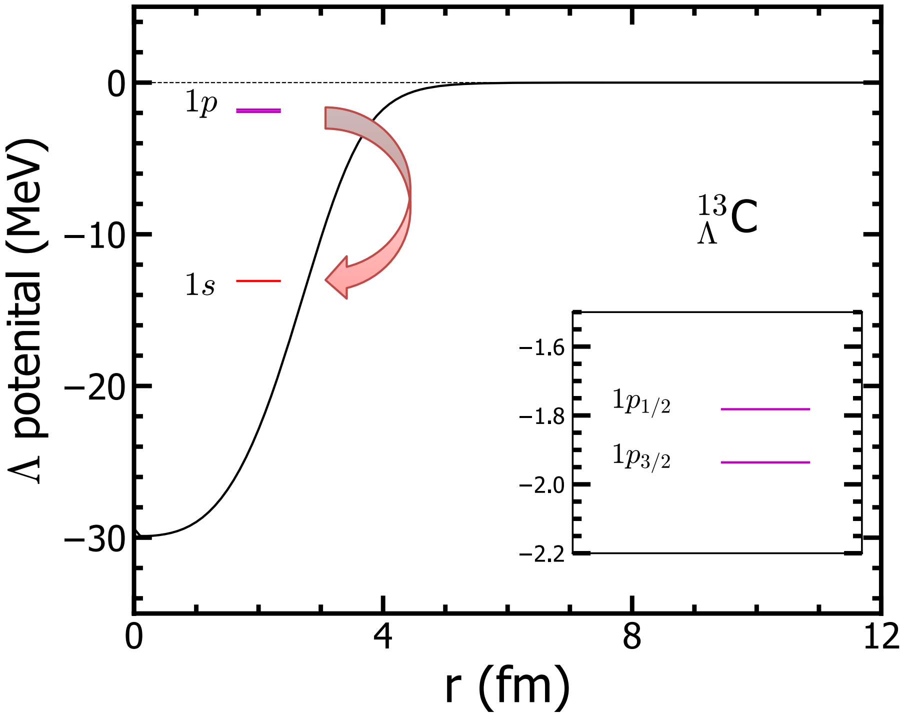
stable nuclei ~300 nuclei
unstable nuclei observed so far ~2700 nuclei
drip-lines (limit of existence)（theoretical predictions) ~8000 nuclei



J Erler *et al.* Nature 486, 509 (2012)

Hyperon halo orbits in C hypernuclei

★ E1 transition from $1p$ to $1s$ state



$$B(EI; J_i \to J_f) = \frac{1}{2J_i+1} |\langle f || \hat{Q}_I || i \rangle|^2$$

$$\langle f || \hat{Q}_I || i \rangle = \langle f || e r^I Y_I(\theta, \varphi) || i \rangle$$

$$B(E1; J_i \to J_f) = \frac{3e_\Lambda^2}{4\pi} \langle f | r | i \rangle^2 (2j_f+1) \begin{pmatrix} j_f & 1 & j_i \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}^2$$

$$\langle f | r | i \rangle = \int_0^\infty R_{f,\Lambda}(r) r R_{i,\Lambda}(r) dr.$$

$$e_\Lambda^{(E1)} = -Z M_\Lambda e / (A M_N + M_\Lambda)$$

$^{13}_{\Lambda}$C Cal. **Λ** ($1p_{1/2} \to 1s_{1/2}$) : B(E1) = 0.103 e^2 fm^2

^{11}Be Exp. n ($\frac{1}{2}^+ \to \frac{1}{2}^-$) : B(E1) = 0.099 ± 0.010 e^2 fm^2

T. Nakamura et al., Phys. Lett. B 394, 11 (1997).

Hyperon halo orbits in Zr hypernuclei

- Hartree-Fock-Bogoliubov (HFB) model

NN interaction: SkI4 P.-G. Reinhard and H. Flocard, NPA 584, 467 (1995).

NN pairing: $$\Delta(\boldsymbol{r}) = \frac{1}{2} V_0 \left[1 - \eta \left(\frac{\rho_q(\boldsymbol{r})}{\rho_0} \right)^{\alpha} \right] \tilde{\rho}(\boldsymbol{r}), \quad q = n \text{ or } p.$$

V_0=-300 MeV fm^3, η=0.5, α=0.5, ρ_0=0.16 fm^{-3}, E_{cut}=70 MeV M. Grasso et al. PRC 74(2006)64317.

N**Λ** interaction: LY5r Y. Z., H. Sagawa, and E. Hiyama, Phys. Rev. C 103, 034321 (2021)

ΛΛ interaction: SΛΛ1 $\lambda_0 = -312.6$ MeV fm^3 D. E. Lanskoy, PRC 58, 3351 (1998).

to reproduce the data of NAGARA event: $^{6}_{\Lambda\Lambda}$He

K. Nakazawa and H. Takahashi, PTPS 185, 335 (2010).

SΛΛ1r $\lambda_0 = -50.0$ MeV fm^3 $\lambda_1 = 57.5$ MeV fm^5 $\lambda_2 = 0$

Table 1. The calculated double-Λ binding energy $B^{\text{cal.}}_{\Lambda\Lambda}$ and the gain energy $\Delta B^{\text{cal.}}_{\Lambda\Lambda}$ for several light double-Λ hypernuclei together with the experimental data [35,37]. All the energies are in MeV.

Nuclide	$B^{\text{cal.}}_{\Lambda\Lambda}$	$B^{\text{expt.}}_{\Lambda\Lambda}$	$\Delta B^{\text{cal.}}_{\Lambda\Lambda}$	$\Delta B^{\text{expt.}}_{\Lambda\Lambda}$
$^{6}_{\Lambda\Lambda}$He	7.564	6.91 ± 0.16 (NAGARA)	0.536	0.67 ± 0.17
$^{11}_{\Lambda\Lambda}$Be	18.594	20.86 ± 3.06 (MIKAGE)	0.478	2.64 ± 3.09
		22.12 ± 2.67 (MIKAGE)		3.90 ± 2.71
		20.83 ± 1.27 (HIDA)		2.61 ± 1.34
		19.07 ± 0.11 (MINO)		1.87 ± 0.37
$^{13}_{\Lambda\Lambda}$B	22.221	23.3 ± 0.7 (KEK-E176)	0.435	0.6 ± 0.8

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$$

$$B_{\Lambda} = B(^{A+1}_{\Lambda}Z) - B(^{A}Z)$$

$$B_{\Lambda\Lambda} = B(^{A+2}_{\Lambda\Lambda}Z) - B(^{A}Z)$$

Hyperon halo orbits in Zr hypernuclei

Table 2 Bulk properties of Zr isotopes with one or two Λ hyperons: rms radius $r^n_{\rm rms}, r^p_{\rm rms}, r^\Lambda_{\rm rms}$ (fm), the average pairing gap Δ_n, Δ_p (MeV), single- or double-Λ binding energies $B_{\Lambda(\Lambda)}$ (MeV), and double-Λ gain energy $\Delta B_{\Lambda\Lambda}$ (MeV). The Λ hyperon(s) is placed in the ground state $1s_{1/2}$ orbit.

	$r^n_{\rm rms}$	$r^p_{\rm rms}$	$r^\Lambda_{\rm rms}$	Δ_n	Δ_p	$B_{\Lambda(\Lambda)}$	$\Delta B_{\Lambda\Lambda}$
^{90}Zr	4.2674	4.1588		0.000	0.915		
$^{91}_{\Lambda}$Zr	4.2608	4.1515	3.1346	0.000	0.901	23.468	
$^{92}_{\Lambda\Lambda}$Zr	4.2543	4.1445	3.1197	0.000	0.889	47.123	0.188
^{114}Zr	4.8000	4.4112		0.953	0.796		
$^{115}_{\Lambda}$Zr	4.7934	4.4036	3.3059	0.953	0.785	23.868	
$^{116}_{\Lambda\Lambda}$Zr	4.7870	4.3962	3.2897	0.952	0.773	47.905	0.169
^{136}Zr	5.4649	4.5415		0.620	0.602		
$^{137}_{\Lambda}$Zr	5.4585	4.5328	3.4003	0.593	0.579	24.074	
$^{138}_{\Lambda\Lambda}$Zr	5.4520	4.5244	3.3823	0.568	0.556	48.315	0.168

much smaller than $^{6}_{\Lambda\Lambda}$He

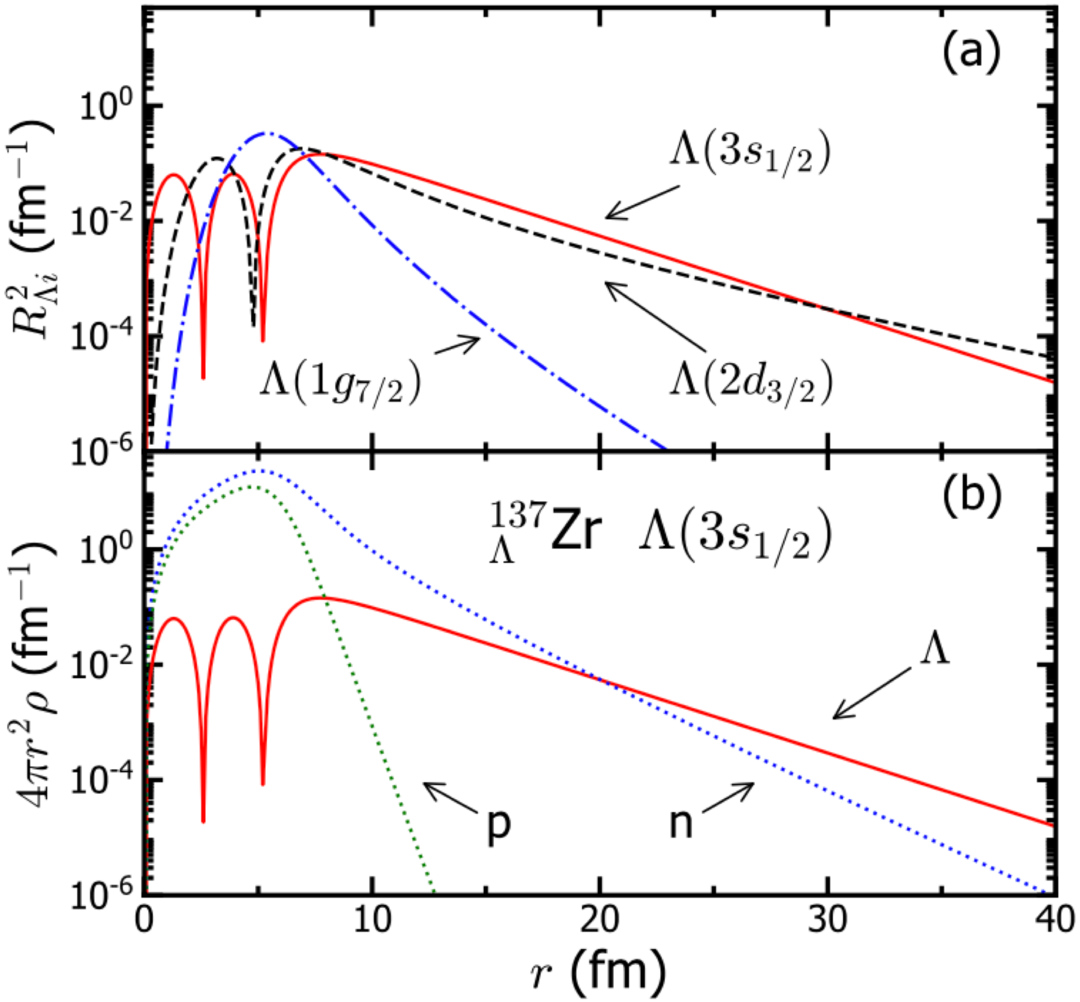
DDDI nn (pp) pairing are included by the HFB calculation

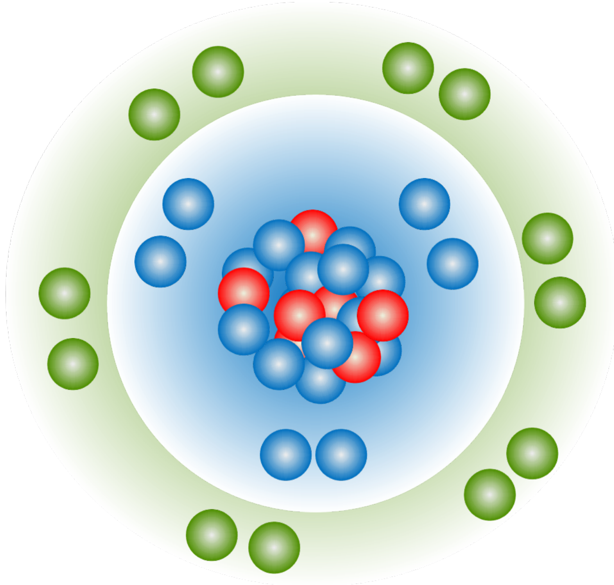
$$\Delta(\boldsymbol{r}) = \frac{1}{2} V_0 \left[1 - \eta \left(\frac{\rho_q(\boldsymbol{r})}{\rho_0}\right)^{\alpha}\right] \tilde{\rho}(\boldsymbol{r}), \quad q = n \text{ or } p.$$

V_0=-300 MeV fm^3, η=0.5, α=0.5, ρ_0=0.16 fm^{-3}, E_{cut}=70 MeV

M Grasso, S Yoshida, N Sandulescu, and N Van Giai. PRC 74(2006)64317.

Hyperon halo orbits in Zr hypernuclei





Giant Hyperon halo outside the giant neutron halo

Y. Z., H. Sagawa, and E. Hiyama, Prog. Theor. Exp. Phys. **2022** 023D01 (2022)

Possible hyperon halo in O hypernuclei

λ_2= -100 MeV fm^5