PDF of Deuteron-like Di-baryon system from Lattice QCD

CLQCD

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in collaboration with

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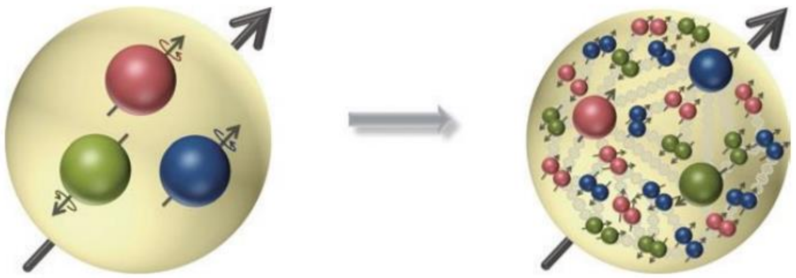
The 23rd International Conference on Few-Body Problems in Physics (FB23)

Outline

- Introduction & Motivation
- Lattice Simulation
- Numerical Result
- Summary

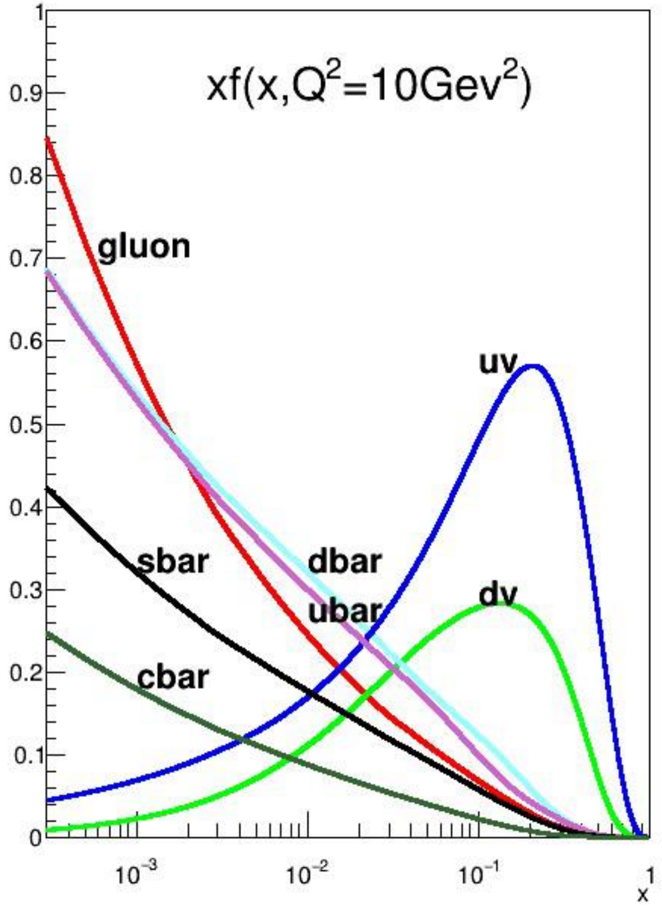
Introduction & Motivation

Parton model（Feynman， 1972）



A schematic illustration of the nucleon internal structure at different energy scales

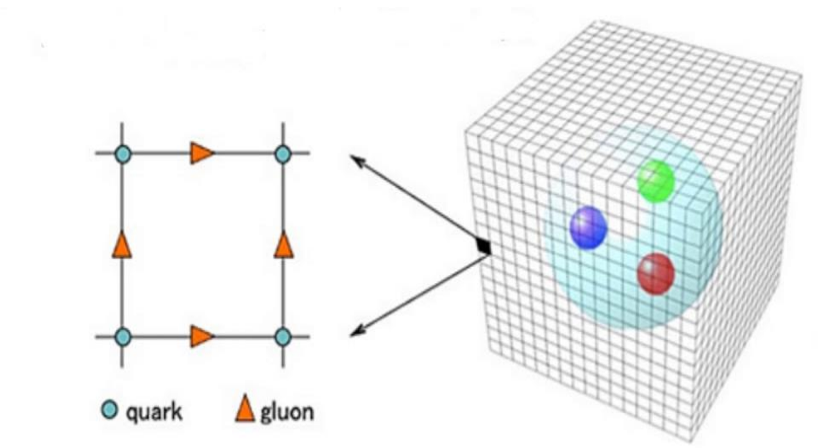
- Nucleon: relativistic bound state of partons (quarks and gluons) ;
- Parton distribution functions (PDFs) : possibility density of parton carrying momentum fraction x.



Parton distribution function (PDF)

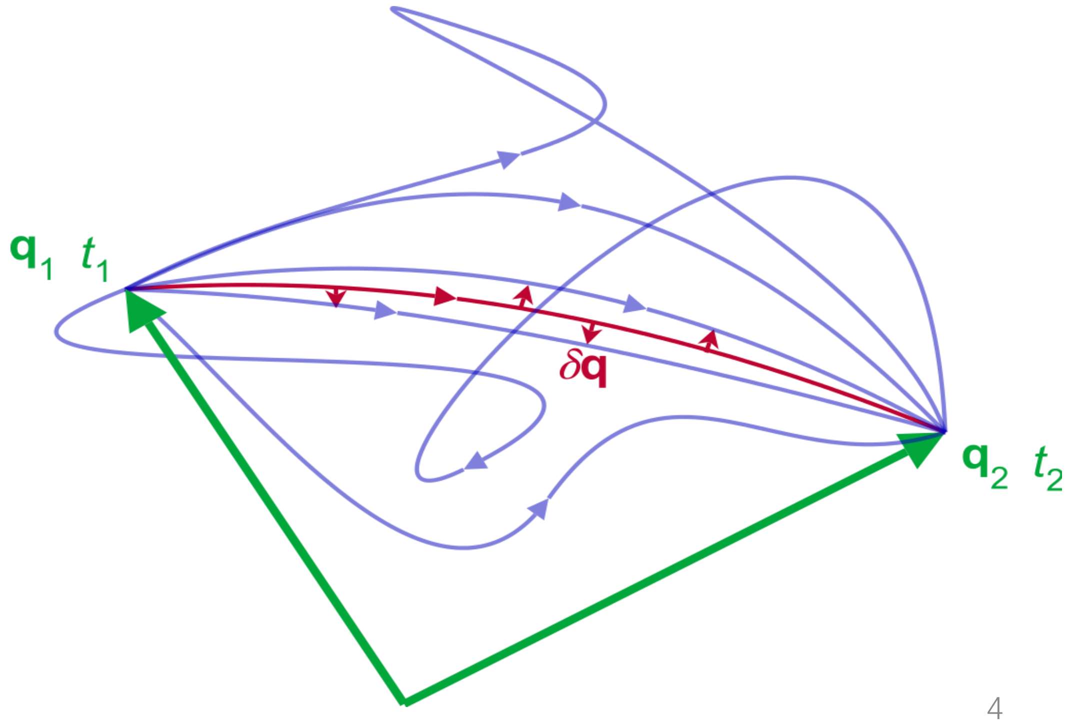
Introduction & Motivation

Lattice QCD (K.G.Wilson ,1974)



- Systematical non-perturbative QCD approach based on first-principle.
- Simulation in 4-dimensional Euclidian space using super computer.

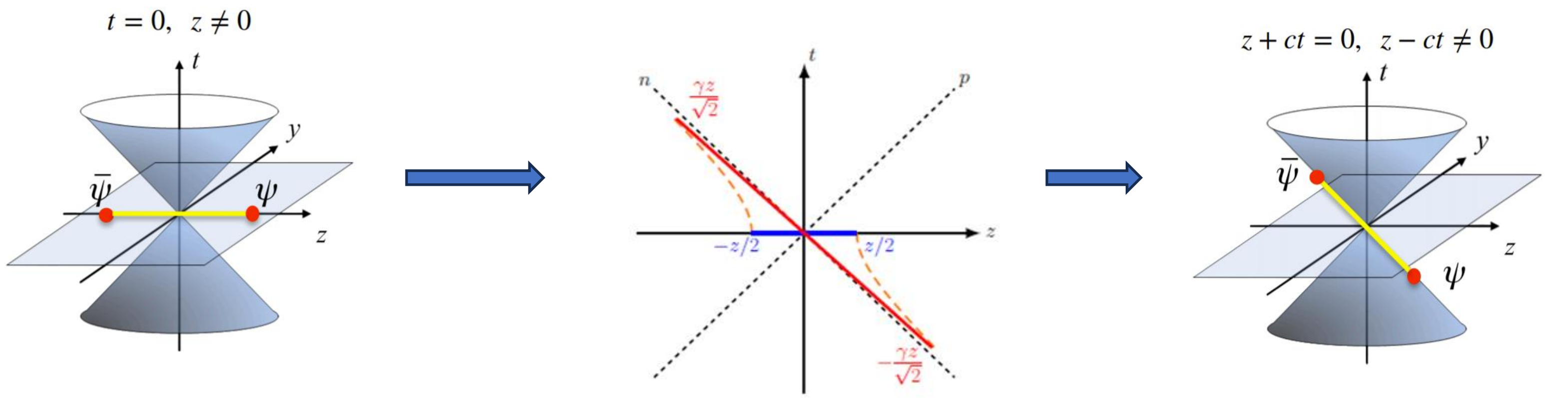
- One **configuration** is one possible path in the path integration containing the gauge link values at each time-space point



Introduction & Motivation

- Light-cone quantities cannot be calculated directly using Lattice QCD approach
- Large momentum effective theory(LaMET)

Phys.Rev.Lett. 110 (2013) 262002



$$\tilde{q}(x) = \int \frac{dz}{4\pi} e^{-ixz\cdot p_z}\langle P|\bar{\psi}(z)\gamma^t U(z,0)\psi(0)|P\rangle$$

Quasi-PDF

Perturbative matching

$$q(x) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^-\cdot p^+}\langle P|\bar{\psi}(\xi^-)\gamma^+ U(\xi^-,0)\psi(0)|P\rangle$$

Light-cone PDF

- Progress using LaMET

  proton unpolarized PDF *Phys.Rev.D 101 (2020) 3, 034020*

  proton transversity PDF *PhysRevLett.131.261901*

  ... ...

Introduction & Motivation

- Deuteron： simplest multi-baryon bound system.
- Binding Energy: 2.22452(20)MeV

Hard to identify deuteron bound state through LQCD calculation!

Phys.Rev.D 107 (2023) 9, 094508

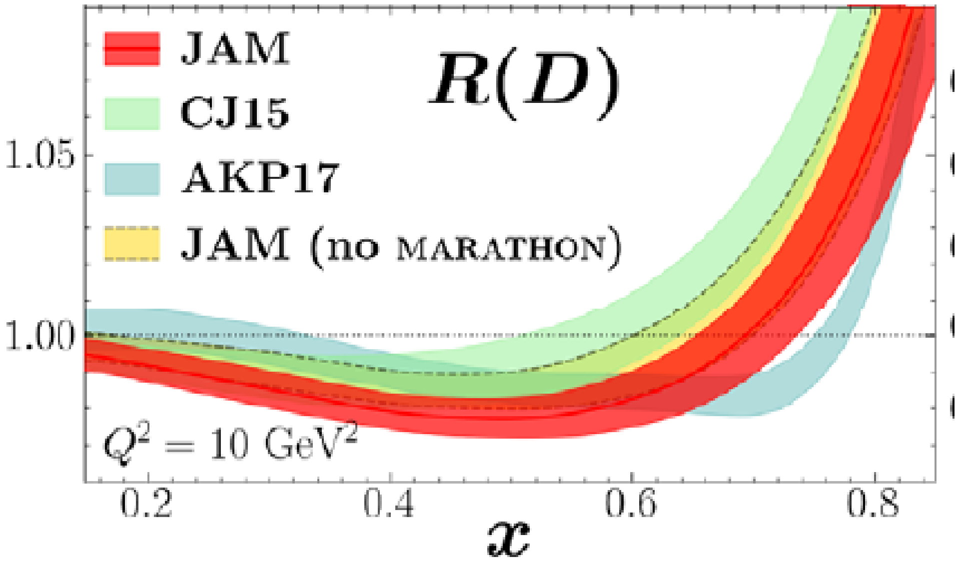
- Quantum Number:

$I = 0, \quad J^P = 1^+$

$L = 0/2, \quad S = 1$

- Calculating the structure of **deuteron-like dibaryon system** may offer an alternative perspective to study the interaction between nucleons.

- EMC effect $\frac{F_2^D(x)}{F_2^P(x)+F_2^N(x)} \neq 1$



Phys.Rev.Lett. 127 (2021) 24

Lattice Simulation

CLQCD

Simulation Setup

PRD109(2024)5,054507

Name	Volume	Lattice Spacing	Beta	m_π(MeV)	m_{η_s}(MeV)	Nconf
C24P29	$24^3 \times 72$	$0.105fm$	6.20	293	659	759
C32P29	$32^3 \times 64$	$0.105fm$	6.20	293	659	870
C24P90	$24^3 \times 72$	$0.105fm$	6.20	941	941	750

- 2+1 flavour ensembles with stout smeared clover fermion action

Lattice Simulation

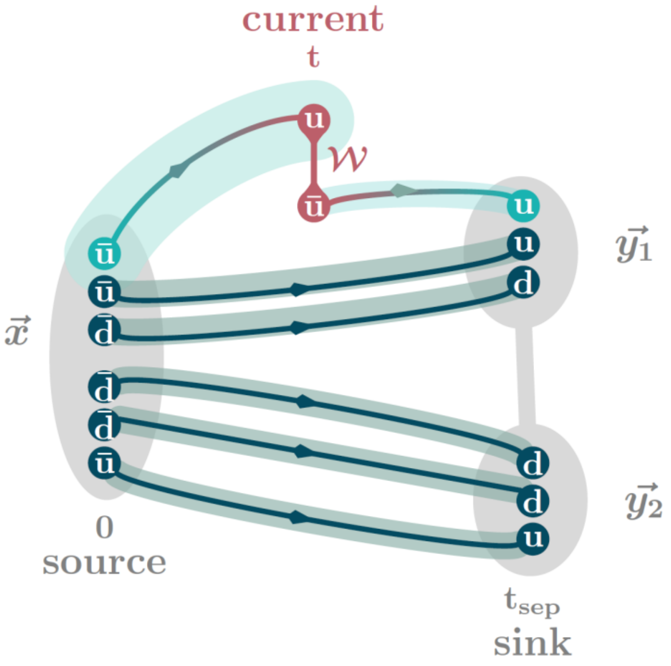
- Two point and three point correlators

$$C^{2\mathrm{pt}}\,(P_z, t) = \langle N(t_{sep}, P_z) | N'^{\dagger}(0, P_z) \rangle$$

$$C^{3\mathrm{pt}}(P_z, z, t) = \left\langle N(t_{sep}, P_z) \middle| \bar{\psi}(t; z)\gamma^{t} U(t; z, 0)\psi(t; 0) \middle| N'^{\dagger}(0, P_z) \right\rangle$$

- Di-baryon operator: combination of proton and neutron operator , which have an overlap with Deuteron's quantum number $\mathrm{I}\ (J^P) = 0\ (1^+)$.
- Correlator of di-baryon system is made up of a hexaquark source and a di-baryon sink

We only consider connected diagrams



Three-point correlator of Deuteron-type dibaryon system

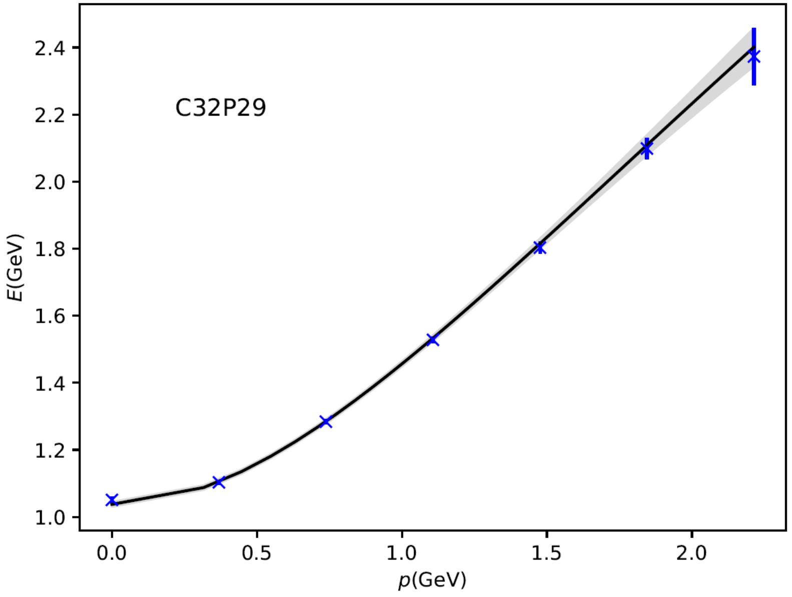
Lattice Simulation

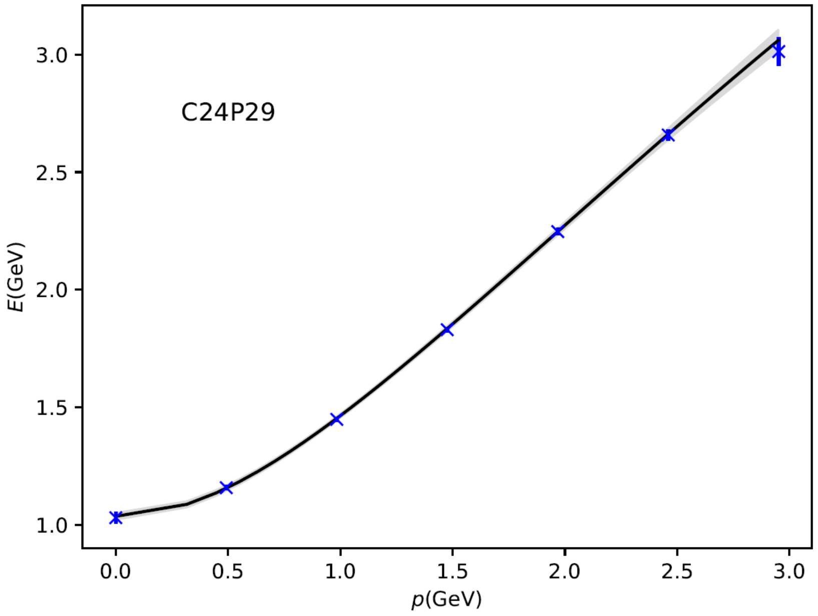
- Dispersion relation of proton

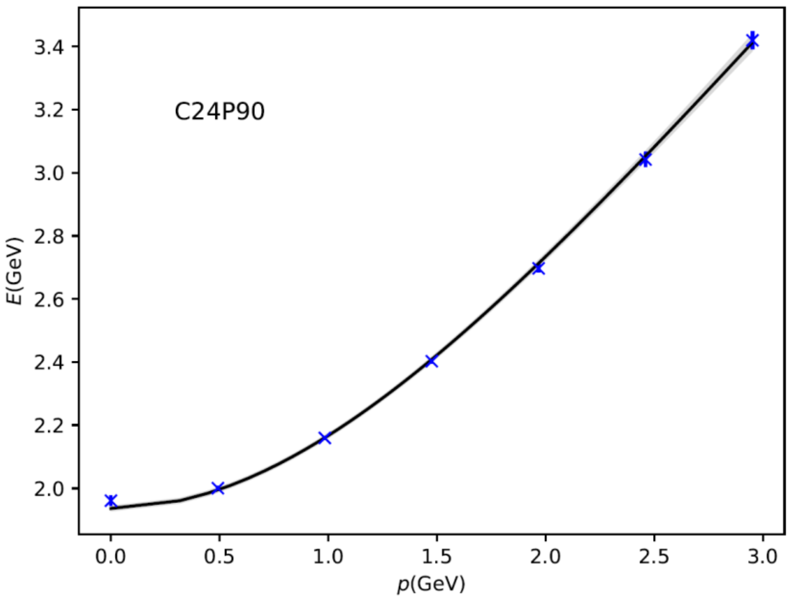
$$C^{2\mathrm{pt}}(P_z, t) = c_4 e^{-E_0(P_z)t}(1 + c_5 e^{-\Delta E t})$$

$$E_0(P_z)^2 = m^2 + c_2 {P_z}^2 + c_3 a^2 {P_z}^4$$

Ensemble	c_2	c_3	$\chi^2/d.o.f.$
C32P29	1.067(29)	−0.022(50)	1.43
C24P29	1.085(28)	−0.015(19)	0.33
C24P90	0.953(14)	−0.0054(99)	0.81



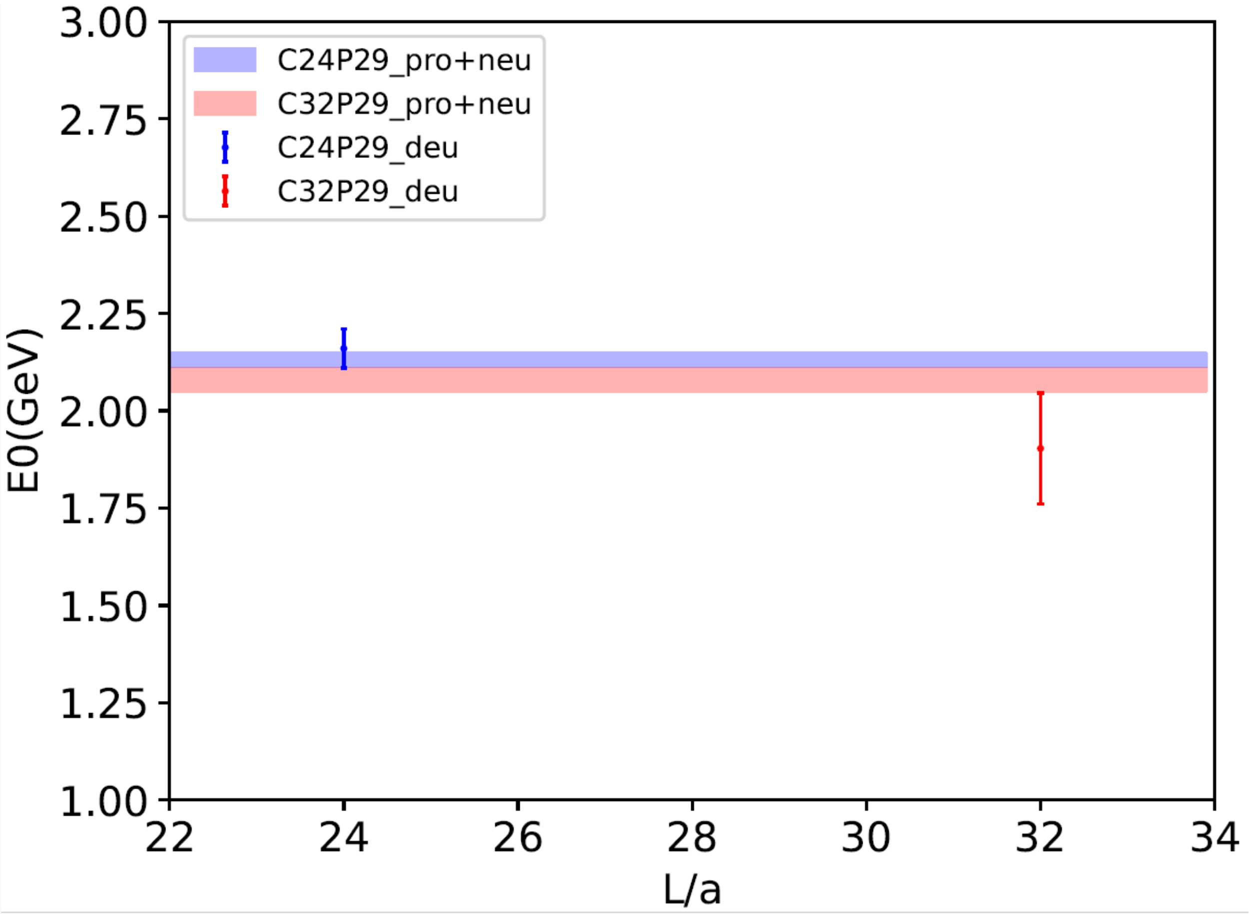




Lattice Simulation

- Mass difference between dibaryon system and free nucleons

$$C^{2\text{pt}}(P_z = 0, t) = c_4 e^{-E_0(t)}(1 + c_5 e^{-\Delta E t})$$



Hard to decide if it is bound state.

Lattice Simulation

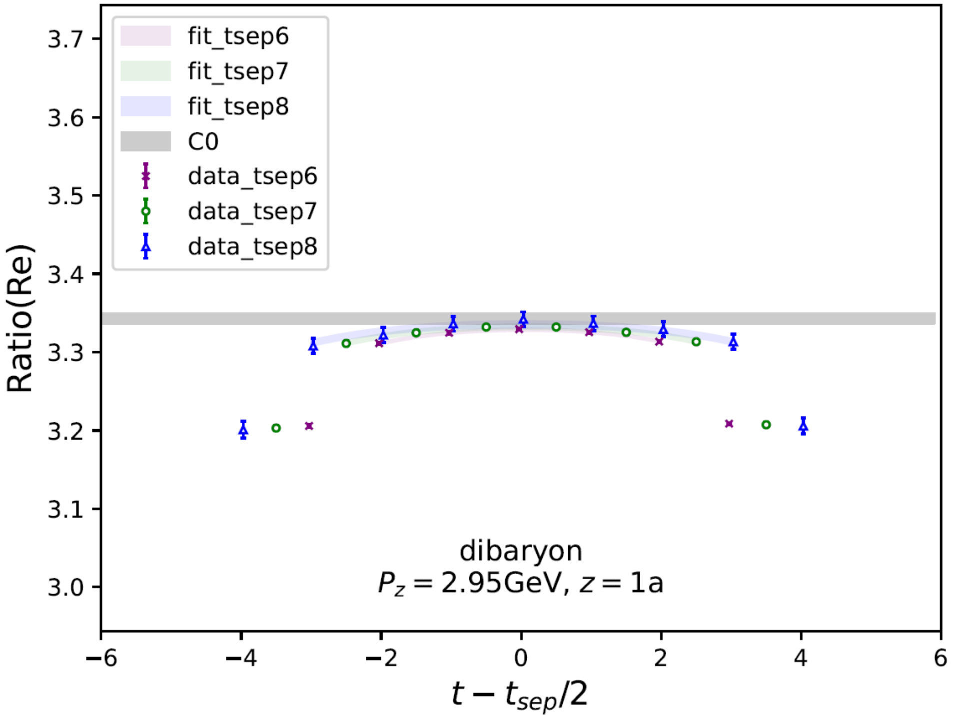
- Joint fitting to obtain ground state bare matrix elements $\tilde{h}^B(z)$

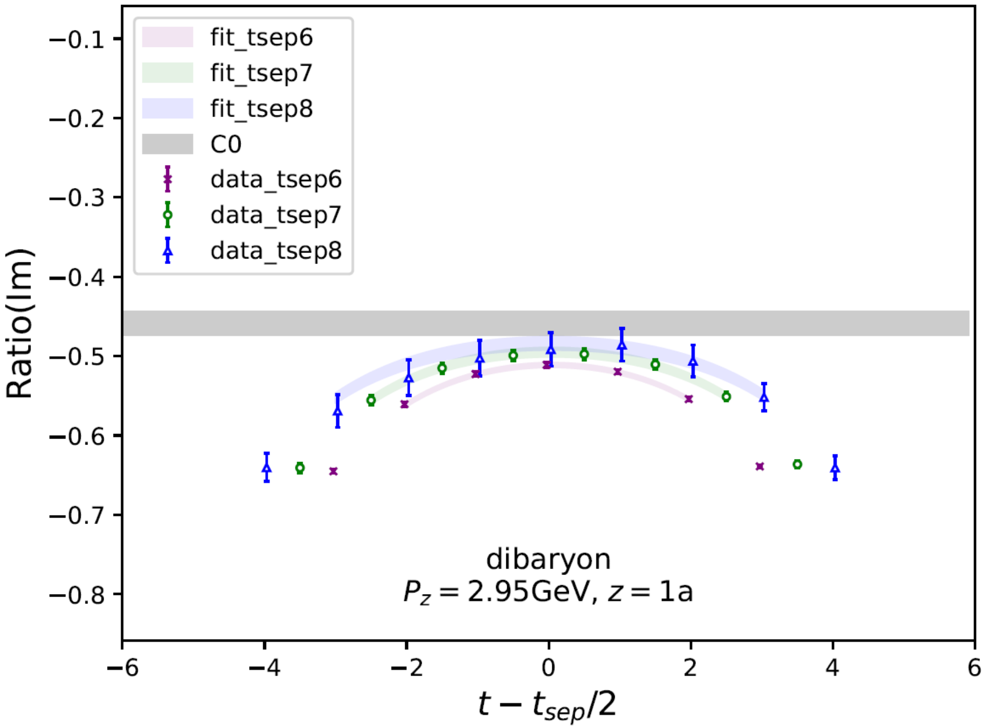
$$C^{2\mathrm{pt}}(P_z,t) = c_4 e^{-E_0 t}\left(1 + c_5 e^{-\Delta E t}\right)$$

$$R_\Gamma^{3\mathrm{pt}}(P_z,t,t_{sep}) \equiv \frac{C_\Gamma^{3\mathrm{pt}}(P_z,t,t_{sep})}{C^{2\mathrm{pt}}(P_z,t_{sep})}$$

bare matrix element

$$\approx \frac{\tilde{h}^B(z,P_z) + c_1 e^{-\Delta E(t_{sep}-t)} + c_2 e^{-\Delta E t} + c_3 e^{-\Delta E t_{sep}}}{1 + c_5 e^{-\Delta E t_{sep}}}$$





Lattice Simulation

Renormalization in hybrid scheme

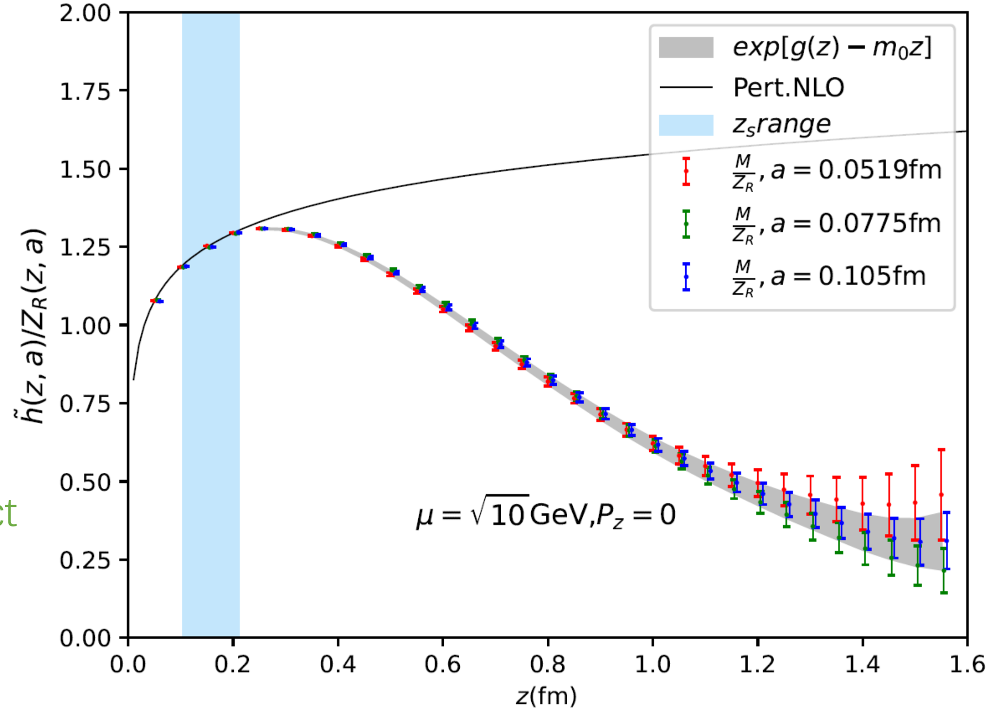
$$\tilde{h}_R(z) = \frac{\tilde{h}(z,P_z,1/a)}{\tilde{h}^{\pi}(z,P_z=0,1/a)}\theta(z_s-|z|) + \eta_s \frac{\tilde{h}(z,P_z,1/a)}{Z_R(z,1/a)}\theta(|z|-z_s)$$

Self-renormalization factor：renormalon ambiguity

linear divergence — discretization effect

$$Z_R(z,1/a) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{QCD}]} + m_0 z + f(z)a^2 + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}\right] + \ln\left[1+\frac{d}{\ln(a\Lambda_{QCD})}\right]\right\}$$

logarithmic UV divergence



The comparison of self-renormalized matrix elements of pion with the perturbative one-loop $\overline{\mathrm{MS}}$ results

The self renormalization factor is extracted by fitting the bare matrix elements of pion:

At short range: $\frac{\tilde{h}^{\pi}(z,P_z=0,1/a)}{Z_R(z,1/a)} = Z_{\overline{MS}}(z,\mu)$, where $Z_{\overline{MS}}(z,\mu)$ is perturbative one-loop $\overline{\mathrm{MS}}$ result

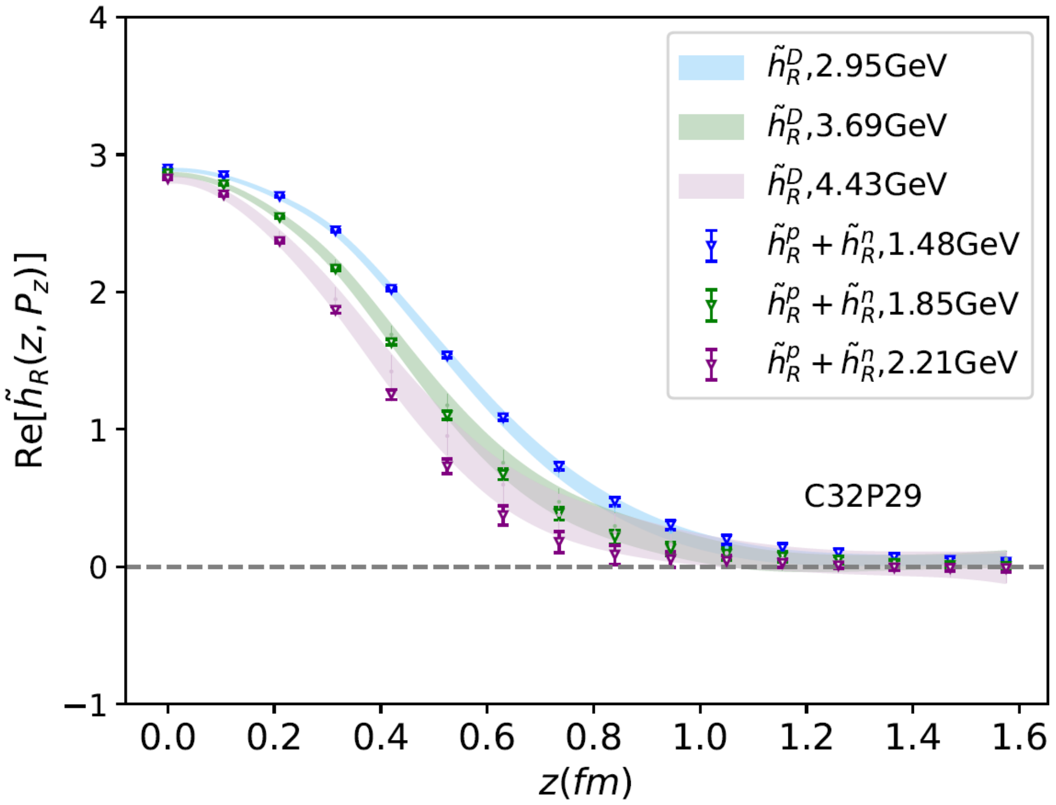
Nucl.Phys.B 964 (2021) 115311

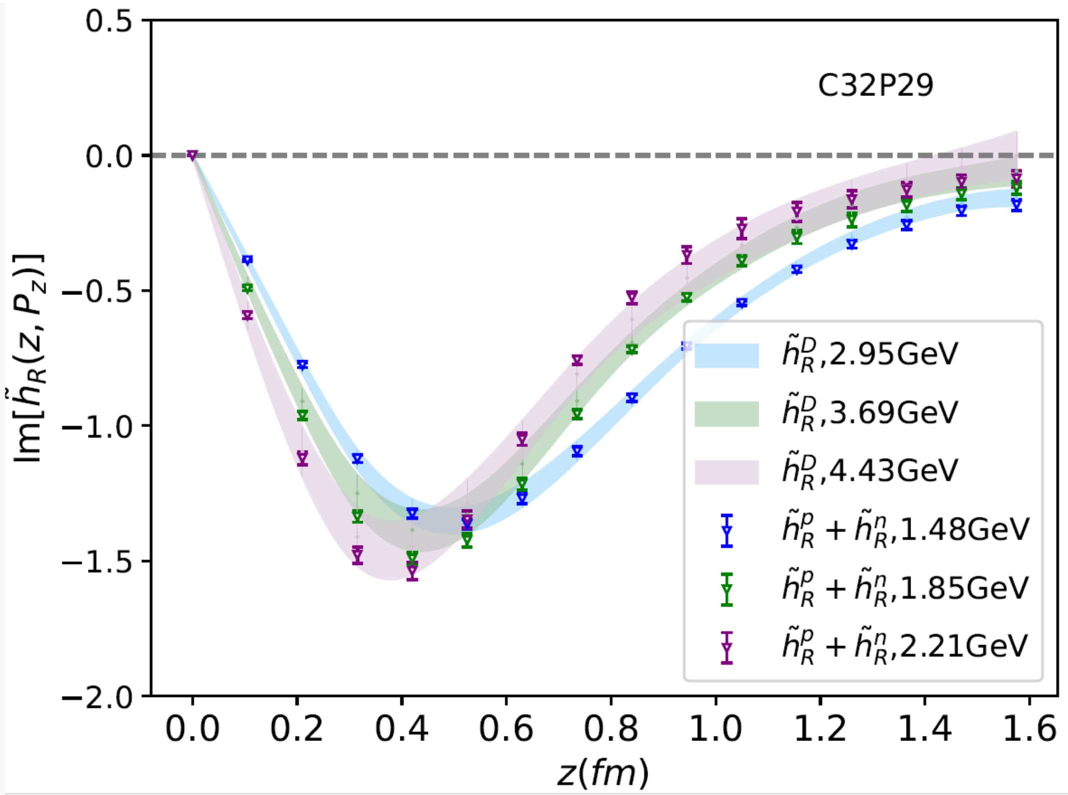
Nucl.Phys.B 969 (2021) 115443

Lattice Simulation

Renormalized matrix elements (hybrid scheme， $\mu = \sqrt{10}$GeV)

Name	Volume	m_π(MeV)
C32P29	$32^3 \times 64$	293



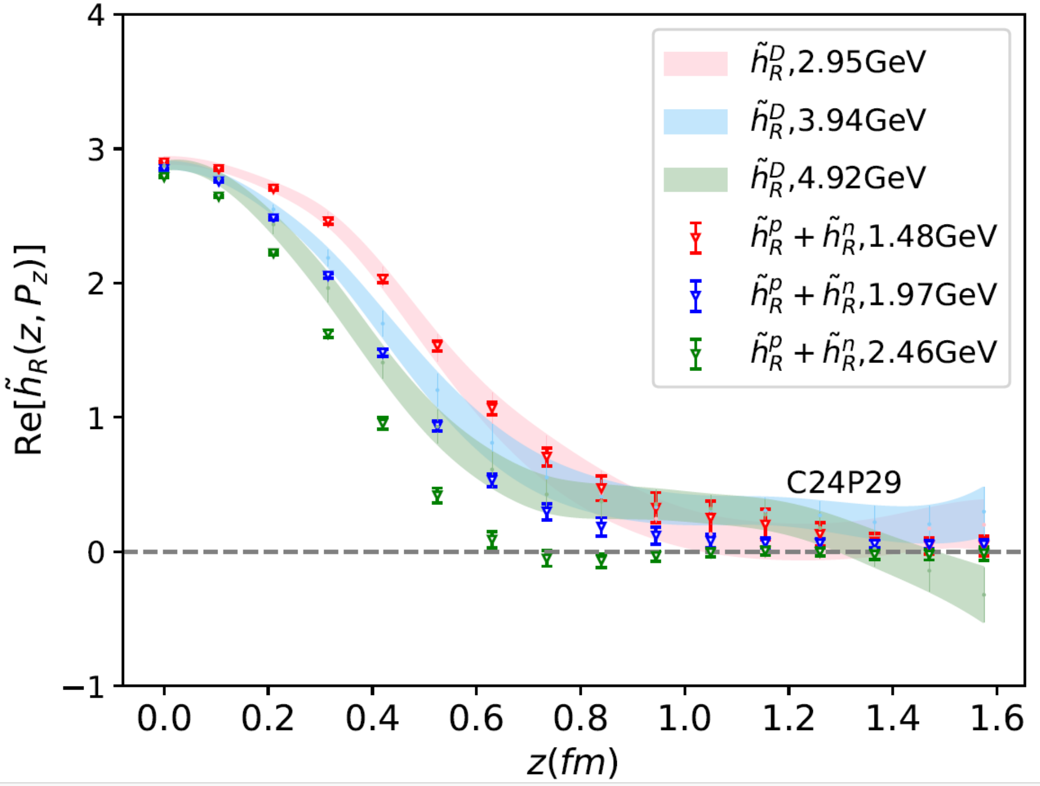


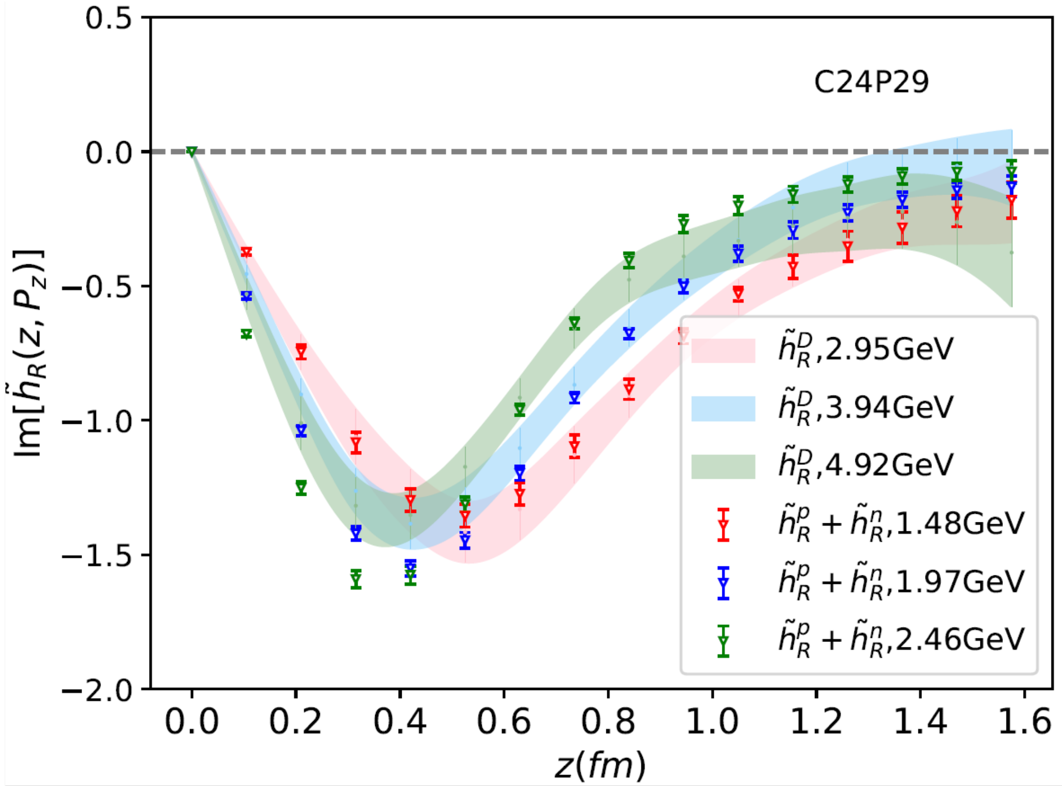
- The results of dibaryon system is compared with that of free nucleons.

Lattice Simulation

Renormalized matrix elements (hybrid scheme, $\mu = \sqrt{10}\text{GeV}$)

Name	Volume	m_π(MeV)
C24P29	$24^3 \times 72$	293



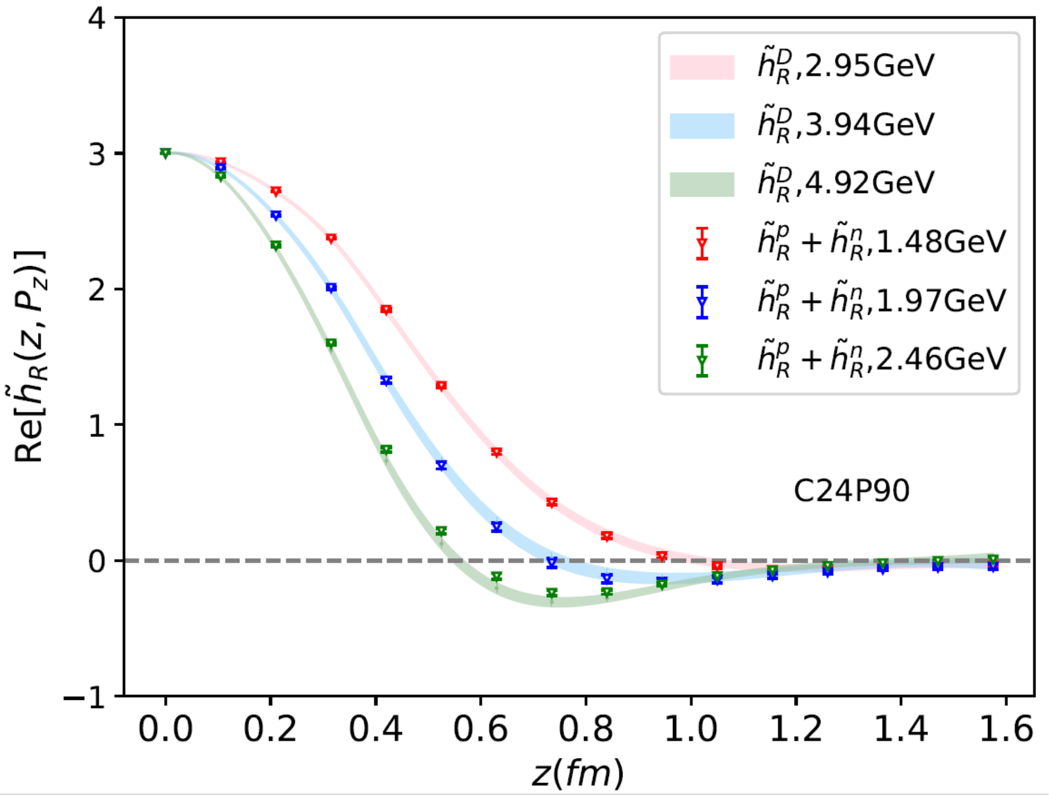


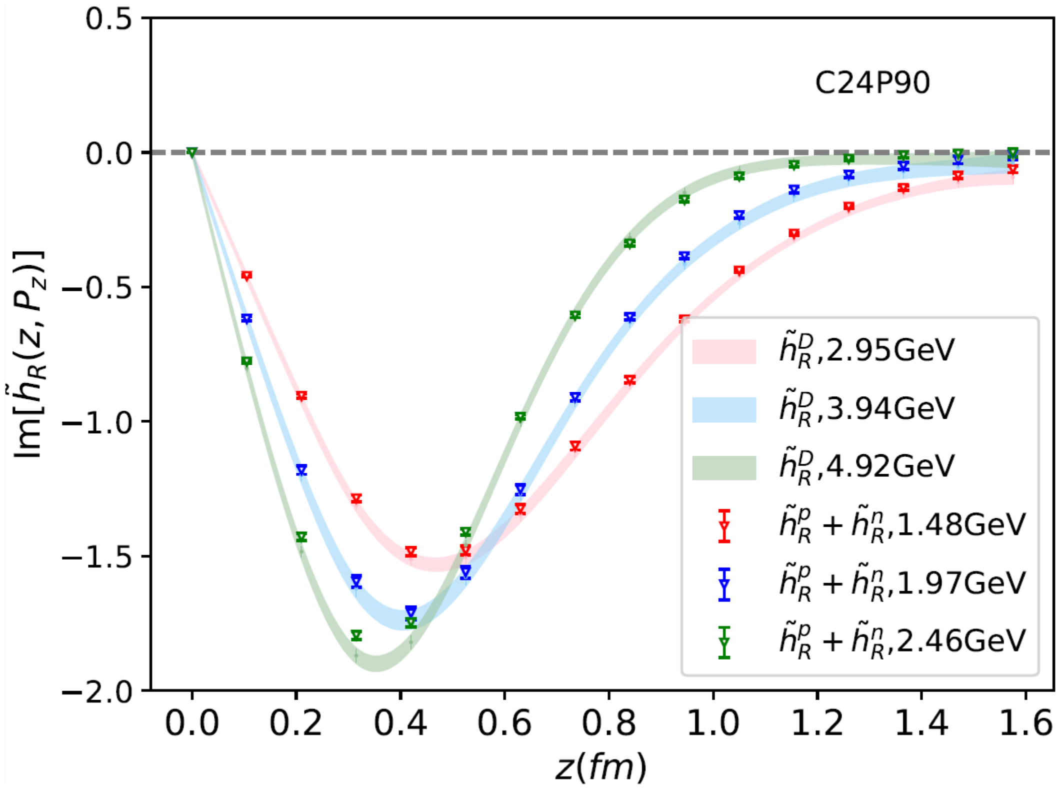
- The results of dibaryon system is compared with that of free nucleons.

Lattice Simulation

Renormalized matrix elements (hybrid scheme, $\mu = \sqrt{10}\text{GeV}$)

Name	Volume	m_π(MeV)
C24P90	$24^3 \times 72$	941





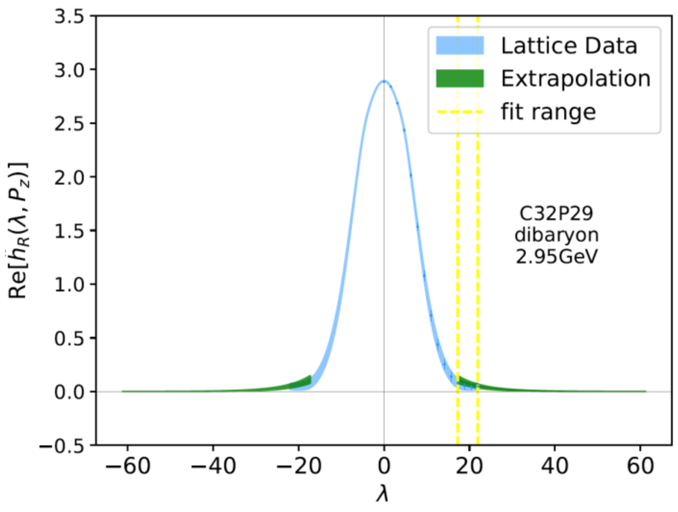
- The results of dibaryon system is compared with that of free nucleons.

Lattice Simulation

Large lambda extrapolation:

- Fourier transformation requires results in the whole coordinate space
- At large $\lambda = zP_z$,

$$\tilde{h}(\lambda) = \left[\frac{c_1}{(i\lambda)^{d_1}} + e^{-i\lambda}\frac{c_2}{(-i\lambda)^{d_2}}\right] e^{-\frac{\lambda}{\lambda_0}}$$



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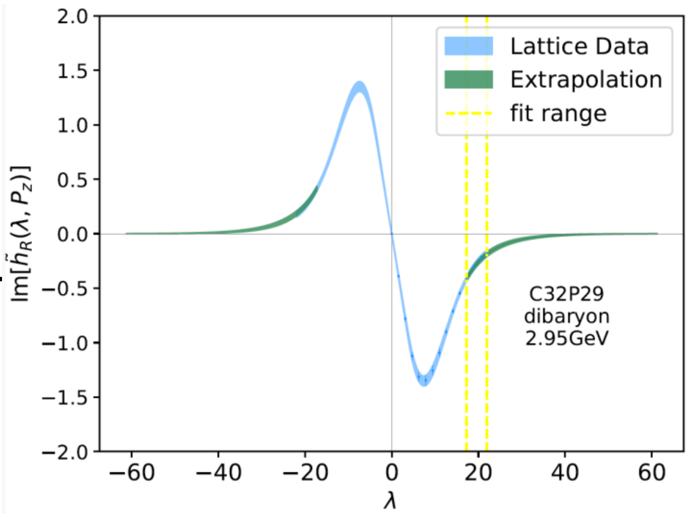
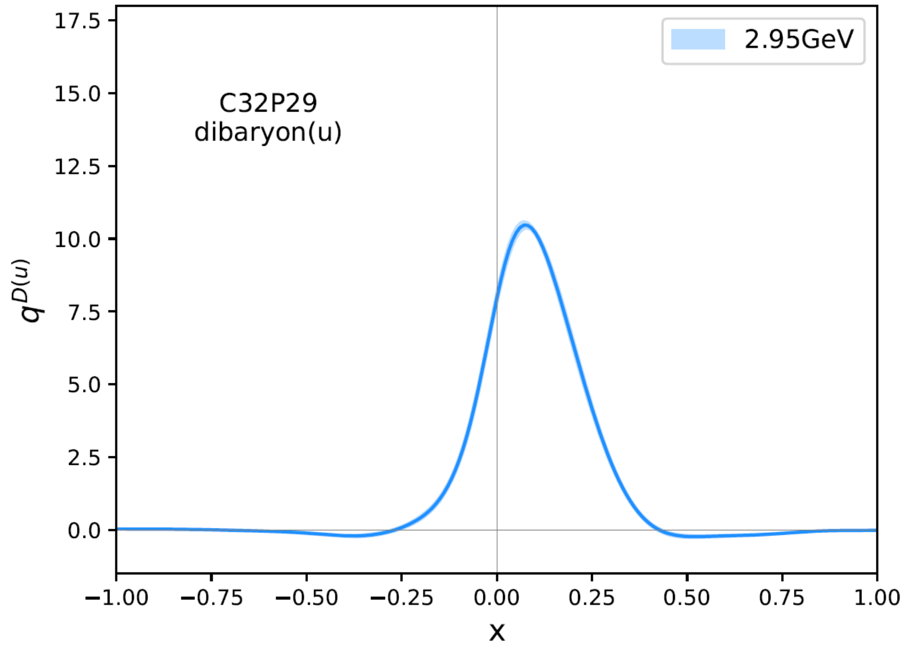


Renormalized matrix elements

Fourier transform:

$$\tilde{q}(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda}\tilde{h}(\lambda)$$



quasi-PDF

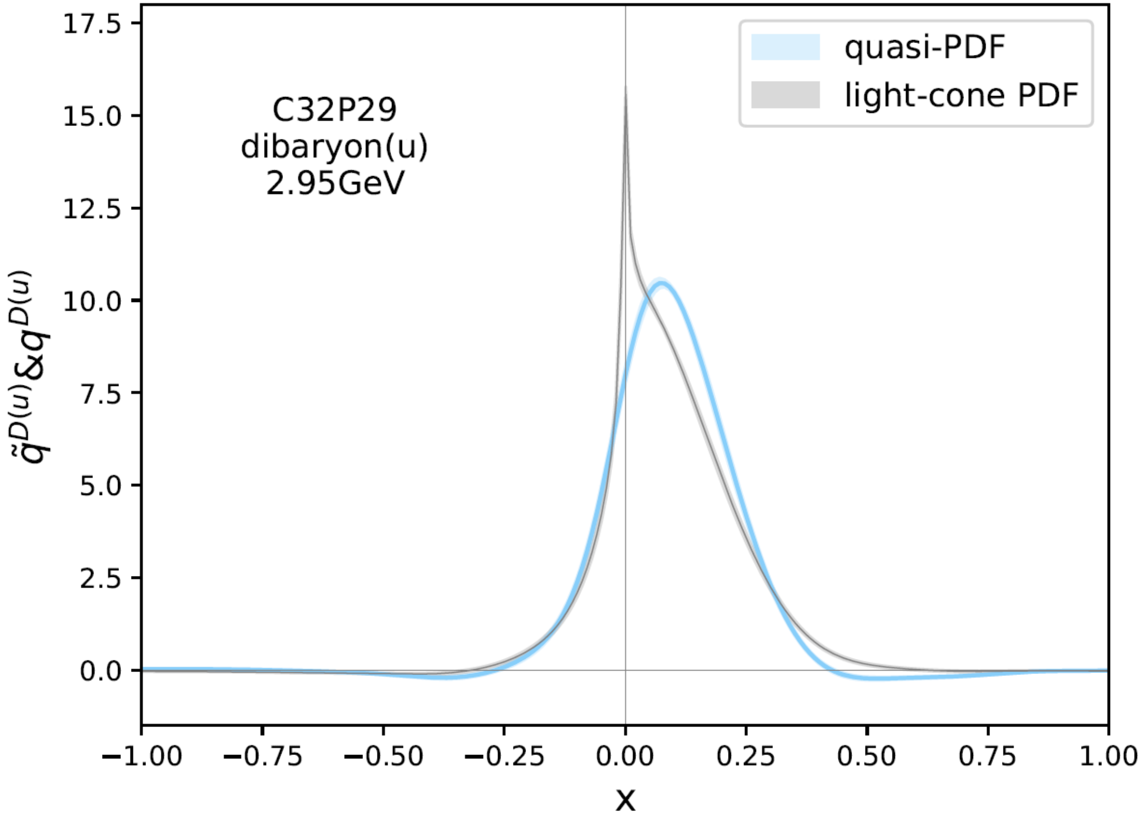
Lattice Simulation

- Matching to infinite light-cone PDF:

matching kernel

$$\tilde{q}(x,\mu) = \int \frac{\mathrm{d}y}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(\mu, y) + O\left(\frac{\mu^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2}\right),$$

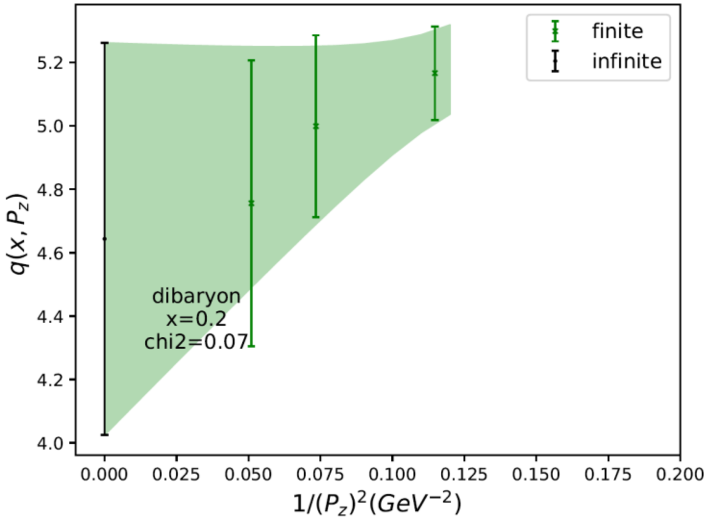
Quasi- PDF light-cone PDF

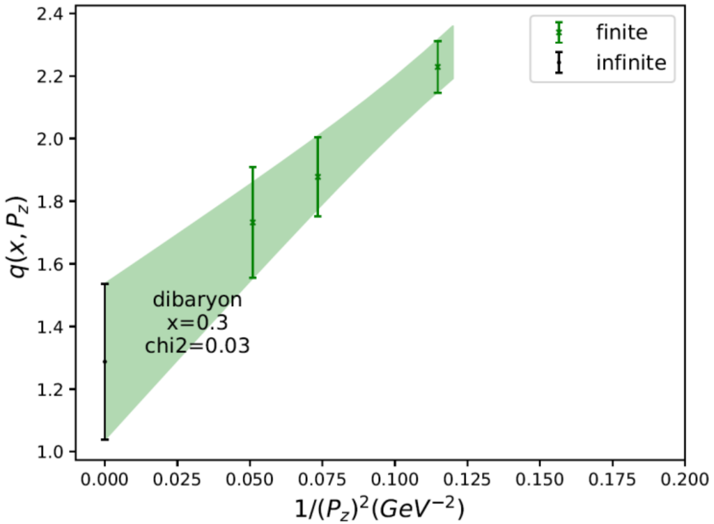


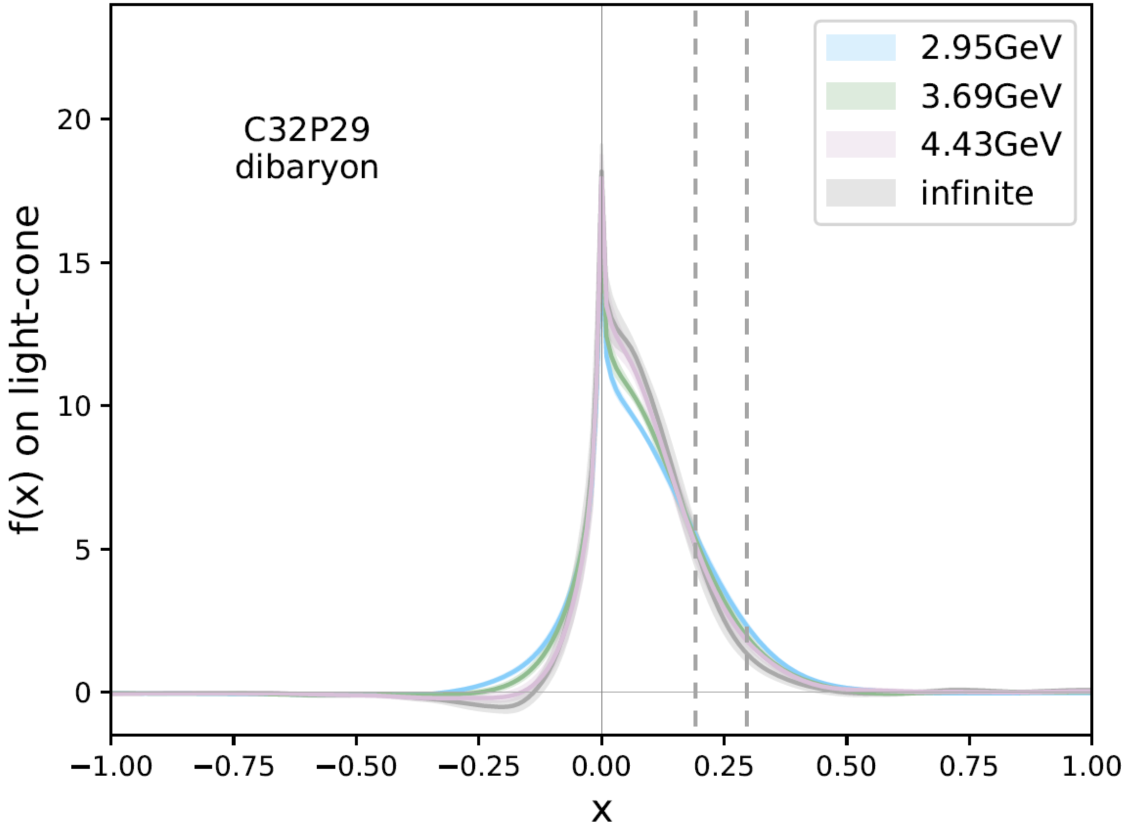
quasi-PDF & light-cone PDF

Lattice Simulation

- Momentum extrapolation: $q(x,\mu,P_z) = q(x,\mu,P_z)\Big|_{Pz\to\infty} + \frac{d_0}{(P_z)^2}$



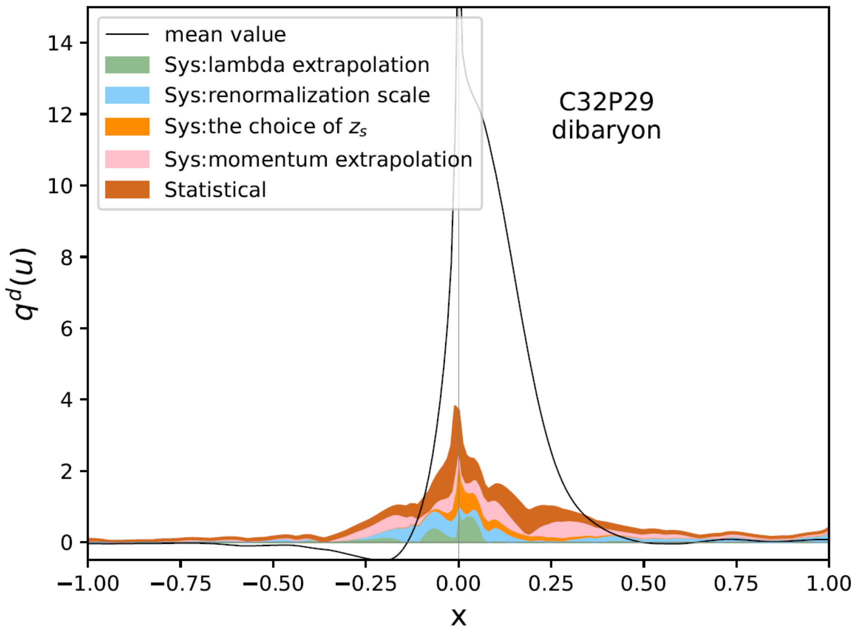


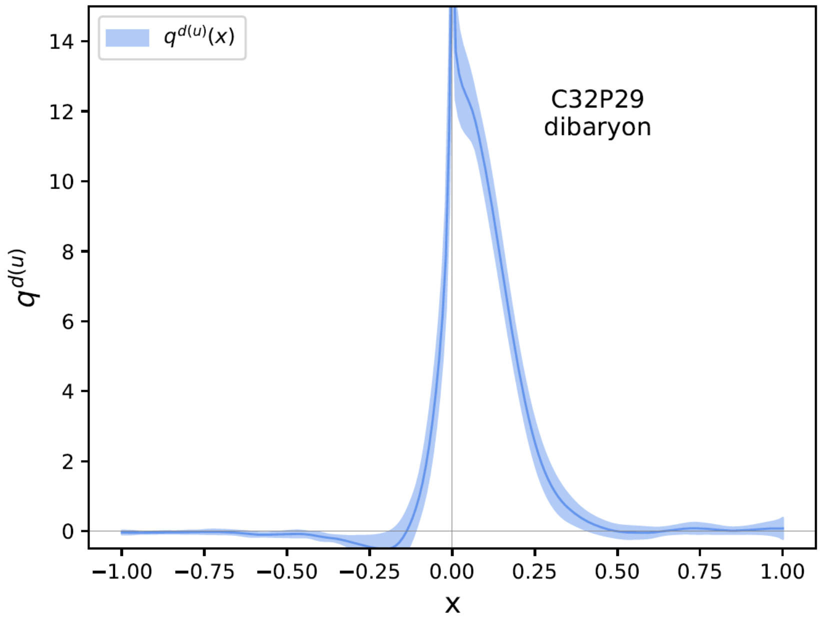


Numerical Result

Evaluating systematic errors:

- Total error: $E=\sqrt{E_{mom}{}^2+E_{lam}{}^2+E_{\mu}{}^2++E_{z_s}{}^2+E_{sta}{}^2}$

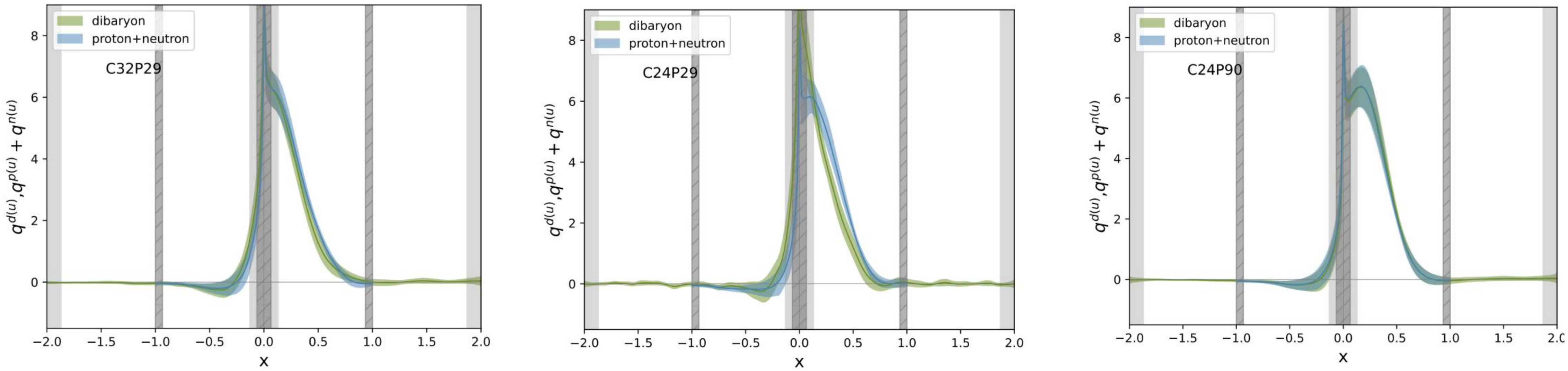




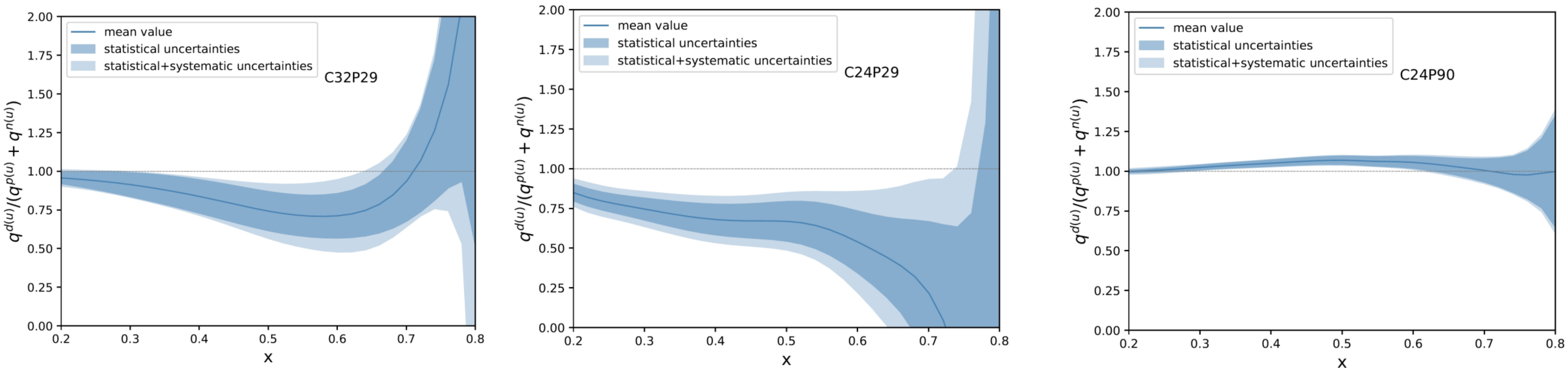
- E_{lam} from lambda extrapolation : difference between different fitting range of extrapolation
- E_{μ} from renormalization scale μ: difference between $\sqrt{10}$ GeV and 2GeV .
- E_{z_s} from the choice of z_s: difference between $z_s=0.21\ fm$ and $0.105\ fm$
- E_{mom} from momentum extrapolation : difference between infinite momentum and largest momentum case.

Numerical Result

Light-cone PDF of dibaryon and the sum of proton and neutron :



Ratio of dibaryon to the sum of proton and neutron:



Summary

➤ **We calculated unpolarized PDF of deuteron type dibaryon system and compared with free nucleon using LaMET ：**

- Calculation at single lattice spacing a=0.105fm, two different pion mass 293MeV&940MeV;
- State-of-art renormalization, matching and extrapolation ;
- Statistic errors are under control .

Thank you!