Resolving negative cross section of quarkonium hadroproduction using soft gluon factorization

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Based on: Chen, Ma, Meng, Phys.Rev.D 108 (2023) 1, 014003

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Outline

I. Introduction

II. Soft gluon factorization

III. Phenomenological studies

IV. Summary

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Introduction

➢ Heavy quarkonium

- Bound state of $Q\overline{Q}$ pair under strong interaction

  the simplest system in QCD: two-body problem

- Non-relativistic system: $v^2 << 1$

  Charmonium: m~**1.5GeV,** $v^2 \approx 0.3$

  Bottomonium: m~**4.5GeV,** $v^2 \approx 0.1$

- Multiple well-separated scales :

  quark mass: m, **momentum:** mv, **energy:** mv^2

  $m \gg mv \gg mv^2 \approx \Lambda_{QCD}$

- Involving both pert. and nonpert. physics

➢ NRQCD factorization **Bodwin, Braaten, Lepage, PRD, 1995**

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle$$

$d\hat{\sigma}_n$: production of a heavy quark pair in state $n({}^{2S+1}L_J^{[c]})$.

$\langle\mathcal{O}_n^H\rangle$: the hadronization of $Q\bar{Q}(n)$ to H;

can be ordered in powers of v;

universality.

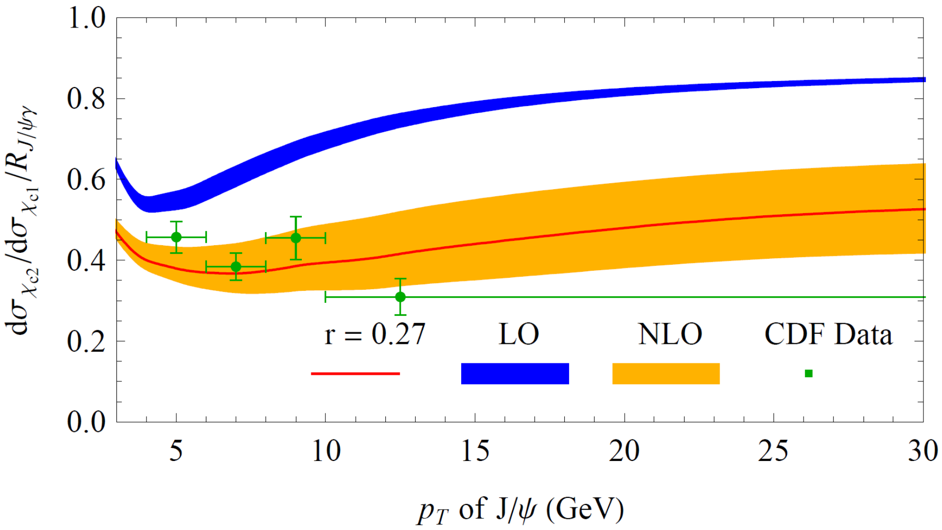
➢ Achievement:χ_c production

- **The ratio $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$**

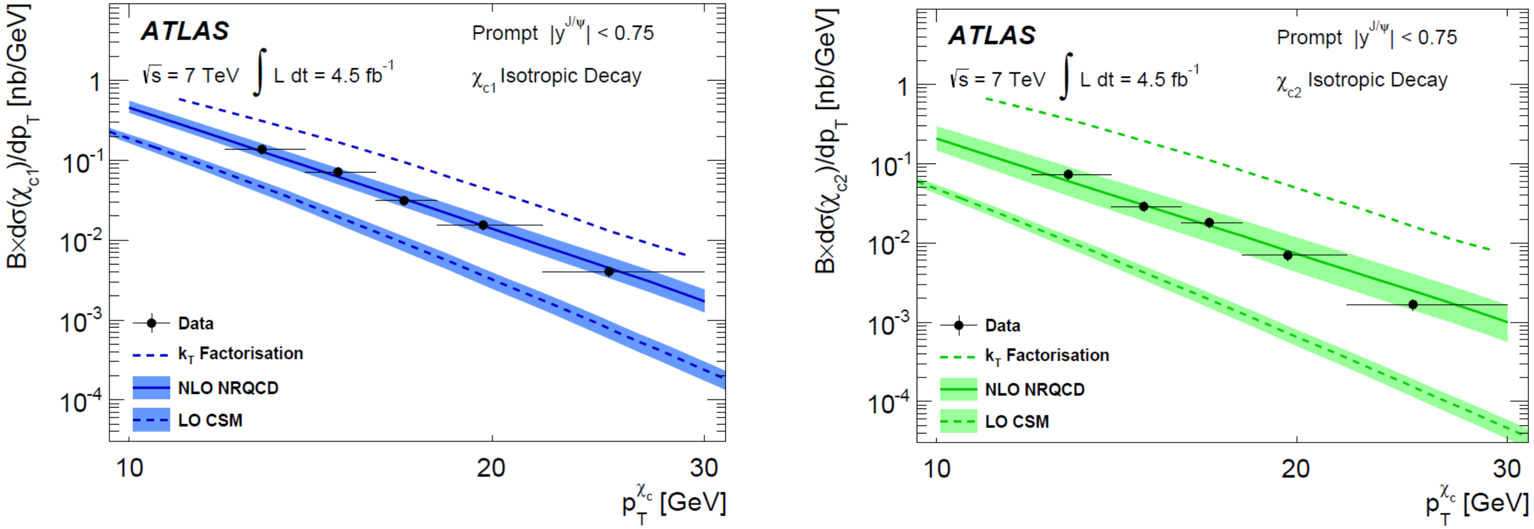
  CEM predicts: $R_{\chi_c} = 5/3$

  LO NRQCD: $R_{\chi_c} = 5/3$

Ma, Wang, Chao, 1002.3987

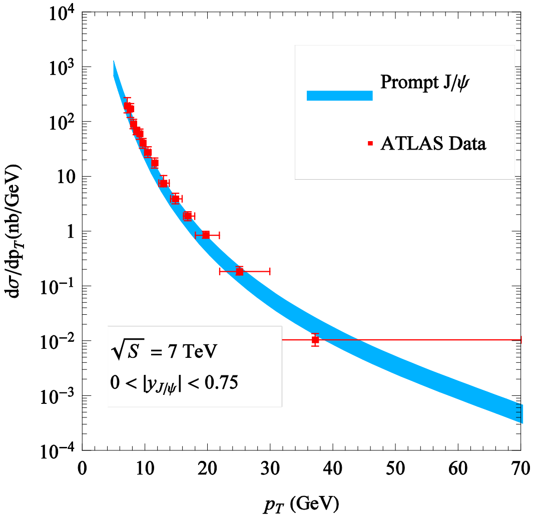


- **The differential cross sections** ATLAS, 1404.7035

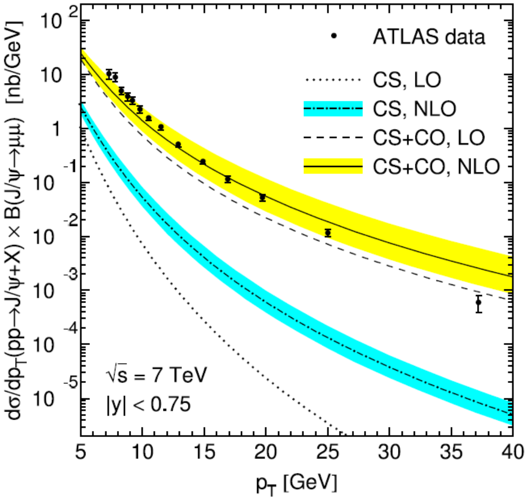


➢ Achievement: explain $\psi(nS)$ production

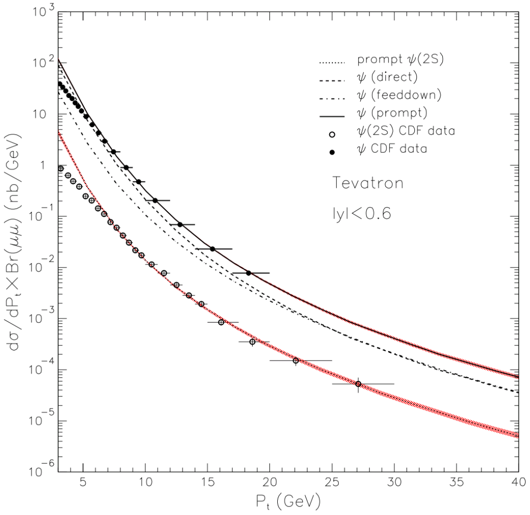
Ma, Wang, Chao, 1012.1030



Butenschoen, Kniehl, 1105.0820

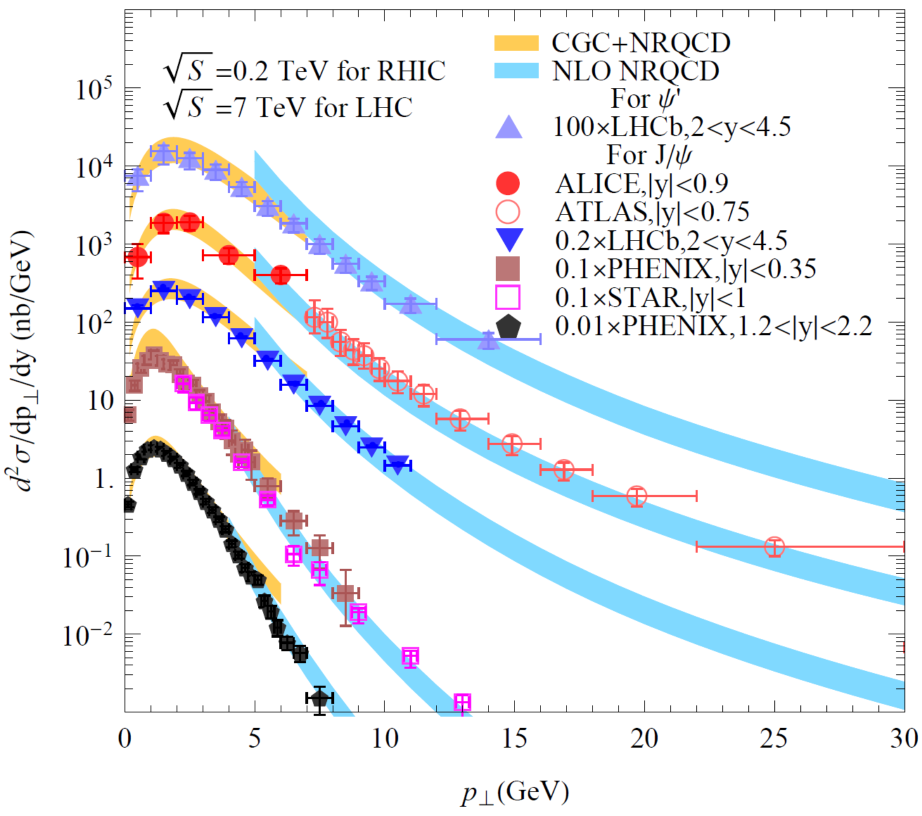


Gong, Wan, Wang, Zhang, 1205.6682



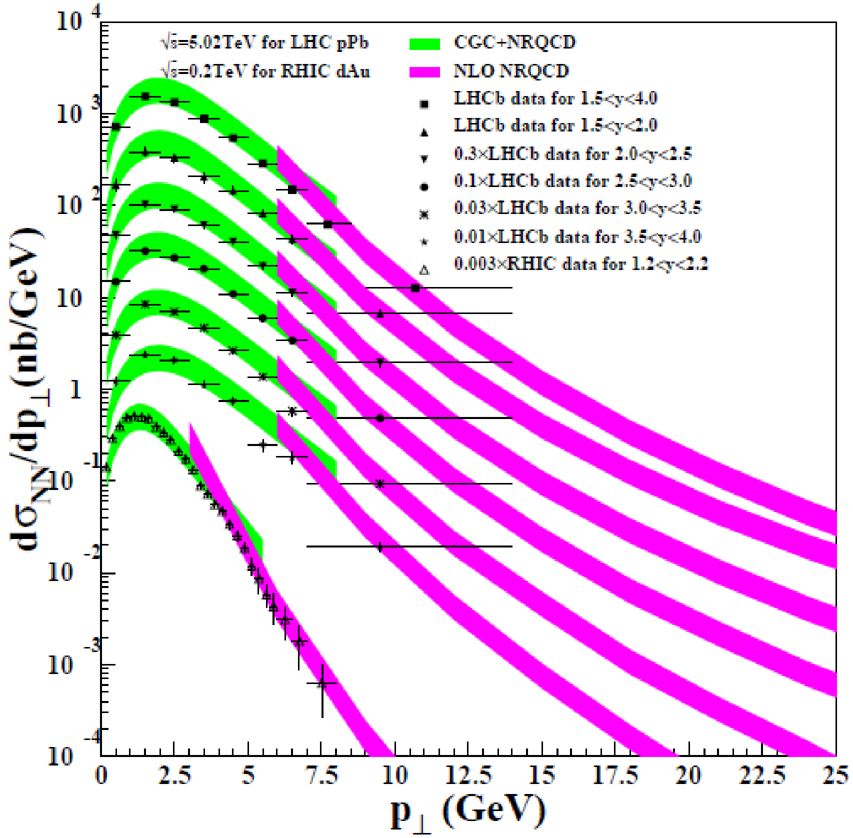
➢ Achievement: comprehensive description of $\psi(nS)$ production (CGC+NRQCD)

Ma, Venugopalan, 1408.4075



ψ(nS) **in p+p collisions**

Ma, Venugopalan, Zhang, 1503.07772



J/ψ **in p+A collisions**

➢ Difficulty : polarization puzzle

- **Dominated by $^3S_1^{[8]}$, LO NRQCD predicts transversely polarized ψ(nS) at high p_T, contradicts with Tevatron and LHC data**

CDF, 0704.0638

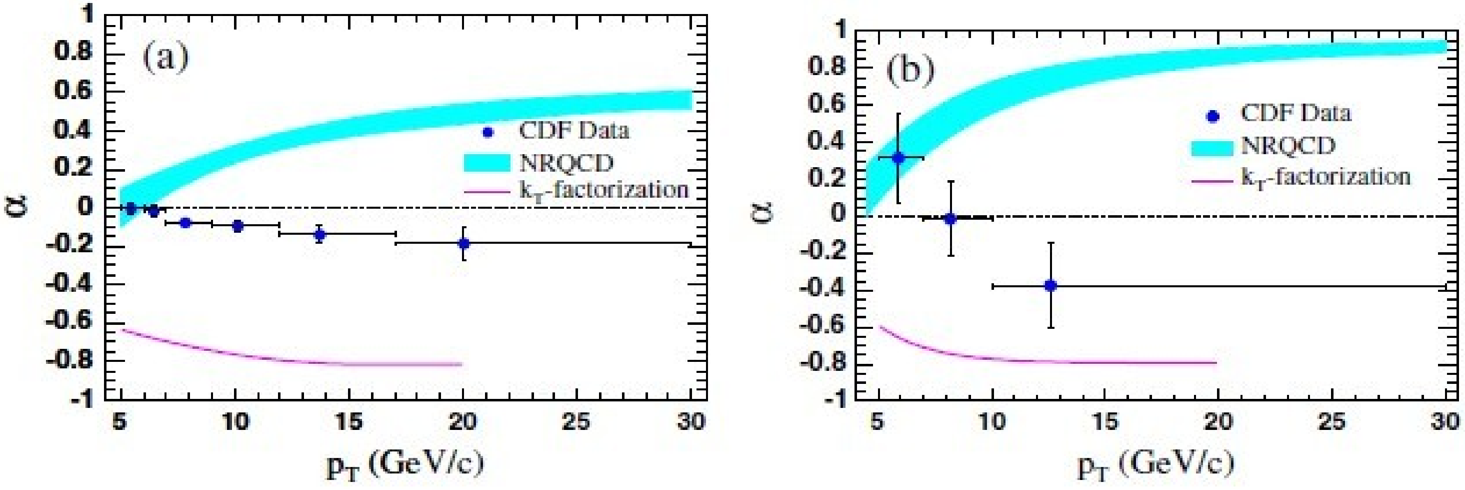
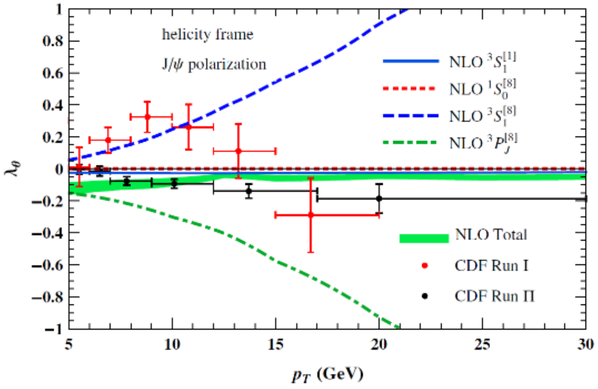
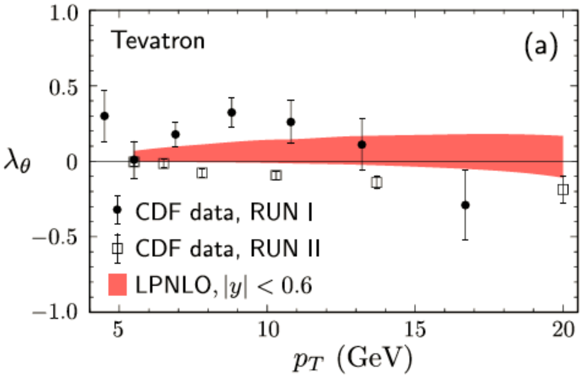


FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

- J/ψ **at NLO: transverse polarization largely canceled between** ${}^3S_1^{[8]}$ **and** ${}^3P_J^{[8]}$

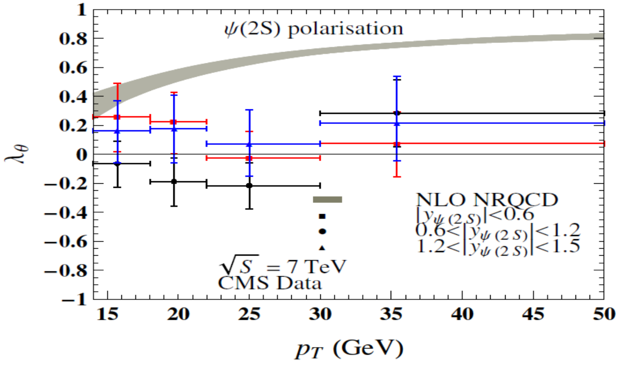


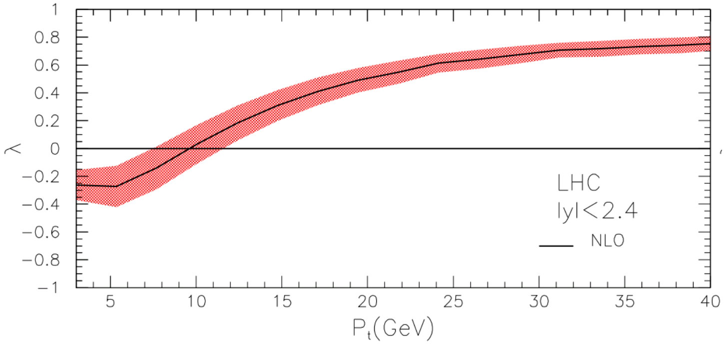


Chao,Ma,Shao,Wang, Zhang,1201.2675

Bodwin, Chung, Kim, Lee, 1403.3612

- $\psi(2S)$**: cancelation weak, hard to understand**





Shao, Han, Ma, Meng, Zhang, Chao, 1411.3300

Gong, Wan, Wang, Zhang, 1205.6682

➢ Difficulty : universality problem

☐ Fit J/ψ yield data at Tevatron with $p_T > 7$ GeV

- **Due to p_T^{-4} and p_T^{-6} behaviors, constrain two combinations**
- $M_0 = \langle O\left({}^1S_0^{[8]}\right)\rangle + 3.9\,\langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2 \approx (7.4 \pm 1.9)\times 10^{-2}\,\mathbf{GeV^3}$
- $M_1 = \langle O\left({}^3S_1^{[8]}\right)\rangle - 0.56\,\langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2 \approx (0.05 \pm 0.02) \times 10^{-2}\,\mathbf{GeV^3}$

Ma, Wang, Chao, 1009.3655

☐ Upper bound from Belle total cross section

$$M_0 < 0.02\mathrm{GeV}^3$$

Zhang, Ma, Wang, Chao, 0911.2166

☐ Global fit **Butenschoen, Kniehl, 1105.0820**

- **Including Belle, LEP, HERA, RHIC, Tevatron, LHC**
- **Total of 194 data points from 26 data sets**
- **Exclude $p_T < 3\ GeV$ pp data and $p_T < 1\ GeV$ ep data**

$$\chi^2_{\mathrm{d.o.f.}} = 725/194 = 3.74$$

- **No universality of NRQCD LDMEs!**

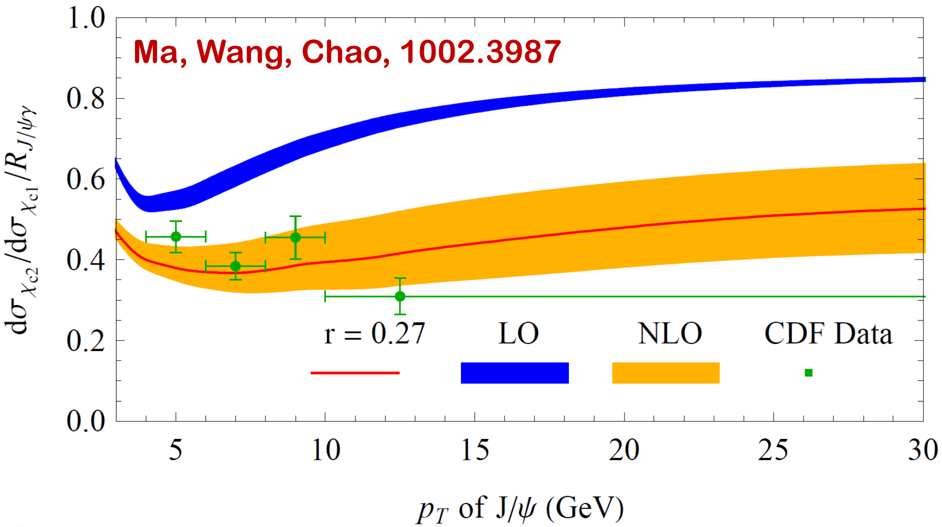
➢ Difficulty : negative cross sections

☐ Explain χ_{cJ} production

- The ratio $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$

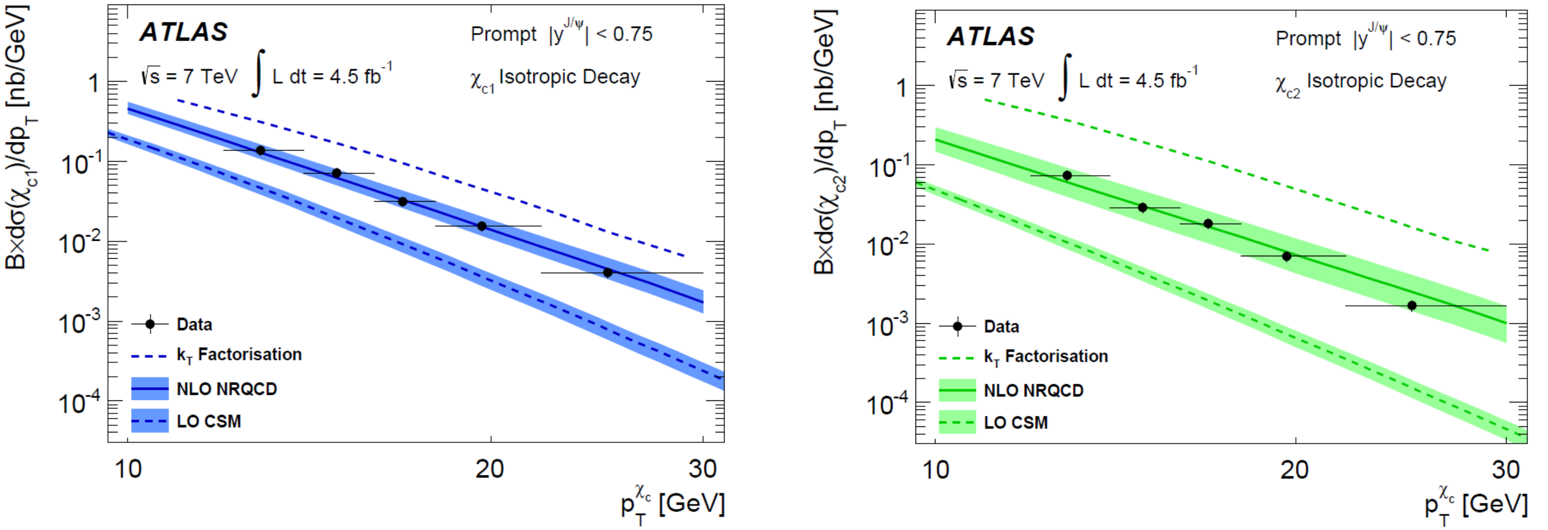
  CEM predicts: $R_{\chi_c} = 5/3$

  LO NRQCD: $R_{\chi_c} = 5/3$


Ma, Wang, Chao, 1002.3987

- The differential cross sections

ATLAS, 1404.7035

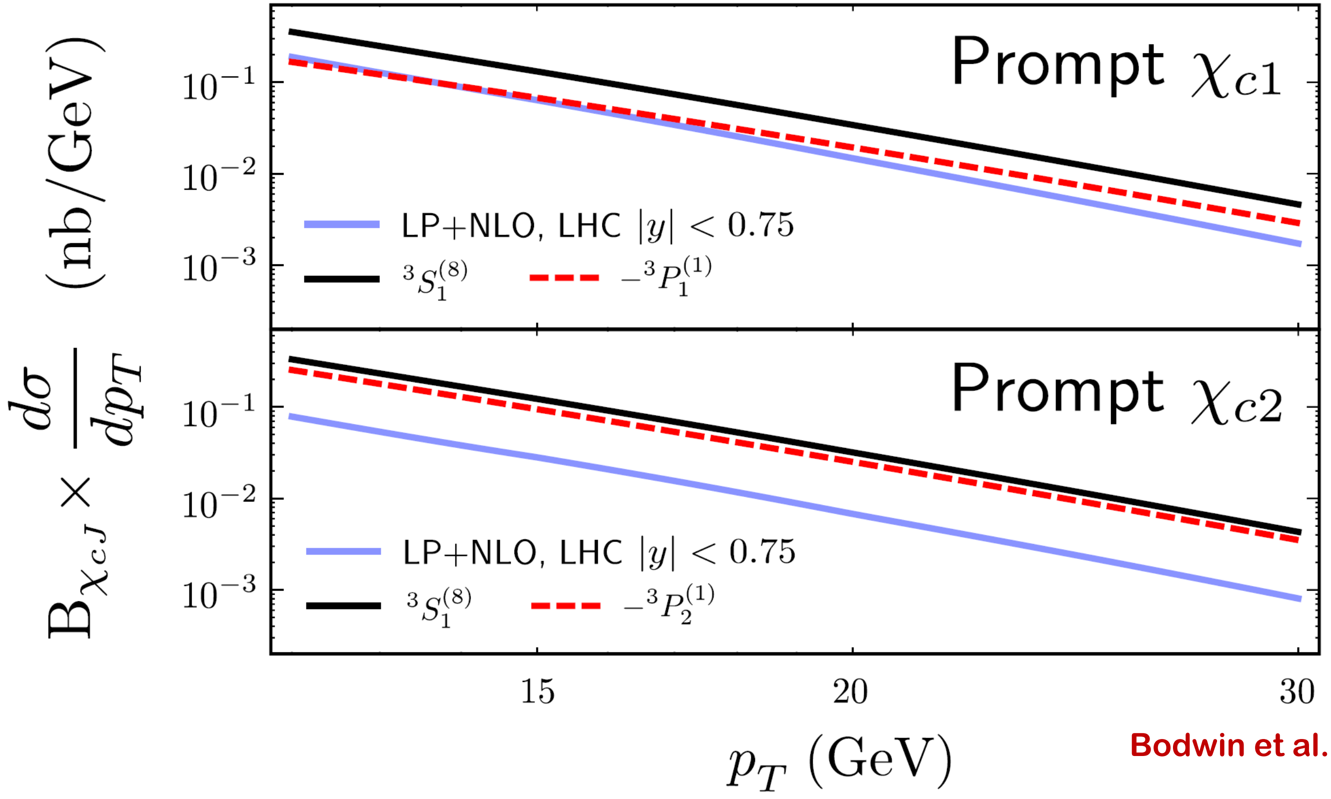


☐ There are substantial cancellations between $^3\boldsymbol{S}_1^{[8]}$ and $^3\mathrm{P}_J^{[1]}$

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[^3S_1^{[8]}]\langle\mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle + (2J+1)d\hat{\sigma}[^3P_J^{[1]}]\frac{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle}{m_c^2}$$

Positive

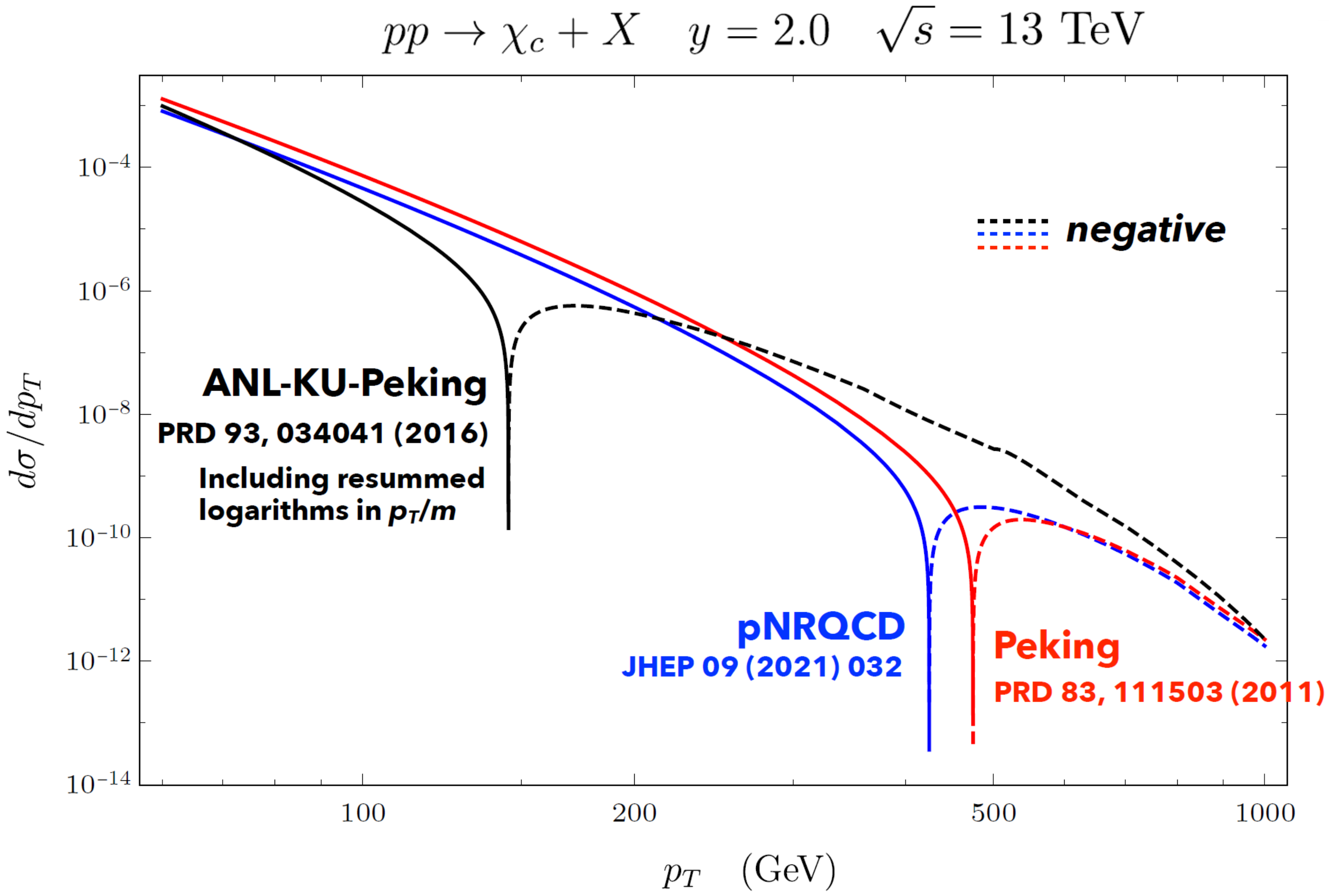
Negative



Bodwin et al., 1509.07904

- **Perturbation unstable**

- **Cross sections turn negative at large p_T**



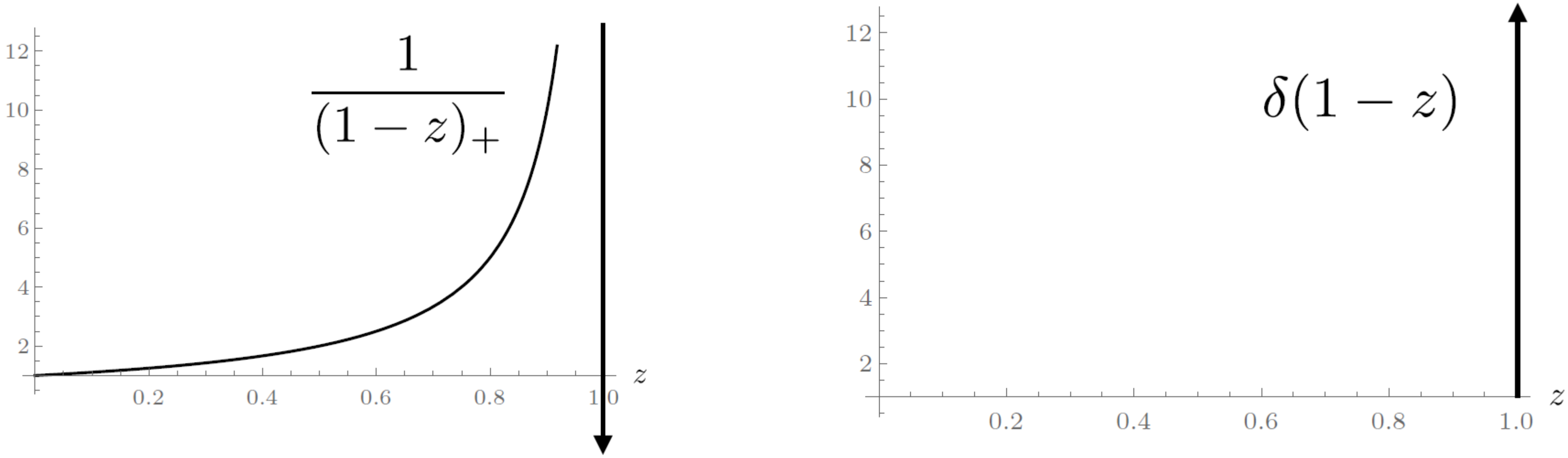
Hee Sok Chung, talk at The 15th International Workshop on Heavy Quarkonium

Why?

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[{}^3S_1^{[8]}]\langle\mathcal{O}^{\chi_{c0}}({}^3S_1^{[8]})\rangle + (2J+1)d\hat{\sigma}[{}^3P_J^{[1]}]\frac{\langle\mathcal{O}^{\chi_{c0}}({}^3P_0^{[1]})\rangle}{m_c^2}$$

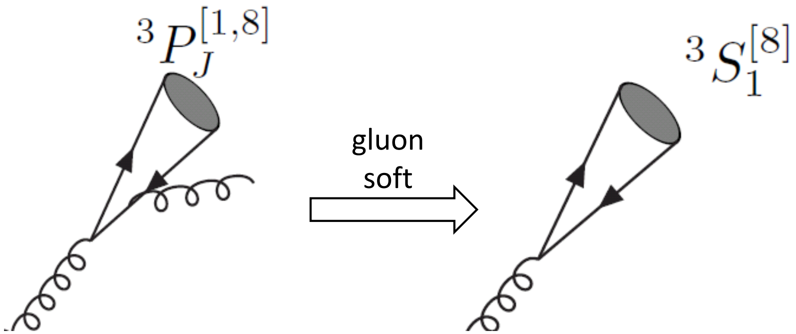
$$d\hat{\sigma}[{}^3P_J^{[1]}] = d\hat{\sigma}_g \otimes \left\{0\times\alpha_s + \frac{2\alpha_s^2}{27N_c m_c^5}\left[\left(\frac{Q_J}{2J+1} - \log\frac{\Lambda}{2m_c}\right)\delta(1-z) + \boxed{\frac{z}{(1-z)_+}} + \frac{P_J(z)}{2J+1}\right]\right\}$$

$$d\hat{\sigma}[{}^3S_1^{[8]}] = d\hat{\sigma}_g \otimes \frac{\pi\alpha_s}{24m_c^3}\boxed{\delta(1-z)}$$



Hee Sok Chung, talk at The 15th International Workshop on Heavy Quarkonium

- **Cross section at very large p_T will depend strongly on $z \to 1$ behavior of FFs**



- **Soft gluon in P-wave: factorized to S-wave matrix element**
- **Plus functions: remnants of the infrared subtraction in matching the $^3\mathrm{P}_J^{[1]}$ SDCs**
- **Subtraction scheme: at *zero momentum*, which contributes the largest production rate. Over subtracted!**
- **Solution: soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in v.**

☐ Soft gluon factorization: resum a dominant series of power corrections (kinematic effects) and log corrections **Ma, Chao, 1703.08402; Chen, Ma, 2005.08786.**

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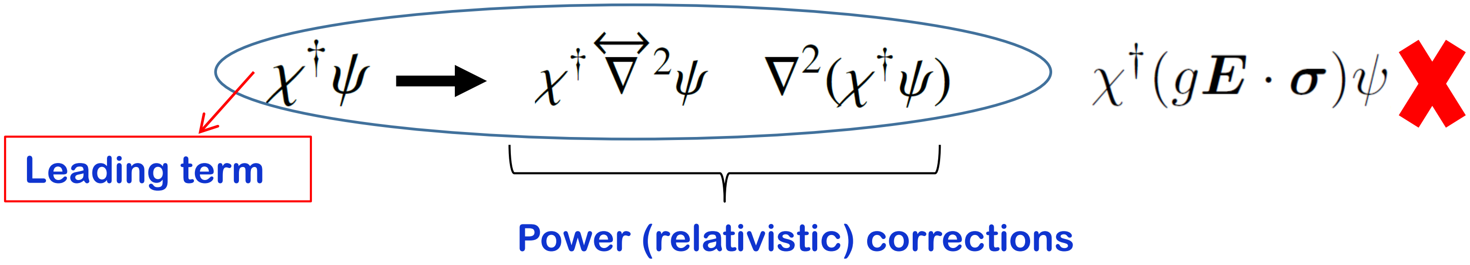
Soft gluon factorization

➢ From NRQCD to SGF

Ma, Chao, 1703.08402;
Chen, Ma, 2005.08786.

- ☐ To resum the series of relativistic corrections originated from kinematic effects in NRQCD
- ☐ Beginning from $\chi^\dagger\psi$, one can construct powers suppressed operators



$$\chi^\dagger\psi \longrightarrow \underbrace{\chi^\dagger\overleftrightarrow{\nabla}^2\psi \quad \nabla^2(\chi^\dagger\psi)}_{\text{Power (relativistic) corrections}} \qquad \chi^\dagger(g\boldsymbol{E}\cdot\boldsymbol{\sigma})\psi \ ✗$$

Leading term

- ☐ Equation of motion

$$\left(iD_0 - \frac{\boldsymbol{D}^2}{2m} + \cdots\right)\psi = 0$$

- ☐ Ignoring gluon field, replace D by ∇

Use EOM to remove relative derivatives

$$\langle H+X|\nabla_0^{n_1}\nabla^{2n_2}(\chi^\dagger\psi)|0\rangle$$ **(inclusive processes)**

Using integration by parts

- **Remove operators unless** $n_1 = n_2 = 0$
- **Matching coefficients are functions of:** $P_H^2, P_H \cdot P_X, P_X^2$

Factorization

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} \approx \sum_n \int \frac{d^4 P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

- $n = {}^{2S+1}L_J^{[c]}$
- **P: momentum of** $Q\bar{Q}$

- $\mathcal{H}_n$: **perturbatively calculable hard parts**
- $F_{n\to H}$: **nonperturbative soft gluon distributions (SGDs)**
- **UV renormalization scale is suppressed**

➢ FFs in SGF

- $D_{f\to H}$: **single parton FFs**
- $\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$: **double parton FFs**
- $\hat{z} = z/x$

$$\begin{aligned} &D_{f\to H}(z,\mu_0) \\ &\quad = \sum_{n,n'} \int \frac{\mathrm{d}x}{x} \hat{D}_{f\to Q\bar{Q}[nn']}(\hat{z}; M_H/x, m_Q, \mu_0, \mu_\Lambda) \\ &\quad\quad \times F_{[nn']\to H}(x, M_H, m_Q, \mu_\Lambda), \end{aligned} \tag{2a}$$

$$\begin{aligned} &\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta,\zeta',\mu_0) \\ &\quad = \sum_{n,n'} \int \frac{\mathrm{d}x}{x} \hat{\mathcal{D}}_{[Q\bar{Q}(\kappa)]\to Q\bar{Q}[nn']}(\hat{z},\zeta,\zeta'; M_H/x, m_Q, \mu_0, \mu_\Lambda) \\ &\quad\quad \times F_{[nn']\to H}(x, M_H, m_Q, \mu_\Lambda), \end{aligned} \tag{2b}$$

➢Soft gluon distributions (SGDs)

☐ Operator definition

- **Expectation values of bilocal operators in QCD vacuum**

$$F_{[nn']\to\psi}(x, M_\psi, m_c, \mu_f) = P_\psi^+ \int \frac{db^-}{2\pi} e^{-iP_\psi^+ b^-/x} \langle 0|[\bar{\Psi}\mathcal{K}_n\Psi]^\dagger(0)[a_\psi^\dagger a_\psi][\bar{\Psi}\mathcal{K}_{n'}\Psi](b^-)|0\rangle_{\mathrm{S}},$$

with

$$a_H^\dagger a_H = \sum_X \sum_{J_z^H} |H+X\rangle\langle H+X|$$

$$\mathcal{K}_n(rb) = \frac{\sqrt{M_H}}{M_H+2m}\,\frac{M_H+\not{P}_H}{2M_H}\Gamma_n\frac{M_H-\not{P}_H}{2M_H}\,\mathcal{C}^{[c]}$$

Spin project operators:

$$\Gamma_n = \sum_{L_z,S_z}\langle L, L_z; S, S_z|J, J_z\rangle\Gamma^o_{LL_z}\Gamma^s_{SS_z}$$

Color project operators:

$$\mathcal{C}^{[1]} = \frac{\mathbf{1}_c}{\sqrt{N_c}} \qquad \mathcal{C}^{[8]} = \sqrt{2}t^{\bar{a}}\,\Phi^{(A)}_{a\bar{a}}(rb)$$

☐ Gauge link

Nayak, Qiu, Sterman, 0509021

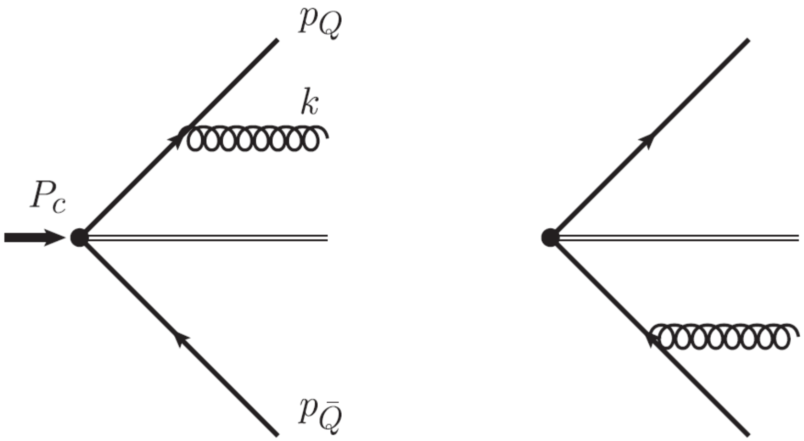
$$\Phi_l(rb^-) = \mathcal{P}\exp\left[-ig_s\int_0^\infty \mathrm{d}\xi\, l\cdot A(rb^-+\xi l)\right],$$

☐ Evaluated in ***small*** region

- **Subscript “S”: evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass**

➢Matching the hard parts

☐ P-wave

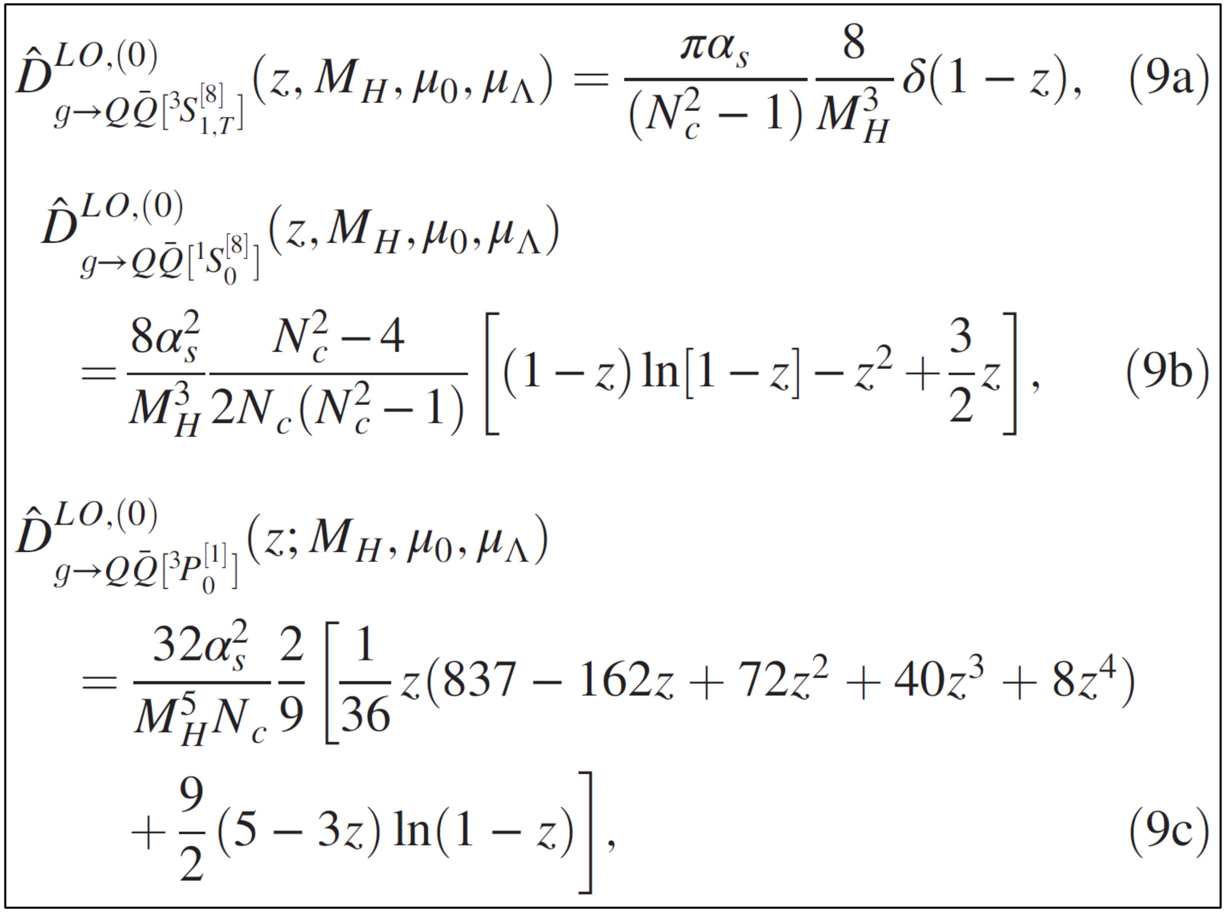


(a) (b)

$$F^{LO}_{[^3S^{[1]}_{1,T}]\to Q\bar{Q}[^3P^{[1]}_0]}(x,M_H,m_Q,\mu_\Lambda) = \frac{\alpha_s}{M_H^2\pi}\frac{N_c^2-1}{N_c}\frac{8}{9}\left[\left(-\frac{1}{\epsilon_{\mathrm{IR}}}-\ln\frac{4\pi\mu_c^2 e^{-\gamma_E}}{M_H^2}-\frac{1}{6}\right)\times\delta(1-x)+2x\frac{1}{(1-x)_+}\right]+\mathcal{O}(\boldsymbol{q}^2),$$

$$D^{LO}_{g\to Q\bar{Q}[^3P^{[1]}_0]}(z;M_H,m_Q,\mu_0) = \frac{32\alpha_s^2\mu_c^{2\epsilon}}{M_H^5 N_c}\frac{2}{9}\left[\left(-\frac{1}{\epsilon_{\mathrm{IR}}}-\frac{1}{6}-\ln\frac{4\pi\mu_c^2 e^{-\gamma_E}}{M_H^2}\right)\delta(1-z) + \frac{z(26z^2-111z+93)}{4}\frac{1}{(1-z)_+}+\frac{9(5-3z)}{2}\times\ln(1-z)\right]+\mathcal{O}(\boldsymbol{q}^2), \qquad \text{(A18a)}$$

☐ Short distance hard parts at LO



$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[^3S^{[8]}_{1,T}]}(z, M_H, \mu_0, \mu_\Lambda) = \frac{\pi\alpha_s}{(N_c^2-1)}\frac{8}{M_H^3}\delta(1-z), \quad (9a)$$

$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[^1S^{[8]}_{0}]}(z, M_H, \mu_0, \mu_\Lambda) = \frac{8\alpha_s^2}{M_H^3}\frac{N_c^2-4}{2N_c(N_c^2-1)}\left[(1-z)\ln[1-z]-z^2+\frac{3}{2}z\right], \quad (9b)$$

$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[^3P^{[1]}_{0}]}(z; M_H, \mu_0, \mu_\Lambda) = \frac{32\alpha_s^2}{M_H^5 N_c}\frac{2}{9}\left[\frac{1}{36}z(837-162z+72z^2+40z^3+8z^4) + \frac{9}{2}(5-3z)\ln(1-z)\right], \quad (9c)$$

- **The P-wave short distance hard parts do not include terms proportional to plus distributions**

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Phenomenological studies

➢Collinear factorization

☐ Heavy quarkonium production at large p_T

$$\mathrm{d}\sigma_{A+B\to H+X}(p) \approx \sum_{i,j} f_{i/A}(x_1,\mu_F) f_{j/B}(x_2,\mu_F)\Big\{\sum_f D_{f\to H}(z,\mu_F)\otimes \mathrm{d}\hat{\sigma}_{i+j\to f+X}(\hat{P}/z,\mu_F)$$
$$+\sum_{\kappa} \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta,\zeta',\mu_F)\otimes \mathrm{d}\hat{\sigma}_{i+j\to[Q\bar{Q}(\kappa)]+X}(\hat{P}(1\pm\zeta)/2z,\hat{P}(1\pm\zeta')/2z,\mu_F)\Big\},$$

☐ Factorization of FFs

- **SGF**
- **NRQCD factorization**

☐ Nonperturbative model for SGDs

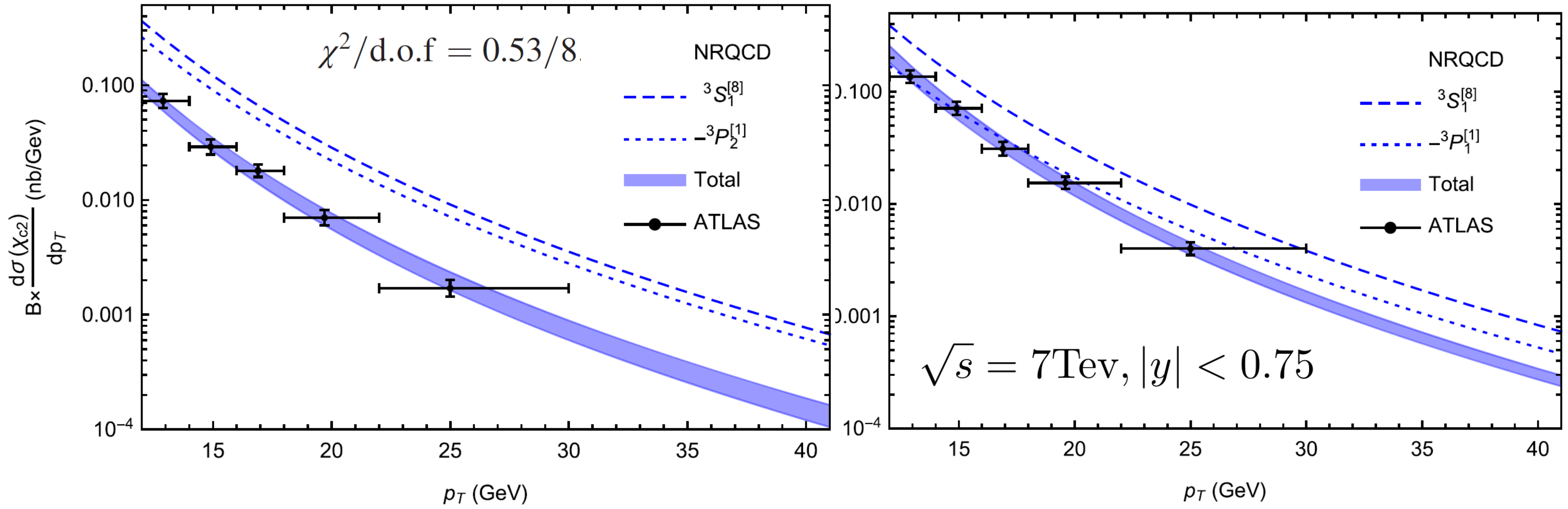
$$F^{\mathrm{mod}}(x) = \frac{N^H\Gamma(M_H b/\bar{\Lambda})(1-x)^{b-1}x^{M_H b/\bar{\Lambda}-b-1}}{\Gamma(M_H b/\bar{\Lambda}-b)\Gamma(b)}$$

- N^H**: the normalization,** $N^H[n]\approx\langle\mathcal{O}^H(n)\rangle$
- $\bar{\Lambda}$: **the average radiated momentum in the hadronization process**
- b: **related to the second moment of model function**

Production of χ_{cJ}

NRQCD factorization

- **The fitted cross sections compared with ATLAS data**



- **Define the ratio**

$$r(\chi_{c0}) \equiv \frac{\langle \mathcal{O}^{\chi_{c0}}({}^3S_1^{[8]})\rangle}{\langle \mathcal{O}^{\chi_{c0}}({}^3P_0^{[1]})\rangle / m_c^2},$$

- **The cross sections**

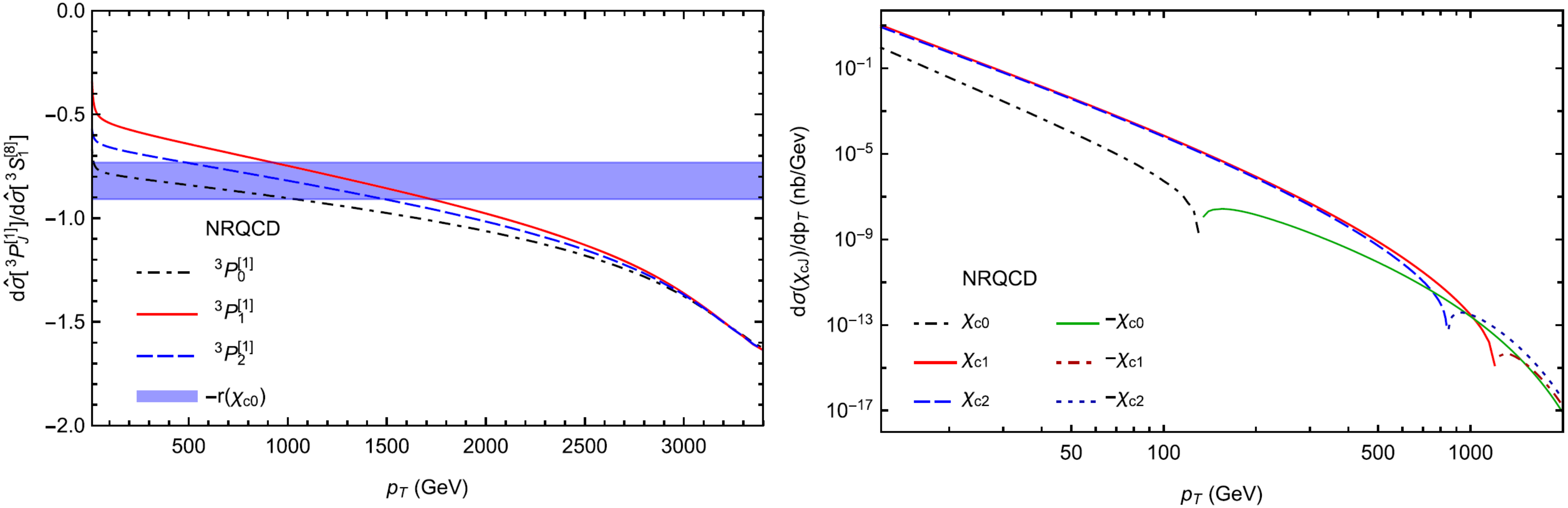
$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[{}^3S_1^{[8]}]\frac{\langle \mathcal{O}^{\chi_{c0}}({}^3P_0^{[1]})\rangle}{m_c^2}\left[r(\chi_{c0}) + \frac{d\hat{\sigma}[{}^3P_J^{[1]}]}{d\hat{\sigma}[{}^3S_1^{[8]}]}\right].$$

- **To achieve a positive cross section, it is necessary to have**

$$\frac{d\hat{\sigma}[{}^3P_J^{[1]}]}{d\hat{\sigma}[{}^3S_1^{[8]}]} > -r(\chi_{c0}).$$

- **Left: comparison between the ratios and $-r(\chi_{c0})$**

  Right: the p_T distributions when the LDMEs take the central values



- **The ratios fall below the lower bound of $-r(\chi_{c0})$ at very large p_T**

SGF

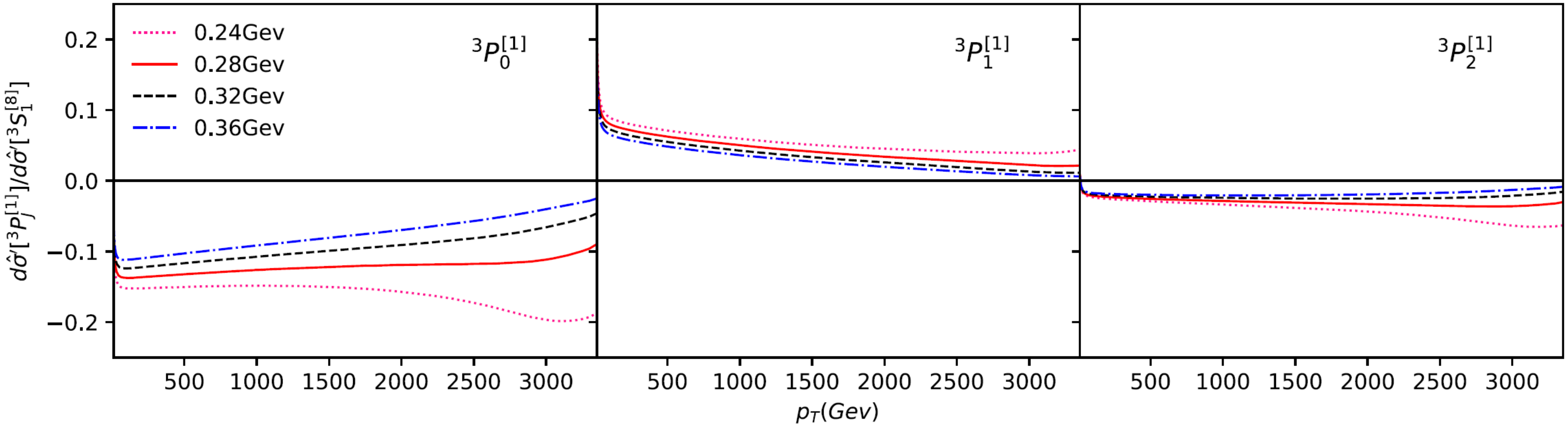
- **The cross sections**

$$d\sigma(\chi_{cJ}) =(2J+1)d\hat{\sigma}'[^3S_1^{[8]}]\frac{N^{\chi_{c0}}[^3P_0^{[1]}]}{m_c^2}\left[r'(\chi_{c0})+\frac{d\hat{\sigma}'[^3P_J^{[1]}]}{d\hat{\sigma}'[^3S_1^{[8]}]}\right].$$

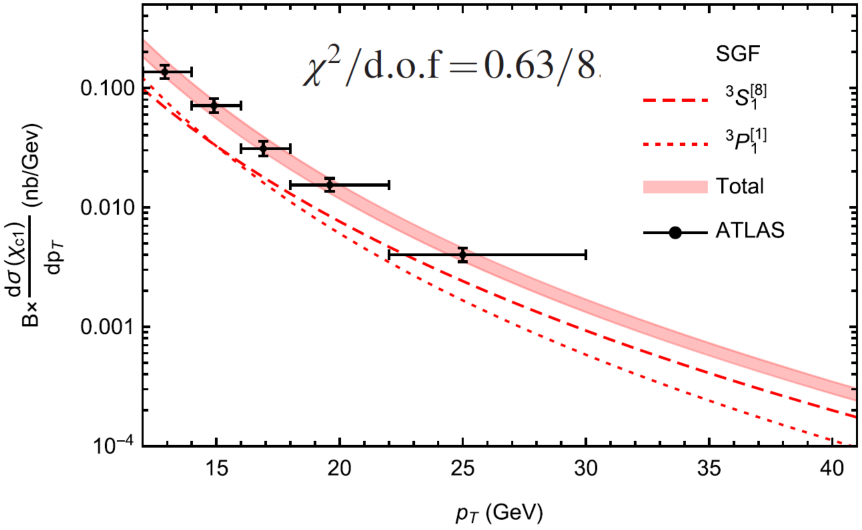
with

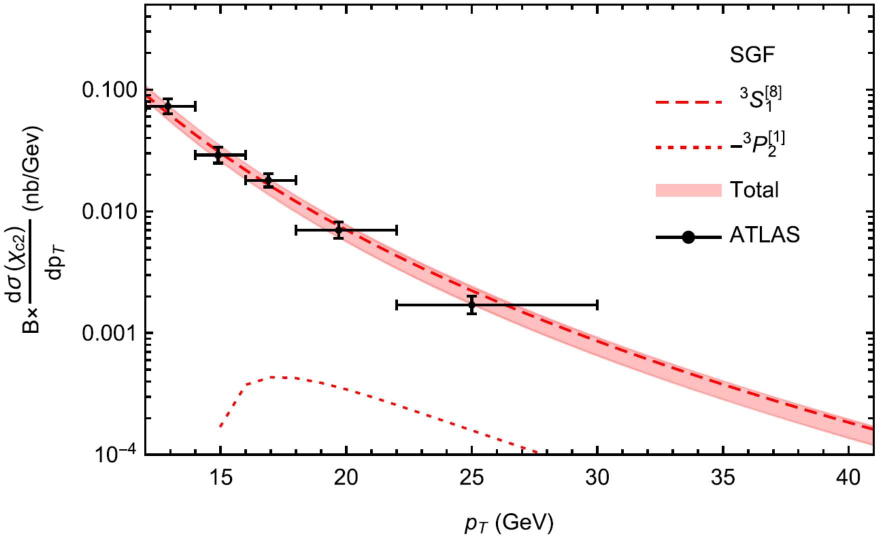
$$r'(\chi_{c0}) \equiv \frac{N^{\chi_{c0}}[^3S_1^{[8]}]}{N^{\chi_{c0}}[^3P_0^{[1]}]/m_c^2}.$$

- $d\hat{\sigma}'[^3P_J^{[1]}]/d\hat{\sigma}'[^3S_1^{[8]}]$ **is sensitive to the parameters** $\bar{\Lambda}$
- **Fix** $\bar{\Lambda}\left[{}^3S_1^{[8]}\right] = 0.4\mathrm{Gev}$ **and vary** $\bar{\Lambda}\left[{}^3\mathrm{P}_J^{[1]}\right] = 0.36, 0.32, 0.28, 0.24\mathrm{Gev}$



- **A constraint relation is suggested:** $\bar{\Lambda}[{}^3P_J^{[1]}] \geq 0.7\bar{\Lambda}[{}^3S_1^{[8]}]$
- **We set** $\bar{\Lambda}\left[{}^3S_1^{[8]}\right] = 0.4\mathrm{Gev}$ **and** $\bar{\Lambda}\left[{}^3\mathrm{P}_J^{[1]}\right] = 0.3\mathrm{Gev}$
- **The fitted cross sections compared with ATLAS data**

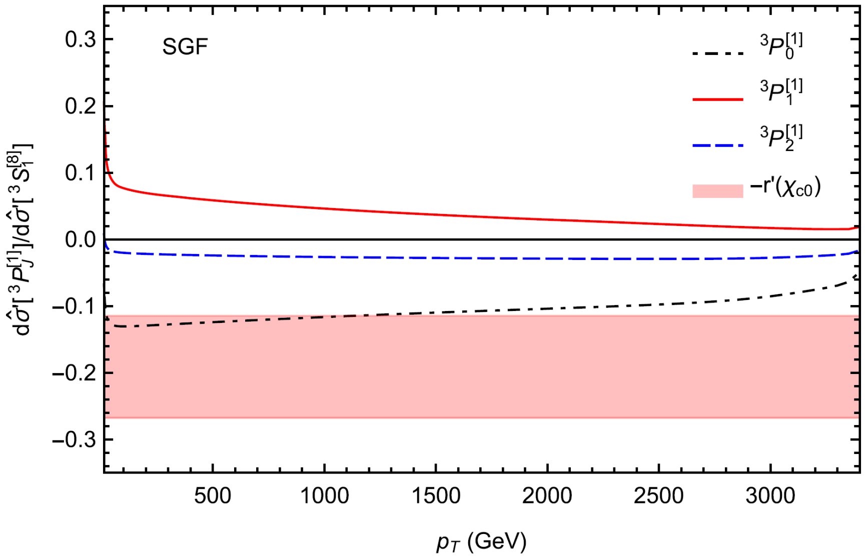


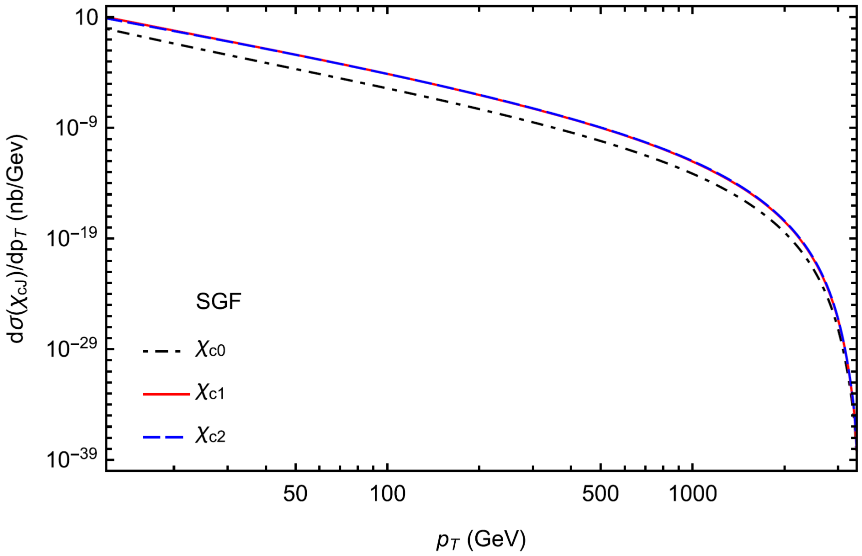


- **The fit to experimental data is as good as that in NRQCD factorization**

- **Left: comparison between the ratios and $-r'(\chi_{c0})$**

  Right: the p_T distributions when the parameters take the central values





- **There is a wide range of $r'(\chi_{c0})$ in which the ratios is larger than $-r'(\chi_{c0})$**
- **The negative cross section problem is resolved in SGF**

Outline

I. Introduction

II. Soft gluon factorization

III. Phenomenological studies

IV. Summary

Summary

- We studied the hadroproduction of χ_{cJ} using the SGF and NRQCD factorization;
- We confirm that the NRQCD predictions for χ_{cJ} production rates at the LHC turn negative at sufficiently large p_T ;
- Our results show that the fit to experimental data in SGF is as good as that in NRQCD factorization;
- Our results show that the negative cross section problem in NRQCD can be resolved in SGF;
- It will be very useful to apply SGF to study the polarizations of ψ(ns) production at LHC in the future.

Thank you!