重味物理前沿论坛研讨会

26 November 2023

RD(*) anomaly: theoretical overview

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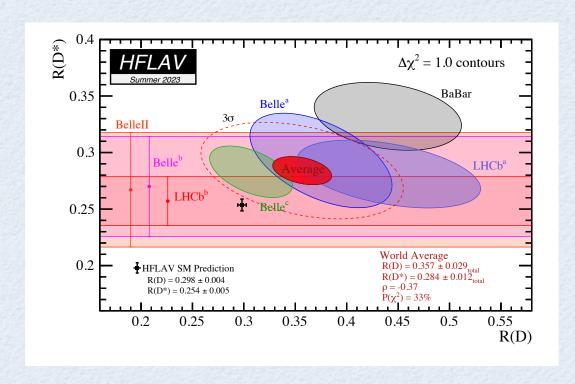
Ryoutaro Watanabe



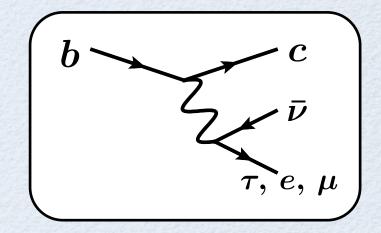
"Anomaly"

has been observed since 2012 in $\,R_{D^{(*)}}=rac{\mathcal{B}(ar{B} o D^{(*)} auar{ u})}{\mathcal{B}(ar{B} o D^{(*)}\ellar{ u})}$

Measurements:



SM process:



Official consensus:

- ~ 3σ deviation from the "official" SM value
- Some NP interpretations are possible, but not conclusive

RD(*): experiments

Experiment	R_{D^*}	R_D	Correlation
BaBar (2012)	$0.332 \pm 0.024 \pm 0.018$	$0.440 \pm 0.058 \pm 0.042$	-0.31
Belle (2015)	$0.293 \pm 0.038 \pm 0.015$	$0.375 \pm 0.064 \pm 0.026$	-0.50
Belle (2016)	$0.270 \pm 0.035^{+0.028}_{-0.025}$		_
Belle (2019)	$0.283 \pm 0.018 \pm 0.014$	$0.307 \pm 0.037 \pm 0.016$	-0.52
LHCb (2015)	$0.336 \pm 0.027 \pm 0.030$	_	_
LHCb (2017)	$0.280 \pm 0.018 \pm 0.029$	_	
Previous average	0.297 ± 0.013	0.338 ± 0.030	-0.39
LHCb (2022)	$0.280 \pm 0.018 \pm 0.024$	$0.441 \pm 0.060 \pm 0.066$	-0.43
New average	0.284 ± 0.013	0.356 ± 0.029	-0.37

Latest status:

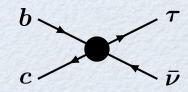
Belle (2019) → no update in 4 years / LHCb run1 (2017) → updated in 2022

Waiting lists:

CMS with "B-parking" / Belle II (first result → in 2023!) / LHCb run2

RD(*): usual interpretations

NP EFT:
$$\mathcal{L}_X = 2\sqrt{2}G_F V_{cb} C_X^{ au} (ar{c} \, \Gamma b) (ar{ au} \, \Gamma'
u)$$



— Solutions to the RD(*) anomaly

$$C_{\mathrm{VLL}}^{\tau} \approx 0.09$$

$$C_{\mathrm{VRL}}^{\tau} \approx 0.42i$$

$$C_T^{\tau} \approx 0.15 + i \, 0.19$$

$$(ar{c}\gamma^{\mu}P_Lb)(ar{\ell}\gamma_{\mu}P_L
u)$$

$$(ar{c}\gamma^{\mu}P_Rb)(ar{\ell}\gamma_{\mu}P_L
u)$$

$$(ar{c}\gamma^{\mu}P_Lb)(ar{\ell}\gamma_{\mu}P_L
u) \qquad (ar{c}\gamma^{\mu}P_Rb)(ar{\ell}\gamma_{\mu}P_L
u) \qquad (ar{c}\sigma^{\mu\nu}P_Lb)(ar{\ell}\sigma_{\mu\nu}P_L
u)$$

$$C_{\mathrm{SLL}}^{ au} pprox -0.82 + 0.78i$$

Right-handed neutrino scenarios are skipped here:

$$(ar{c}P_Lb)(ar{\ell}P_L
u)$$

1802.01732, 1804.04135, 1804.04642, 1807.04753, 1811.04496

— Models of the mediator particle

Vector boson (W'): $C_{\text{VLL}}^{\tau}, C_{\text{VRL}}^{\tau}$

→ SU(2) model inevitably includes Z' that is very constrained due to tree-level FCNC

Charged Higgs: $C_{\rm SLL}^{\tau}$

→ typical models (type-I, II) do not give desired SLL and so type-III is the last hope

RD(*): usual interpretations

Leptoquarks (LQ): S_1 , R_2 , U_1

- $S_1(\bar{3}, 1, 1/3)$ scalar: $C_{VLL}, C_{SLL} = -4C_T \approx 0.13$
 - → VLL & SLL-T type couplings are independent and both has the solution
 - → S1-S3 mixture was discussed for RK

1703.09226

$$R_2(3, 2, 7/6)$$
 scalar: $C_{SLL} = +4C_T \approx 0.40 i$

→ could be related to GUT and neutrino mass generation

1701 08322

$$U_1(3,1,2/3)$$
 vector: C_{VLL} , C_{SLL}

1709.00692, 1808.07492, 1812.01603, 2103.11889

- → VLL and SLL are independent unless UV is discussed
- → Famous Pati-Salam UV induces Z' that has to be managed (model dependent)
- ightharpoonup Another UV from U(2) flavor symmetry gives $\,C_{
 m SLL} = -2\,e^{I\phi}\,C_{
 m VLL}$

Content

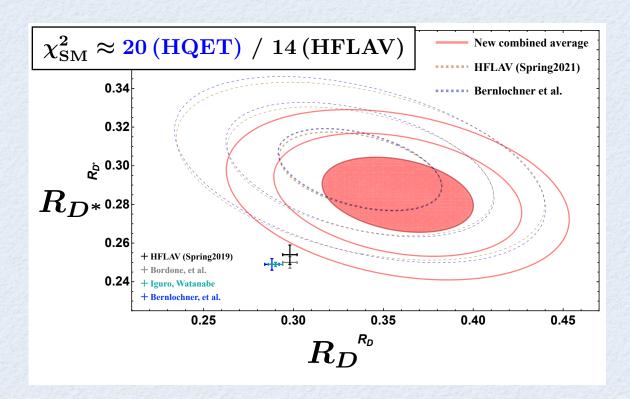
- Status on related observables/measurements to RD(*)
- Impacts on the NP solutions
- SM predictions

— NP in the light lepton modes?

— Flavor signals: B_c , Λ_b , Tau polarizations

- Collider signals: Tau + missing (+b jet)

SM predictions (Form Factors)



BGL parameterization:

+ HFLAV (Spring2019)

HQET parameterization:

- + EPJC80(2020)74 [3/2/1 model]
- + JHEP08(2020)006 [3/2/1 model]
- + Phys, Rev. D106(2022)096015 [CLN base]

Why do we have different SM values?:

- FF shape fit is still unstable → FF model dependences
- Lattice was available only for B → D until last year

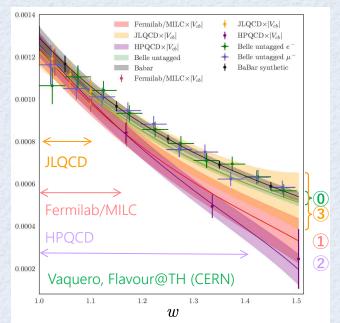
New lattice results for B → D*!!:

— Fermilab-MILC (2022 published) / HPQCD (2023) / JLQCD (2023)

SM predictions (Form Factors)

New lattice results for B → D*!!:

— Fermilab-MILC (2022 published) / HPQCD (2022) / JLQCD (2023)



JLQCD:

good consistency with exp / small recoil bins

HPQCD:

steeper slope / large recoil bins

- **0** extracted from Belle data
- 3 JLQCD
- 1 Fermilab-MILC
- 2 HPQCD

deviations

New issue: how should we combine the lattice results?

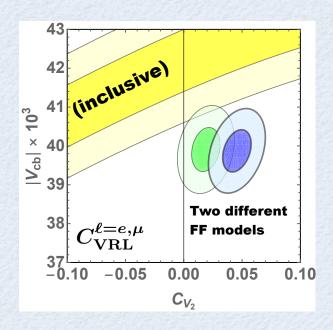
Effect on RD*:

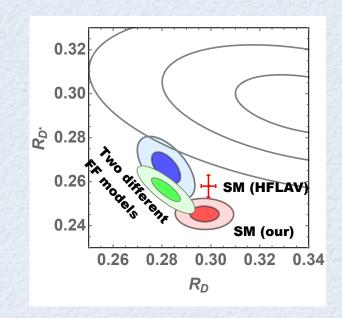
[0.252(22) JLQCD] vs [0.279(13) HPQCD] vs [0.265(13) MILC]

- JLQCD consistent with the present status [0.254(5) HFLAV]
- HPQCD & MILC larger value / reducing the B anomaly [0.285(13) exp]

NP in the light lepton modes?

Simultaneous fit of FF + Vcb + NP in $B o D^{(*)} \mu
u \,, \; D^{(*)} e
u$





2004.10208 (RW)

- assuming LFU type NP in e/ μ : $C_X^e = C_X^\mu$
- taking Belle full angular data (2017,2018) & all available theory
- NP can be hidden behind the Vcb measurement
 - ightharpoonup possible size is < 5% of the "SM size" $\equiv 2\sqrt{2}G_FV_{cb}$
- Impact on RD(*), NP in denominator, is mild
 - → RD* increases while RD decreases in case of VRL type NP

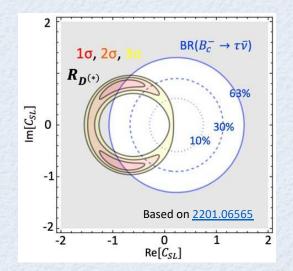
(1) Bc lifetime

excluded the scalar NP solution (SLL):

— Difference in experiment/theory is room for NP contribution hep-ph/9601249, 1611.06676

$$[au_{B_c}^{
m exp}pprox 0.5
m ps] \
m vs. \ [0.4
m ps < au_{B_c}^{
m th} < 0.7
m ps] \ \Rightarrow \ {
m Br}(B_c
ightarrow {
m induced by NP}) < {
m 30\%} \ au
onumber \ au$$

- → killed all the scalar NP solutions to the anomaly
- The present calculation (OPE) is sensitive to charm mass input
 - → 1811.09603 pointed out a conservative bound should be < 6 0%
 - → 2105.02988 provides update concerning charm mass: th. could reach <1.0ps (<50%)
 - → theory calculation is not conclusive, need further update...



2201,06565

- This update significantly affects the SLL scenario
 - → Scalar type solution revived, but on the edge!
 - **→ Type-III charged Higgs is now viable!**
 - → Good news for several LQ scenarios as well

(2) Ab decay

Another R proposal from b-baryon: $R_{\Lambda_c}=\mathcal{B}(\Lambda_b o \Lambda_c\, au\,
u)ig/\mathcal{B}(\Lambda_b o \Lambda_c\,\ell\,
u)$

- light lepton modes were measured by DELPHI/CDF/LHCb since 2004
- the first result for tau together with R was reported by LHCb in this year!

2201.03497

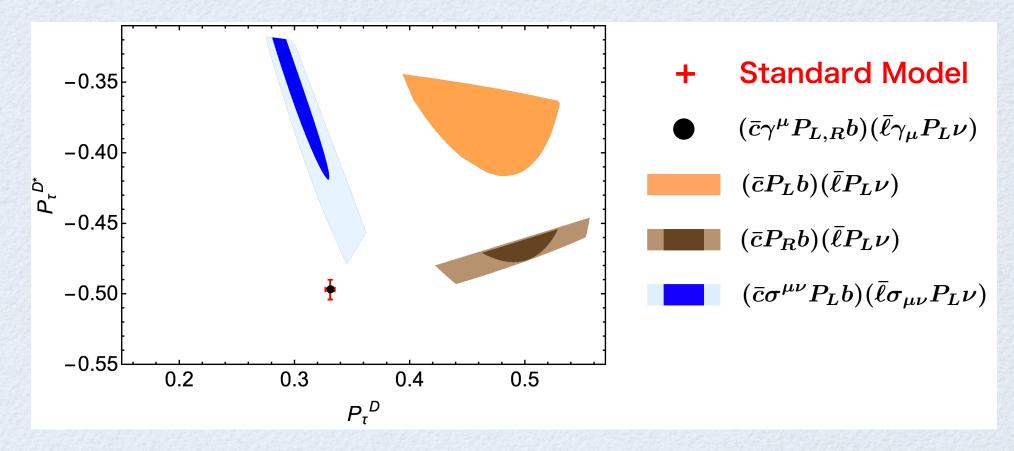
LHCb (2022):
$$0.242 \pm 0.026 \pm 0.04 \pm 0.059$$
 \Leftrightarrow SM (2018): 0.324 ± 0.004

Heavy Quark Symmetry ensures sum rule:
$$\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\rm SM}}=0.28\frac{R_D}{R_D^{\rm SM}}+0.72\frac{R_{D^*}}{R_{D^*}^{\rm SM}}+\delta$$

1811.09603, 1905.08253

- forall can be negligible under any NP existence as long as $|C_T| \ll 1$
 - ightharpoonup Recall the T solution: $|C_T|pprox |0.15+i\,0.19|=0.24 \;\; \Rightarrow \delta=-0.03$
- measured RD(*) provides model-independent fit: $R_{\Lambda_c}^{
 m fit}=0.380\pm0.013\pm0.005$
 - **→** is another index to test the anomaly
 - → IOW, this R cannot identify NP but is a unique value for every NP solution and for SM
 - → the measured RAc is not consistent with the RD(*) anomaly: potentially another problem

(3) Tau spin polarization



- NP solutions for RD(*) anomaly predict distinct signals
 - → could identify T/SLL/SRL solution (blue/yellow/cyan)
 - **→** Current experimental measurement

Belle (2017) 1709.00129

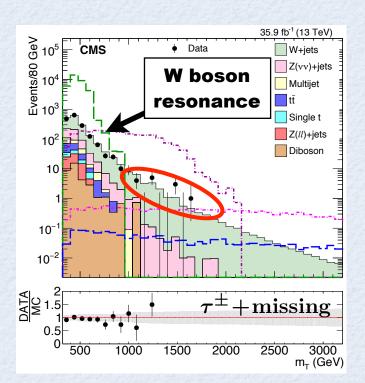
D* mode: $P_{ au, {
m exp}}^{D^*} = -0.38 \pm 0.51_{-0.16}^{+0.21}$

D mode: not measured yet

Collider signals

W boson resonance:

- has been observed with missing transverse mass
- its tail can be interpreted as NP contribution responsible for the RD(*) anomaly
- minimal NP process is $bc \rightarrow tv$
 - → W' is severely constrained: < 2TeV excluded (bc PDF suppressed) / < 5TeV (SSM)
 - **⇒ EFT** based analysis is also available and gives very crucial bound



1811,07920

— competitive with the NP solutions that require large WCs:

$$\begin{split} |C_{\rm VLL}^{\rm LHC\text{-}EFT}| &< 0.32 & \Leftrightarrow & C_{\rm VLL}^{R_{D}(*)} \approx 0.09 \\ |C_{\rm VRL}^{\rm LHC\text{-}EFT}| &< 0.33 & \Leftrightarrow & C_{\rm VRL}^{R_{D}(*)} \approx 0.42 \, i \\ |C_{T}^{\rm LHC\text{-}EFT}| &< 0.20 & \Leftrightarrow & |C_{T}^{R_{D}(*)}| \approx |0.15 + i \, 0.19| = 0.24 \\ |C_{\rm SLL}^{\rm LHC\text{-}EFT}| &< 0.32 & \Leftrightarrow & |C_{\rm SLL}^{R_{D}(*)}| \approx |-0.82 + i \, 0.78| = 1.13 \end{split}$$

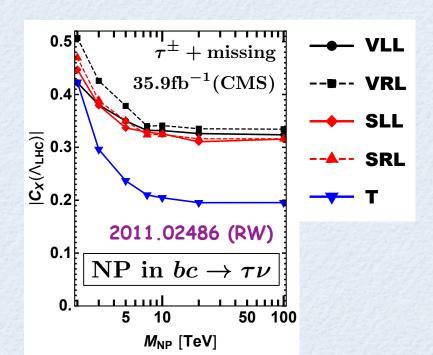
- Charged Higgs is very excluded, but has an exception
 - → tail pT < 500GeV is less sensitive to NP signal
 - ⇒ mass window 180GeV < mH < 400GeV is not accessible

Collider signals

t-channel case:

- EFT approximation is not good at high-mT
 - ⇒ if NP mass is close to mT bin ~ 1TeV applicable for bound
 - → In particular, it overestimates the signal in the case of t-channel
 - → Large t(<0) generates large mT and reduces the contribution

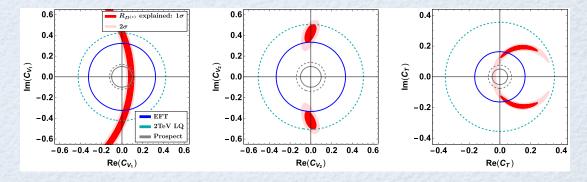
$$-\text{ex}) \,\, \mathcal{L}_U = h_U^{ij} \left(\bar{q}_L^i \gamma^\mu \ell_L^j \right) U_\mu + \text{h.c.} \quad \Rightarrow \quad \frac{h_U^{b\tau} \cdot h_U^{c\nu}}{t - m_{\mathrm{LQ}}^2} \neq - \, \frac{h_U^{b\tau} \cdot h_U^{c\nu}}{m_{\mathrm{LQ}}^2} \equiv C_{\mathrm{VLL}}$$



Proper bound for t-channel NP:

- **→ 2TeV** LQ: EFT bound is 40~100% overestimated
- → 5TeV LQ: 10~20% overestimated
- → T solution is still viable in the case of LQ type

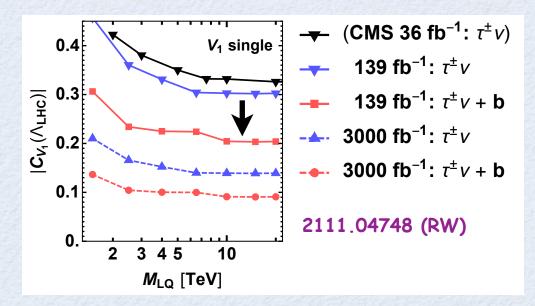
$$|C_T^{ ext{LHC-LQ}}| < extbf{0.42} \; \Leftrightarrow \; |C_T^{R_{D^{(*)}}}| pprox |0.15 + i\,0.19| = 0.24$$

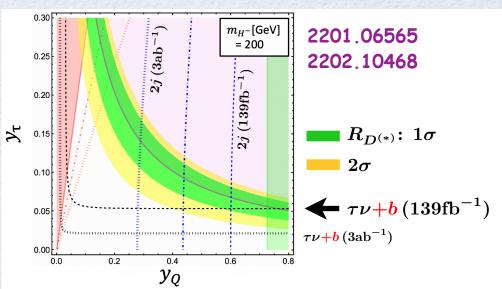


Collider signals

proposal of improvement:

- Requiring additional b-jet greatly reduces the SM background 2008.07541
 - ightharpoonup comes from $gq
 ightharpoonup b \, \ell \,
 u$ (q=u,c) suppressed by $|Vqb|^2$ in the SM
 - ⇒ simulation shows +b search could improve the LHC bound by ~50%
 - ightharpoonup 3ab^-1 LHC could reach the VLL solution: $|C_{\mathrm{VLL}}^{\mathrm{3ab}^{-1}} + b| \lesssim 0.1$
- τν+b search can also access mH < 400GeV (out of range for τν search)</p>
 - ⇒ suppressing trigger rate could reach up to 180GeV
 - ⇒ simulation shows 139fb^-1 data is sufficient to test the SLL solution for RD(*)





Summary

- SM predictions
 - New lattice form factor calculations bring impacts on the SM values
- NP in the light lepton modes?
 - NP hidden in the Vcb measurement is possible (< 5%), but impact on RD(*) is limited
- Flavor signals: B_c , Λ_b , Tau polarizations
 - Bc → TV has great potential to pin down the RD(*) solution CEPC has a crucial role!
 - RAc has model-independent sum rule with RD(*), and gives another index for the anomaly
- Collider signals: Tau + missing (+ b jet)
 - High-pT (>500GeV) tail is sensitive to NP responsible for RD(*), and already competitive
 - EFT bounds already excluded some RD(*) solutions, while t-channel bounds more milder
 - Additional b-jet tag will improve the collider bound and reach 10% precision

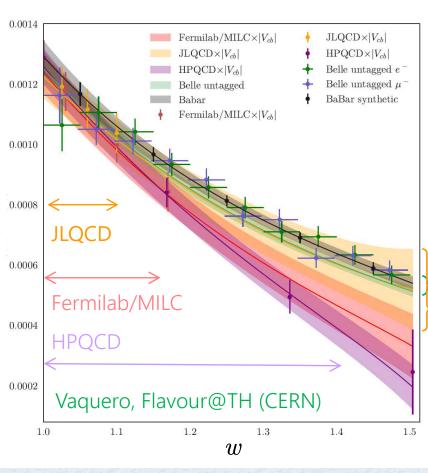
Backup

Lattice results competition

tension on $B \rightarrow D^*\ell v$

Vaquero [WG1+2] Tue 18:03 Colguhoun [WG2+3] Thu 18:50

$$\left| \eta_{EW} V_{cb} \, \mathcal{F}
ight|^{\, 2} \quad \mathcal{F}^{\, 2} \! \propto \! \left[2 rac{1 - 2wr + r^{\, 2}}{\left(1 - r
ight)^{\, 2}} \left\{ \! 1 + rac{w - 1}{w + 1} rac{R_{\! 1}^{\, 2}}{w + 1}
ight\} + \left\{ \! 1 + rac{w - 1}{1 - r} \left(1 - rac{R_{\! 2}}{1}
ight) \!
ight\}^{\, 2}
ight] \! h_{A_{\! 1}}^{\, 2} \; R_{\! 1} \! = \! rac{h_{V}}{h_{A_{\! 1}}}, \; \; R_{\! 2} \! = \! rac{h_{A_{\! 3}} + rh_{A_{\! 2}}}{h_{A_{\! 1}}} \; \left[\! \frac{1 - 2wr + r^{\, 2}}{w + 1} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! \right] \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! \right] \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! \right] \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! \right] \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! \right] \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! \right) \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{R_{\! 2}}{w + 1}
ight) \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{h_{A_{\! 1}}} \left(1 - rac{W_{\! 1}}{w + 1}
ight) \! r_{\! 1}^{\, 2} + \left[\! \frac{1 - 2wr + r^{\, 2}}{w + 1} \left(1 - rac{W_{\! 1}}{w + 1}
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- (0) Belle and BaBar data
- 1) Fermilab/MILC : steeper slope?
 - + χ^2 /dof ~ 1.5 to fit w/ exp data
- 2 HPQCD : even steeper slope!
 - + siginificant tension with exp $(\ell=e, \mu)$ at medium/large w
 - $+ |V_{cb}| = 44.2(1.8) \times 10^{-3}$ from total Γ
- JLQCD: good consistency w/ exp
- tension on R_2 (?) [Belle 2301.07529, Jung Flavour@TH]
- \bigcirc small recoils [JLQCD, Fermilab/MILC] \Leftrightarrow larger ap [HPQCD]
- \Rightarrow "safe" extension to large w: JLQCD; Fermilab/MILC $a^{-1} \sim 6.6$ GeV

(2) Bc decay

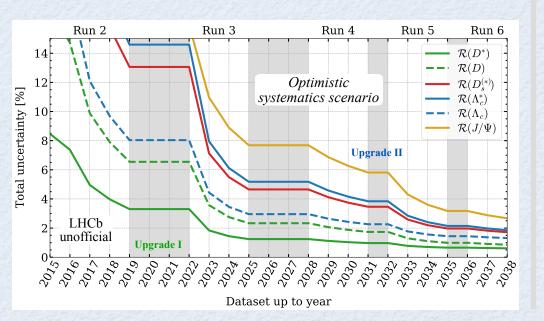
The "R" observable for Bc: $R_{J/\psi}={\cal B}(B_c o J/\!\psi\, au\,
u)\Big/{\cal B}(B_c o J/\!\psi\,\mu\,
u)$

 \Leftrightarrow

1711.05623

LHCb (2017):
$$0.71 \pm 0.17 \pm 0.18$$

- Update is planned in the LHCb roadmap
 - → error could go into 8% in 5 years
- Sufficiently crucial for the RD(*) anomaly
 - → NP prediction on RJψ can be tested



SM (2017): 0.28 ± 0.05 1709.08644 SM (2019): 0.24 ± 0.01 1901.08368

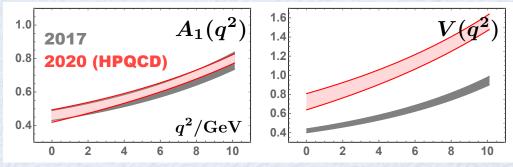
— FF updated:

2007.06957

2204.04357

- → QCD (2017)/ SR (2019) / lattice (2020)
- → deviations affected the SM value

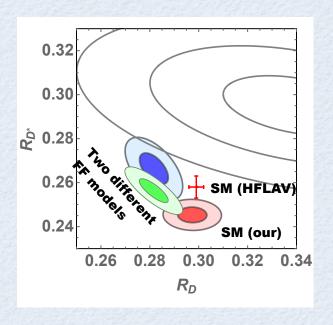
SM (2022): 0.258 ± 0.004



- NP prediction from the RD(*) solution:
 - ⇒ ex) VLL solution predicts 0.28-0.29
 - → Summary given later

NP in the light lepton modes?

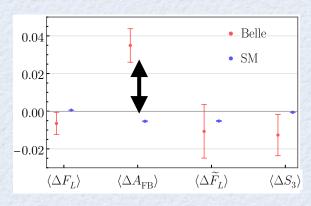
(1) Simultaneous fit of FF + Vcb + NP in $B o D^{(*)} \mu u \,, \; D^{(*)} e u$



2004.10208 (RW)

- assuming LFU type NP in e/ μ $C_X^e = C_X^\mu$
- taking Belle full angular data (2017,2018) & all available theory
 - → processes usually used to measure Vcb
- NP can be hidden behind the Vcb measurement
 - ightharpoonup possible size is < 5% of the "SM size" $\equiv 2\sqrt{2}G_FV_{cb}$
- Impact on RD(*), NP in denominator, is mild
 - → RD* increases while RD decreases in case of VRL type NP

(2) New anomaly in angular obs. $\Delta A_{ m FB} = A_{ m FB}(D^*\mu u) - A_{ m FB}(D^*e u)$

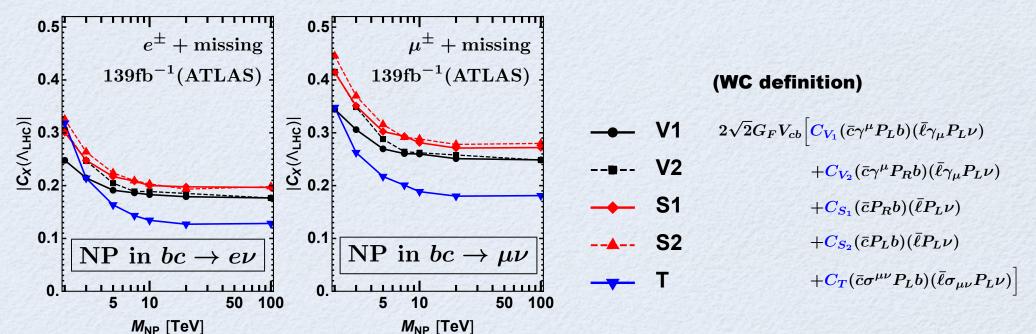


2104.02094, 2203.07189

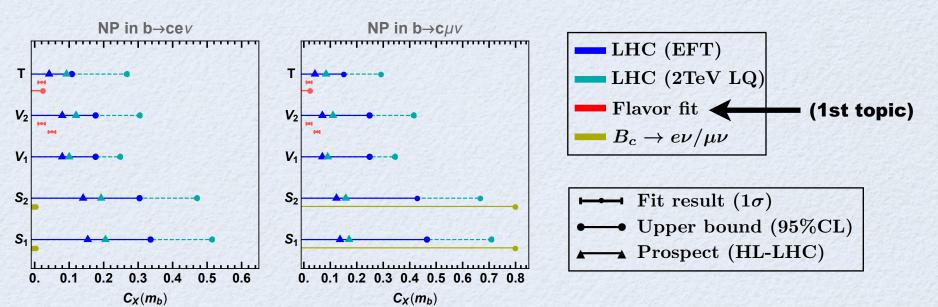
- using Belle 2018 data, angular asymmetries can be constructed
- "anomaly" was observed in the FB asymmetry between e/µ
 - → Single NP operators difficult / Tuned NP couplings needed
 - ⇒ Impact on RD(*) is very limited since Br(e/µ) = 1 ± 0.01

Mediator (LQ) mass dependence:

Result 1/2

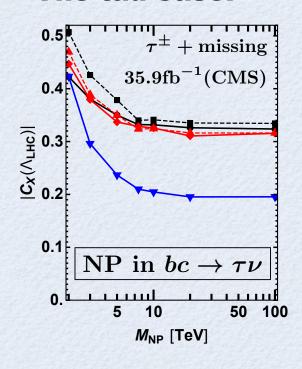


Impact on Flavor (Vcb+NP fit):



The tau case:

Result 2/2

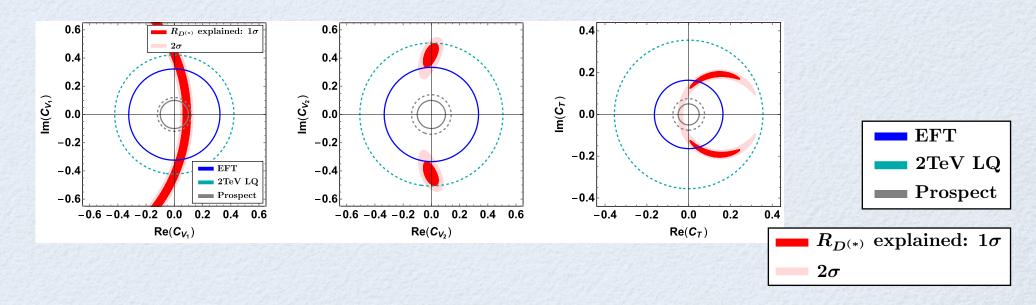


(Summary)

2TeV LQ: EFT bound is 40~100% overestimated

5TeV LQ: 10~20% overestimated

Impact on Flavor (RD(*) anomaly):



+ b-jet tag

Requiring additional b-jet greatly reduces the SM background

$$\left|\ell^{\pm}\nu+b\right|_{\mathrm{SM}} \;\;\Rightarrow\;\; gq o b\ell \nu \;\; (q=u,c) \;\;\Rightarrow\;\; \left|V_{ub,cb}\right|^{2} \; \mathrm{suppression}$$

Improvement ①: stronger bound is simply expected

— can look into detail of the U1-LQ model = SM-like vector operator

$$\mathcal{L}_U = m{h}_U^{ij} \left(ar{q}_L^i \gamma^\mu \ell_L^j
ight) U_\mu + ext{h.c.} \qquad C_{V_1} \equiv -rac{h_U^{b au} \cdot h_U^{c
u}}{m_{ ext{LQ}}^2}, \; \; ext{but indeed} \; m{h}_U^{c
u} = m{h}_U^{s\ell}$$

$$\ell^{\pm}
uigg|_{U_1 ext{-LQ}} \;\; \Rightarrow \;\; cb, {\color{red}cs} \to \ell
u \;\; \Rightarrow \;\; ext{The C_{V_1} bound is valid only if $h^{b au}_U\gg h^{c
u}_U$ for $U_1 ext{-LQ}$}$$

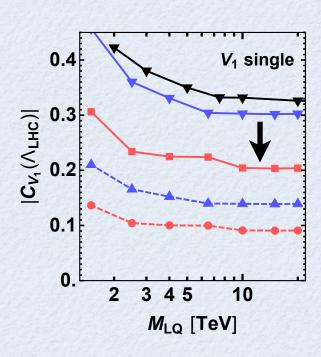
$$\ell^{\pm}\nu + b\Big|_{U_1\text{-LQ}} \Rightarrow cg \to b\ell\nu \Rightarrow \text{no } s \text{ quark, (but could be mis-tagged)}$$

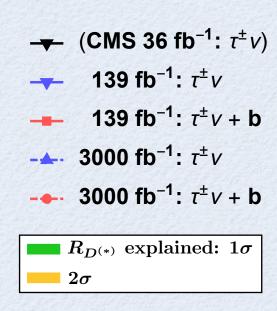
Improvement 2: complementary bound on the two couplings

+ b-jet tag

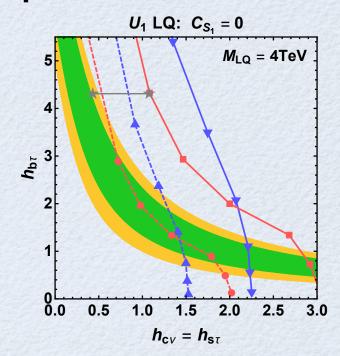
(BG/Signal events generated & simulated: details skipped)

Improvement 1:





Improvement 2:



Observations:

- +b search improves the bound by ~50%
- +b search at HL_LHC can achieve Cx~0.1, i.e. 10% NP effect
- Given the LQ mass, the two couplings (not combination) are constrained

FF parameterization

CLN

Caprini, Lellouch, Neubert (1997)

- "Traditional" parameterization based on HQET
- Form Factors are approximated and related with each other

Cons: parameterization is valid only up to $1/m_Q$ correction

Comparison: inclusive decay has no $(1/m_Q)^1$ but starts from $(1/m_Q)^2$

BGL

Boyd, Grinstein, Lebed (1997)

- "General" parameterization with minimum requirement
- Each Form Factor involves independent parameters

Cons: FFs in New Physics involve new unknown parameters

FF parameterization

- √ "general HQET" Jung, Straub (2018), Bordone, Jung, Dyk (2019)
 - general HQET based parameterization
 - includes higher order corrections at the cost of larger parameter set
- NNLO could be competitive to NLO because $(\Lambda/m_c)^2 \sim (\Lambda/m_b)^1$
- Including NNLO is also a fair comparison with inclusive mode Pros:

√ Modeling

HQET property: one LO / three NLO / six NNLO Isgur-Wise functions

Parameterization: ex) $\xi(w) \equiv \sum_{n=0}^{N_{
m LO}} a_{m{\xi}}^{(n)} z^n$ Truncation order: arbitrary

Two proposed modelings for the truncation orders: * CLN is naively (3/0/-)

$$(N_{
m LO}/N_{
m NLO}/N_{
m NNLO}) = egin{cases} (3/2/1) &
ightarrow {f 23} \ parameters! \ (2/1/0) &
ightarrow {f 13} \ parameters! \end{cases}$$

Tau Polarization in LQ

