重味物理前沿研讨会

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Some aspects of hadronic τ decays in resonance chiral theory

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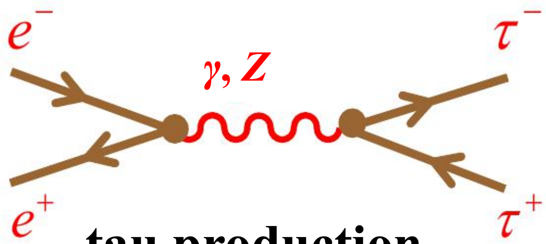
Mini-overview of hadronic tau decays and Resonance Chiral Theory



tau production

γ exchange dominates at tau-charm factory.

Z exchange dominates at CEPC.

Number of taus produced at e^+e^- colliders:

ALEPH: ~3×10^5

BaBar /Belle: ~1×10^9

Belle-II: ~5×10^{10}

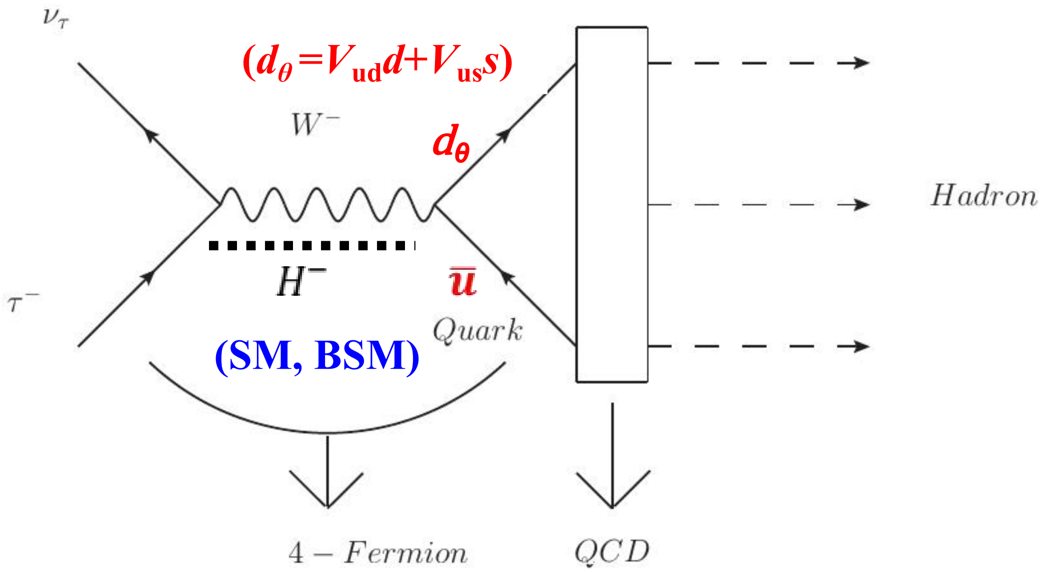
CEPC (Tera-*Z* factory): ~ 3×10^{10}

STCF: ~4×10^{10} (around 10% at threshold)

Tau provides broad interests for particle physics:

- ✓ **Precision tests for electroweak sector: V_{CKM}, lepton universality, g-2, ...**
- ✓ **Stong interactions: α_s, hadron resonances, chiral symmetry, ...**
- ✓ **Possible discoveries for new physics: cLFV, CPV, ...**

Sketch for hadronic tau decays (similar for leptonic decays by dropping QCD part)



Theoretical tools: SM EFT + Chiral EFT

- **SM EFT → LEFT** [Cirigliano et al, '10] [Y.Liao et al., '21] [F.Z.Chen et al, '22] ... .

$$\mathcal{L}_{\rm eff}=-\frac{G_\mu V_{uD}}{\sqrt{2}}\Big[\Big(1+\epsilon_L^{D\ell}\Big)\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1-\gamma_5)D+\epsilon_R^{D\ell}\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1+\gamma_5)D$$
$$+\bar{\ell}(1-\gamma_5)\nu_\ell\cdot\bar{u}\Big[\epsilon_S^{D\ell}-\epsilon_P^{D\ell}\gamma_5\Big]D+\frac{1}{4}\hat{\epsilon}_T^{D\ell}\bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)D\Big]+\text{h.c.},$$

ε_X parameterize various new physics at high energy scale

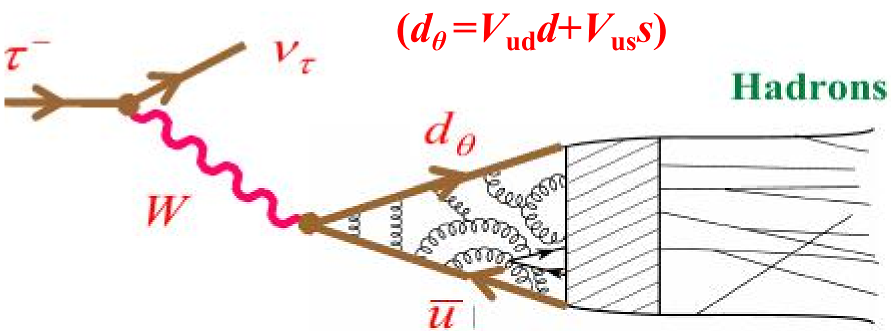
- **Chiral EFT** [Gasser,Leutwyler,'83 '84]

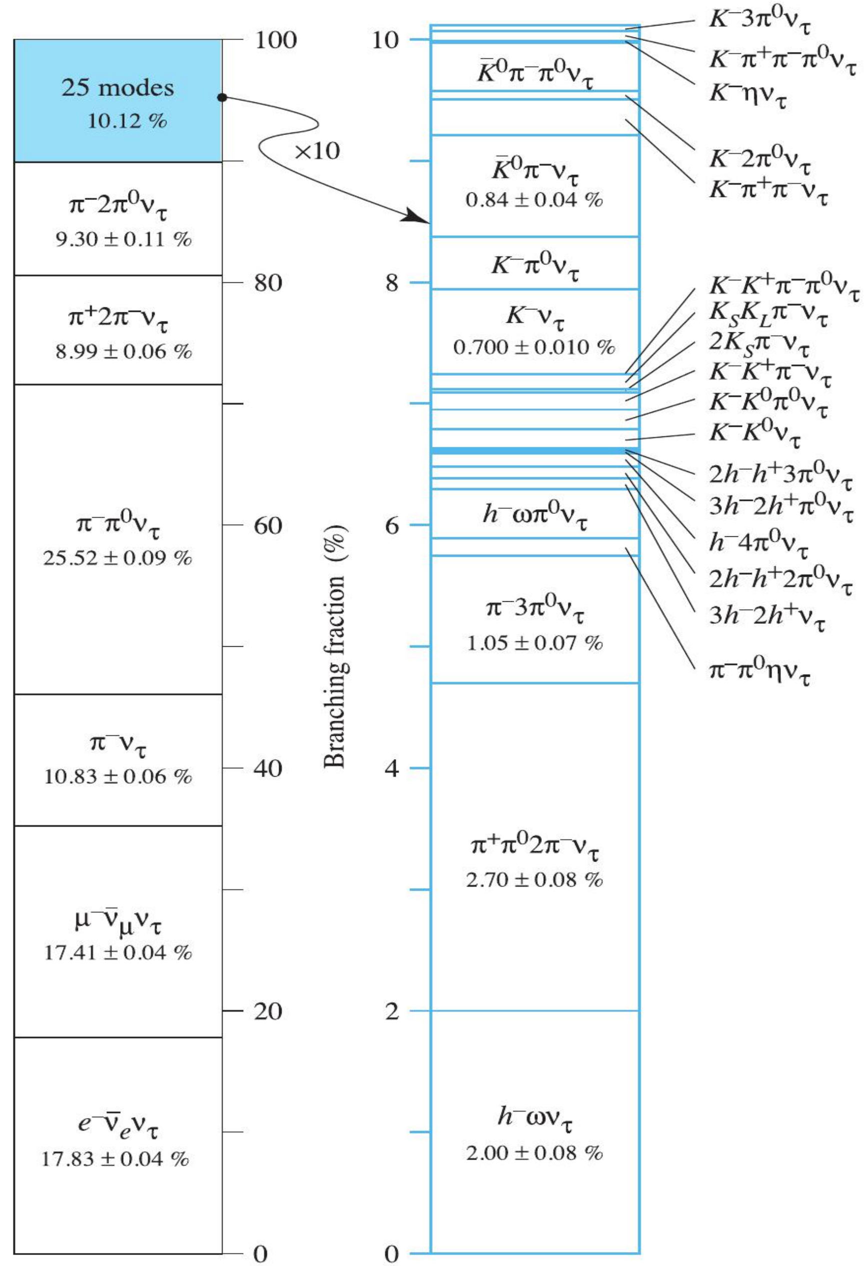
$$\mathcal{L}_2=\frac{F^2}{4}\langle u_\mu u^\mu+\chi_+\rangle$$

$\mathcal{O}(p^4)$:

$$\mathcal{L}_4^{\chi PT} = L_1\langle u_\mu u^\mu\rangle^2 + L_2\langle u_\mu u^\nu\rangle\langle u^\mu u_\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle + L_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle$$
$$+L_5\langle u_\mu u^\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 + \frac{L_8}{2}\langle\chi_+^2+\chi_-^2\rangle+\cdots$$

Hadronic decays: a unique feature for tau lepton



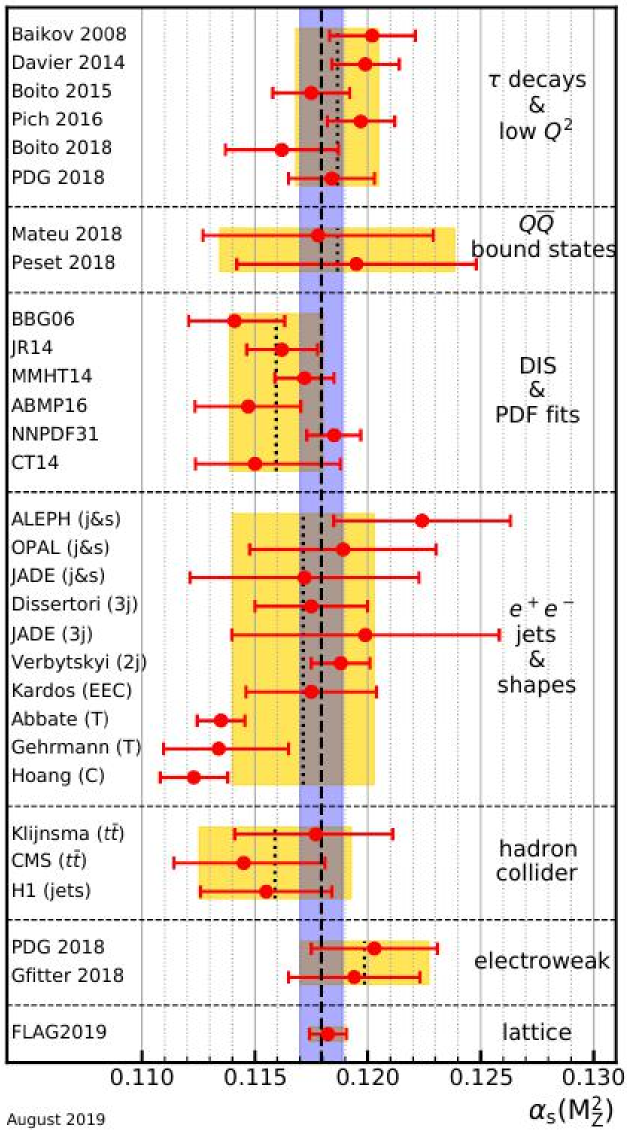


Valuable laboratory to study:

fundamental parameters &

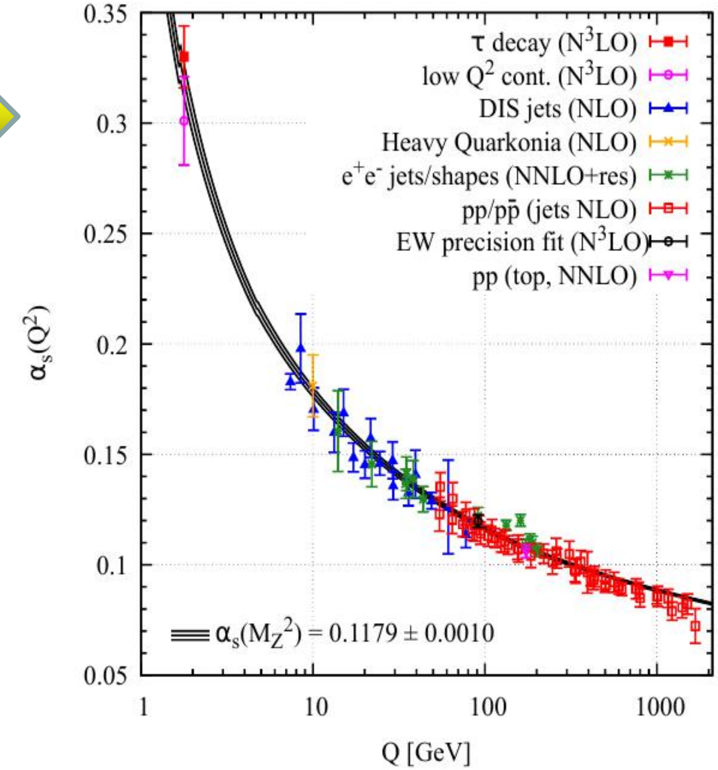
rich hadron phenomenologies

Strong coupling of QCD: α_s



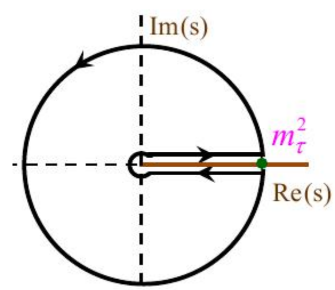
PDG

Hadronic tau decay: an invaluable source to test the QCD prediction of $\alpha_s(Q^2)$ below 2 GeV.



$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \,\text{hadrons})}{\Gamma(\tau \to \nu_\tau e \nu_e)} = \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \omega_J(s)\, \mathrm{Im}\Pi^J(s)$$

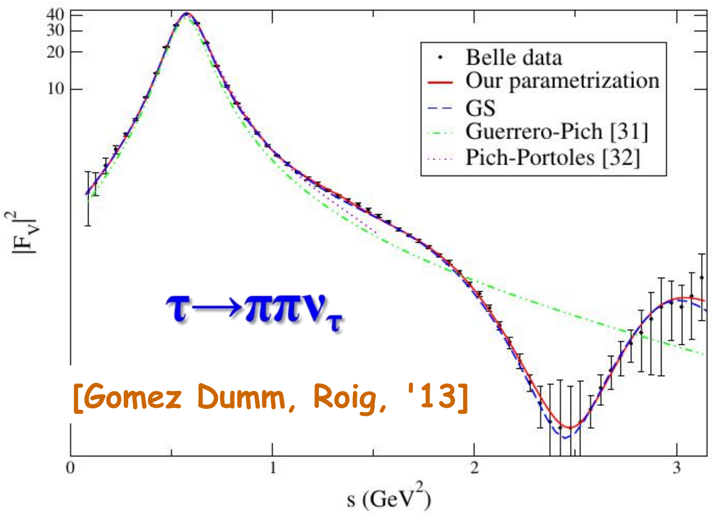
Exp

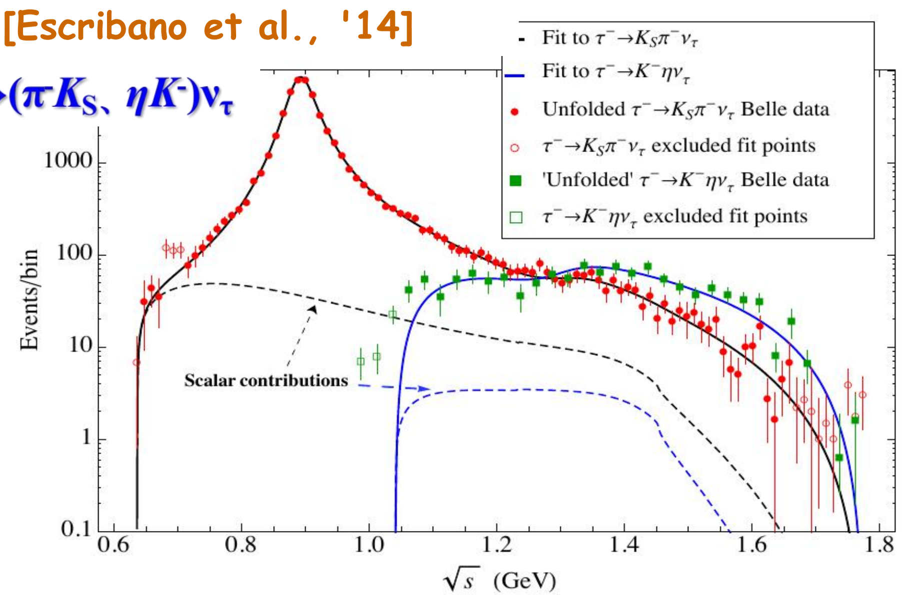


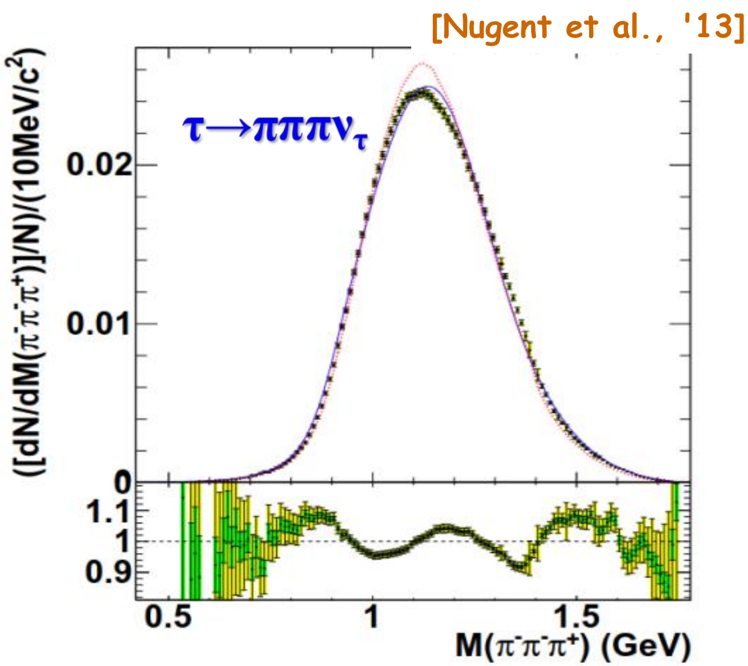
$$R_\tau = \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \omega_J(s)\, \Pi^J(s)$$

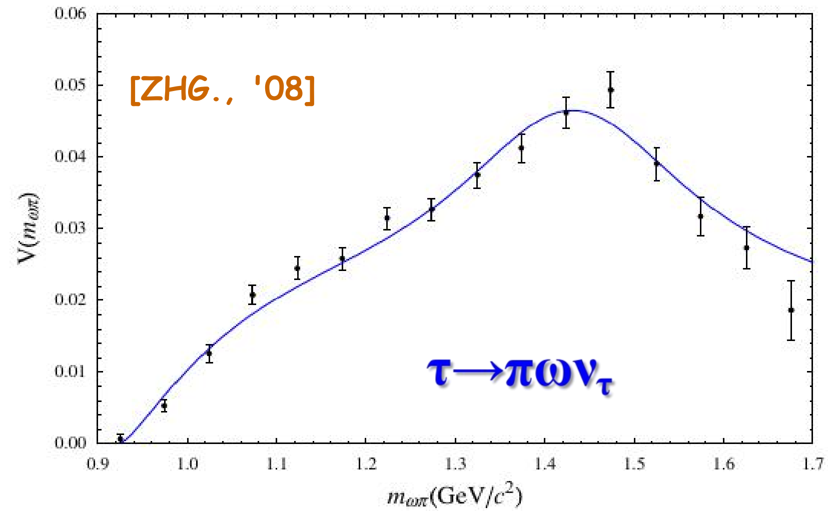
$$\Pi^J(s) \overset{\text{OPE}}{=\!=} \sum_D \frac{C_D^J(s, \alpha_s(\mu), \mu)\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

Invariant-mass spectra for exclusive decays



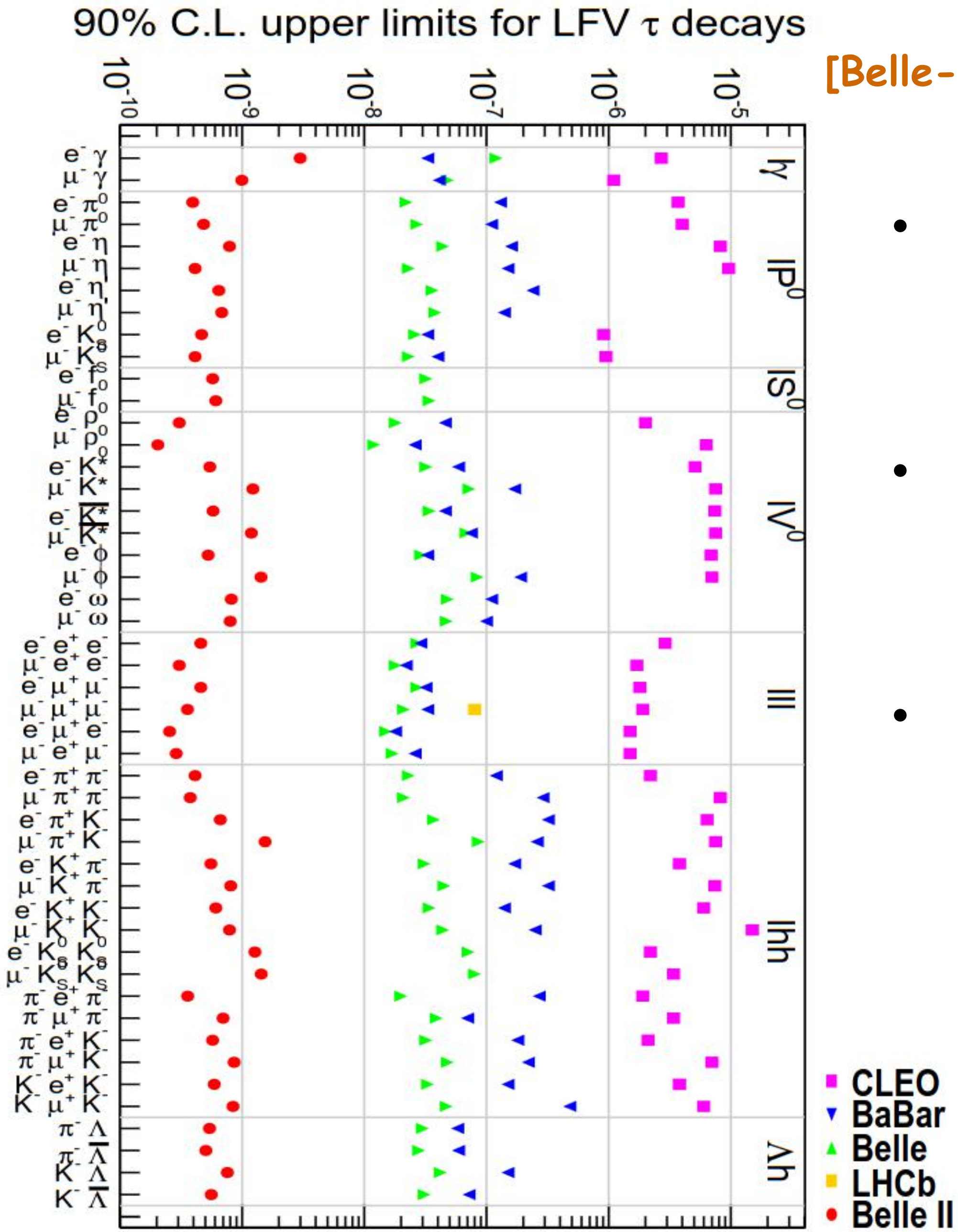






Hadron properties, Form factors, Chiral dynamics, … …

➢ Charged lepton flavor violation in tau decays



[Belle-II, '22]

- **Not only statistic but also systematic uncertainties are important in $\tau \to l\,\gamma$**

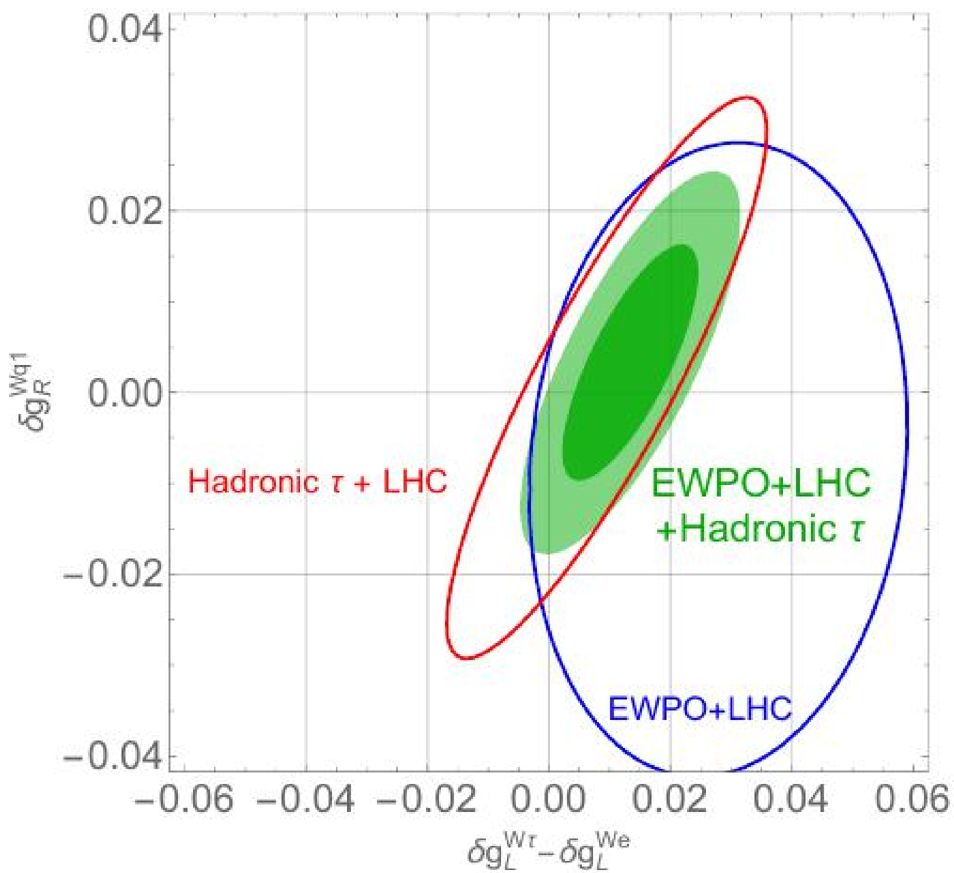
- **Clean backgroud makes $\tau \to l\,l'\,l''$ one of the best channels to search for LFV signals.**

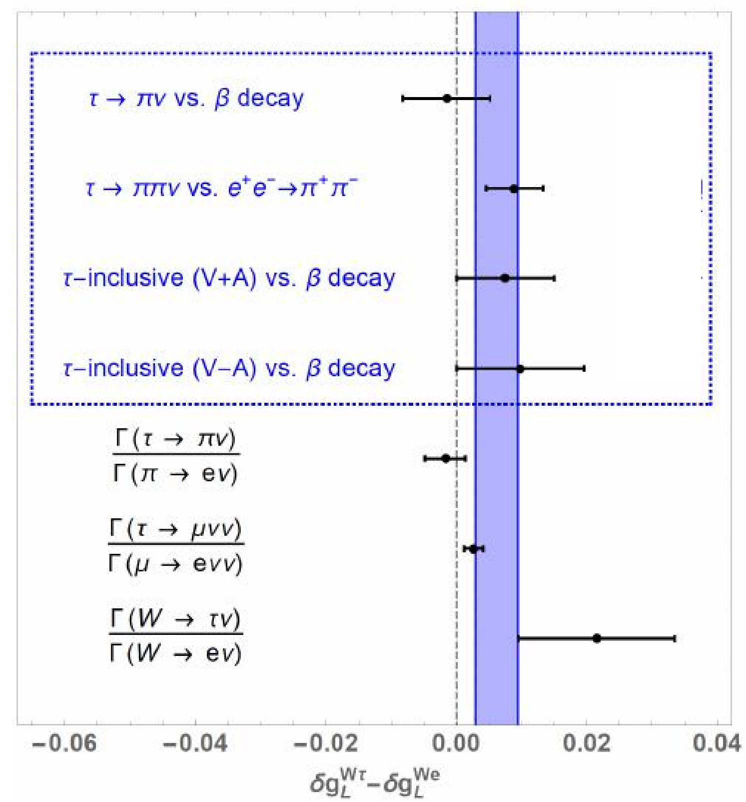
- **$\tau \to l + hadrons$ provides a different laboratory to probe different LFV origins, comparing with the pure leptonic processes.**

➢ **Powerful tool to constrain new physics: combination of hadronic tau data + LHC data**

$$\mathcal{L}_{\rm eff} = -\frac{G_\mu V_{uD}}{\sqrt{2}}\Bigg[\left(1+\epsilon_L^{D\ell}\right)\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1-\gamma_5)D + \epsilon_R^{D\ell}\;\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1+\gamma_5)D$$
$$+\;\bar{\ell}(1-\gamma_5)\nu_\ell\cdot\bar{u}\left[\epsilon_S^{D\ell}-\epsilon_P^{D\ell}\gamma_5\right]D + \frac{1}{4}\hat{\epsilon}_T^{D\ell}\;\bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)D\Bigg] + \text{h.c.},$$

[Cirigliano et al., PRL'18]





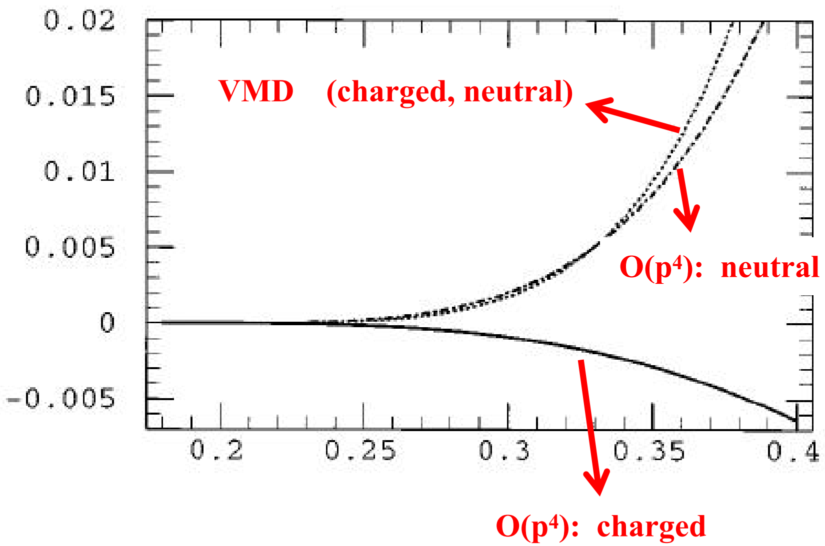
Chiral symmetry is RELEVANT to tau decays

Example: $\tau \rightarrow \nu_\tau \pi\pi\pi$ transition amplitudes in the low energy region
VMD models do not automatically respect chiral symmetry.

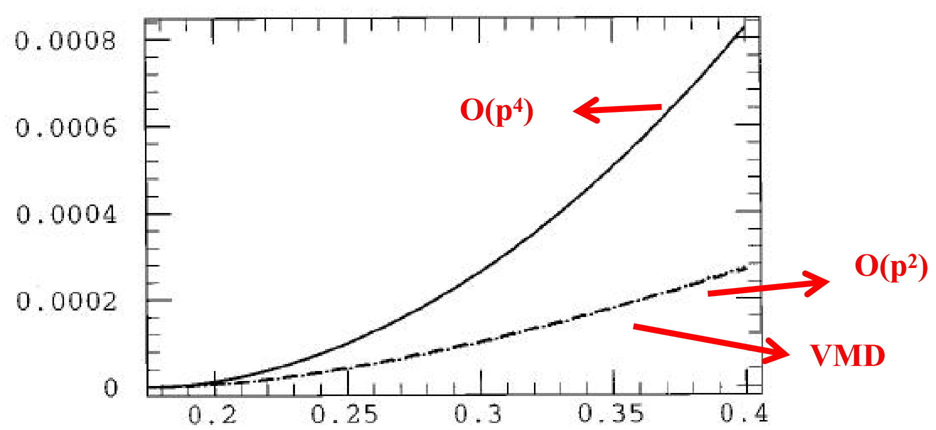
$$J_\alpha = -i\frac{2\sqrt{2}}{3f_\pi}\mathrm{BW}_a(Q^2)(B_\rho(s_2)V_{1\alpha} + B_\rho(s_1)V_{2\alpha})$$

[Kuhn, Santamaria, ZPC'90]

W_D structure function



W_{SA} structure function (neutral channel)



[Colangelo, et al., PRD'96]

- **Resonance chiral theory implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.**

Resonance chiral theory (RχT)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

$$\text{Resonances :} \qquad R \stackrel{G}{\Longrightarrow} h\, R\, h^\dagger, \qquad h \in H$$

$$\text{pNGB and external sources :} \qquad X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}$$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$\begin{aligned}
\mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle\,, \\
\mathcal{L}_{2A} &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle\,, \\
\mathcal{L}_{2S} &= c_d\langle\, S u_\mu u^\mu \,\rangle + c_m\langle\, S\chi_+ \,\rangle\,, \\
\mathcal{L}_{2P} &= i d_m\langle\, P\chi_- \,\rangle.
\end{aligned}$$

Operators beyond minimal
[Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA}\langle[V^{\mu\nu}, A_{\mu\nu}]\chi_-\rangle + \dots\,,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$\begin{aligned}
\mathcal{L}_{VVP} &= d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \dots\,, \\
\mathcal{L}_{VJP} &= \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \dots\,,
\end{aligned}$$

QCD dynamics in RχT

- Low energy QCD: implemented from the construction of RχT

- Intermediate energy: explicit resonance states

- High energy information: to match the same physical objects in RχT and QCD, $\langle\, J(x_n)\cdot\cdot J(0)\,\rangle^{\mathrm{R}\chi\mathrm{T}} = \langle\, J(x_n)\cdot\cdot J(0)\,\rangle^{\mathrm{QCD}}$.

  For example: $\pi\pi$ vector form factor

$$
\begin{aligned}
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{R}\chi\mathrm{T}} &= 1+\frac{F_V G_V}{F^2}\frac{q^2}{M_V^2-q^2}\,, \\
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{QCD}} &\to 0, \qquad \text{for } q^2\to\infty
\end{aligned}
$$

  This leads to

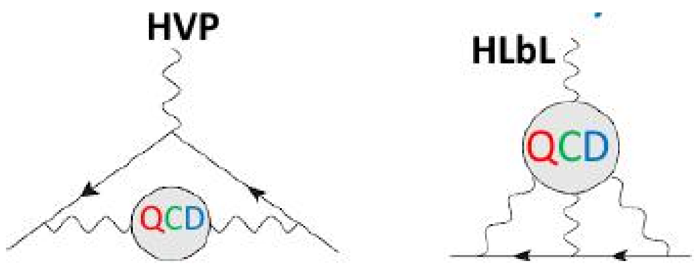
$$
[\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{R}\chi\mathrm{T}} = [\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{QCD}} \Longrightarrow F_V G_V = F^2
$$

Phenomenologies in $\tau \rightarrow \pi\pi\gamma\nu_\tau$

Why to focus on $\tau \to \pi\pi\gamma\nu_\tau$

- **Relevance to precise determination of a_μ**

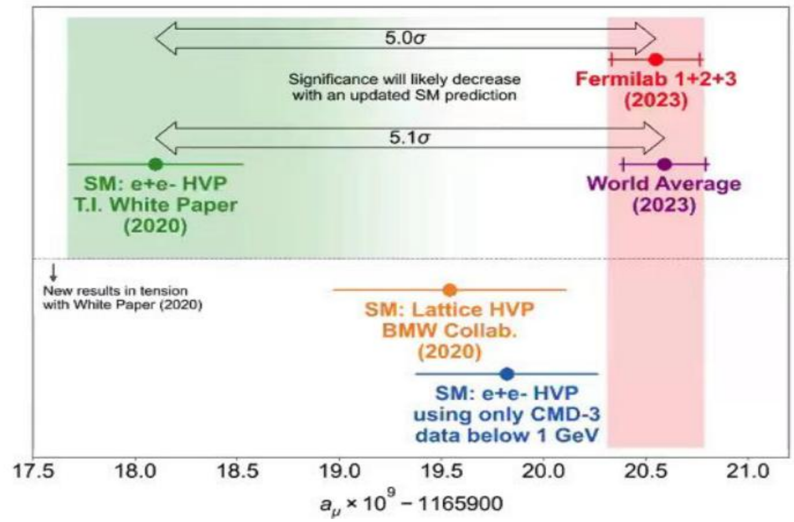
SM uncertainty dominated by



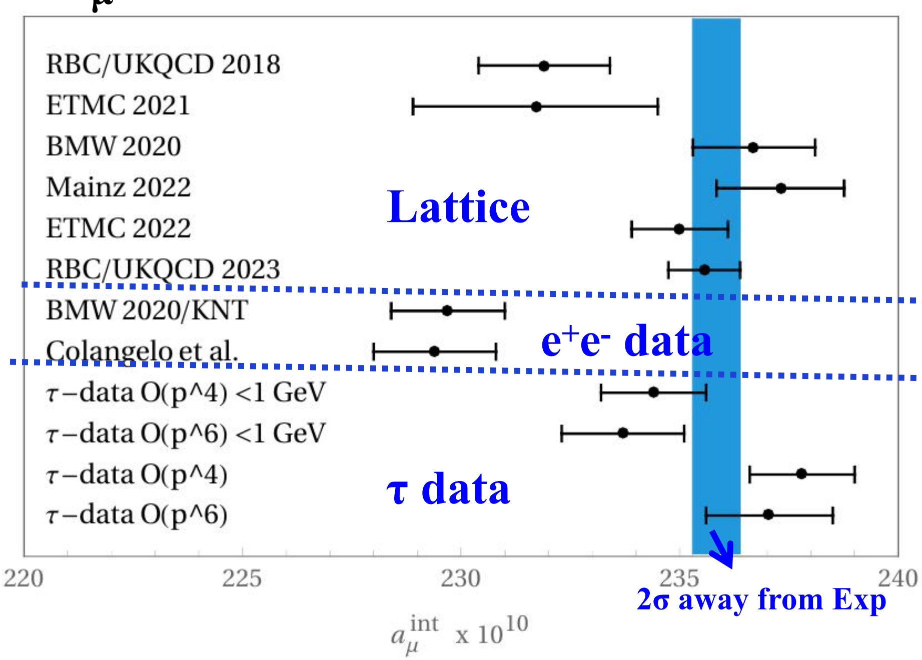
↓

Dominated by $\pi\pi$ (> ~75%)

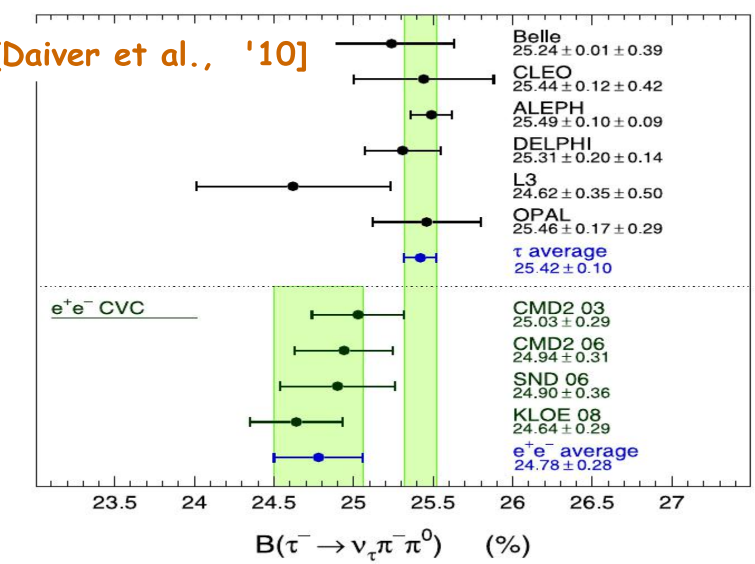
[Muon g-2, '23]

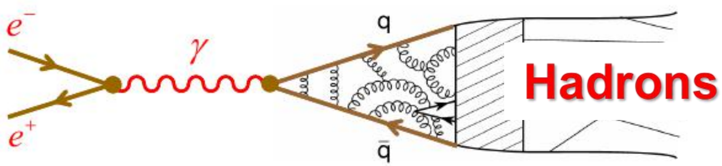


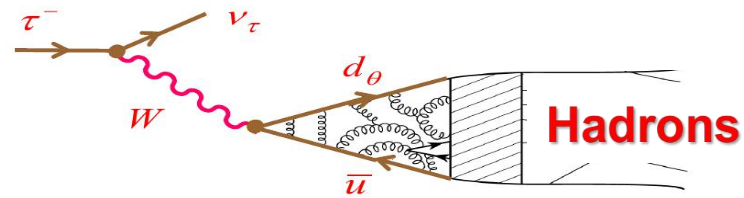
$a_\mu^{HVP,LO}$ [Masjuan et al., '23]



[Daiver et al., '10]







❖ **Key problem in the matching:** **isospin breaking (IB) effects**

IB corrections to a_μ

[Cirigliano et al., JHEP'02]

$$\Delta a_\mu^{\mathrm{vacpol}} = \frac{1}{4\pi^3}\int_{4M_\pi^2}^{t_{\max}} dt K(t) \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma_{\pi\pi[\gamma]}}{dt}\right] \times \left(\frac{R_{\mathrm{IB}}(t)}{S_{\mathrm{EW}}} - 1\right)$$

$$R_{\mathrm{IB}}(t) = \frac{1}{G_{\mathrm{EM}}(t)}\,\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}}\,\left|\frac{F_V(t)}{f_+(t)}\right|^2$$

EM corrections — **Kinematics** — **IB effects in Form Factors**

$G_{\mathrm{EM}}(t) \sim$ **virtual photon + real photon**

Photon loops in $\tau \to \pi\pi\nu_\tau$

Radiative decays: $\tau \to \pi\pi\gamma\nu_\tau$

[Miranda,Roig, PRD'20]

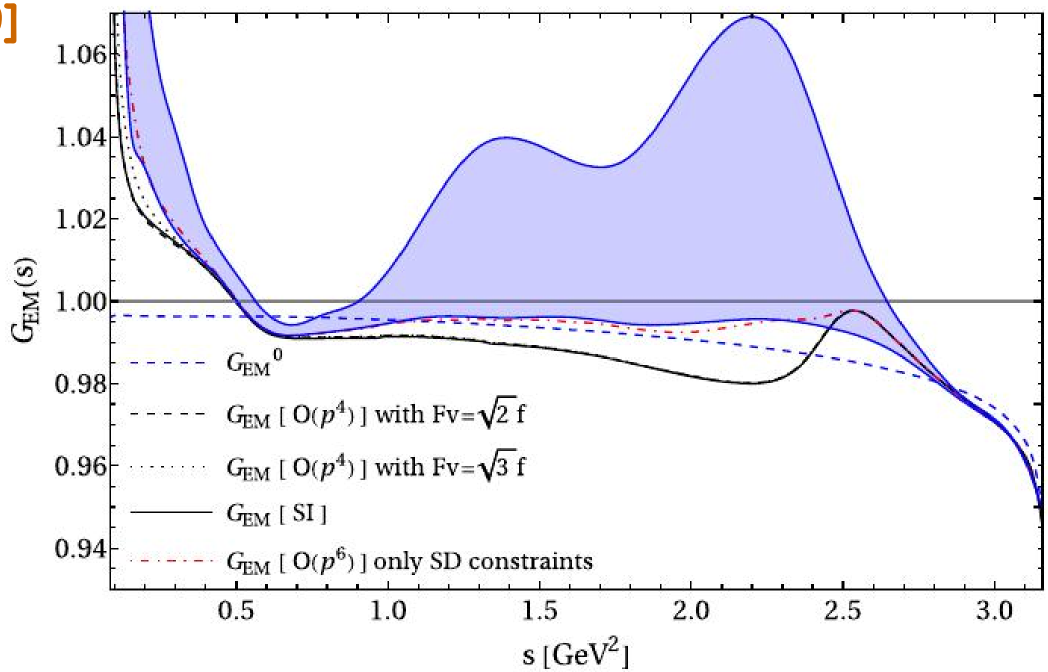


TABLE IV. Contributions to $\Delta a_\mu^{\mathrm{HVP,LO}}$ in units of 10^{-11} using the dispersive representation of the form factor. From the two evaluations labeled $\mathcal{O}(p^4)$, the left (right) one corresponds to $F_V = \sqrt{2}F$ ($F_V = \sqrt{3}F$).

$[s_1, s_2]$	$\Delta a_{\mu,\mathrm{G_{EM}^{(0)}}}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,\mathrm{SI}}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[\mathcal{O}(p^4)]}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[\mathcal{O}(p^4)]}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[SD]}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[\mathcal{O}(p^6)]}^{\mathrm{HVP,LO}}$
$[4m_\pi^2, 1\ \mathrm{GeV}^2]$	+17.8	−11.0	−11.3	−17.0	−32.4	−74.8 ± 44.0
$[4m_\pi^2, 2\ \mathrm{GeV}^2]$	+18.3	−10.1	−10.3	−16.0	−31.9	−75.9 ± 45.5
$[4m_\pi^2, 3\ \mathrm{GeV}^2]$	+18.4	−10.0	−10.2	−15.9	−31.9	−75.9 ± 45.6
$[4m_\pi^2, m_\tau^2]$	+18.4	−10.0	−10.2	−15.9	−31.9	−75.9 ± 45.6

Referenced value using the tau data to calculate a_μ

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = (12.5 \pm 6.0) \times 10^{-10}$$

➢ CP violation in tau decays

$$A_{CP} = \frac{\Gamma(\tau^- \to \nu_\tau H) - \Gamma(\tau^+ \to \nu_\tau \bar{H})}{\Gamma(\tau^- \to \nu_\tau H) + \Gamma(\tau^+ \to \nu_\tau \bar{H})}$$

Intensive discussions on tau -> Ks pi nu

$$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$\approx (0.36 \pm 0.01)\%$$

SM prediction

$$\left(-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}}\right)\%$$

BaBar

[Bigi et al., PLB'05] [Grossman et al., JHEP'12] [Lees et al., PRD'12]

[Cirigliano et al., PRL'18] [Rendo et al., PRD'19] [Chen et al., PRD'19 JHEP'20]

Other types of CPV observables: T-odd triple-product asymmety

A typical T-odd kinematical variable:

$$\xi \equiv \varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma \overset{\text{rest frame}}{\underset{\text{of particle } a}{=\!=\!=}} \vec{b} \cdot (\vec{c} \times \vec{d})\, m_a / s_a$$

a*, *b*, *c*, *d*: either *momentum* or *spin

T transformation $(t \to -t, \vec{p} \to -\vec{p}, \cdots)$: $\bar{\xi} \to -\xi$

- **When spin is involved, measurement of polarization is needed.**

  [Nelson, et al., PRD'94] [Tsai, PRD'95] [Datta, PRD'07] ...

- **When focusing on the situation with four momenta, *i.e.***

$$\xi = \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \overset{\text{rest frame}}{\underset{\text{of particle } 1}{=\!=\!=}} \vec{p_2} \cdot (\vec{p_3} \times \vec{p_4})\, m_1$$

In this case, there should be at least four particles in the final state!

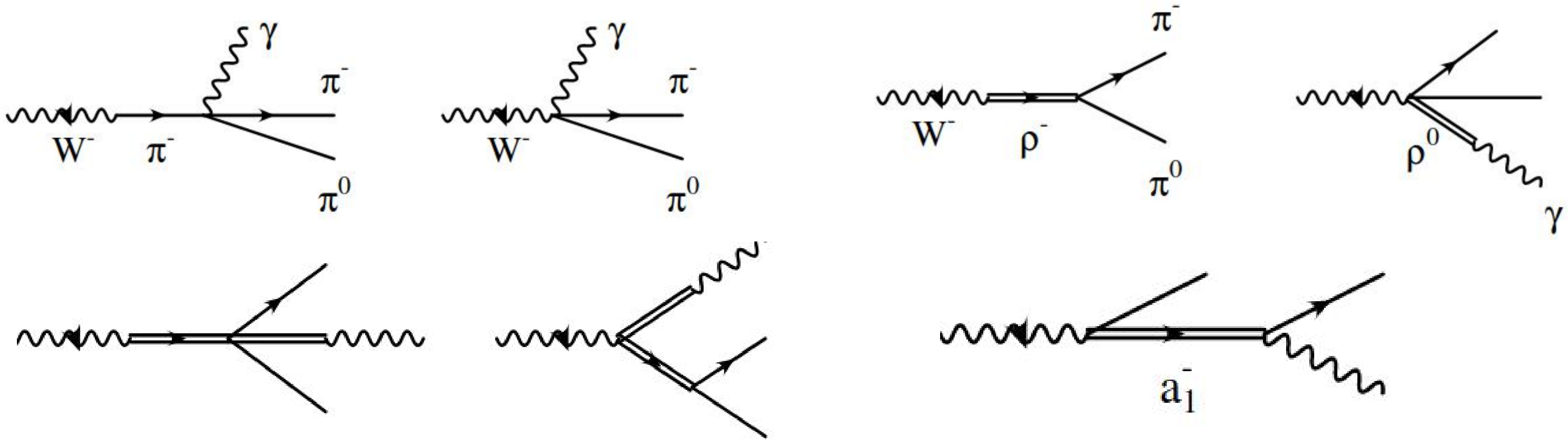
- **Pro: Strong phase is not necessary for a CPV phenomenon using TPA.**

  Con: TPA could also be caused by the final-state interactions!

$\tau \to \pi\pi\gamma\nu_\tau$: good place to probe T-odd triple-product asymmety

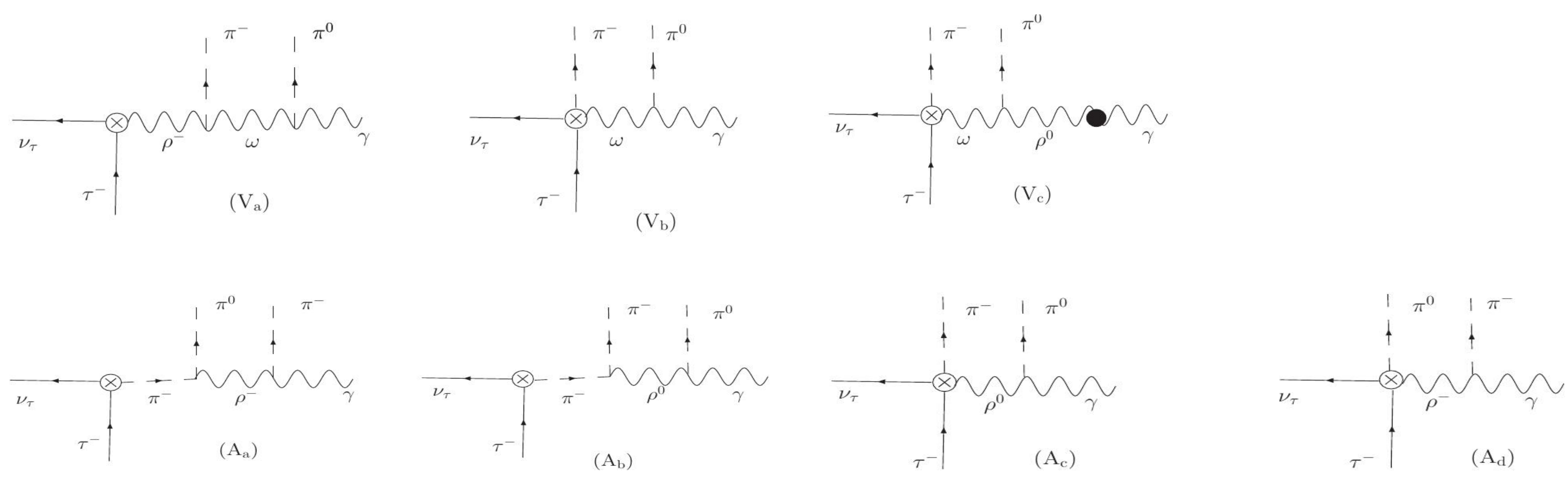
Minimal RChT contributions to $\tau \to \pi\pi\gamma\nu_\tau$

[Cirigliano et al., JHEP'02]



Contributions from VVP and VJP operators in RChT

[Chen, Duan, ZHG , JHEP'22]



High energy contraints to the resonance couplings

$$\int d^4x \int d^4y e^{i(p\cdot x+q\cdot y)} \langle 0|T[V_\mu^a(x)V_\nu^b(y)P^c(0)]|0\rangle$$
$$= d^{abc}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \Pi_{\rm VVP}(p^2,q^2,r^2),$$

$$\lim_{\lambda\to\infty}\Pi^{(8)}_{\rm VVP}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$=\lim_{\lambda\to\infty}\Pi^{(0)}_{\rm VVP}[(\lambda p)^2,(\lambda q)^2,(\lambda p+\lambda q)^2]$$
$$=-\frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2}[1+\mathcal{O}(\alpha_S)]+O\left(\frac{1}{\lambda^6}\right).$$

$$c_1+4c_3=0 \qquad c_1-c_2+c_5=0 \qquad c_5-c_6=\frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}$$

$$d_1+8d_2=-\frac{N_c M_V^2}{(8\pi F_V)^2}+\frac{F^2}{4F_V^2} \qquad d_3=-\frac{N_c M_V^2}{(8\pi F_V)^2}+\frac{F^2}{8F_V^2}$$

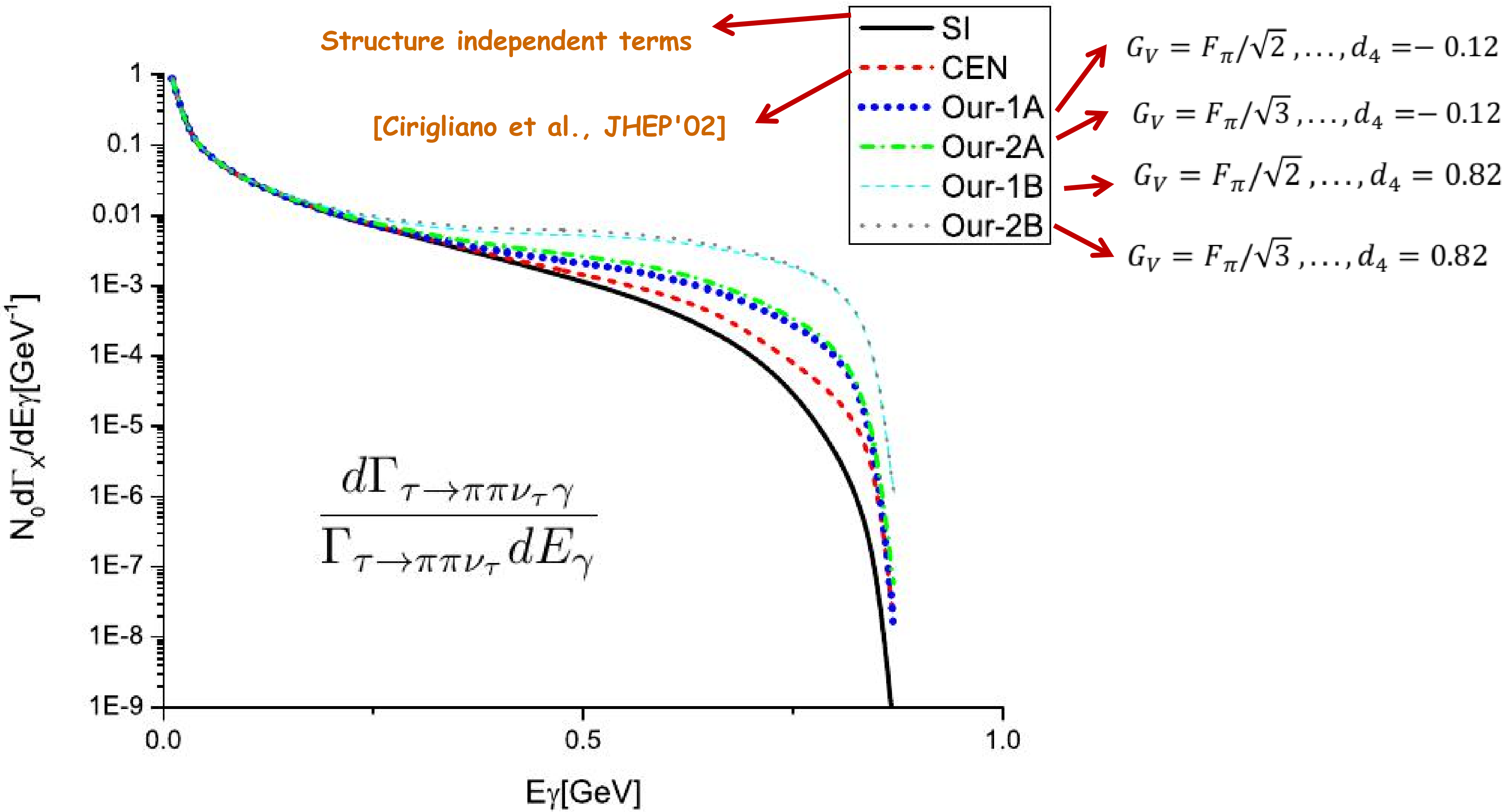
Other constraints from scattering and form factors

$$F_A=F_\pi, \quad F_V=\sqrt{2}F_\pi, \quad G_V=F_\pi/\sqrt{2}.$$

Or

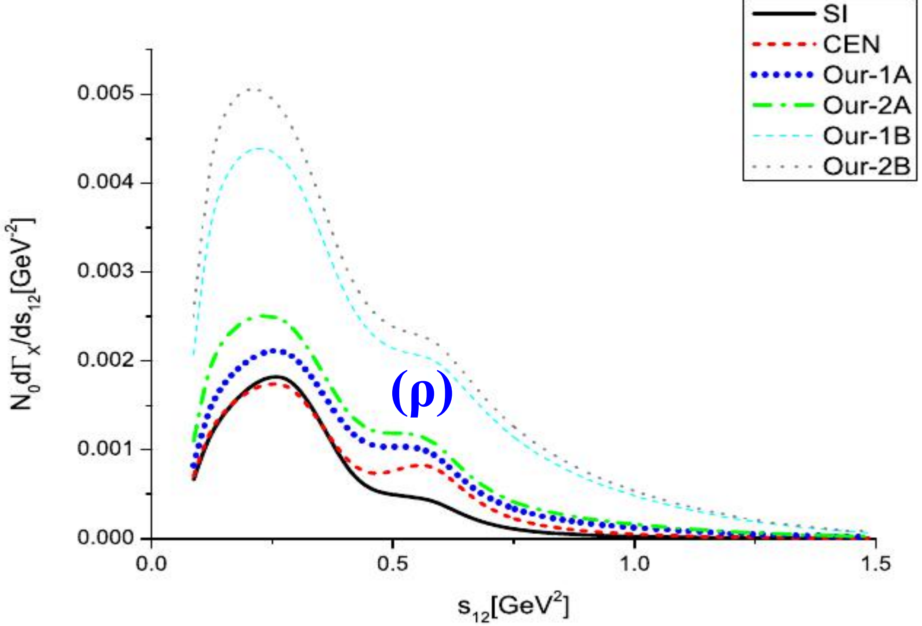
$$F_A=\sqrt{2}F_\pi, \quad F_V=\sqrt{3}F_\pi, \quad G_V=F_\pi/\sqrt{3}$$

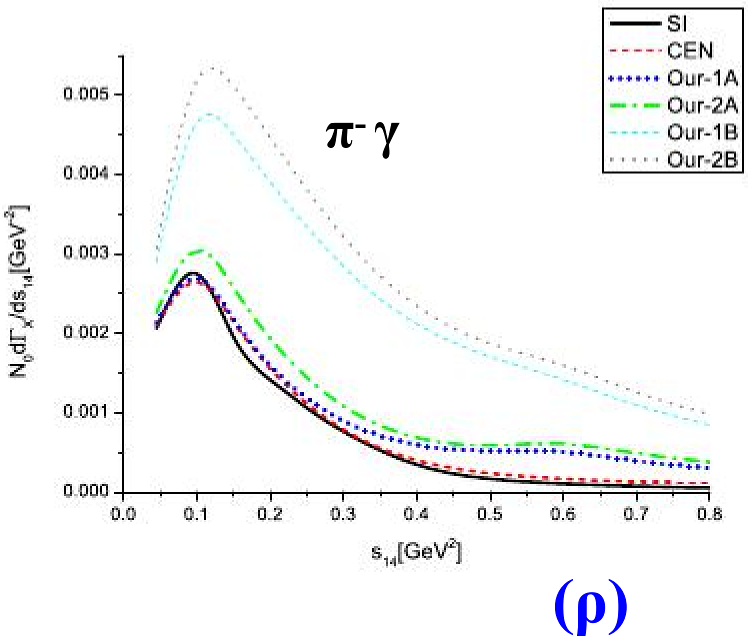
Differential decay widths as a function of photon energies

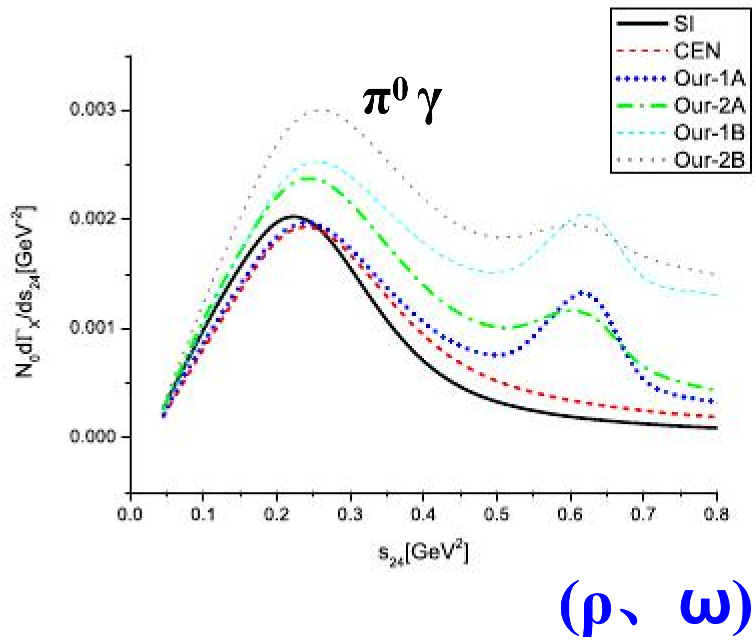


❖ **When the photon energy cutoff is around 300 MeV, the absolute branching ratio is predicted to be around 10^{-4} and it has a good chance to be well measured at Belle-II, STCF, CEPC, ...**

Invariant-mass distributions of the $\pi\pi$ system

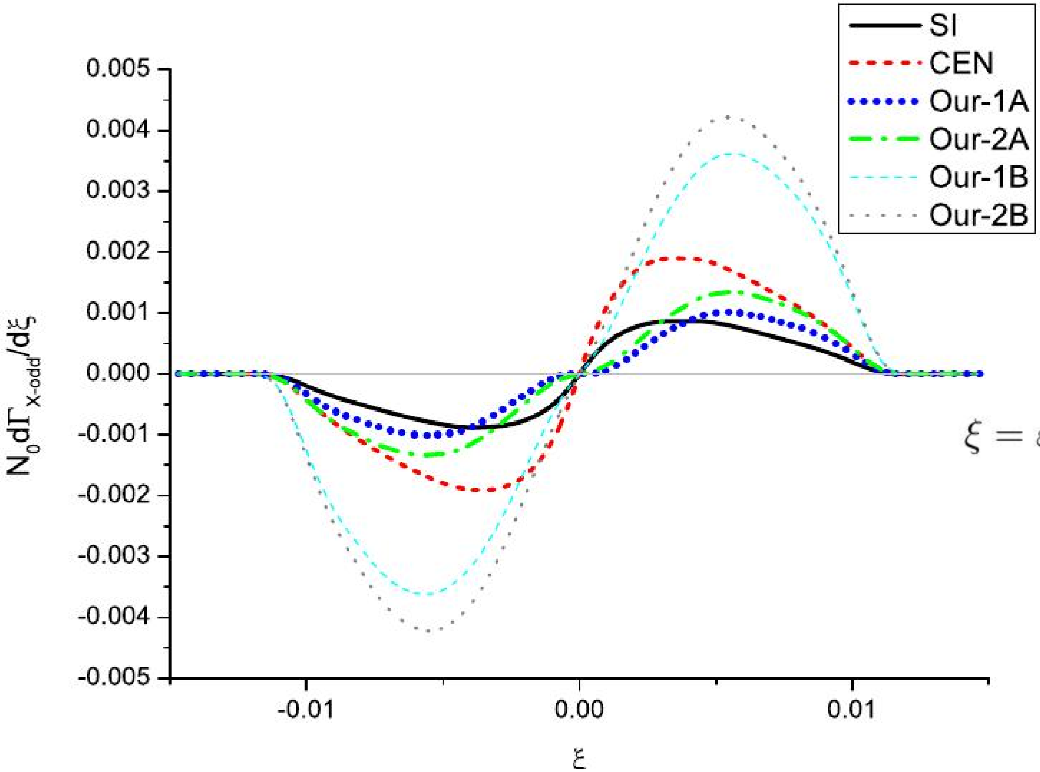






Predicitons of the T-odd asymmetry distribution in $\tau \to \pi\pi\gamma\nu_\tau$

[Chen,Duan,ZHG,JHEP'22]



$$\tau^-(P) \to \pi^-(p_1)\pi^0(p_2)\nu_\tau(q)\gamma(k)$$

$$\xi = \varepsilon_{\mu\nu\rho\sigma}P^\mu k^\nu p_1^\rho p_2^\sigma/m_\tau^4 \;\underset{\text{of }\tau}{\overset{\text{rest frame}}{=\!=\!=}}\; \vec{k}\cdot(\vec{p}_1\times\vec{p}_2)/m_\tau^3$$

$$A_\xi = \frac{N_+ - N_-}{N_+ + N_-} \quad \text{with} \quad N_\pm = \int_{\xi \gtrless 0} \mathrm{d}\Gamma$$

E_γ^{cut}	A_ξ(Our-1A)	A_ξ(Our-2A)	A_ξ(Our-1B)	A_ξ(Our-2B)
100 MeV	1.2/1.7/1.0/1.6	1.3/1.8/0.98/1.4	1.6/1.7/1.4/1.6	1.7/1.8/1.3/1.4
300 MeV	1.5/2.6/1.0/2.2	1.6/2.5/0.73/1.6	2.3/2.6/2.0/2.2	2.4/2.5/1.7/1.6
500 MeV	0.98/1.4/0.58/0.88	0.91/1.4/0.68/0.43	2.1/1.4/1.8/8.8	2.1/1.4/1.5/4.3

(numbers are multiplied by 10^{-2})

- **The magnitudes of A_ξ for $\tau \to \pi\pi\gamma\nu_\tau$ are around two orders larger than those in $K_{l3\gamma}$. It has the good chance to be measured in Belle-II、STCF、CEPC ... .**

Prospects of revealing the genuine CPV signals

- **CPV signals can be probed by taking the differences of A_ξ in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$ and $\tau^+ \to \pi^+\pi^0\gamma\nu_\tau$**

$$A_\xi = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \qquad \overline{A}_{\bar\xi} = \frac{\overline{\Gamma}_+ - \overline{\Gamma}_-}{\overline{\Gamma}_+ + \overline{\Gamma}_-}$$

$$\overline{\Gamma}_+ = \frac{(2\pi)^4}{2m_\tau}\int_{\bar\xi>0} \mathrm{d}\Phi\,(\overline{\hat{M}_0} + \bar\xi\overline{\hat{M}_1}), \qquad \overline{\Gamma}_- = \frac{(2\pi)^4}{2m_\tau}\int_{\bar\xi<0} \mathrm{d}\Phi\,(\overline{\hat{M}_0} + \bar\xi\overline{\hat{M}_1})$$

$$\mathcal{M} = e\,G_F\,V_{ud}^*\epsilon^{*\mu}(k)\,\Big\{(1+\mathbf{g_V})F_\nu\bar{u}(q)\gamma^\nu(1-\gamma_5)(m_\tau + \not{P} - \not{k})\gamma_\mu u(P)$$
$$+\big[(1+\mathbf{g_V})V_{\mu\nu} - (1-\mathbf{g_A})A_{\mu\nu}\big]\bar{u}(q)\gamma^\nu(1-\gamma_5)u(P)\Big\}$$

$$\mathcal{A}_\xi = A_\xi - \overline{A}_{\bar\xi} \supset \mathrm{Im}(\mathbf{g_V^*g_A})\mathrm{Re}[F_V(t/u)^*A_i],\ \mathrm{Im}(\mathbf{g_V^*g_A})\mathrm{Re}(V_j^*A_i)$$

- **Generally speaking, sizable hadronic contributions are also expected to enhance the CPV signals in $\tau \to \pi\pi\gamma\nu_\tau$.**

- **TPA in other types of τ decays could be also possible.**

Summary

Tau offers a laboratory for a broad range of interesting topics:

- **Precision tests of SM: CKM, α_s, m_τ, lepton universality, ...**
- **Hadron interactions: light-flavor resonances, chiral symmetry, form factors, second-class currents, ...**
- **BSM tests:**

  CPV (rate asym., triple-product asym.)

  LFV (lepton/radiative decays, hadron decays)

  ... ...

- **Resonance chiral theory offers a systematical tool to sudy the tau decays.**
  E.g., rich phenomenologies in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$: photon spectrum, $\pi^-\pi^0$, $\pi^-\gamma$, $\pi^0\gamma$ spectra, the triple-product T-odd asymmetries.

Thanks for your patience!