Accessing B-meson LCDA in LaMET

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Outline

- **Introduction to B-meson LCDA**
  1. Importance of B-meson LCDA
  2. Definition and properties of B-meson LCDA
- **Introduction to LaMET**
  1. Introduction of Larger Momentum Effective Theory (LaMET)
  2. Recent progress in the frame of LaMET
- **Our work about B-meson LCDA**
  1. Matching between B-meson quasi-DA and LCDA
  2. Renormalization of B-meson quasi-DA
  3. Inverse and logarithmic moments
- **Proposals for future research**

Why B-meson LCDA important?

- Understanding the general properties of power expansion in EFTs (HQET, SCET, NRQCD)

- Understanding the strong interaction dynamics of heavy quark decays.

  - ✓ $B \to \pi\,\pi$ Phys. Rev. Lett. 83, 1914 (1999) 1402 citations in INSPIRE
  - ✓ $B \to \pi\,K$ Nucl. Phys. B 606, 245 (2001) 1163 citations in INSPIRE
  - ✓ $B \to \pi\,D$ Phys. Rev. D 69, 112002 (2004) 399 citations in INSPIRE

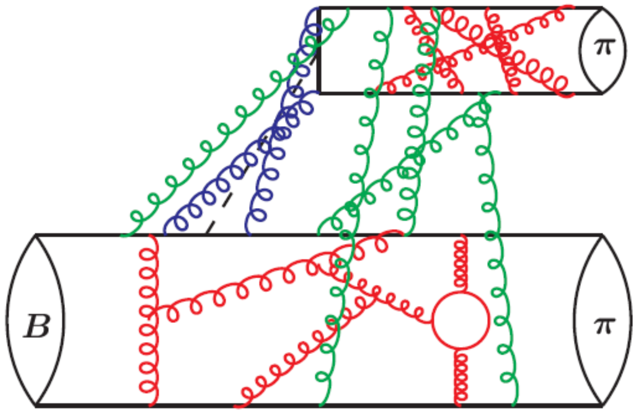
  "The hadronic matrix elements that enter B meson decays into two light mesons can be computed from first principles … and expressed in terms of form factors and meson light-cone distribution amplitudes."

- Accurate measurement of standard model parameters: V_{ub} and V_{cb}

  - ✓ $B \to \pi\,\ell\,\nu$ Phys. Lett. B 633, 61 (2006) 211 citations in INSPIRE
  - ✓ $B \to D\,\ell\,\nu$ Phys. Rev. D 92, 5 (2015) 381 citations in INSPIRE

  "The uncertainty in their prediction is dominated by the uncertainty in the light-cone distribution amplitudes (LCDAs) of the B- and π-mesons."

- Understanding the strong interaction dynamics of heavy quark decays.

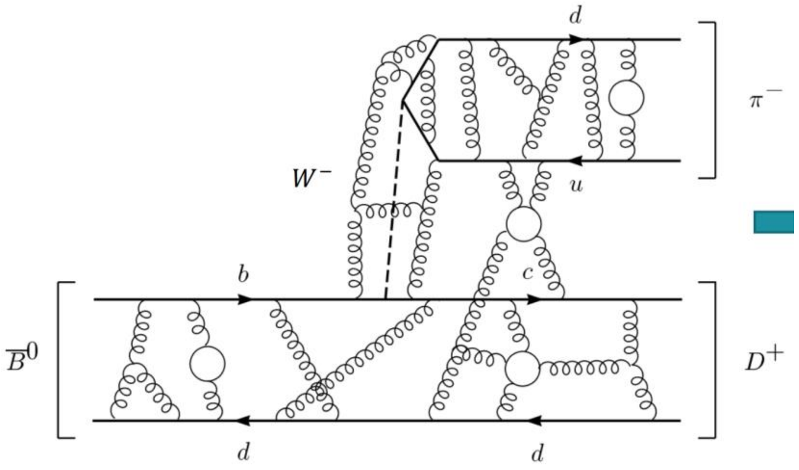


$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

$B \to \pi$ form factor

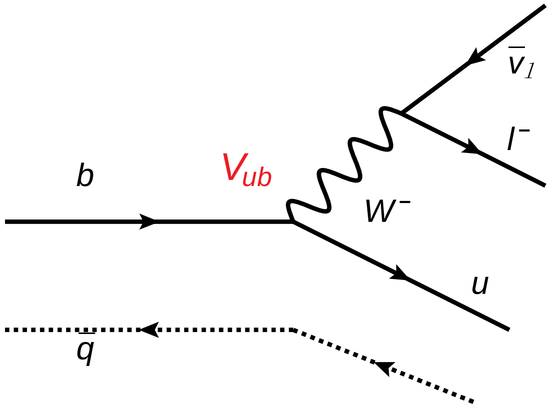
Hard kernel

B-meson LCDA



"The significance of the factorization formula is that all leading-power nonperturbative effects in the $B \to \pi\,\pi$ amplitudes can be absorbed into the form factor and the light-cone wave functions."

- Accurate measurement of standard model parameters: V_{ub} and V_{cb}.



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- The light-ray HQET matrix element *[Grozin, Neubert, 1997; Beneke, Feldmann, 2000]*

$$\langle 0\,|\bar{q}_\beta(z)[z,0]h_{v\alpha}(0)|\,\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\phi}_B^+(t,\mu) + \frac{\tilde{\phi}_B^-(t,\mu) - \tilde{\phi}_B^+(t,\mu)}{t}\not{z}\right\}\gamma_5\right]_{\alpha\beta}.$$

We assume that $z^2 = 0$, define $t = v.z$ and the path-ordered exponential

$$[z,0] = \mathrm{P}\exp\left(ig_s\int_{z_2}^{z_1} dz^\mu A_\mu(z)\right).$$

The prefactor is chosen in such a way that for $z = 0$ one obtains

$$\langle 0|\bar{q}_\beta\,[\gamma^\mu\gamma_5]_{\beta\alpha}\,b_\alpha|\bar{B}(v)\rangle = if_B m_B v^\mu\,.$$

If $\tilde{\phi}_B^+(t=0) = \tilde{\phi}_B^-(t=0)$=1.

Leading twist

Sub-leading twist

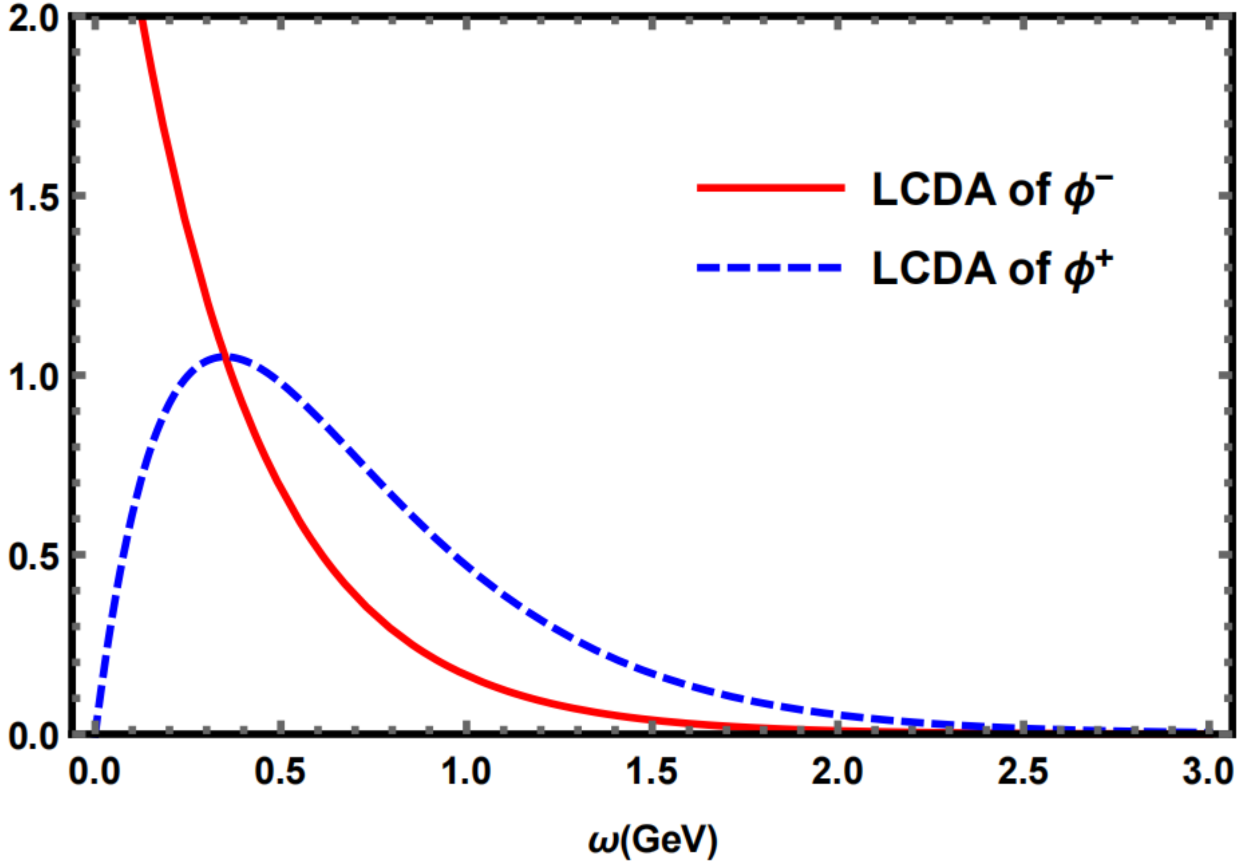
- For a meson with an arbitrary velocity in the $n_+ - n_-$ plane, this formula becomes

$$\langle 0\,|\bar{q}_\beta(z)[z,0]h_{v\alpha}(0)|\,\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{8}\left\{\left[\tilde{\phi}_B^+(t,\mu)v_+\gamma_- + \tilde{\phi}_B^-(t,\mu)v_-\gamma_+\right]\gamma_5\right\}_{\alpha\beta}.$$

- The exponential models of B-meson LCDA in momentum space

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}} , \qquad \phi_B^-(\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}} .$$

[Phys. Rev. D 55 (1997) 272-290] [Phys. Rev. D 69, 034014(2004)]



- Evolution equations of $\tilde{\phi}_+^B$ and $\tilde{\phi}_-^B$ [Lange, Neubert, 2003; Bell, Feldmann, 2008]

$$\frac{d}{d\ln\mu}\phi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega' \gamma_+^{(1)}(\omega,\omega',\mu)\,\phi_B^+(\omega',\mu) + \mathcal{O}(\alpha_s^2)\ .$$

$$\frac{d}{d\ln\mu}\phi_B^-(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega' \gamma_-^{(1)}(\omega,\omega',\mu)\,\phi_B^-(\omega',\mu) + \mathcal{O}(\alpha_s^2)\ .$$

Where

$$\gamma_+^{(1)}(\omega,\omega',\mu) = \left(\Gamma_{\rm cusp}^{(1)}\ \ln\frac{\mu}{\omega} - 2\right)\delta(\omega-\omega') - \Gamma_{\rm cusp}^{(1)}\,\omega\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right]_+\ .$$

$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)}(\omega,\omega';\mu) - \Gamma_{\rm cusp}^{(1)}\frac{\theta(\omega'-\omega)}{\omega'}\ .$$

- Solution of evolution equations. [Bell, Feldmann, YMW and Yip, 2013; Braun, Manashov, 2014]

- RG equations of $\phi_B^+(\omega,\mu)$ at two-loops. [Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]

- RG equations of the higher-twist B-meson distribution amplitudes. [Braun, Ji, Manashov, 2017]

- NNLO QCD correction to relevant hadronic B-meson decays. [Bell, Beneke, Huber, Li, 2020]

- Precise determination of the B-meson LCDAs
  - ✓ They are the dominant uncertainties for predictions of exclusive B-meson decays.
  - ✓ Progress is mainly concentrated on the perturbative aspect.
  - ✓ The studies on the shape of B-meson LCDAs are quite model dependent.

- How could we overcome this bottleneck?

- LQCD provides a systematic ab initio calculations of the non-perturbative strong interactions.

- We need a new idea, the recently developed Large Momentum Effective Theory may help us.

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Large Momentum Effective Theory

X. Ji, Phys. Rev. Lett. 110 (2013)   *X. Ji, Sci.China Phys.Mech.Astron. 57 (2014)*

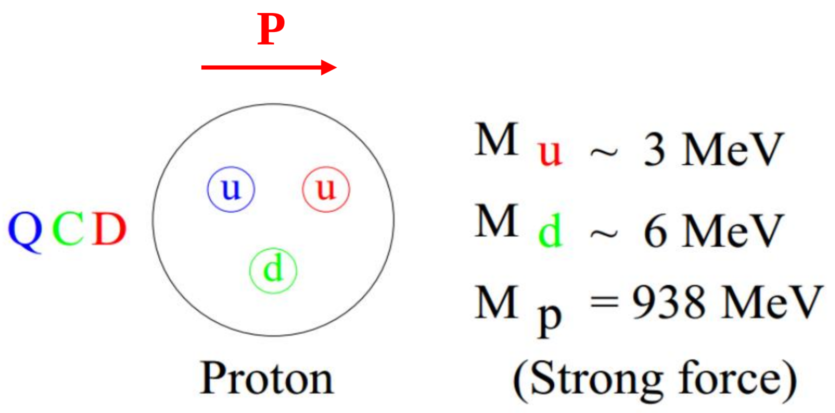
- With this approach, parton physics can be extracted using effective field theory (EFT) methods from the physical properties of the proton at a moderately-large momentum.

- Thus, the theory has been named as **large-momentum effective theory** (LaMET).

- LaMET is not merely a theoretical trick, but is based on an important physical insight by Feynman.

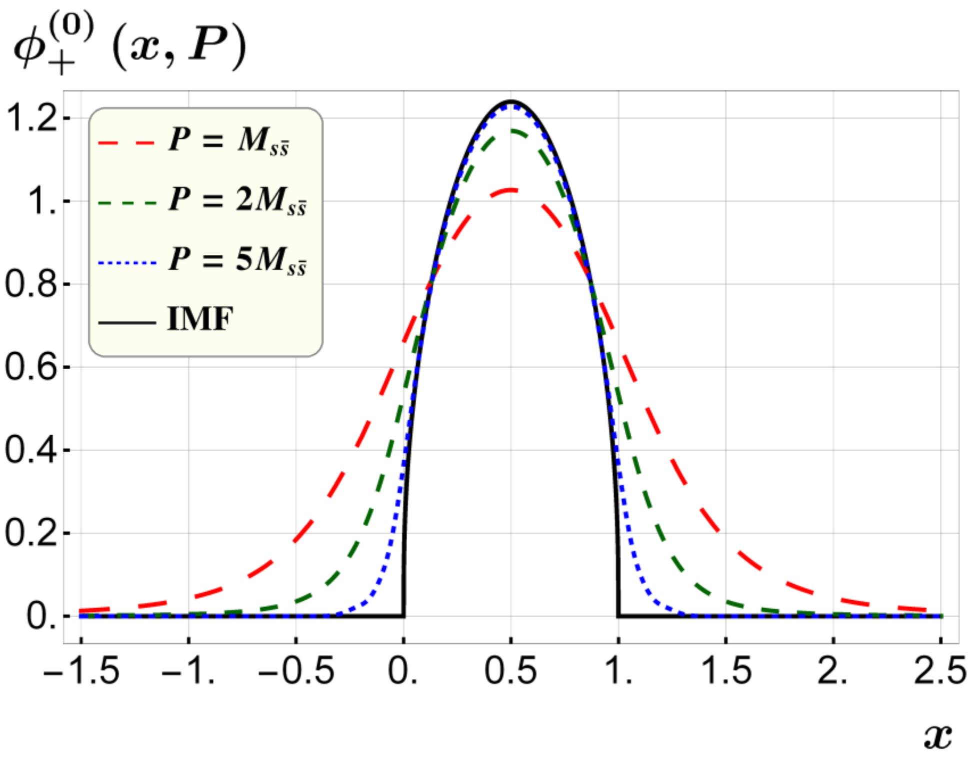
  Naive parton model: The structure of the proton shall be approximately independent of its momentum so long as it is much larger than a typical strong-interaction scale Λ. For example, the quark momentum distribution in the proton at $P = |\vec{P}| = 5$ GeV shall not be very different from that at P = 50 GeV or P = 5 TeV.

- One might call this phenomenon Large-momentum symmetry.



- Assuming this symmetry, Feynman replaced the protons probed at different large momenta in high energy scattering with the one at the infinite momentum $P = \infty$, therefore the idealized concepts of a proton in the infinite-momentum frame (IMF) and its constituents — partons — were born.

- It is generally expected that the large momentum limit of the proton state exists and is smooth, and some small parameters such as Λ/P^z control the limiting process. This is true in certain simple QFT models such as ’t Hooft model. [Ji, Liu, Liu, Zhang, Zhao, 2021]



Wave function amplitudes of a meson in the ’t Hooft model at different external momenta. [Jia, Liang, Li, Xiong, 2017]

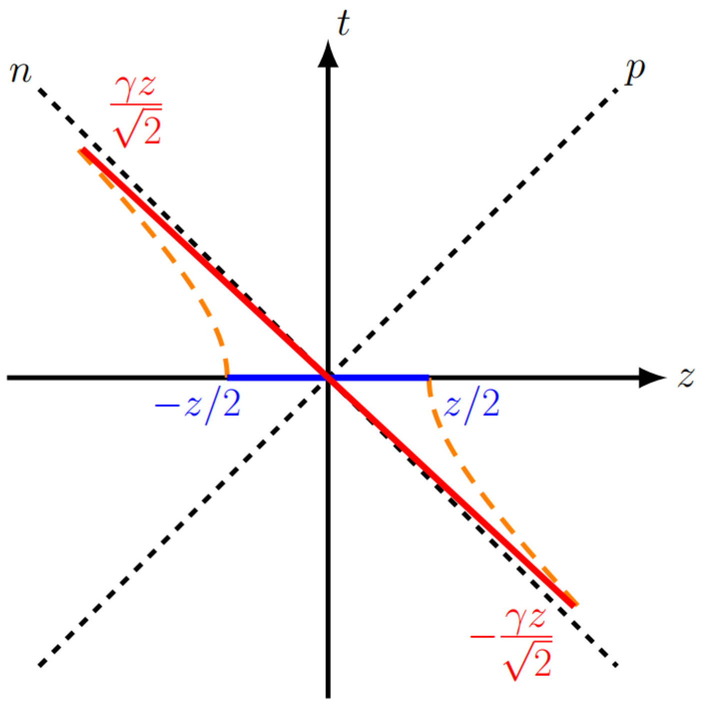
This is the type of examples that Feynman’s intuition applies.

- Along this line of thought, could we build a lattice calculable quantity which will approach the light-cone quantities when boosted in large P limit?

$$\tilde{q}\left(x,\mu^2,P^z\right)=\int\frac{dz}{4\pi}e^{izk^z}\langle P|\bar{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z\left(z'\right)\right)\psi(0)|P\rangle$$

$$q\left(x,\mu^2\right)=\int\frac{d\xi^-}{4\pi}e^{-ix\xi^-P^+}\langle P|\bar{\psi}\left(\xi^-\right)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+\left(\eta^-\right)\right)\psi(0)|P\rangle$$

The momentum distribution defined above has been called *quasi-PDF*, but in reality it is a physical momentum distribution in a proton of momentum *P* .



The PDFs describe the probability distributions of quarks and gluon inside nucleon.

- **However, in QFTs, UV divergences bring in complications.**

  ✓ The physically relevant one is clearly $\Lambda_{UV} \gg P \to \infty$, as discussed in the previous subsection.

  ✓ Historically, It was found that taking $P \to \infty$ by ignoring the UV divergences considerably simplifies the perturbation theory rules.

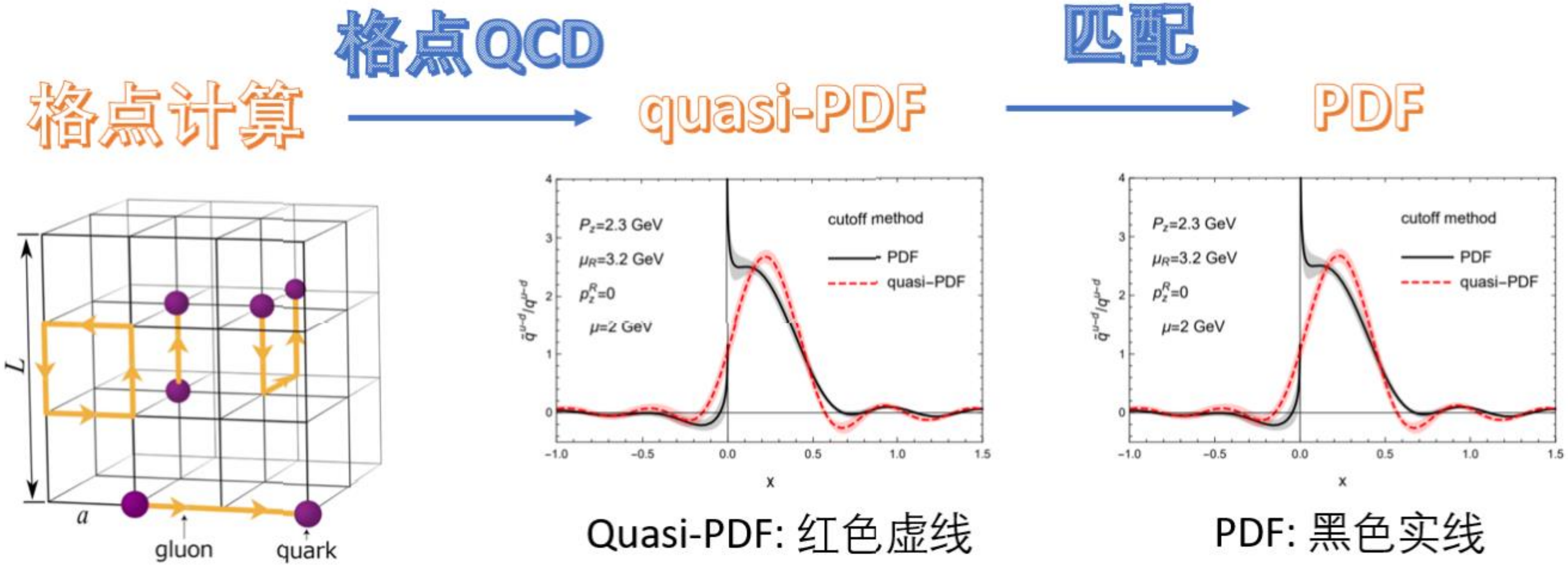
- It is the "naive" limit, $P \gg \Lambda_{UV} \to \infty$, that corresponds to Feynman's parton model, and hence we name the resulting theory as "effective field theory" for partons.

- In asymptotically free theories such as QCD, differences (or discontinuities) in taking the limits of $P \gg \Lambda_{UV}$ and $\Lambda_{UV} \gg P \to \infty$ are perturbatively calculable, as only the high-momentum modes matter. The differences are called **matching coefficients.**

- **Setp1：** Constructing lattice calculable ME, choosing an appropriate renormalization scheme.

- **Step2：** Lattice calculations.

- **Step3：** Extracting the light-cone physics from the lattice ME (Matching).



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LaMET

X. Ji, Phys. Rev. Lett. 110 (2013) 630 citations in INSPIRE

Theoretical research

Quark PDF:

- ✓ X. Xiong, X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. D 90, 014051 (2014);
- ✓ L. B. Chen, W. Wang and R. Zhu, Phys. Rev. Lett. 126, 072002 (2021);

Gluon PDF:

- ✓ J. H. Zhang, X. Ji, A. Schafer, W. Wang, S. Zhao, Phys. Rev. Lett. 122, 142001 (2019);
- ✓ W. Wang and S. Zhao JHEP 05 (2018);

GPD:

- ✓ Y. S. Liu, W. Wang, J. Xu, Q. A. Zhang, J. H. Zhang, S. Zhao and Y. Zhao, Phys. Rev. D 100, 034006 (2019);

LCDA for light-meson:

- ✓ J. Xu, Q. A. Zhang and S. Zhao, Phys. Rev. D 97, 114026 (2018)

……

Lattice simulation

Quark PDF:

- ✓ H. W. Lin, J. W. Chen, S. D. Cohen and X. Ji, Phys. Rev. D 91, 054510 (2015) ;
- ✓ J. Green, K. Jansen and F. Steffens, Phys. Rev. Lett. 121, 022004 (2018);

Gluon PDF:

- ✓ Z. Y. Fan, Y. B. Yang, A. Anthony, H. W. Lin and K. F. Liu, Phys. Rev. Lett. 121, 242001 (2018);

GPD:

- ✓ J. W. Chen, H. W. Lin and J. H. Zhang, Nucl. Phys. B 952, 114940 (2020);

LCDA for light-meson:

- ✓ J. H. Zhang, J. W. Chen, X. Ji, L. Jin and H. W. Lin, Phys. Rev. D 95, 094514 (2017);
- ✓ J. H. Zhang et al. [LP3 Collaboration], Nucl. Phys. B 939, 429 (2019);

……

Related theories

Pseudodistribution:

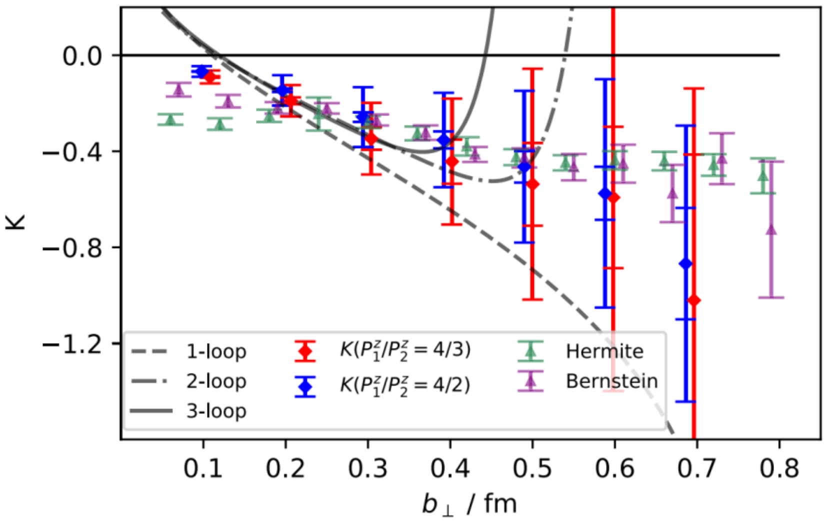
- ✓ A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017);
- ✓ K. Orginos, A. V. Radyushkin, J. Karpie and S. Zafeiropoulos, Phys. Rev. D 96, 094503 (2017);
- ✓ S. Zhao and A. V. Radyushkin, Phys. Rev. D 103, 054022 (2021);
- ✓ I. Balitsky, W. Morris, A. V. Radyushkin, JHEP 02, 193 (2022);

Lattice cross sections:

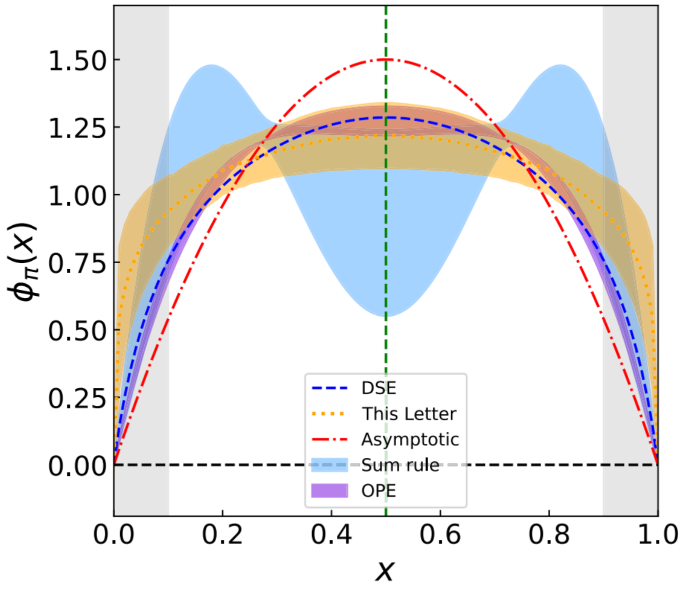
- ✓ Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 120, 022003 (2018);
- ✓ R. S. Sufian, J. Karpie, C. Egerer, K. Orginos and J. W. Qiu, Phys. Rev. D 99, 074507 (2019);
- ✓ Z. Y. Li, Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 126, 072001 (2021);
- ✓ J. Bringewatt, N. Sato, W. Melnitchouk, J. W. Qiu and F. Steffens et al., Phys. Rev. D 103, 016003 (2021);

……

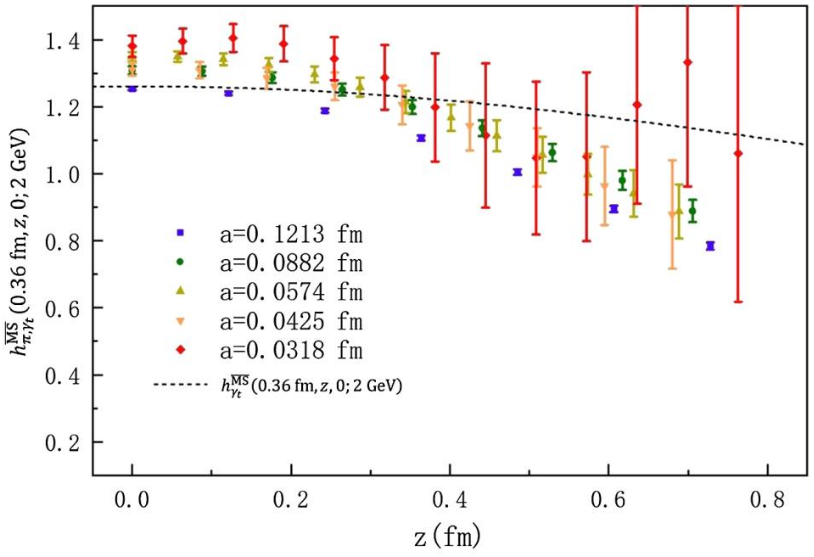
Lattice Parton Collaboration (LPC)



Q. A. Zhang et al.
Phys. Rev. Lett. 125, 19 (2020)



J. Hua et al.
Phys. Rev. Lett. 129, 13 (2022)



K. Zhang et al.
Phys. Rev. Lett. 129, 8 (2022)

Our works

- **Light-meson**
  - ✓ Matching Coefficient — *Phys.Rev.D 97, 114026 (2018)* — *Phys.Rev.D 100, 034006 (2019)*
  - ✓ Renormalization — *Phys.Rev.D 99, 094036 (2019)*
  - ✓ Lattice simulation — *Phys.Rev.Lett. 129, 132001 (2022)* — *Phys.Rev.Lett. 127, 6, 062002 (2021)*
- **B-meson**
  - ✓ Leading-twist LCDA — *Phys.Rev.D 102, 011502 (2020)* — *Phys.Rev.D 106, 114019 (2022)*
  - ✓ Inverse and logarithmic moment — *Phys.Rev.D 106, L011503 (2022)* — *Has been Done*
  - ✓ Subleading-twist LCDA — *arXiv: 2308.13977 [hep-ph]*

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B-meson LCDA

The leading-twist LCDA $\phi_B^+(\eta,\mu)$ in coordinate space is defined by the renormalized HQET matrix element of a light-ray operator.

$$\langle 0|\,(\bar{q}_s\,W_c)\,(\eta\,\bar{n})\;\; \bar{\not{n}}\,\gamma_5\;\left(W_c^\dagger\,h_v\right)(0)|\bar{B}(v)\rangle = i\tilde{f}_B(\mu)\,m_B\,\tilde{\phi}_B^+(\eta,\mu)$$

Applying the Fourier transformation for $\phi_B^+(\eta,\mu)$ leads to the momentum-space distribution function

$$\phi_B^+(\omega,\mu) = \frac{1}{2\,\pi}\int_{-\infty}^{+\infty} d\eta\, e^{i\,\bar{n}\cdot v\,\omega\,\eta}\,\tilde{\phi}_B^+(\eta,\mu)$$

Following the construction presented above, we will employ the following B-meson quasi-distribution amplitude

$$i\tilde{f}_B(\mu)\,m_B\,\varphi_B^+(\xi,\mu) = \int_{-\infty}^{+\infty}\frac{d\tau}{2\,\pi}\,e^{i\,n_z\cdot v\,\xi\,\tau}\langle 0|\,(\bar{q}_s\,W_c)\,(\tau\,n_z)\;\; \not{n}_z\,\gamma_5\;\left(W_c^\dagger\,h_v\right)(0)|\bar{B}(v)\rangle$$

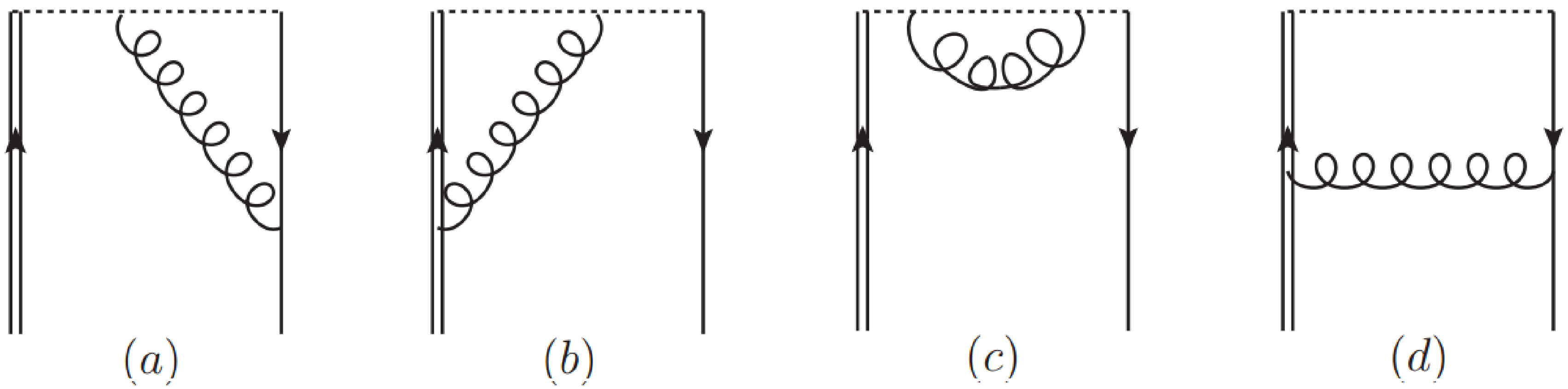
Here $n_z = (0,0,0,1)$.

Matching

We now proceed to determine the perturbative matching coefficient function entering the hard-collinear factorization formula

$$\varphi_B^+(\xi,\mu) = \int_0^\infty d\omega\, \underline{H(\xi,\omega,n_z\cdot v,\mu)}\,\phi_B^+(\omega,\mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{n_z\cdot v\,\xi}\right)$$

Matching coefficient got by calculating these diagrams below



- The matching coefficient is

$$
\begin{aligned}
&H(\xi,\,\omega,\,n_z\cdot v,\,\mu) = \delta(\xi-\omega)\\
&+\frac{\alpha_s(\mu)\,C_F}{4\,\pi}\left\{\frac{1}{\omega-\xi}\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\theta(-\xi)\;\theta(\omega)\right.\\
&+\left\{\frac{1}{\omega-\xi}\left[3-2\left(1+\frac{2\,\xi}{\omega}\right)\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\left(\ln\left(\frac{\omega-\xi}{\xi}\right)+1\right)\right]\right\}_{\oplus}\\
&\times\theta(\xi)\;\theta(\omega-\xi)\\
&+\left\{\frac{1}{\xi-\omega}\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\xi-\omega)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\right\}_{\oplus}\theta(\omega)\;\theta(\xi-\omega)\\
&\left.+2\left[\ln^2\frac{\mu}{n_z\cdot v\;\xi}-3\,\ln\frac{\mu}{n_z\cdot v\;\xi}+f(a)\right]\delta(\xi-\omega)\right\}
\end{aligned}
$$

$$
\begin{aligned}
f(a) \;=\;& \ln\frac{a^2}{4(a-1)^3}\,\ln\frac{\mu}{n_z\cdot v\;\xi}+\ln(a-1)\,\ln\frac{8\,(a-1)}{a}\\
&+\mathrm{Li}_2(1-a)+\ln a\,\ln\left(\frac{a}{4}\right)-\frac{1}{2}\ln(a-1)\\
&+\ln(8\,a)+\ln^2 2+\frac{\pi^2}{8}-3
\end{aligned}
$$

Perspectives for lattice calculations

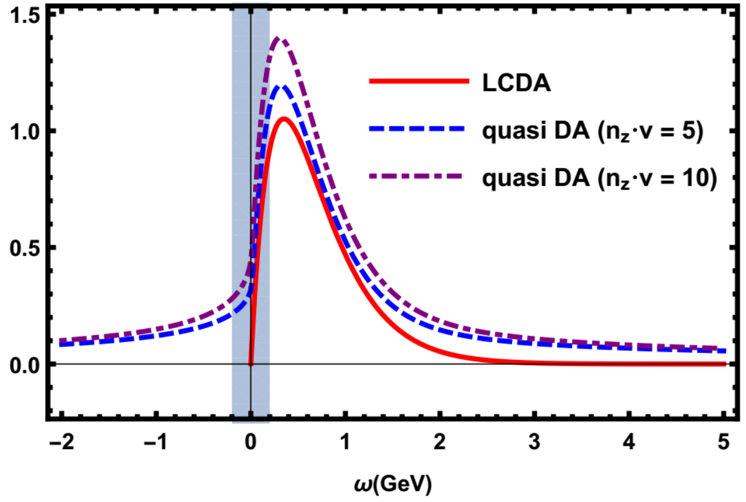
- It will be instructive to understand the characteristic feature of $\varphi_B^+(\xi,\mu)$ with distinct nonperturbative models of $\phi_B^+(\omega,\mu)$.
- Taking advantage of the two phenomenological models motivated by the HQET sum rule calculation at leading order and at next-to-leading order

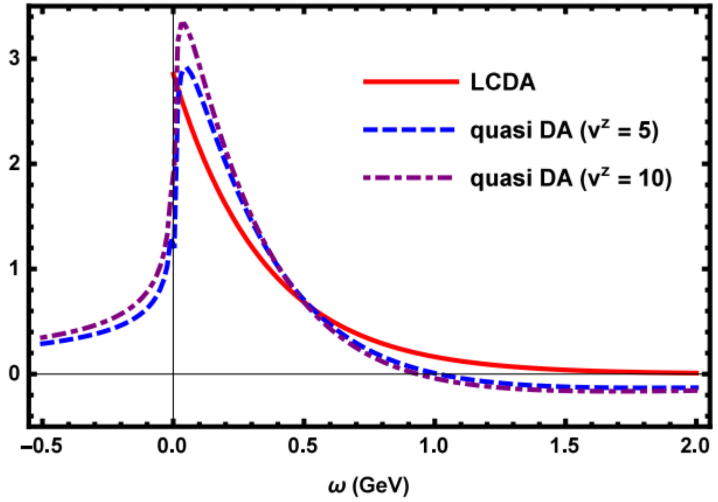
$$\phi^+_{B,\,\mathrm{I}}(\omega,\,\mu=1.5\,\mathrm{GeV}) = \frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

$$\phi^+_{B,\,\mathrm{II}}(\omega,\,\mu=1.5\,\mathrm{GeV}) = \left[\frac{1}{k^2+1} - \frac{2\,(\sigma_B^{(1)}-1)}{\pi^2}\ln k\right] \times \frac{4}{\pi\,\omega_0}\,\frac{k}{k^2+1},\quad k=\frac{\omega}{1\,\mathrm{GeV}},$$

$$\phi^-_{B,\mathrm{I}}(\omega) = \frac{1}{\lambda_B}e^{-\omega/\lambda_B}$$

$$\phi^-_{B,\mathrm{II}}(\omega,\mu) = -\frac{2}{\pi\lambda_B}\left(\frac{\omega\mu}{\omega^2+\mu^2} + \arctan\frac{\omega}{\mu} - \frac{\pi}{2} + \frac{4\,(\sigma_B-1)}{\pi^2}\left\{\mathrm{Im}\left[\mathrm{Li}_2(\frac{i\omega}{\mu})\right] - \arctan\frac{\omega}{\mu}\ln\frac{\omega}{\mu}\right\}\right)$$





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- The multiplicative renormalizability of the constructed quasi-HQET operator to all orders in perturbation theory has been demonstrated.

- This enables a nonperturbative renormalization such as the RI/MOM scheme.

Quasi-DA

$$i\tilde{f}_B(\tilde{\mu})m_B\varphi_B^+(\xi,\tilde{\mu}) = \int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{in_z\cdot v\xi\tau}\left\langle 0\left|\left(\bar{q}W_c\right)\left(\tau n_z\right)\not{n}_z\gamma_5\left(W_c^\dagger h_v\right)(0)\right|\bar{B}(v)\right\rangle.$$

Here $\tilde{\mu}$ is a renormalization scale for the quasidistribution amplitude whose definition depends on the renormalization scheme.

- The RI/MOM renormalization factor Z_{OM} is determined nonperturbatively on the lattice by imposing the condition that the quantum corrections of the correlator in an off-shell quark state vanish at the scales $k^2 = -\mu_R^2$ and $k^z = k_R^z$,

$$Z_{\mathrm{OM}}^{-1}\left(\tau,k_R^z,\mu_R,\Lambda\right)\left\langle 0\left|\left(\bar{q}W_c\right)\left(\tau n_z\right)\not{n}_z\gamma_5\left(W_c^\dagger h_v\right)(0)\right|b\bar{q}(k)\right\rangle\Big|_{\substack{k^2=-\mu_2^2\\ k^z=k_R^R}} = \left\langle 0\left|\left(\bar{q}W_c\right)\left(\tau n_z\right)\not{n}_z\gamma_5\left(W_c^\dagger h_v\right)(0)\right|b\bar{q}\left(k_R\right)\right\rangle\Big|_{\mathrm{tree}},$$

here $\tilde{\mu}$ is the renormalization scale. Hereafter we simply denote $\{\tilde{\mu}\} = \{k^2 = -\mu_R^2, k^z = k_R^z\}$.

- The factorization formula is

$$\varphi_B^+(\xi,\tilde{\mu}) = \int_0^\infty d\omega H\left(\xi,\omega,n_z\cdot v,\mu,\{\tilde{\mu}\}\right)\phi_B^+(\omega,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{n_z\cdot v\xi}\right),$$

The $\overline{MS}$ renormalization of ϕ_B^+ has already been done

The RI/MOM renormalization of φ_B^+

We denote the bare correlator for the B meson on the lattice as

$$\tilde{h}_B\left(\tau,k^z,1/\epsilon\right) = \left\langle 0\left|\left(\bar{q}_s W_c\right)\left(\tau n_z\right)\not{n}_z\gamma_5\left(W_c^\dagger h_v\right)(0)\right| b\bar{q}(k)\right\rangle ,$$

which is renormalized as

$$\tilde{h}_R\left(\tau,k^z,\{\tilde{\mu}\}\right) = Z_{\rm OM}^{-1}(\tau,\{\tilde{\mu}\},1/\epsilon)\tilde{h}_B\left(\tau,k^z,1/\epsilon\right) .$$

Fourier transform it into ξ space to obtain the distribution

$$\tilde{\mathcal{F}}\left(\xi,k^z,\{\tilde{\mu}\}\right) = \int\frac{d\tau}{2\pi}e^{iv^z\xi\tau}\tilde{h}_R\left(\tau,k^z,\{\tilde{\mu}\}\right) ,$$

The local correspondence is

$$\tilde{\mathcal{V}}\left(k^z,\{\tilde{\mu}\}\right) = \tilde{h}_R\left(\tau=0,k^z,\{\tilde{\mu}\}\right) .$$

$$\varphi_B^+(\xi,\tilde{\mu}) = v^z\int\frac{d\tau}{2\pi}e^{iv^z\xi\tau}\frac{\tilde{h}_R\left(\tau,k^z,\{\tilde{\mu}\}\right)}{\tilde{h}_R\left(\tau=0,k^z,\{\tilde{\mu}\}\right)} .$$

- The renormalized quasi-DA can be written as

$$\varphi_B^{+(1)}(\xi,\tilde{\mu}) = v^z \int \frac{d\tau}{2\pi} e^{iv^z\xi\tau} \left\{ \left(\frac{Z_{\rm OM}^{-1}(\tau,\{\tilde{\mu}\},1/\epsilon)}{Z_{\rm OM}^{-1}(0,\{\tilde{\mu}\},1/\epsilon)} \right)^{(1)} \left(\frac{\tilde{h}(\tau,k^z)}{\tilde{h}(0,k^z)} \right)^{(0)} + \left(\frac{Z_{\rm OM}^{-1}(\tau,\{\tilde{\mu}\})}{Z_{\rm OM}^{-1}(0,\{\tilde{\mu}\})} \right)^{(0)} \left(\frac{\tilde{h}_B(\tau,k^z,1/\epsilon)}{\tilde{h}_B(0,k^z,1/\epsilon)} \right)^{(1)} \right\}$$

$$= -v^z \int \frac{d\tau}{2\pi} e^{iv^z\xi\tau} \int d\xi' e^{-i\tau(v^z\xi'-k_R^z)} \varphi_{B,\rm CT}^{+(1)}(\xi',\{\tilde{\mu}\}) e^{-ik^z\tau} + \varphi_{B,\rm bare}^{+(1)}(\xi,k^z)$$

$$= \varphi_{B,\rm bare}^{+(1)}(\xi,k^z) - \varphi_{B,\rm CT}^{+(1)}(\xi+\tilde{k}_R-\tilde{k},r_R).$$

$$H\left(\xi,\omega,v^z,\mu,\{\tilde{\mu}\}\right) = \delta(\xi-\omega) + g_1(\xi,\omega,\mu) - g_2(\xi,\omega,\{\tilde{\mu}\}) + \frac{\alpha_s C_F}{4\pi} \ln v^z \left(3 + 4\ln\frac{a-1}{a}\right) \delta(\xi-\omega)\,.$$

$$g_1(\xi,\omega,\mu) = \frac{\alpha_s C_F}{4\pi} \begin{cases} \frac{1}{\omega(\omega-\xi)}\left(\omega - 2\xi\ln\frac{-\xi}{\omega-\xi}\right) & (\xi<0) \\ \left[\frac{1}{\omega(\omega-\xi)}\left(\omega - 2\xi + 2\xi\ln\frac{4v^2\xi(\omega-\xi)}{\mu^2}\right)\right]_{\oplus} & (0<\xi<\omega) \\ \left[\frac{1}{\omega(\omega-\xi)}\left(-\omega + 2\omega\ln\frac{\mu^2}{(\xi-\omega)^2} + 2\xi\ln\frac{\xi}{\xi-\omega}\right)\right]_{\oplus} & (\xi>\omega) \end{cases}\,.$$

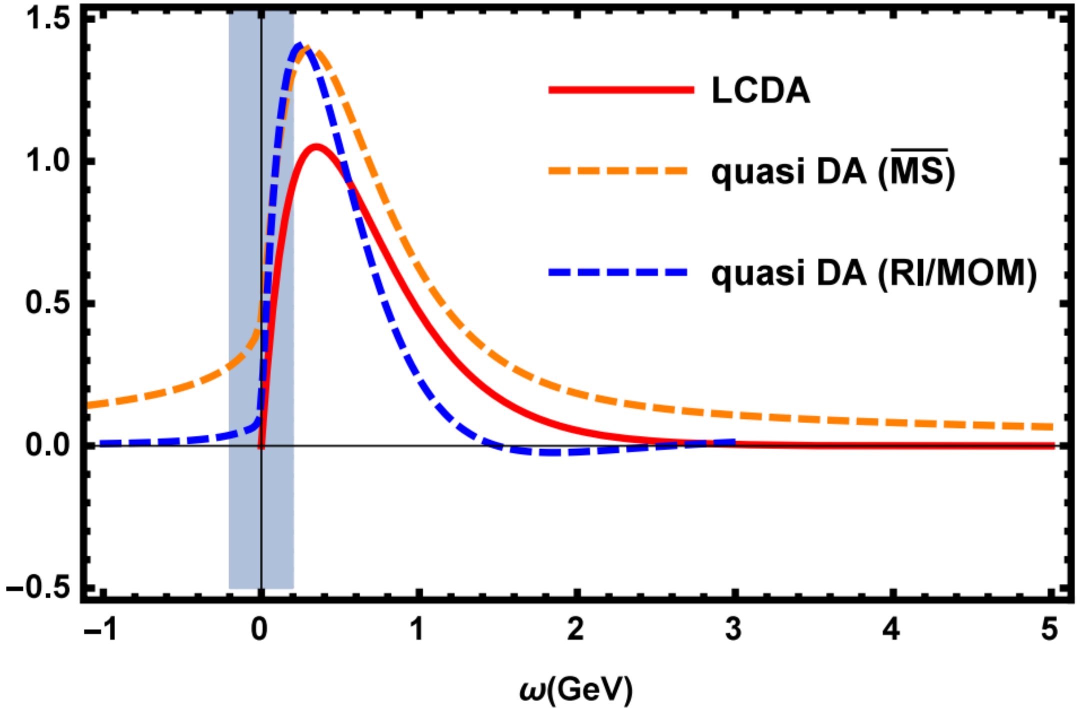
$$g_2(\xi,\omega,\{\tilde{\mu}\}) = \frac{\alpha_s C_F}{4\pi} \begin{cases} -\frac{1}{\omega-\xi} & \left(\xi<\omega-\tilde{k}_R\right) \\ \left[\frac{1}{2k_R^z\sqrt{1-r_R}(\omega-\xi)}\left(-2\sqrt{1-r_R}\left(k_R^z+2v^z(\xi-\omega)\right) - \left(4v^z(\xi-\omega) - k_R^z(r_R-4)\right)\ln\frac{2-2\sqrt{1-r_R}-r_R}{r_R}\right)\right]_{\oplus} & \left(\omega-\tilde{k}_R<\xi<\omega\right) \\ \left[\frac{1}{\omega-\xi}\right]_{\oplus} & (\xi>\omega). \end{cases}\,.$$

- Comparison with the results of φ_B^+ in $\overline{\text{MS}}$ and RI/MOM schemes respectively.

- Taking advantage of a well-known phenomenological model of ϕ_B^+ motivated by the HQET sum rule

$$\phi_B^+(\omega,\, \mu = 1.5\,\text{GeV}) \quad = \quad \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0}\,, \qquad \omega_0 = 350\,\text{MeV}$$

We have taken $k_R^z = 2$ GeV, $\mu = 1.5$ GeV, $r_R = 2$ and $v^z = 10$.



Outline

- **Introduction to B-meson LCDA**
  1. Importance of B-meson LCDA
  2. Definition and properties of B-meson LCDA
- **Introduction to LaMET**
  1. Introduction of Larger Momentum Effective Theory (LaMET)
  2. Recent progress in the frame of LaMET
- **Our work about B-meson LCDA**
  1. Matching between B-meson quasi-DA and LCDA
  2. Renormalization of B-meson quasi-DA
  3. Inverse and logarithmic moments
- **Proposals for future research**

- The inverse moment (IM) is an indispensable part of many factorization theorems in B physics, such as B-meson radiative decay and $B \to P\ V$ form factors.

- It will be of phenomenological significance to study the IM of quasi-DA. *Y. Yia and X. Xiong, Phys. Rev. D 94, 9 (2016)*

- We perform a study on the structure of the inverse moment (IM) of quasidistributions, by taking B-meson quasi-DA as an example.

- The IM of LCDA

$$\lambda_B^{-1}(\mu) \equiv \int_{-\infty}^{\infty} d\omega \frac{\phi_B^+(\omega,\mu)}{\omega}. \quad \longrightarrow \quad \tilde{\lambda}_B^{-1}(v_3,\mu) \equiv \int_{-\infty}^{\infty} d\omega \frac{\varphi_B^+(\omega, v_3,\mu)}{\omega}.$$

- The operator definition of IM of B-meson quasi-DA is

$$\tilde{\lambda}_B^{-1}(v_3,\mu)\Big|_{+i\epsilon} = \frac{1}{v_0 F(\mu)} \int_0^{-\infty} d\xi \left\langle 0 \left| \bar{q}\left(\frac{\xi}{v_3} n\right) S\left(\frac{\xi}{v_3} n, 0\right) \gamma_0 \gamma_5 h_v(0) \right| \bar{B}(v) \right\rangle .$$

- Because the inverse moment is defined with the matrix element of an equal-time operator, it can be simulated by lattice QCD directly, without calculating the whole quasi-DA first.

- After renormalization, we have the RGE for IMs of LCDA and quasi-DA

$$\mu\frac{d}{d\mu}\lambda_B^{-1}(\mu) = -\frac{\alpha_s(\mu)}{2\pi}C_F\left[2\lambda_B^{-1}(\mu)\sigma_1(\mu) - \lambda_B^{-1}(\mu)\right] .$$

Not multiplicative renormalized

$$\mu\frac{d}{d\mu}\tilde{\lambda}_B^{-1}(v_3,\mu) = -\frac{\alpha_s(\mu)}{2\pi}C_F\left(2\ln 2iv_3 + 1\right)\tilde{\lambda}_B^{-1}(v_3,\mu) .$$

Multiplicative renormalized

$$\sigma_n(\mu) = \lambda_B(\mu)\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\ln^n\frac{\mu}{\omega}\phi_B^+(\omega,\mu) .$$

- There is a matching formula between IMs.

$$\tilde{\lambda}_B(v_3,\mu_Q) = C_0\left(v_3,\frac{\mu_Q}{\mu_L}\right)\lambda_B(\mu_L) + \sum_{n=1}C_n\left(v_3,\frac{\mu_Q}{\mu_L}\right)\sigma_n(\mu_L) + \mathcal{O}(1/v_3) .$$

Where

$$C_n\left(v_3,\frac{\mu_Q}{\mu_L}\right) = \sum_{m=0}\left(\frac{\alpha_s(\mu)}{2\pi}C_F\right)^m C_n^{(m)}\left(v_3,\frac{\mu_Q}{\mu_L}\right) .$$

- At leading order, $C_0^{(0)} = 1$ and $C_n^{(0)} = 0$ for $n \geq 1$. The next-to-leading order corrections of the hard coefficients are listed below,

$$
\begin{cases}
C_0^{(1)}\left(v_3, \frac{\mu_Q}{\mu_L}\right) = (2\ln 2iv_3 + 1)\ln\frac{\mu_Q}{\mu_L} \\
\qquad - \ln 2iv_3\,(\ln 2iv_3 + 3) - \frac{5\pi^2}{24} + 3 \\
C_1^{(1)}\left(v_3, \frac{\mu_Q}{\mu_L}\right) = 2\ln 2iv_3 + 3 \\
C_2^{(1)}\left(v_3, \frac{\mu_Q}{\mu_L}\right) = -1, \\
C_n^{(1)}\left(v_3, \frac{\mu_Q}{\mu_L}\right) = 0 \quad (n \geq 3).
\end{cases}
\quad .
$$

- The RGE for the first IM of quasi-DA and the correct form of matching relation in LaMET have been presented.

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Two-loop corrections to matching coefficient

$$\varphi_B^+(\xi,\mu) = \int_0^\infty d\omega\, H(\xi,\omega,n_z\cdot v,\mu)\,\phi_B^+(\omega,\mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z\cdot v\,\xi}\right)$$

"Two-loop evolution equation for the B-meson distribution amplitude" [Braun, Ji, Manashov, 2019]

To be calculated

- **Status for quark PDF**

L.-B. Chen, W.Wang and R.Zhu, Phys.Rev.Lett. 126 (2021)

Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, Phys.Rev.Lett. 126 (2021)

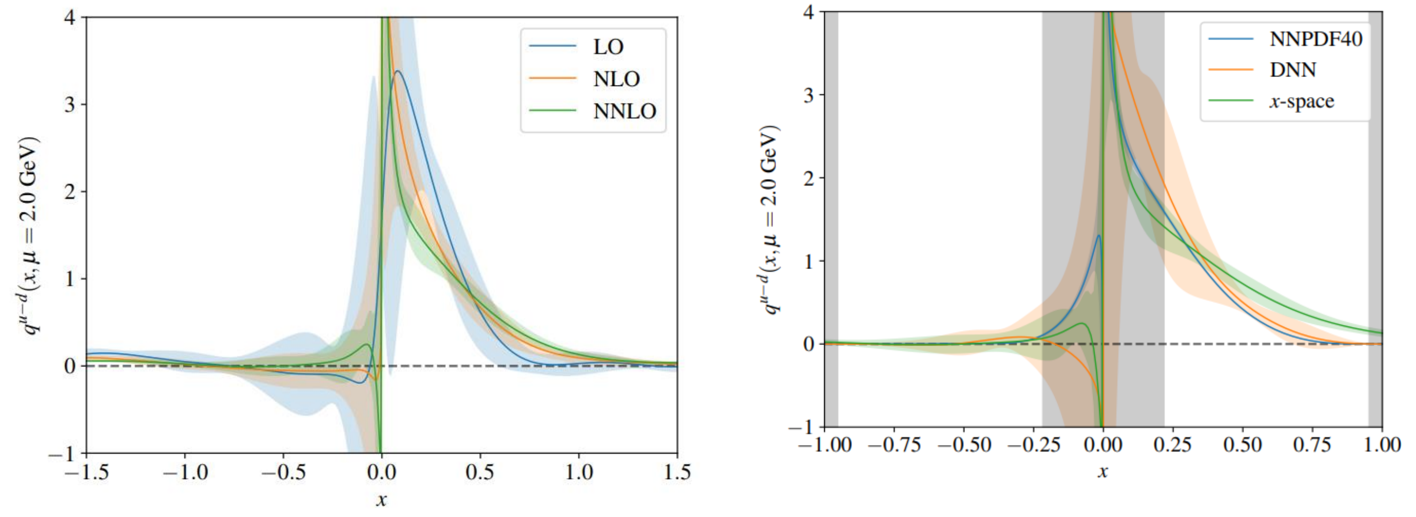

FIG. 15: Dependence of the x-space matched isovector-quark PDF on the perturbative order used in the matching kernel. The results shown use the largest value of momentum computed in this work (i.e. $P_z = 1.53$ GeV).

FIG. 17: Comparison of the x-dependence of the isovector-quark PDF from the global analysis of NNPDF4.0, the DNN, and x-space matching. The gray bands correspond to the regions of x where we do not rigorously trust the results of LaMET.

Phys. Rev. D 107, 074509 (2023)

Power corrections

- **quark-PDF**

$$\tilde{q}(y, P^z, \mu) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) q(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{[(1-y)P^z]^2}\right),$$

- **Light-meson**

$$\tilde{\mathcal{F}}(\Gamma, x, P^z, \tilde{\mu}) = \int_0^1 dy \tilde{C}_\Gamma\left(x, y, \frac{\tilde{\mu}}{\mu}, \frac{P^z}{\mu}\right) \mathcal{F}(\bar{\Gamma}, y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right),$$

- **B-meson**

$$\varphi_B^+(\xi, \mu) = \int_0^\infty d\omega\, H(\xi, \omega, n_z \cdot v, \mu) \phi_B^+(\omega, \mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z \cdot v \xi}\right).$$

One-loop:

Phys.Rev.D 98, 056004 (2018)

Two-loop:

Phys.Rev.Lett. 126 (2021)

However, no power correction considered

Accessing B-meson LCDA through other approaches

Related theories

Pseudodistribution:

- ✓ A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017);
- ✓ K. Orginos, A. V. Radyushkin, J. Karpie and S. Zafeiropoulos, Phys. Rev. D 96, 094503 (2017);
- ✓ S. Zhao and A. V. Radyushkin, Phys. Rev. D 103, 054022 (2021);
- ✓ I. Balitsky, W. Morris, A. V. Radyushkin, JHEP 02, 193 (2022);

Lattice cross sections:

- ✓ Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 120, 022003 (2018);
- ✓ R. S. Sufian, J. Karpie, C. Egerer, K. Orginos and J. W. Qiu, Phys. Rev. D 99, 074507 (2019);
- ✓ Z. Y. Li, Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 126, 072001 (2021);
- ✓ J. Bringewatt, N. Sato, W. Melnitchouk, J. W. Qiu and F. Steffens et al., Phys. Rev. D 103, 016003 (2021);

……

Phys.Rev.D 103, 054022 (2021):
"B-meson Ioffe-time distribution amplitude at short distances"

JHEP 03, 086 (2023) :
Non-singlet quark helicity PDFs of the nucleon from pseudo-distributions

Phys.Rev.D 103, 016003 (2021) :
Confronting lattice parton distributions with global QCD analysis

Lattice simulation

- **Many progresses**

ALPHA Collaboration

Alpha collaboration

- ✓ Form factor for $B_s \to K\ell\nu$ (2016)
- ✓ Precision $B^*B\pi$ coupling (2021)

Fermilab Lattice and MILC collaborations

- ✓ $|V_{ub}|$ from $B \to \pi\ell\nu$ (2015)

HPQCD Collaboration

- ✓ Form factor for $B \to D\ell\nu$ (2015)
- ✓ Form factors for B-to-light meson (2021)

RBC/UKQCD

- ✓ Semileptonic $B_s \to K\ell\nu$ (2023)

- **No results from** LPC
  - Difficulties
  - **Please stay tuned for our subsequent research progress!**

郑州大学
ZHENGZHOU UNIVERSITY

Thanks！

Welcome to Zhengzhou!