DiPion light-cone distribution amplitudes and the B_{l4}, D_{l4} decays

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November 25, 2023

Overview

Take DiPion as the example

- Light-cone distribution amplitudes
- Double expansion and the coefficient $B_{nl}^{(I)}(s)$

DiPion LCDAs in B_{l4} and D_{l4} decays

- $B \to \pi\pi$ form factors
- $D_s \to [\pi^+\pi^-]_{\rm S}\, e^+\nu_e$

Conclusion and Prospect

DiPion

Why DiPion ?

- $|V_{ub}|$ tension $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2022]

‡ $|V_{ub}|_{\rm in} = (4.13 \pm 0.25) \times 10^{-3}, |V_{ub}|_{\rm ex} = (3.70 \pm 0.16) \times 10^{-3}$, $\sim 2.5\sigma$

‡ enlarge the set of exclusive processes to determine $|V_{ub}|$, a candidate is $B \to \rho l \nu$

△ ρ is reconstructed by $\pi\pi$ invariant mass spectral, width effect/nonresonant contribution ?

△ the underlying consideration is $B \to \pi\pi l \bar{\nu}_l$ (B_{l4}) [Faller 2014]

- V_{cs} issue $|V_{cs}| = 0.975 \pm 0.006$

‡ $|V_{cs}|_{\rm semilep} = 0.972 \pm 0.007, |V_{cs}|_{\rm leptonic} = 0.984 \pm 0.012,$ $\sim 1.5\sigma$ derivation

‡ $\sim 3\sigma$ tension two years ago, 0.939 ± 0.038 and 0.992 ± 0.012 [PDG 2020, 2021]

‡ new channels like semileptonic $D_s^{(*)}$ decays are highly anticipated

△ $D_s \to f_0 l \nu$, large uncertainty due to the width and complicate structure

△ $D_s^* \to \phi l \nu$, small branching fraction

- **Introduce DiPion LCDAs to describe the width effects (resonance contribution and nonresonant background) in heavy flavor decays**

- Phenomena: Multibody decays, $B \to [\pi\pi, K\pi]\, l\bar{\nu}$, $B \to D\pi l \bar{\nu}$ and LFU ...

Pion LCDAs

Study Pion DAs in the light-cone dominated processes

- DA is expressed by the MEs of gauge invariant non-local operators

$$\langle 0|\bar{u}(x)\Gamma[x,-x]d(-x)|\pi^-(P)\rangle$$

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt(x-y)_\mu A^\mu(tx+\bar{t}y)\right]$$

- Introduce light-like vectors p and z: $P^2 = m_\pi^2,\ p^2 = 0,\ z^2 = 0$

△ $P \to p$ in the limit $m_\pi^2 = 0$ and $x \to z$ for $x^2 = 0$

△ expansion in power of large momentum transfer is governed by contributions from small transversal separations x^2 between constituents

$$z_\mu = x_\mu - \frac{P_\mu}{m_\rho^2}\left[xP - \sqrt{(xP)^2 - x^2 m_\pi^2}\right] = x_\mu\left[1 - \frac{x^2 m_\pi^2}{4(z\cdot p)^2} - \frac{p_\mu}{2}\frac{x^2}{z\cdot p} + \mathcal{O}(x^4)\right]$$

$$p_\mu = P_\mu - \frac{z_\mu}{2}\frac{m_\pi^2}{p\cdot z} \quad \Rightarrow \quad z\cdot P = z\cdot p = \left[(xP)^2 - x^2 m_\pi^2\right]^{1/2}$$

△ Projector onto the directions orthogonal to p and z $\quad g_{\mu\nu}^{\perp} = g_{\mu\nu} - \frac{1}{p\cdot z}\left(p_\mu z_\nu + p_\nu z_\mu\right)$

△ For an arbitrary Lorentz vector a_μ and b_μ, $\quad a_z \equiv a_\mu z^\mu,\quad a_p \equiv a_\mu p^\mu,\quad b_{\mu z} \equiv b_{\mu\nu} z^\nu,\quad \cdots$

Pion LCDAs

- Introduce LCDAs by the MEs of non-local operators on the light-cone

$$\langle 0|\bar{u}(z)\Gamma[z,-z]d(-z)|\pi^-(P)\rangle \propto \phi_t(u,\mu)$$

- Up to (power) twist 3 of lowest Fock wave function $\zeta = 2u - 1,\ m_0^\pi = \frac{m_\pi^2}{m_0 + m_d}$

$$\langle 0|\bar{u}(z)\gamma_z\gamma_5 d(-z)|\pi^-(P)\rangle = if_\pi p_z \int_0^1 du\, e^{i\zeta p\cdot z}\phi(u,\mu)$$

$$\langle 0|\bar{u}(z)i\gamma_5 d(-z)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta p\cdot z}\phi^p(u,\mu)$$

$$\langle 0|\bar{u}(z)i\sigma_{\mu\nu}\gamma_5 d(-z)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}\left(p_\mu z_\nu - p_\nu z_\mu\right)\int_0^1 du\, e^{i\zeta p\cdot z}\phi^\sigma(u,\mu)$$

- LCDAs are dimensionless functions of u and renormalization scale μ

△ describe the probability amplitudes to find the π in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ the nonlocal operators on the lhs are renormalized at scale μ with the factor $Z_2(\mu)$

$\phi_2(u,\mu) = Z_2(\mu)\int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{\rm BS}(u,k_\perp)$

△ decay constant $\langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle = if_\pi p_\mu$ △ normalization $\int_0^1 du\, \Phi(u) = 1$

Pion LCDAs

- three sources of the power suppressed contributions to exclusive processes in QCD
  - ‡ bad component (wrong spin projection) in the wave function
  - ‡ transversal motion of q ($\bar{q}$) in the leading twist components
  - ‡ higher Fock states with additional gluons and/or $q\bar{q}$ pairs

- define the LCDAs with the Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi \int_0^1 du\, e^{i\zeta P\cdot x}\left[iP_\mu\left(\phi(u)+\frac{x^2}{4}g_1(u,\mu)\right)\right.$$
$$\left.+\left(x_\mu-\frac{x^2 P_\mu}{2P\cdot x}\right)g_2(u,\mu)\right]$$

$$\langle 0|\bar{u}(x)i\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta P\cdot x}\phi^p(u,\mu)$$

$$\langle 0|\bar{u}(x)i\sigma_{\mu\nu}\gamma_5 d(-x)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}\left(P_\mu x_\nu - P_\nu x_\mu\right)\int_0^1 du\, e^{i\zeta P\cdot x}\phi^\sigma(u,\mu)$$

Pion LCDAs

Conformal spin and collinear twist definition

[Braun & Korchemsky & Müller 2003]

- A convenient tool to study DAs is provided by conformal expansion
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD* *under conformal trans.* *VS rotation symmetry*
- *longitudinal* $\otimes$ *transversal* dofs *VS angular* $\otimes$ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group
  collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$

Pion LCDAs

$$\langle 0|\bar{u}_i(0)d_j(z)|\pi^-(p)\rangle = -\frac{i}{4}\int_0^1 du e^{-iup\cdot z}\left[\not{p}\gamma_5\phi_\pi(u) + \gamma_5\phi_\pi^p(u) + \gamma_5(1-\not{n}\not{\bar{n}})\phi_\pi^t(u)\right]_{ji}$$

$$\phi_\pi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu)C_n^{3/2}(u)$$

$$\phi_\pi^p(u,\mu) = \frac{m_0^\pi(\mu)}{p^+}\left[1 + 30\eta_{3\pi}C_2^{1/2}(u) - 3\eta_{3\pi}\omega_{3\pi}C_4^{1/2}(u)\right]$$

$$\phi_\pi^\sigma(u) = \frac{m_0^\pi(\mu)}{p^+}6u(1-u)\left[1 + 5\eta_{3\pi}C_2^{3/2}(u)\right]$$

- $\phi(x)$ and $\phi^{p,t}(u)$ are the twist two and twist three LCDAs, twist four $\cdots$
- $a_0^\pi = f_\pi$, $a_{n\geqslant 2}^\pi(\mu_0)$ and $m_0^\pi(\mu_0)$ obtained by non-pert. theory/lattice QCD
- μ dependences in a_n^π the integration over the transversal dof

  [Brodsky & Lepage1980, Balitsky & Braun1988]
- $C_n(u)$ are Gegenbauer polynomials $\sim$ Jacobi Polynomials $P_n^{j_1,j_2}\left(\frac{\overleftrightarrow{D}_+}{\overleftrightarrow{\partial}_+}\right)$ in the local collinear conformal expansion longitudinal dof

  [Lepage & Brodsky 1979, 80, Efremov & Radyushkin 1980, Braun & Filyanov 1990]
- Great achievements (high precision) in F_π, $B \to \pi$ et.al processes

DiPion LCDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle\pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\,k_\mu \int dx\, e^{iuz(k\cdot n)}\,\Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

△ $n^2 = 0$, △ index f, f' respects the (anti-)quark flavor, △ a, b indicates the electric charge

△ coefficient $\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, △ $k = k_1 + k_2$ is the invariant mass of dipion state

△ $\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs

△ DiPion is not a tetraquark state, but a collinear two pion system with nonlocal $q\bar{q}, \cdots$ operators

† Three independent kinematic variables

△ momentum fraction z carried by anti-quark with respecting to the total momentum of DiPion state,

△ longitudinal momentum fraction carried by one of the pions $\zeta = k_1^+/k^+$, $2q\cdot\bar{k}\ (\propto 2\zeta - 1)$ △ k^2

† Normalization conditions

$$\int_0^1 \Phi_{\parallel}^{I=1(0)}(u,\zeta,k^2) = (2\zeta - 1)F_\pi(k^2)$$

$$\int_0^1 dz\,(2z-1)\Phi_{\parallel}^{I=0}(z,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\rm EMT}(k^2)$$

△ $F_\pi^{em}(0) = 1$, △ $F_\pi^{\rm EMT}(0) = 1$,

△ $M_2^{(\pi)}$ is the momentum fraction carried by quarks in the pion associated to the usual quark distribution

DiPion LCDAs

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\text{even}}^{\infty}\sum_{l=1,\text{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\text{odd}}^{\infty}\sum_{l=0,\text{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π,ρ,f_0 mesons

$$B_{n\ell}(k^2,\mu) = B_{n\ell}(k^2,\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{[\gamma_n^{(0)}-\gamma_0^{(0)}]/[2\beta_0]}, \quad \gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k}-\frac{3}{4}-\frac{1}{2(n+1)(n+2)}\right)$$

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel,I=1}(0) = a_n^\pi\,, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel,I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing

△ express the moments of SPDs in terms of $B_{nl}(k^2)$ in the forward limit as

$$M_{N=\text{odd}}^\pi = \frac{3}{2}\frac{N+1}{2N+1}B_{N-1,N}^{I=1}(0), \quad M_{N=\text{even}}^\pi = 3\frac{N+1}{2N+1}B_{N-1,N}^{I=0}(0)$$

DiPion LCDAs

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0

△ relation between the a_n^ρ and the coefficients $B_{n\ell}$

$$a_n^\rho = B_{n1}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} c_m^{n1} m_\rho^{2m}\right], \quad c_m^{(n1)} = \frac{1}{m!}\frac{d^m}{dk^{2m}}\left[\ln B_{n1}(0) - \ln B_{01}(0)\right]$$

△ f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ by $\qquad \langle \pi(k_1)\pi(k_2)|\rho\rangle = g_{\rho\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha$

$$f_\rho^{\parallel} = \frac{\sqrt{2}\,\Gamma_\rho\,\mathrm{Im}B_{01}^{\parallel}(m_\rho^2)}{g_{\rho\pi\pi}}, \quad f_\rho^{\perp} = \frac{\sqrt{2}\,\Gamma_\rho\, m_\rho\,\mathrm{Im}B_{01}^{\perp}(m_\rho^2)}{g_{\rho\pi\pi}\, f_{2\pi}^{\perp}}$$

- The subtraction constants of $B_{n\ell}(s)$

(nl)	$B_{n\ell}^{\parallel}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B_{n\ell}^{\parallel}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B_{n\ell}^{\perp}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^π, a_2^ρ [SC 2019, 2023]

DiPion LCDAs

- How to describe **the evolution from** $4m_\pi^2$ **to large invariant mass** $k^2 \sim \mathcal{O}(m_c^2)$?

- Watson theorem of π-π scattering amplitudes

  △ implies an intuitive way to express the imaginary part of 2πDAs

  △ leads to the Omnés solution of N-subtracted DR for the coefficients

$$B^I_{n\ell}(k^2) = B^I_{n\ell}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B^I_{n\ell}(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds\, \frac{\delta^I_\ell(s)}{s^N(s-k^2-i0)}\right]$$

- **2πDAs in a wide range of energies is given by δ^I_ℓ and a few subtraction constants**

- Above discussions are all at leading twist level, subleading twist LCDAs are not known yet

‡ Would the chiral EFT help us to set down $B_{nl}(k^2 \sim 4m_\pi^2)$ for both leading and subleading twist LCDAs ?

$B \to \pi\pi$ form factors

$D_s \to [\pi\pi]_{\mathrm{S}}\, e^+ \nu_e$

$B \to \pi\pi$ form factors

- Probing different exclusive $b \to u$ processes help in the V_{ub} determination
- B_{l4} decays have rich observables, nontrivial tests of SM [Faller 2014]

- $B \to \pi\pi l\bar{\nu}_l$ has already been measured, mainly its resonant part $B \to \rho l\bar{\nu}_l$
  $(1.58 \pm 0.11) \times 10^{-4}$ [CLEO 2000, BABAR 2011, Belle 2013]
- First measurement of the branching fraction of $B^+ \to \pi^+\pi^- l^+\bar{\nu}_l$
  $(2.3 \pm 0.4) \times 10^{-4}$ [Belle 2020]
- More detailed data on $B \to \pi\pi l\bar{\nu}$ observables are expected from Belle II

- Dynamics is governed by the $B \to \pi\pi$ form factors, a big task for the practitioners of QCD-based methods
- How large of ρ contribution in $B \to \pi\pi$ form factors ?

- First Lattice QCD study of the $B \to \pi\pi l\bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [arXiv:2212.08833 [hep-lat]]

$B \to \pi\pi$ form factors

$$i\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma$$
$$+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right)$$
$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)$$

- $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function
- $q\cdot k = (m_B^2 - q^2 - k^2)/2$ and $q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta - 1)$
- $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

- Starting with the correlation function

$$\begin{aligned} F_\mu(k_1,k_2,q) &= i\int d^4x e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle \\ &\downarrow \textit{Lorentz decomposition} \\ &\equiv \varepsilon_{\mu\nu\rho\sigma}q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})} \end{aligned}$$

$B \to \pi\pi$ form factors

- Take $F^l_\perp(q^2, k^2, \zeta)$ as an example

$$\frac{F^l_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2 f^\perp_{2\pi}\,(2\zeta-1)} \int_{u_0}^1 \frac{du}{u}\,\Phi^l_\perp(u,\zeta,k^2)\, e^{-\frac{s(u)+m_B^2}{M^2}}$$

- Definition of the Chiral-odd DiPion LCDAs

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(xn)\sigma_{\mu\nu}\tau q_{f'}(0)|0\rangle = \kappa_{ab}\,\frac{2i}{f^\perp_{2\pi}}\,\frac{k_{1\mu}k_{2\nu} - k_{2\mu}k_{1\nu}}{2\zeta-1}\int dx\, e^{izx(k\cdot n)}\,\Phi^{ab,ff'}_\perp(z,\zeta,k^2)$$

- Partial wave expansion

$$F_{\perp,\parallel}(k^2,q^2,\zeta) = \sum_\ell \sqrt{2\ell+1}\, F^{(\ell)}_{\perp,\parallel}(k^2,q^2)\,\frac{P^{(1)}_\ell(\cos\theta_\pi)}{\sin\theta_\pi}$$

- Using the orthogonality relation of the Legender polynomials

$$F^{(\ell)}_\perp(k^2,q^2) = \frac{\sqrt{k^2}}{\sqrt{2} f^\perp_{2\pi}}\,\frac{\sqrt{\lambda_B}\, m_b}{m_B^2 f_B}\, e^{\frac{m_B^2}{M^2}} \sum_{n=0,2,\cdots}\ \sum_{\ell'=1,3}^{n+1} I_{\ell\ell'}\, B^\perp_{n\ell'}(k^2,\mu) J^\perp_n(q^2,k^2,M^2,s_0^B).$$

$$I_{\ell\ell'} \equiv -\frac{\sqrt{2\ell+1}(\ell-1!)}{2(\ell+1)!}\int_{-1}^1 \frac{dz}{z}\sqrt{1-z^2}\,P^{(1)}_\ell(z) P^{(0)}_{\ell'}(z),$$

$$J^\perp_n(q^2,k^2,M^2,s_0^B) = \int_{u_0}^1 du e^{\frac{-s}{M^2}}\, 6(1-u) C^{3/2}_n(2u-1).$$

$B \to \pi\pi$ form factors [Hambrock 2016, Cheng 2017, Cheng 2019]

- $I_{\ell\ell'} = 0$ when $\ell > \ell'$, $I_{11} = 1/\sqrt{3}$, $I_{13} = -1/\sqrt{3}$, $I_{15} = 4/(5\sqrt{3})$
- $\ell' = 1$, asymptotic DAs, $P-$wave term remains in the DiPion LCDAs

- Short-distance part of the correlation: $\mu = 3$ GeV without NLO correction
- $f_B = 207^{+9}_{-17}$ MeV [P. Gelhausen 2013,2014]
- $M^2 = 16.0 \pm 4.0$ GeV^2 corresponding to $s_0^B = 37.5 \pm 2.5$ GeV^2

- How large of $P-$wave contribution to $B \to \pi\pi$ FFs $(\ell = 1)$?

$$R_\ell \equiv \frac{F_\perp^{(\ell>1)}(k^2, q^2)}{F_\perp^{(\ell=1)}(k^2, q^2)}$$

- How much ρ contained in $P-$wave $B \to \pi\pi$ FFs $(\ell = 1, \ell' = 1)$?
- Hadronic dispersion relation for the P-wave part of $B \to \pi\pi$ form factors

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = \langle \pi^+(k_1)\pi^0(k_2)|\rho\rangle\langle\rho|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle + \cdots$$

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \cdots$$

△ $\rho \to 2\pi$ strong coupling $g_{\rho\pi\pi} = 5.96 \pm 0.04 \Leftarrow$ the energy dependent total width of ρ

$B \to \pi\pi$ form factors

- And the leading twist $B \to \rho$ form factor obtained from ρ-LCDAs under the zero-width approximation

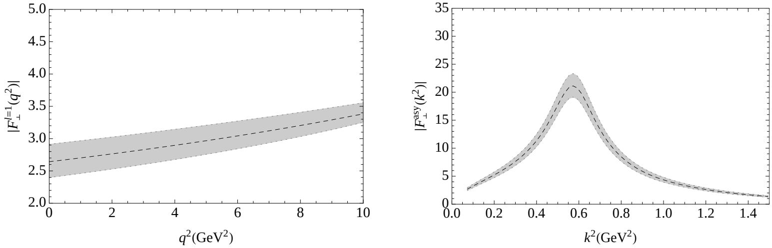
$$V^{B\to\rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp \int_{u_0}^1 \frac{du}{u} \phi_\perp^{(\rho)}(u)\, e^{m_B^2/M^2} e^{(-q^2\bar{u}+m_\rho^2 u\bar{u})/(uM^2)}$$

$$i\langle\rho^+(k)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = i\varepsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}q^\beta k^\gamma \frac{V(q^2)}{m_B+m_\rho} + \epsilon_\nu^*\,(m_B+m_\rho)\,A_1(q^2)$$

$$-(2k+q)_\nu\,(\epsilon^*\cdot q)\,\frac{A_2(q^2)}{m_B+m_\rho} - q_\nu\,(\epsilon^*\cdot q)\,\frac{2m_\rho^2}{q^2}\left[A_3(q^2)-A_0(q^2)\right]$$

- ρ meson: $a_2^\perp = 0.2 \pm 0.1$, $a_{n>2} = 0$, $f_\rho^\perp = 160 \pm 10$ MeV

$B \to \pi\pi$ form factors [Hambrock 2016, Cheng 2017, Cheng 2019]



- $|F_\perp^{\ell=1}(q^2, 4m_\pi^2)|$ (left) and asymptotic $|F_\perp^{\ell=1,\ell'=1}(0.1, k^2)|$ (right)

- At the current accuracy: LO and leading twist of 2πDAs
- High partial waves give few percent contributions to $B \to \pi\pi$ form factors
- ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

- $|F_\perp^{\ell=1}(q^2, 4m_\pi^2)|$ obtained from 2πDAs shows about 30% smaller than it obtained from B-meson LCDAs, high twist contributions ? Uncertainty of B-meson LCDAs ?

$D_s \to (f_0 \to) [\pi\pi]_S e^+ \nu_e$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons

△ $D_{(s)} \to a_0 e^+ \nu$[BESIII 18, 21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII 19], $D_s \to f_0(\to \pi^+\pi^-) e^+ \nu$[CLEO 09]
△ $\mathcal{B}$ of $D_s \to f_0(\to \pi^0\pi^0, K_s K_s) e^+ \nu$[BESIII 22], $D_s \to f_0(\to \pi^+\pi^-) e^+ \nu$ form factor[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

△ isospin symmetry expectation $\mathcal{B}(f_0 \to \pi^+\pi^-)/\mathcal{B}(f_0 \to \pi^0\pi^0) = 2$, possible ρ^0 pollution
△ $f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$

- Theoretical consideration $\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2$

- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1^2 \beta_\pi(s)}{|m_S^2 - s + i\,(g_1^2 \beta_\pi(s)) + g_2^2 \beta_K(s))\,|^2}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}} q^2}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

- $D_s \to f_0$ ffs to $D_s \to [\pi\pi]_S$ ffs

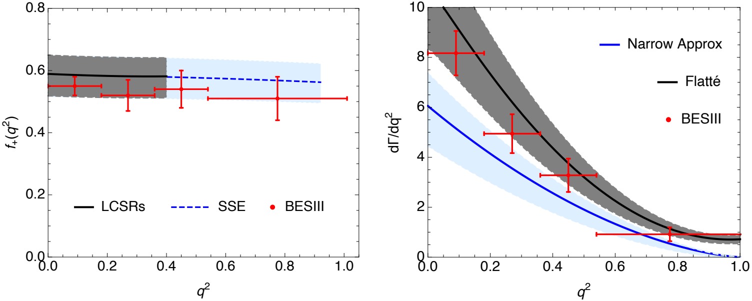
$D_s \to (f_0 \to) [\pi\pi]_S \, e^+ \nu_e$ [Cheng 2023]

- Definitions of $D_s \to f_0$ form factors

$$\langle f_0(p_1)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p)\rangle = -i\left[f_+(q^2)\,(p+p_1)_\mu + f_-(q^2)q_\mu\right]$$

- Form factor and the differential decay width

△ $M^2 = 5.0 \pm 0.5\ \text{GeV}^2$ and $s_0 = 6.0 \pm 0.5\ \text{GeV}^2$, △ $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV used in the previous LCSRs [Colangelo 2010], △ $a_1^{s/\sigma}$ term contributions are considered for the first time , △ f_0 is not a pure $\bar{s}s$ state, the mixing angle is chosen at $20^\circ \pm 10^\circ$

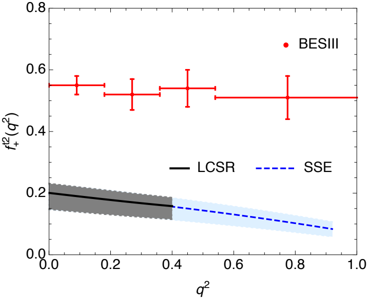


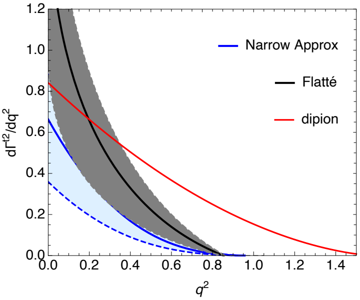
$D_s \to (f_0 \to) \, [\pi\pi]_{\mathrm{S}} \, e^+ \nu_e$ [Cheng 2023]

- Definitions of $D_s \to [\pi\pi]_{\mathrm{S}}$ form factors

$$\langle [\pi(k_1)\pi(k_2)]_{\mathrm{S}} \, | \bar{s}\gamma_\mu(1-\gamma_5)c | D_s^+(p)\rangle = -iF_t(q^2, s, \zeta)k_\mu^t - iF_0(q^2, s, \zeta)k_\mu^0 - iF_{\|}(q^2, s, \zeta)k_\mu^{\|}$$

- Form factor and the differential decay width at leading twist [SC 2023]





- subleading twist LCDAs give dominate contribution in $D_s \to [\pi\pi]_{\mathrm{S}}$ transition
- different in B/Z case where the leading twist is dominate/overwhelming
- shows relatively moderate evolution with larger allowed momentum transfer
- further measurements would help us to understand the dipion system, ρ, f_0

Conclusion and Prospect

- The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons

- Possible improvement study in the CKM determinations and the flavor anomalies
- A new booster on the accurate calculation in flavor physics ?

Thank you for your patience.