

$B \rightarrow D$ form factors beyond leading power and extraction of $|V_{cb}|$ and $R(D)$

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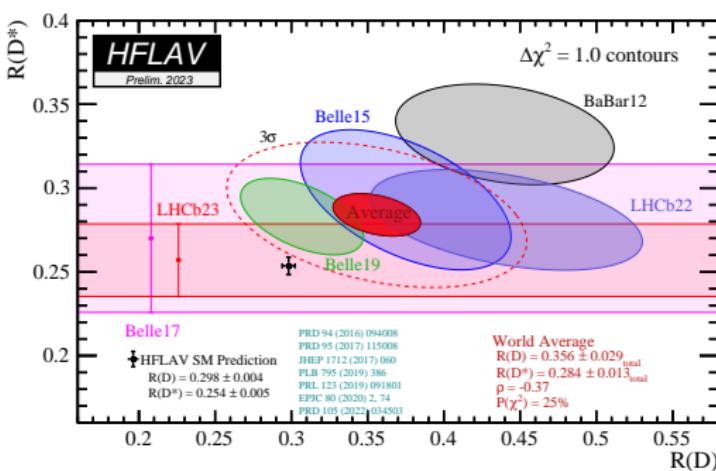
Outline

- * Introduction to $B \rightarrow D\ell\nu_\ell$ decay
- * Factorization formula of $B \rightarrow D$ form factors at **leading power**
- * **Subleading power corrections** to $B \rightarrow D$ form factors
- * Numeric applications

Introduction

New physics: $R(D^{(*)})$ anomalies $R(D) - R(D^*) \sim 3.2\sigma$

$$R(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)}, \quad R(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^*\ell\nu_\ell)}$$



BaBar: [1205.5442]

Evidence for an excess of $\bar{B} \rightarrow D^{(*)}\tau^- \bar{\nu}_\tau$ decays

BaBar Collaboration • J.P. Lees (Annecy, LAPP) et al. (May, 2012)

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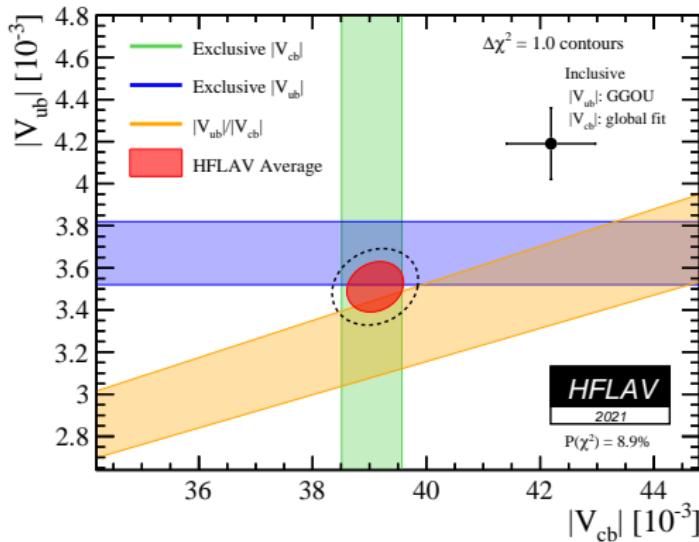
Belle: [1910.05864]

LHCb: [2302.02886]

Belle-II, HL-LHC

Introduction

$|V_{cb}|$: about 3σ tension between exclusive and inclusive results.



- Inclusive: $B \rightarrow X_c \ell \nu_\ell$
CLEO: [\[hep-ex/0403052\]](#)
BaBar: [\[0908.0415\]](#)
Belle: [\[hep-ex/0610012\]](#)
- Exclusive: $B \rightarrow D \ell \nu_\ell$
CLEO: [\[hep-ex/0203032\]](#)
BaBar: [\[0712.3503\]](#)
Belle: [\[1510.03657\]](#)
Belle-II: [\[2008.07198\]](#)

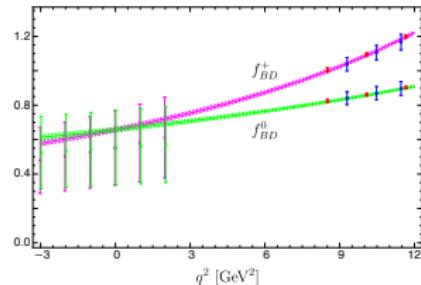
$$\frac{d\Gamma(B \rightarrow D \ell \nu_\ell)}{dq^2} = |V_{cb}|^2 \left[C_{f+}(q^2) |f_{BD}^+(q^2)|^2 + C_{f0}(q^2) |f_{BD}^0(q^2)|^2 \right]$$

$B \rightarrow D$ form factors

$$\langle D(p) | \bar{c} \gamma_\mu b | \bar{B}(p_B) \rangle = f_{BD}^+(q^2) \left[2p_\mu + \left(1 - \frac{m_B^2 - m_D^2}{q^2} \right) q_\mu \right] + f_{BD}^0(q^2) \frac{m_B^2 - m_D^2}{q^2} q_\mu$$

- D -meson small recoil region

- HQET: $1/m_b$, $1/m_c$ corrections [Bernlochner, Ligeti et al. 1703.05330], [Bigi, Gambino and Schacht, 1707.09509]
- LQCD: 2+1 flavor [MILC 1503.07237], [HPQCD 1505.03925], [JLQCD 1912.11770]



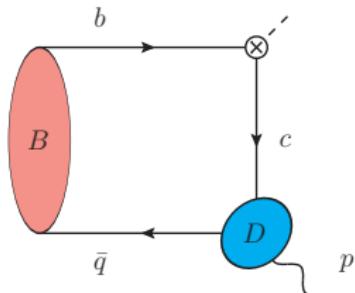
- D -meson large recoil region

- SCET, QCD Factorization: endpoint singularity [Beneke and Feldmann, 0311335]
- PQCD: NLO [Fan, Wang, Cheng and Xiao, 1301.6246], [Fan, Xiao, Wang and Li, 1505.07169], [Hu, Jin and Xiao, 1912.03918]
- LCSR: LP [Wang, YBW, Shen and Lü, 1701.06810], [Zhang et al. 1709.02226] LP+LO, NLP (3P LCDA) [Gubernari, Kokulu and Dyk, 1811.00983] LP+NLO, NLP [Gao, Huber, Ji, Wang, Wang and YBW, 2112.12674]

$B \rightarrow D$ form factors in LCSR

Two-point correlation function [Colangelo and Khodjamirian, hep-ph/0010175]

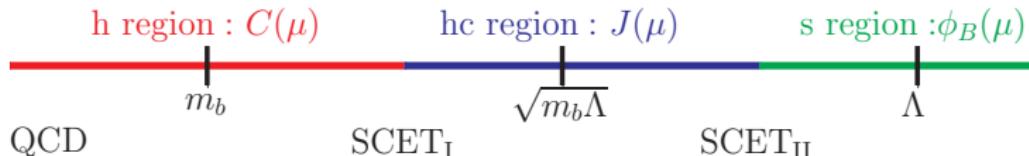
$$\Pi = i \int d^4x e^{ip \cdot x} \langle 0 | T\{\bar{q}(x) \not{p} \gamma_5 c(x), \bar{c}(0) \gamma_\mu b(0)\} | \bar{B}(p_B) \rangle$$



- Hadronic level: $\sum_n |n\rangle\langle n| \Rightarrow f_D \times f_{BD}^{+,0} + \text{cont. sta.}$
- Partonic level: factorization formula for Π_p
- Parton-hadron duality: $f_{BD}^{+,0} = (\Pi_p - \text{cont. sta.})/f_D$
- Borel transformation to avoid the endpoint divergence

Leading power

- hard (h) scale $\mathcal{O}(m_b)$, soft (s) scale $\mathcal{O}(\Lambda)$
- hard-collinear (hc) scale $\mathcal{O}(\sqrt{m_b \Lambda})$: $m_c \sim \sqrt{m_b \Lambda}, \sqrt{k_s \cdot k_c}$



Method of regions [Beneke and Smirnov, hep-ph/9711391]

SCET [Bauer, Fleming, Pirjol and Stewart, hep-ph/0011336], [Beneke, Chapovsky, Diehl and Feldmann, hep-ph/0206152]

$$\Pi_p = C(\mu) \otimes J(\mu) \otimes \phi_B(\mu)$$

Resummation of large logs: RG evolution



Subleading power corrections

Collider physics: refactorization of endpoint divergence

- $H \rightarrow \gamma\gamma$: [Liu and Neubert, 1912.08818], [Liu, Mecaj, Neubert and Wang, 2009.06779]
- DIS, DY: [Beneke et al., 2205.04479]

Power corrections are numerically important in B decays, $\lambda = \Lambda/m_b$

$$\lambda \sim \alpha_s(\mu) \sim 20\% \Rightarrow \text{NLP@LO} \sim \text{LP@NLO}$$

- * Higher Fock states of the B meson: $|B\rangle \Rightarrow |b\bar{q}g\rangle$
- * Charm-quark expansion: $p \sim hc$ and $k \sim s$

$$\frac{(\not{p} - \not{k}) + m_c}{(\not{p} - \not{k})^2 - m_c^2} = \frac{1}{\not{n} \cdot \hat{p}} \left\{ \frac{\not{n}}{2} + \underbrace{\frac{\not{n} \cdot \not{p}}{m_c} \left[\frac{\not{n}}{2} + \dots \right]}_{\mathcal{O}(\lambda)} + \dots \right\}$$

- * Heavy-quark expansion: QCD \rightarrow HQET

$$b = h_v + \frac{i\not{D}_\perp}{2m_b} h_v + \dots, \quad i\not{D}_\perp/m_b \sim \mathcal{O}(\lambda)$$

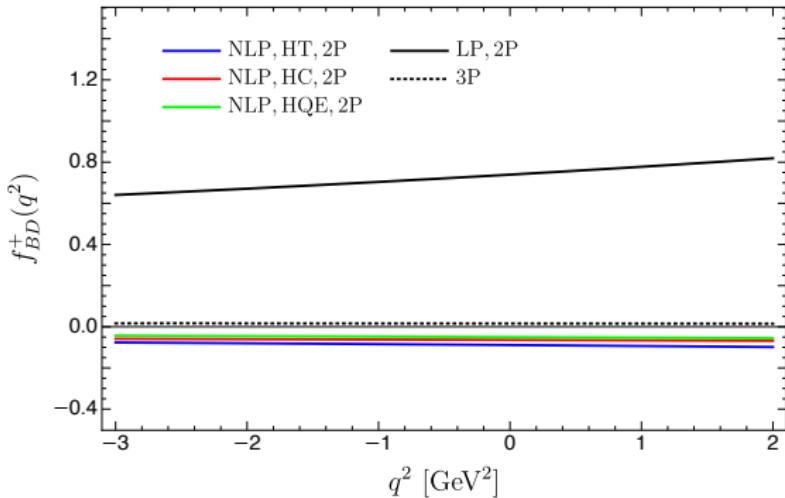
Numerical results

Large recoil region:

$$\mu \in [1.0, 2.0] \text{ GeV}$$

$C(\mu)$: NLL

$\phi_B(\mu)$: LL

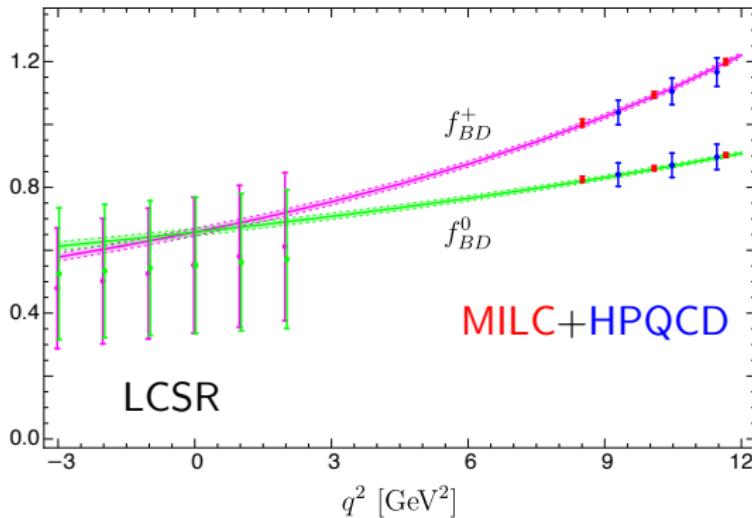


LP	NLP, HT	NLP, HC exp.	NLP, HQ exp.
0.74	-0.09	-0.06	-0.05

NLP: $\sim 20\%$

f_{BD}^+ decrease, $|V_{cb}|$ increase

Numerical results



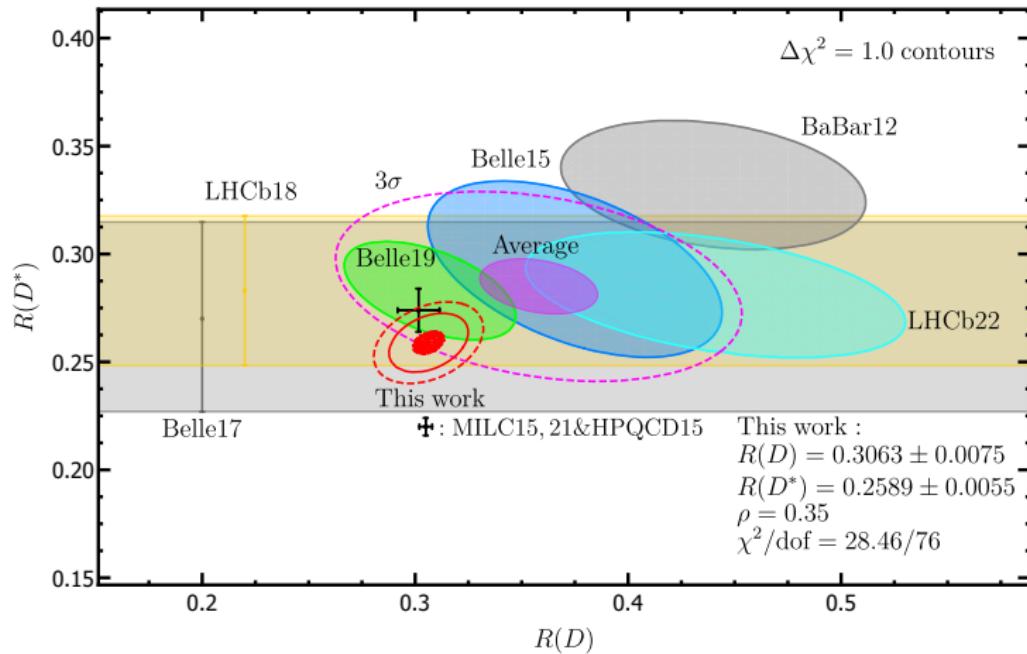
$$R(D): 1.98\sigma \rightarrow 1.85\sigma$$

$$|V_{cb}| = \begin{cases} (42.19 \pm 0.78) & |_{\text{incl.}} \\ (39.10 \pm 0.50) & |_{\text{excl.}} \end{cases}$$

	LCSR	Lattice	HQET	This work
$ V_{cb} \times 10^3$	40.3(0.8)	39.36(68)	39.3(1.0)	$40.2^{+0.6}_{-0.5} _{\text{th}}^{+1.4} _{\text{BaBar}}$ $40.9^{+0.6}_{-0.5} _{\text{th}}^{+1.0} _{\text{Belle}}$
$R(D)$	0.297(3)	0.299(11)	0.299(3)	0.302(3)

Numerical results

$\bar{B}_{(s)} \rightarrow D_{(s)}^{(*)} \ell \bar{\nu}_\ell$ decays+LQCD results: $3.2\sigma \rightarrow 2.5\sigma$



[Cui, Huang, Wang and Zhao, 2301.12391]

Summary

- * Introduction to the $B \rightarrow D\ell\nu_\ell$ decay
- * $B \rightarrow D$ form factors with LCSR at large recoil
 - LP factorization formula: $C(\mu) \otimes J(\mu) \otimes \phi_B(\mu)$
 - NLP power corrections
 - B -meson higher Fock states
 - Charm-quark expansion
 - Heavy-quark expansion
- * Extraction of $|V_{cb}|$ and $R(D)$: LCSR+LQCD

Thank you!