手征有效场论在轻强子研究中的一些应用

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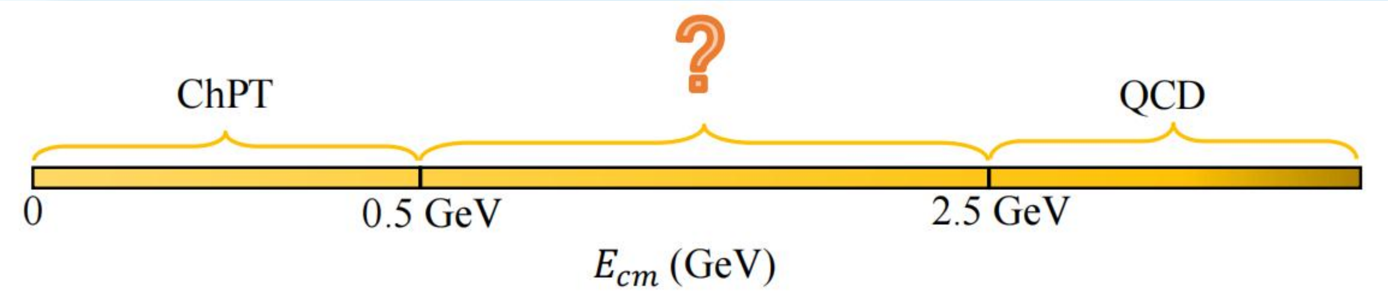
Outline

1. Introduction
2. Some techniques for ChEFT
3. $\pi\pi$，πK scatterings
4. light scalar mesons、g-2
5. Summary

1. Introduction

- QCD works above Λ_{QCD}
- ChPT works near the threshold
- 0.5-2.5 GeV: plenty of resonances
- RChT: introducing heavier resonances as new degrees of freedom
- ChEFT for baryons



Why $\pi\pi$, πK scatterings?

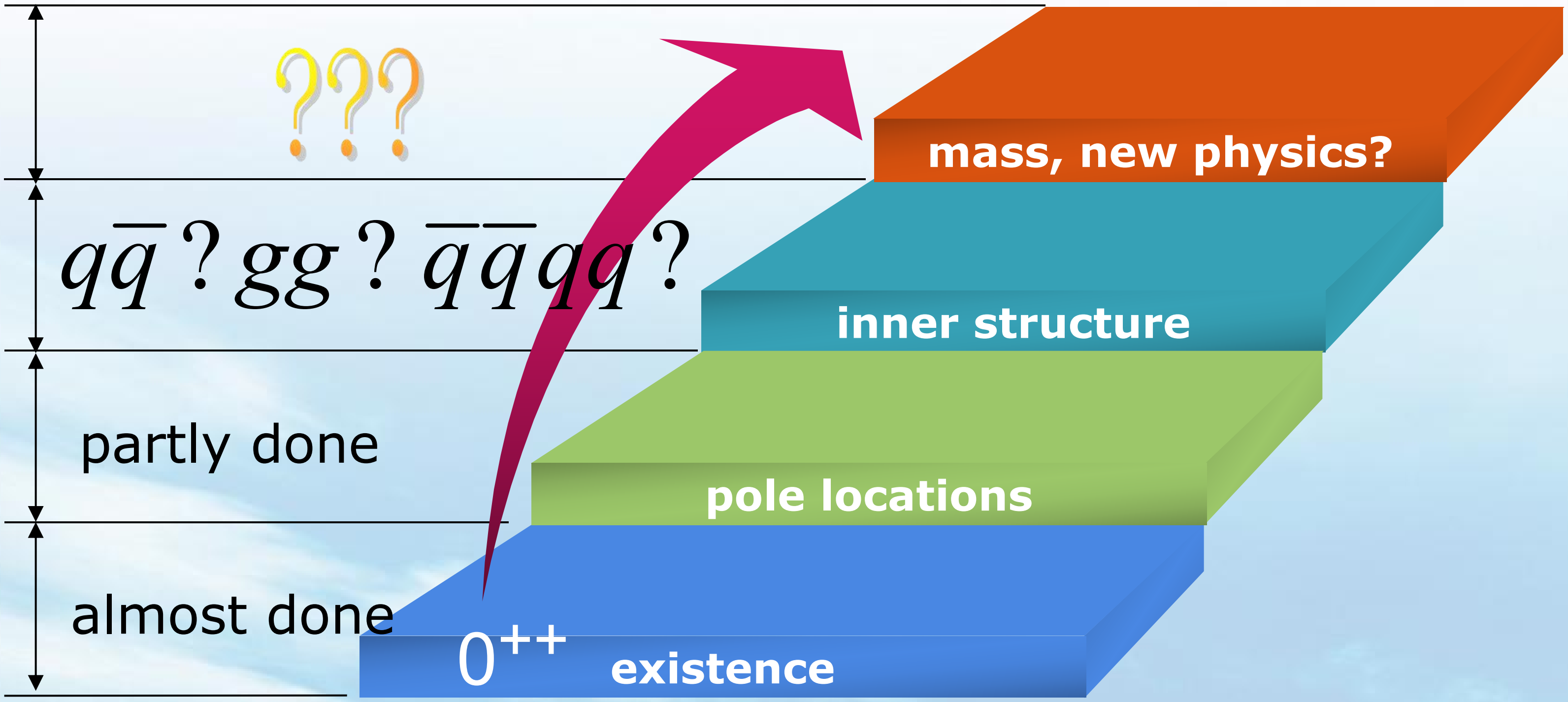
- **Both experimental and theoretical efforts for more than 50 years.**

- **Direct rich physics:**
  - **precise test of ChPT**
  - **quark masses and Chiral condensate**
  - **resonances**

- **Relevant to processes including $\pi\pi$, πK final states**
  - **Y(4260)→J/ψππ, ψ'→ϒππ, J/ψ→ϒππ,**

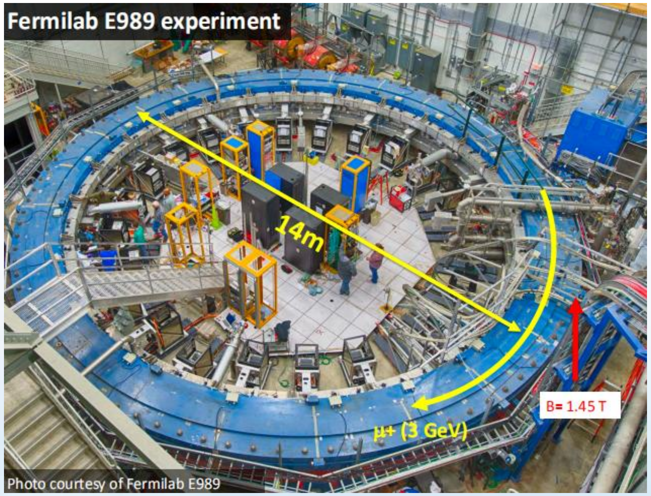
Scalars

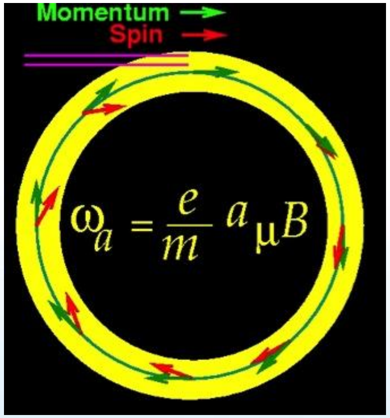
- Scalars: the same quantum number as that of QCD vaccum



muon g-2

- The most precise indicator of new physics

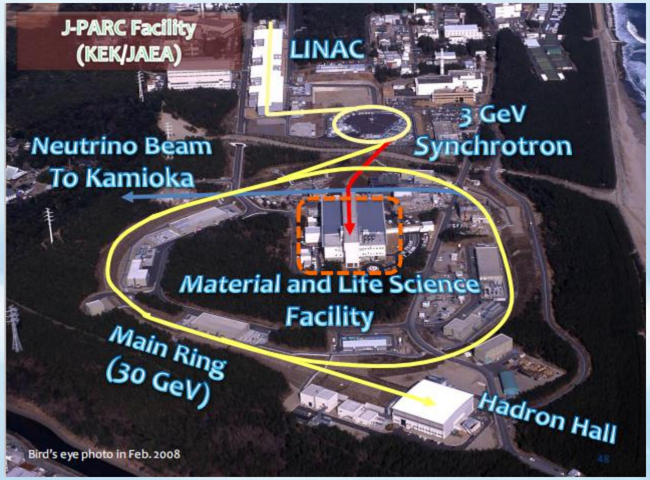




$$a = \frac{g-2}{2} \qquad \vec{\mu}_S = g\frac{q}{2m}\vec{S}$$

$$a_\mu = \frac{\omega_a/\omega_p}{\omega_a/\omega_p - \mu_\mu/\mu_p}$$

Tsutomu Mibe, talk at g-2 Theory Initiative



FNAL

Run1: only 6% of full statistics used, 2021

Run2-3: analyzing, factor 2 improvment, 2023

Run4: 13 times as large as BNL's

Run5: 20 times as large as BNL's

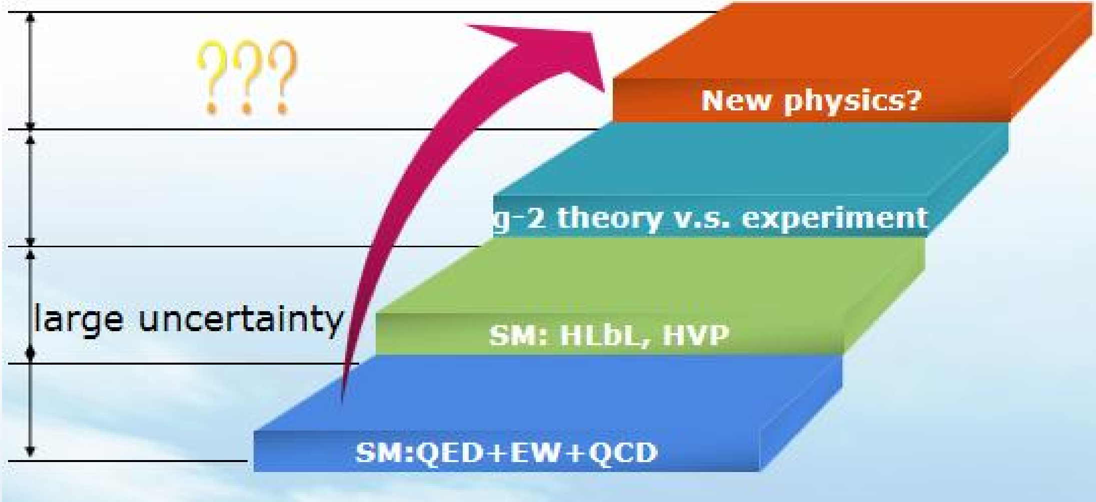
J-PARC

BNL E821 → J-PARC E3

g-2: 0.46 ppm → 0.37 ppm (→0.1ppm)

50 times as large as BNL's

uncertainty from SM



$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{QCD}}$$

- HVP, HLbL?

	values ($\times 10^{-11}$)
QED	116584718.931(104)
EW	153.6(1.0)
HVP	6845(40)
HLBL	92(18)
SM	116591810(43)
exp.(BNL)	116592089(63)
exp.(FNAL)	116592040(54)
exp.(avg.)	116592061(41)
a_μ^{SM}-a_μ^{exp}	251(59)

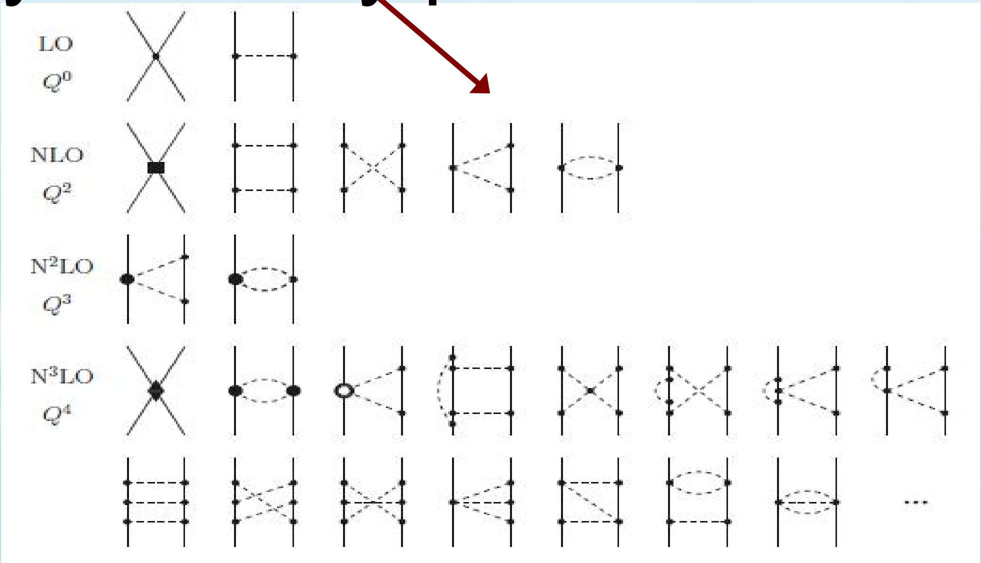
Phys.Rev.Lett.126, 141801 (2021)
Phys.Rev.D 73, 072003 (2006).

Phys.Rept.887(2020)1

Science and Technology Cooperation Program in High Energy Physics. This review benefited from discussions with O. Catà, N. Christ, L.Y. Dai, H. Davoudiasl, S. Fayer, S. Ganguly, A. Gasparian, S. Hashimoto, T. Iijima, K. Kampf, D. Kawall, I. Larin, Z. Pagel, M. Petschlies, A. Rebhan, K. Schilcher, K. Shimomura, E. Shintani, D. Steffen, S. Tracz, C. Tu, and T. Yamazaki.

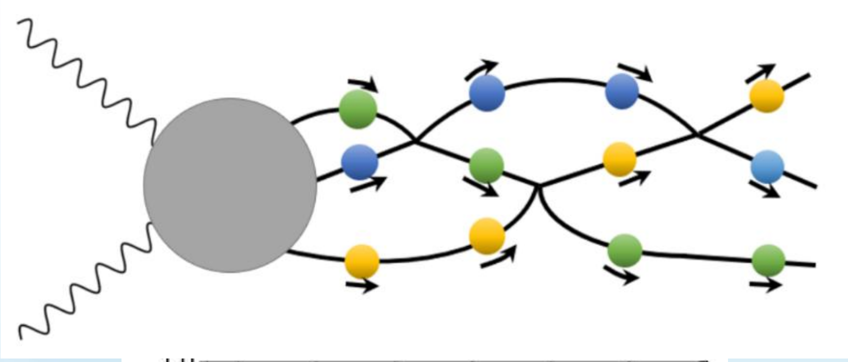
2. Some techniques for ChEFT

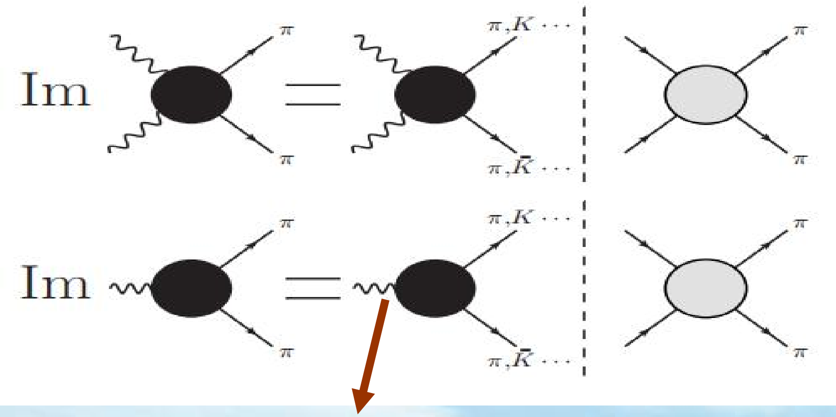
- ChEFT
  - Unrenormalizable theory, works only in low eneergy region
  - Power-counting for some ChEFTs?
  - Unknown LECs
- Improve the accuracy of theory prediction?



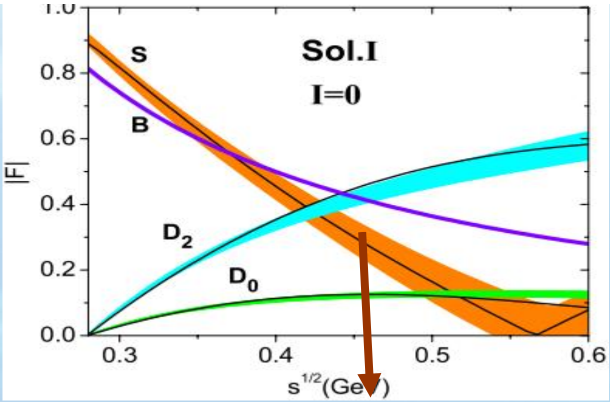
unrenormalizable: ChEFT+FSI

- The re-scattering of hadrons: FSI
- The Born term could be 'enhanced' by FSI
- FSI tools: KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*





Yao, Dai, Zheng, Zhou, RPP84(2021)076201



Dai&Pennington, PRD90 (2014) 036004

Power-counting for RChT?

- 1/Nc expansion,
  - loop diagrams are suppressed
  - uncertainty ~1/3

- 'chiral counting' by integrating out resonances
  - Those generating $O(p^6)$ ChPT Lagrangians

$$\langle R_a \chi(p^4)\rangle, \langle R_a R_b \chi(p^2)\rangle \text{ and } \langle R_a R_b R_c\rangle.$$

Dai *et.al.*, PRD99 (2019) 114015

Unkown LECs?

- Experimental determination
- Matching with QCD Green functions
  - RChT should give the same high energy behavior as that of QCD
- Extend GF to the unphysical region of LQCD?

LECs: Matching GFs

- Matching GF between QCD and ChEFT in the high energy region, using large Nc and OPE.

$$\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{i(p_1\cdot x + p_2\cdot y)} \langle 0|T\left\{S^i(0)A^j_\mu(x)A^k_\nu(y)\right\}|0\rangle$$

$$\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{i(p_1\cdot x + p_2\cdot y)} \langle 0|T\left\{S^i(0)V^j_\mu(x)V^k_\nu(y)\right\}|0\rangle$$

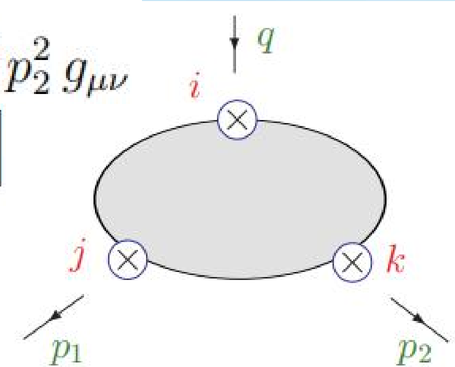
- Ward identity, etc.

$$\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = d^{ijk}B_0\left[-2F^2\frac{(p_1)_\mu(p_2)_\nu}{p_1^2p_2^2} + \mathcal{F}_A\left(p_1^2,p_2^2,q^2\right)P_{\mu\nu} + \mathcal{G}_A\left(p_1^2,p_2^2,q^2\right)Q_{\mu\nu}\right]$$

$$P_{\mu\nu} = (p_2)_\mu(p_1)_\nu - p_1\cdot p_2\, g_{\mu\nu}\,,$$

$$Q_{\mu\nu} = p_1^2(p_2)_\mu(p_2)_\nu + p_2^2(p_1)_\mu(p_1)_\nu - p_1\cdot p_2\,(p_1)_\mu(p_2)_\nu - p_1^2p_2^2 g_{\mu\nu}$$

$$\lim_{\lambda\to\infty}\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu}(\lambda p_1,\lambda p_2) = -2\,d^{ijk}B_0F^2\frac{1}{\lambda^2}\frac{1}{p_1^2p_2^2q^2}\left[q^2(p_1)_\mu(p_2)_\nu + Q_{\mu\nu} - p_1\cdot p_2\,P_{\mu\nu}\right]$$



Dai *et.al.*, PRD99 (2019) 114015

SAA matching

- constrains

$$\hat{L}_5 = \hat{C}_{12} = \hat{C}_{80} = \hat{C}_{85} = 0,$$
$$\lambda_6^A = \lambda_{16}^A = \lambda_{12}^S = \lambda_{16}^S = 0,$$
$$\lambda_6^{AA} = -\frac{F^2}{16 F_A^2},$$
$$\lambda_1^{SA} = \frac{1}{\sqrt{2} F_A}\left(c_d - \frac{F^2}{8 c_m}\right),$$
$$\lambda_2^{SA} = -\frac{c_d}{2\sqrt{2} F_A}.$$

- 15 couplings, 4 of them remain λ_{17}^A λ_{17}^S λ_{18}^S λ^{SAA}
- also from $\Pi_{SS-PP}^{ij}(t)$ $F_S^{ij}(t)$, one can knows three more couplings, only 1 remain

$$\lambda_{17}^S = \lambda_{18}^S = 0,$$
$$\lambda_{17}^A = 0,$$

V. Cirigliano, et.al., NPB753 (2006) 179

G. Ecker, PLB223 (1989) 425

3. ππ，πK scatterings

- For ππ scattering amplitudes, it can be parametrized as

$$T_J^I(s) = (s - z_J^I)^{n_J} f_J^I(s) e^{i\varphi^{IJ}(s)}$$

Adler zero, threshold

- Writting dispersion relation on the reduced amplitude, one has.

$$T_J^I(s) = T_J^I(s_0)\left(\frac{s - z_J^I}{s_0 - z_J^I}\right)^{n_J} \Omega_L^{IJ}(s)\Omega_R^{IJ}(s)$$

ChPT

- s_0 could be chosen as 0 for simplicity.

correlation between l.h.c and r.h.c

- The Omnes function of the l.h.c satisfy the relation in the elastic region.

$$\Omega_L^{IJ}(s) = -\frac{\mathrm{Im}[\Omega_R^{IJ}(s)^{-1}](s_0 - z_J^I)^{n_J}}{\rho(s)T_J^I(s_0)(s - z_J^I)^{n_J}}$$

conformal mapping for phases, only 2 terms

unitarity

- the Omnes function of the r.h.c is known
  - experiment data
  - constraints from Roy-like equations
  - ChPT amplitudes in the low energy region could be used to constrain the l.h.c.

r.h.c.

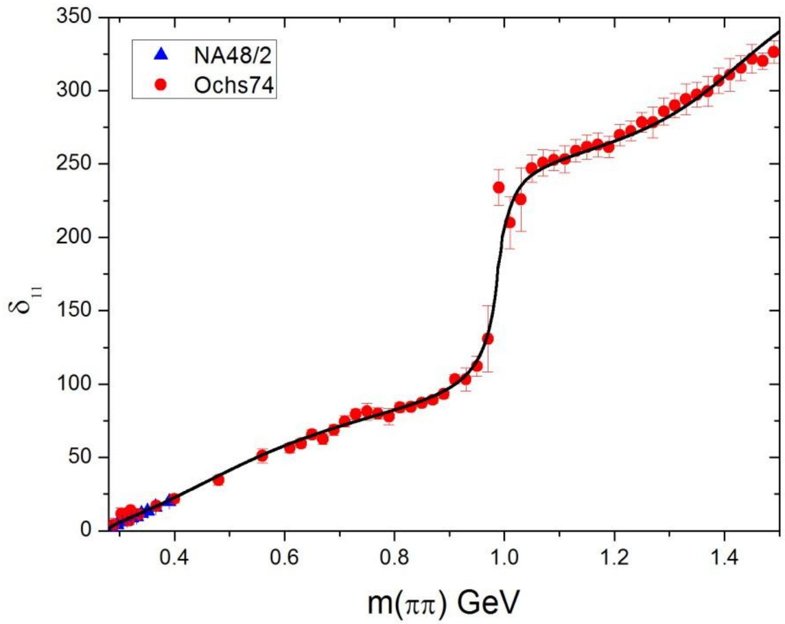
- $\pi\pi$ - KK **scattering inputs**
  - K-matrix to represent S and D partial waves
  - Data on Phase shifts and inelasticities of $\pi\pi$ - KK coupled channel scattering.
  - BABAR's Dalitz plot analysis of $D_s^+ \to (\pi^+\pi^-)\pi^+$ and $D_s^+ \to (K^+K^-)\pi^+$ process. BES's analysis on $J/\psi \to \pi^+\pi^- \phi$ and $J/\psi \to K^+K^-\phi$.
  - Dispersion analysis:
    - T-matrix of $\pi\pi$ scattering by CFDIV .
    - $\pi\pi \to KK$ amplitudes given by Roy-Steiner Equation.

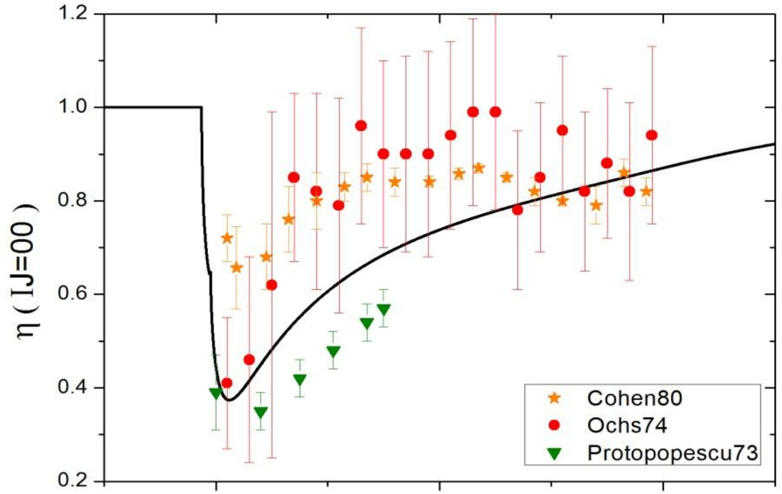
Descotes *et.al*
EPJC33 (2004) 409

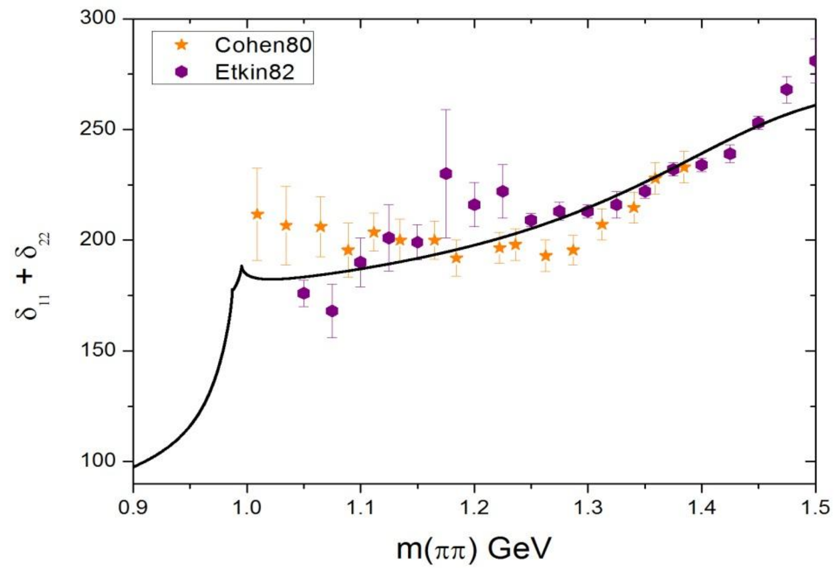
Pelaez *et al.*
PRD83 (2011) 074004

Data: phase shift and inelasticity

- ππ→ππ,KK phase shift and inelasticity

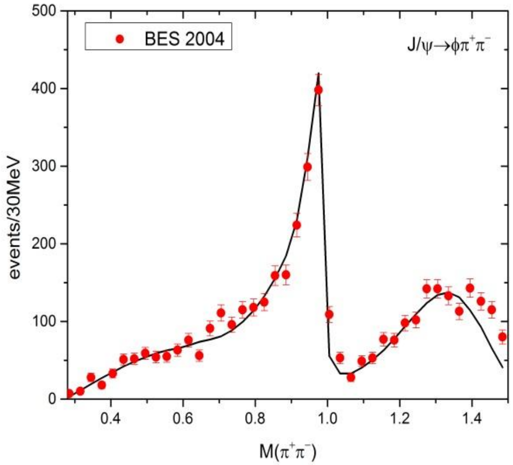


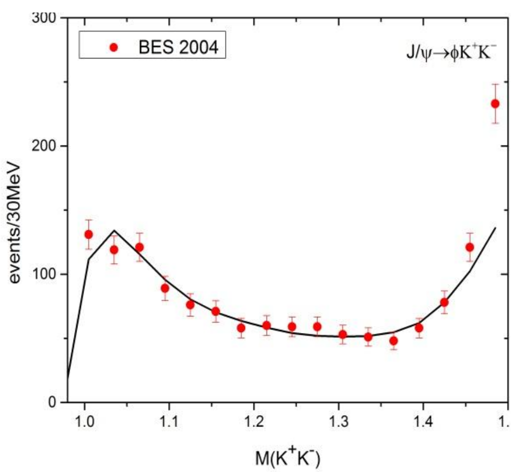


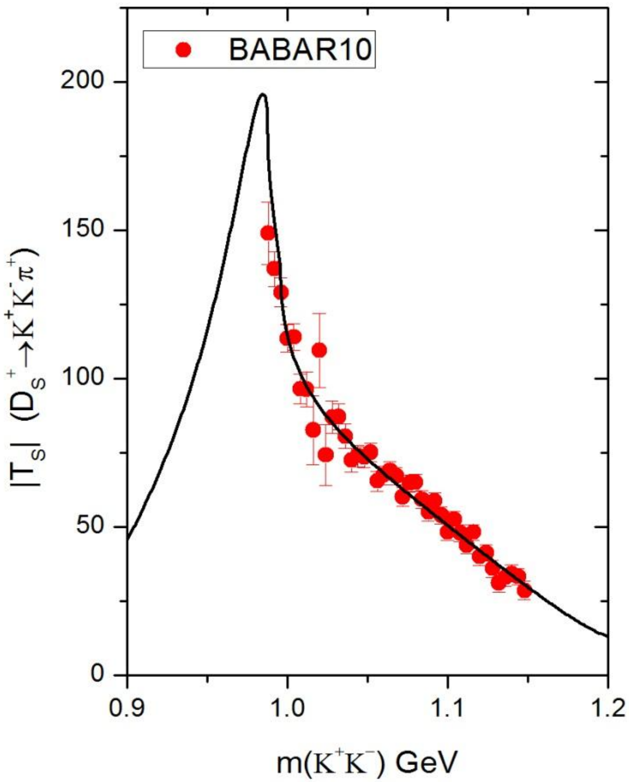


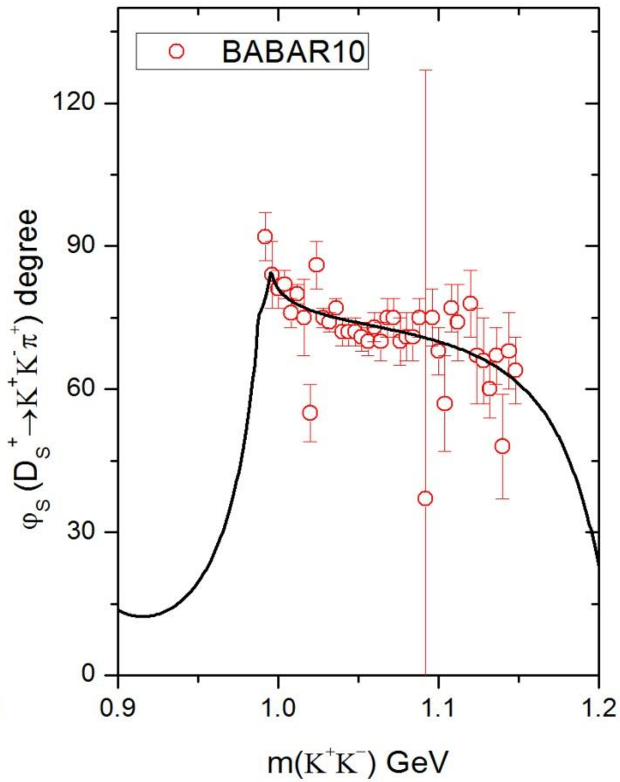
BABAR && BESS

- **ππ - KK scattering inputs**
  - KK threshold region is important as it is around $f_0(980)$.



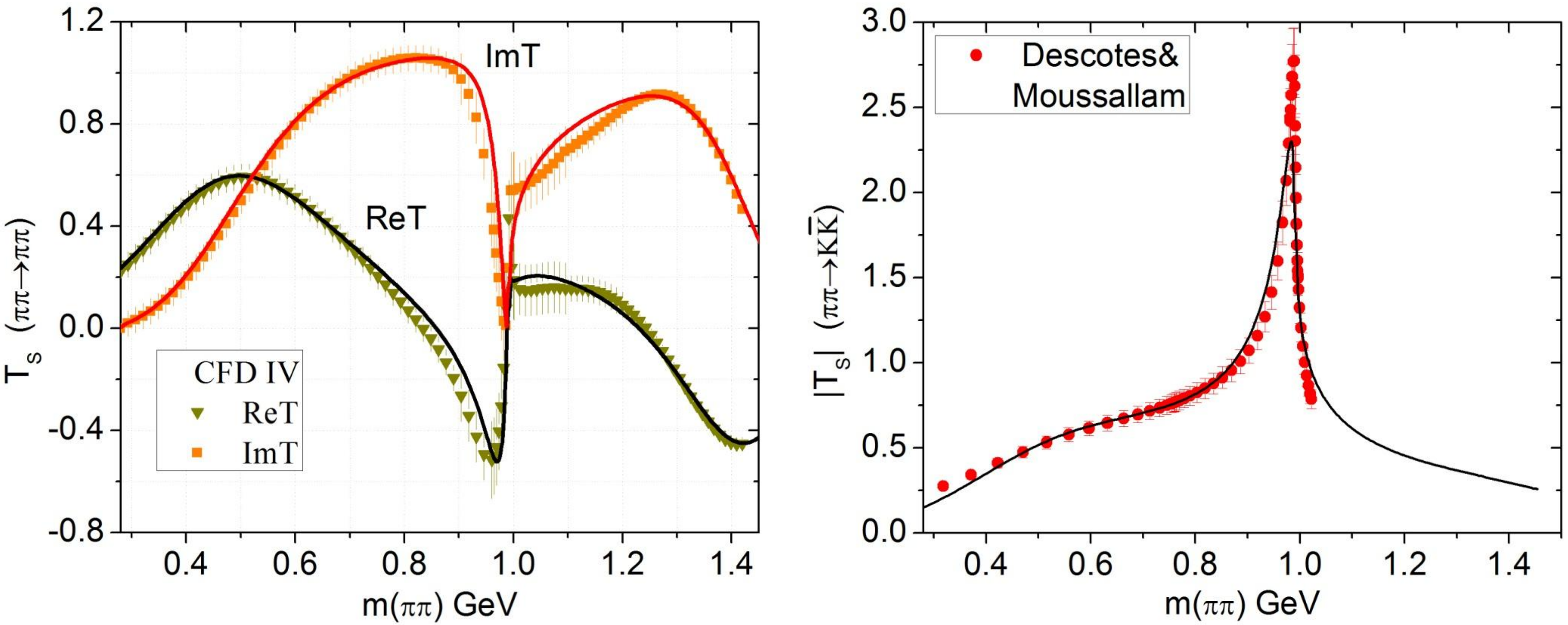




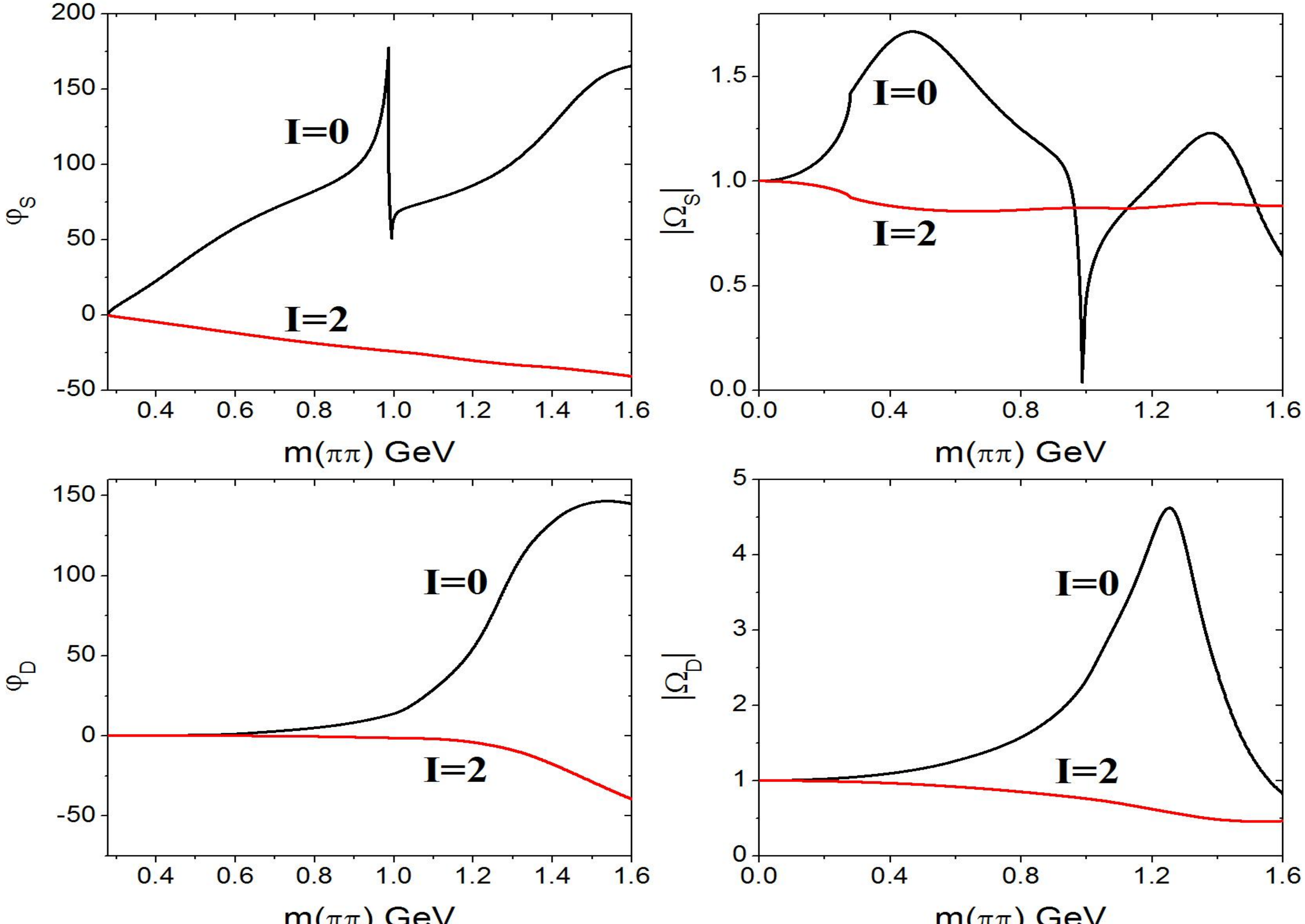


Dispersion analysis constraints

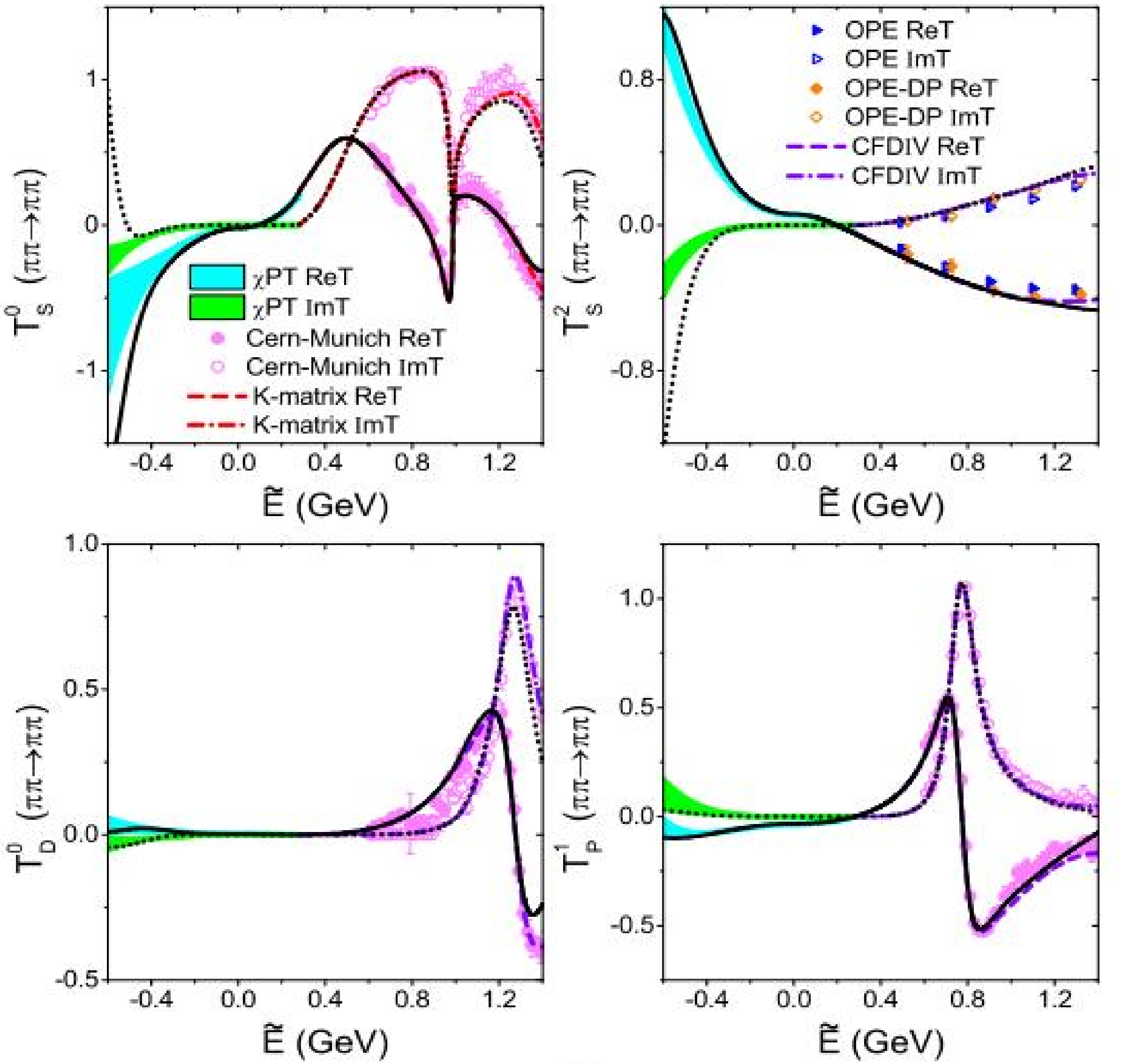
- Roy-like equations take crossing symmetry, unitarity into account



r.h.c. Phases and Omnes function



our ππ amplitudes



- The single channel unitarity is kept
- the left hand amplitudes in the low energy region are well constrained

poles

State	Case	pole locations (MeV)	$g_{f\pi\pi} = \|g_{f\pi\pi}\|e^{i\phi}$ $\|g_{f\pi\pi}\|$ (GeV)	ϕ (°)
$\sigma/f_0(500)$	A	$432.5 - i269.8$	0.46	−77
	B	$442.7 - i270.5$	0.48	−74
	C	$438.2 - i270.6$	0.47	−75
$f_0(980)$	A	$997.5 - i19.0$	0.25	−81
	B	$997.6 - i21.6$	0.27	−83
	C	$997.6 - i20.5$	0.26	−82
$f_2(1270)$	A	$1260.9 - i111.2$	0.55	−10
	B	$1294.1 - i57.9$	0.52	11
	C	$1266.0 - i99.5$	0.54	−8
$\rho(770)$	A	$761.1 - i70.6$	0.34	−12
	B	$763.0 - i73.3$	0.35	−11
	C	$761.3 - i71.7$	0.34	−12
$2S$ v.s.	A	29.8	9.8×10^{-3}	90
	B	29.8	9.8×10^{-3}	90
	C	32.3	11.0×10^{-3}	90

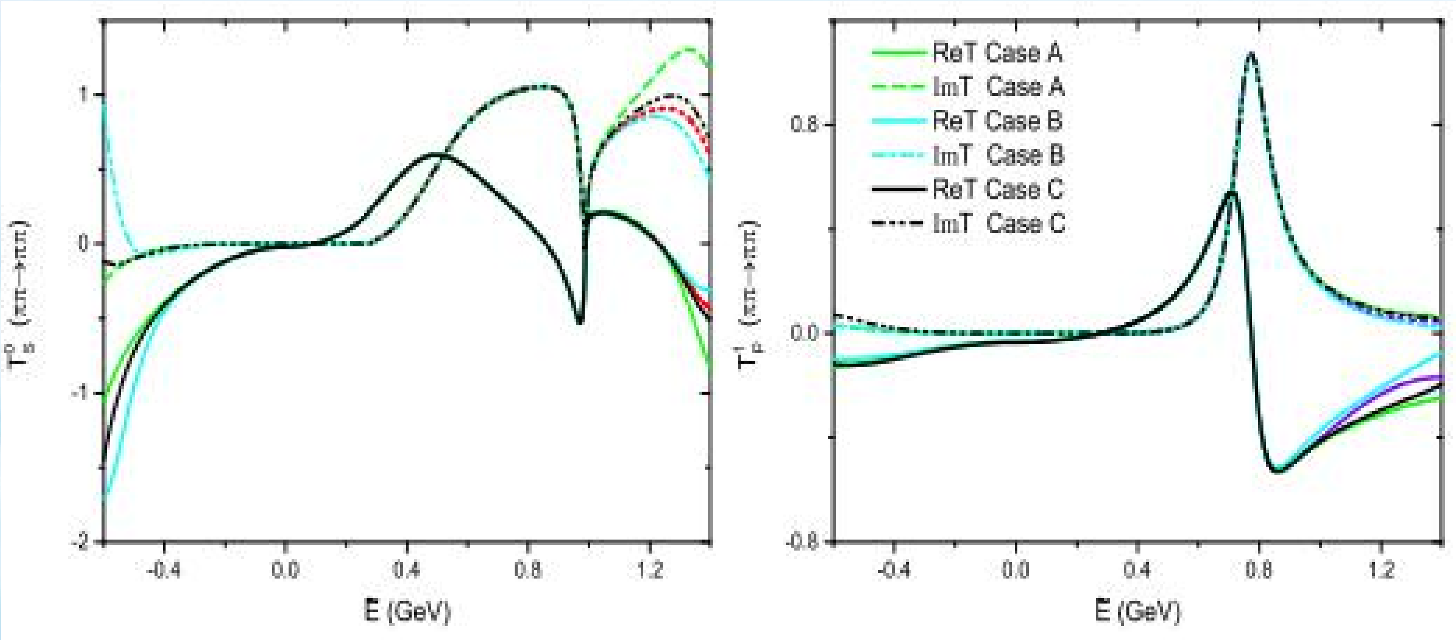
- pole locations and couplings
- phase of couplings of scalars are close to -90. Vector and tensor's are close to 0

Q. Ang et.al., CTP 36 (2001) 563, UChPT

- Re-confirm of the virtual state in isospin 2 S-wave: only depends on the sign of scattering length, Adler zero and analyticty

Correlation between cuts and poles

- To discuss cut's contribution, usually people delete the cuts and estimate the variation of the poles.
- Problem: Without cuts, the unitarity is violated and the continuation (within unitarity) is not valid any more.
- Ours: the solution of l.h.c in different Cases are different, but the unitarity in each Case is kept.



correlation between cuts and poles

$$\mathcal{R}_{\mathrm{Im}T_J^I} = \frac{1}{N}\sum_{n=1}^{N}\frac{|\Delta \mathrm{Im}T_J^I(s_n)|}{|\mathrm{Im}T_J^I(s_n)|}$$

$$\mathcal{R}_{pole} = \frac{|\Delta \mathrm{Re}\sqrt{s_p}| + |\Delta \mathrm{Im}\sqrt{s_p}|}{|\sqrt{s_p}|}$$

$$C_{pole} = \frac{\mathcal{R}_{pole}}{\mathcal{R}_{\mathrm{Im}T_J^I}}$$

		Case A	Case B	Case C
l.h.c.	$\mathcal{R}_{\mathrm{Im_L}T_S^0}$	171%	126%	45%
	C_σ^L	2.01%	1.70%	2.87%
	$C_{f_0(980)}^L$	0.08%	0.10%	0.04%
	$\mathcal{R}_{\mathrm{Im_L}T_P^1}$	41%	20%	61%
	C_ρ^L	0.63%	1.75%	1.41%
r.h.c.	$\mathcal{R}_{\mathrm{Im_R}T_S^0}$	1.70%	0.64%	1.36%
	C_σ^R	387%	328%	189%
	$C_{f_0(980)}^R$	30%	142%	55%
	$\mathcal{R}_{\mathrm{Im_R}T_P^1}$	6.6%	10.0%	4.4%
	C_ρ^R	17.6%	6.4%	28.2%

- The poles are more sensitive to the r.h.c rather than the l.h.c.
- σ is much more sensitive to the r.h.c than that of f980 and ρ , since it is farther away from the real axis.
- Tested in πK, πη scattering?
- Generalized to inelastic scattering?

3、Property of scalars

- Inner structure?
- The simplest way, satisfying intuition: For a molecule, its mass should increase/decrease as that of the constituent hadrons!
- How to make sure the trend of the amplitudes is right in unphysical region?
- In the physical region, constrained by data and also ensured by ChEFT.

$$
\begin{aligned}
\mathscr{L}_2 = &\ \frac{f_0^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U + \mathscr{M}(U + U^\dagger)\rangle, \\
\mathscr{L}_4 = &\ L_1\langle \partial_\mu U^\dagger \partial^\mu U\rangle^2 + L_2\langle \partial_\mu U^\dagger \partial_\nu U\rangle\langle \partial^\mu U^\dagger \partial^\nu U\rangle \\
&+ L_3\langle \partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\rangle + L_4\langle \partial_\mu U^\dagger \partial^\mu U\rangle\langle U^\dagger \mathscr{M} + \mathscr{M}^\dagger U\rangle \\
&+ L_5\langle \partial_\mu U^\dagger \partial^\mu U(U^\dagger \mathscr{M} + \mathscr{M}^\dagger U)\rangle + L_6\langle U^\dagger \mathscr{M} + \mathscr{M}^\dagger U\rangle^2 \\
&+ L_7\langle U^\dagger \mathscr{M} - \mathscr{M}^\dagger U\rangle^2 + L_8\langle U^\dagger \mathscr{M} U^\dagger \mathscr{M} + \mathscr{M}^\dagger U \mathscr{M}^\dagger U\rangle,
\end{aligned}
$$

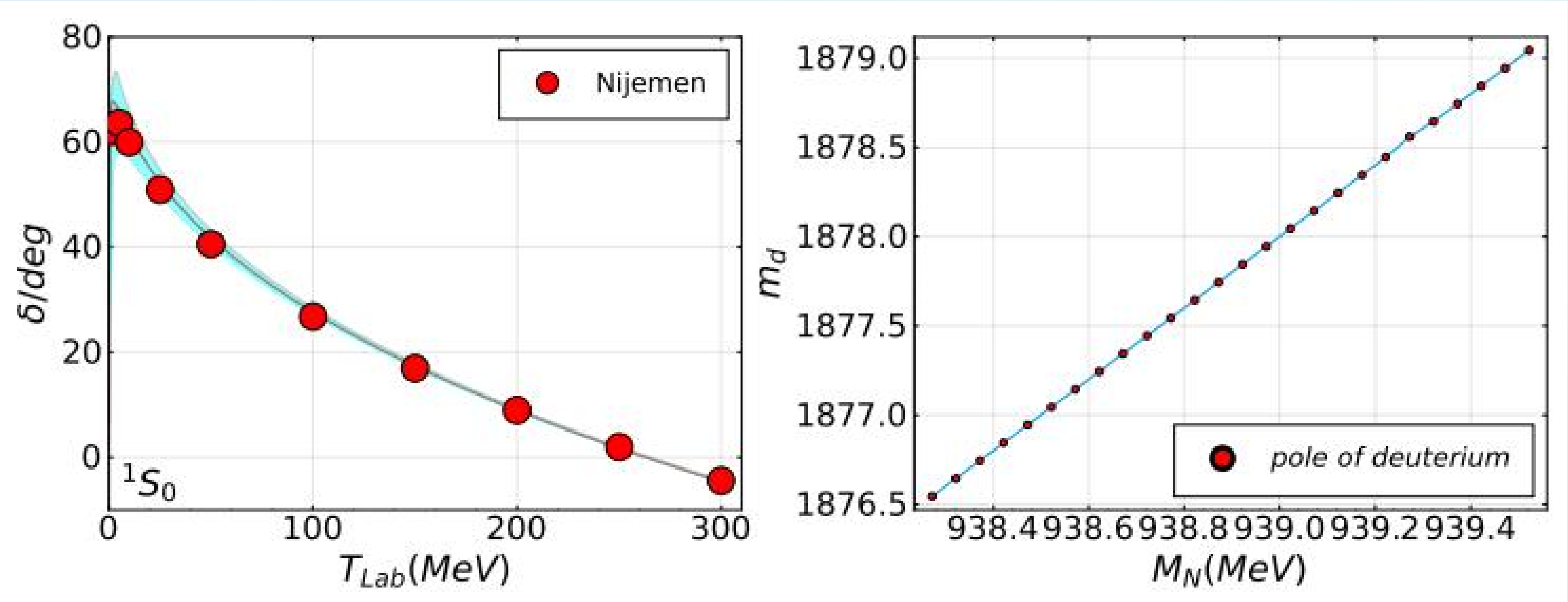
$$
\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}
$$

ChEFT

- Supplies dynamics
- Isospin symmetry: The mass difference between charged and neutral particles is ignored in ChEFT
- Describe the physics in low energy region successfully
- Isospin symmetry is good
  for strong interactions!

deuteron

- Deuteron: Maybe the only undoubted molecule.
- Varying the masses within the range allowed by isospin symmetry. The amplitudes still fit rather well to the ‘data’.
- Mass of deuteron increases as that of nucleons.

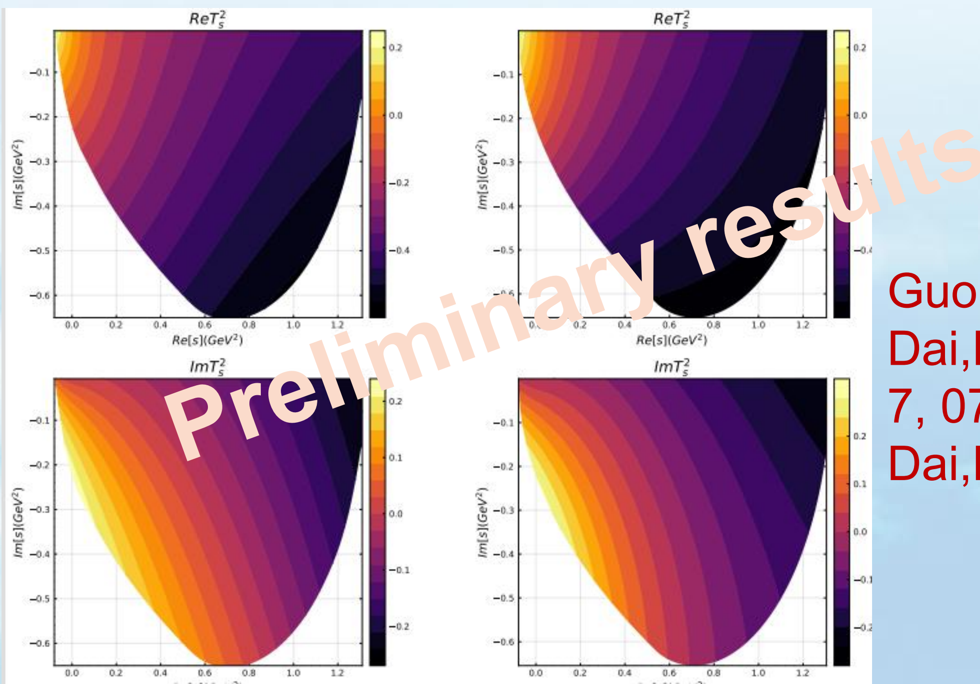


amplitudes

- ChPT for the dynamics
- Unitarization to restore unitarity

$$T^{(I,J)} = T_2^{(I,J)} \cdot [T_2^{(I,J)} - T_4^{(I,J)}]^{-1} T_2^{(I,J)}$$

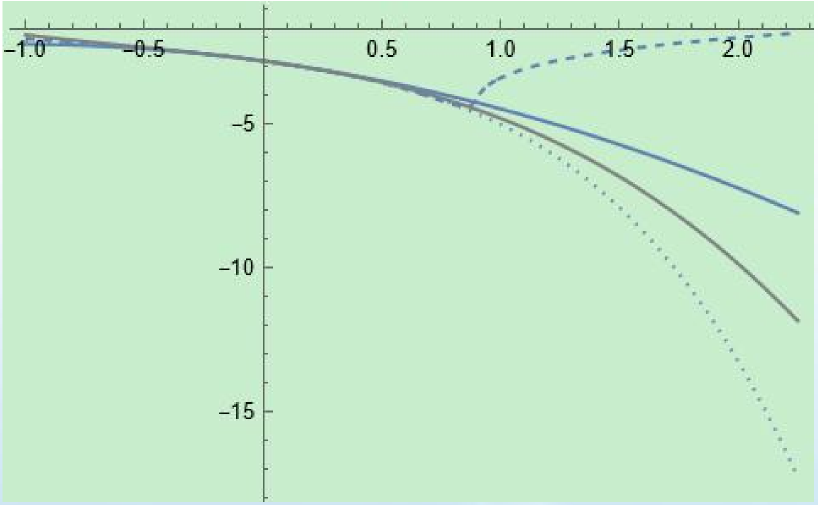
- Fitting Roy's amplitudes in the complex plane to include part of crossing symmetry



Guo, Yang, Dai, in preparation;
Dai,Kang,Meissner, PRD 98 (2018) 7, 074033;
Dai,Meissner, PLB 783 (2018) 294

amplitudes

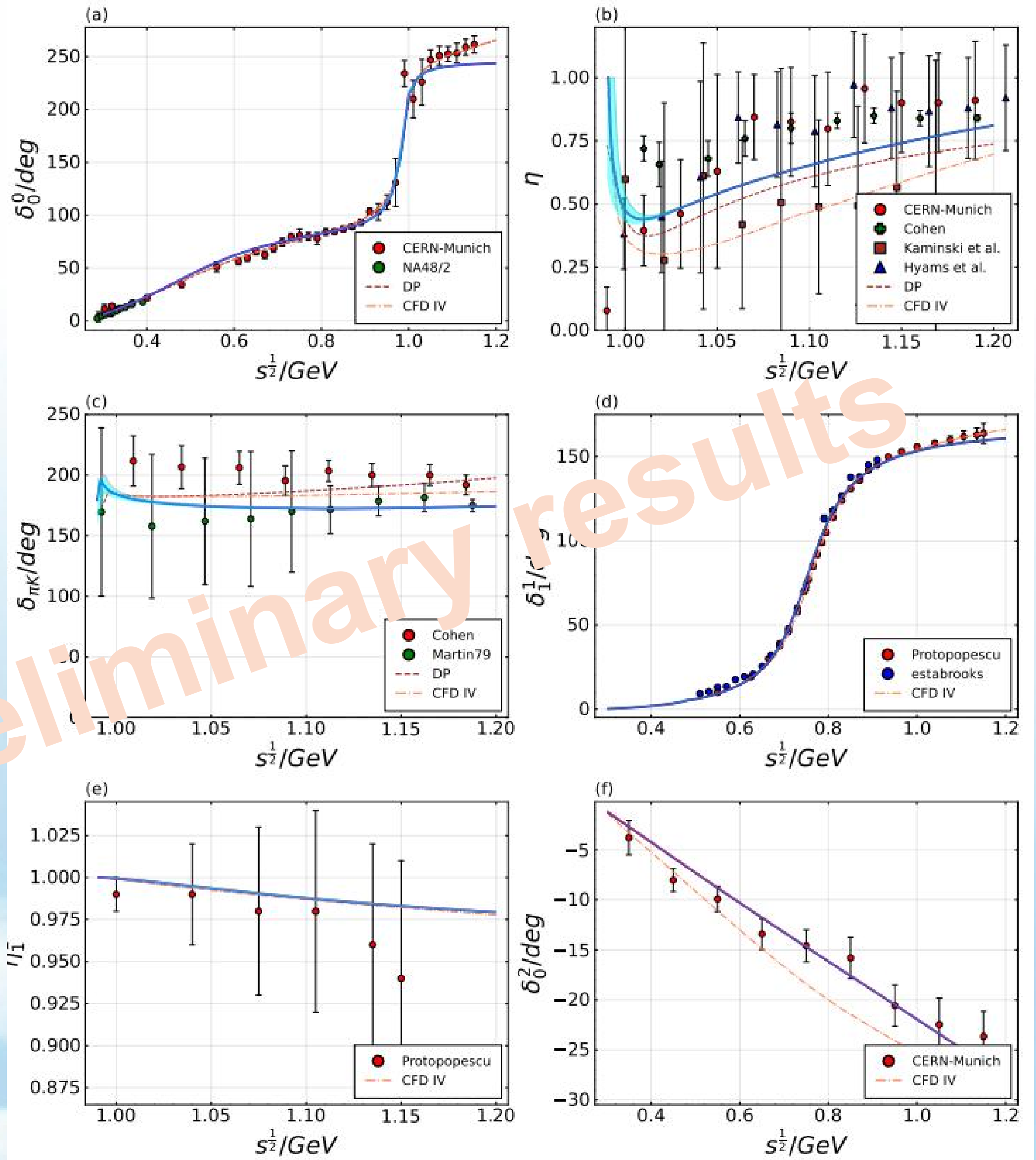
- l.h.c. caused by KK scattering is removed, to strictly restore unitarity



- Random forest method is applied to get more reliable LECs from minimum χ^2

scalars

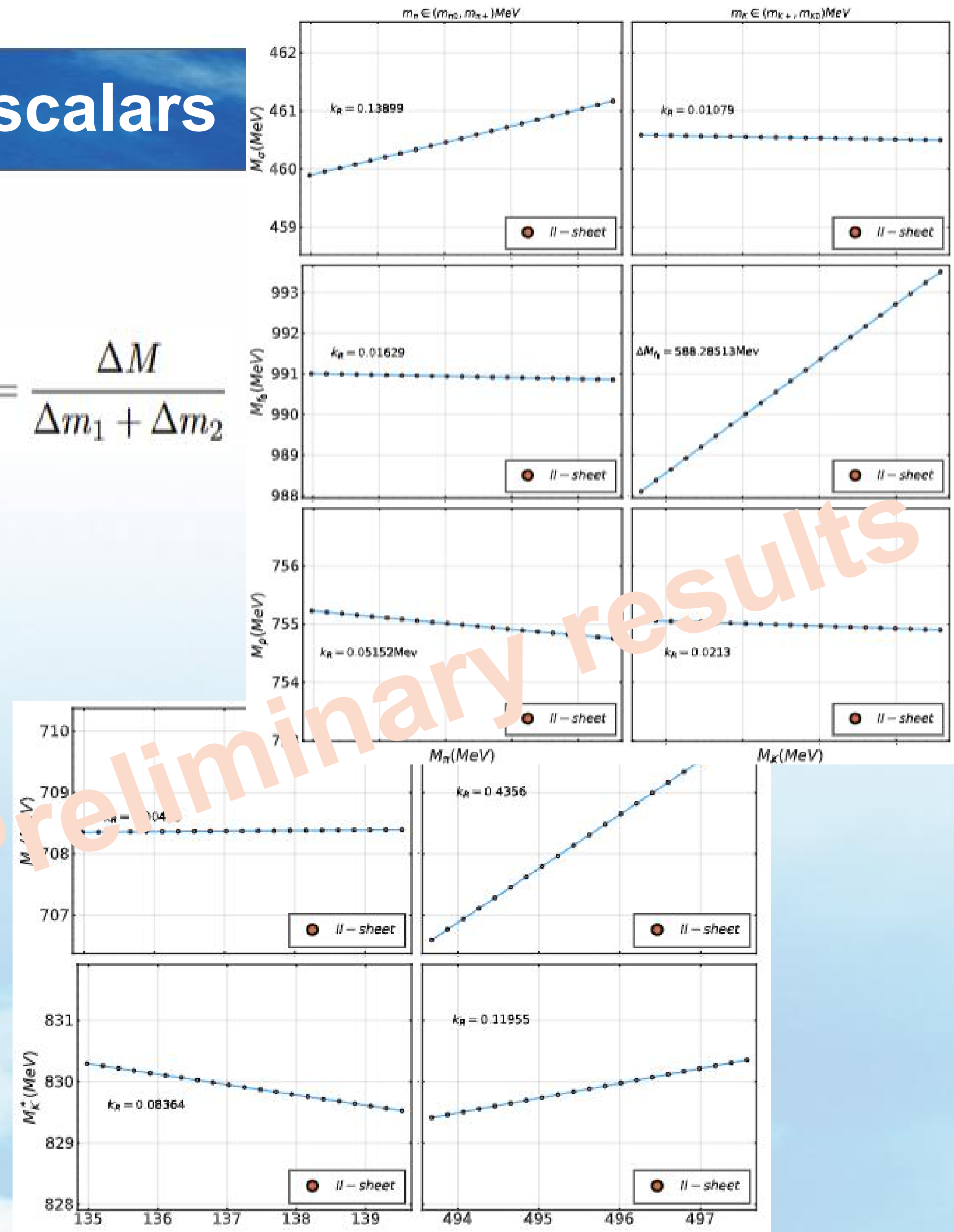
- Varying the masses of pseudoscalars, the amplitudes are almost not changed



scalars

- Pole counting:
  - σ RS-II, III;
  - $f_0(980)$, RS-II
  - ρ(770), RS-II, RS-III

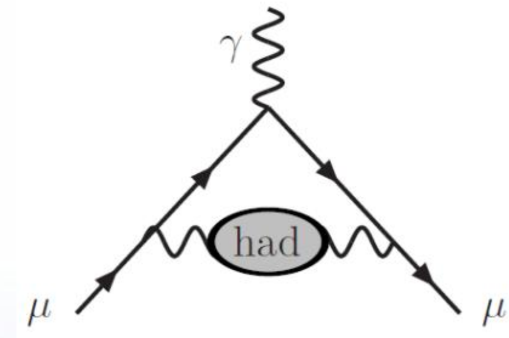
$$k_R = \frac{\Delta M}{\Delta m_1 + \Delta m_2}$$



- σ, not ordinary qq, not molecule

- $f_0(980)$, dominated by KK molecule!

4. muon g-2: HVP



- LQCD
- Data-driven
- Amplitude analysis? dispersive approach, ChEFT, etc.
  - Only one physical amplitude!
  - It should satisfy the fundamental QFT principles
  - It should be compatible with the exp results

- Improved data-driven: RChT+FSI
  - resonances included as new degrees of freedom
  - Construct Lagrangians by discrete and chiral symmetries
  - Matching with QCD, DRs to reduce LECs
  - 1/Nc expansion
  - including FSI

$$R \equiv \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i \phi_R^i$$

$$\mathcal{L}_{\rm kin}^R = -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} \rangle + \frac{M_R^2}{4} \langle R_{\mu\nu} R^{\mu\nu} \rangle \,, \qquad R = V, A \,,$$

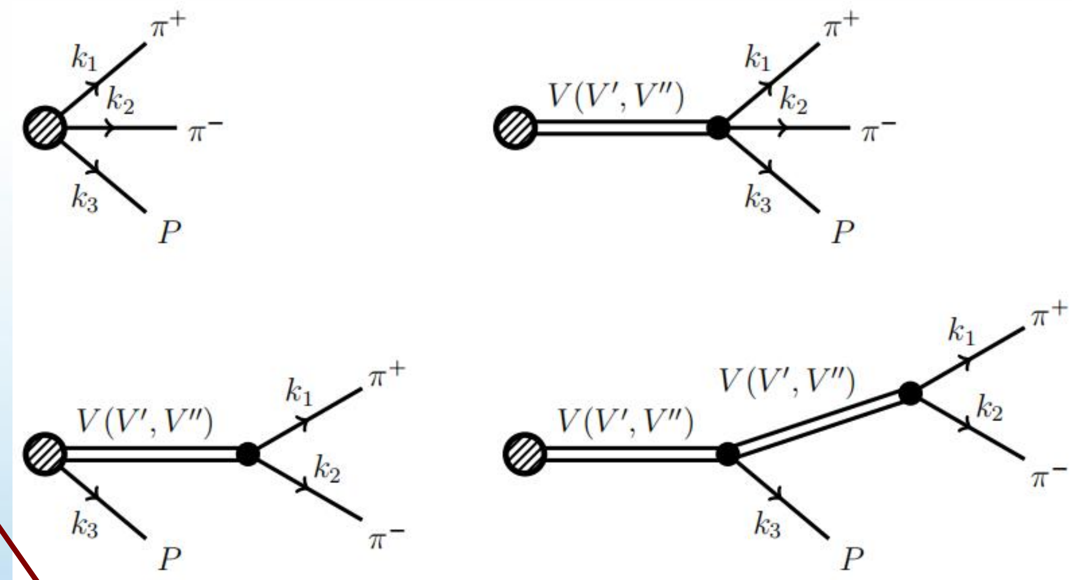
$$\mathcal{L}_{\rm kin}^R = \frac{1}{2}\langle \nabla^\mu R \nabla_\nu R - M_R^2 R^2 \rangle \,, \qquad R = S, P \,.$$

Operator $\mathcal{O}_i^{SS}$	Operator $\mathcal{O}_i^{PP}$
$\langle S S u_\mu u^\mu \rangle$	$\langle P P u_\mu u^\mu \rangle$
$\langle S u_\mu S u^\mu \rangle$	$\langle P u_\mu P u^\mu \rangle$
$\langle S S \chi_+ \rangle$	$\langle P P \chi_+ \rangle$

Dai et.al., PRD 99 (2019) 114015;
Guo et.al., JHEP 06 (2007) 030;

Building amplitudes

- RChT in the resonance region, excited states?

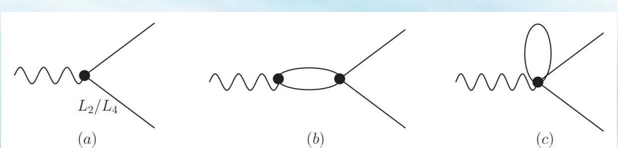


- V', V'' has the same topologies as the ground states

- ππ-KK FSI part by matching with Omens functions and ChPT

$$\frac{1}{M_V^2 - x} \to \frac{1}{M_V^2 - x} + \frac{\beta'_\pi}{M_{V'}^2 - x} + \frac{\beta''_\pi}{M_{V''}^2 - x}$$

Guerrero, et.al., PLB 412 (1997) 382



Dai, et.al., PRD88 (2013) 056001

Building amplitudes

- We give a combined analysis on several channels:

$$\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\eta \quad \pi^0\gamma \text{ and } \eta\gamma.$$

- ρ-ω mixing, origined from Gasser&Leutwyler's

$$F_V^\pi = \left(1 + \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\rho, \Gamma_\rho, Q^2) + \beta'_{\pi\pi} BW(M_{\rho'}, \Gamma_{\rho'}, Q^2) + \beta''_{\pi\pi} BW(M_{\rho''}, \Gamma_{\rho''}, Q^2)\right) \left(\frac{1}{\sqrt{3}} \sin\theta_V \sin\delta^\rho + \cos\delta\right)\cos\delta - \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\omega, \Gamma_\omega, Q^2) + \beta'_{\pi\pi} BW(M_{\omega'}, \Gamma_{\omega'}, Q^2) + \beta'''_{\pi\pi} BW(M_{\omega''}, \Gamma_{\omega''}, Q^2)\right)\left(\frac{1}{\sqrt{3}} \sin\theta_V \cos\delta - \sin\delta^\omega\right)\sin\delta^\omega\right) \exp\left[\frac{-s}{96\pi^2 F^2}\left(\mathrm{Re}\left[A[m_\pi, M_\rho, Q^2] + \frac{1}{2} A[m_K, M_\rho, Q^2]\right]\right)\right]$$

- Not much freedom for Fit

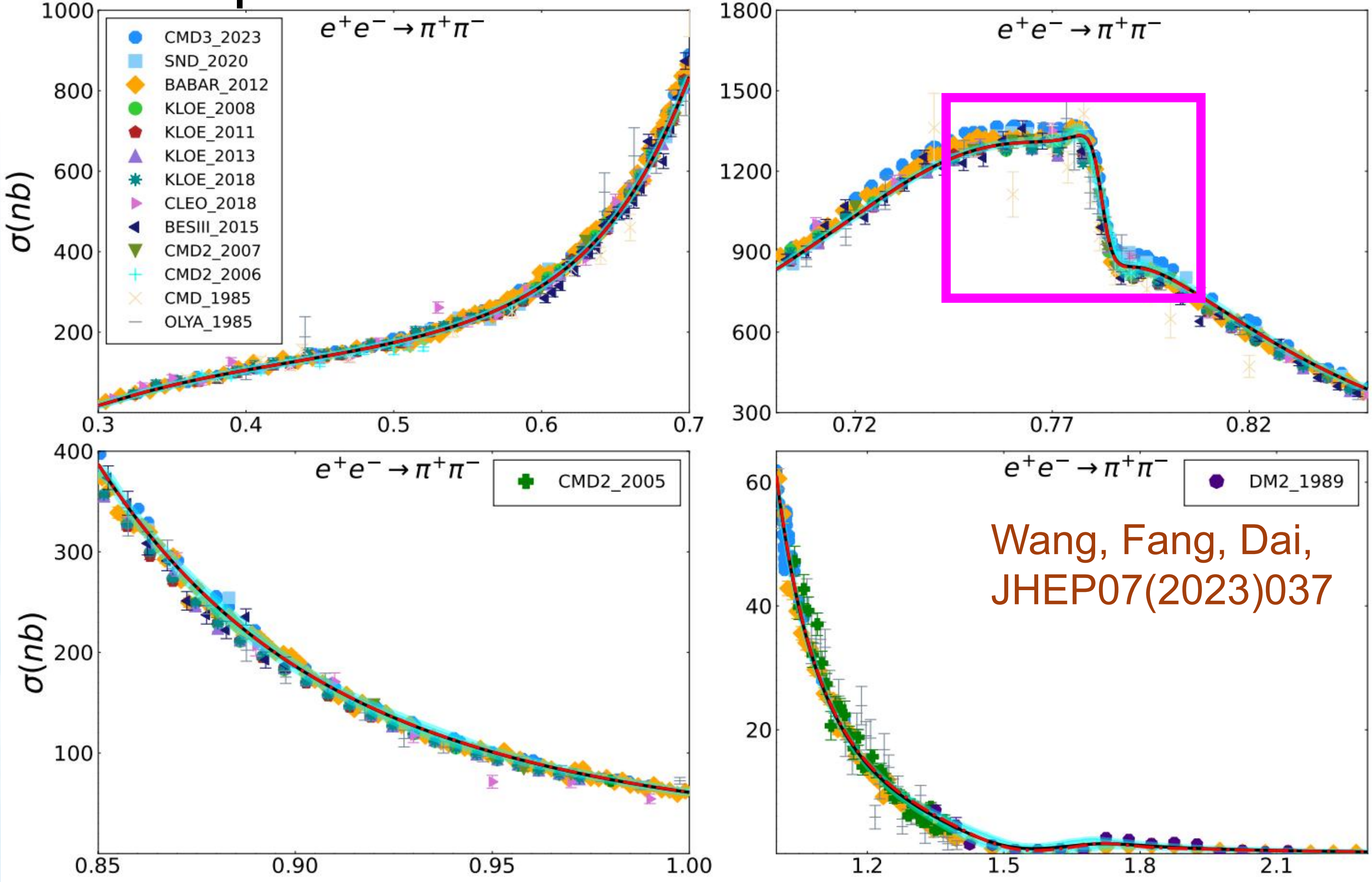
It is 1, from QCD as well as disersion relation constraints

Gasser&Leutwyler, Phys.Rept.87 (1982) 77

Guerrero&Pich, PLB 412 (1997) 382

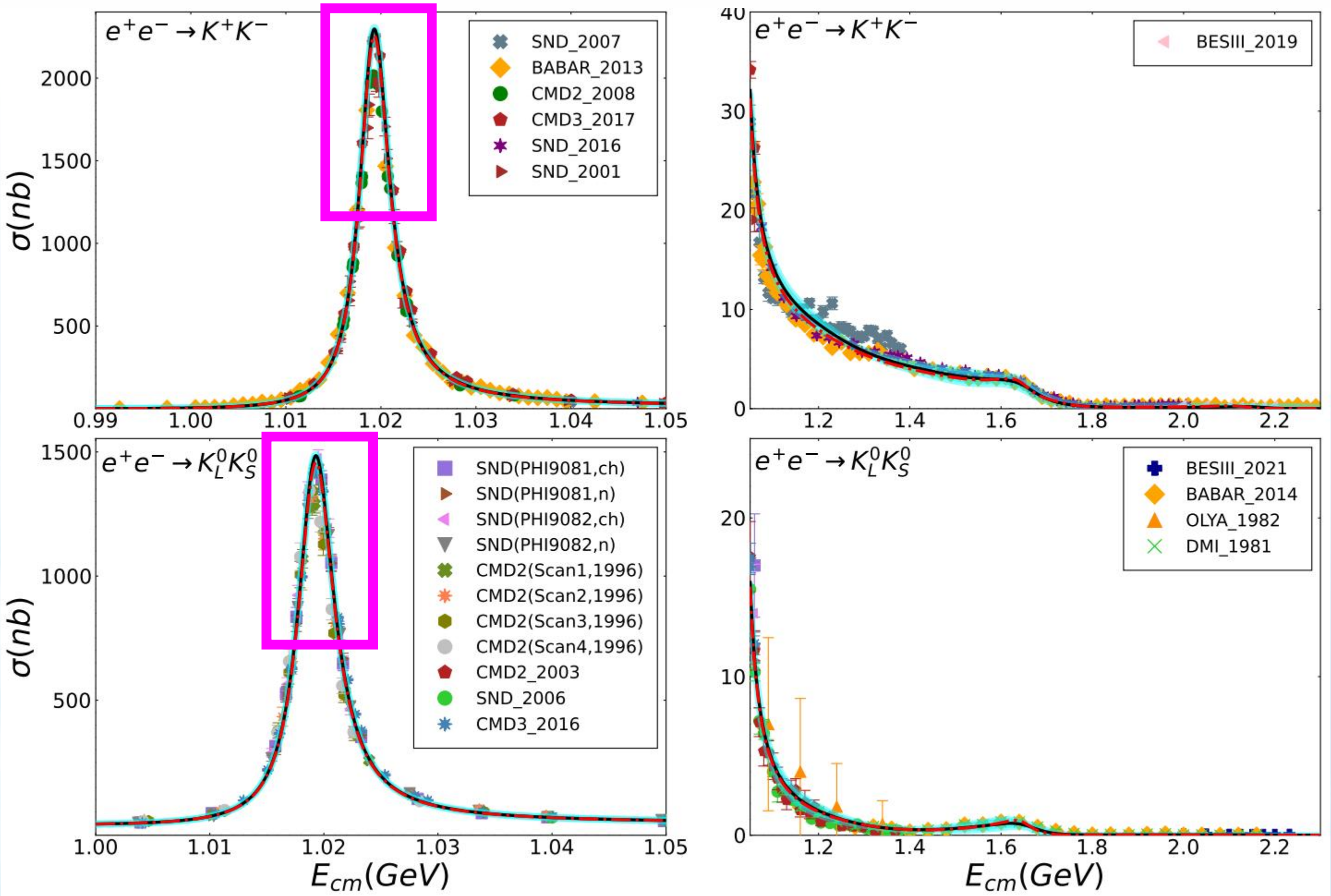
$\pi\pi$

- $\pi\pi$: Now closer to KLOE and BESIII's
- Latest exp: CMD-3



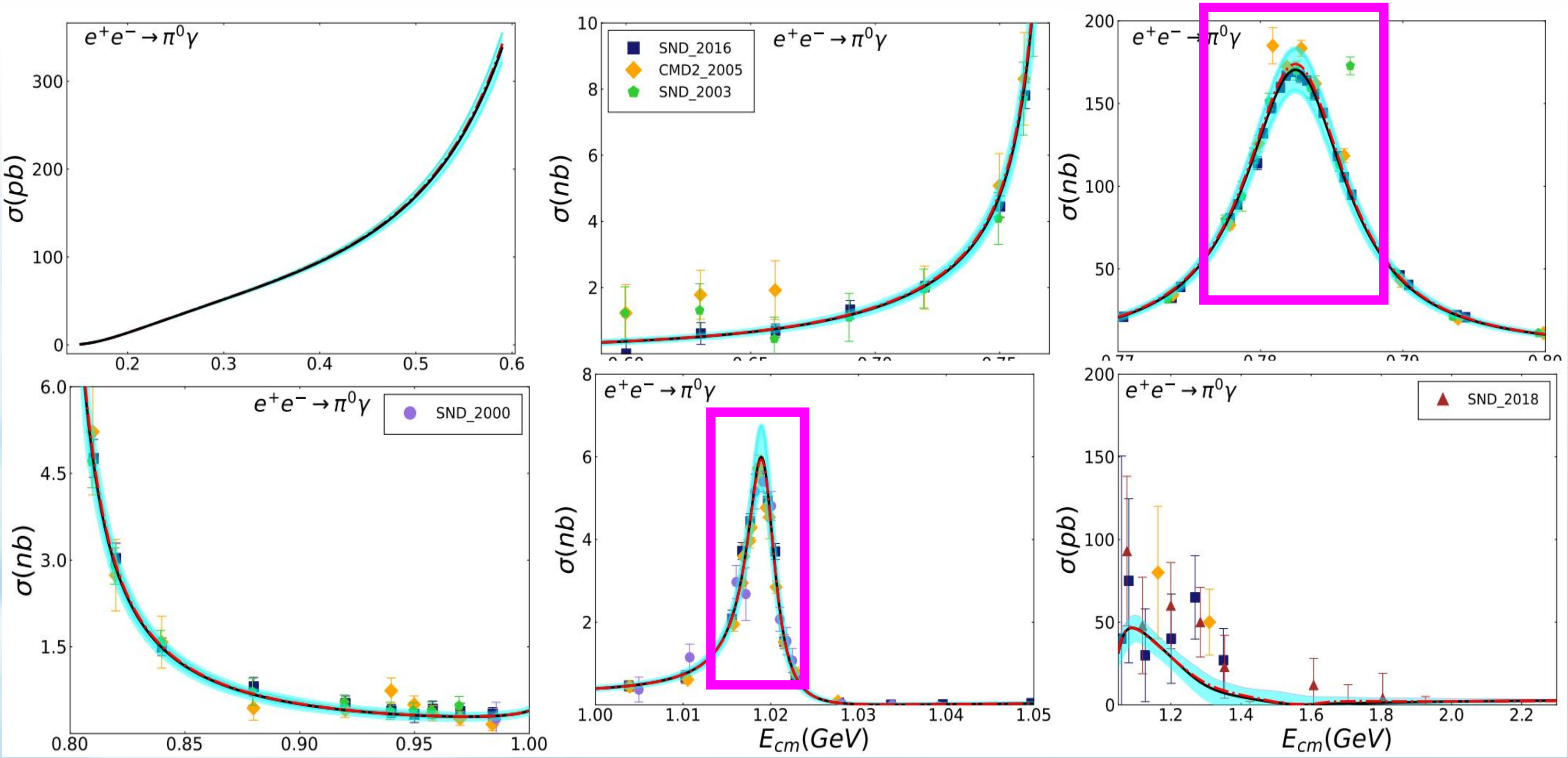
KK

- KK: data in the ϕ 'peak' have large discrepancy
- K_LK_S: further direct constraints on $\pi\pi$, KK channels



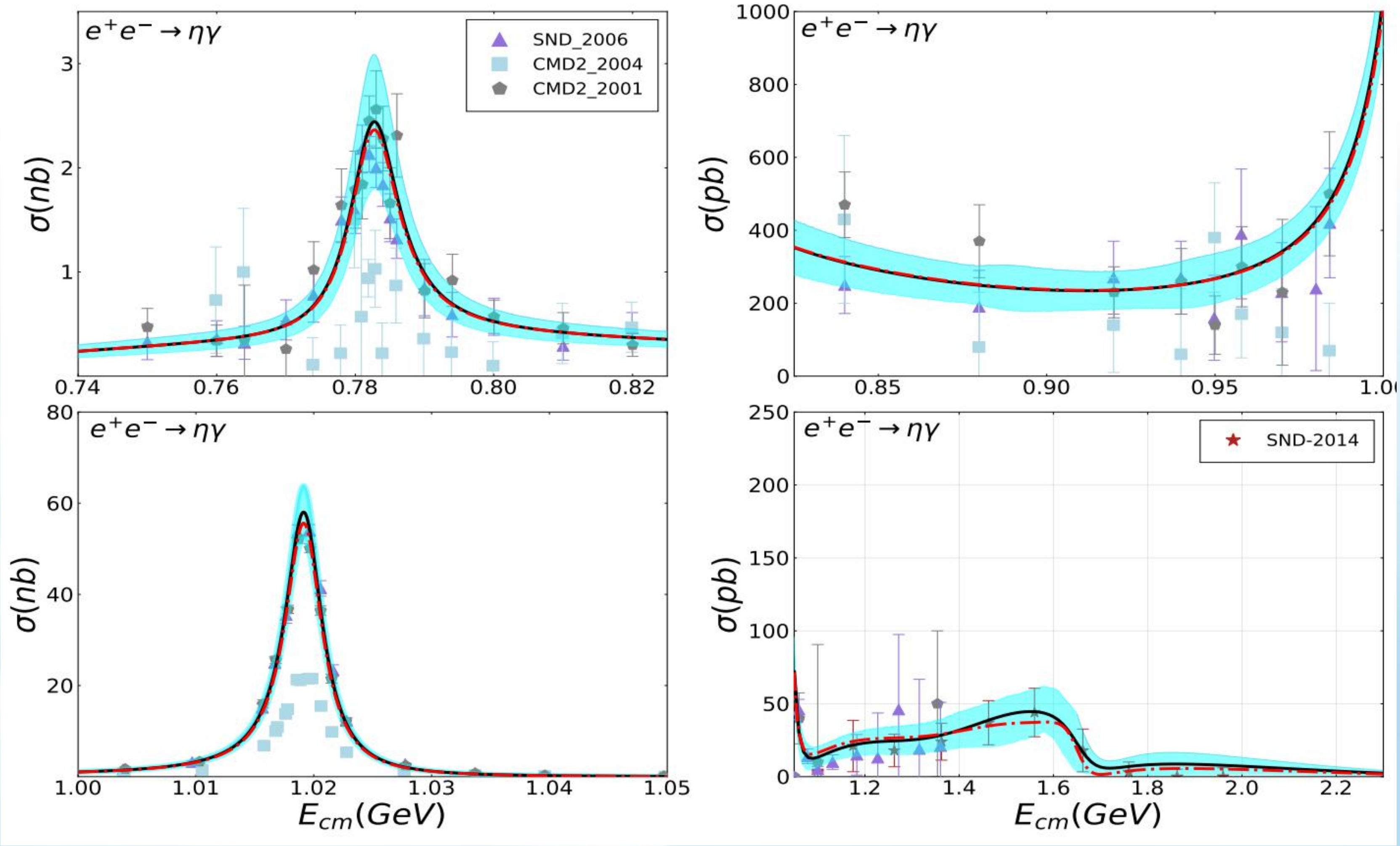
πγ

- πγ: helps to constrain ππ, KK channels: ρ, ω, ϕ



ηγ

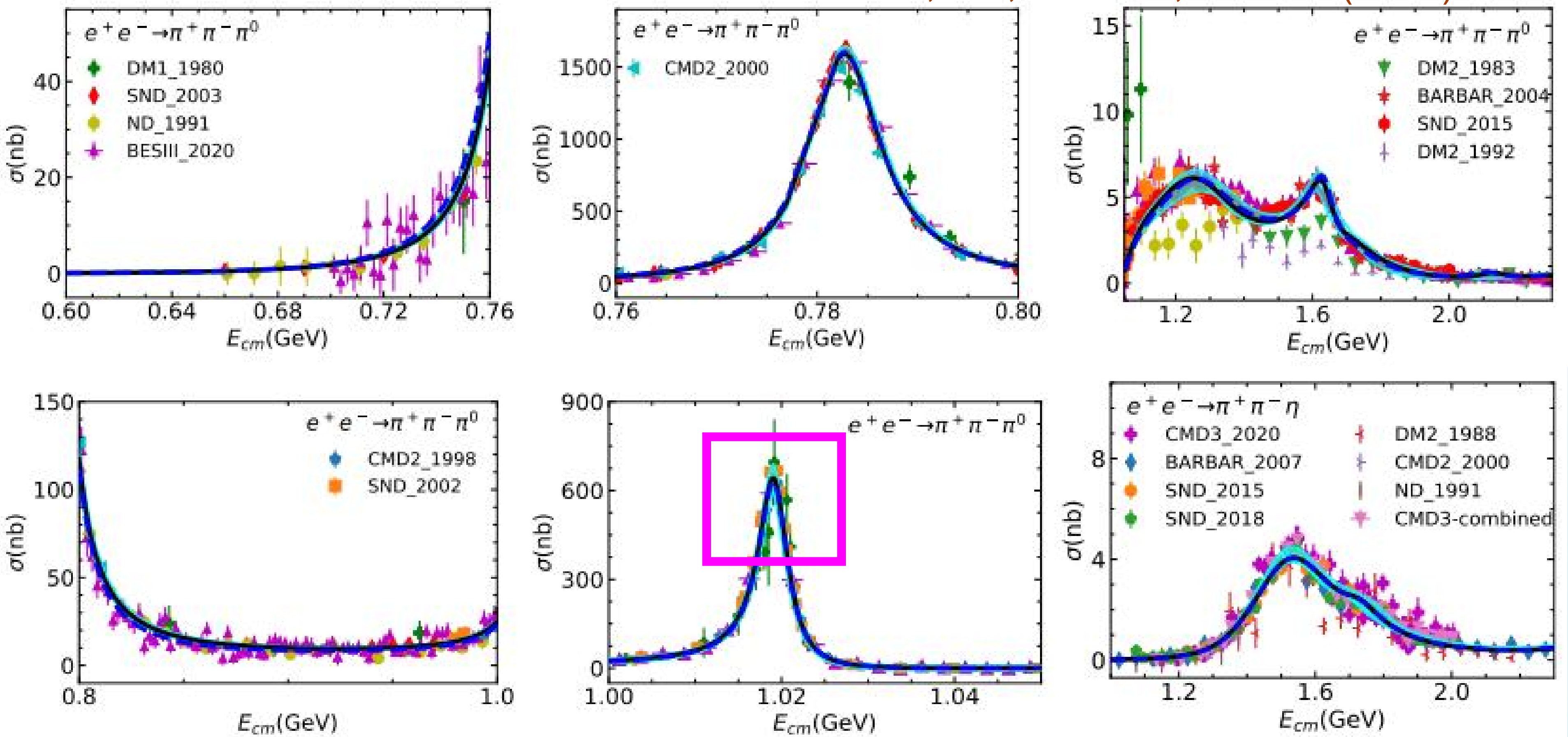
- ηγ: helps to constrain KK, and parameters of ρ, ω, ϕ



$\pi\pi\pi$, $\pi\pi\eta$

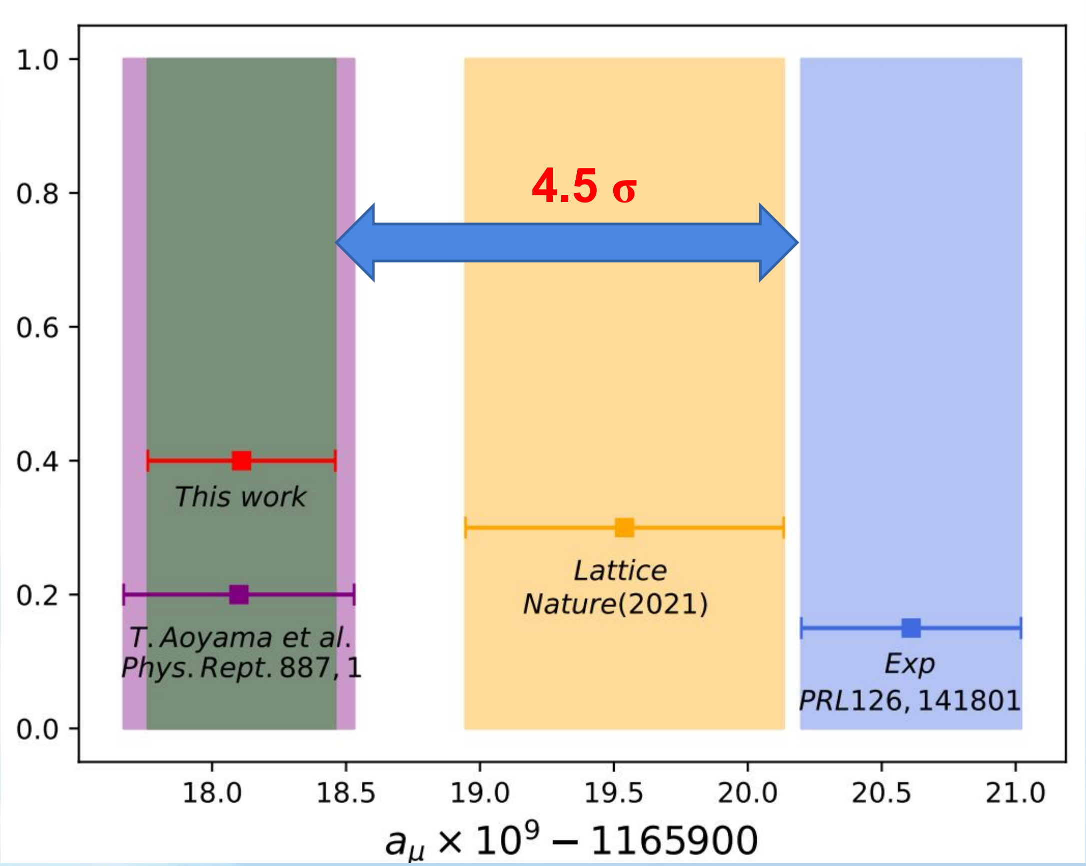
- $\pi\pi\pi$: needs more precise data in the ω ϕ region
- $\pi\pi\eta$: check our model

Qin, Dai, Portoles, JHEP03(2021)092



HVP

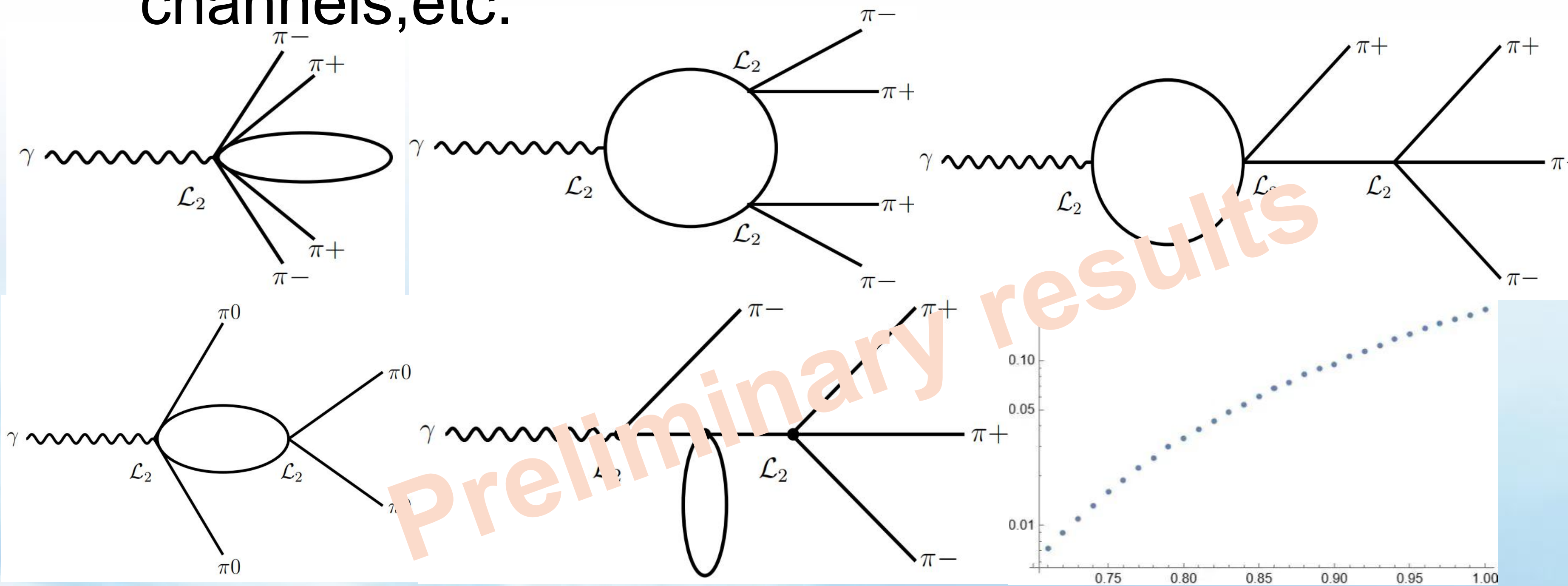
- Ours: a_μ=11659181.1 ±3.5 × 10^{-11}
- It differs 4.5σ from latest experiment's



Wang, Fang, Dai

Four body final states?

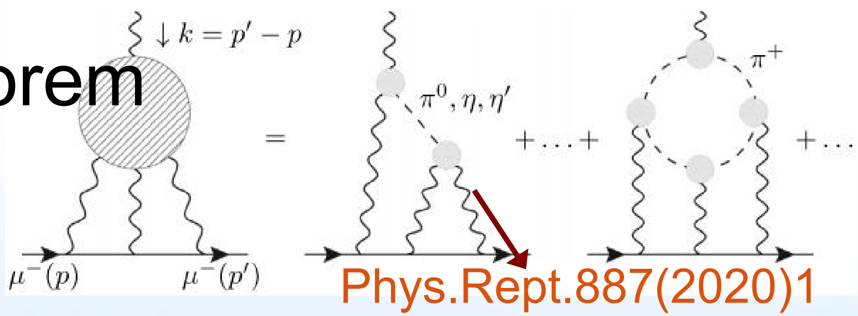
- Future experiments? BESIII,CMD-3...
- Four body final states are important: ππππ, ππKK channels,etc.

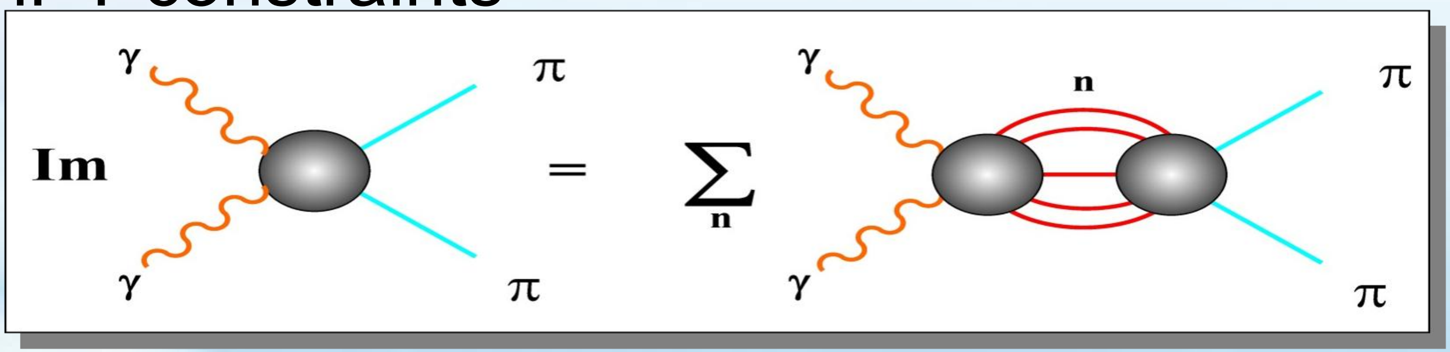


- ChPT's << data, in resonance energy region
- FSI?
- Resonances?

HLBL

- γγ→MM contributes significantly to HLbL sumrule
- Final State Interaction Theorem
- Dispersion relations
- ChPT constraints



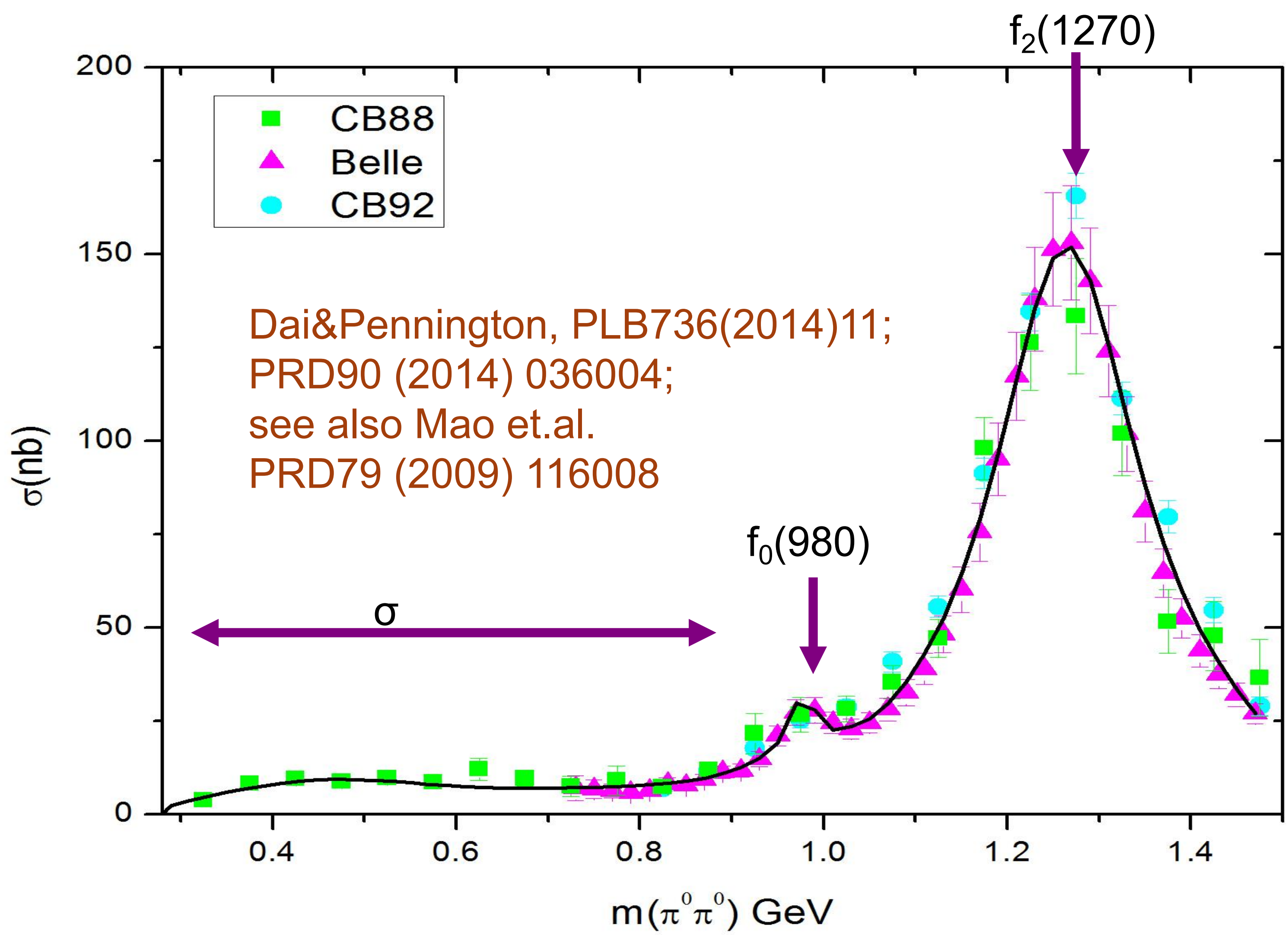


$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right] \Omega^I_{00}(s')^{-1}}{s'^2 (s' - s)}$$

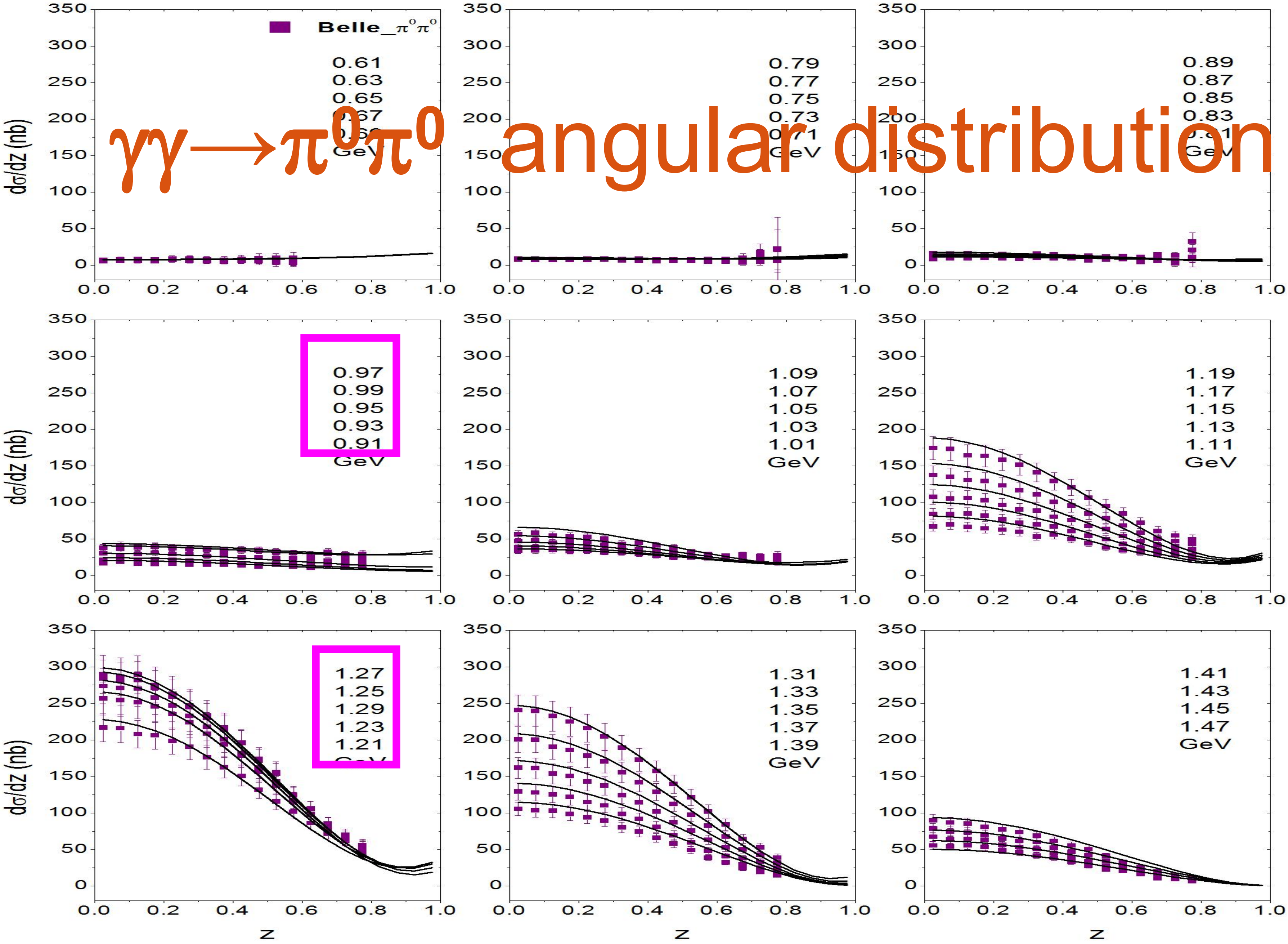
$$- \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_R ds' \frac{\mathcal{B}^I_{00}(s')\, \mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2 (s' - s)}$$

Solved by ChPT

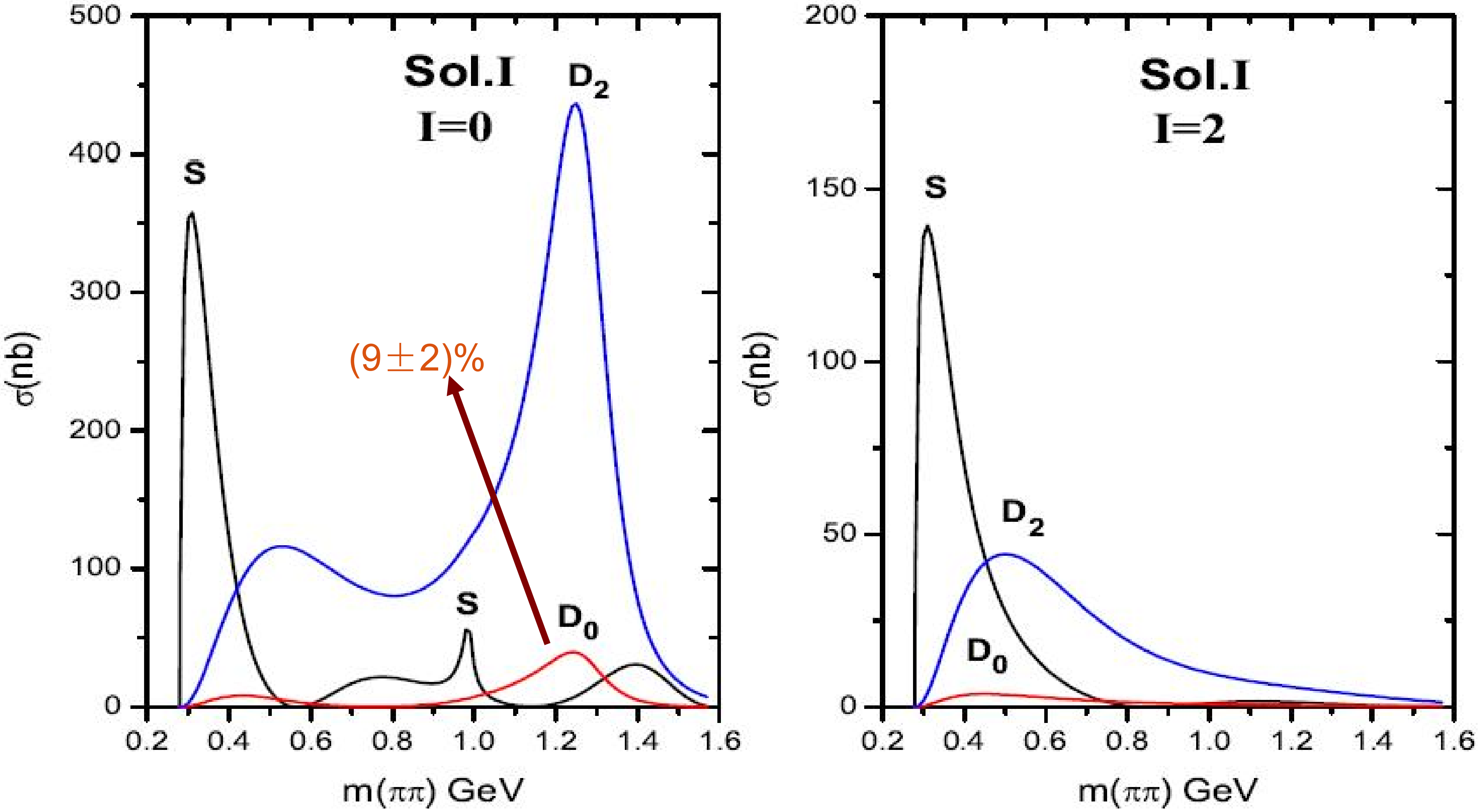
$\gamma\gamma \to \pi^0\pi^0$ integrated cross section



The angular distribution is helpful to seperate each partial wave.

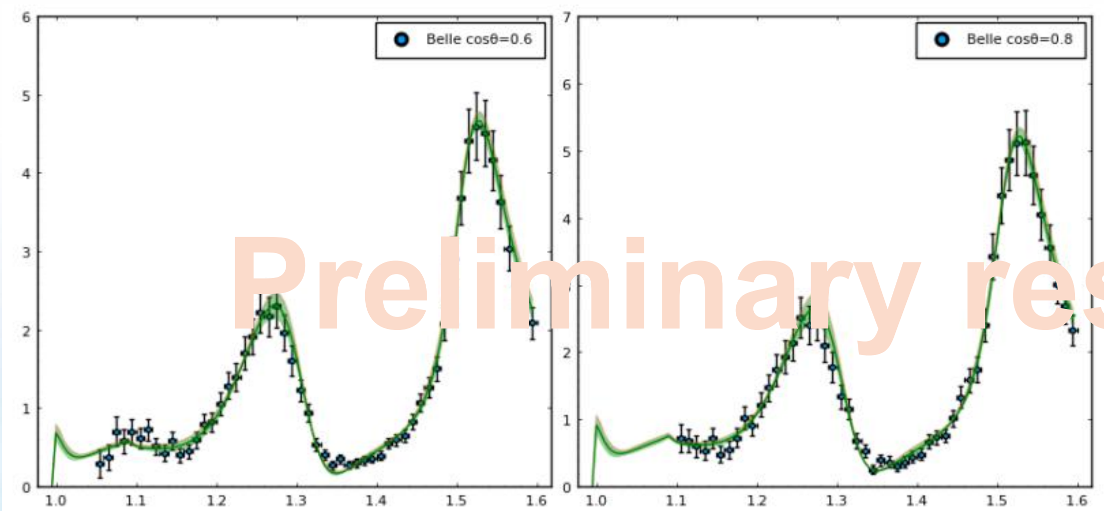


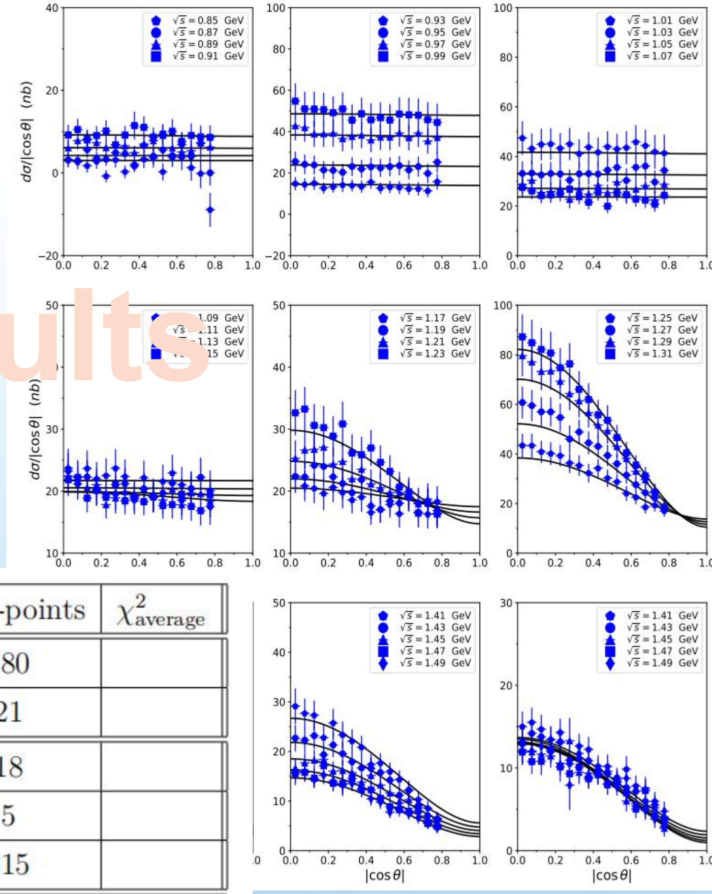
$\gamma\gamma\to\pi\pi$ individual partial waves



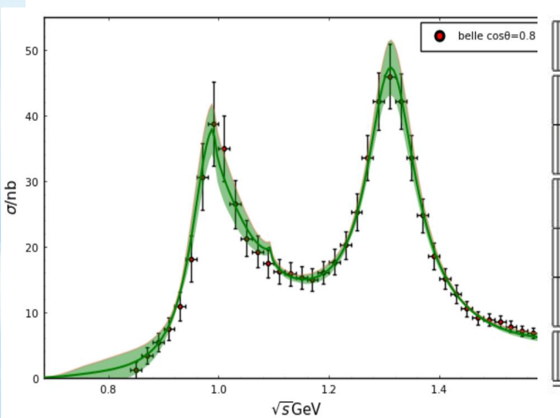
Other $\gamma\gamma$ collisions

- πη-KK-πη' coupled channel scatterings





Preliminary results



Experiment	Process	Data-points	χ^2_{average}
Belle/Crystal ball	$\gamma\gamma \to \pi^0\eta$	680	
CB(AGS)/A2 MAMI-B	$\eta \to \pi^0\gamma\gamma$	21	
TPC/Argus/Belle	$\gamma\gamma \to K^+K^-$	18	
TASSO/CELLO	$\gamma\gamma \to \bar{K}^0K^0$	5	
Belle	$\gamma\gamma \to \bar{K}^0_SK^0_S$	315	
BESIII	$\eta' \to \pi^0\gamma\gamma$	13	

- $a_0(980)$?
- HLBL constraints for I=1amplitudes

Kuang, Dai *et.al.*, in preparation

Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero.
- 4π channel's contribution is significant for HLBL
- I=0:150–200 nb, I=2: 50nb

evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25* †	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

BESIII? BelleII?

Dai&Pennington, PRD95 (2017) 056007;

4、Summary

ChEFT	ChEFT supplies dynamics of hadron interactions. Combined with FSI, they can be powerful tool.
$\pi\pi$, πK	ChEFT+FSI can give reliable $\pi\pi$, πK scattering amplitudes, with them one can accurately extract the poles
scalars	Analyticity+unitarity+Chiral symmetry can given strong constraints on the scalars, we propose a way to study their inner structure
g-2	Our g-2 has a significant discrepancy with the latest FNAL's. Processes of multi-body channels (both HVP and HLBL) needs to be studied: $\pi\pi\pi\pi$, $\pi\pi$KK?
Next?	Improving ChEFT+FSI? LQCD in the unphysical region, or some new thoughts?

Thank You For your patience !