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# A New Framework on Global Analysis of Fragmentation Functions

JHEP 2023, 108 (2023) with J. Gao, X.M. Shen & B. Zhou  
PRL 132, 261903 (2024) with J. Gao, X.M. Shen, H. Xing & Y. Zhao  
arXiv:2407.04422 with J. Gao, X.M. Shen, H. Xing & Y. Zhao

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# Outline

- 1 Introduction
- 2 Automatic Calculation of Hadron Cross Sections at NLO
- 3 Global Analysis of FFs to Light Charged Hadrons
- 4 Applications
- 5 Summary and Prospects

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# Single Inclusive Hadron Production

- In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), semi-inclusive DIS (SIDIS), pp collisions

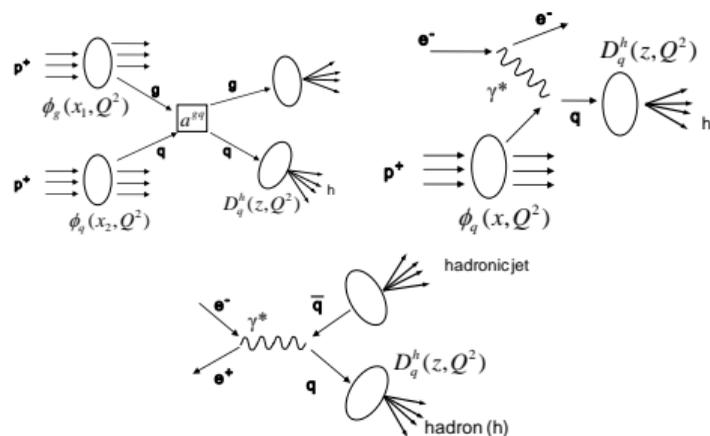


Figure: Illustration of hadron production in  $pp$ , SIDIS, and SIA processes

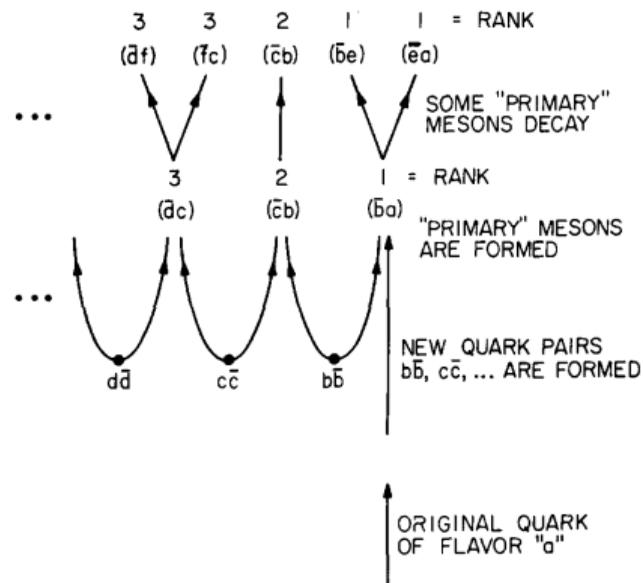
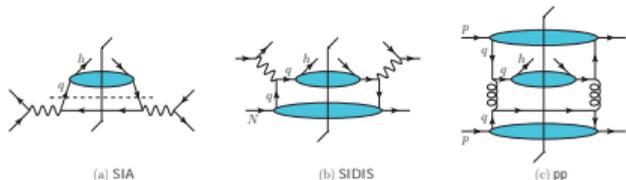


Figure: Cascade decay ansatz (R.D. Field & R.P. Feynman)

# QCD Collinear Factorization

- QCD factorization theorem enables the separation of the perturbatively calculable part of the cross section from the non-perturbative part which involves initial and final state hadrons. [J. C. Collins, D. E. Soper, G. Sterman]



- Hard scattering processes are independent of hadrons involved. Coefficient functions are perturbatively calculable.
- PDFs/FFs are non-perturbative part that describe the inner structure of hadrons or parton-hadron transition. They are universal and can be fitted from data.
- Evolution of FFs w.r.t. scale is governed by DGLAP equation.

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dz} = \frac{1}{\sum_q e_q^2} (2F_1^h(z, Q^2) + F_L^h(z, Q^2))$$

$$2F_1^h(z, Q^2) = \sum_q e_q^2 \left( D_1^{h/q}(z, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} (C_1^q \otimes D_1^{h/q} + C_1^g \otimes D_1^{h/g})(z, Q^2) \right)$$

$$F_L^h(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_q e_q^2 (C_L^q \otimes D_1^{h/q} + C_L^g \otimes D_1^{h/g})(z, Q^2).$$

$$\frac{d^3\sigma^{\ell p \rightarrow \ell hX}}{dx dy dz} = \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \left( \frac{1+(1-y)^2}{y} 2F_1^h(x, z, Q^2) + \frac{2(1-y)}{y} F_L^h(x, z, Q^2) \right)$$

$$2F_1^h(x, z, Q^2) = \sum_q e_q^2 \left( f_1^{q/p} D_1^{h/q} + \frac{\alpha_s(Q^2)}{2\pi} (f_1^{q/p} \otimes C_1^{qq} \otimes D_1^{h/q} + f_1^{q/p} \otimes C_1^{gq} \otimes D_1^{h/g} + f_1^{g/p} \otimes C_1^{qq} \otimes D_1^{h/q} + f_1^{g/p} \otimes C_1^{gq} \otimes D_1^{h/g}) \right),$$

$$F_L^h(x, z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_q e_q^2 (f_1^{q/p} \otimes C_L^{qq} \otimes D_1^{h/q} + f_1^{q/p} \otimes C_L^{gq} \otimes D_1^{h/g} + f_1^{g/p} \otimes C_L^{qq} \otimes D_1^{h/q} + f_1^{g/p} \otimes C_L^{gq} \otimes D_1^{h/g})$$

$$\frac{\partial}{\partial \ln Q^2} D_i^h(z, Q) = \sum_j P_{ji}(z, \alpha_s(Q)) \otimes D_j(z, Q), \quad i, j = q, \bar{q}, g$$

Operator Definition:

$$D_1^{h/q}(z) = \frac{z}{4} \int_X \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \text{Tr} \left[ \langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right].$$

# Global Data and Analysis

- Fragmentation measurements from various colliders (LHC, RHIC, LEP, PETRA, PEP, SLC, HERA, SPS) are available.

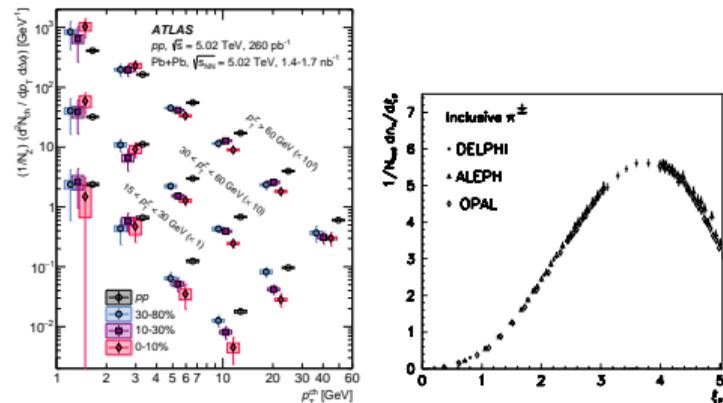
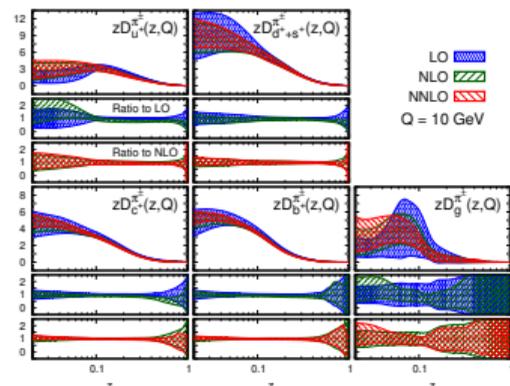


Figure: ATLAS 5.02 TeV [2008.09811] (left), DELPHI/ALEPH/OPAL 91.2 GeV[EPJC 5 (1998) 585-620] (right)

- Existing analyses include DSS, HKNS, AKK, NNFF, MAPFF, and JAM.
- Most are at NLO accuracy. Coefficients for SIDIS at NNLO have only recently become available, and NNLO calculations for  $pp$  collision processes are absent.



Introduction Figure: NNFF at 10 GeV

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# FMNLO

FMNLO is a framework for automatically calculating the hadron cross-section of arbitrary processes up to NLO precision

- ▶ Based on a hybrid scheme of phase space slicing and local subtraction method
- ▶ Supports arbitrary processes via the interface of MG5\_aMC@NLO, including  $pp$ ,  $ep$ , SIA, BSM
- ▶ Efficient: after the interpolation grid is generated, it can be convoluted with various FFs to obtain observables.
- ▶ Extensible, linked to LHAPDF or fortran subroutines
- ▶ More info on arXiv: 2401.02781, 2407.04422, <https://fmnlo.sjtu.edu.cn/~fmnlo>

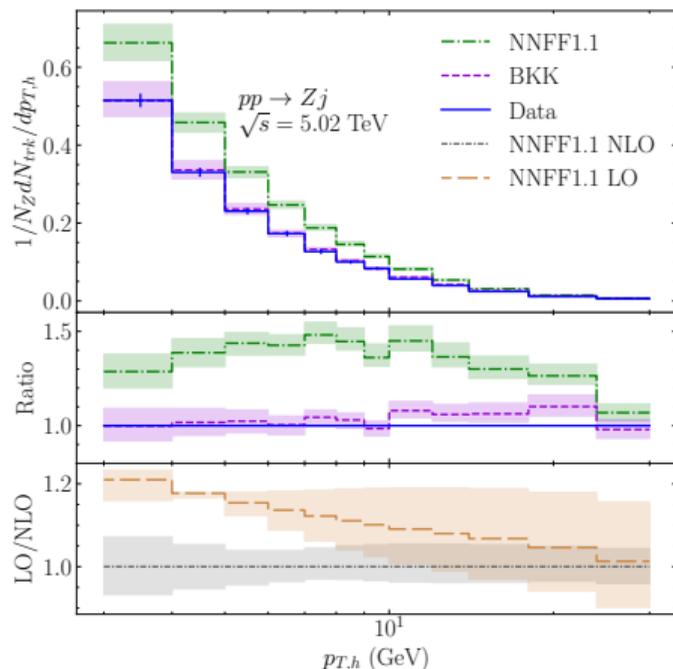


Figure: [C.Liu,J.Gao,X.Shen,B.Zhou, 2305.14620]

# FMNLO Tutorial

Step 1: Generation of interpolation grids: navigate to `mgen` folder and edit `proc.run`, run `./mgen.sh`

```
1 # main input for generation of NLO
  ↳ fragmentation grid file by MG5
2 process A210304377
3 # subgrids with name tags
4 grid pp_pt
5 obs 2
6 cut 0.02
7 ptz1 30.0
8 ptz2 10000.0
9 # in MG5 format
10 set lpp1 1
11 set lpp2 1
12 set ebeam1 2510.0
13 set ebeam2 2510.0
14 set lhaid 13100
15 set iseed 11
16 set muR_over_ref 1.0
17 set muF_over_ref 1.0
18 end
```

Step 2: Convolution with FFs: navigate to `data` folder and edit `input.card`, run `./fmnlo` and get outputs

```
1 # loop for D fun (1/2 -> LO/NLO) | evo for D fun (0/1 -> internal/hoppet)
2 # followed by >=1/0 -> internal/LHAPDF | FFID | FFmember
3 2 0
4 0 NNFF11_HadronSum_nlo 00
5 # normalization | grid file | binnig file
6 # 0/1/2 -> absolute dis./normalized to corresponding order/leading order
7 # can include multiple entries in several lines
8 1 1 "../grid/A210304377_pp_pt.fmg" "../grid/2103-04377-pt.Bin"
```

1	ID(1/x dx/dpTh)	pTd[GeV]	pTu[GeV]	LO*{1,0.5,2}
2	↳ NLO*{1,0.5,2}	NLO/LO		
2	1 4.00000E+00	5.00000E+00	5.39764E-01	... 4.61013E-01 ... 0.854 ...
3	2 5.00000E+00	6.00000E+00	3.82521E-01	... 3.32960E-01 ... 0.870 ...
4	3 6.00000E+00	7.00000E+00	2.80537E-01	... 2.47702E-01 ... 0.883 ...
5	4 7.00000E+00	8.00000E+00	2.10826E-01	... 1.88229E-01 ... 0.893 ...
6	5 8.00000E+00	9.00000E+00	1.61447E-01	... 1.45418E-01 ... 0.901 ...
7	6 9.00000E+00	1.00000E+01	1.25590E-01	... 1.13940E-01 ... 0.907 ...
8	7 1.00000E+01	1.20000E+01	8.91022E-02	... 8.14961E-02 ... 0.915 ...
9	8 1.20000E+01	1.40000E+01	5.81090E-02	... 5.36035E-02 ... 0.922 ...
10	9 1.40000E+01	1.80000E+01	3.35533E-02	... 3.12641E-02 ... 0.932 ...
11	10 1.80000E+01	2.40000E+01	1.51170E-02	... 1.43453E-02 ... 0.949 ...
12	11 2.40000E+01	3.00000E+01	6.48118E-03	... 6.33102E-03 ... 0.977 ...
13				

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# Data Selection

- A wide range of data on fragmentation to pions, kaons, protons, and charged hadrons is incorporated in the fit. The hadron-in-jet data is included in the global analysis for the first time, including inclusive hadron production, Z-tagged jets, photon-tagged jets, and dijets measurements

exp.	$\sqrt{s}(\text{TeV})$	luminosity	hadrons	final states	$R_j$	cuts for jets/hadron	observable	$N_{\text{pt}}$
ATLAS	5.02	25 pb <sup>-1</sup>	$h^\pm$	$\gamma + j$	0.4	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}$	$\frac{1}{N_{\text{jet}}} \frac{dN_{h,j}}{d\phi_{T,h}}$	6
CMS	5.02	27.4 pb <sup>-1</sup>	$h^\pm$	$\gamma + j$	0.3	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{h,j}}{d\phi}$	4
ATLAS	5.02	260 pb <sup>-1</sup>	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{4}\pi$	$\frac{1}{n_Z} \frac{dN_{h,j}}{d\phi_{T,h}}$	9
CMS	5.02	320 pb <sup>-1</sup>	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{8}\pi$	$\frac{1}{n_Z} \frac{dN_{h,j}}{d\phi_{T,h}}$	11
LHCb	13	1.64 fb <sup>-1</sup>	$\pi^\pm, K^\pm, p/\bar{p}$	$Z + j$	0.5	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{n_Z} \frac{dN_{h,j}}{d\phi}$	20
ATLAS	5.02	25 pb <sup>-1</sup>	$h^\pm$	inc. jet	0.4	-	$\frac{1}{N_{\text{jet}}} \frac{dN_{h,j}}{d\phi}$	63
ATLAS	7	36 pb <sup>-1</sup>	$h^\pm$	inc. jet	0.6	$\Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{h,j}}{d\phi}$	103
ATLAS	13	33 fb <sup>-1</sup>	$h^\pm$	dijet	0.4	$p_T^{\text{had}}/p_T^{\text{bhad}} < 1.5$	$\frac{1}{N_{\text{jet}}} \frac{dN_{h,j}}{d\phi}$	280

exp.	$\sqrt{s_{NN}}(\text{TeV})$	# events (million)	$p_{T,h}$	hadrons	observable	$N_{\text{pt}}$
ALICE	13	40-60(pp)	[2, 20] GeV	$\pi, K, p, K_S^0$	$K/\pi, p/\pi, K_S^0/\pi$	49
ALICE	7	150(pp)	[3, 20] GeV	$\pi, K, p$	13TeV/7TeV for $\pi, K, p$	37
ALICE	5.02	120(pp)	[2, 20] GeV	$\pi, K, p$	$K/\pi, p/\pi$	34
ALICE	2.76	40(pp)	[2, 20] GeV	$\pi, K, p$	$K/\pi, p/\pi$	27
STAR	0.2	14(pp)	[3, 15] GeV	$\pi, K, p, K_S^0$	$K/\pi, p/\pi^+, p/\pi^-, K_S^0/\pi, \pi^-/\pi^+, K^-/K^+$	60

Table: [C.Liu,J.Gao,X.Shen,H.Xing,Y.Zhao,2407.04422]

- Other data include ratios of inclusive hadron production rates in pp collisions, SIA data (with/without heavy-flavor tagging)<sup>†</sup>, mostly at the Z-pole, and hadron production in SIDIS from HERA and COMPASS

exp.	$\sqrt{s}(\text{GeV})$	luminosity	kinematic cuts	hadrons	obs	$N_{\text{pt}}$
H1	318	44 pb <sup>-1</sup>	$Q^2 \in [175, 20000] \text{ GeV}^2$	$h^\pm$	$D \equiv \frac{1}{N} \frac{dn_{h,\pm}}{dx_F}$	16
H1	318	44 pb <sup>-1</sup>	$Q^2 \in [175, 8000] \text{ GeV}^2$	$h^\pm$	$A \equiv \frac{D^+ - D^-}{D^+ + D^-}$	14
ZEUS	300,318	440 pb <sup>-1</sup>	$Q^2 \in [160, 40960] \text{ GeV}^2$	$h^\pm$	$D$	32
COMPASS06	17.3	540 pb <sup>-1</sup>	$x \in [0.14, 0.4], y \in [0.3, 0.5]$	$\pi, K, h$	$\frac{dM^h}{d\phi}$	124
COMPASS16	17.3	-	$x \in [0.14, 0.4], y \in [0.3, 0.5]$	$\pi, K, p$	$\frac{dM^h}{d\phi}$	97

exp.	$\sqrt{s}$	lum.(n <sub>Z</sub> )	final states	hadrons	$N_{\text{pt}}$
OPAL	$m_Z$	780 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm$	20
ALEPH	$m_Z$	520 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	42
DELPHI	$m_Z$	1 400 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	39
			$Z \rightarrow b\bar{b}$	$\pi^\pm, K^\pm, p(\bar{p})$	39
	$m_Z$	400 000	$Z \rightarrow q\bar{q}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
			$Z \rightarrow b\bar{b}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
			$Z \rightarrow c\bar{c}$	$\pi^\pm, K^\pm, p(\bar{p})$	66
TASSO	34GeV	77 pb <sup>-1</sup>	inc. had.	$\pi^\pm, K^\pm, p(\bar{p})$	3
TASSO	44GeV	34 pb <sup>-1</sup>	inc. had.	$\pi^\pm, \pi^0$	5
TPC	29GeV	70 pb <sup>-1</sup>	inc. had.	$\pi^\pm, K^\pm$	12
OPAL	201.7GeV	433 pb <sup>-1</sup>	inc. had.	$h^\pm$	17
DELPHI	189GeV	157.7 pb <sup>-1</sup>	inc. had.	$\pi^\pm, K^\pm, p(\bar{p})$	9

# Parametrization of Fragmentation Functions

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^m a_{i,n}^h (\sqrt{z})^n\right)$$

parton-to- $\pi^+$	favored	$\alpha$	$\beta$	$a_0$	$a_1$	$a_2$	d.o.f.
$u$	Y						5
$d \simeq u$	Y	-	-		-	-	1
$\bar{u} = d$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
$g$	N		F				4
parton-to- $K^+$	favored	$\alpha$	$\beta$	$a_0$	$a_1$	$a_2$	d.o.f.
$u$	Y					x	4
$\bar{s} \simeq u$	Y	-	-		-	x	1
$\bar{u} = d = d = s$	N					x	4
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
$g$	N		F			x	3
parton-to- $p$	favored	$\alpha$	$\beta$	$a_0$	$a_1$	$a_2$	d.o.f.
$u = 2d$	Y					x	4
$\bar{u} = d = s = \bar{s}$	N				x	x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
$g$	N		F			x	3

Table:

[C.Liu, J. Gao, X. Shen, H. Xing, Y. Zhao, 2407.04422]

- ▶ Choose  $Q_0 = 5$  GeV and zero mass scheme for heavy quarks with  $n_f = 5$
- ▶ The degree of polynomial is increased until no significant improvements are observed
- ▶ Approximate ( $\simeq$ ) or exact ( $=$ ) flavor symmetry among favored (anti-)quarks or unfavored light (anti-)quarks are assumed, resulting in 63 free parameters in total.
- ▶ CT14 NLO parton distribution functions are used for calculations involving initial hadrons
- ▶ A stringent cut ( $p_T > 4$  GeV,  $z > 0.01$ ) is applied to data to ensure the validity of factorization theorem.
- ▶ Theoretical calculations are performed using the FMNLO framework at NLO accuracy. Uncertainties are estimated using Hessian methods [hep-ph/0101032] and various combinations of fragmentation and factorization scales.

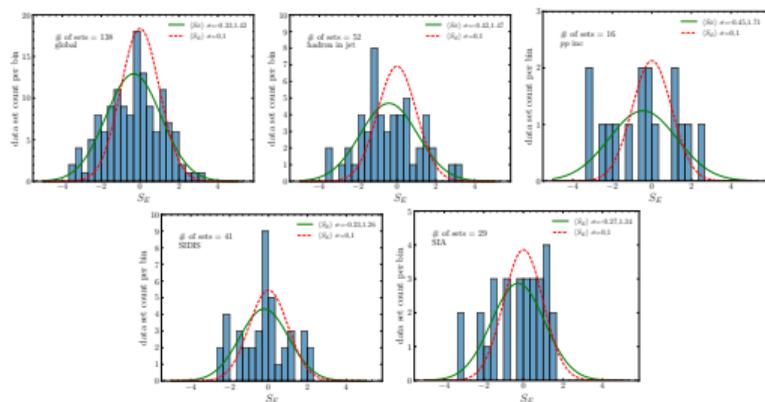
# Fit Quality

Experiments	$N_{pt}$	$\chi^2$	$\chi^2/N_{pt}$
ATLAS 5.02 TeV $\gamma + j$	6	9.6	1.61
CMS 5.02 TeV $\gamma + j$	4	11.1	2.78
ATLAS 5.02 TeV $Z + h$	9	22.2	2.47
CMS 5.02 TeV $Z + h$	11	6.2	0.56
LHCb 13 TeV $Z + j$	20	30.6	1.53
ATLAS 5.02 TeV inc. jet	63	67.9	1.08
ATLAS 7 TeV inc. jet	103	91.3	0.89
ATLAS 13 TeV dijet	280	191.6	0.68
<b>pp hadron in jet sum</b>	<b>496</b>	<b>430.5</b>	<b>0.87</b>
ALICE 13 TeV	49	45.0	0.92
ALICE 7 TeV	37	36.3	0.98
ALICE 5.02 TeV	34	37.5	1.10
ALICE 2.76 TeV	27	31.8	1.18
STAR 200 GeV	60	42.2	0.70
<b>pp inclusive sum</b>	<b>207</b>	<b>192.8</b>	<b>0.93</b>
H1 $\dagger$	16	12.5	0.78
H1 (asy.) $\dagger$	14	12.2	0.87
ZEUS $\dagger$	32	65.5	2.05
COMPASS 06 ( $D$ )	124	107.3	0.87
COMPASS 16 ( $p$ )	97	56.8	0.59
<b>SIDIS sum</b>	<b>283</b>	<b>254.4</b>	<b>0.90</b>
OPAL $Z \rightarrow q\bar{q}$	20	16.3	0.81
ALEPH $Z \rightarrow q\bar{q}$	42	31.4	0.75
DELPHI $Z \rightarrow q\bar{q}$	39	12.5	0.32
DELPHI $Z \rightarrow b\bar{b}$	39	23.9	0.61
SLD $Z \rightarrow q\bar{q}$	66	53.0	0.8
SLD $Z \rightarrow b\bar{b}$	66	82.0	1.24
SLD $Z \rightarrow c\bar{c}$	66	76.5	1.16
TASSO 34 GeV inc. had.	3	2.7	0.9
TASSO 44 GeV inc. had.	5	4.3	0.86
TPC 29 GeV inc. had.	12	11.6	0.97
OPAL (202 GeV) inc. had. $\dagger$	17	24.2	1.42
DELPHI (189 GeV) inc. had.	9	15.3	1.70
<b>SIA sum</b>	<b>384</b>	<b>353.8</b>	<b>0.92</b>
<b>Global total</b>	<b>1370</b>	<b>1231.5</b>	<b>0.90</b>

- ▶ A best-fit with good agreements to the global data sets (1370 points in total) are found with  $\chi^2/N_{pt} = 0.90$
- ▶ The  $\chi^2/N_{pt}$  values are 0.93, 0.87, 0.90, and 0.92 for the data groups of inclusive hadron production, jet fragmentation in pp collisions, inclusive hadron production from SIA and SIDIS, respectively.

Table:  $\chi^2$  table for individual set [2407.04422]

# Fit Quality



- Distribution of  $S_E$  for all subsets closely resemble Gaussian distribution with  $\sigma = 1.42$  and  $\mu = -0.33$ , which motivates a choice of tolerance of  $\Delta\chi^2 \approx 2$  in our determination of uncertainties of the FFs with the Hessian method [hep-ph/0101032]

Figure:  $S_E$  distribution for total/ $pp$  hadron-in-jet/ $pp$  inclusive/SIDIS/SIA process  
 [C.Liu,J.Gao,X.Shen,H.Xing,Y.Zhao,2407.04422]

$$S_E = \frac{(18N_{pt})^{3/2}}{18N_{pt}+1} \left\{ \frac{6}{6-\ln(\chi^2/N_{pt})} - \frac{9N_{pt}-1}{9N_{pt}} \right\}$$

# NPC23 FFs

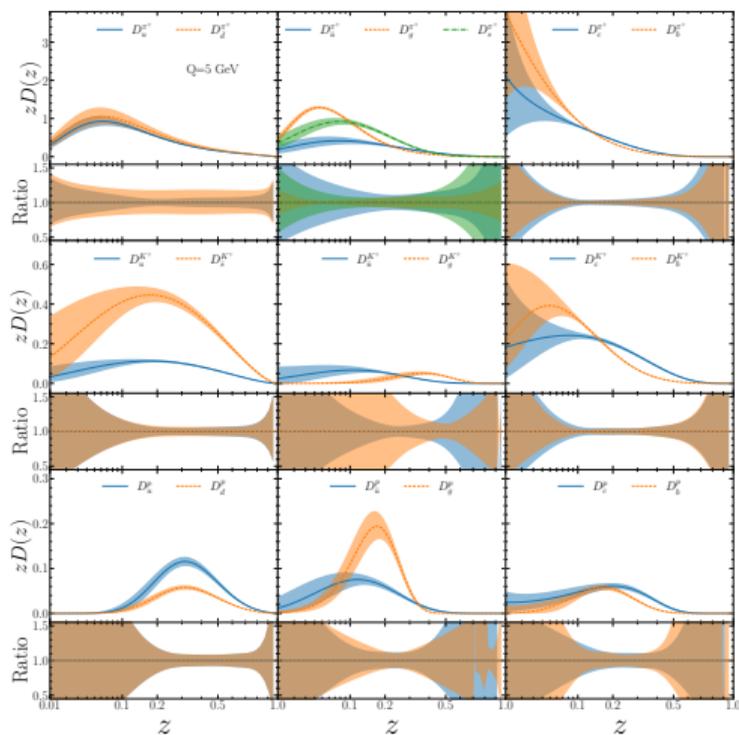


Figure: FFs from various partons to light charged hadrons ( $\pi^+$ ,  $K^+$ ,  $p$ ) at 5 GeV [2407.04422]

ChongYang Liu

- ▶ FFs to charged hadrons are well constrained for momentum fractions  $z \sim 0.1 - 0.7$
- ▶ Gluon to pion FFs show uncertainties of 3%, 4%, and 8% at  $z = 0.05, 0.1$ , and  $0.3$ , respectively.
- ▶  $u$ -quark to pion, kaon, and proton FFs have uncertainties of 4%, 4%, and 7% at  $z = 0.3$ , respectively.
- ▶ The FFs from heavy quarks ( $c$  and  $b$ ) are well constrained for  $z$  between  $0.1 \sim 0.5$  due to tagged data in SIA at Z pole
- ▶ Strange quark to pion FFs can be larger than  $d$  quark to pion FFs, possibly due to pulls from different SIA measurements.
- ▶ Gluon FFs are well-constrained due to the inclusion of hadron-in-jet data from LHC

# Test on Momentum Sum Rule

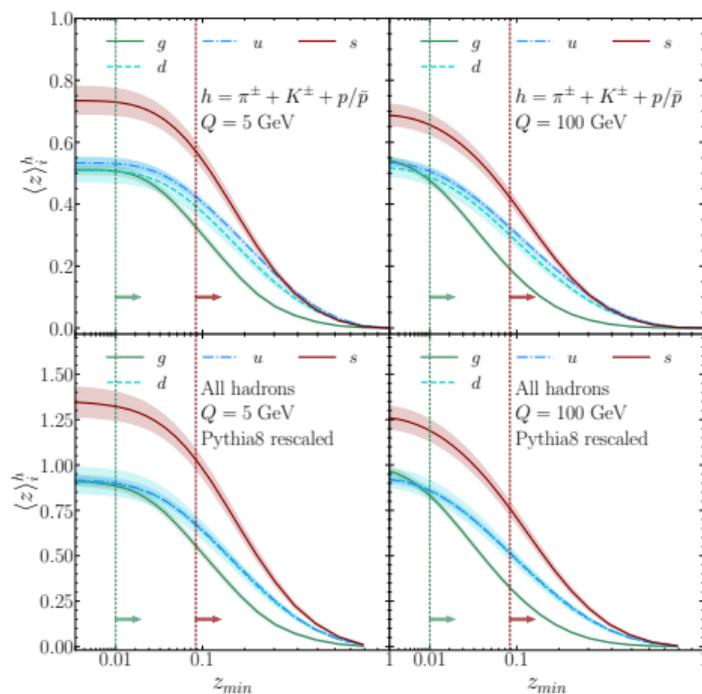


Figure: Momentum fraction carried by light charged hadrons and all hadrons

[C.Liu,J.Gao,X.Shen,H.Xing,Y.Zhao,2401.02781]

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► Momentum Sum Rule:  

$$\sum_h \int_0^1 dz z D_i^h(z, Q) = 1.$$

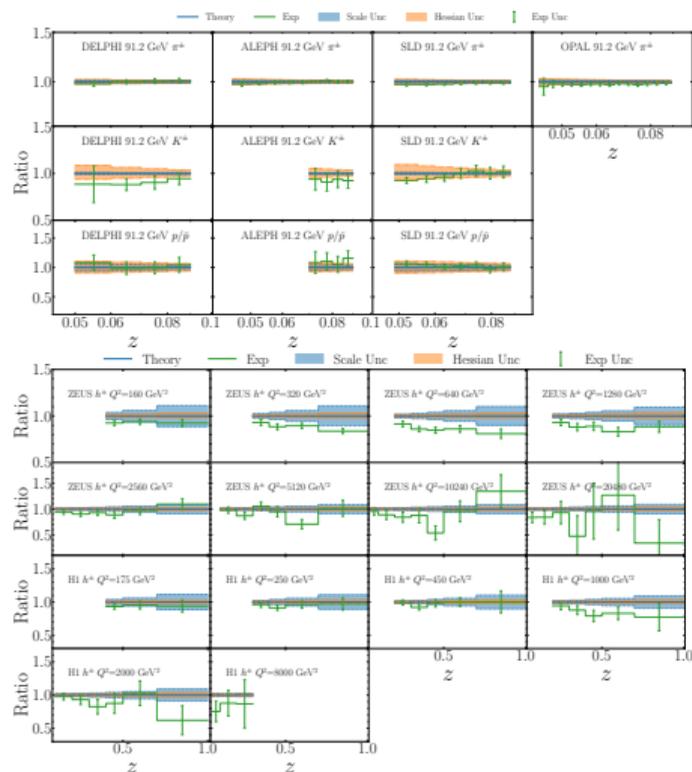
► It's important to check the validation of these fundamental properties by a data-driven analysis.[2309.03346]

► Total momentum fraction of parton  $i$  carried by hadron  $h$ :  $\langle z \rangle_i^h = \int_{z_{min}}^1 dz z D_i^h(z, Q).$

► We extrapolate  $\langle z \rangle_i^h$  to small  $z$  and rescale to the momentum carried by all hadrons. Rescale factors of light charged hadrons( $\pi^\pm, K^\pm, p/(\bar{p})$ ) to all hadrons are obtained from Pythia8 simulation.

► For strange quarks the values can be well above 1 due to both the tension mentioned earlier and the limited coverage of data.

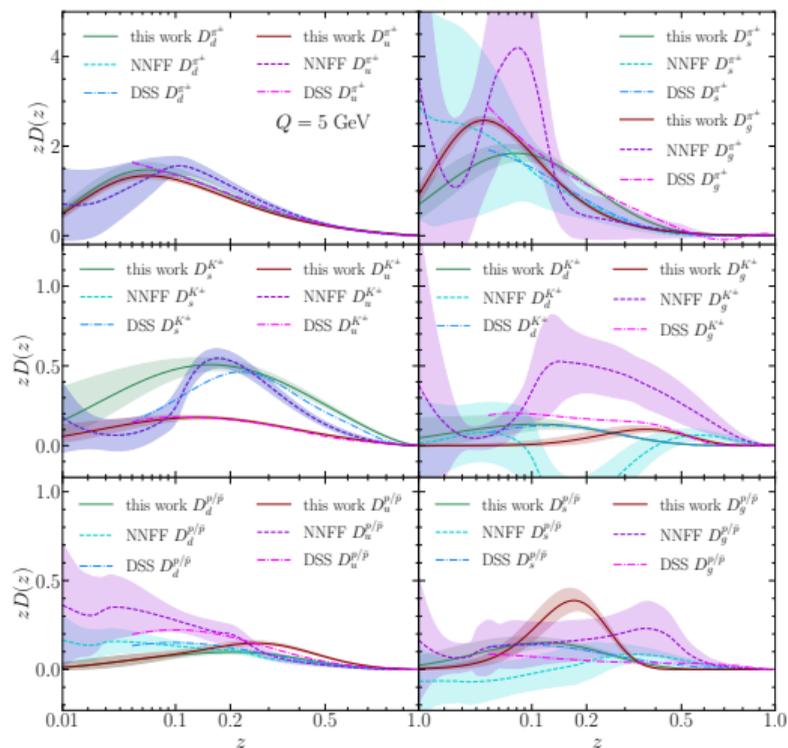
# Extrapolation to Small $z$ Range



- We show a comparison of predictions to data excluded by kinematic cuts (upper) and find good agreement down to  $z = 0.05$ , which indicates the QCD factorization is still valid for SIA data down to  $z \sim 0.05$
- Similar comparisons are performed with SIDIS data (lower), and the consistency is found down to  $E_h \sim 2 - 4 \text{ GeV}$ .

Figure: [C.Liu,J.Gao,X.Shen,H.Xing,Y.Zhao,2407.04422]

# Comparison with Other Groups



- Reasonable agreement can be observed between our results and DSS for FFs of  $u$  and  $d$  quarks to  $\pi^\pm$ , and of  $u$  quark to  $K^\pm$ .
- Large discrepancies are found for FFs to protons and for FFs of gluon to all three charged hadrons.
- The high precision of gluon FFs is mostly due to jet fragmentation data at the LHC.
- The newly added data on proton production from SIDIS and pp collisions lead to better flavor separation and thus the differences observed for FFs to protons.

Figure:

[C.Liu, J.Gao, X.Shen, H.Xing, Y.Zhao, 2401.02781]

Chong Yang Liu

# Outline

- 1 Introduction
- 2 Automatic Calculation of Hadron Cross Sections at NLO
- 3 Global Analysis of FFs to Light Charged Hadrons
- 4 Applications**
- 5 Summary and Prospects

# Heavy Nuclei Collision

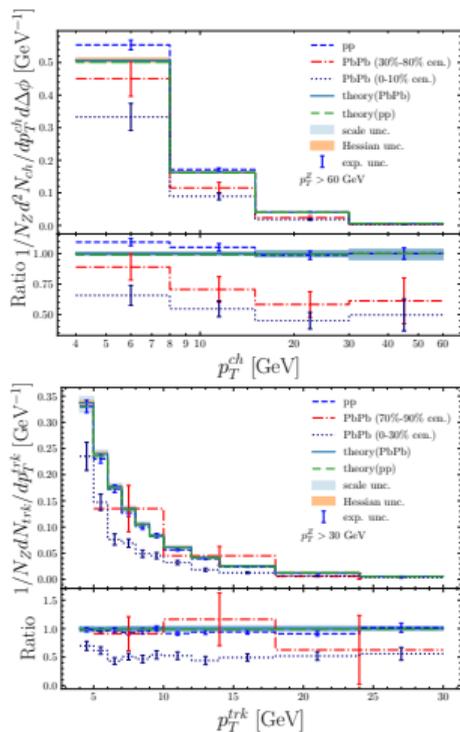
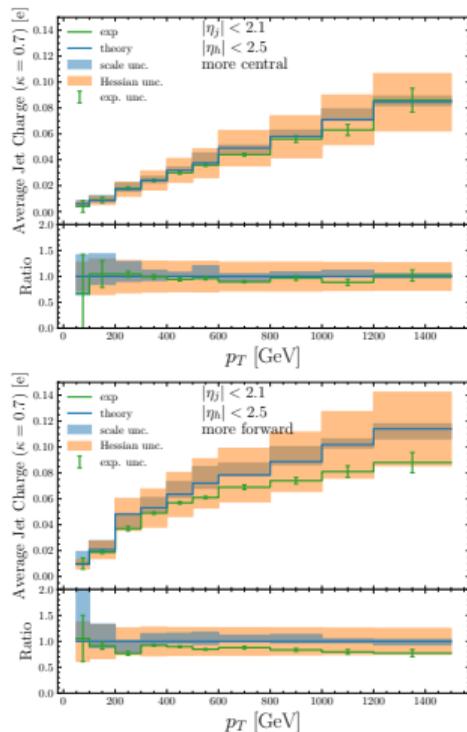


Figure:  
[C.Liu,J.Gao,X.Shen,H.Xing,Y.Zhao,2407.04422]

- ▶ By comparing reference cross sections with experiments, medium effects can be studied.
- ▶ nCTEQ15 PDFs are used for PDF inputs of  $Pb$  nuclei
- ▶  $PbPb$  reference cross sections are close to  $pp$  reference cross sections, as changes in PDF lead to small corrections in the flavor composition of final state jets
- ▶ Measurements of central  $PbPb$  collisions cross section are suppressed by 50% compared to the reference cross sections.
- ▶ Measurements of peripheral collisions (lower) align with reference cross sections within uncertainties.

# Jet Charge



- ▶ Jet charge:  $Q_J = \sum_{i \in J} \left( \frac{p_{T,i}}{p_{T,J}} \right)^\kappa Q_i$
- ▶ Our predictions from best-fit FFs agree well with the ATLAS measurements on the more central jet
- ▶ Our predictions are higher by 10% ~ 20% compared to the ATLAS measurements on the more forward jet.
- ▶ The Hessian uncertainties from FFs are about 30% for all  $p_T$  ranges considered, much larger than both the experimental uncertainties and the scale variations
- ▶ Current or future data from LHC measurements on jet charges can place further stringent constraints on FFs

Figure:

[C.Liu,J.Gao,X.Shen,H.Xing,Y.Zhao,2407.04422]

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# Summary and Prospects

- ▶ Determining various non-perturbative inputs of QCD is essential for precision programs at the LHC and upcoming Electron-Ion Colliders, as well as for understanding QCD confinement.
- ▶ FMNLO is a program that can be used to calculate hadron production cross sections at NLO for arbitrary hard process. The computationally intensive hard interaction part is saved as grids which greatly improve efficiency.
- ▶ We perform a joint determination of FF to light charged hadrons ( $\pi^+$ ,  $K^+$ ,  $p$ ) with carefully selected global data from SIA, SIDIS and  $pp$  collisions, leading to a well determined fragmentation functions but discrepancies with previous fit are also found. We further apply the FFs to the prediction of heavy nuclei collision and jet charge measurements.
- ▶ We plan to extend the identified hadrons to include neutral hadrons ( $\pi^0$ ,  $\eta$ ,  $K_S^0$ ,  $\Lambda$ ).
- ▶ Upgrade the accuracy of the program to NNLO

**Thank you for your attention!**