

Compton Scattering at the High Intensity Gamma-Ray Source (HIGS)

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Shandong University

The 12th Workshop on Hadron Physics and Opportunities Worldwide

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- **Nucleon polarizabilities**
- **Compton scattering experiments at HIGS**
 - The HIGS facility
 - Experimental apparatus
- **Results for nuclear Compton scattering**
 - from the proton, deuteron, ^4He , and ^3He
- **Outlook**

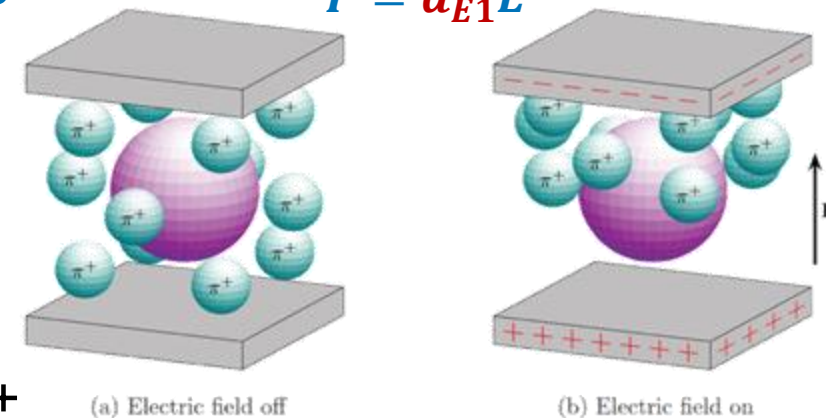
- Nucleon **electromagnetic (EM) structure constants**
- Characterize the **response of the charged constituents of a nucleon to an external EM field**
 - α_{E1} : charged pion-cloud dynamics
 - β_{M1} : pion charge current dynamics (diamagnetic) + constituent quarks dynamics (paramagnetic)

- Primarily accessed via **Compton Scattering**
- Theoretical approaches: **χ EFT, LQCD**, DR, models
- Balding sum rule

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi} \int_{\omega_0}^{\infty} \frac{\sigma_{tot}(\omega')}{\omega'^2 - \omega^2} d\omega'$$

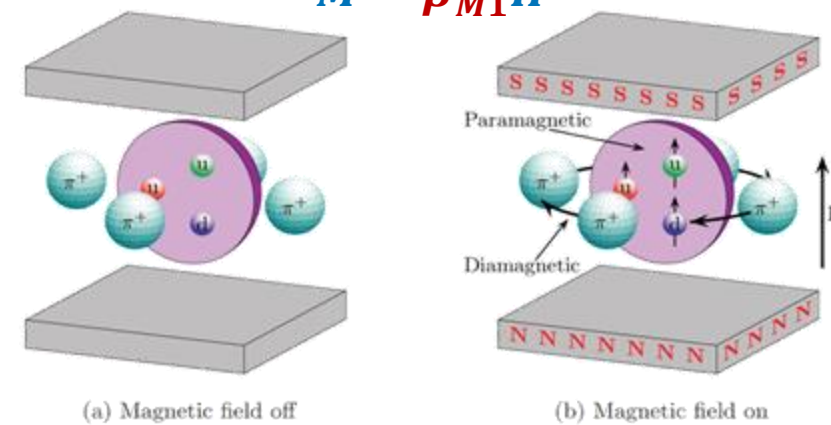
Electric polarizability (α_{E1})

$$\vec{P} = \alpha_{E1} \vec{E}$$



Magnetic polarizability (β_{M1})

$$\vec{M} = \beta_{M1} \vec{H}$$



(Graphs credited to P. Martel)

Effective probe for nucleon polarizabilities

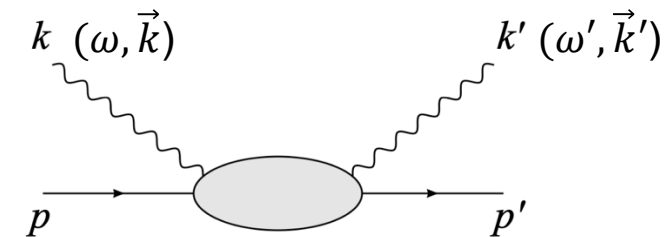
- Low-energy expansion of the differential cross section:

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{1}{2} \left(\frac{e^2 Z^2}{M_N} \right)^2 \left(\frac{\omega'}{\omega} \right)^2 [1 + g(\omega^2, \kappa)] \\ & - \left(\frac{e^2 Z^2}{4\pi M_N} \right) \left(\frac{\omega'}{\omega} \right)^2 (\omega\omega') \left[\frac{1}{2} (\alpha + \beta)(1 + \cos\theta)^2 + \frac{1}{2} (\alpha - \beta)(1 - \cos\theta)^2 \right] \\ & + f(\omega^3, \gamma_1, \gamma_2, \gamma_3, \gamma_4) \\ & + \mathcal{O}(\omega^4) \end{aligned}$$

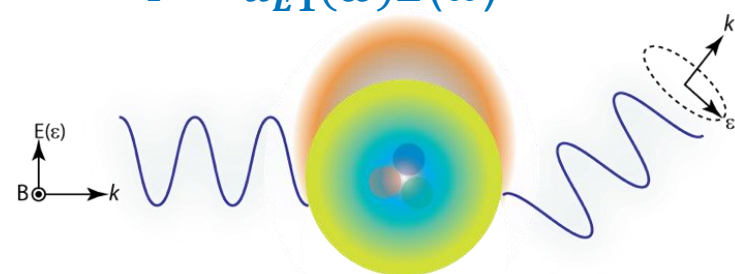
eZ : nucleon charge

M_N : nucleon mass

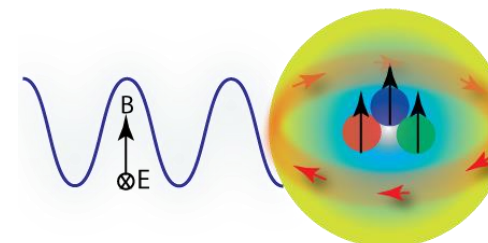
κ : anomalous magnetic moment



$$\vec{P} = \alpha_{E1}(\omega) \vec{E}(\omega)$$



$$\vec{M} = \beta_{M1}(\omega) \vec{H}(\omega)$$



Effective probe for nucleon polarizabilities

- Low-energy expansion of the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left(\frac{e^2 Z^2}{M_N} \right)^2 \left(\frac{\omega'}{\omega} \right)^2 [1 + g(\omega^2, \kappa)]$$

Born term (nucleons are assumed as point-like particles)

$$- \left(\frac{e^2 Z^2}{4\pi M_N} \right) \left(\frac{\omega'}{\omega} \right)^2 (\omega\omega') \left[\frac{1}{2} (\alpha + \beta) (1 + \cos\theta)^2 + \frac{1}{2} (\alpha - \beta) (1 - \cos\theta)^2 \right]$$

$$+ f(\omega^3, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$$

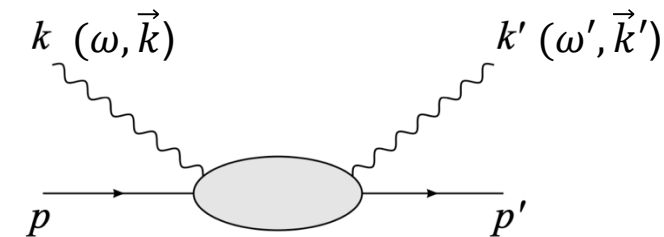
Electromagnetic dipole polarizabilities

$$+ \mathcal{O}(\omega^4)$$

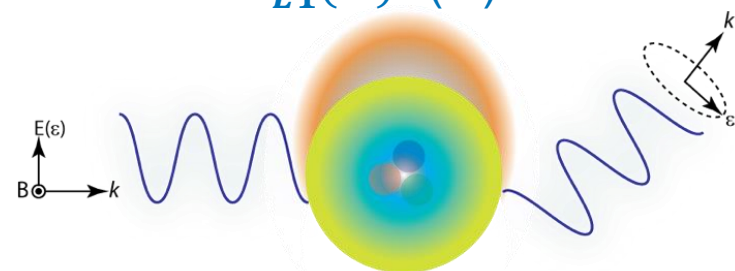
Spin polarizabilities

eZ : nucleon charge
 M_N : nucleon mass
 κ : anomalous magnetic moment

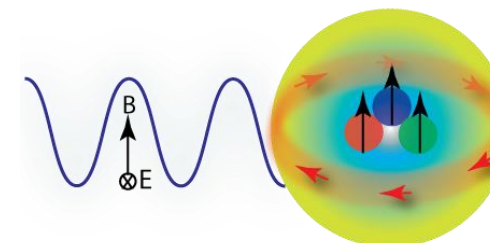
To extract α and β
 ✓ Measure the forward and backward Compton scattering cross sections



$$\vec{P} = \alpha_{E1}(\omega) \vec{E}(\omega)$$



$$\vec{M} = \beta_{M1}(\omega) \vec{H}(\omega)$$



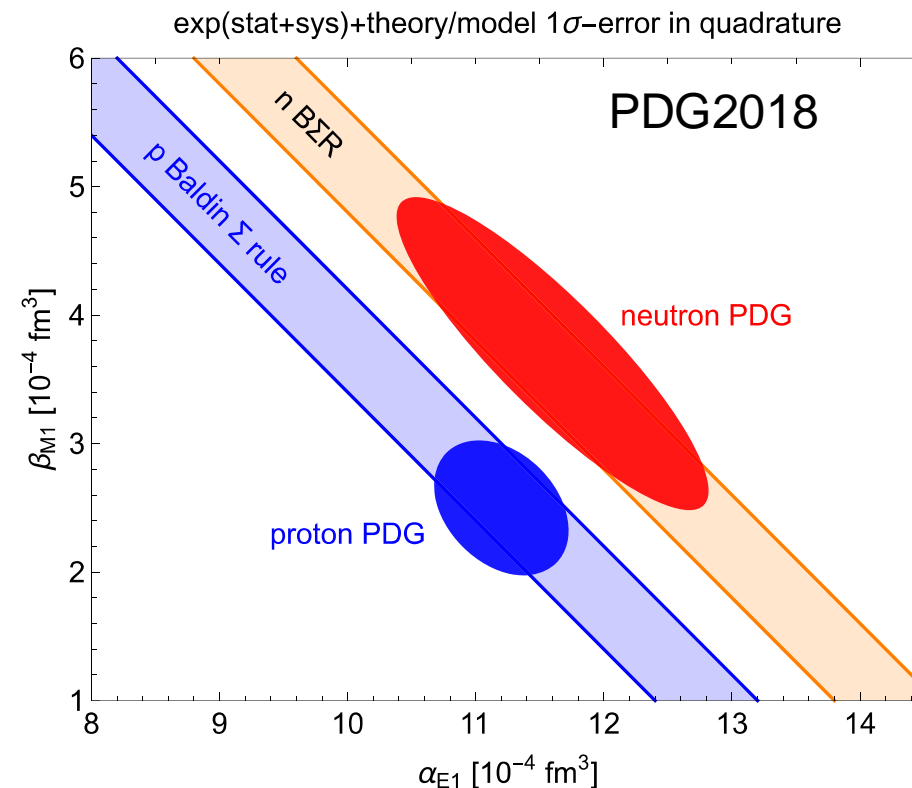
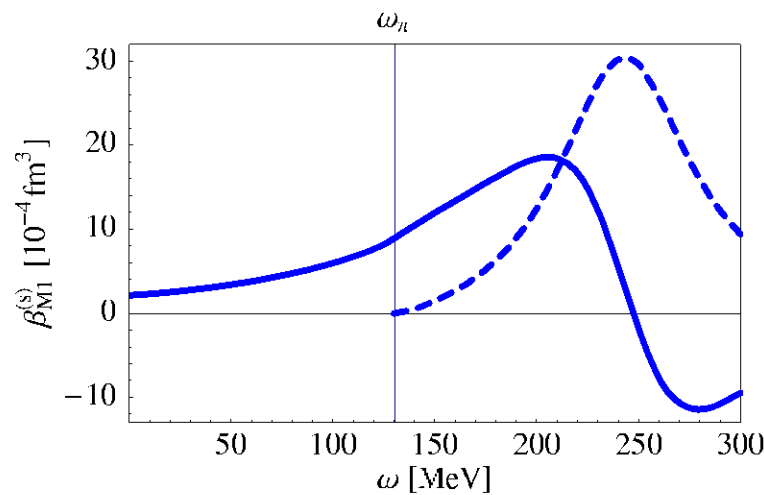
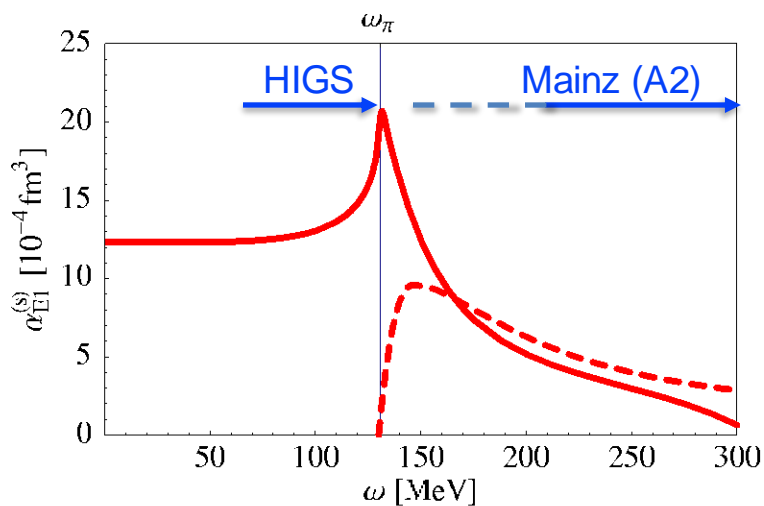
- **Proton** EM polarizabilities relatively well determined using **liquid hydrogen targets**
- **Neutron** measurements always harder
 - Charge neutral
 - α_{E1} and β_{M1} appear at the order of ω^4
 - No stable free neutron target
 - use **light nuclear targets** (D, He, Li,...)
 - model dependent

$$\alpha_{E1}^{(p)} = 11.2 \pm 0.4$$

$$\beta_{M1}^{(p)} = 2.5 \pm 0.4$$

$$\alpha_{E1}^{(n)} = 11.8 \pm 1.1$$

$$\beta_{M1}^{(n)} = 3.7 \pm 1.2$$



Baldin sum rule

$$\alpha_{E1}^{(p)} + \beta_{M1}^{(p)} = 13.8 \pm 0.4$$

$$\alpha_{E1}^{(n)} + \beta_{M1}^{(n)} = 15.2 \pm 0.4$$

R. P. Hildebrandt, H.W. Griesshammer, T.R. Hemmert and B. Pasquini, Eur. Phys. J. A **20**, 293 (2004)

From US 2023 Long Range Plan for Nuclear Science:

A consortium of 13 university-based accelerator laboratories, known collectively as the **Association for Research at University Nuclear Accelerators (ARUNA)**



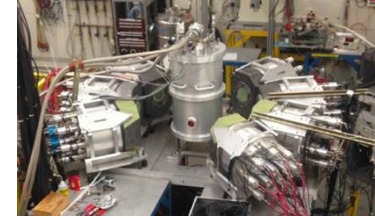
Figure 9.7. The unique ARUNA facilities are distributed throughout the country in 11 states: Florida, Kentucky, Indiana, Massachusetts, Michigan, New York, North Carolina, Ohio, Texas, Virginia, and Washington [44].

University (Figure 9.7). In addition to providing a high level of hands-on training in every aspect of an experiment, the accelerator facilities at these institutions provide unique beam and research capabilities that are often not available elsewhere. These facilities add an element of agility to US low-energy nuclear physics research by offering flexibility in scheduling and quick response to research developments and challenges. Importantly, ARUNA facilities are cost-effective to operate, enabling beam time to be devoted to a project for a long duration as is often required in nuclear astrophysics, where cross sections are low, and in fundamental symmetries, where high statistics and extensive studies of systematics are required. The diversity of approaches provided by these laboratories is a critical asset of the field, and ARUNA laboratories provide a highly creative, flexible, stimulating, and supportive scientific environment with many opportunities for students to acquire the essential skills necessary for them to become a well-trained nuclear workforce. Scientists at ARUNA facilities pursue research in nuclear astrophysics, low-energy nuclear physics, fundamental symmetries, and a rapidly growing number of nuclear physics applications that build bridges to other research communities.

High Intensity Gamma-Ray Source (HIGS)

Most intense Compton γ -ray source in the world

- Energy range: 1-120 MeV
- Energy resolution: $\Delta E/E \sim 5\%$ (selectable by collimation)
- Intensity: $>10^7 \gamma/s$ on target
- Polarization: $>95\%$ linear and circ.

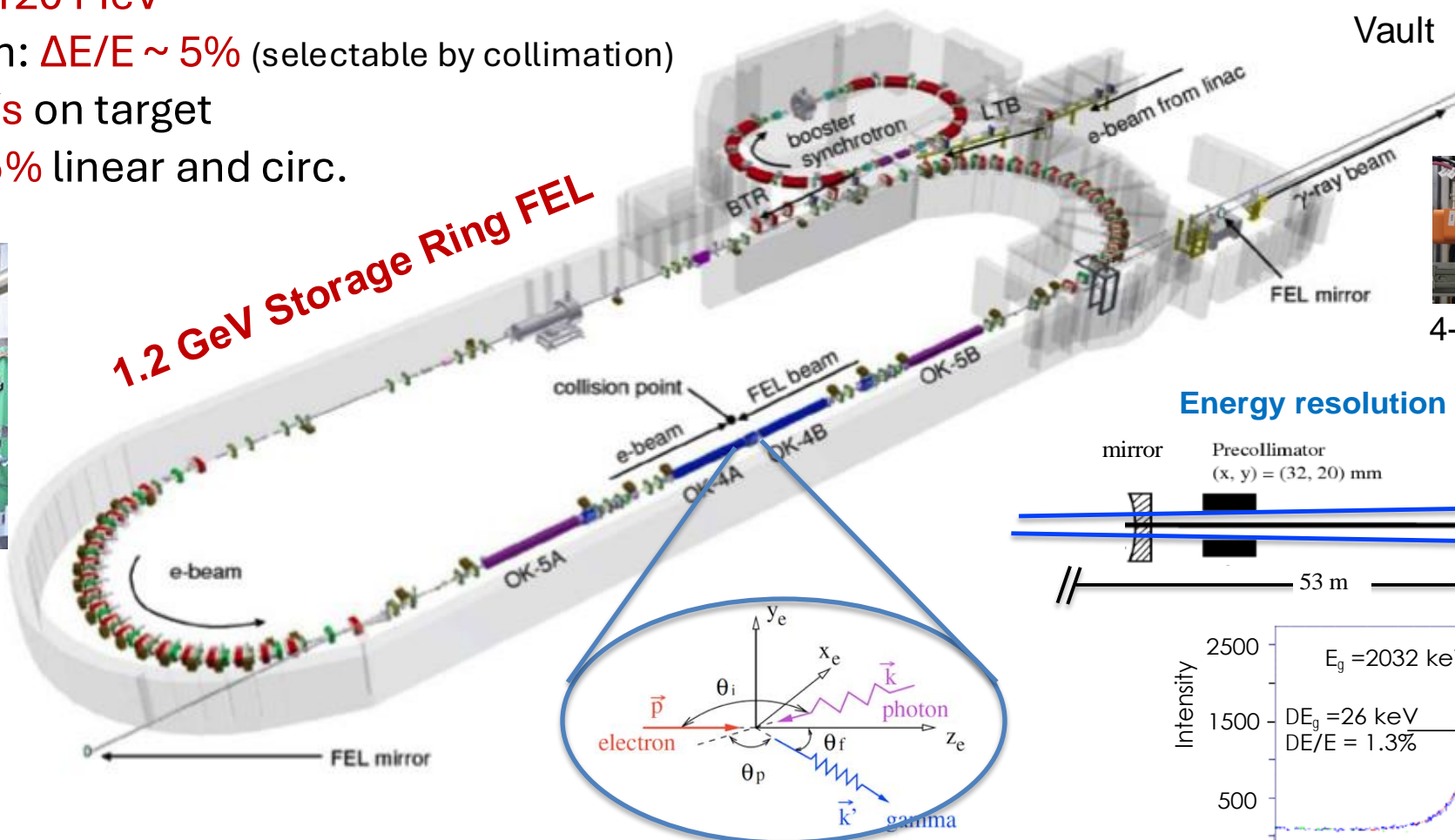


Gamma Vault



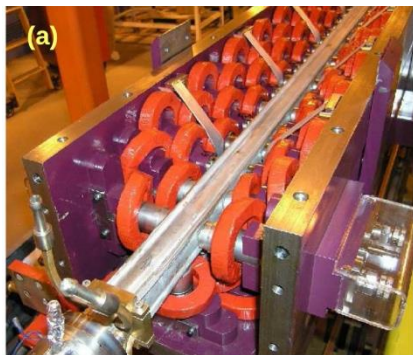
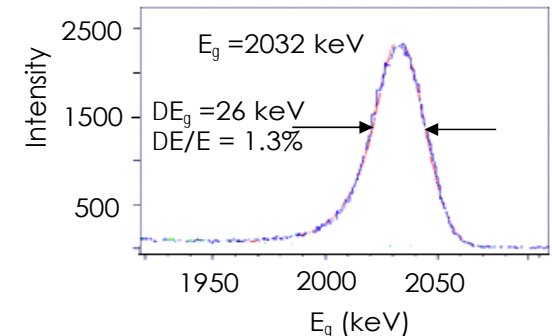
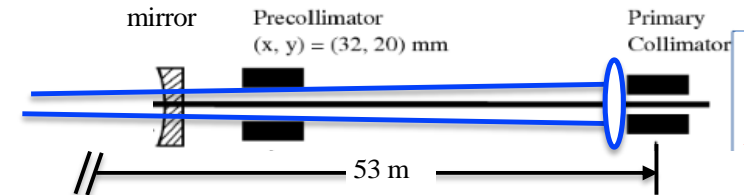
4-paddle flux monitor

1.2 GeV Storage Ring FEL



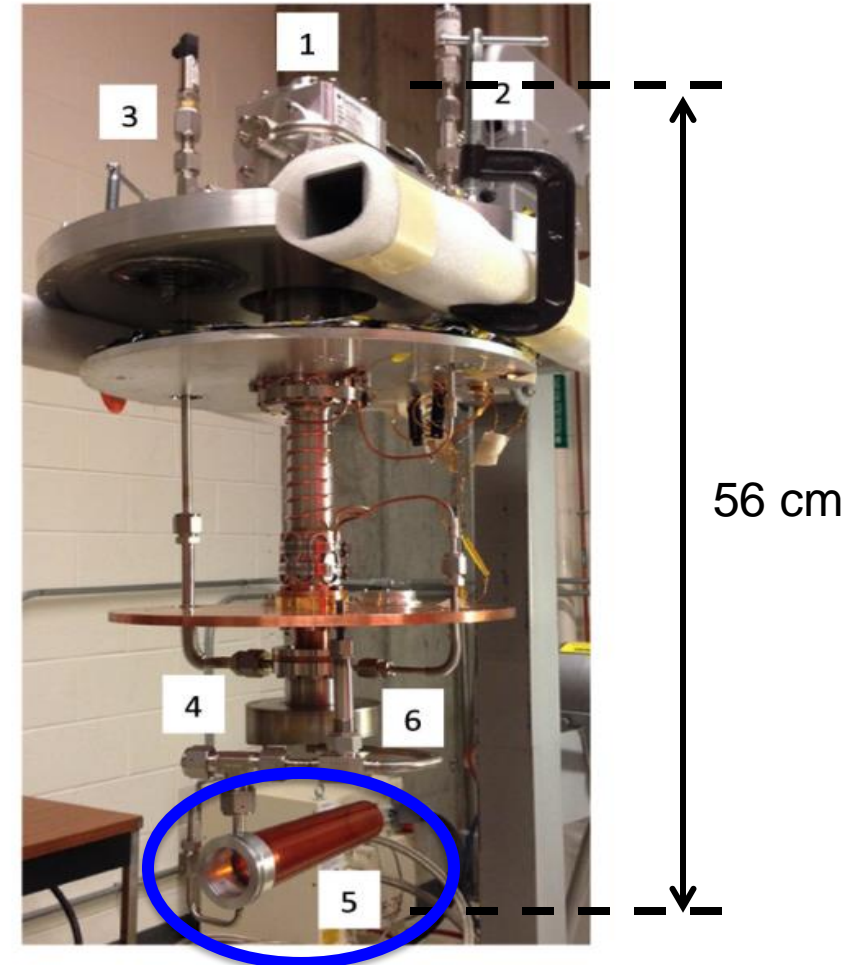
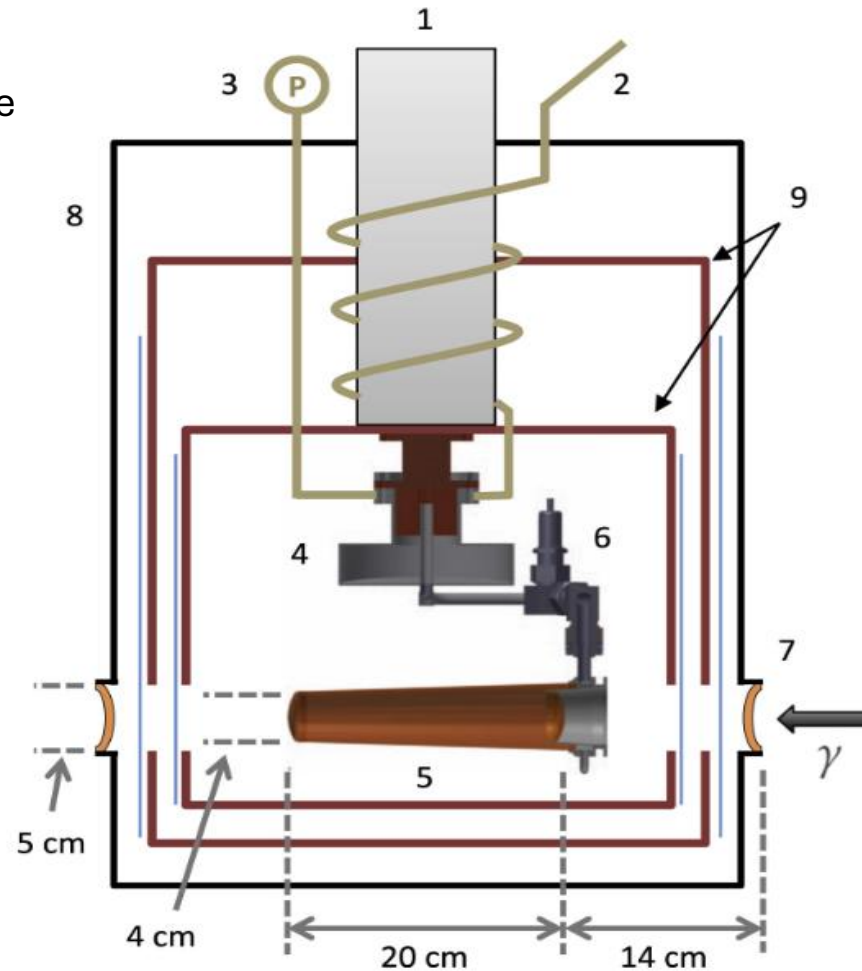
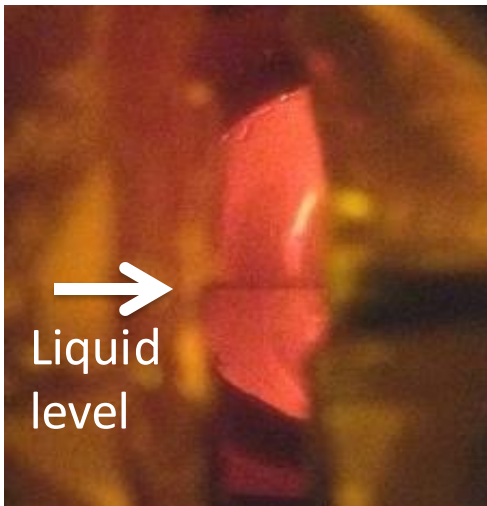
Compton backscattering to produce γ -ray photons

Energy resolution by collimation



Cryogenic Target for Liquid H, D and ^4He

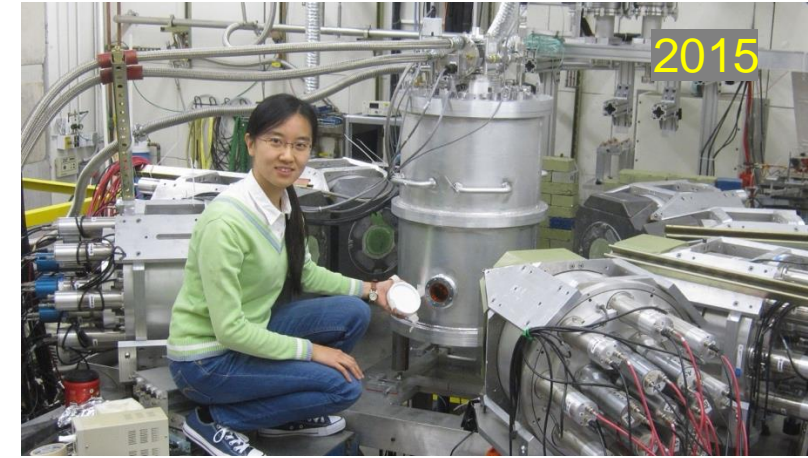
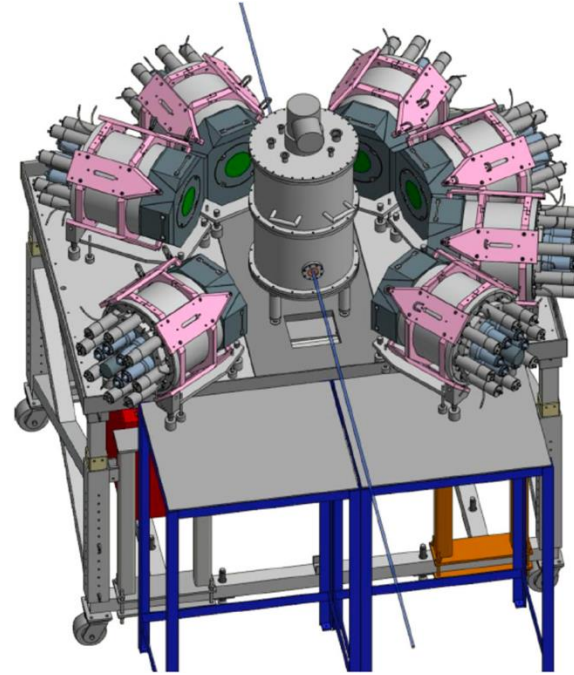
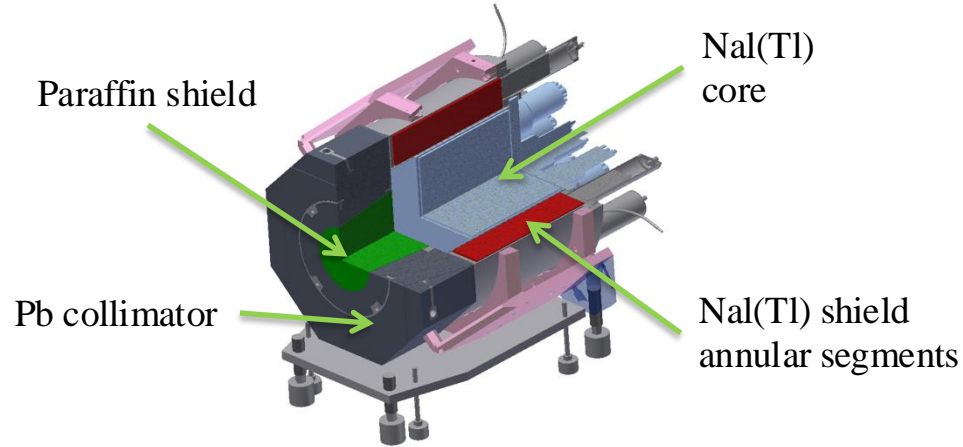
1. Cryocooler
2. Room-temperature gas inlet
3. Vent line with pressure gauge
4. Condenser
5. Target cell
6. Outlet valve
7. Kapton window
8. Aluminum can
9. Thermal shield



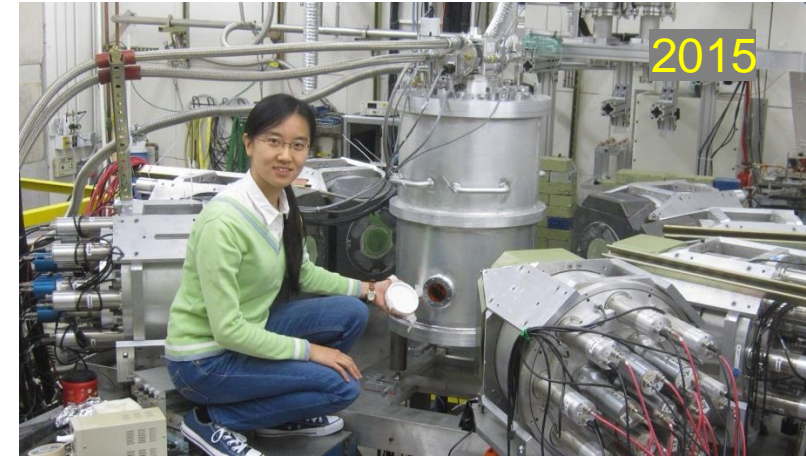
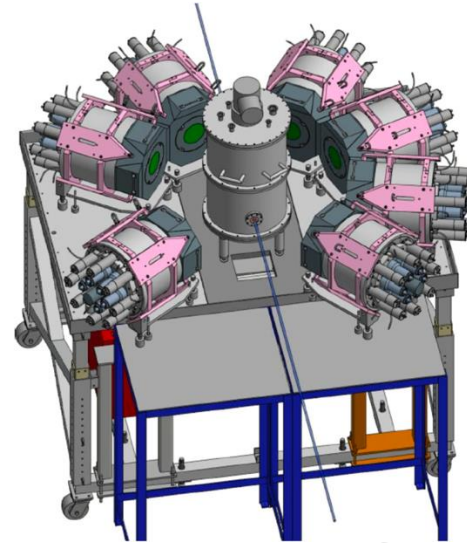
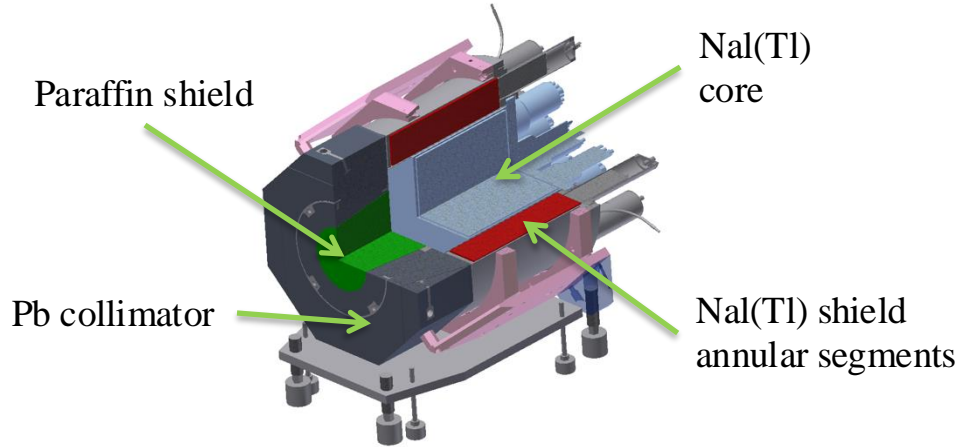
Target cell

D. P. Kendellen *et al.*, Nucl. Instrum. and Meth. A 840, 174–180 (2016)

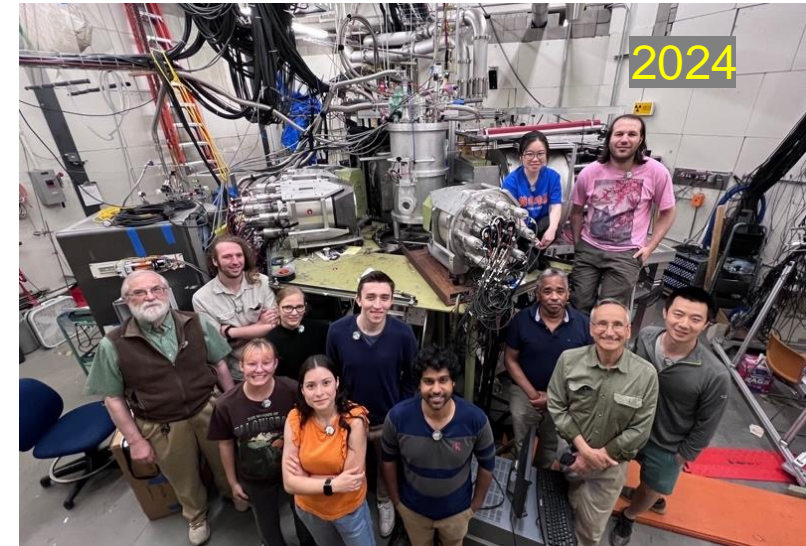
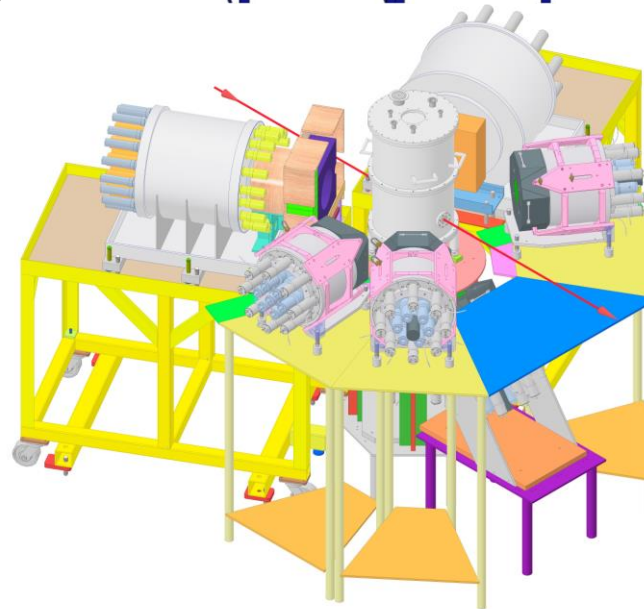
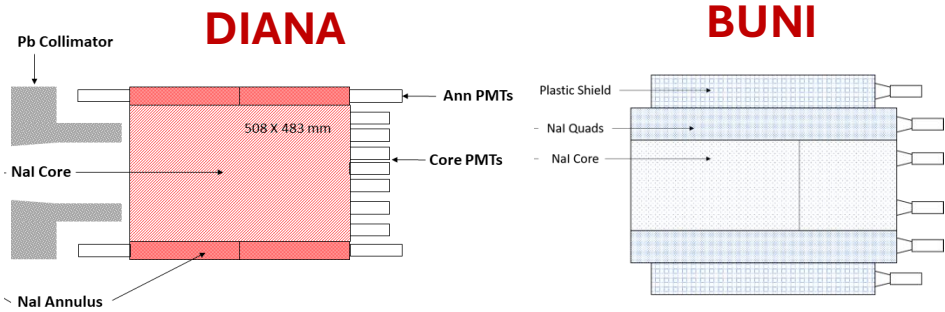
➤ Eight 10" x 12" NaI(Tl) detectors:
High Intensity NaI Detector Array (HINDA)



➤ Eight 10" x 12" NaI(Tl) detectors:
High Intensity NaI Detector Array (HINDA)



➤ Two larger NaI(Tl) detectors:



Compton Scattering Runs at HIGS

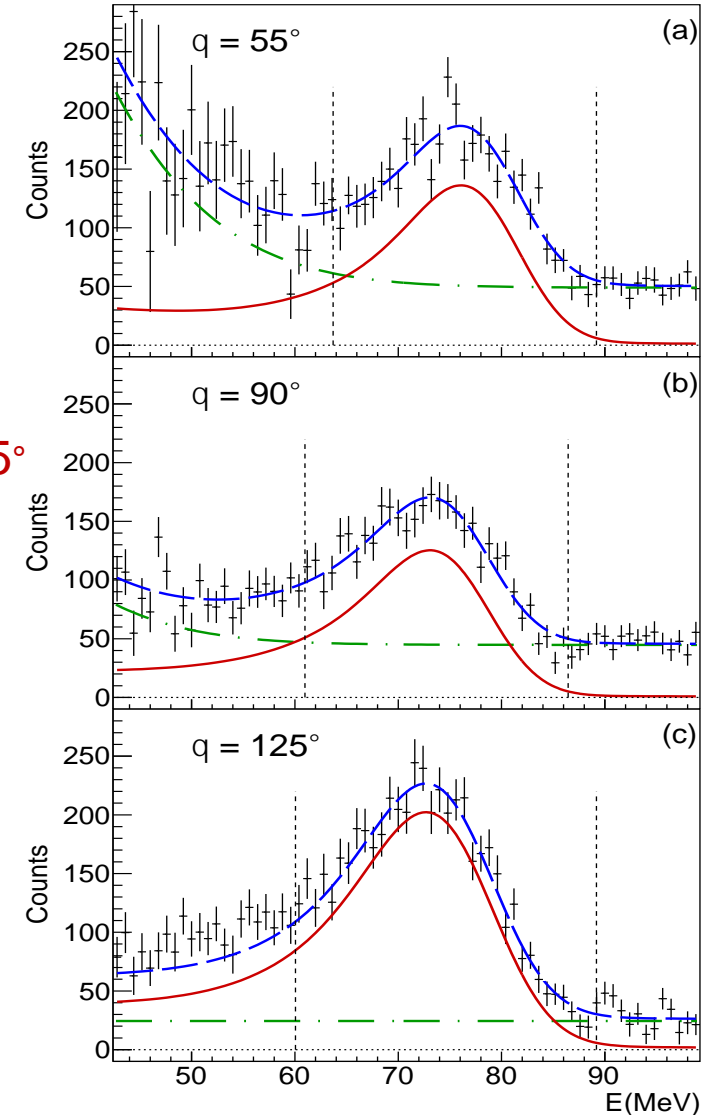
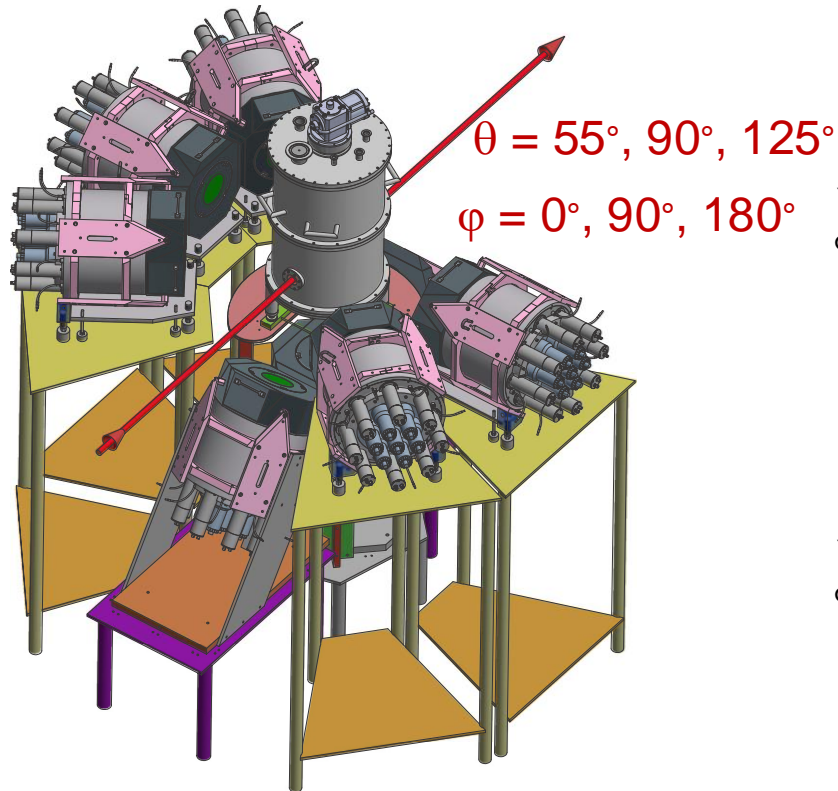
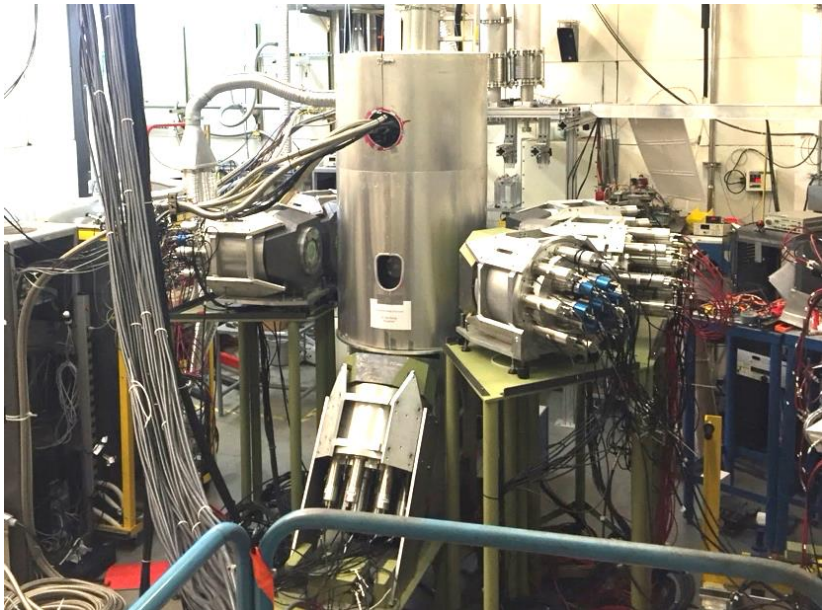
Year	Target	Beam Energy (MeV)	Beam Polarization	Run Hours
2015	Helium-4	61	Circular	54
2016	Deuteron	65	Circular	304
2016	Deuteron	85	Circular	268
2017	Helium-4	81	Circular	110
2017	Proton	81	Circular	107
2017	Proton	83	Linear	144
2021	Deuteron	61	Circular	227
2022	Deuteron	85	Circular	285
2024	Helium-3	60	Circular	210
2024	Helium-3	100	Circular	340

PHYSICAL REVIEW LETTERS **128**, 132502 (2022)

Proton Compton Scattering from Linearly Polarized Gamma Rays

X. Li^{1,2,*} M. W. Ahmed,^{2,3} A. Banu,⁴ C. Bartram,^{2,5} B. Crowe,^{2,3} E. J. Downie,⁶ M. Emamian,² G. Feldman,⁶ H. Gao,^{1,2} D. Godagama,⁷ H. W. Griebhammer,^{6,1} C. R. Howell,^{1,2} H. J. Karwowski,^{2,5} D. P. Kendellen,^{1,2} M. A. Kovash,⁷ K. K. H. Leung,^{1,2,8} D. M. Markoff,^{2,3} J. A. McGovern,⁹ S. Mikhailov,² R. E. Pywell,¹⁰ M. H. Sikora,^{6,2} J. A. Silano,^{2,5} R. S. Sosa,³ M. C. Spraker,¹¹ G. Swift,² P. Wallace,² H. R. Weller,^{1,2} C. S. Whisnant,⁴ Y. K. Wu,^{1,2} and Z. W. Zhao^{1,2}

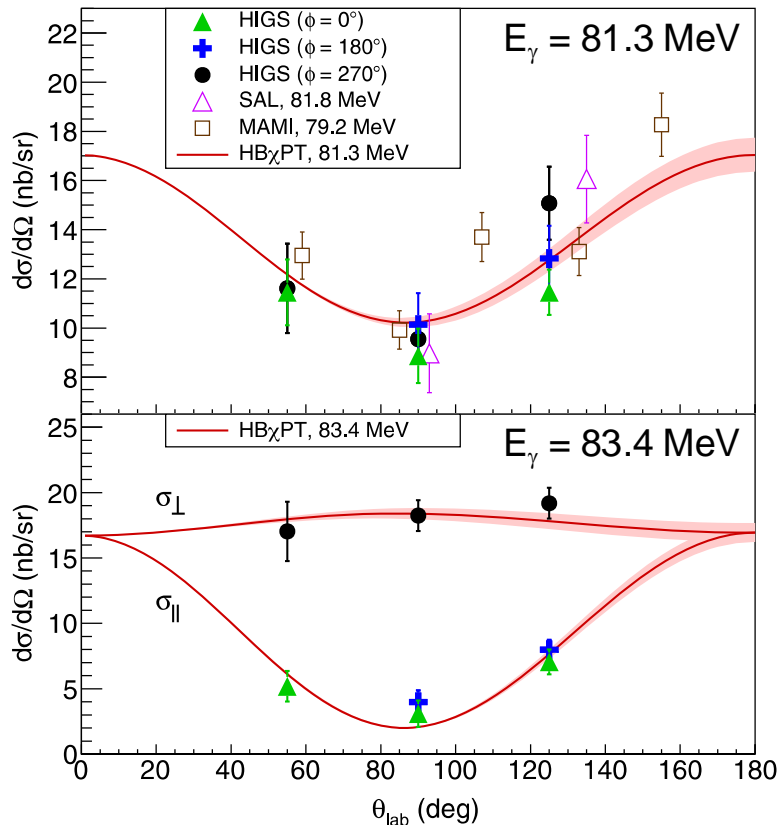
Circular pol.: $E_\gamma = 81.3 \text{ MeV}$ $\frac{d\sigma(\theta)}{d\Omega}$
Linear pol.: $E_\gamma = 83.4 \text{ MeV}$ $\Sigma_3(\theta)$



PHYSICAL REVIEW LETTERS **128**, 132502 (2022)

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Without BSR applied

$$\alpha_{E1}^p = 15.4 \pm 1.8_{\text{stat}},$$

$$\beta_{M1}^p = 2.1 \pm 2.0_{\text{stat}},$$

PDG2018 values

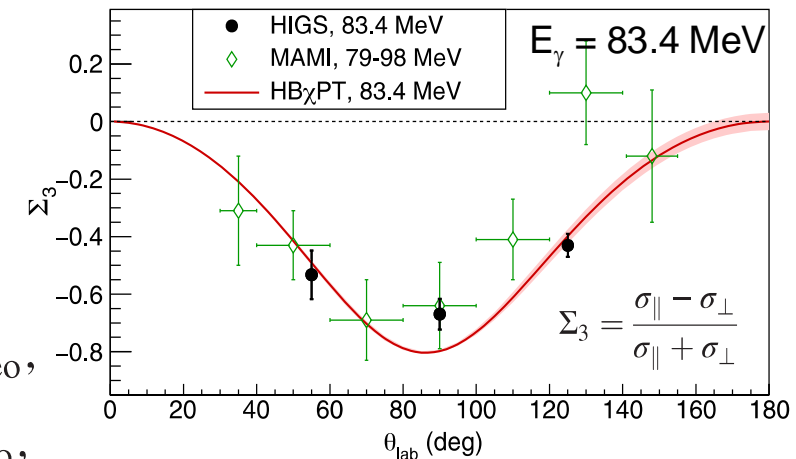
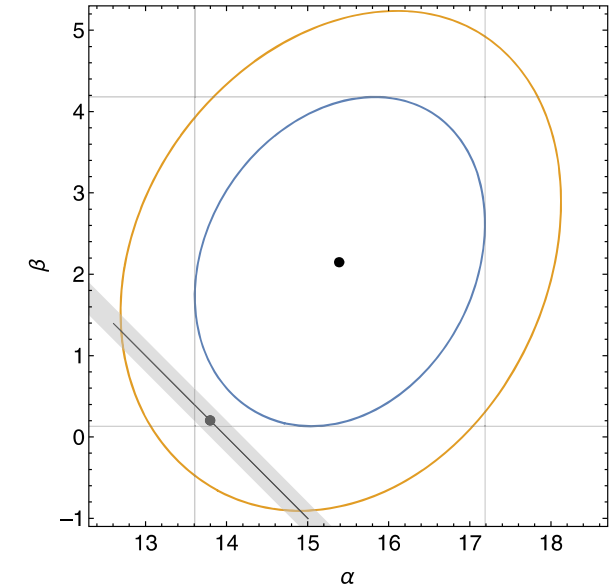
$$\alpha_{E1}^{(p)} = 11.2 \pm 0.4$$

$$\beta_{M1}^{(p)} = 2.5 \pm 0.4$$

With BSR applied

$$\alpha_{E1}^p = 13.8 \pm 1.2_{\text{stat}} \pm 0.1_{\text{BSR}} \pm 0.3_{\text{theo}},$$

$$\beta_{M1}^p = 0.2 \mp 1.2_{\text{stat}} \pm 0.1_{\text{BSR}} \mp 0.3_{\text{theo}},$$



PHYSICAL REVIEW C **96**, 055209 (2017)



Compton scattering from ^4He at 61 MeV

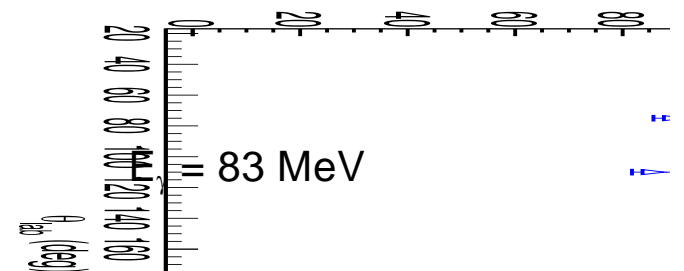
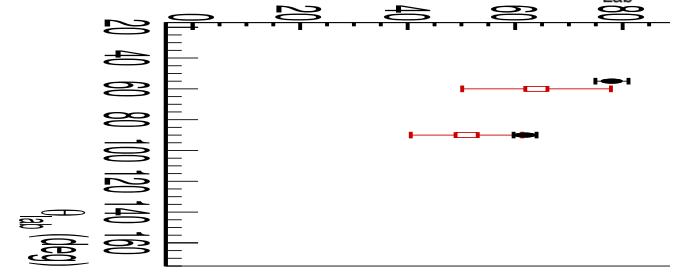
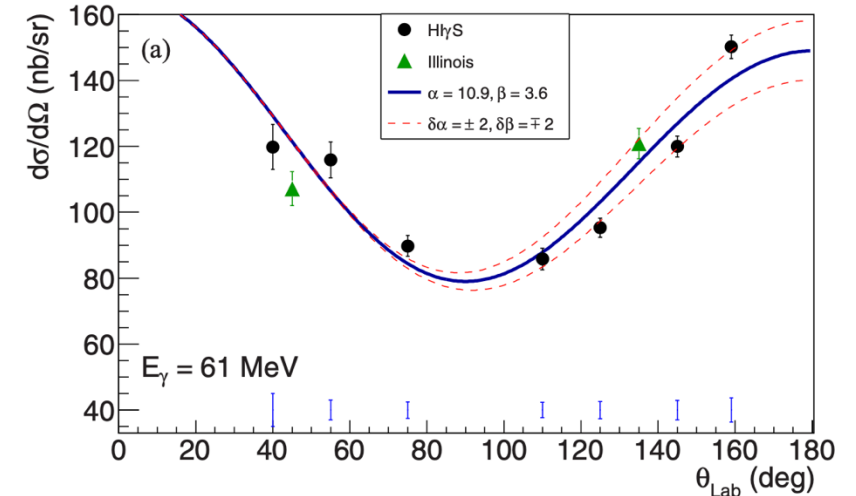
M. H. Sikora,^{1,2,*} M. W. Ahmed,^{1,2,3} A. Banu,⁴ C. Bartram,^{2,5} B. Crowe,^{2,3} E. J. Downie,⁶ G. Feldman,⁶ H. Gao,^{1,2} H. W. Griebhammer,⁶ H. Hao,² C. R. Howell,² H. J. Karwowski,^{2,5} D. P. Kendellen,^{1,2} M. A. Kovash,⁷ X. Li,^{1,2} D. M. Markoff,^{2,3} S. Mikhailov,² V. Popov,² R. E. Pywell,⁸ J. A. Silano,^{2,5} M. C. Spraker,⁹ P. Wallace,² H. R. Weller,^{1,2} C. S. Whisnant,⁴ Y. K. Wu,^{1,2} W. Xiong,^{1,2} X. Yan,^{1,2} and Z. W. Zhao^{1,2}

PHYSICAL REVIEW C **101**, 034618 (2020)

Compton scattering from ^4He at the TUNL HI γ S facility

X. Li^{1,2,*} M. W. Ahmed,^{1,2,3} A. Banu,⁴ C. Bartram,^{2,5} B. Crowe,^{2,3} E. J. Downie,⁶ M. Emamian,² G. Feldman,⁶ H. Gao,^{1,2} D. Godagama,⁷ H. W. Griebhammer,^{6,1} C. R. Howell,^{1,2} H. J. Karwowski,^{2,5} D. P. Kendellen,^{1,2} M. A. Kovash,⁷ K. K. H. Leung,^{2,8} D. Markoff,^{2,3} S. Mikhailov,² R. E. Pywell,⁹ M. H. Sikora,^{6,2} J. A. Silano,^{2,5} R. S. Sosa,³ M. C. Spraker,¹⁰ G. Swift,² P. Wallace,² H. R. Weller,^{1,2} C. S. Whisnant,⁴ Y. K. Wu,^{1,2} and Z. W. Zhao^{1,2}

- **Fore-aft asymmetry at higher energies** indicates a strong sensitivity to subnuclear effects
- **α and β for the neutron** to be extracted from the high precision ^4He data with upcoming new χEFT calculation



Elastic and Inelastic Compton Scattering from Deuterium at 65 and 85 MeV

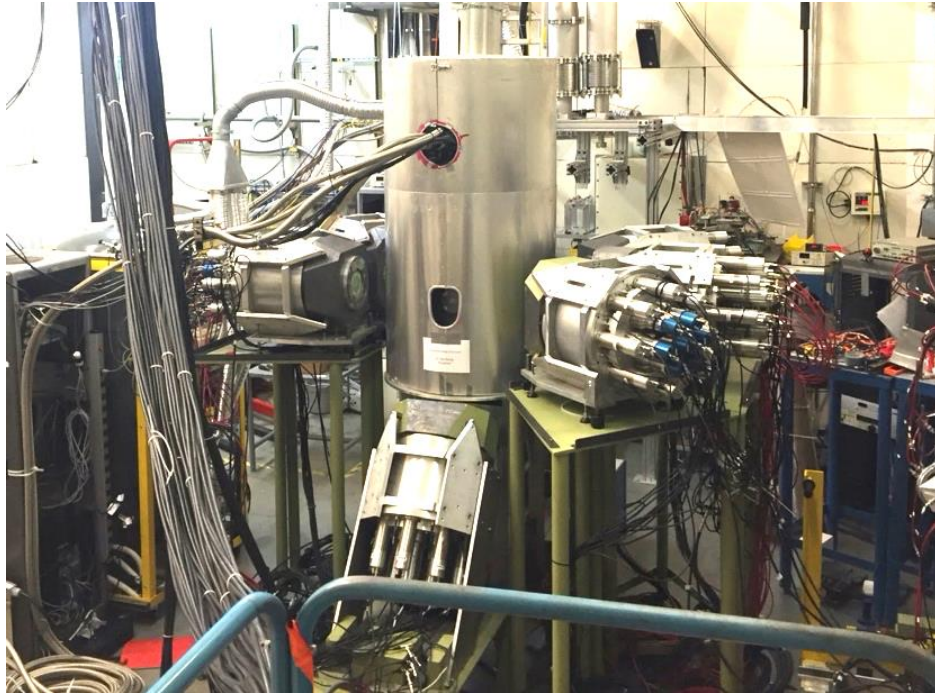
Mohammad Ahmed (Spokesperson)

Department of Mathematics and Physics, North Carolina Central University, Durham, NC 27707, 919-530-6100, ahmed2@nccu.edu

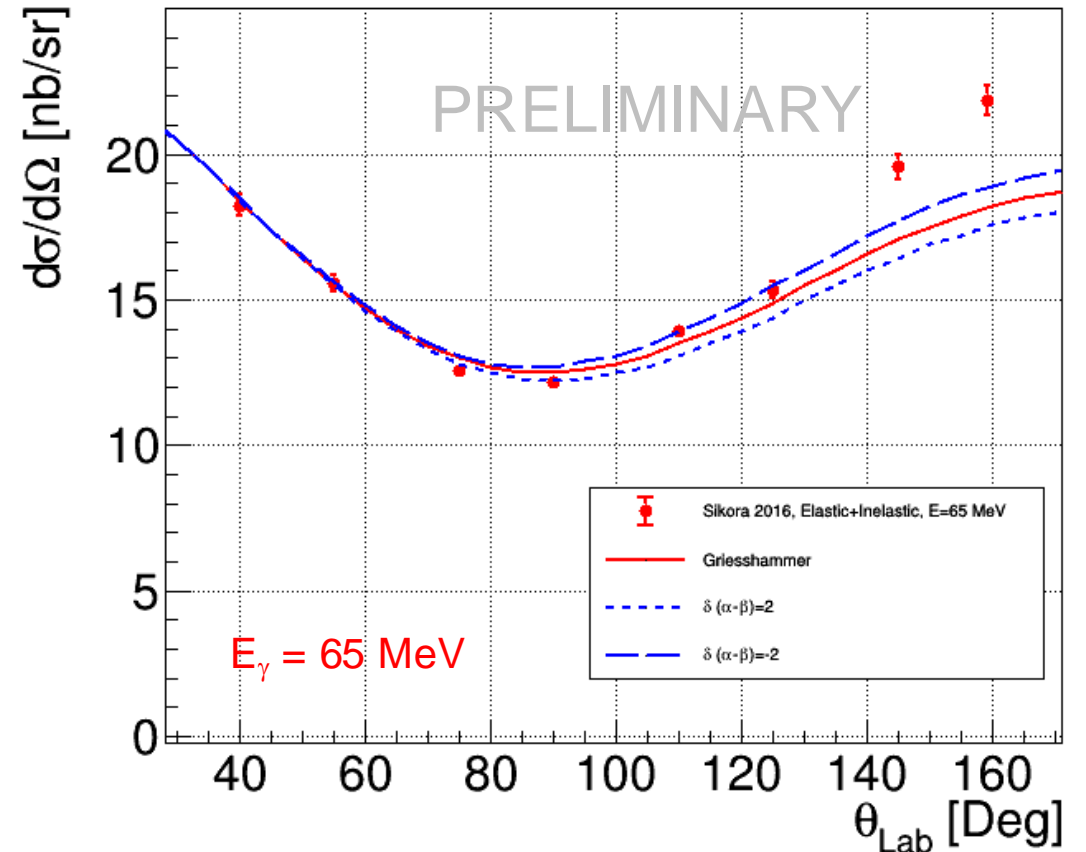
Michael A. Kovash (Spokesperson)

Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506-0055, 859-257-1150, kovash@pa.uky.edu

Compton@HIγS Collaboration



Deuteron measurement: need high-resolution detectors to separate elastic and inelastic scattering channels (2.2 MeV apart)

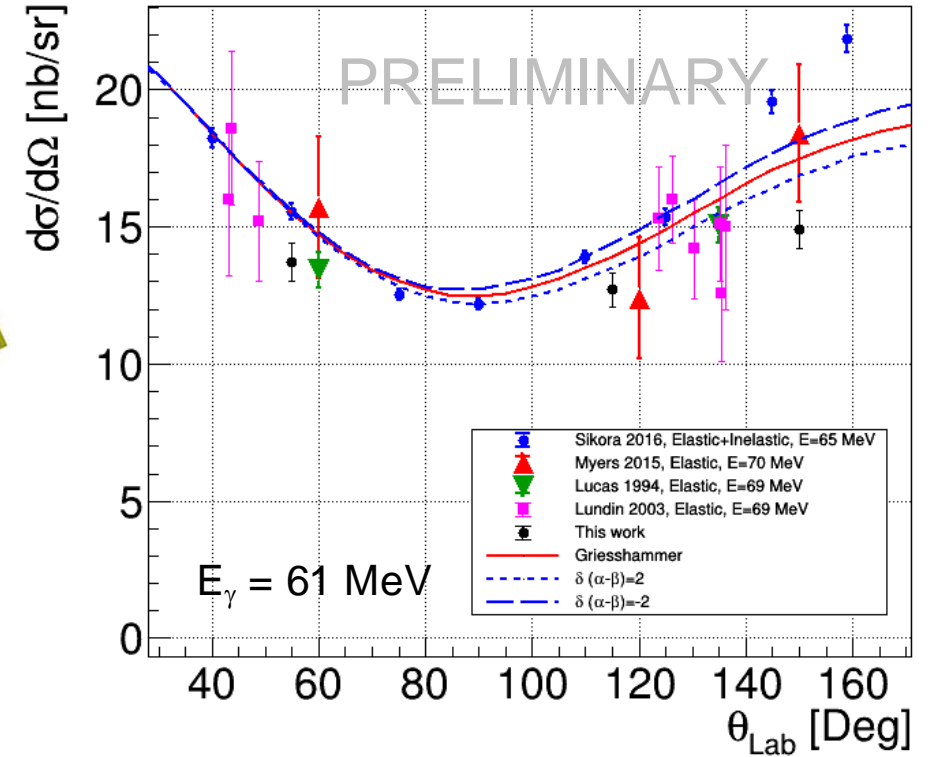
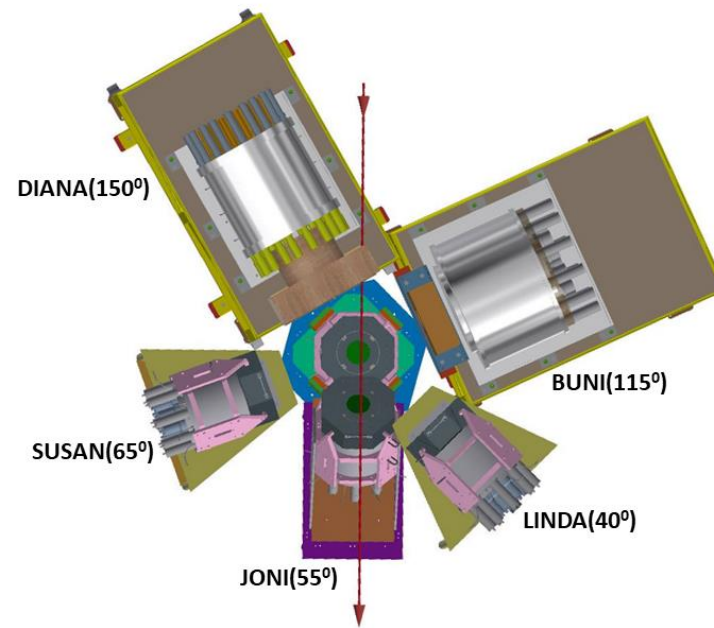
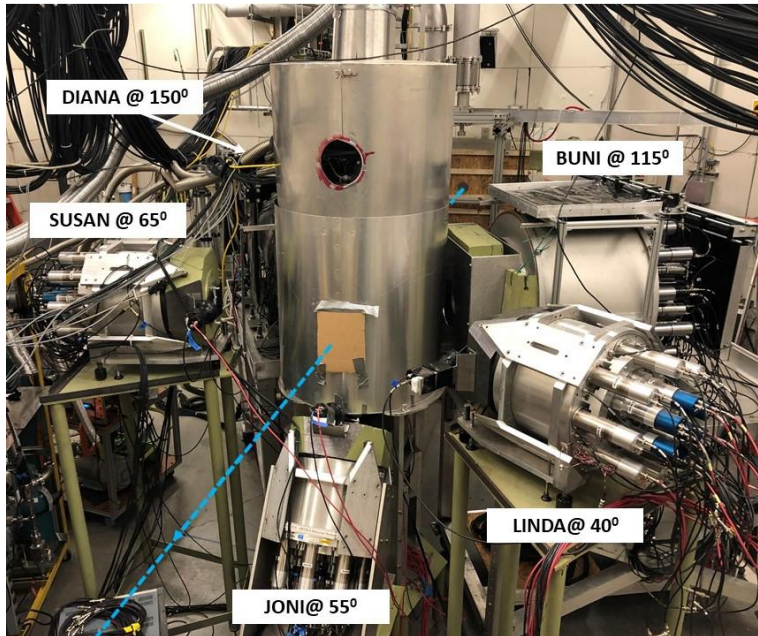


Compton Scattering from Deuteron

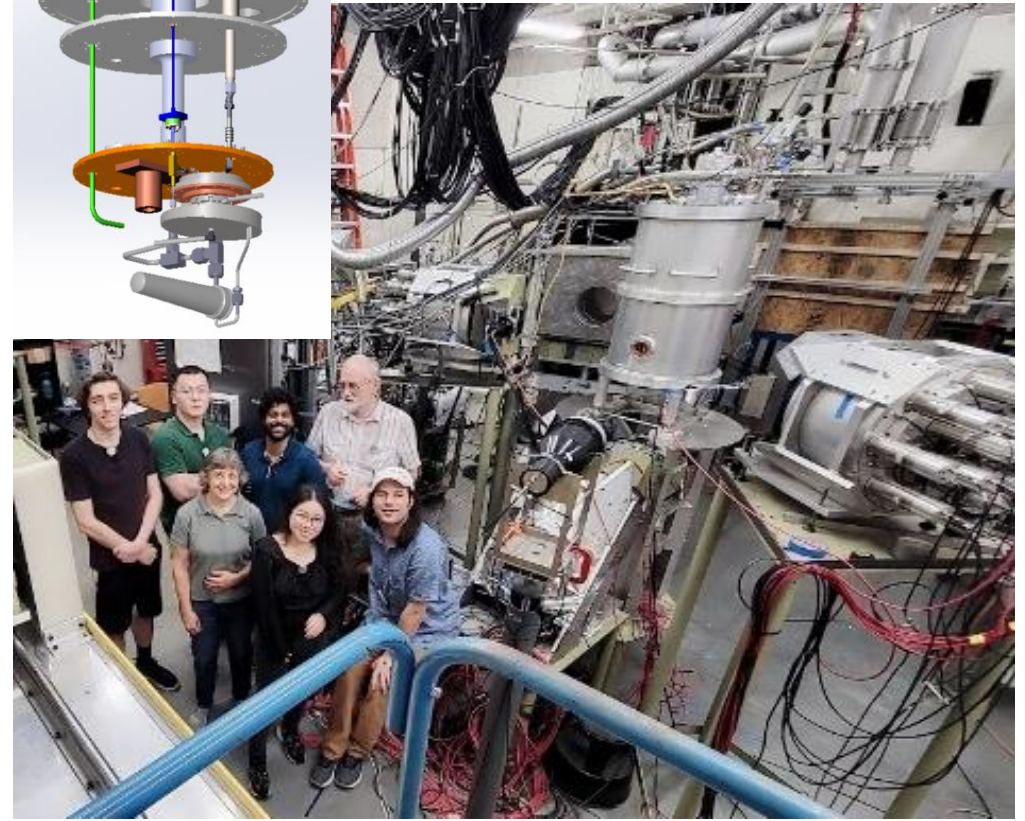
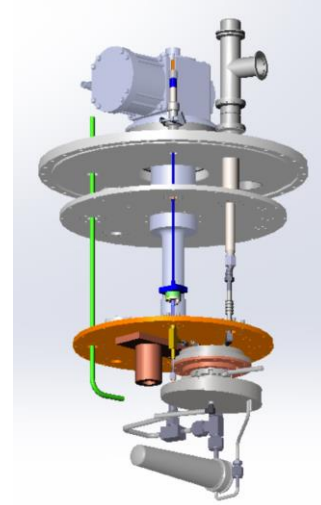
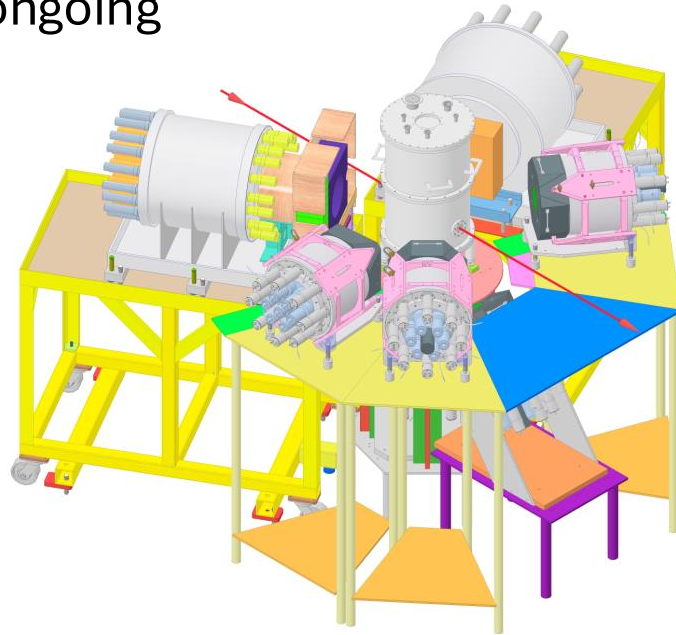
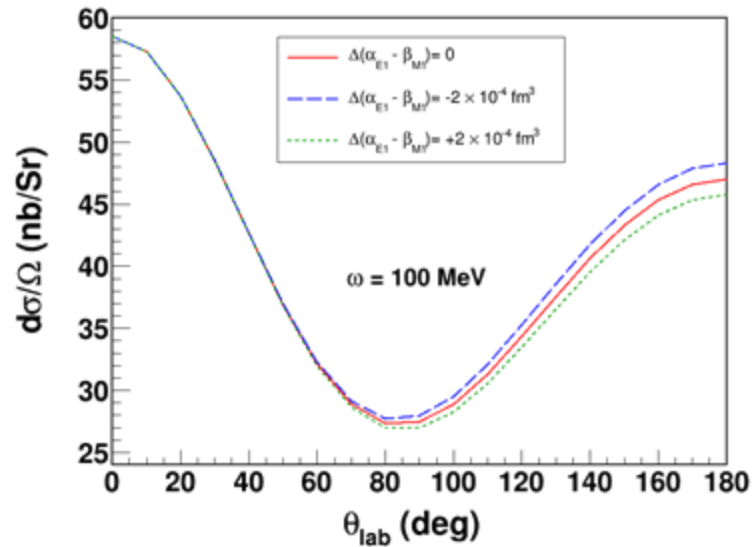
Elastic and Inelastic Compton Scattering from Deuterium at 61 MeV

Ph.D Thesis

Danula Godagama, University of Kentucky



- New gas handling system for 1.7 K liquid ^3He target
- Circularly polarized photon beam
 - 60 MeV data taking completed
 - 100 MeV data taking ongoing



- χ EFT calculation available for ^3He Compton scattering
- Aim to extract neutron EM polarizabilities from the 100 MeV data with uncertainties $< 0.7 \times 10^{-4} \text{ fm}^3$

$$\frac{d\sigma}{d\Omega} = \Phi^2 |T|^2$$

Spin polarizabilities

$$\gamma_{E1E1} = -\gamma_1 - \gamma_3,$$

$$\gamma_{M1M1} = \gamma_4,$$

$$\gamma_{M1E2} = \gamma_2 + \gamma_4,$$

$$\gamma_{E1M2} = \gamma_3.$$

**Spin-dependent
scattering amplitude**

$$\begin{aligned} T(\omega, z) = & A_1(\omega, z) (\vec{\epsilon}'^* \cdot \vec{\epsilon}) \\ & + A_2(\omega, z) (\vec{\epsilon}'^* \cdot \hat{k}) (\vec{\epsilon} \cdot \hat{k}') \\ & + iA_3(\omega, z) \vec{\sigma} \cdot (\vec{\epsilon}'^* \times \vec{\epsilon}) \\ & + iA_4(\omega, z) \vec{\sigma} \cdot (\hat{k}' \times \hat{k}) (\vec{\epsilon}'^* \cdot \vec{\epsilon}) \\ & + iA_5(\omega, z) \vec{\sigma} \cdot [(\vec{\epsilon}'^* \times \hat{k}) (\vec{\epsilon} \cdot \hat{k}') - (\vec{\epsilon} \times \hat{k}') (\vec{\epsilon}'^* \cdot \hat{k})] \\ & + iA_6(\omega, z) \vec{\sigma} \cdot [(\vec{\epsilon}'^* \times \hat{k}') (\vec{\epsilon} \cdot \hat{k}) - (\vec{\epsilon} \times \hat{k}) (\vec{\epsilon}'^* \cdot \hat{k}')] \end{aligned}$$

$$A_1(\omega, z) = -\frac{Z^2 e^2}{M} + \frac{e^2 \omega^2}{4M^3} \left((Z + \kappa)^2 (1 + z) - Z^2 \right) (1 - z) + 4\pi\omega^2 (\alpha_{E1} + z\beta_{M1}) + \mathcal{O}(\omega^4),$$

$$A_2(\omega, z) = \frac{e^2 \omega^2}{4M^3} \kappa (2Z + \kappa) z - 4\pi\omega^2 \beta_{M1} + \mathcal{O}(\omega^4),$$

$$A_3(\omega, z) = \frac{e^2 \omega}{2M^2} \left(Z(Z + 2\kappa) - (Z + \kappa)^2 z \right) + A_3^{\pi^0} + 4\pi\omega^3 \left(\underline{\gamma_1} - (\underline{\gamma_2} + 2\underline{\gamma_4}) z \right) + \mathcal{O}(\omega^5),$$

$$A_4(\omega, z) = -\frac{e^2 \omega}{2M^2} (Z + \kappa)^2 + 4\pi\omega^3 \underline{\gamma_2} + \mathcal{O}(\omega^5),$$

$$A_5(\omega, z) = \frac{e^2 \omega}{2M^2} (Z + \kappa)^2 + A_5^{\pi^0} + 4\pi\omega^3 \underline{\gamma_4} + \mathcal{O}(\omega^5),$$

$$A_6(\omega, z) = -\frac{e^2 \omega}{2M^2} Z(Z + \kappa) + A_6^{\pi^0} + 4\pi\omega^3 \underline{\gamma_3} + \mathcal{O}(\omega^5),$$

HIGS Scientific Program: Determination of the spin-dependent polarizabilities

Polarized photon beam and polarized target required

$$\Sigma_{2x} = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

$$\Sigma_{2z} = \frac{\sigma^{\rightarrow} - \sigma^{\leftarrow}}{\sigma^{\rightarrow} + \sigma^{\leftarrow}}$$

$$\Sigma_3 = \frac{\sigma^{\parallel} - \sigma^{\perp}}{\sigma^{\parallel} + \sigma^{\perp}}$$

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

$$\gamma_{\pi} = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

$$\gamma_0 = (-1.00 \pm 0.13) \times 10^{-4} \text{ fm}^4$$

J. Ahrens *et al.* (GDH/A2), PRL **87**, 022003 (2001);
H. Dutz *et al.* (GDH), PRL **91**, 192001 (2003).

Recent result from MAMI

$$\gamma_{\pi} = (-38.7 \pm 1.8) \times 10^{-4} \text{ fm}^4$$

M. Camen *et al.*, PRC **65**, 032202(R) (2022).

Beam Polarization

$i = 1$: unpolarized

$i = 2$: circular polarization

$i = 3$: linear polarization

Recent and Future Measurements:

Observable	Proton	Neutron: ^2H	Neutron: ^3He	Neutron: ^4He
$d\sigma/d\Omega$	MAMI/ HIGS	HIGS	HIGS	HIGS
Σ_3	MAMI/ HIGS			
Σ_{2x}	MAMI HIGS	HIGS		
Σ_{2z}	MAMI HIGS	HIGS		

HIGS unpolarized targets
HIGS liquid ^3He target
HIGS with polarized target capability

Courtesy of Calvin R. Howell

- **Reduce uncertainty in the neutron scalar polarizabilities**
 - The goal is to reduce the uncertainties to be on par with the proton
 - Perform high-precision cross-section and Σ_3 Compton-scattering measurements on ^2H , ^3He and ^4He at $E_\gamma = 100$ to 130 MeV (e.g., map out $\alpha^n(\omega)$ over the π production threshold cusp)
- **Map out proton scalar polarizabilities over the unitary cusp**
 - Perform cross-section and Σ_3 Compton-scattering cross-section measurements on the proton at $E_\gamma = 100$ to 150 MeV
- **Determine proton spin polarizabilities up to pion-production threshold**
 - Measure asymmetry data (Σ_3 , Σ_{2z} and Σ_{2x}) at energies $E_\gamma = 100$ to 130 MeV; complement data from Mainz
 - Use several χ EFT calculations for reliable assessment of model dependence
- **Determine the neutron spin polarizabilities**
 - Measure asymmetry data (Σ_{2z} and Σ_{2x}) at energies $E_\gamma = 100$ to 300 MeV for Compton-scattering on polarized ^2H and ^3He targets; $E_\gamma = 100 - 150$ MeV at HIGS and $E_\gamma = 250 - 300$ MeV at Mainz

Courtesy of Calvin R. Howell

- **Reduce uncertainty in the neutron scalar polarizabilities**
 - The goal is to reduce the uncertainties to be on par with the proton
 - Perform high-precision cross-section and Σ_3 Compton-scattering measurements on ^2H , ^3He and ^4He at $E_\gamma = 100$ to 130 MeV (e.g., map out $\alpha^n(\omega)$ over the π production threshold cusp)
- **Map out proton scalar polarizabilities over the unitary cusp**
 - Perform cross-section and Σ_3 Compton-scattering cross-section measurements on the proton at $E_\gamma = 100$ to 150 MeV
- **Establish a Cryogenic Polarized Proton Program**
 - Obtain funding for a TUNL staff position in low-temperature physics to lead polarized target R&D program
 - Obtain funding to build polarized target technical infrastructure
- **Improve determination of proton spin polarizabilities**
 - Measure asymmetry data (Σ_{2z} and Σ_{2x}) at energies $E_\gamma = 100$ to 130 MeV; complement data from Mainz at $E_\gamma = 260 - 310$ MeV
 - Use several χ EFT calculations for reliable assessment of model dependence
- **Determine the neutron spin polarizabilities**
 - Measure asymmetry data (S_{2z} and S_{2x}) at energies $E_\gamma = 100$ to 130 MeV for Compton-scattering on polarized ^2H at $E_\gamma = 100 - 130$ MeV

HIGS Compton Collaboration

Groups from 13 institutions: 10 USA + 3 international

- 1) **Duke Univ.:** H. Gao, C.R. Howell
- 2) **GWU:** E. Downie, J. Feldman, H. Griesshammer
- 3) **James Madison Univ.:** A. Banu and S. Whisant
- 4) **Montclair State University:** K. Leung
- 5) **Mount Alison Univ.,** David Hornidge
- 6) **NC Central Univ.:** M.W. Ahmed, B. Crowe, D. Markoff
- 7) **North Georgia State Univ.:** M. Spraker
- 8) **Ohio Univ.:** D. Phillips
- 9) **Univ. Kentucky:** M. Kovash
- 10) **Univ. Manchester:** J.A. McGovern
- 11) **UNC-Chapel Hill:** H. Karwowski
- 12) **Univ. Mass. - Amherst:** R. Miskimen
- 13) **Univ. Saskatchewan:** R. Pywell

Blue font = TUNL consortium institution

Red font = Theorist

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Thanks for your attention!