Quark and hadron masses from CLQCD ensembles

Yi-Bo Yang

University of Chinese Academy of Sciences

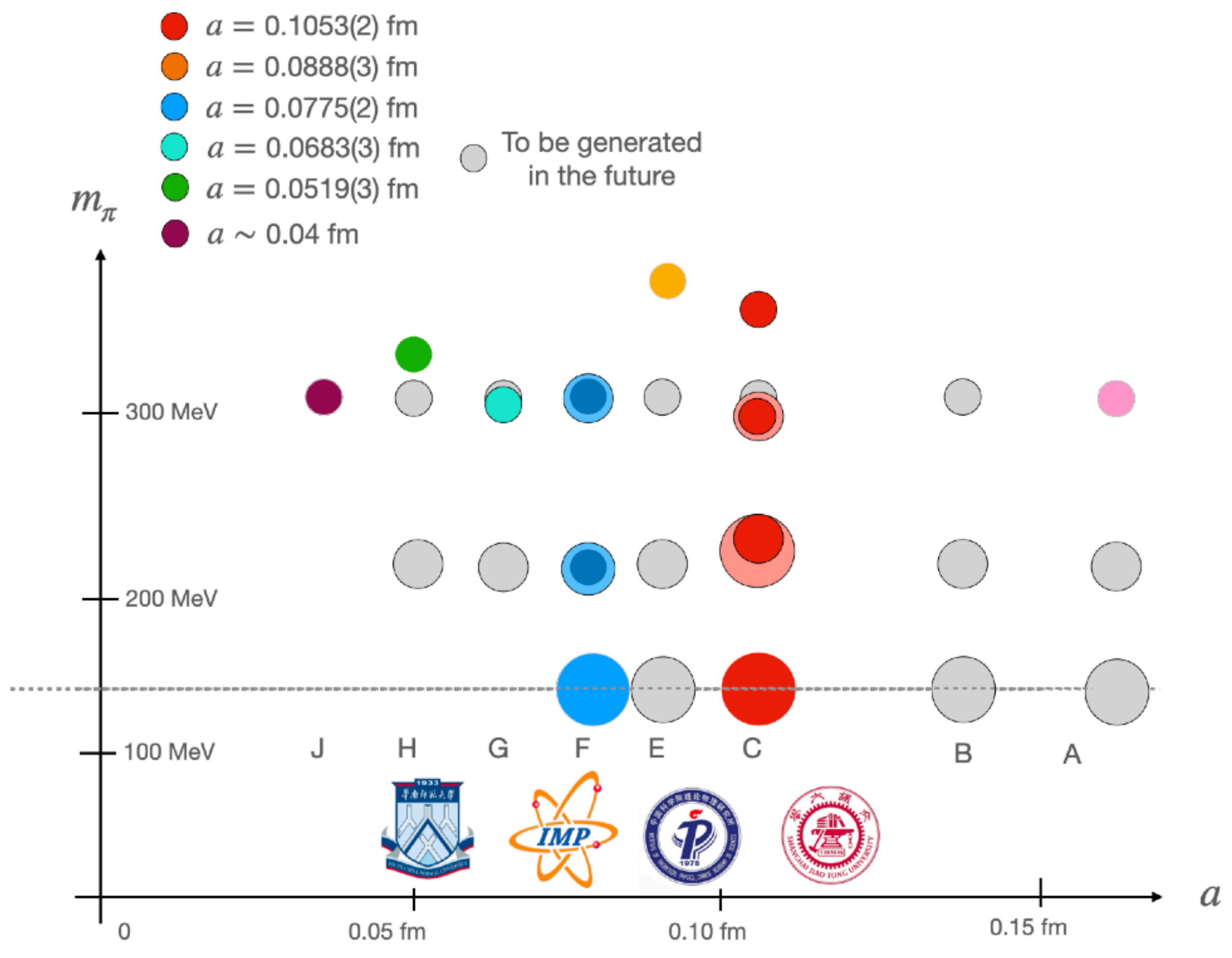
ICTP-AP
International Centre for Theoretical Physics Asia-Pacific
国际理论物理中心-亚太地区

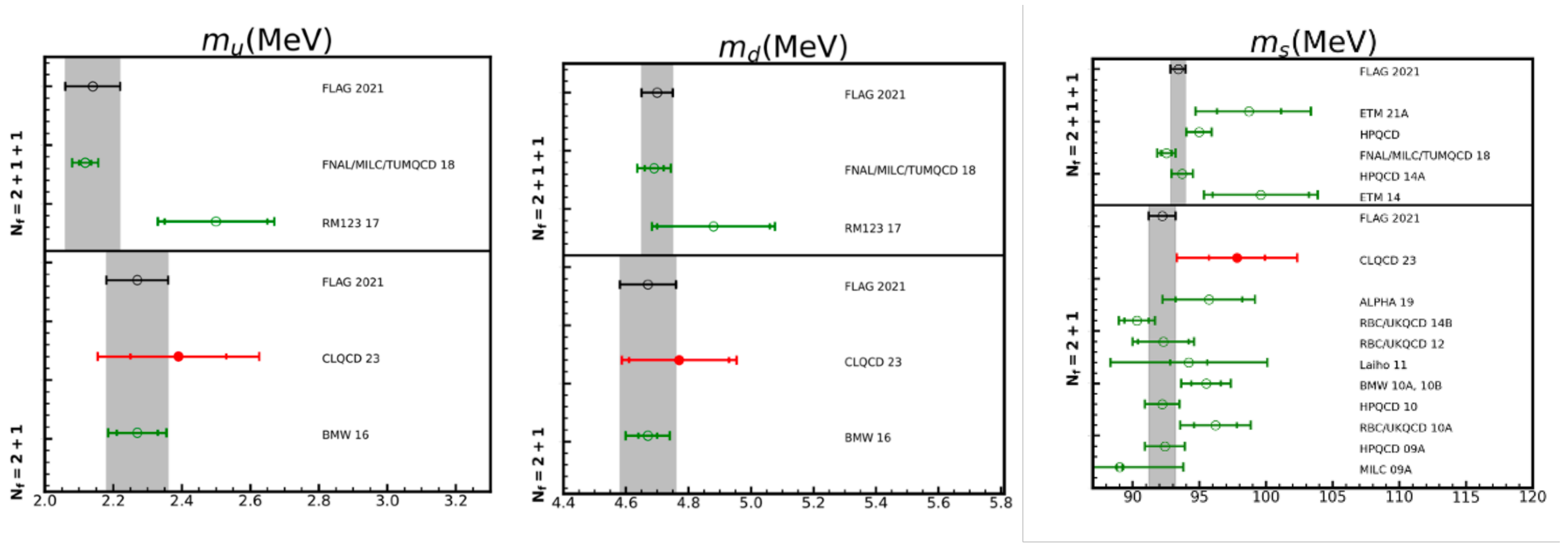
With Zhi-Cheng Hu, Bo-Lun Hu, Ji-Hao Wang, Hai-Yang Du, and et.al.,
For CLQCD

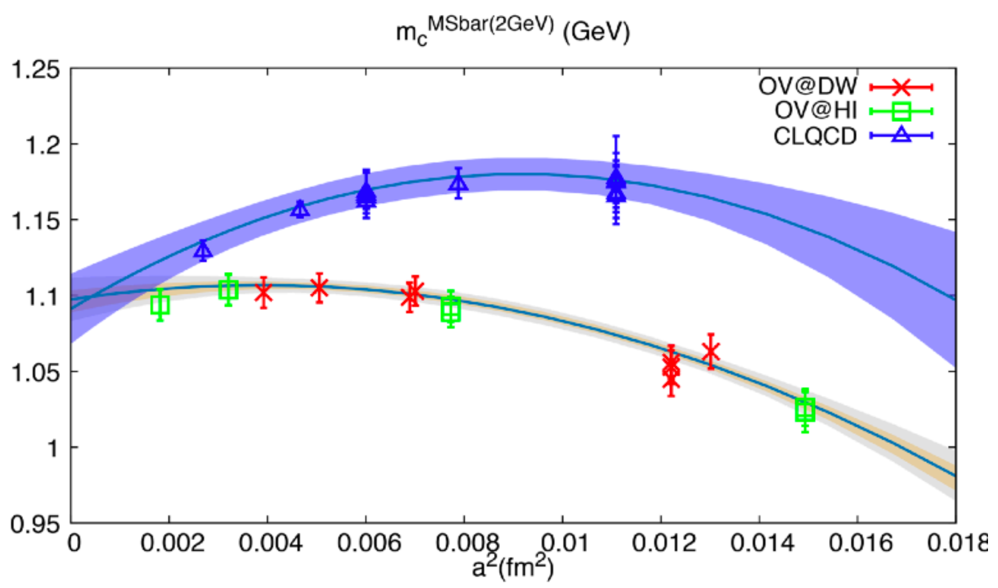
CLQCD

Outline

- **LQCD background and CLQCD ensembles**

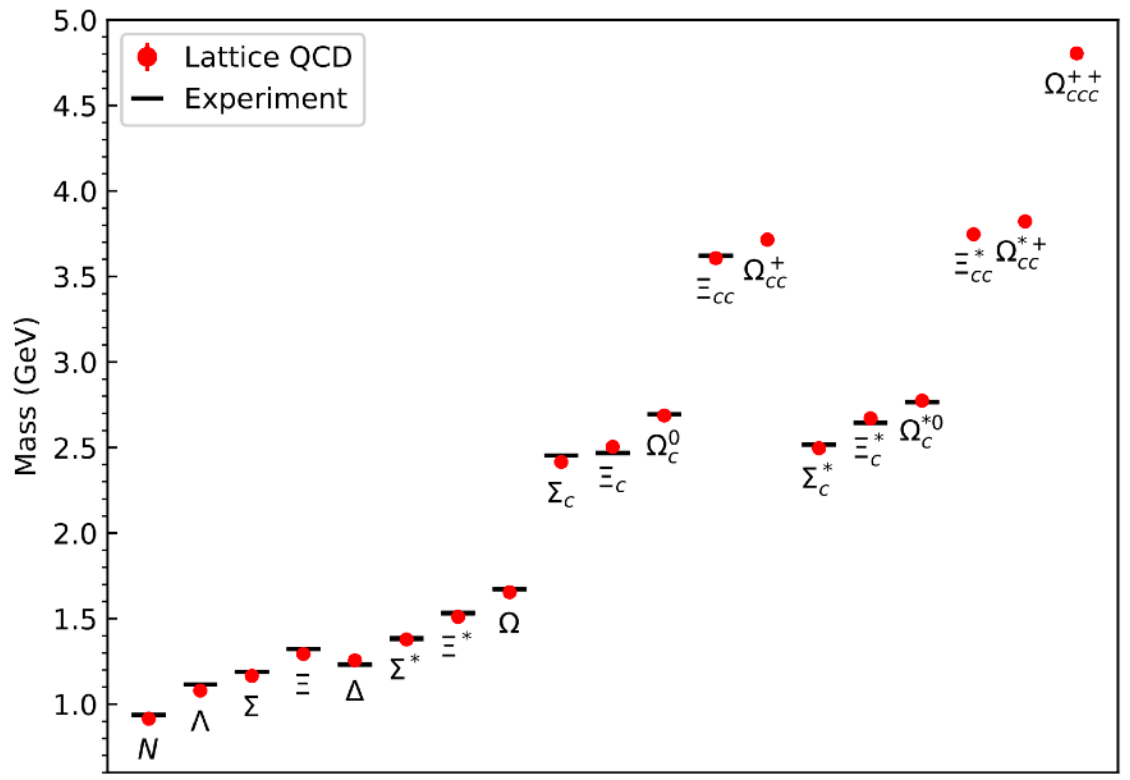


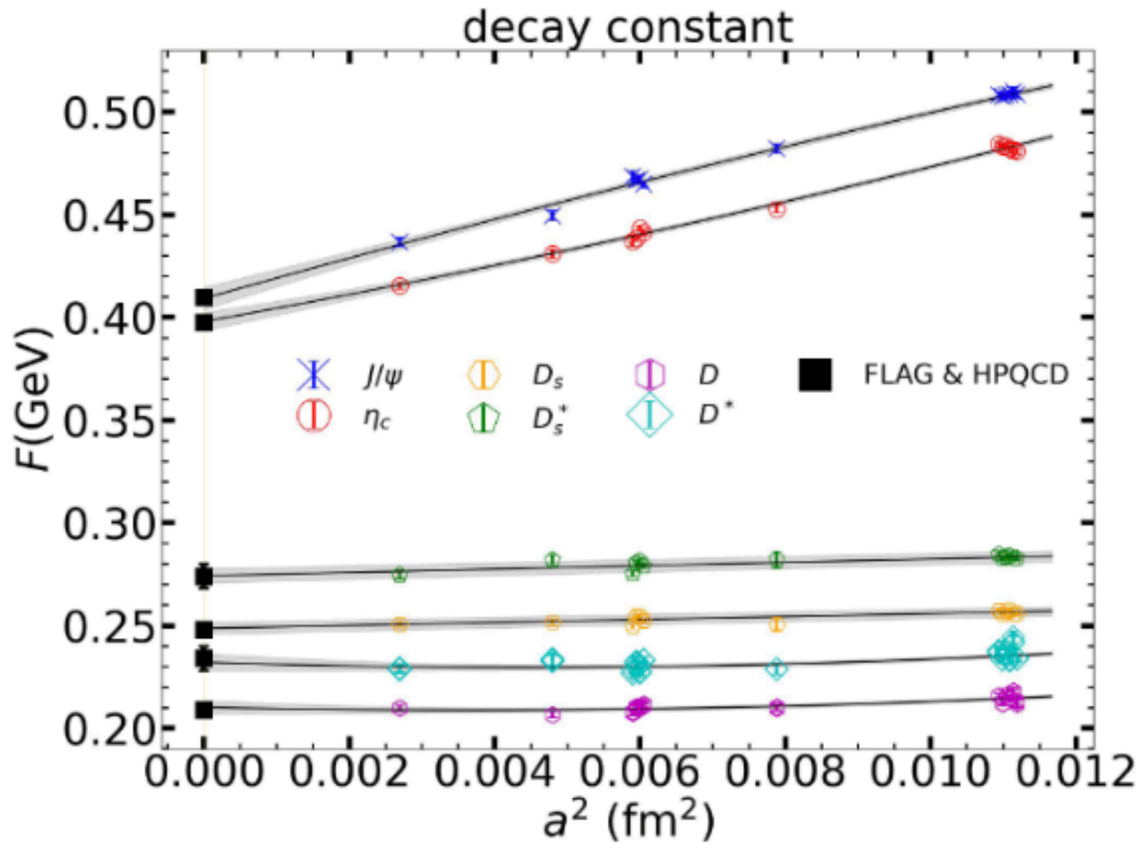




- Quark mass determinations

- Hadron masses and decay constants





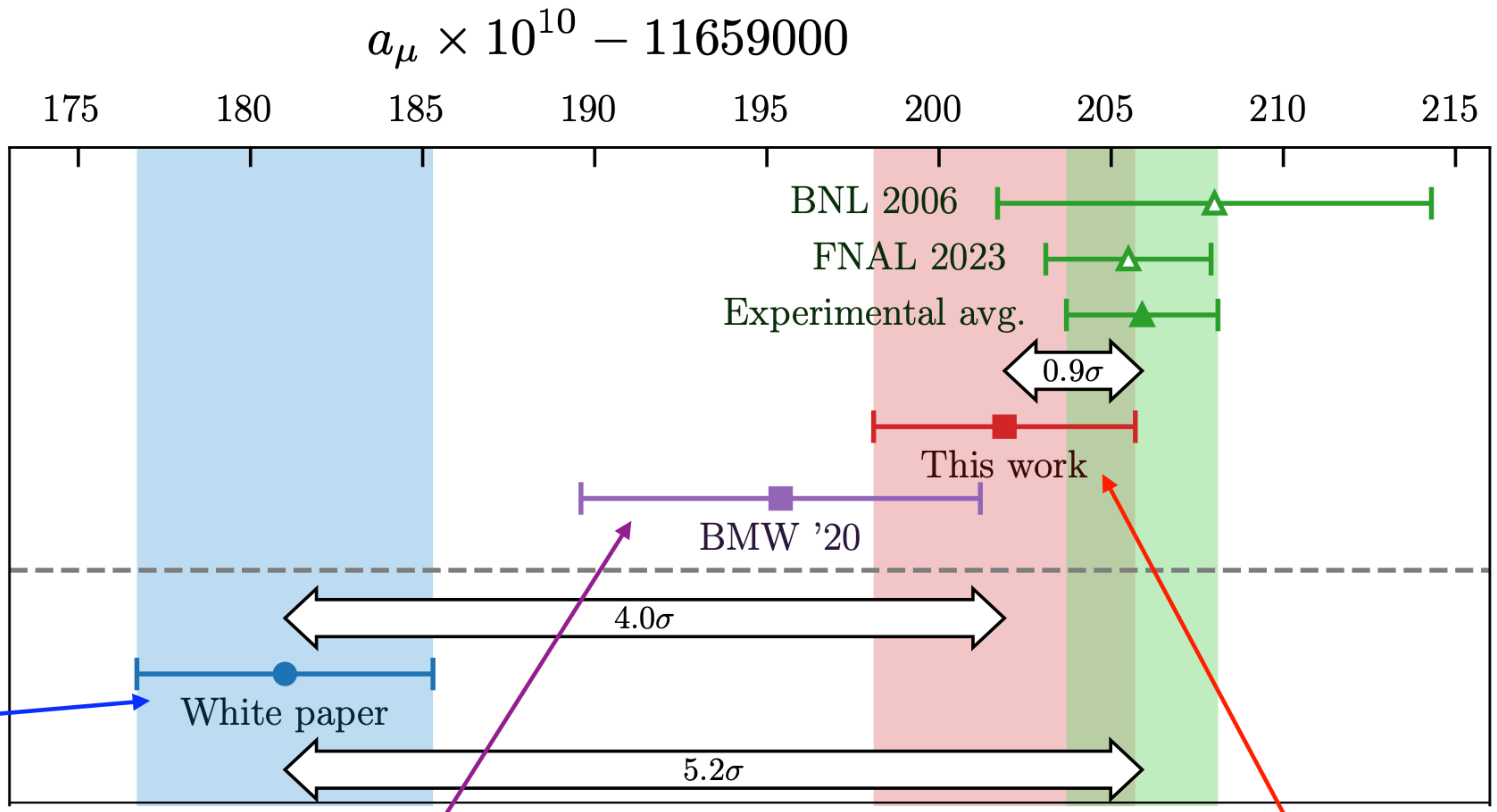
Lattice QCD

Most recent HYP prediction for muon g-2

BMW/DMZ24, 2407.10913

adds 0.048fm ensemble, reduces finite L/T error. Uses data-driven for large-t tail. Blinded analysis.

WP20 data-driven: 693.1(4.0)



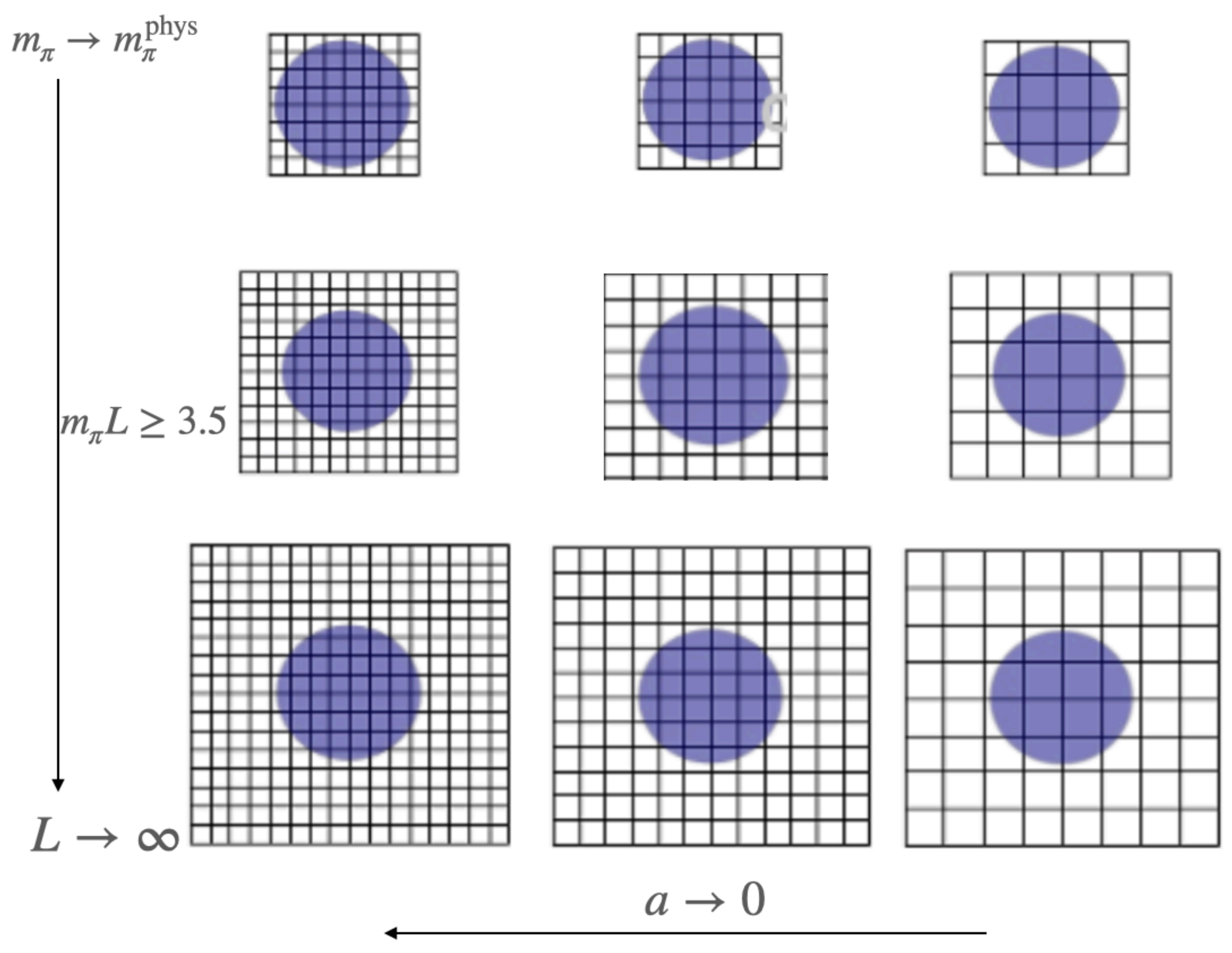
BMW20: $10^{10} a_\mu^{\mathrm{LOHVP}} = 707.5(5.5)$

BMW/DMZ24: $10^{10} a_\mu^{\mathrm{LOHVP}} = 714.1(3.3)$

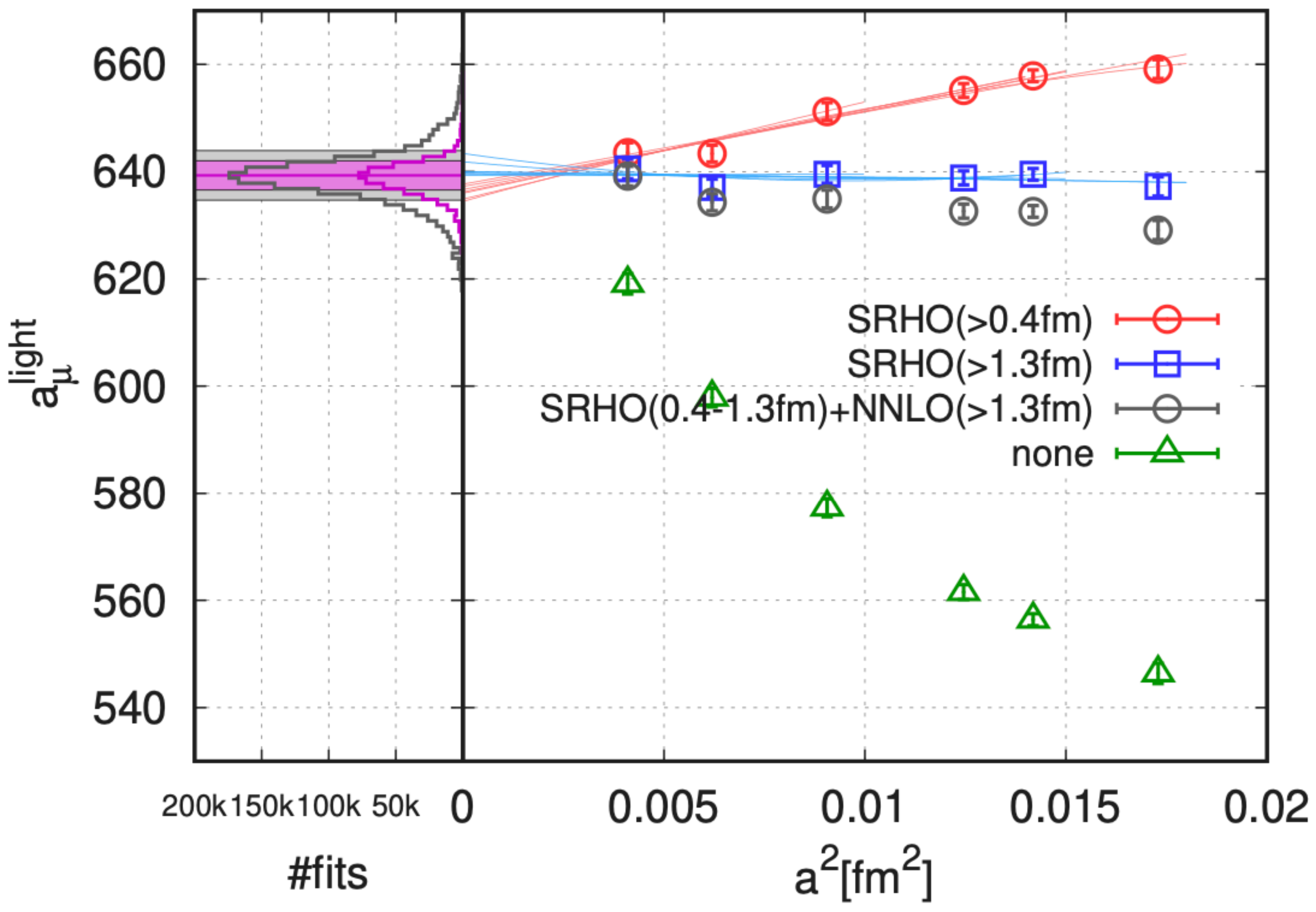
Accurate LQCD predictions require:

- Continuum extrapolation;
- Physical quark masses
- Infinite volume extrapolation;
- QED correction (QCD+QED) and iso-spin breaking effect $\mathcal{O}(m_d - m_u)$;.

Lattice QCD



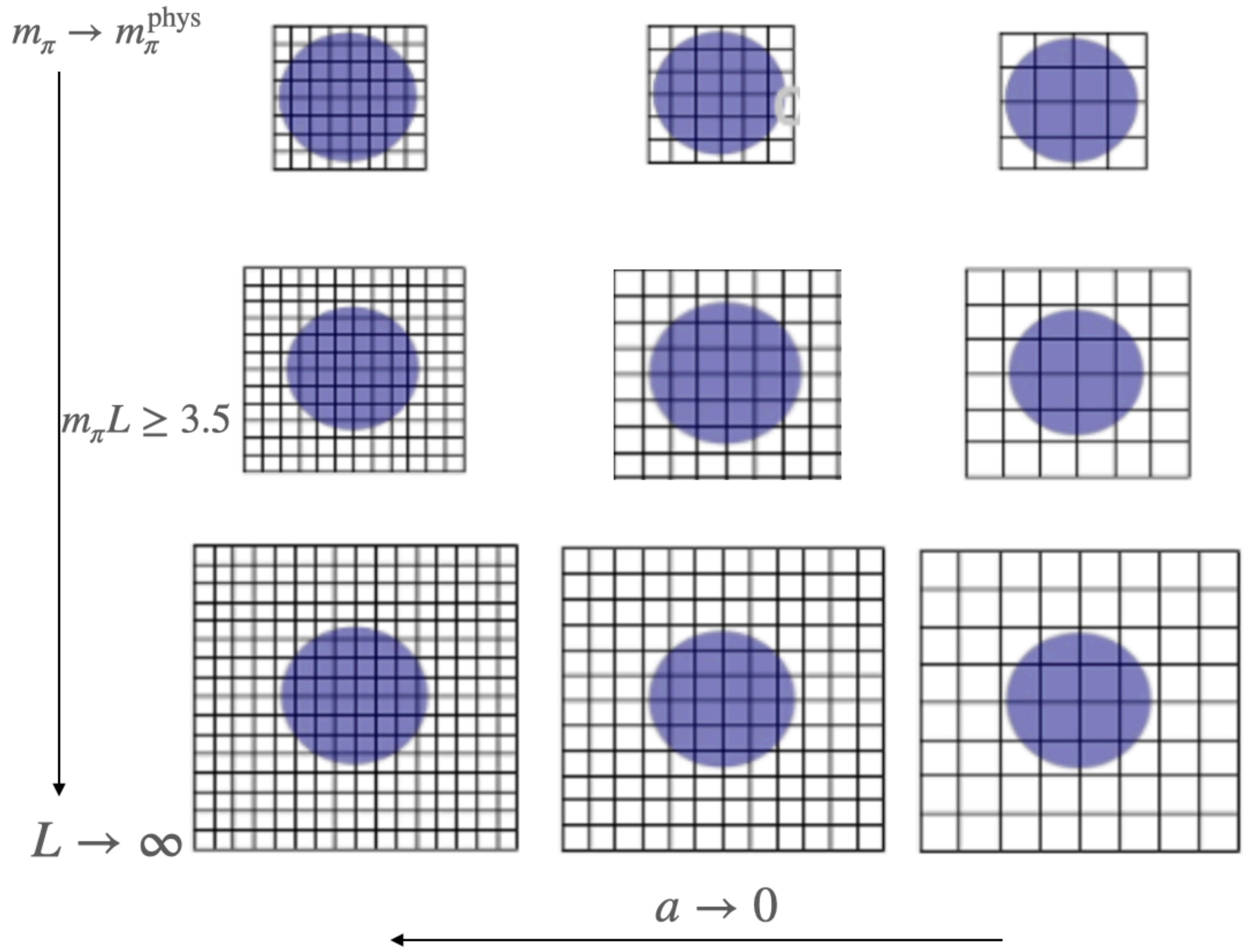
Discretization error



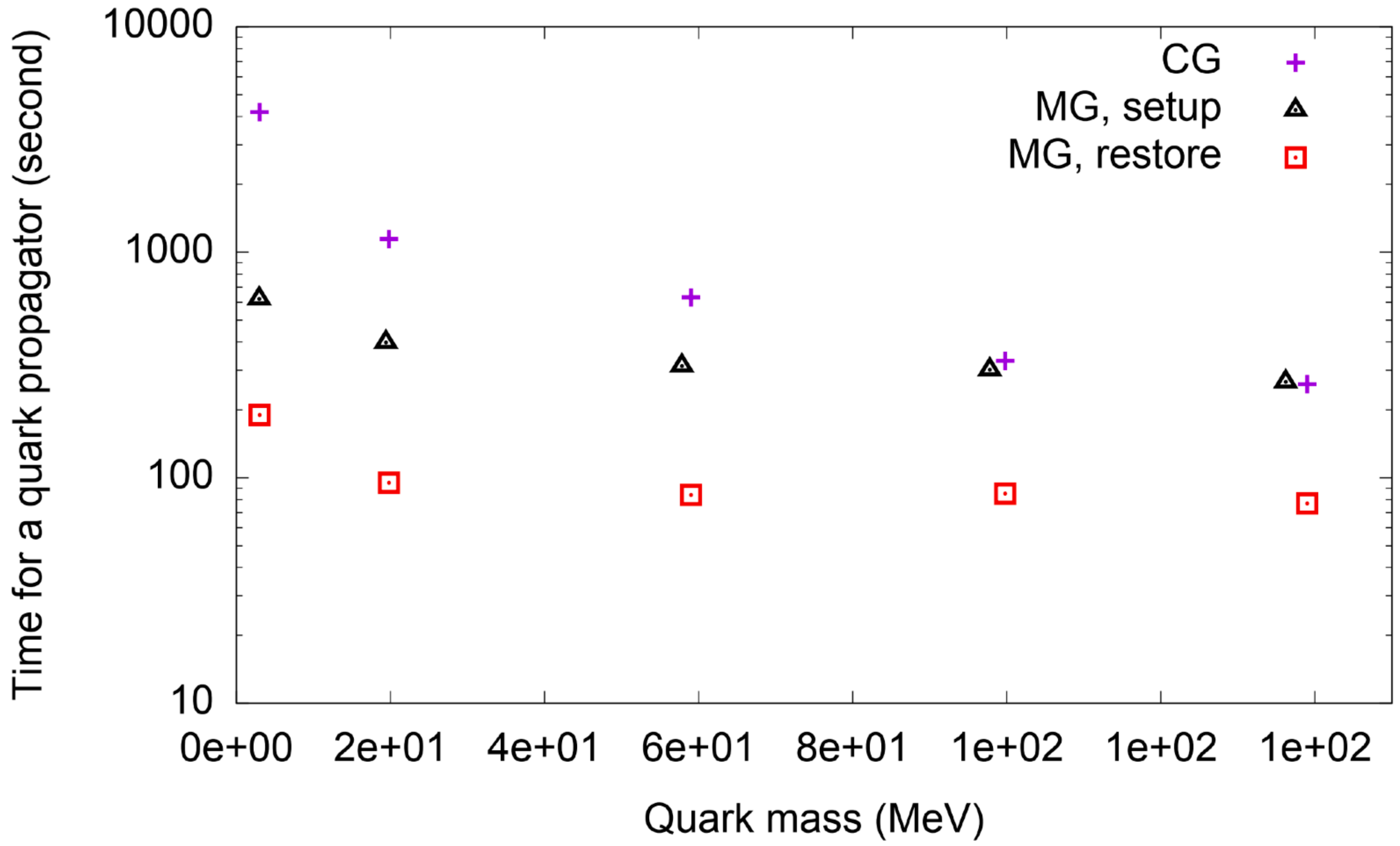
BMWc, Nature 593(2021)51

- Lattice calculation will suffer from the discretization error, which is usually $\mathcal{O}(a^2\Lambda_{QCD}^2)$.
- If we reduce the lattice spacing a by a factor of 2, the cost of calculation will increase by a factor of at least 16.
- The current FLAG "green star" requires at least three lattice spacings and at least two points below 0.1 fm and a range of lattice spacings satisfying $a_{\rm max}^2/a_{\rm min}^2 \geq 2$.

Lattice QCD



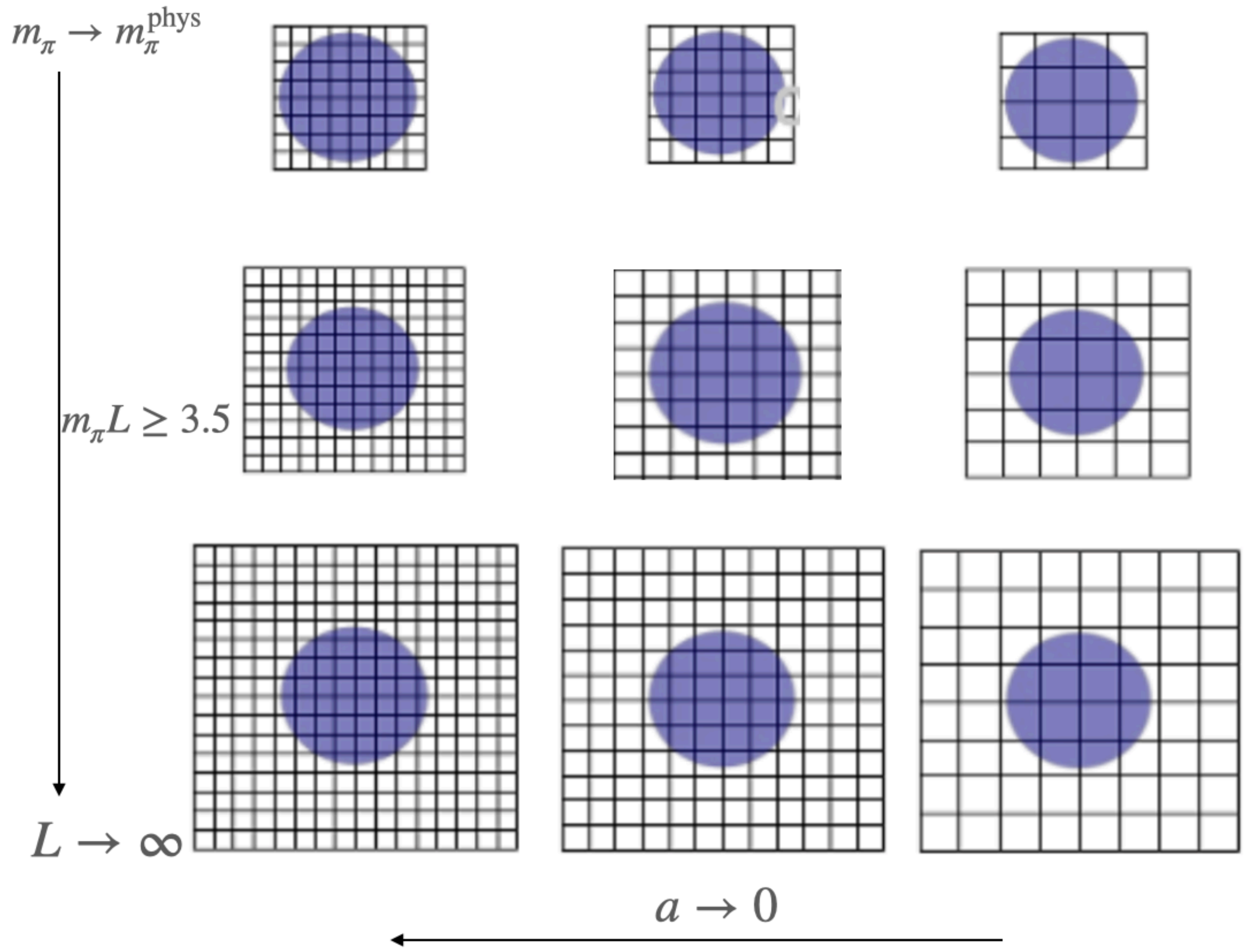
Chiral extrapolation



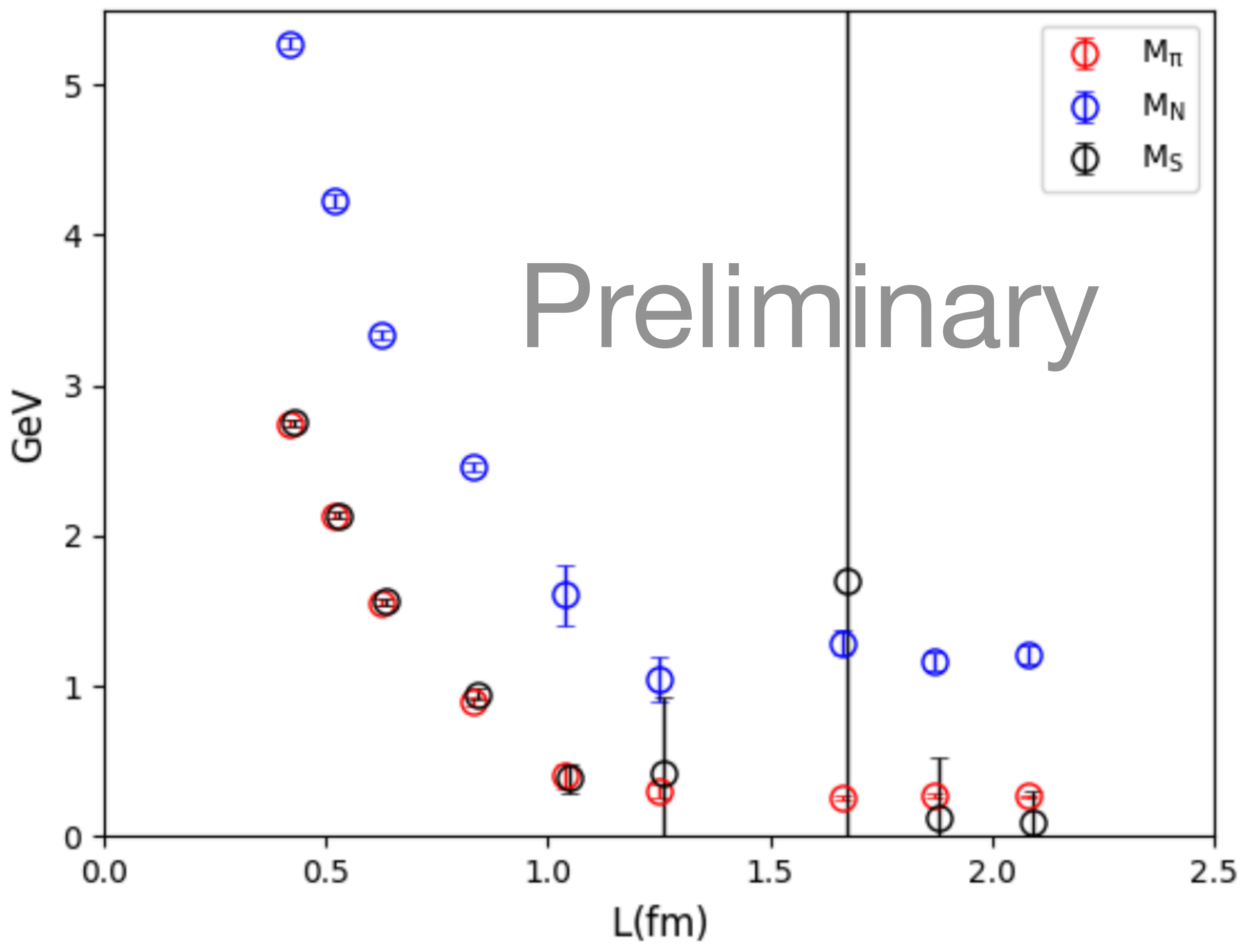
- The cost to simulate light quark can be an order of magnitude larger than that of the strange quark.
- Non-trivial algorithm likes multigrid can speed up the calculation of the light quark for certain fermion actions.
- The current FLAG "green star" requires $m_{\pi,\min} <$ 200 MeV with at least three m_π in the chiral extrapolation, **or** $m_{\pi,\text{ case1}} = 135 \pm 10$ MeV and $m_{\pi,\text{ case2}} < 200$ MeV.

◦

Lattice QCD



Finite volume effect



- Hadron mass can have very strong dependence on spatial size L, especially when $L \leq \Lambda_{\rm QCD}^{-1}$;
- The finite volume chiral perturbative theory suggest an $e^{-m_\pi L}$ correction when $m_\pi L \geq 3$, it means that the volume required by $m_\pi \sim$ 135 MeV is ~ 11 times of that required by $m_\pi \sim$ 300 MeV.
- The current FLAG "green star" requires $m_\pi L \sim$ 3.2 for $m_\pi \sim$ 135 MeV, **or** at least three volumes.

Lattice QCD

QED and iso-spin breaking effects

$$m_H(m_u, m_d, m_s, m_c, 0, 0, \alpha_{\rm QED}) = m_H(m_l = \frac{m_u + m_d}{2}, m_l, m_s, m_c, a, 1/L, 0)$$
$$+ \sum_i c_{i,a} a^{2i} + \sum_j c_{j,L} f(m_\pi) L^{-j} e^{-m_\pi L} + \ldots$$
$$+ (c_{\rm ISO} + c_{{\rm ISO},a} a^2 + \ldots)(m_d - m_u) + (c_{\rm QED} + c_{{\rm QED},a} a^2 + \ldots)\alpha_{\rm QED} \ldots$$

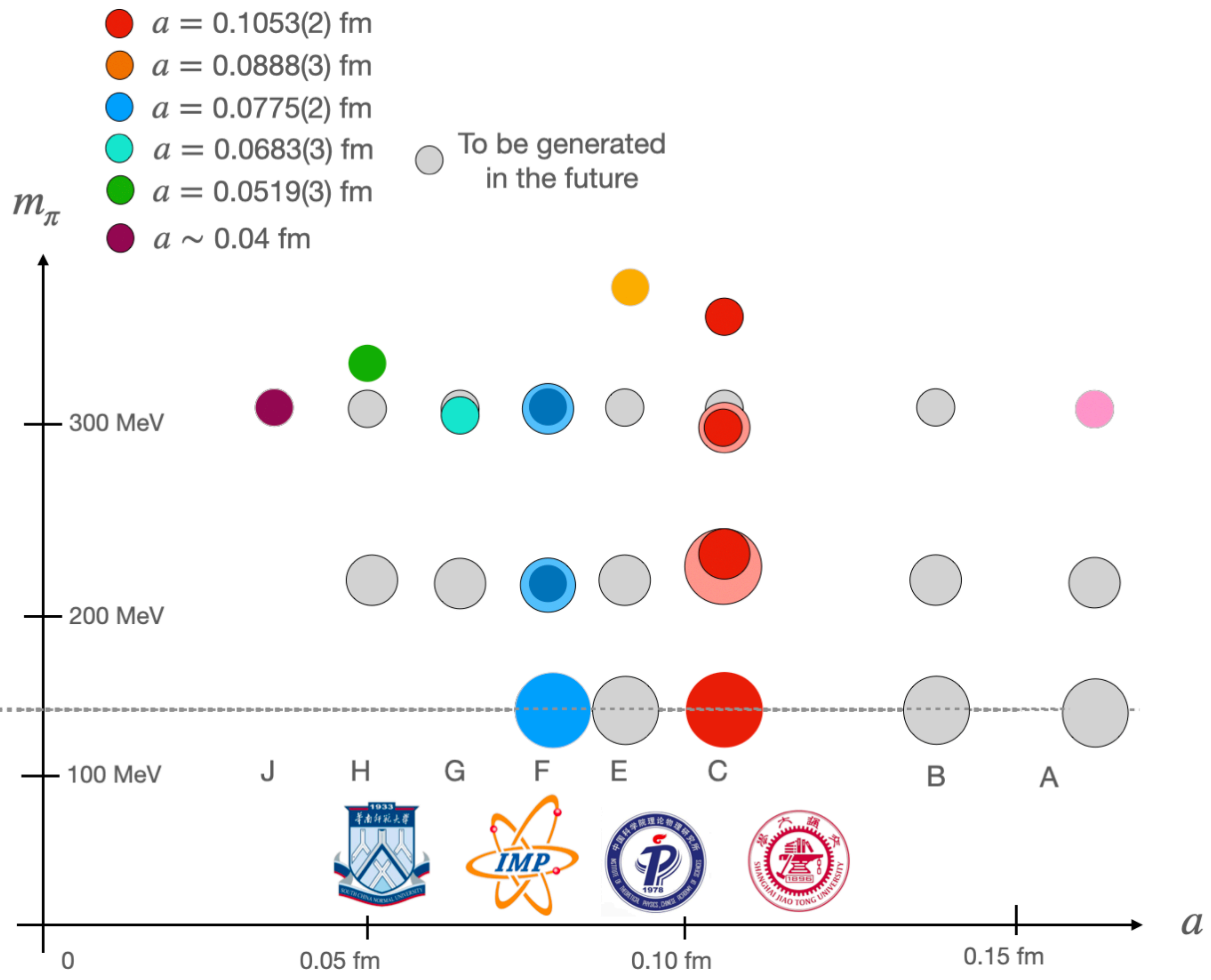
- Both the QED and iso-spin breaking effects are the order of $\alpha \sim \frac{m_d - m_u}{\Lambda_{\rm QCD}} \sim 1\,\%$.
- It is more practical to calculate the results of iso-symmetric pure QCD and extrapolated to continuum and infinite volume limits, physical (iso-symmetric) light, strange and charm quark masses;
- And then add the QED and iso-spin breaking effects at the leading order.

Lattice QCD

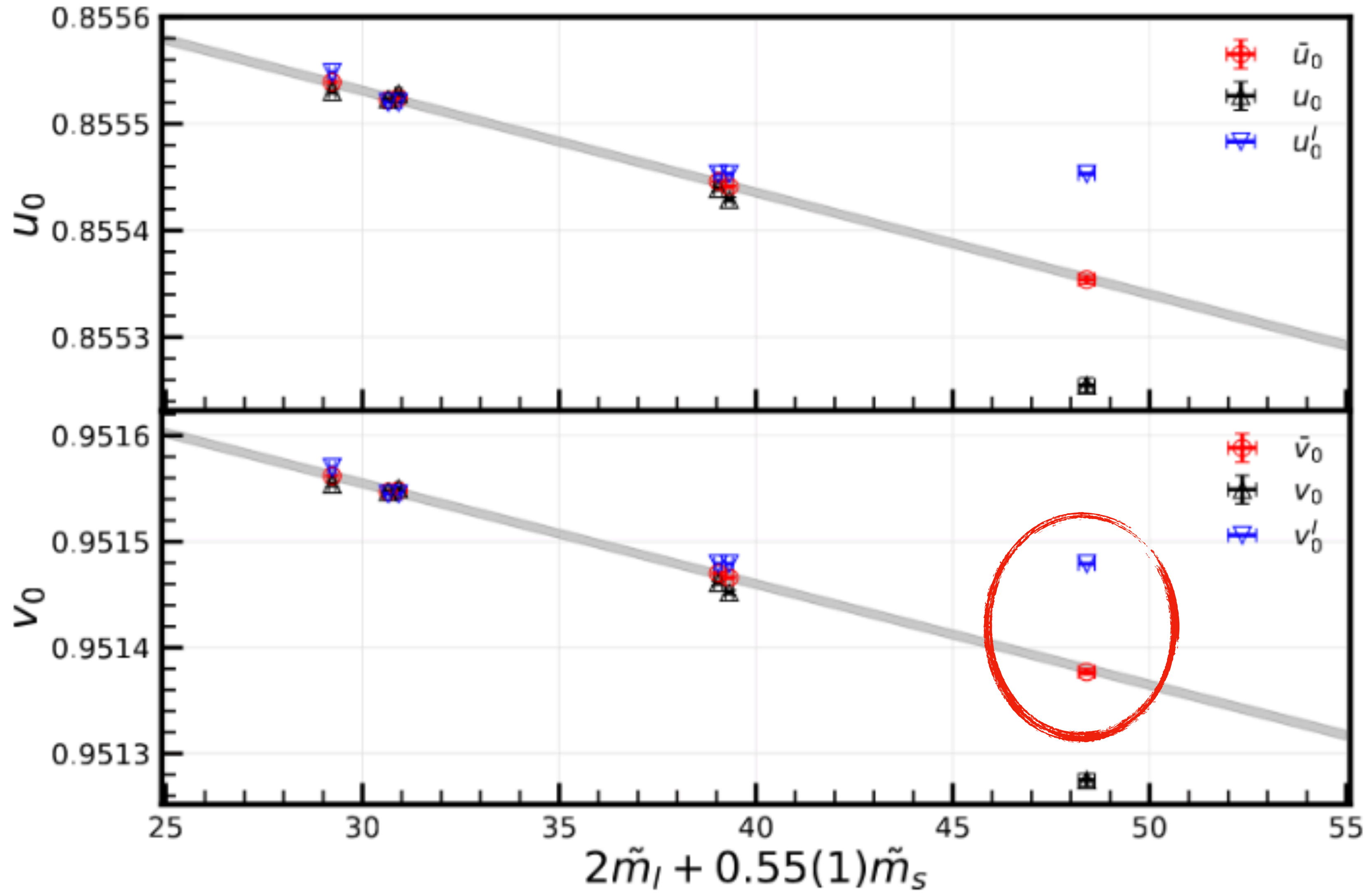
CLQCD ensembles

$$S_g(g_0) = \frac{1}{N_c}\mathrm{Re}\sum_{x,\mu<\nu}\mathrm{Tr}\left[1 - 10/(g_0^2 u_0^4)\left(\mathscr{P}^U_{\mu,\nu}(x) + \frac{1}{20u_0^2}\mathscr{R}^U_{\mu,\nu}(x)\right)\right]$$

$$S_q(m) = \sum_{x,\mu=1,\ldots,4,\eta=\pm} \bar{\psi}(x)\sum\frac{1+\eta\gamma_\mu}{2}V_{\eta\mu}(x)\psi(x+\eta\hat{\mu}a) + \sum_x \psi(x)\left[-(4+ma)\delta_{y,x} + \frac{1}{v_0^3}\sigma^{\mu\nu}g_0F^V_{\mu\nu}\right]\psi(x),$$



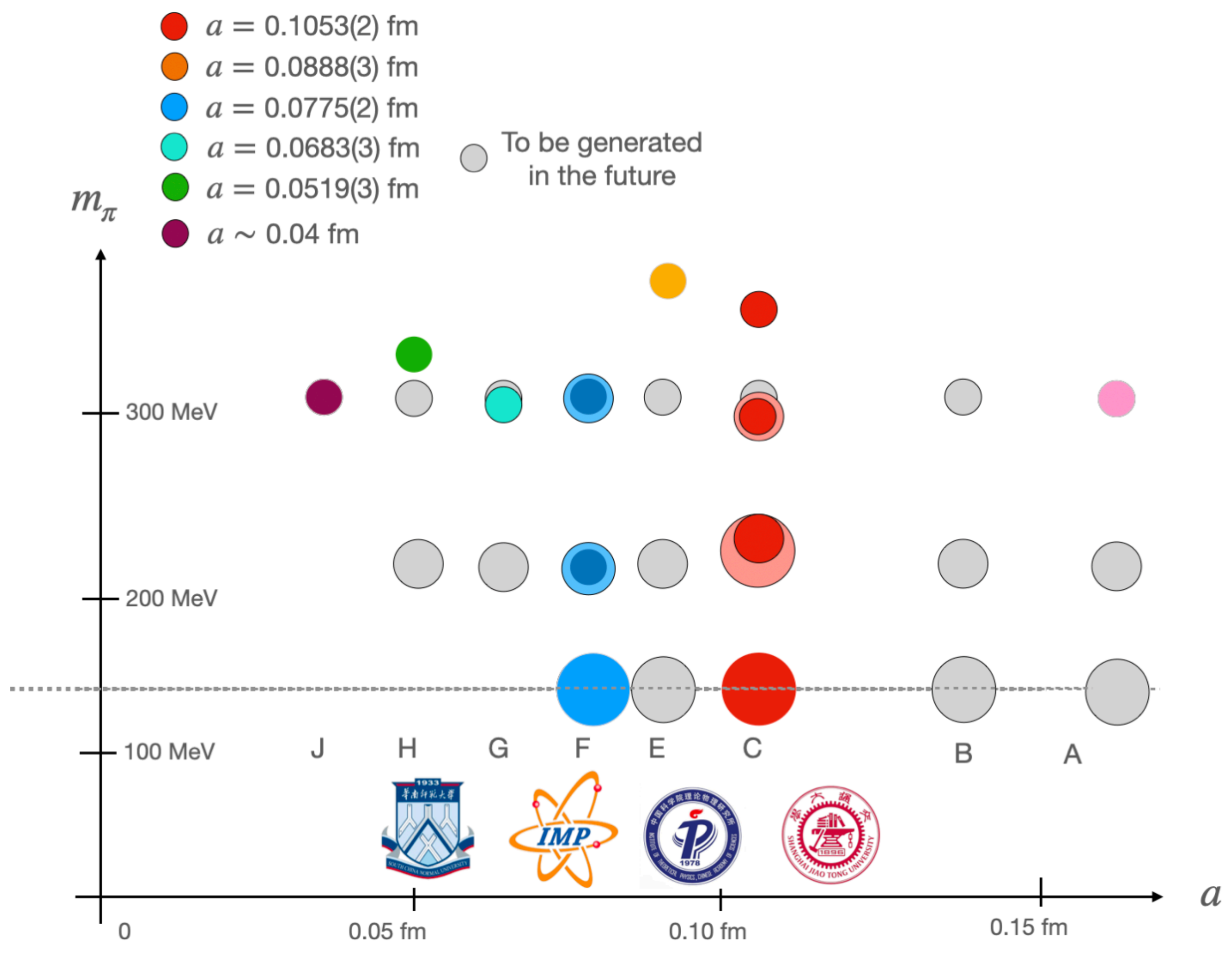
- Tadpole improved Symanzik gauge;
- Tadpole improved Clover fermion;
- Tadpole improvement requires fine-tuning of the tadpole factors u_0 and u_I;
- We tune those factors to the $0.001\ \%$ level, as the mistuning effect can be $\mathcal{O}(100)$ enhanced in the hadron and quark masses.



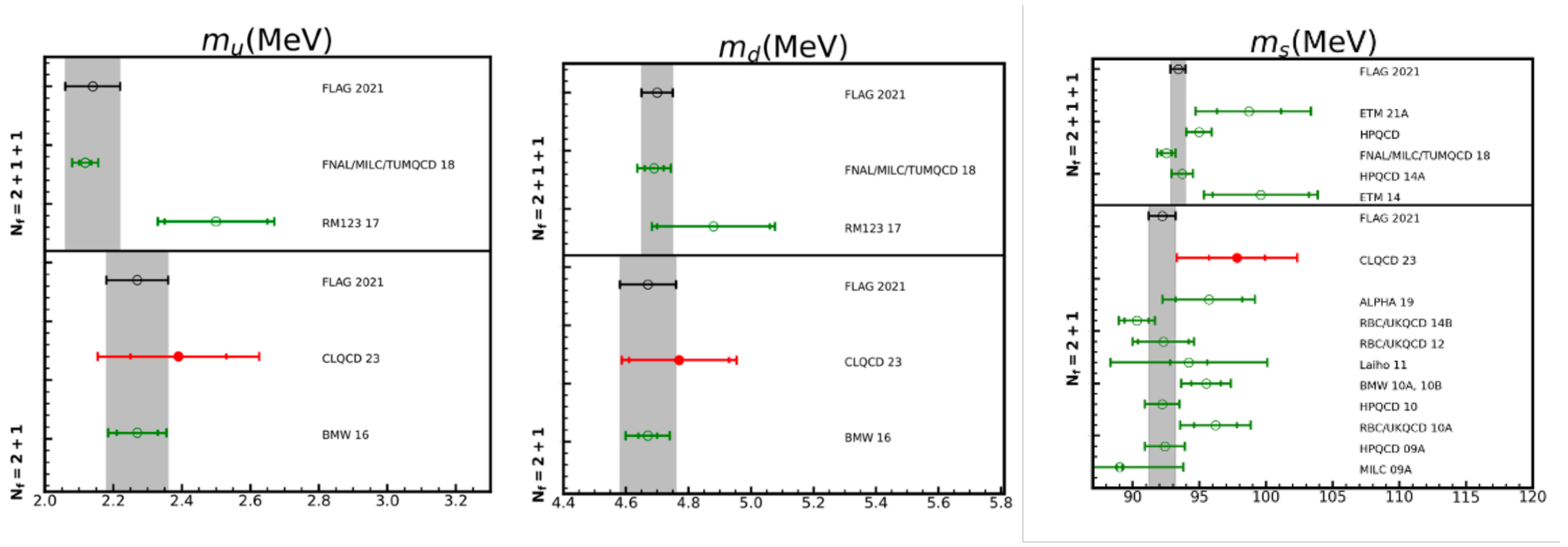
	$\tilde{m}_{\mathrm{PS}}$	$\tilde{m}_l^{\mathrm{PC}}$	$\tilde{f}_{\mathrm{PS}}$	Z_{wp}
$1/(v_0^I)^3$=1.1609	0.1832(12)	0.01191(13)	0.0768(10)	10.50(30)
$1/(\bar{v}_0)^3$=1.1613	0.1822(12)	0.01178(13)	0.0768(11)	10.50(31)
difference	0.0011(01)	0.00013(01)	0.0000(00)	0.00(01)

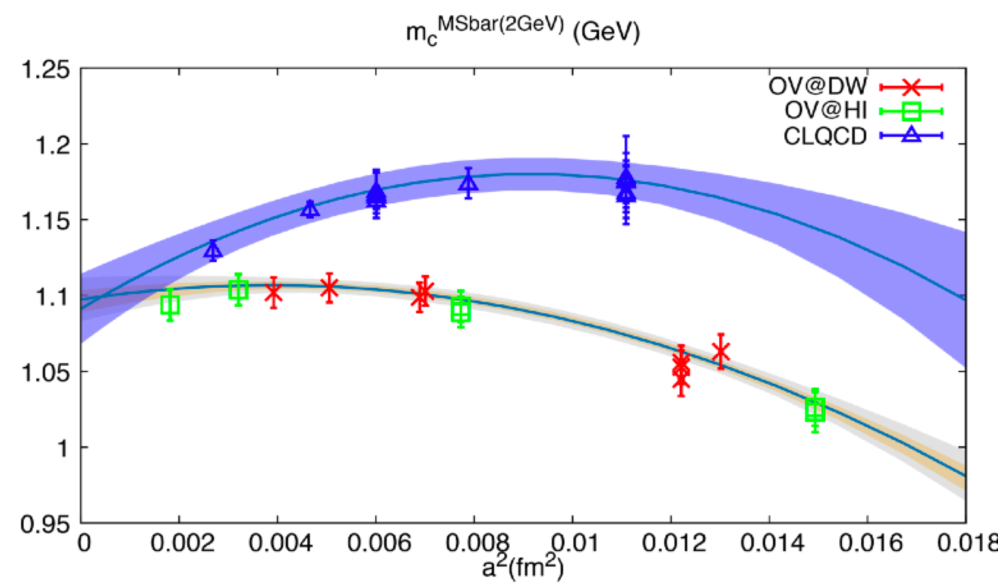
Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Outline



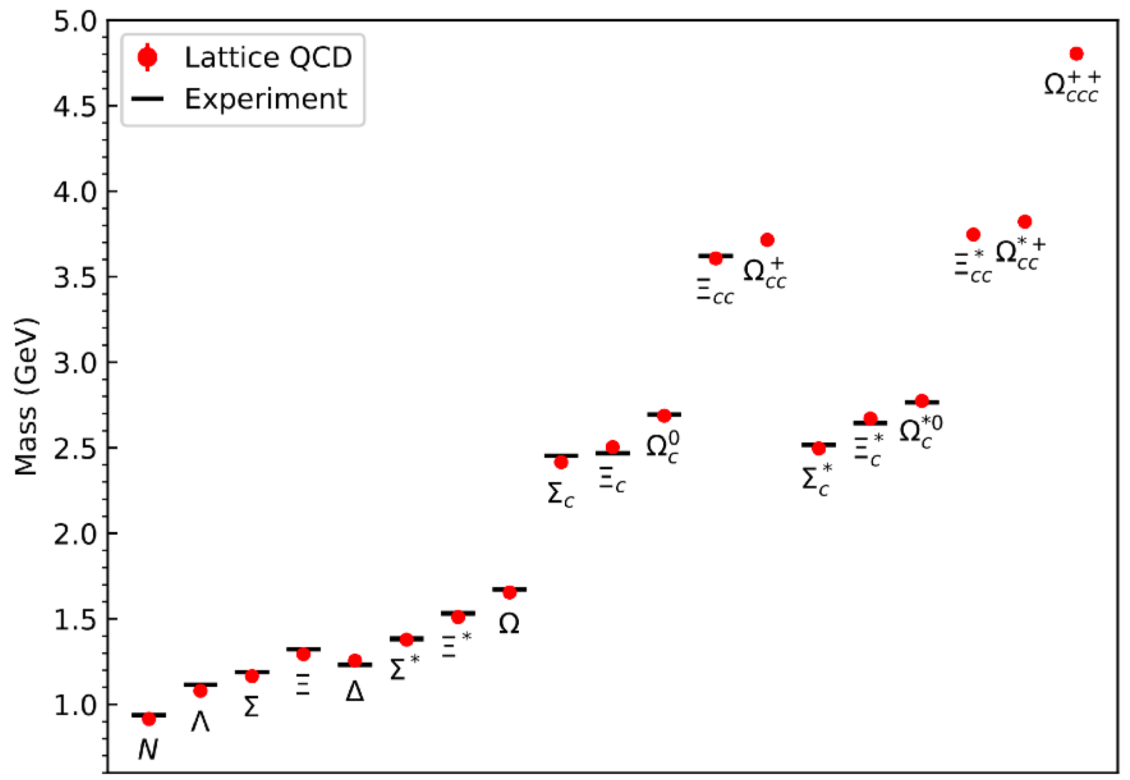
- LQCD background and CLQCD ensembles

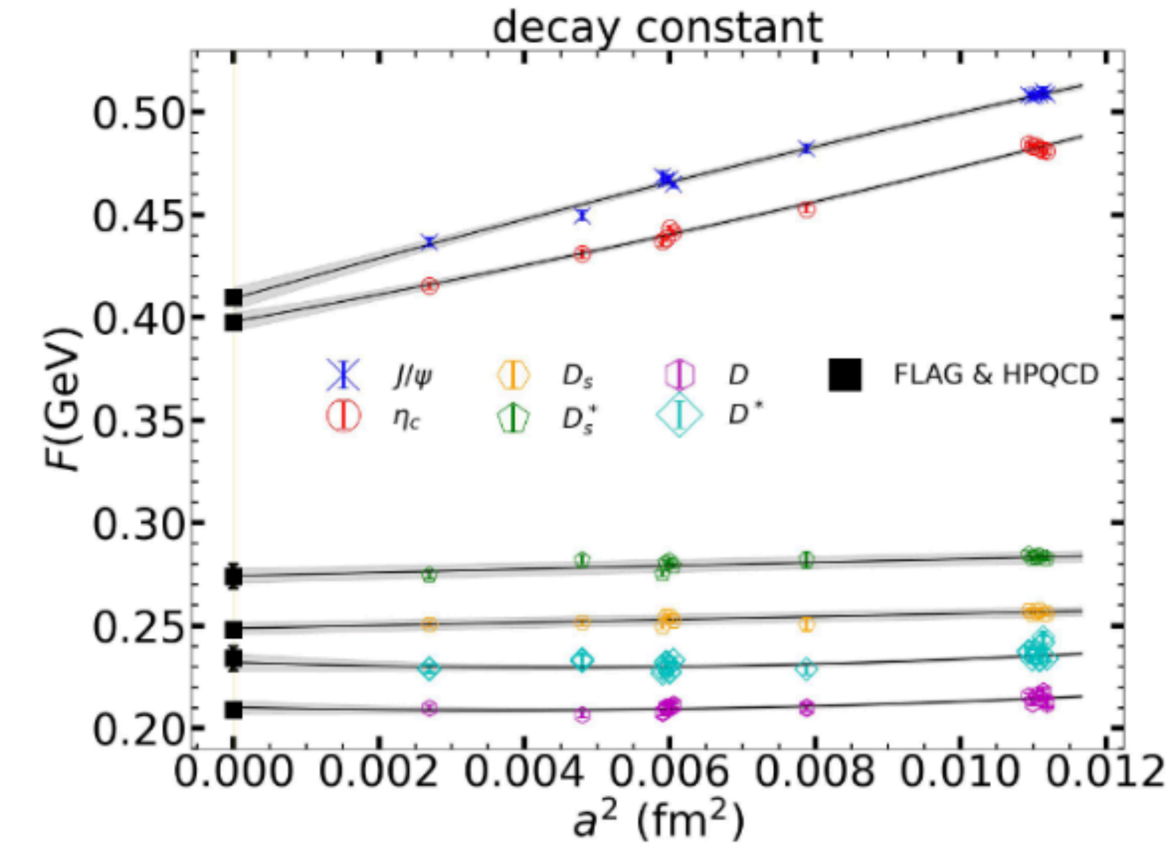




- **Quark mass determinations**

- Hadron masses and decay constants

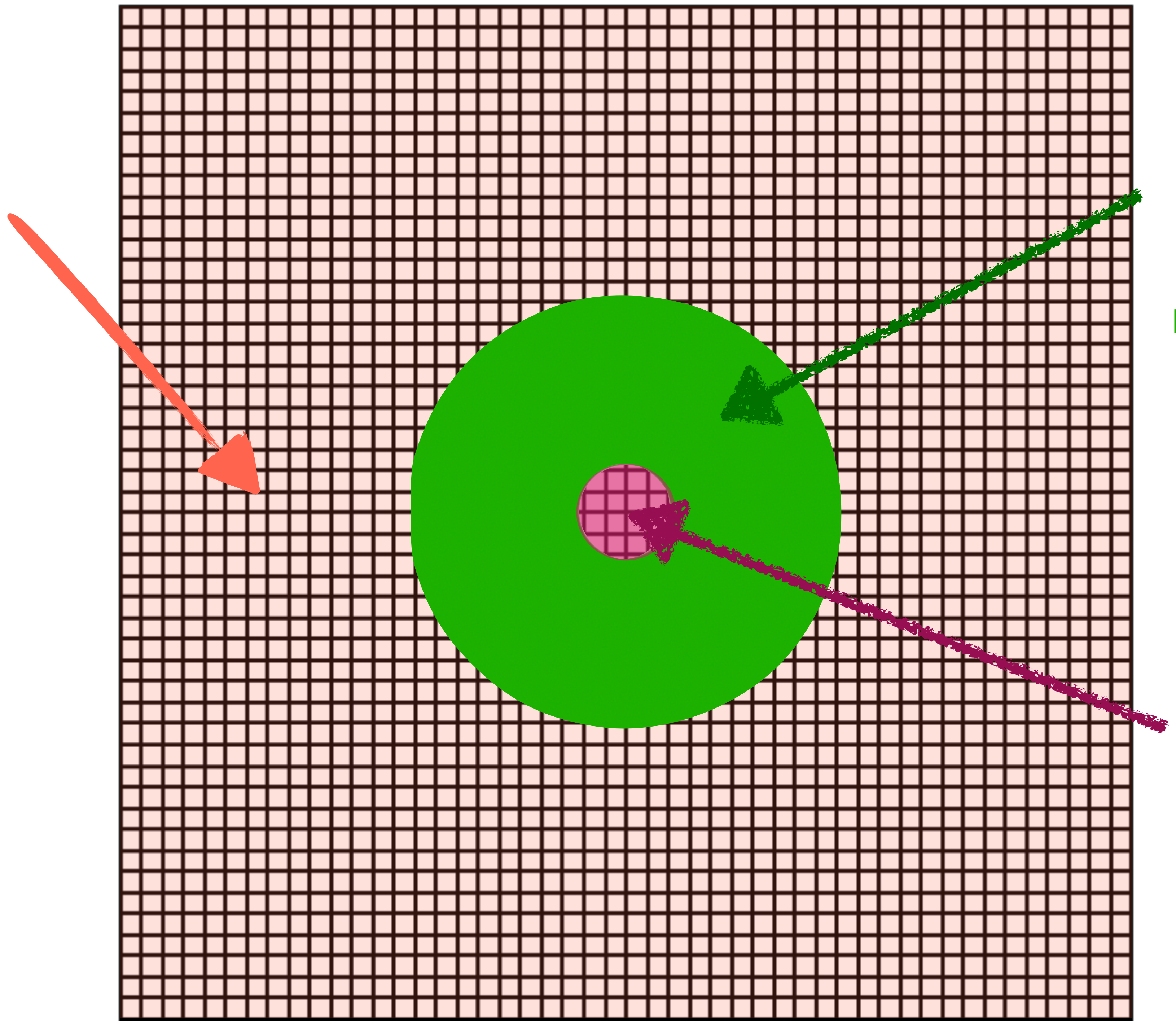




Quark mass

Renormalization through intermediate scheme

$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q, 1/a)}{Z_m^{\mathrm{MOM,Dim}}(Q, \mu, \epsilon)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^m, \alpha_s^n)$$



- The RI/MOM renormalization targets to cancel the $\alpha_s \log(a)$ divergences using the off-shell quark matrix element;
- **Up to the $\mathcal{O}(a^2p^2)$ correction which can be eliminated by the $a^2p^2 \to 0$ extrapolation.**

G. Martinelli, et.al, NPB445(1995)81, arXiv: hep-lat/9411010

Quark mass

Perturbative renormalization

- The RI/MOM renormalization constant of the quark mass under the lattice regularization is:

$$Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi) = (Z_S^{\mathrm{MOM,Lat}}(Q,1/a,\xi))^{-1} = \langle q|\mathcal{O}|q\rangle^{\mathrm{Lat}} = 1 + \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2);$$

- The RI/MOM and $\overline{\mathrm{MS}}$ renormalization constants under the dimensional regularization are:

$$Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi) = \langle q|\mathcal{O}|q\rangle^{\mathrm{Dim}} = 1 + \frac{\alpha_s C_F}{4\pi}[\frac{3}{\tilde{\epsilon}} - 3\log(\frac{Q^2}{\mu^2}) - \xi + 5] + \mathcal{O}(\alpha_s^2);$$

$$Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\tilde{\epsilon}} + \mathcal{O}(\alpha_s^2);$$

- Thus the renormalized quark mass under the $\overline{\mathrm{MS}}$ scheme can be defined by:

$$m_q^{\overline{\mathrm{MS}}}(\mu) = \frac{Z_m^{\mathrm{MOM,Lat}}(Q,1/a,\xi)}{Z_m^{\mathrm{MOM,Dim}}(Q,\mu,\epsilon,\xi)} Z_m^{\overline{\mathrm{MS}},\mathrm{Dim}}(\epsilon) m_q^{\mathrm{Lat}}(1/a) = (1 + \frac{\alpha_s C_F}{4\pi}[-3\log(a^2\mu^2) - 5 + b_S]) m_q^{\mathrm{Lat}}(1/a) + \mathcal{O}(a^{2m}Q^{2m}, \alpha_s^n)\,.$$

Quark mass

Non-Perturbative renormalization

- Obtain the regularization independent renormalization constant non-perturbatively:

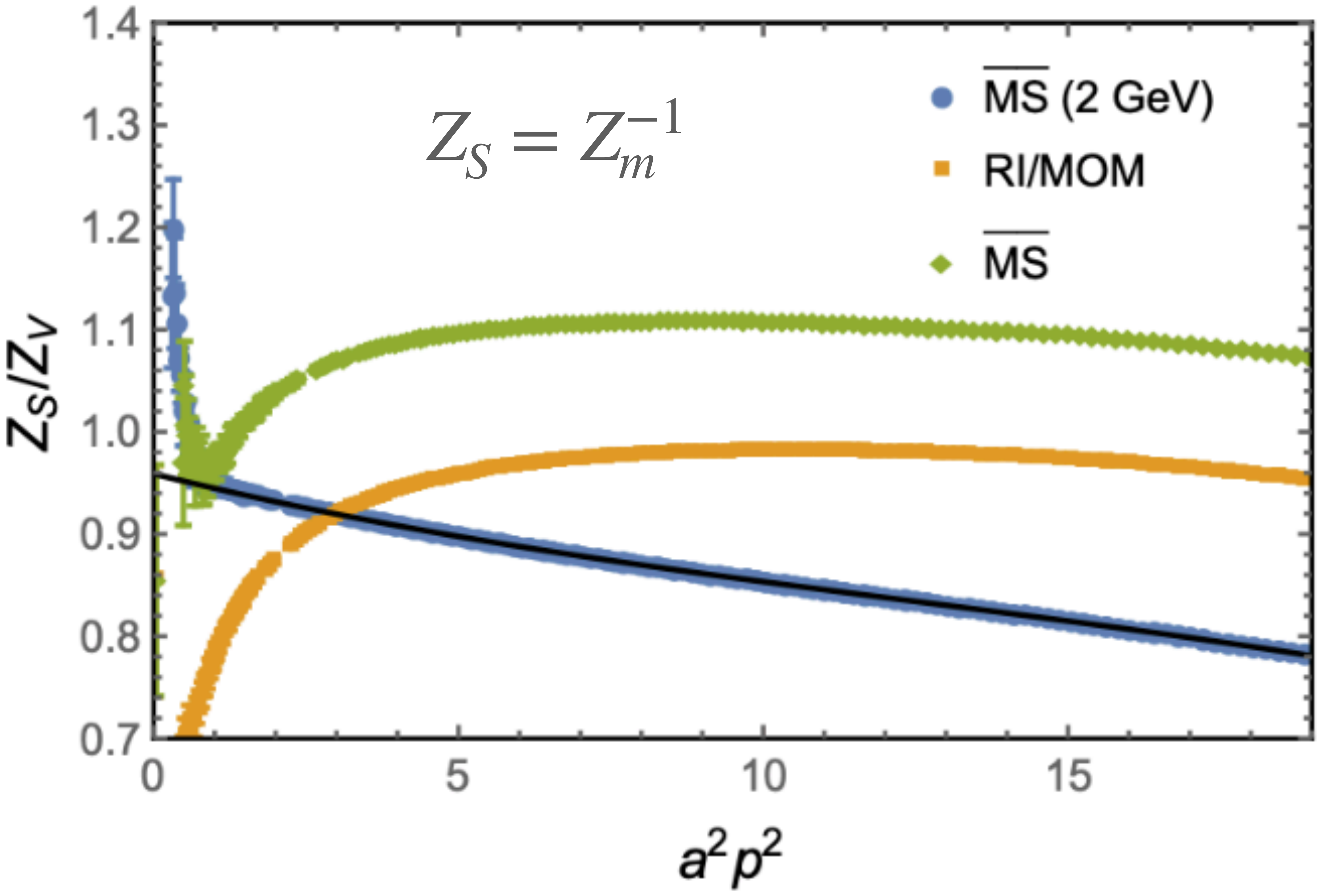
$$Z_S^{\mathrm{MOM}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$

- Calculate the matching coefficient perturbatively and obtain the result at $\overline{\mathrm{MS}}$ scale Q:

$$Z_S^{\mathrm{MS}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$

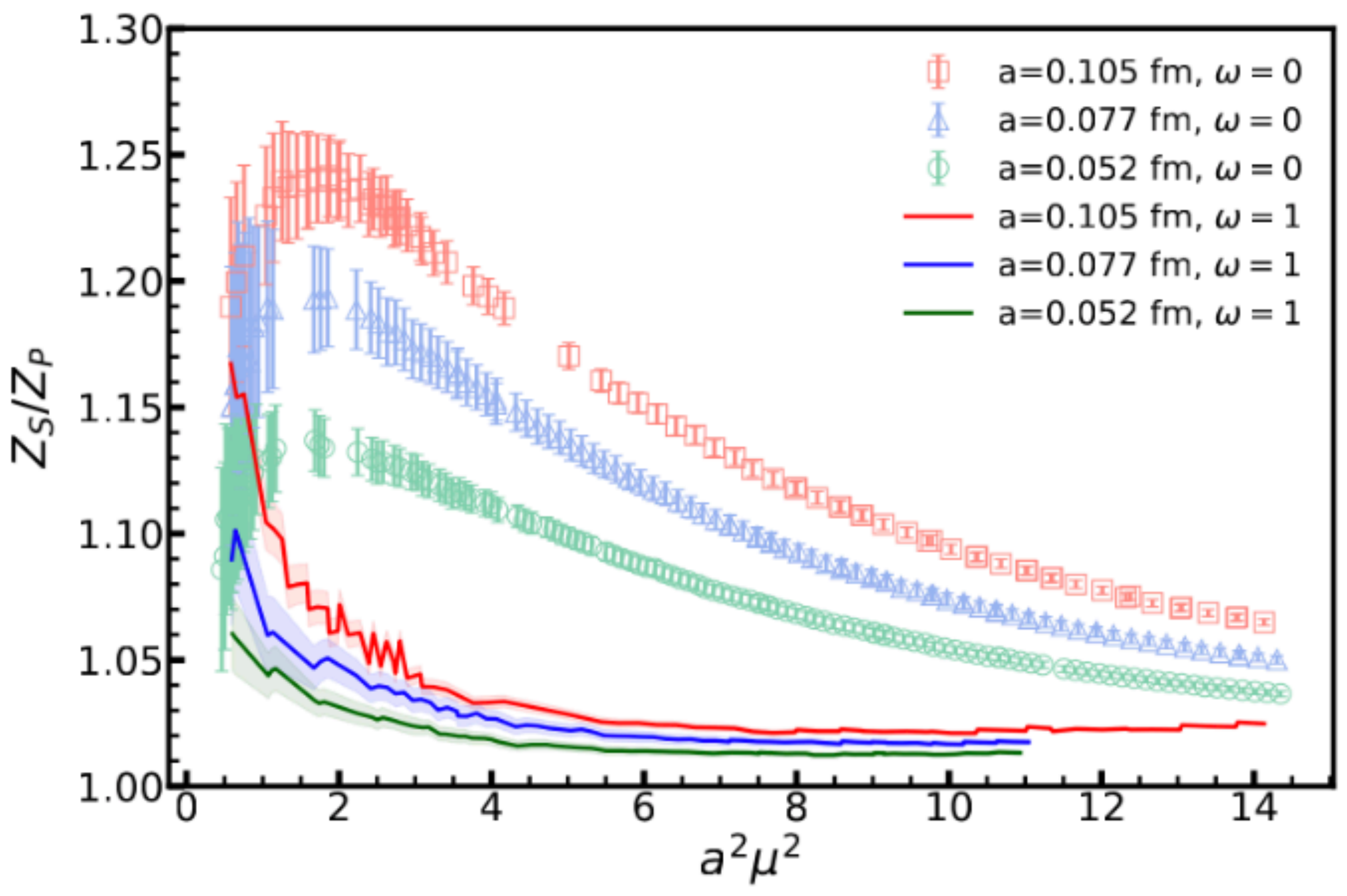
- Obtain the result at $\overline{\mathrm{MS}}$ scale μ with the scale evolution:

$$Z_S^{\overline{\mathrm{MS}}}(\mu,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2\mu^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$



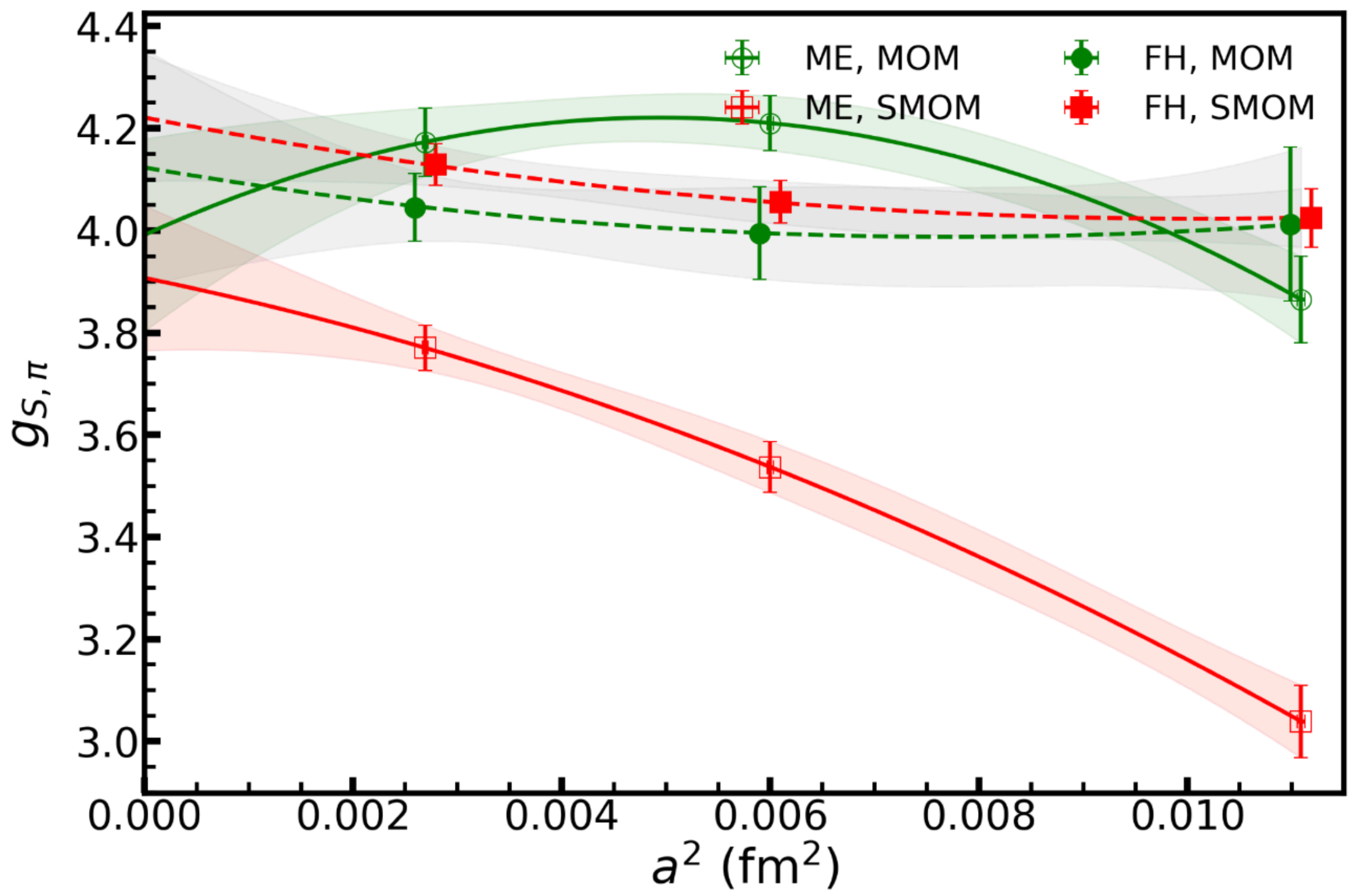
Non-perturbative renormalization

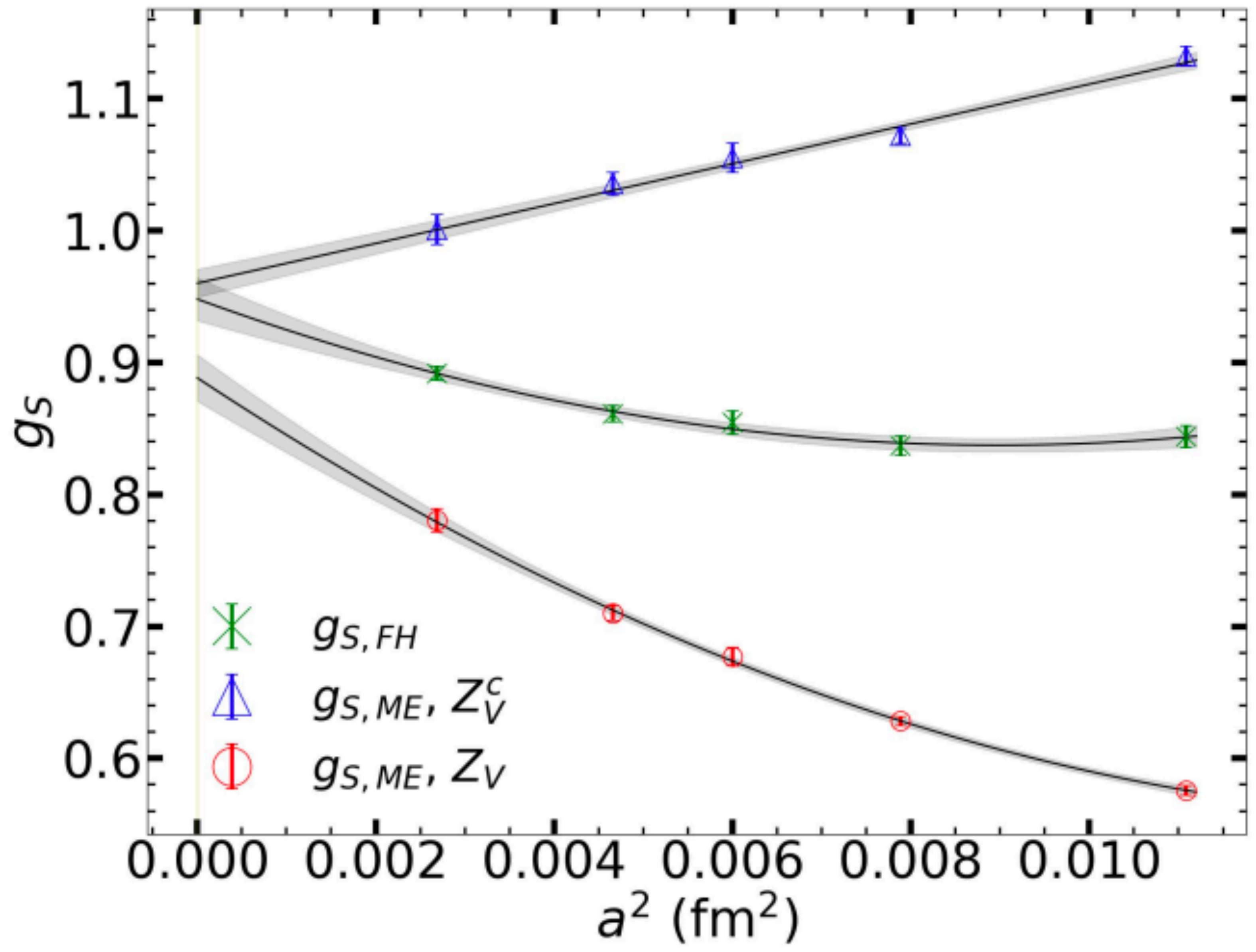
Restore the additive chiral symmetry breaking of Clover fermion



- Clover fermion also shows additional chiral symmetry breaking between Z_S and Z_P;

- Light quark scalar matrix element (ME) from the direct calculation $Z_S\langle\pi\,|\,\bar{q}q\,|\,\pi\rangle$ and Feynman-Hellman (FH) theorem $Z_P/Z_A\dfrac{\partial m_\pi}{\partial m_q^{\mathrm{PC}}}$ are consistent after the renormalization;





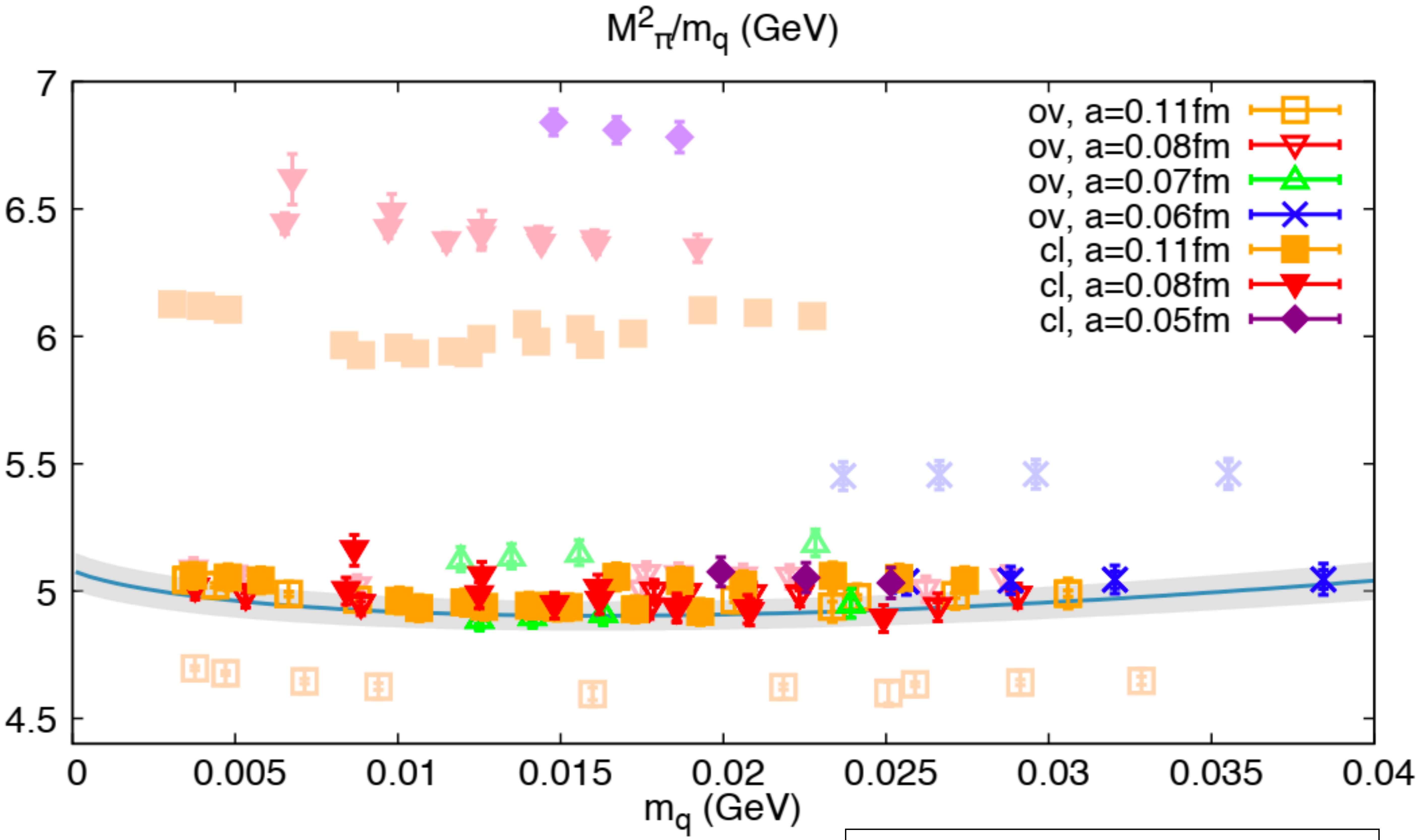
- Charm quark scalar ME from the direct calculation and FH theorem are also consistent after the charm quark improved normalization applied.

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

Renormalized quark masses

Impact of the renormalization



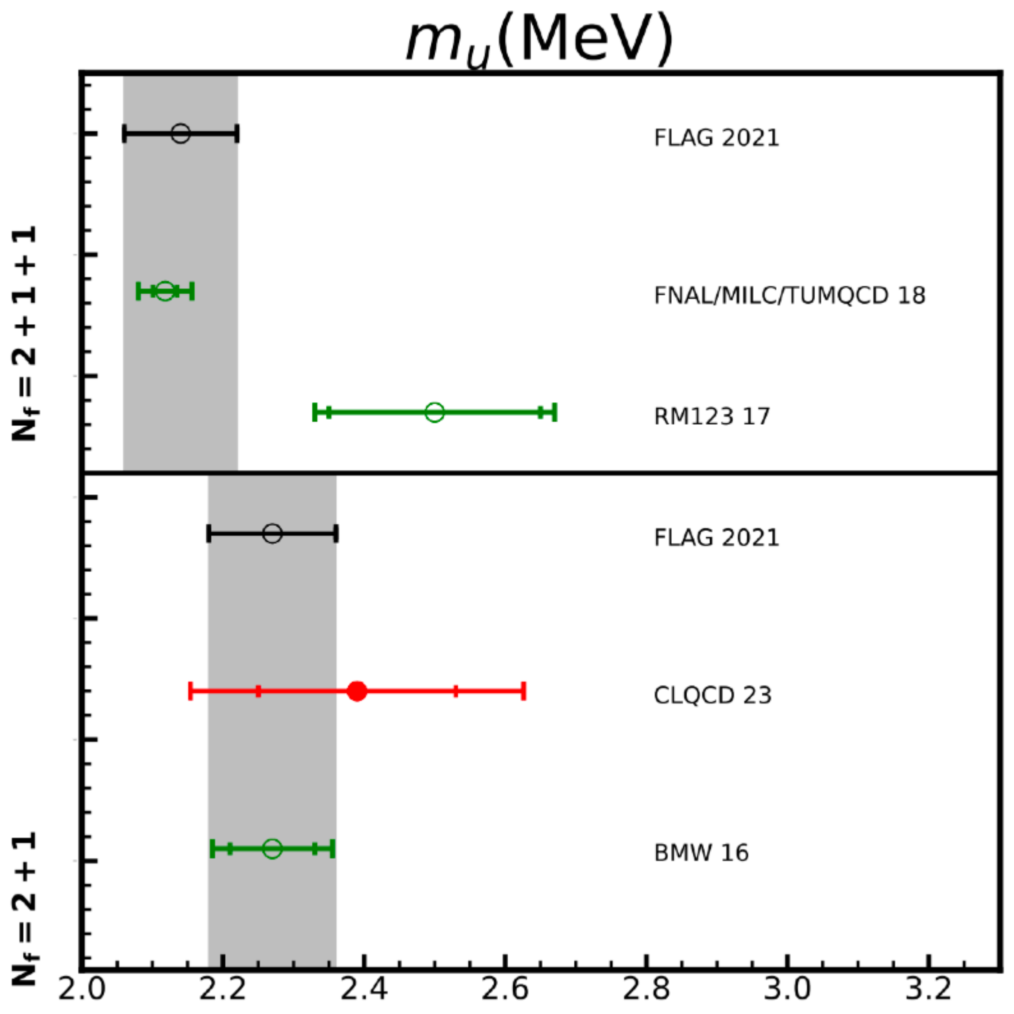
Dian-Jun Zhao, et.al., χQCD, in preparation

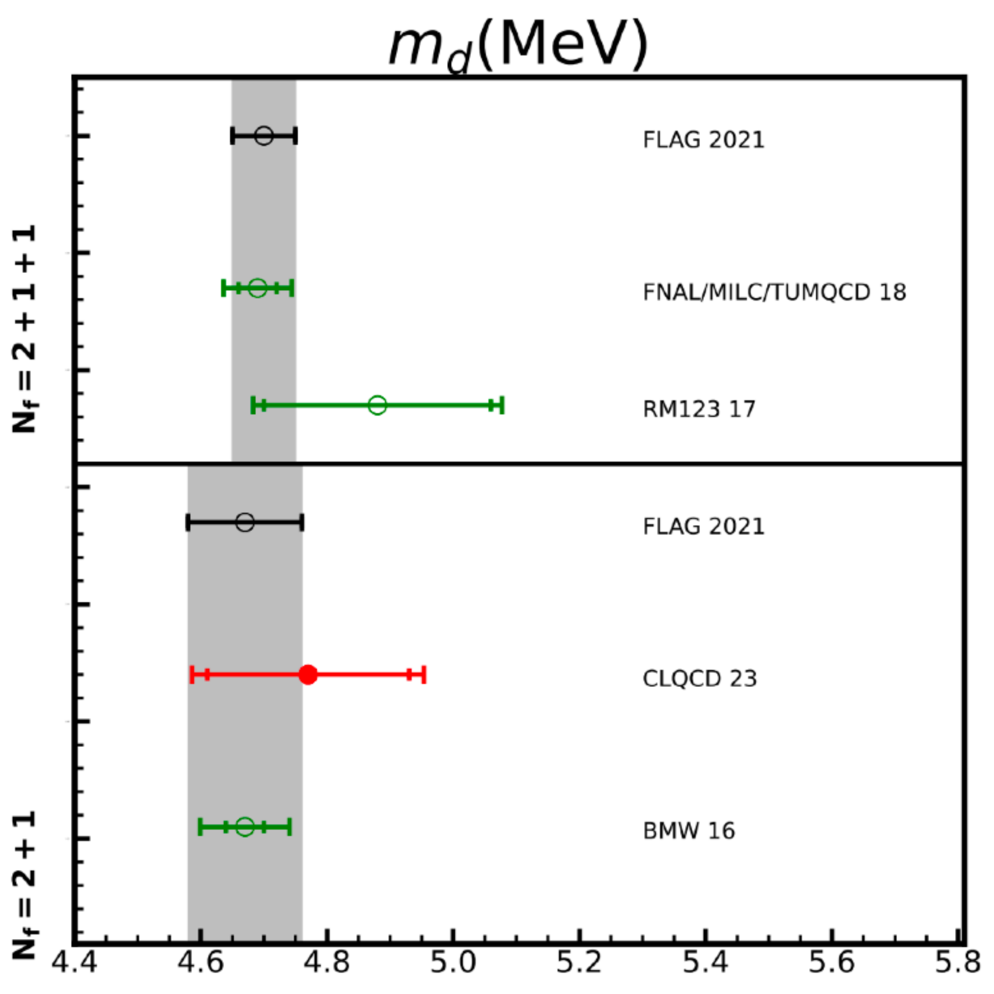
- $m_\pi^2/m_q \sim \Sigma/F^2$ which is insensitive to the quark mass, with the partially quenching effect subtracted;
- The PCAC mass $m_q^{\mathrm{PC}} = \frac{\langle 0|\partial_4 A_4|\mathrm{PS}\rangle}{2\langle 0|P|\mathrm{PS}\rangle}$ has obvious 1/a and action dependences:

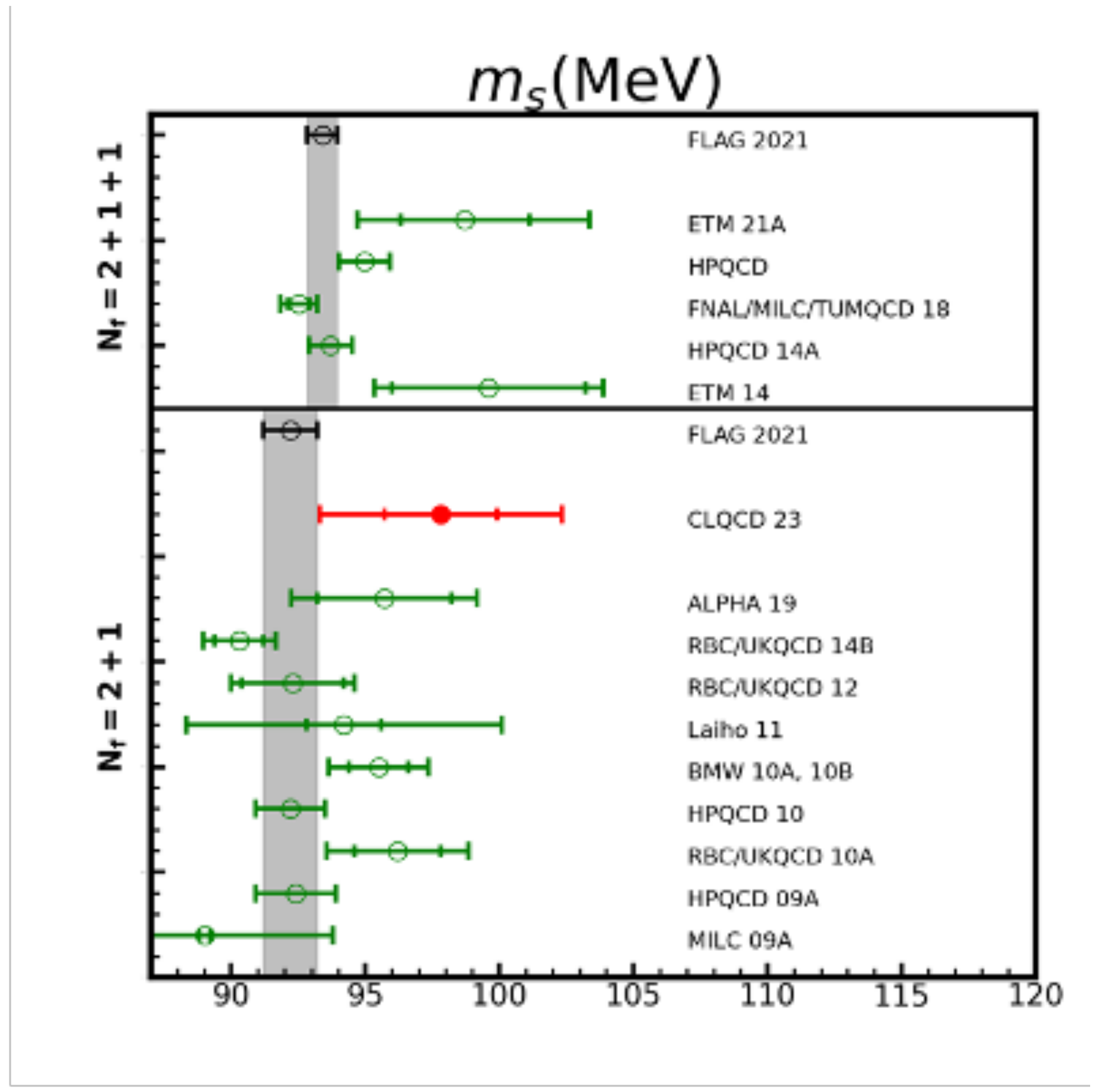
1. Smaller with large intrinsic scale 1/a;
2. Very sensitive to the fermion action.

- RI/MOM renormalization eliminates both the dependences and makes $m_\pi^2/m_q^{\overline{\mathrm{MS}}}$ of all the ensembles on a similar curve.

Quark mass







Determine the pure QCD quark masses

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27 \text{ MeV} = m_{p,\text{QCD}} + 1.00(16) \text{ MeV} + \ldots;$
- $m_n = 939.57 \text{ MeV};$
- $m_\pi^0 = 134.98 \text{ MeV};$
- $m_\pi^+ = 139.57 \text{ MeV} = m_\pi^0 + 4.53(6) \text{ MeV} + \ldots;$

X. Feng, et,al. Phys.Rev.Lett.128(2022)062003

- $m_K^0 = 497.61(1) \text{ MeV} = m_{K,\text{QCD}}^0 + 0.17(02) \text{ MeV} + \ldots;$
- $m_K^+ = 493.68(2) \text{ MeV} = m_{K,\text{QCD}}^+ + 2.24(15) \text{ MeV} + \ldots$

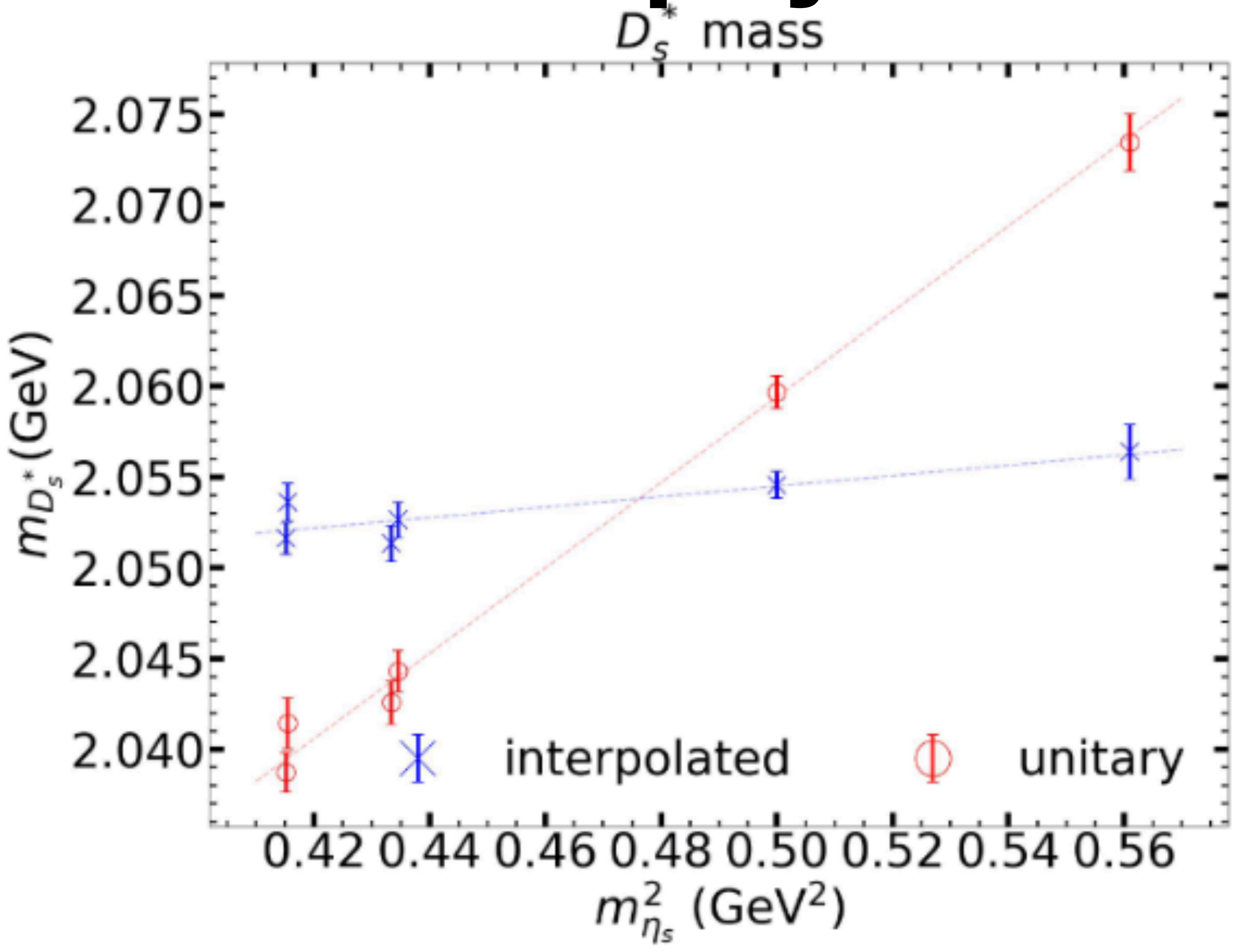
D. Giusti, et,al. PRD95(2017)114504

Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

Quark mass

Toward the charm physics

Symbol	$\hat{\beta}$	a (fm)	u_0	v_0	$\tilde{m}_l^b$	$\tilde{m}_s^b$	$\tilde{L}^3 \times \tilde{T}$	m_π (MeV)	m_{η_s} (MeV)	$\tilde{m}_s^{\rm I}$	$\tilde{m}_c^{\rm I}$	$n_{\rm cfg} \times n_{\rm src}$
C24P34	6.200	0.10530(18)	0.855453	0.951479	-0.2770	-0.2310	$24^3 \times 64$	340.5(1.7)	748.99(73)	-0.2396(2)	0.4080(26)	200×32
C24P29			0.855453	0.951479	-0.2770	-0.2400	$24^3 \times 72$	292.7(1.2)	658.29(65)	-0.2357(2)	0.4168(26)	760×3
C32P29			0.855453	0.951479	-0.2770	-0.2400	$32^3 \times 64$	292.4(1.1)	659.22(41)	-0.2358(2)	0.4158(26)	489×3
C32P23			0.855520	0.951545	-0.2790	-0.2400	$32^3 \times 64$	228.0(1.2)	644.36(45)	-0.2338(2)	0.4198(26)	400×3
C48P23			0.855520	0.951545	-0.2790	-0.2400	$48^3 \times 96$	225.6(0.9)	644.58(62)	-0.2338(2)	0.4214(26)	62×3
C48P14			0.855548	0.951570	-0.2825	-0.2310	$48^3 \times 96$	135.5(1.6)	707.06(44)	-0.2335(2)	0.4212(26)	188×3
E28P35	6.308	0.08877(30)	0.859646	0.954385	-0.2490	-0.2170	$28^3 \times 64$	352.1(1.2)	720.31(94)	-0.2204(3)	0.2707(37)	147×4
F32P30	6.410	0.07750(18)	0.863437	0.956942	-0.2295	-0.2050	$32^3 \times 96$	303.2(1.3)	677.6(1.0)	-0.2039(2)	0.1968(21)	250×3
F48P30			0.863473	0.956984	-0.2295	-0.2050	$48^3 \times 96$	303.4(0.9)	676.32(62)	-0.2038(2)	0.1949(21)	99×3
F32P21			0.863488	0.957017	-0.2320	-0.2050	$32^3 \times 64$	210.9(2.2)	660.27(94)	-0.2024(2)	0.1989(21)	194×3
F48P21			0.863499	0.957006	-0.2320	-0.2050	$48^3 \times 96$	207.2(1.1)	663.39(65)	-0.2026(2)	0.1991(21)	82×12
G36P29	6.498	0.06826(27)	0.866476	0.958910	-0.2150	-0.1926	$36^3 \times 108$	295.1(1.2)	693.2(1.0)	-0.1929(2)	0.1378(28)	68×4
H48P32	6.720	0.05187(26)	0.873378	0.963137	-0.1850	-0.1700	$48^3 \times 144$	317.2(0.9)	695.9(1.3)	-0.1703(2)	0.0533(24)	157×12



$$m_{\eta_s} = 687.4(2.2)\ \text{MeV}$$

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, Phys.Rev.D109 (2024) 054507

$$m_{\eta_s} = 689.89(49)\ \text{MeV}$$

BMWc, Nature 593(2021)51

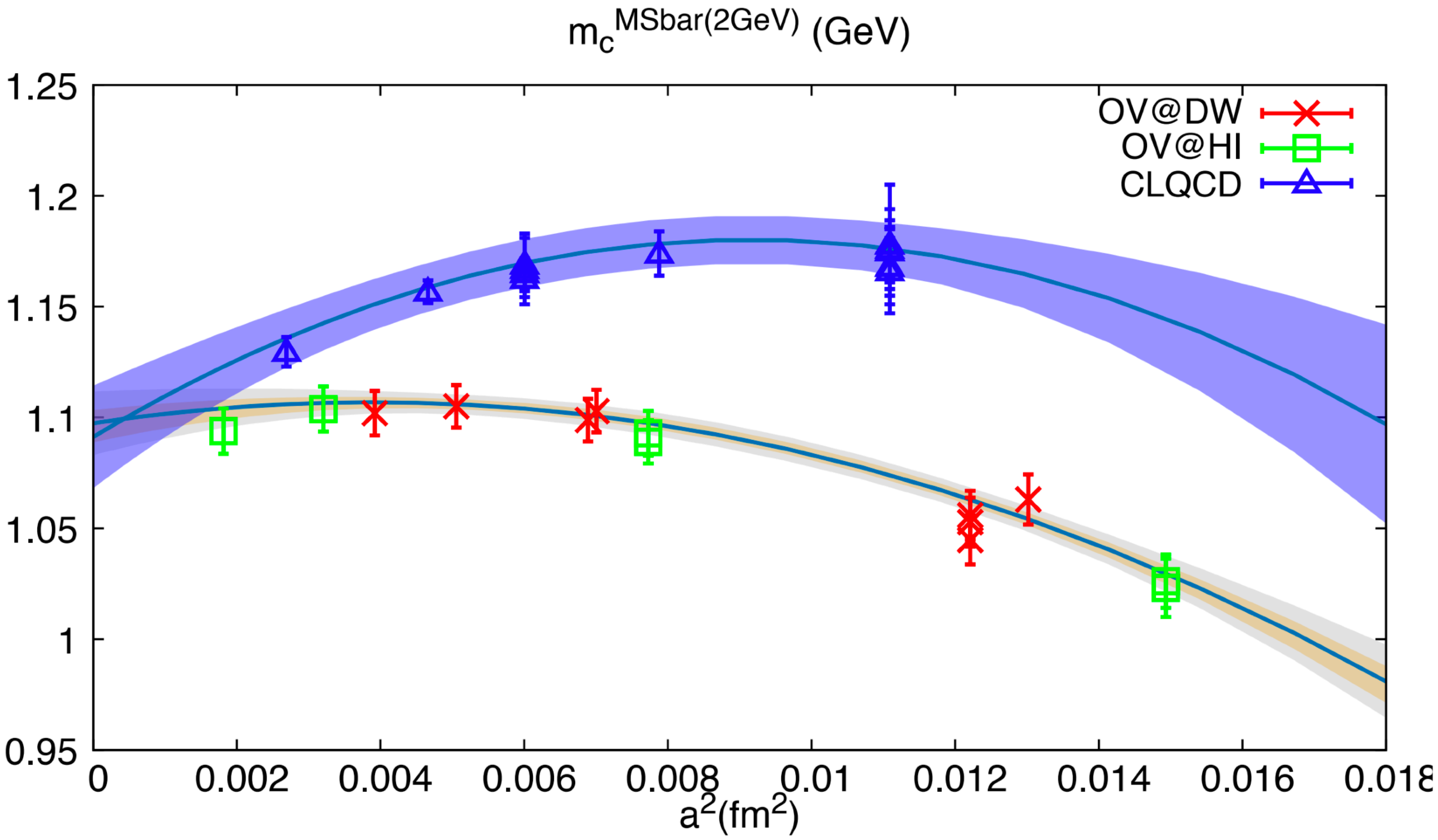
$$m_{D_s}^{\rm QCD} = m_{D_s}^{\rm phys} - \Delta^{\rm QED} m_{D_s} = 1966.7(1.5)\ \text{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 1904.08731

- Use etas to determine the valence strange quark mass;
- Significantly suppress the strange quark mass dependence on each ensemble.
- Use QED-subtracted m_{D_s} mass to determine the pure QCD valence charm quark mass;
- $\Delta^{\rm QED} m_{D_s}$ is determined to be 2.3(4) MeV under the $m^{\overline{\rm MS}}_{q,{\rm QCD+QED}}(2{\rm GeV}) = m^{\overline{\rm MS}}_{q,{\rm QCD}}(2{\rm GeV})$ scheme.
- Eliminate the effects from unphysical light and strange sea quark masses using the joint fit.

Renormalized quark masses

Charm quark mass



Dian-Jun Zhao, et.al., χQCD, in preparation

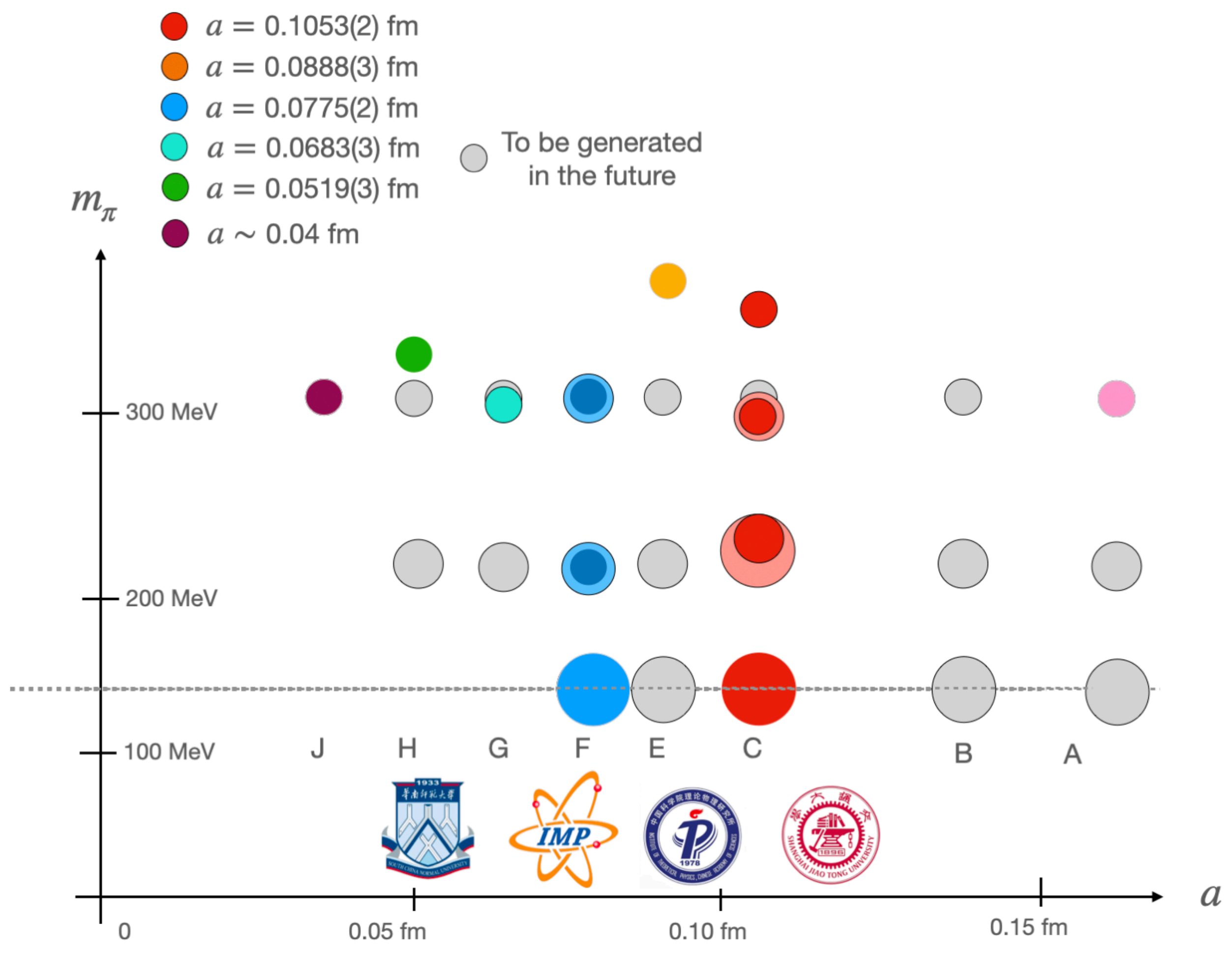
Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

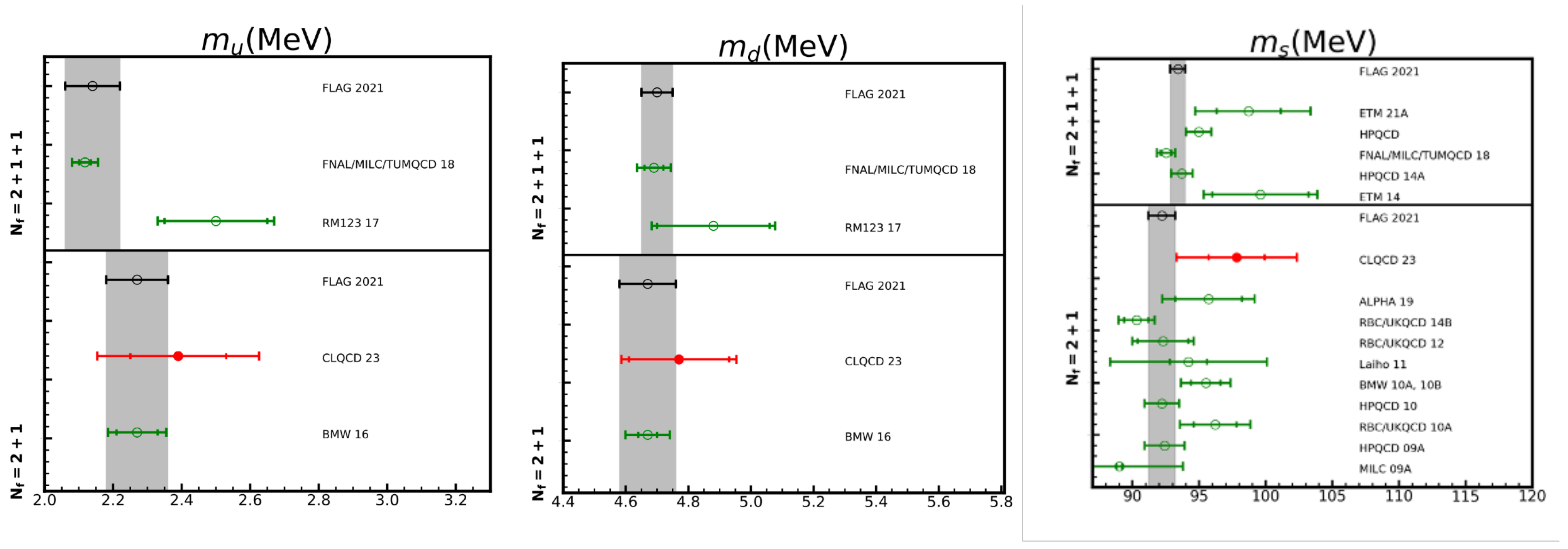
Based on the $a^2 + a^4$ extrapolation:

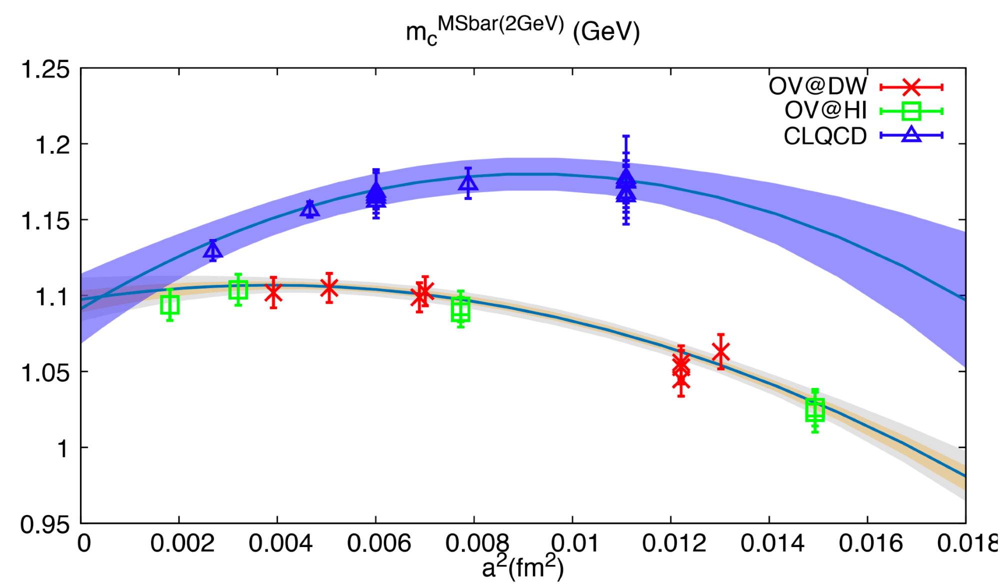
- The prediction based on the Overlap fermion (χ QCD) and also Clover fermion (CLQCD) agrees within 1-2%.
- Such a value is similar to the current lattice averages within ~1%.

Outline

- LQCD background and CLQCD ensembles

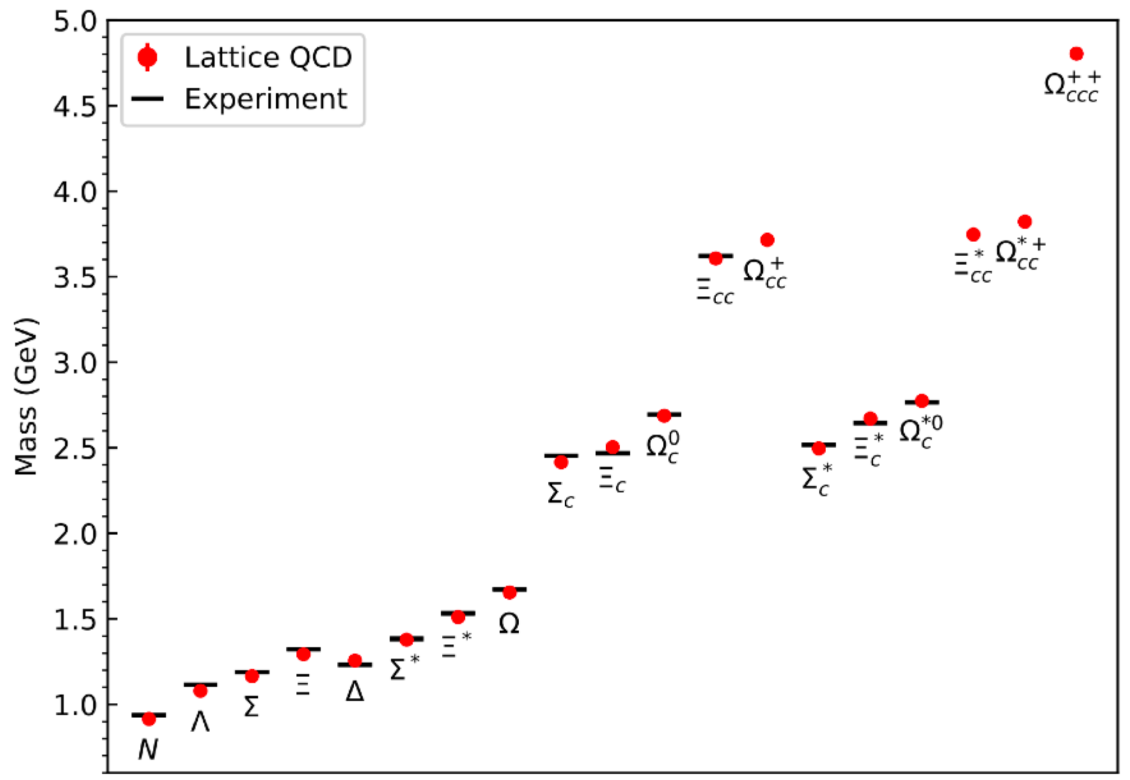


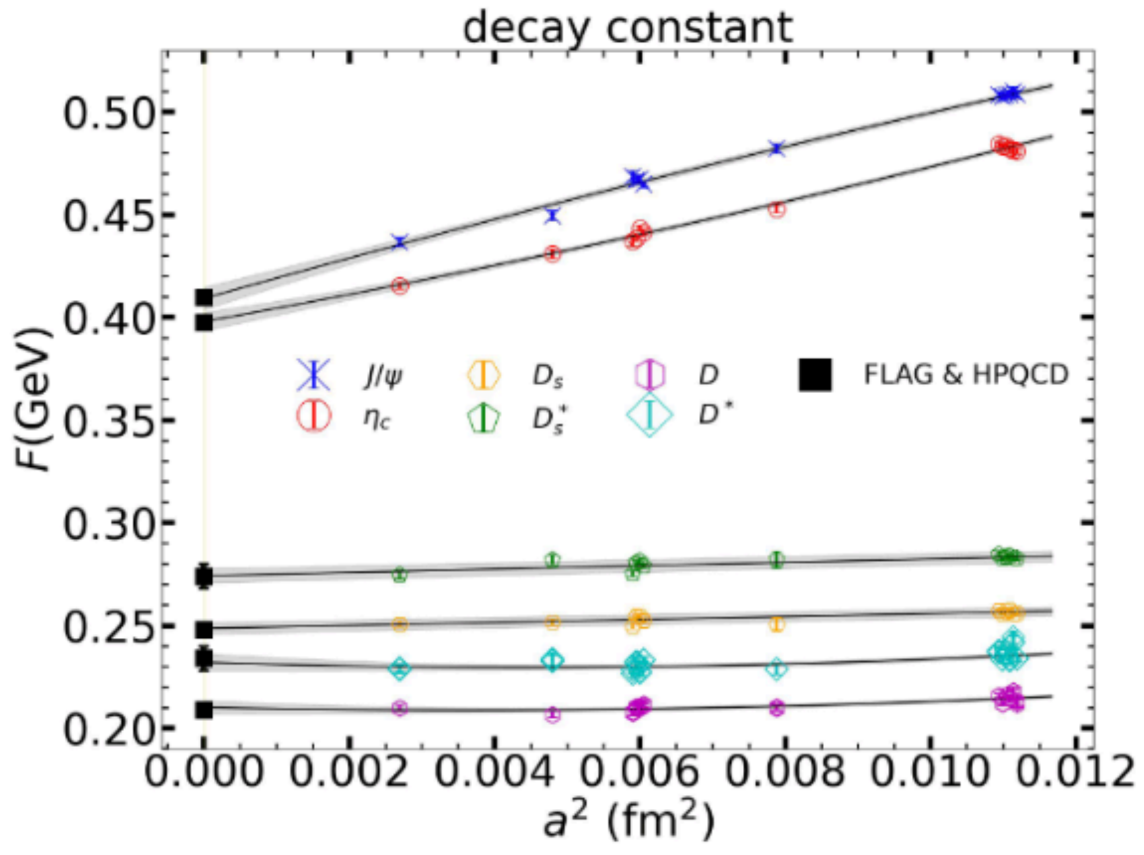




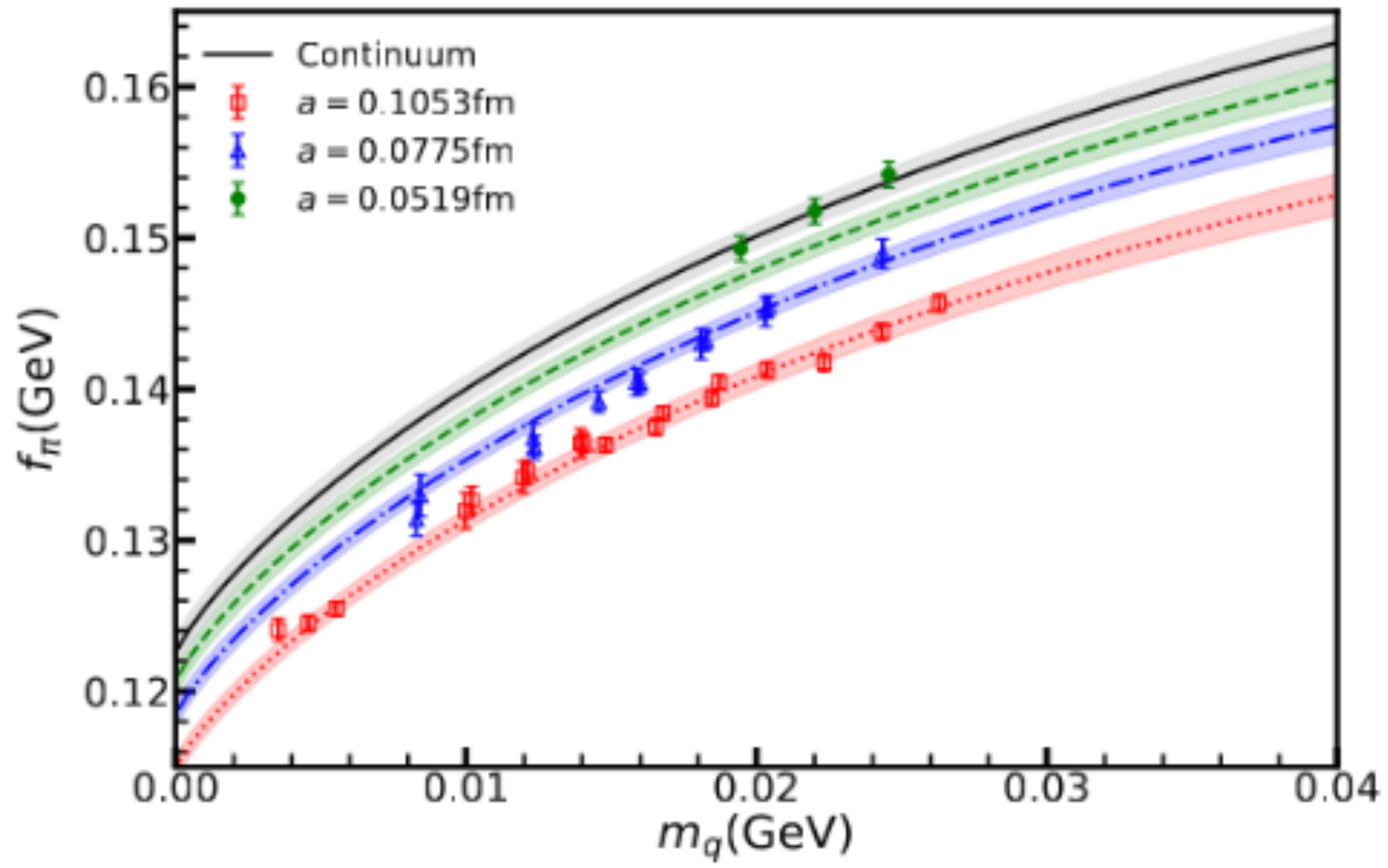
- Quark mass determinations

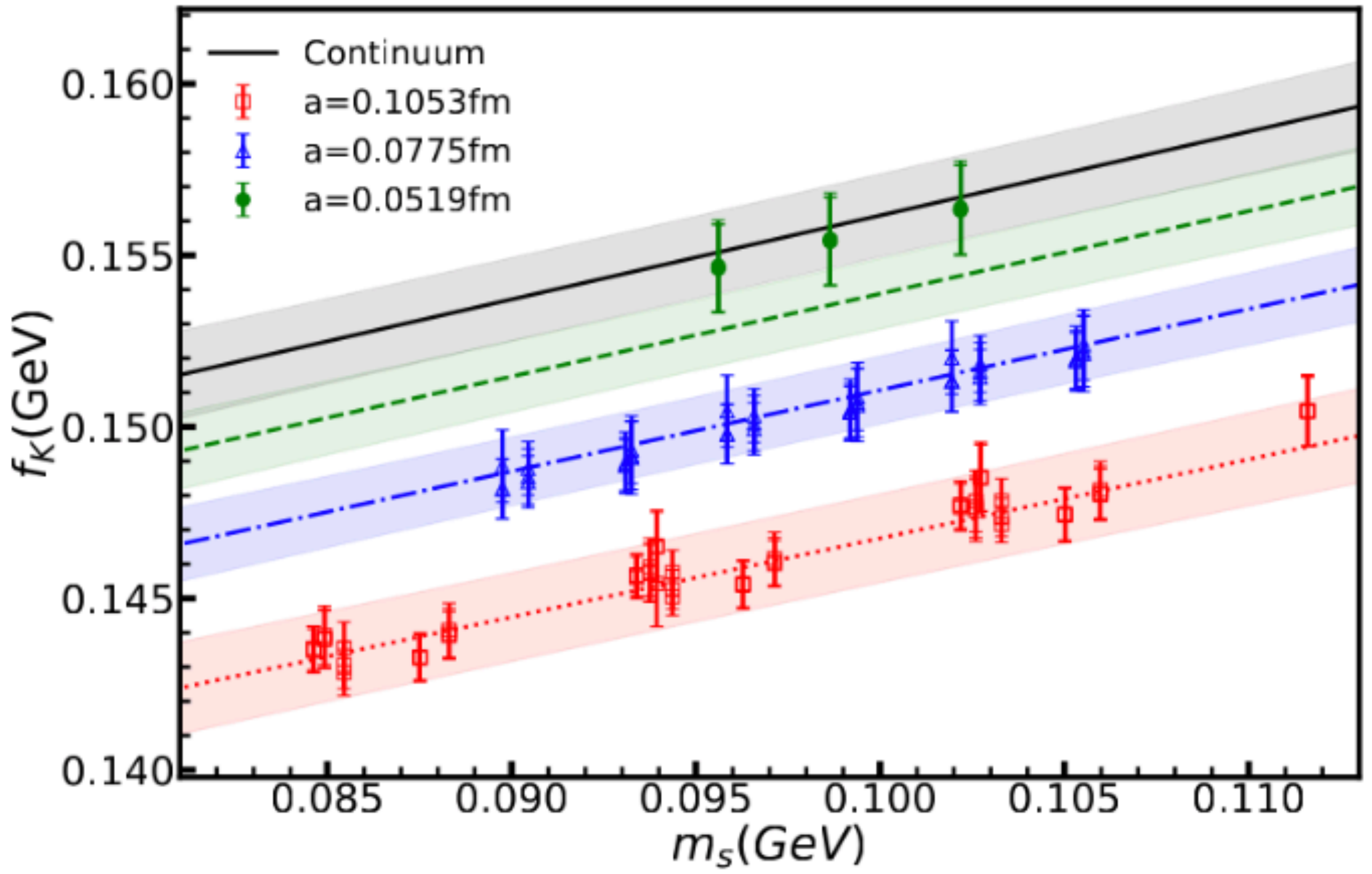
- **Hadron masses and decay constants**





Decay constants





Pion and Kaon cases

$$\frac{f_K}{f_\pi} = 1.1905(68)(15)$$

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\rm exp}(20)_{\rm th}$$

$$|V_{ud}| = 0.9740(03)_{\rm lat}(01)_{\rm ph}$$

$$|V_{us}| = 0.2265(13)_{\rm lat}(03)_{\rm ph}$$

$$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$$

- Additional input is required to determine $|V_{ud(s)}|$ separately and verify the unitarity of CKM.

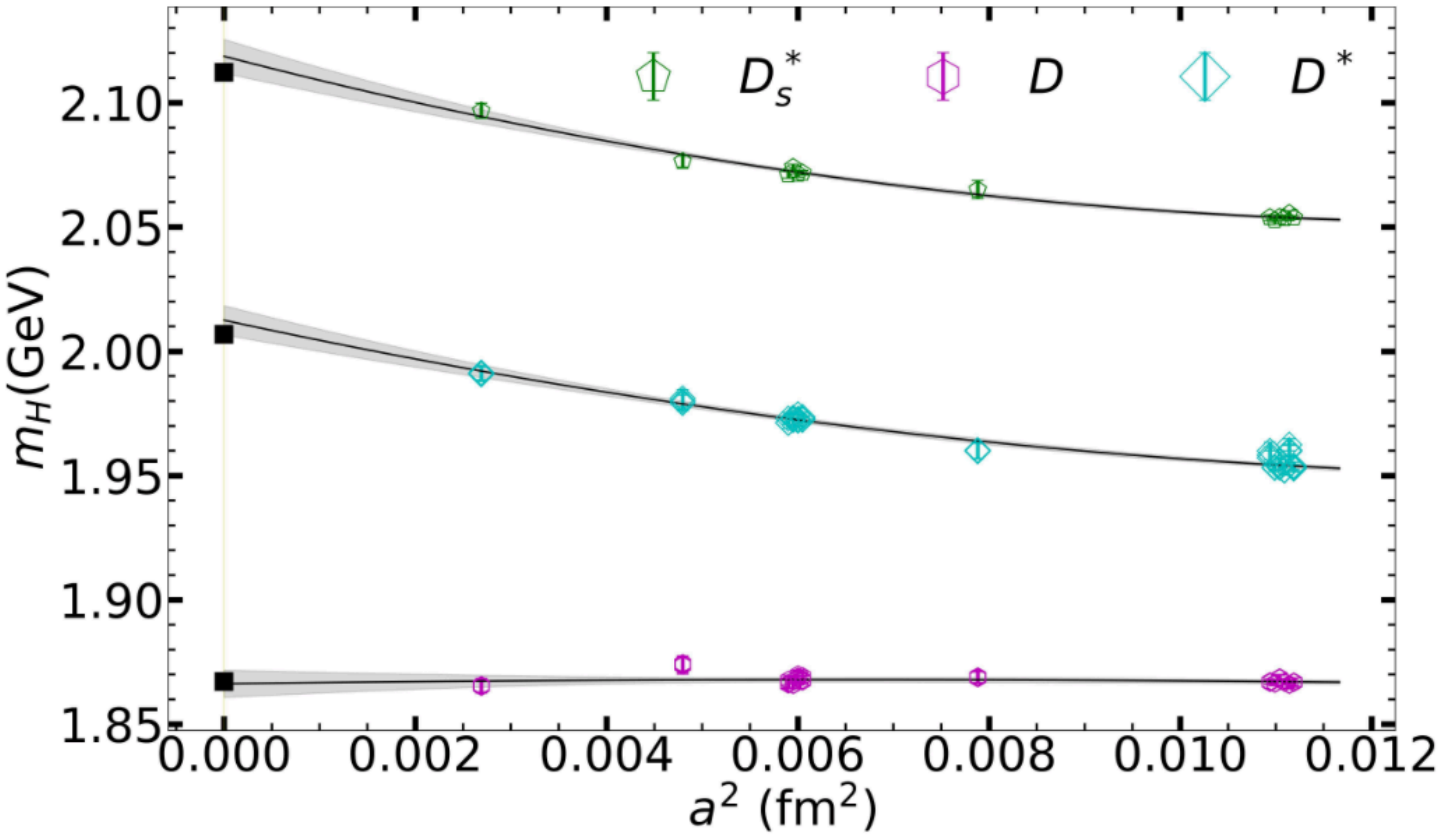
Charmed meson spectrum

Open charm cases

$$m_{D_s}^{\rm QCD} = m_{D_s}^{\rm phys} - \Delta^{\rm QED} m_{D_s} = 1966.7(1.5)\ \mathrm{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- m_D is almost constant at different lattice spacing, with $m_D^{\pm} - m_D^0 = 2.9(3)_{\rm QCD} + 2.4(5)_{\rm QED} = 5.3(3)(5)\ \mathrm{MeV}$;

  RM123, Phys.Rev.D95(2017) 114504

- Agree with the PDG value 4.8(1) MeV well.

- Both m_D^* and $m_{D_s}^*$ have obvious lattice spacing dependence and the continuum extrapolated values agree with PDG well.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

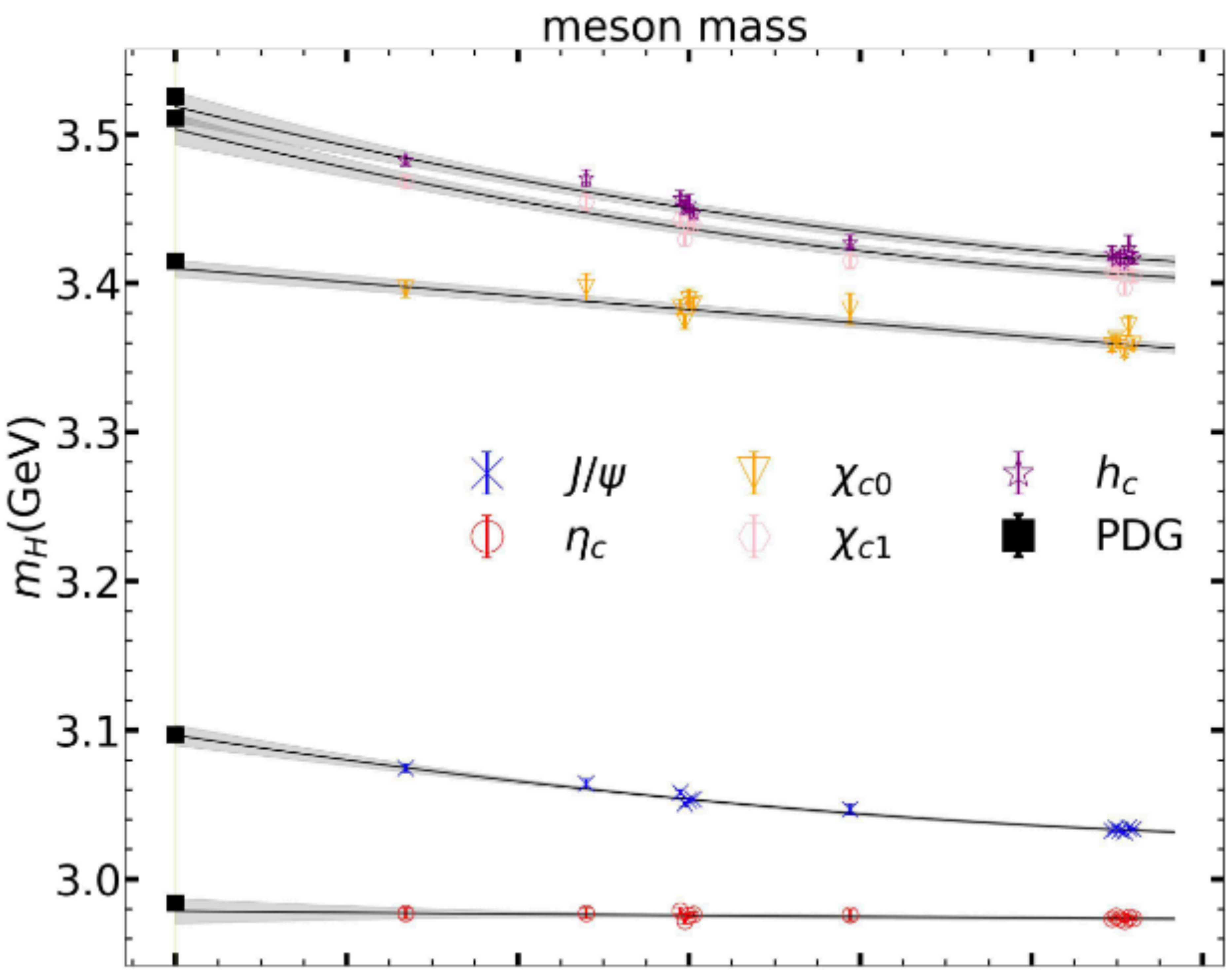
Charmed meson spectrum

charmonium cases

$$m_{D_s}^{\rm QCD} = m_{D_s}^{\rm phys} - \Delta^{\rm QED} m_{D_s} = 1966.7(1.5)\ {\rm MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- $m_{J/\psi}$ agrees with PDG well but m_{η_c} is a few MeV lower;
- $m_{J/\psi} - m_{\eta_c} = 118(3)\ {\rm MeV}$ agree with previous HPQCD pure QCD prediction 119(1) MeV.
- P-wave charmonium masses also agree with PDG well, with $m_{1P} - m_{1S} = 451(11)\ {\rm MeV}$.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

Summary on SM parameters

$m_{\pi^\pm}(m_u, m_d)$ $\longrightarrow$ $(m_u + m_d)^{\overline{\rm MS}(2\ {\rm GeV})}/2 = 3.60(19)\ {\rm MeV}$

$m_{\pi^0} = 134.98\ {\rm MeV}$ $\longrightarrow$ $(m_u + m_d)^{\overline{\rm MS}(2\ {\rm GeV})}/2$

$w_0 = 0.1736(9)\ {\rm fm}$ $\longrightarrow$ $(m_u + m_d)^{\overline{\rm MS}(2\ {\rm GeV})}/2 = 3.60(19)\ {\rm MeV}$

$(m_u + m_d)^{\overline{\rm MS}(2\ {\rm GeV})}/2 = 3.60(19)\ {\rm MeV}$ $\longrightarrow$

$$m_u^{\overline{\rm MS}(2\ {\rm GeV})} = 2.45(30)\ {\rm MeV}$$

$$m_d^{\overline{\rm MS}(2\ {\rm GeV})} = 4.74(14)\ {\rm MeV}$$

$$m_s^{\overline{\rm MS}(2\ {\rm GeV})} = 98.8(5.5)\ {\rm MeV}$$

$m_{K^0}^{\rm QCD} = 497.44(02)\ {\rm MeV}$

$m_{K^\pm}^{\rm QCD} = 491.44(15)\ {\rm MeV}$ $\longrightarrow$ (m_u, m_d, m_s)

$m_K(m_u^{val}, m_d^{val}, m_l^{sea}, m_s^{sea})$ $\longrightarrow$ (m_u, m_d, m_s)

(m_u, m_d, m_s) $\longrightarrow$ $m_c^{\overline{\rm MS}(m_c)} = 1289(17)(01)\ {\rm MeV}$

$m_{D_s}^{\rm QCD} = 1966.7(1.5)\ {\rm MeV}$ $\longrightarrow$ $m_c^{\overline{\rm MS}(m_c)} = 1289(17)(01)\ {\rm MeV}$

$m_{D_s}(m_c^{val}, m_s^{val}, m_l^{sea}, m_s^{sea})$ $\longrightarrow$ $m_c^{\overline{\rm MS}(m_c)}$

$$\frac{f_K}{f_\pi} = 1.1905(70)_{\rm lat}$$

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\rm exp}(20)_{\rm th} \longrightarrow$$

$$|V_{ud}| = 0.9740(03)_{\rm lat}(01)_{\rm ph}$$

$$|V_{us}| = 0.2265(13)_{\rm lat}(03)_{\rm ph}$$

$$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$$

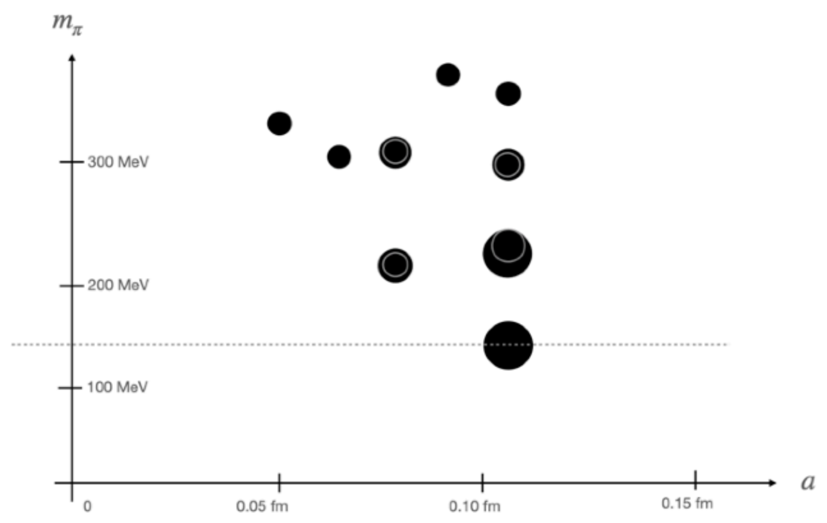
$$f_{D^+} = 0.2102(33)_{\rm lat}\ {\rm MeV}$$

$$f_{D^+}|V_{cd}| = 45.8(1.1)_{\rm exp}\ {\rm MeV} \longrightarrow |V_{cd}| = 0.2179(33)_{\rm lat}(52)_{\rm exp}$$

$$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\rm exp}\ {\rm MeV} \longrightarrow |V_{cs}| = 0.979(11)_{\rm lat}(11)_{\rm exp}$$

$$f_{D_s^+} = 0.2487(28)_{\rm lat}\ {\rm MeV}$$

Baryon spectrum

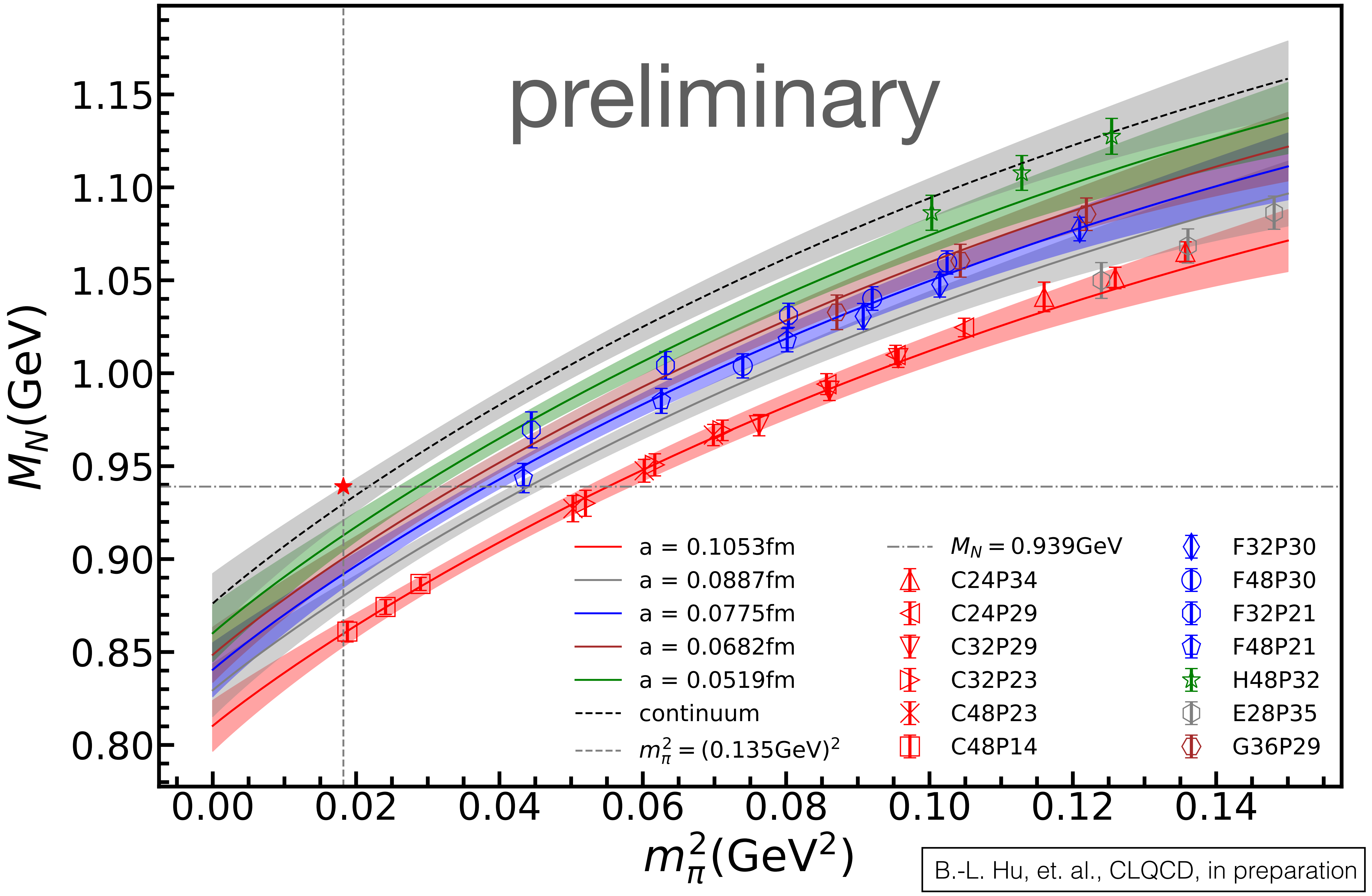


Parameter	Value
M_0	$0.876(16)$
C_1	$2.13(39)$
C_2	$1.39(59)$
C_3	$-6.77(57)$
C_4	$1.85(49)$
C_5	$0.92(38)$
g_A	$0.99(27)$
g_1	$-0.03(51)$
M_{phys}	$0.9296(91)$
χ^2	0.73
Q	0.86

- Sigma term based on FH theorem:
  $\sigma_{\pi N} \equiv m_l \langle p \,|\, \bar{u}u + \bar{d}d \,|\, p \rangle = m_l \frac{\partial M_N}{\partial m_l}$
  =48.8(6.4) MeV;
- Previous Overlap result based on FH theorem:
  $\sigma_{\pi N} = 52(8)$ MeV;
- Previous Overlap result based on direct ME calculation:
  $\sigma_{\pi N} = 46(7)$ MeV.

Nucleon case

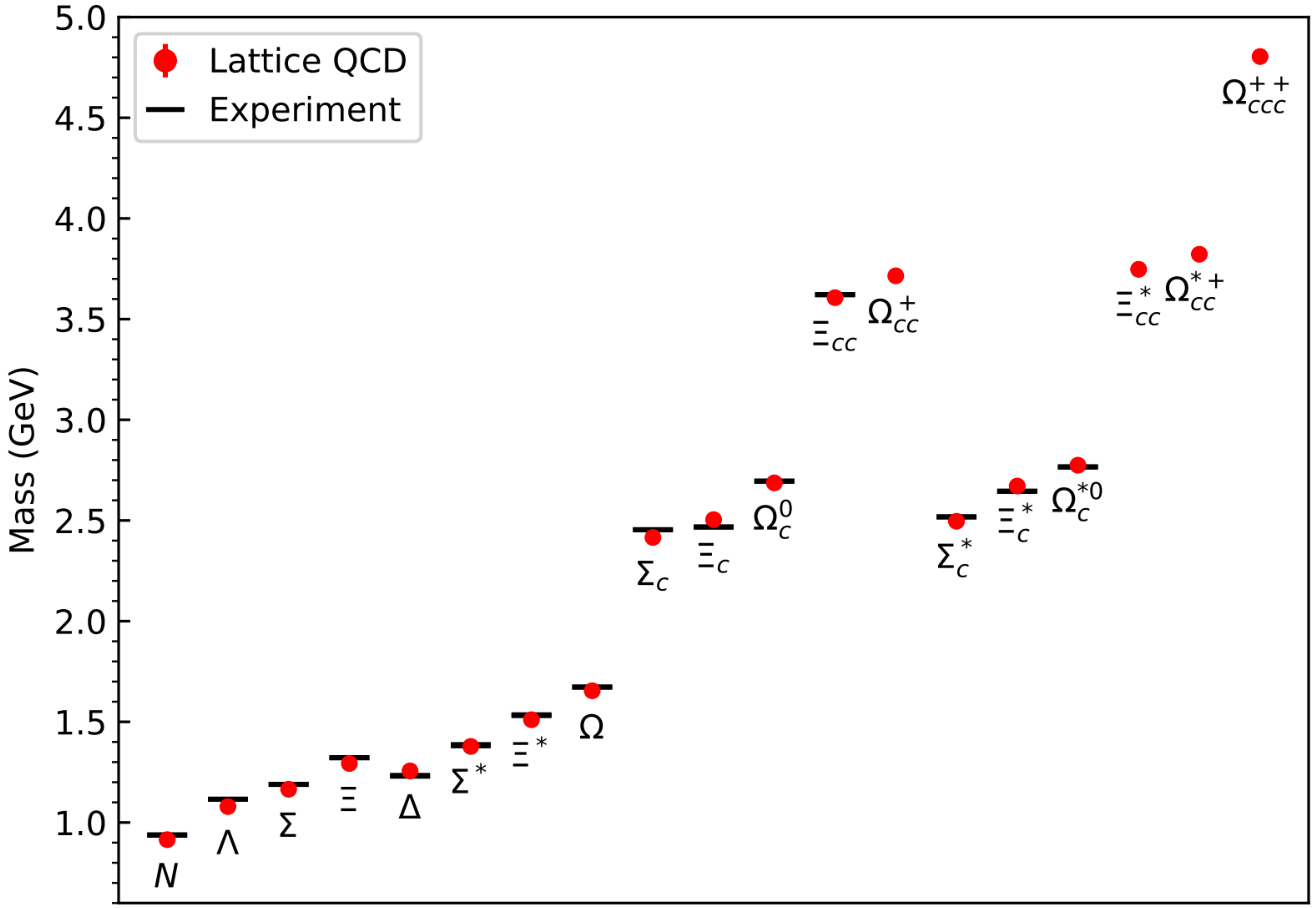
$$M\left(m_\pi^v, m_\pi^{\text{sea}}, m_s^{\text{sea}}, a, L\right) = \Bigg[M_0 + C_1\left(m_\pi^v\right)^2 + C_2\left(m_\pi^{\text{sea}}\right)^2 - \frac{\left(g_A^2 - 4g_Ag_1 - 5g_1^2\right)\pi}{3\left(4\pi f_\pi\right)^2}\left(m_\pi^v\right)^3 - \frac{\left(8g_A^2 + 4g_Ag_1 + 5g_1^2\right)\pi}{3\left(4\pi f_\pi\right)^2}\left(m_\pi^{pq}\right)^3 + C_4\frac{\left(m_\pi^v\right)^2}{L}e^{-m_\pi^v L} + C_5(m_s^{\text{sea}} - m_s^{\text{phys}})\Bigg](1 + C_3 a^2),$$



B.-L. Hu, et. al., CLQCD, in preparation

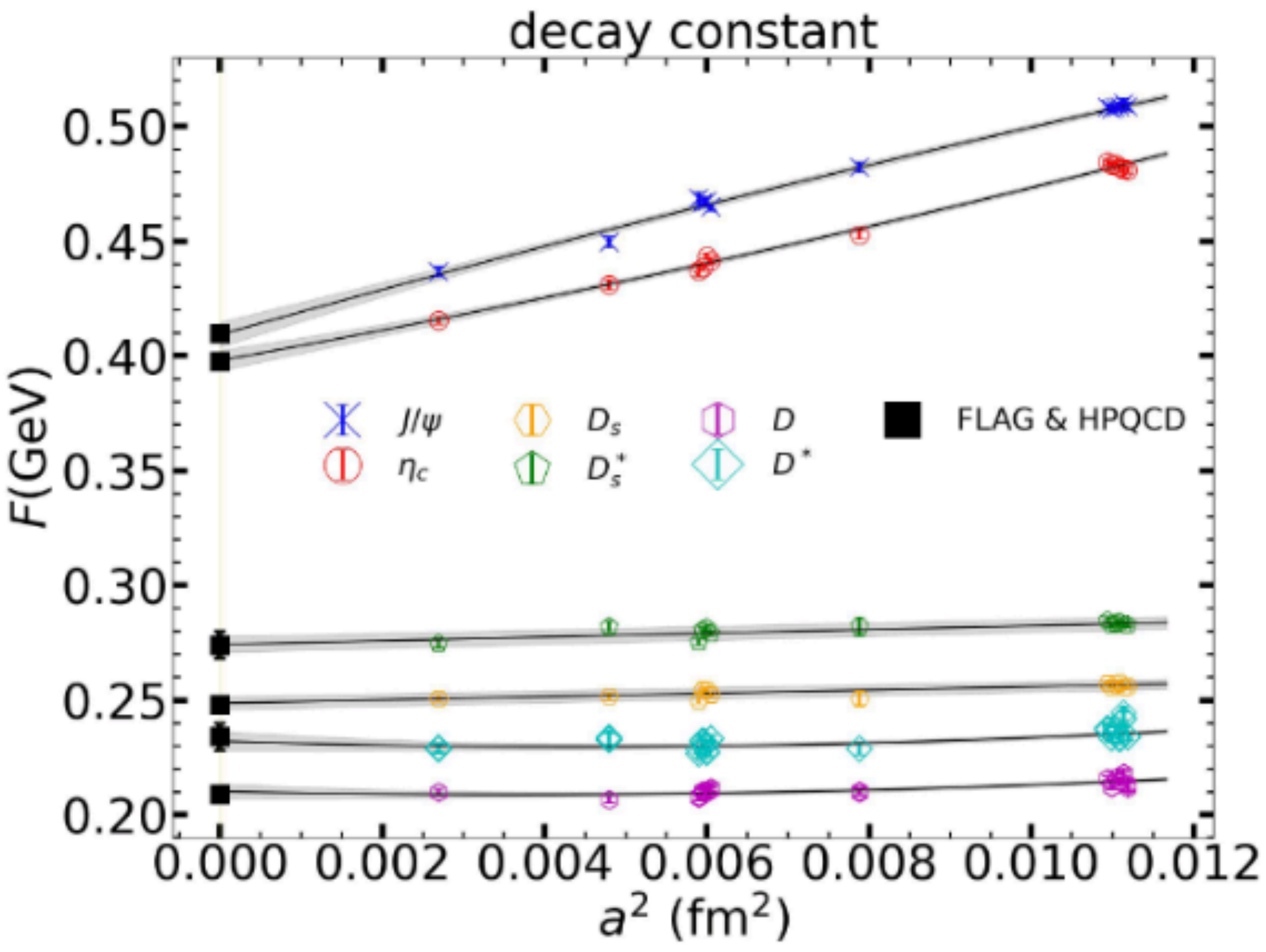
Baryon spectrum

of four light flavors



- Generally agree with the PDG values at 1% level;
- Mass difference of the light Octet and decuplet baryons comes majorly the trace anomaly which is ~300 MeV;
- Trace anomaly contribution to the charmed baryon is under investigation.
- The missing QED effect will be investigated in the near future.

Decay constants



Open charm cases

$f_{D^+} = 0.2102(33)_{\text{lat}}$ MeV

↓

$f_{D^+}|V_{cd}| = 45.8(1.1)_{\text{exp}}$ MeV ⟶ $|V_{cd}| = 0.2179(33)_{\text{lat}}(52)_{\text{exp}}$

$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\text{exp}}$ MeV ⟶ $|V_{cs}| = 0.979(11)_{\text{lat}}(11)_{\text{exp}}$

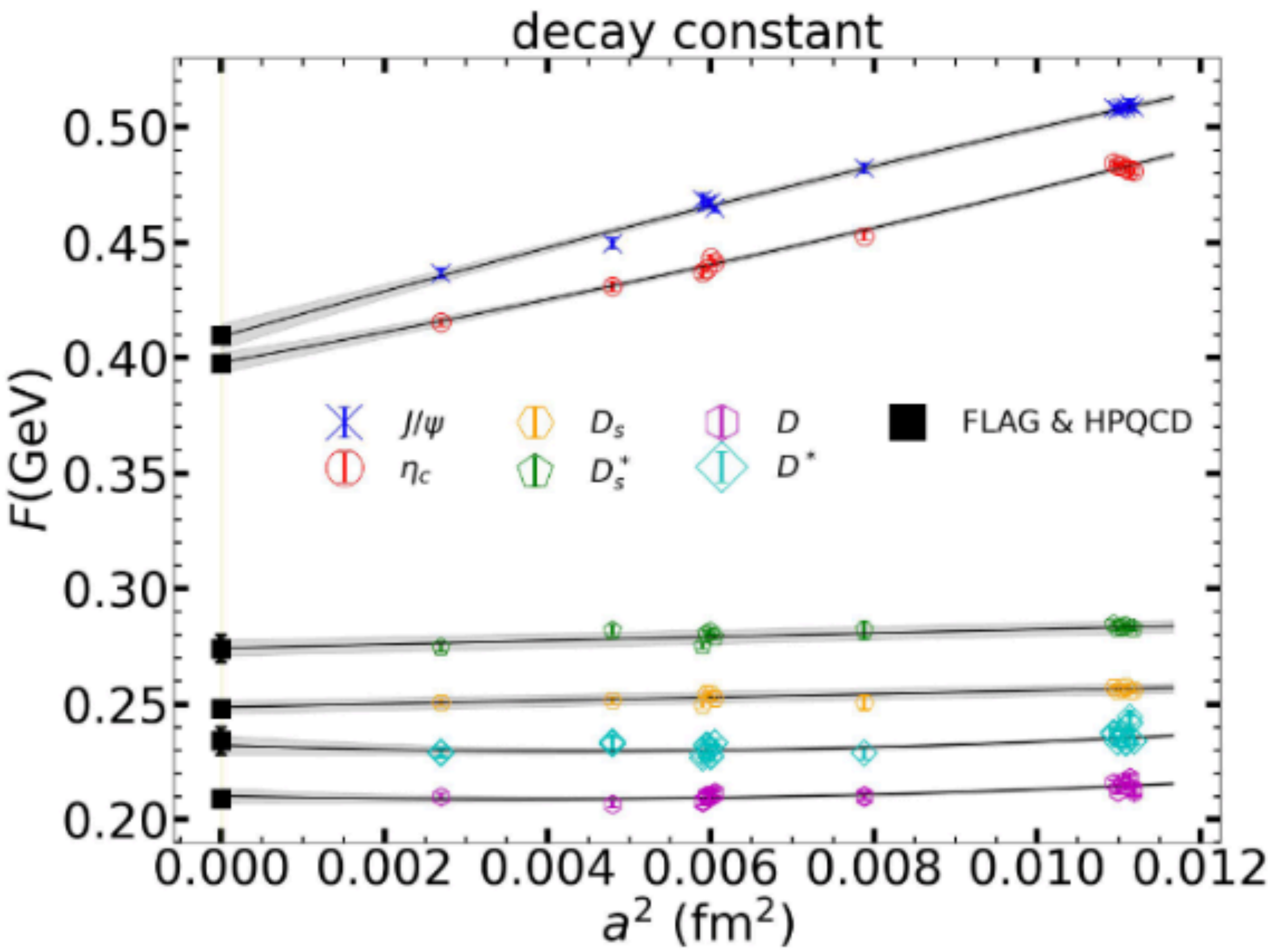
↑

$f_{D_s^+} = 0.2487(28)_{\text{lat}}$ MeV

- Verified the unitarity of CKM matrix elements involving the charm quark: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.008(23)(23)$.
- Also provide the most precise f_{D^*} and $f_{D_s^*}$ so far.

Decay constants

S-wave charmonium

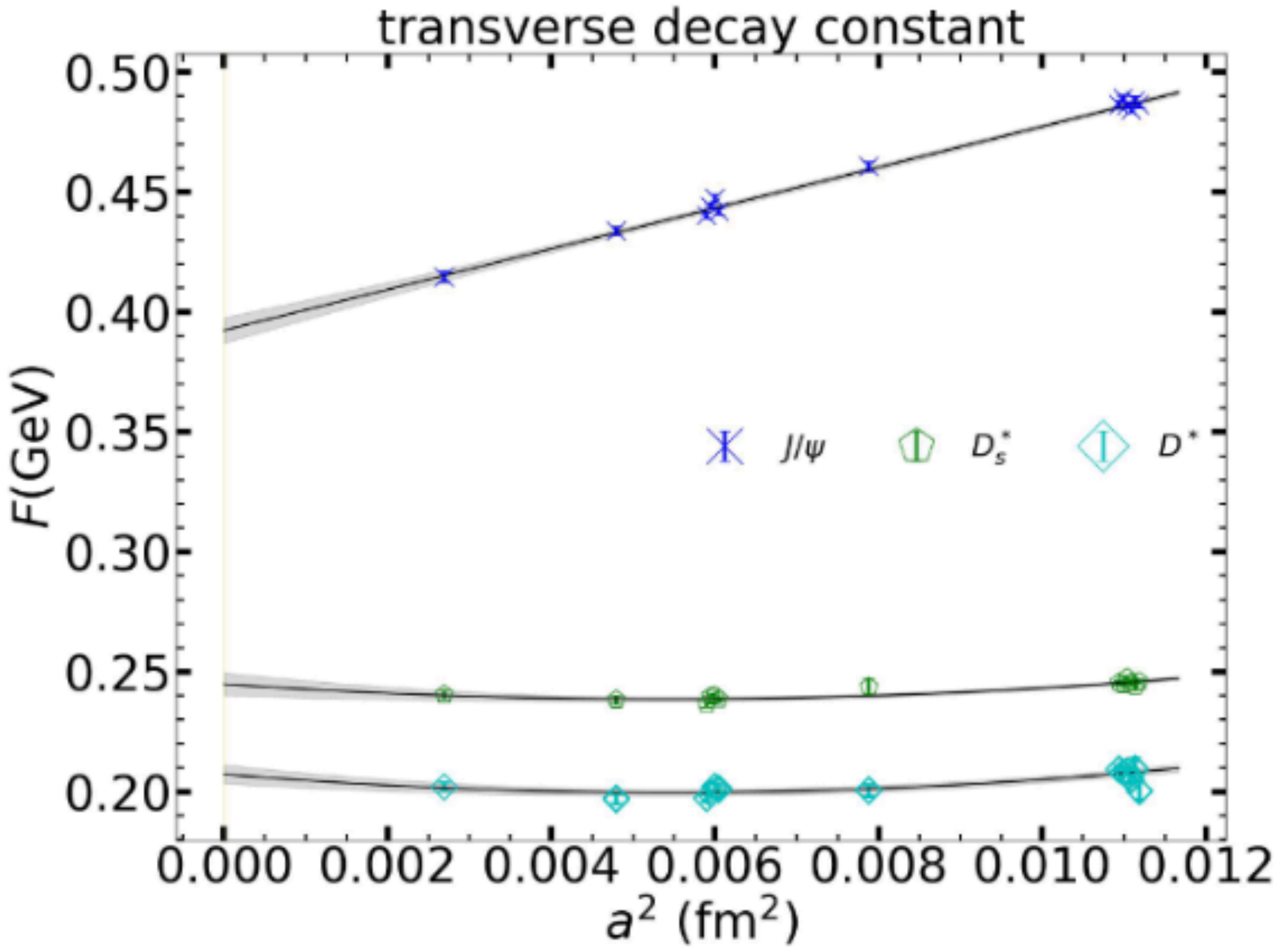


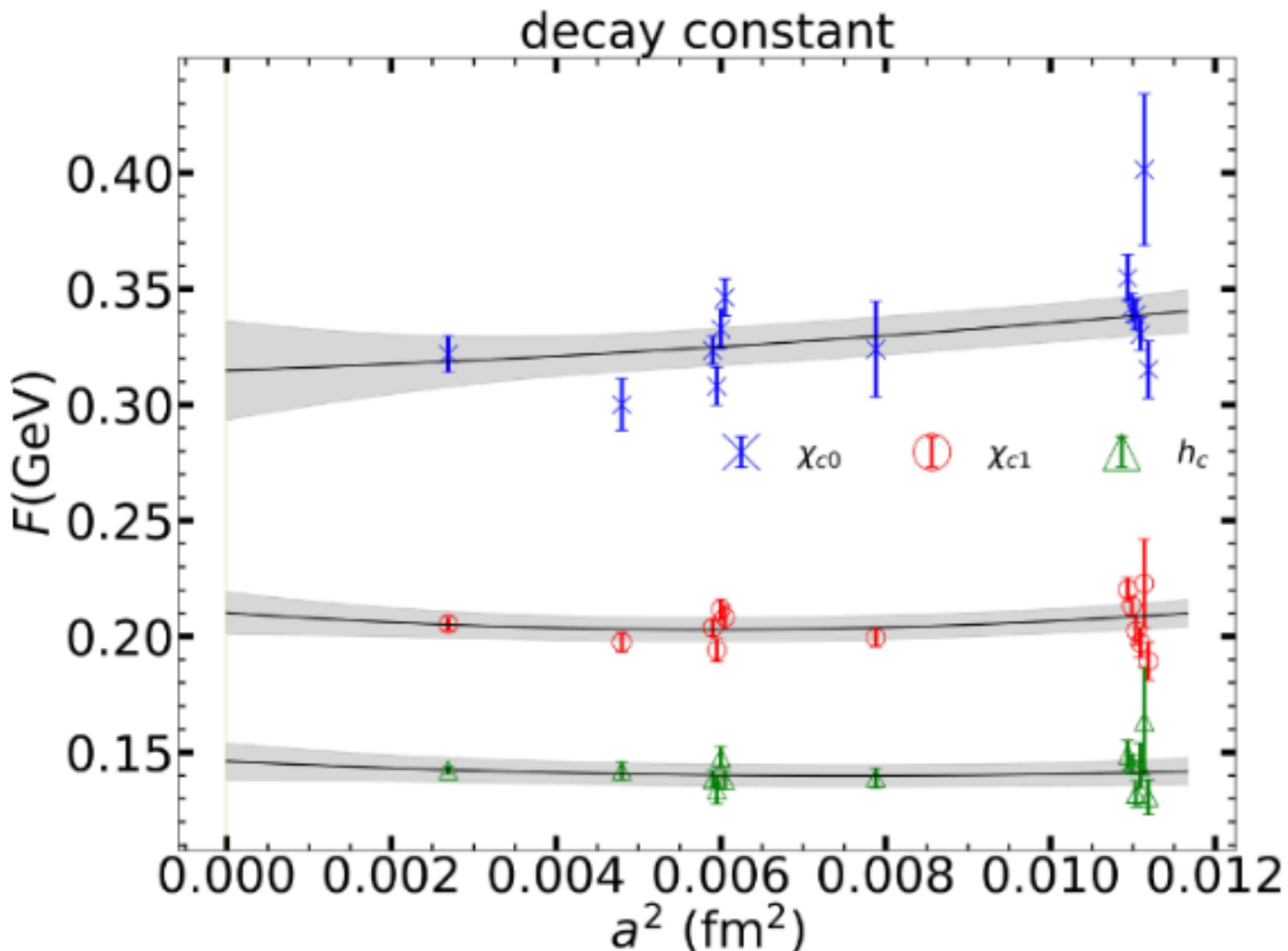
- Our prediction $f_{J/\psi} = 405.9(5.7)$ MeV is consistent with the experimental value 406.5(3.7)(0.5) MeV and also HPQCD prediction 409.6(1.6) MeV;
- We also predict $f_{\eta_c} = 398.1(4.6)$ MeV which is consistent with the HPQCD prediction 397.5(1.0) MeV.

Decay constants

Other decay constants

- We also predict the transverse decay constant of the charmed vector mesons and also the decay constant of the P-wave charmonium, which can be verified by the future experiments.





Summary

- The state-of-the-arts Lattice QCD ensemble should have enough ensembles to approach the continuum, infinite volume and physical quark masses reliably; and the present CLQCD ensembles have been close to this goal.
- Up, down, strange and charm quark masses have been determined at a few percent level;
- The charmed meson and baryon masses are predicted at ~0.3% uncertainty and agree with the experimental values at 1% level.
- More predictions are in progress.