Pion Photoproduction and Resonance Structure

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Hadron Physics

mainly focused on hadron scatterings, spectra, structures, interactions, etc.

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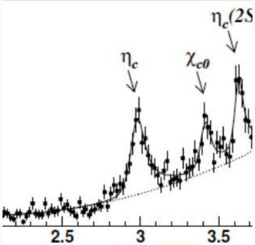

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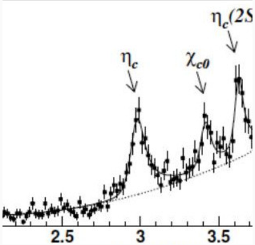

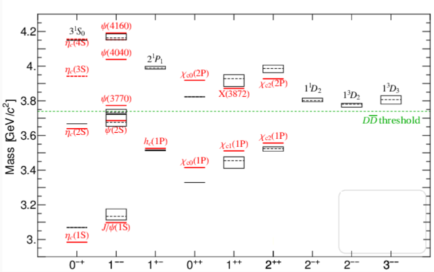

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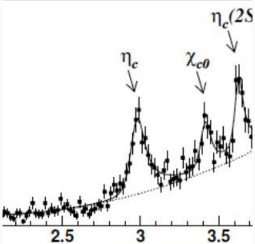

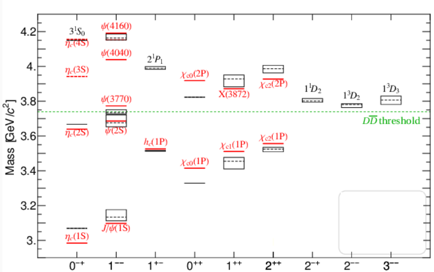

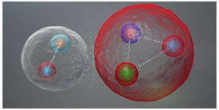

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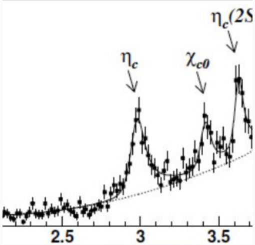

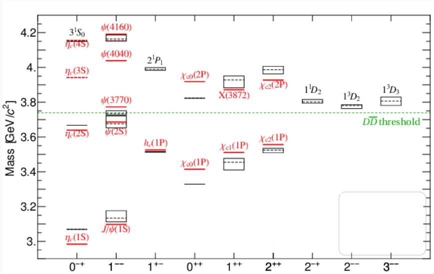

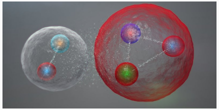

traditional perturbation expansion in series of $(\alpha_s)^n$?

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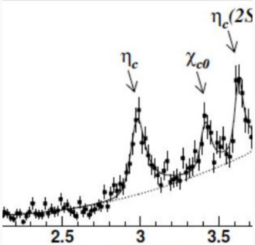

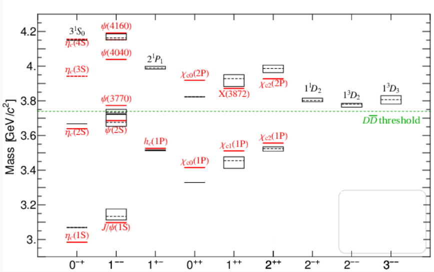

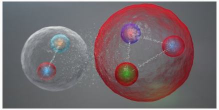

traditional perturbation expansion in series of $(\alpha_s)^n$?

- constituent quark model
- effective field theory
- lattice QCD
- QCD sum rule
- large Nc
- ...

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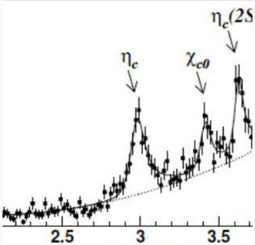

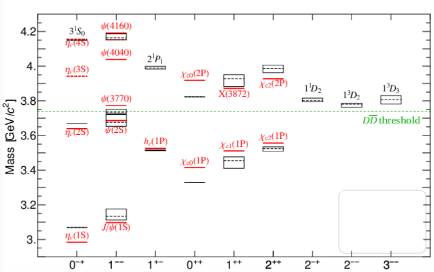

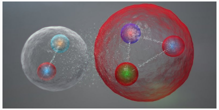

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Lattice QCD

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions

  at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

Connection between Scattering Data and Lattice QCD Data

Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

Lattice QCD Data → Physical Data

- Lüscher Formalisms and extensions:
  Model independent; efficient in single-channel problems
    Spectrum → Phaseshifts;
- Effective Field Theory (EFT), Models, etc
  with low-energy constants fitted by Lattice QCD data

Physical Data → Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

Lattice QCD and Effective Field Theory

Effective field theory deals with extrapolation powerfully.

Finite-volume effect can be studied by discretizing the EFT.

Discrete spacing effects can also be studied with EFT.

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

Hamiltonian Effective Field Theory

Hamiltonian Effective Field Theory (HEFT)

analyses both experimental data at infinite volume

and lattice QCD results at finite volume at the same time.

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  wavefunctions: analyse the structure of the eigenstates on the lattice

- finite-volume and infinite-volume results are connected by the coupling constants etc.

Odd-parity low-lying nucleon excitations with pion photoproduction

$N^*(1535)$ with πN Scattering

$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions

  among the bare baryon N_0^*, πN channel, and ηN channel.

$$G^2_{\pi N; N_0^*}(k) = \frac{3 g^2_{\pi N; N_0^*}}{4\pi^2 f^2} \omega_\pi(k)$$

$$V^S_{\pi N, \pi N}(k, k') = \frac{3 g^S_{\pi N}}{4\pi^2 f^2} \frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)} \frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')}$$

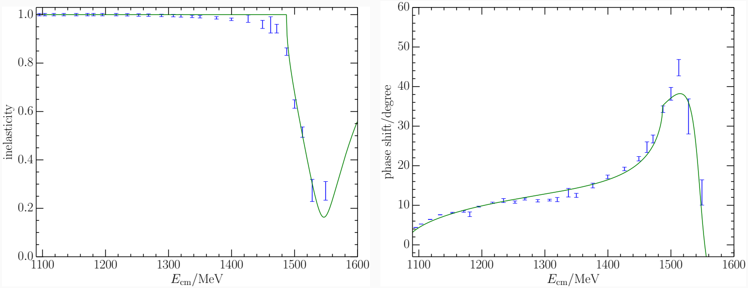
- Phase shifts and inelasticities

  are obtained by solving Bethe-Salpeter equation with the interactions.

$$T_{\alpha,\beta}(k, k'; E) = V_{\alpha,\beta}(k, k') + \sum_\gamma \int q^2 dq$$

$$V_{\alpha,\gamma}(k, q) \frac{1}{E - \sqrt{m^2_{\gamma_1} + q^2} - \sqrt{m^2_{\gamma_2} + q^2} + i\epsilon} T_{\gamma,\beta}(q, k'; E)$$

$N^*(1535)$ with πN scattering at infinite volume



πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

Our Pole: 1531 $\pm$ 29 $-$ i 88 $\pm$ 2 MeV.

Particle Data Group: 1510$\pm$20 $-$ i 85 $\pm$ 40 MeV.

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, Phys. Rev. Lett. 116 (2016) no.8, 082004

Discretization in finite volume

$$H_0 = \mathrm{diag}\{m^0_{N_1}, \omega_{\pi N}(k_0),\ \omega_{\eta N}(k_0), \omega_{\pi N}(k_1),\ \omega_{\eta N}(k_1), ...\},$$

$$H_I = \begin{pmatrix} 0 & \tilde{G}_{\pi N}(k_0) & \tilde{G}_{\eta N}(k_0) & \tilde{G}_{\pi N}(k_1) & \tilde{G}_{\eta N}(k_1) & \dots \\ \tilde{G}_{\pi N}(k_0) & \tilde{V}^S_{\pi N,\pi N}(k_0, k_0) & 0 & \tilde{V}^S_{\pi N,\pi N}(k_0, k_1) & 0 & \dots \\ \tilde{G}_{\eta N}(k_0) & 0 & 0 & 0 & 0 & \dots \\ \tilde{G}_{\pi N}(k_1) & \tilde{V}^S_{\pi N,\pi N}(k_1, k_0) & 0 & \tilde{V}^S_{\pi N,\pi N}(k_1, k_1) & 0 & \dots \\ \tilde{G}_{\eta N}(k_1) & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where

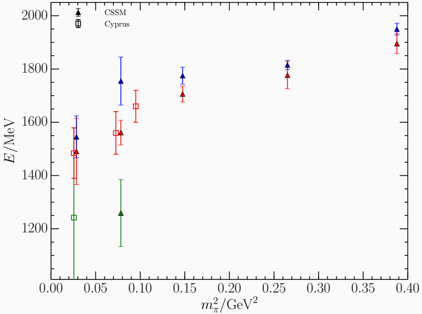
$$\tilde{G}_i(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3/2} G_i(k_n),$$

$$\tilde{V}^S_{i,j}(k_n, k_m) = \frac{\sqrt{C_3(n)C_3(m)}}{4\pi}\left(\frac{2\pi}{L}\right)^3 V^S_{i,j}(k_n, k_m).$$

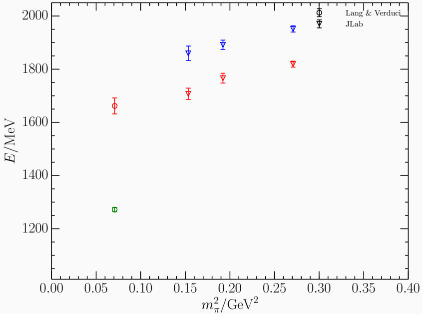
$C_3(n)$ represents the number of summing the squares of three integers to equal n.

Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes



$L \approx 3$ fm

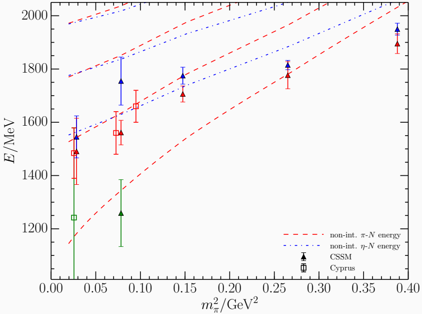


$L \approx 2$ fm

N^* Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



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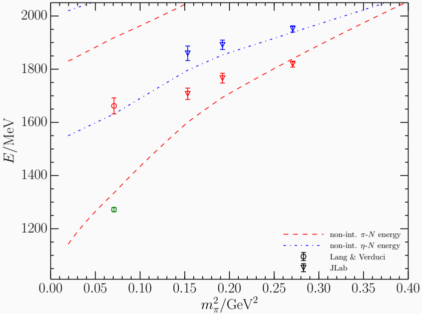

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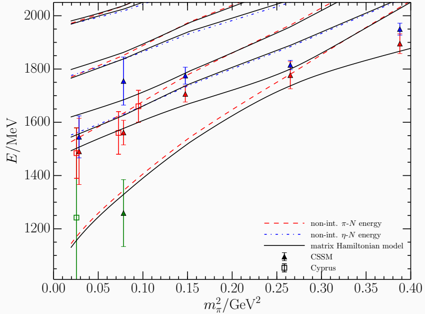
Eigenenergies of Hamiltonian effective field theory



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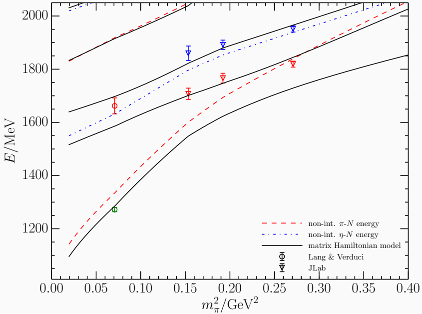

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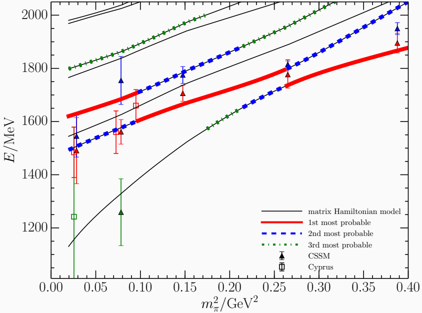
Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes

Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD

We not only provide the mass but also analyze why some states are observed on the lattice



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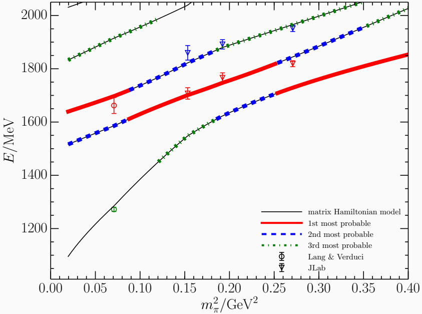
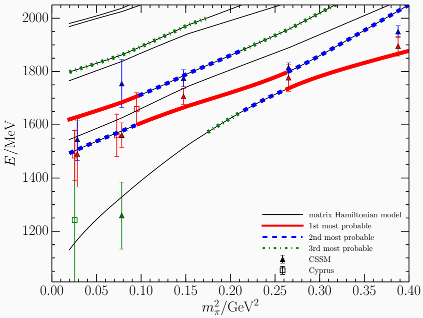


$L \approx 2$ fm

N^* Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Components of Eigenstates with $L \approx 3$ fm

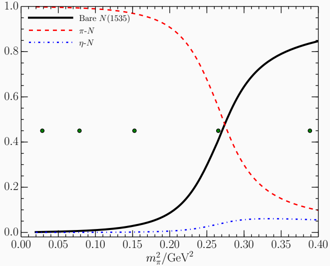


N^* Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

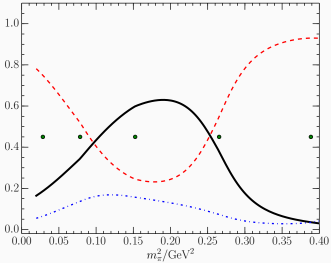
- The 1st eigenstate at light quark masses is mainly πN scattering states.
- The most probable state at physical quark mass is the 4th eigenstate.

  It contains about 60% bare $N^*(1535)$, 20% πN and 20% ηN.

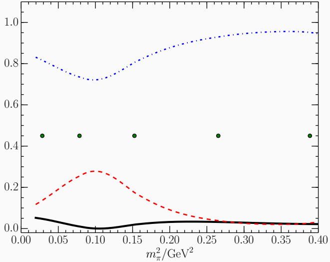
Components of Eigenstates with $L \approx 3$ fm



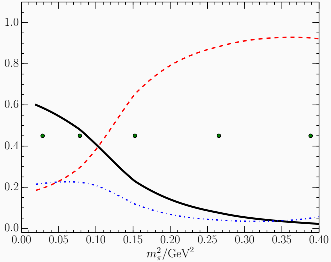
1st eigenstate



2nd eigenstate



3rd eigenstate



4th eigenstate

Pion Photoproduction off Nucleon with Hamiltonian EFT

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- combining
  - $\pi N \to \pi N$
  - lattice QCD data
  - $\gamma + N \to \pi + N$

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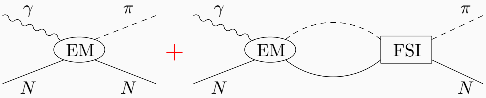

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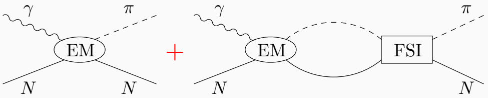

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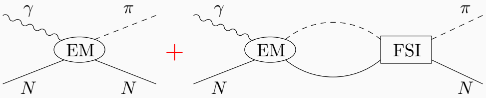

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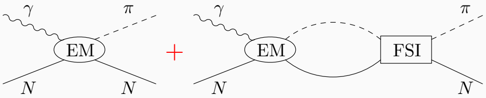

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  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance

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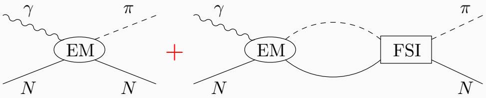

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  - Finite State Interaction (FSI) part has been determined previously
  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance
  - necessities for the photon-nucleus investigation

Electromagnetic Multipoles

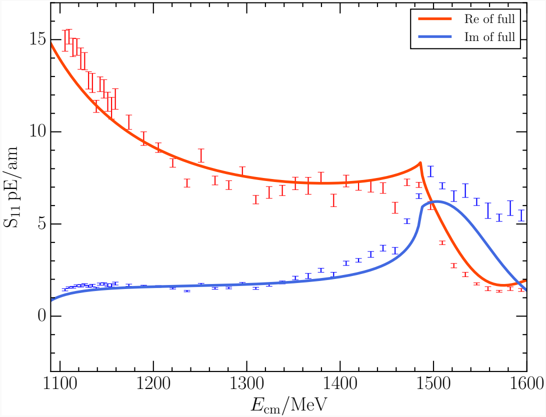
- $|\gamma N\rangle \rightarrow |\phi(\vec{k}),\ N(-\vec{k}, s_z'^N)\rangle$, $\quad k_x, k_y, k_z, s_z'^N$
- $|\gamma N\rangle \rightarrow |\phi\, N; k, J, J_z, L\rangle$, $\quad k,\ J,\ J_z,\ L$
- $|\gamma N\rangle \rightarrow |\phi\, N; k, J, J_z, \lambda_N'\rangle$, $\quad k,\ J,\ J_z, \lambda_N'$

$$V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q) = 2\pi \int_{-1}^{1} \mathrm{d}(\cos\theta) \sum_{s_z'^N} d^J_{\lambda_\gamma-\lambda_N,-\lambda_N'}(\theta) d^{1/2}_{s_z'^N,-\lambda_N'}(\theta)^* \mathcal{M}_{\alpha,\gamma N}(s_z'^N, \lambda_N, \lambda_\gamma; \vec{k}, \vec{q}),$$

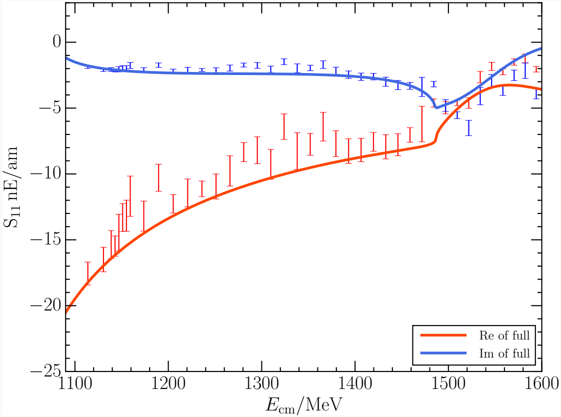
$$V^{JLS;\lambda_\gamma\lambda_N}_{\alpha,\gamma N}(k,q) = \sqrt{\frac{2L+1}{2J+1}} \sum_{\lambda_N'} \langle L, S, 0, -\lambda_N' | J, -\lambda_N' \rangle \times V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q).$$

D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

Electric dipole amplitudes E_{0+}



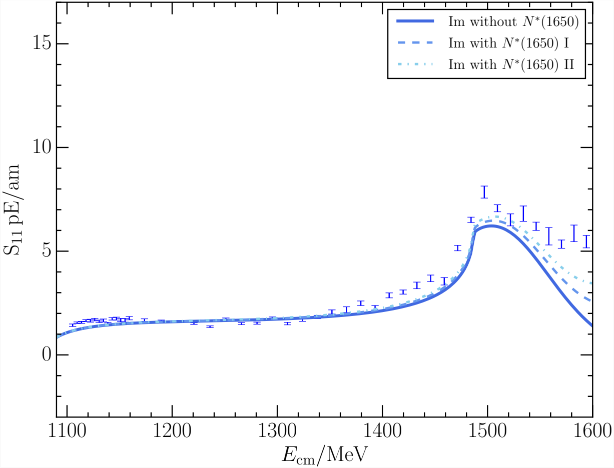
$\gamma p \to \pi N$



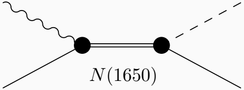
$\gamma n \to \pi N$

D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

Estimation of the $N^*(1650)$ contribution



Estimation:



+u channel

Couplings for the effective vertices are extracted from the decay widths of $N^*(1650)$.

D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

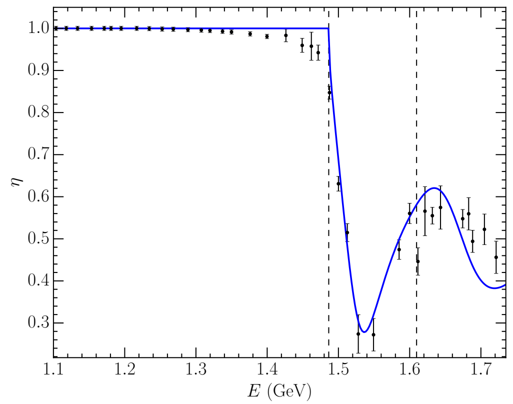
Therefore, we updated our results by

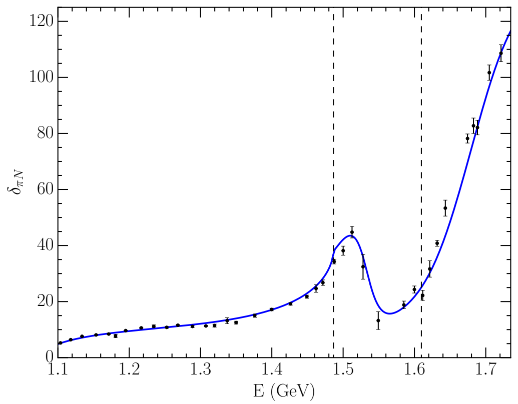
Explicitly including $N^*(1650)$ as well as $N^*(1535)$

Explicitly including $N^*(1650)$ as well as $N^*(1535)$

In Phys. Rev. D 108 (2023) 9, 094519, we consider

- two bare baryon states N_1 and N_2;
- πN, ηN, and $K\Lambda$;
- more experimental data with larger energies (1.60, 1.75) GeV.





Pole positions for $N^*(1535)$ and $N^*(1650)$

In the Particle Data Group (PDG) tables, the poles for the two low-lying odd-parity nucleon resonances are given as

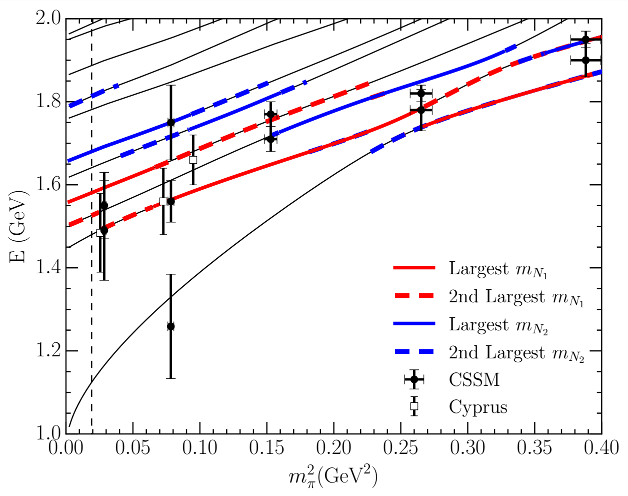
$$E_{N^*(1535)} = 1510 \pm 10 - (65 \pm 10)i \text{ MeV},$$
$$E_{N^*(1650)} = 1655 \pm 15 - (67 \pm 18)i \text{ MeV}.$$

Using HEFT, two poles for $N^*(1535)$ and $N^*(1650)$ in the second Riemann sheet are found at energies

$$E_1 = 1500 - 50i \text{ MeV},$$
$$E_2 = 1658 - 56i \text{ MeV}.$$

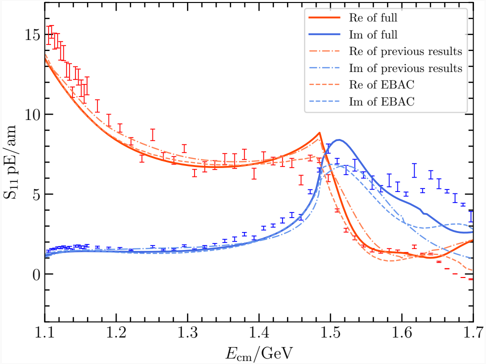
Our results are in excellent agreement with the PDG pole positions.

Finite-volume spectrum



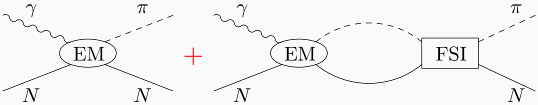
C. D. Abell, D. B. Leinweber, Z.-W. Liu, A. W. Thomas, J.-J. Wu, PRD 108 (2023) 9,094519

Electric dipole amplitudes E_{0+} with two bare states

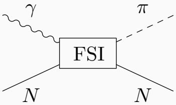


Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, arXiv: 2407.05334.

The bare core in $N^*(1535)$



- If $N^*(1535)$ has no bare core, it would play roles ONLY in finite state interaction



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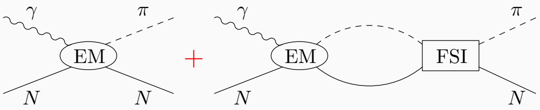

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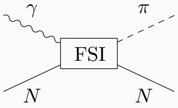
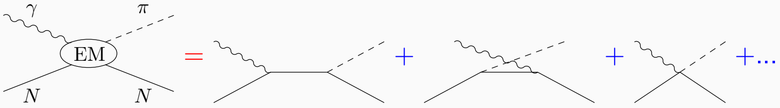
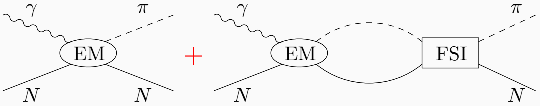


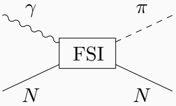
- If with bare core, $N^*(1535)$ also plays roles in electromagnetic potential



The bare core in $N^*(1535)$



- If $N^*(1535)$ has no bare core, it would play roles ONLY in finite state interaction



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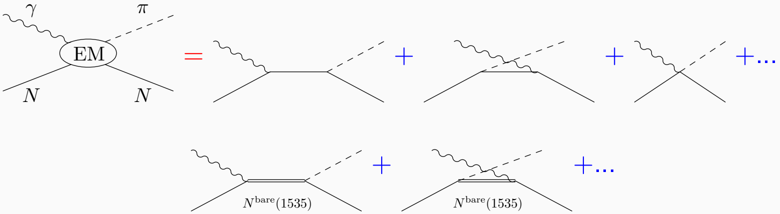
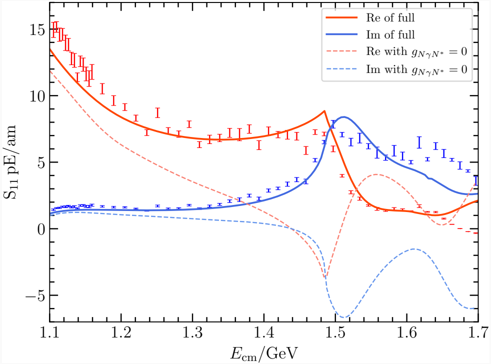


The bare core in $N^*(1535)$ cannot be absent in pion photoproduction



Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, arXiv: 2407.05334.

Low-lying Baryons

Much more scattering data on low-lying baryons, $N^*(1440)$, $N^*(1535)$, $\Lambda(1405)$ compared to those for large-mass resonances or charmed hadrons.

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Naive quark model predicts wrong mass order for $N^*(1440)$ & $N^*(1535)$.

IF: harmonic oscillator form for confinement potential

Then: $E = (2n_r + L + 3/2)\omega$

$N^*(1440)$: $n_r = 1,\ L = 0 \quad \Longrightarrow E = 7/2\omega$

$N^*(1535)$: $n_r = 0,\ L = 1 \quad \Longrightarrow E = 5/2\omega$

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Triquark or pentaquark state?

...

$\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

$$v^I = \sum_{\alpha,\beta} \int d^3\vec{k}\, d^3\vec{k}' \, |\alpha(\vec{k})\rangle \, V^I_{\alpha,\beta}(k,k') \, \langle \beta(\vec{k}')| \, ,$$

$$V_{\alpha,\beta}(k,k') = g_{\alpha,\beta} \frac{\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)} \sqrt{\omega_{\beta_M}(k')}}$$

$|\alpha\rangle = |\pi\Sigma\rangle$, $|\bar{K}N\rangle$, $|\eta\Lambda\rangle$, $|K\Xi\rangle$, $|\pi\Lambda\rangle$

- two scenarios: with or without a bare baryon

$$g^I = \sum_{\alpha,B_0} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle \, G^{I\dagger}_{\alpha,B_0}(k) \, \langle B_0| + |B_0\rangle \, G^I_{\alpha,B_0}(k) \, \langle \alpha(\vec{k})| \right\} ,$$

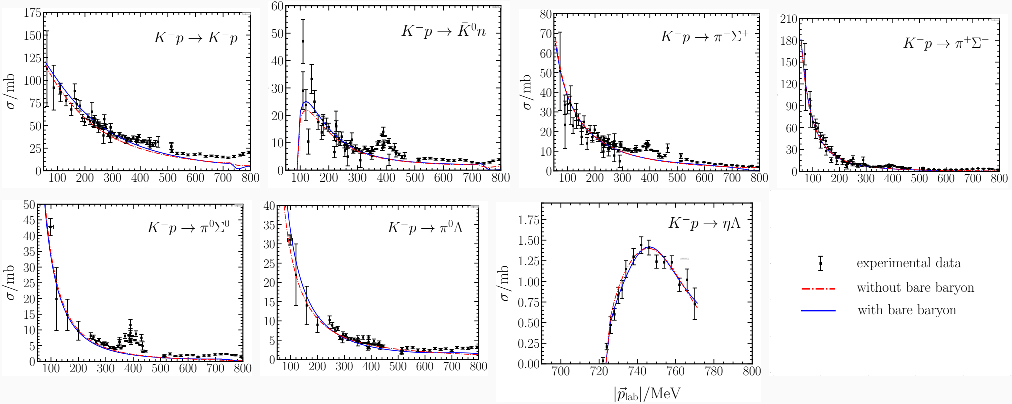
where

$$G^I_{\alpha,B_0}(k) = \frac{\sqrt{3}\, g^I_{\alpha,B_0}}{2\pi f} \sqrt{\omega_\pi(k)} \; u(k).$$

$$H^I_{\rm int} = g^I + v^I .$$

$\Lambda(1405)$ with $K^- p$ scattering

We can fit the cross sections of $K^- p$ well both with and without a bare baryon.

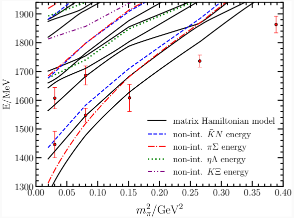


Two-pole structure of $\Lambda(1405)$: $1424 - i67$ MeV, $1428 - i24$ MeV ;

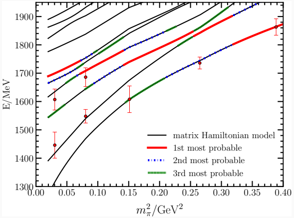
Pole for $\Lambda(1670)$: $1674 - i11$ MeV.

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Spectrum on the Lattice



without a bare baryon



with a bare baryon

Λ Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting the lattice QCD data at large pion masses.
- The bare triquark core is important for $\Lambda(1670)$.
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state.

  containing very little of bare baryon at physical pion mass.

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Comparison with recent BaSc lattice simulations

Λ Spectra with $S = -1$, $I(J^P) = 0(\frac{1}{2}^-)$
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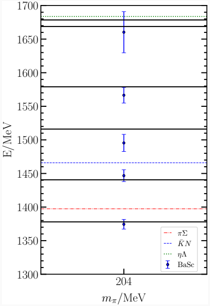
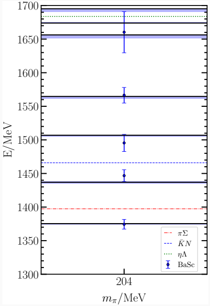


without
a bare baryon



with
a bare baryon

- The BaSc lattice collaboration obtained all HEFT states with multiquark interpolating operators;
- The right HEFT results with bare Λ fit the lattice simulations better;
- The left HEFT results without bare triquark core lose the 1σ consistence with the lattice simulations.

Baryon Scattering (BaSc) Collaboration, Phys.Rev.Lett. 132 (2024) 5, 051901; Phys.Rev.D 109 (2024) 1, 014511

J.-J. Liu, Z.-W. Liu, K. Chen, D. Guo, D. B. Leinweber, X. Liu, A. W. Thomas, Phys. Rev. D 109 (2024) 5, 054025.

Even-parity low-lying nucleon excitations with pion photoproduction

$N^*(1440)$ Resonance

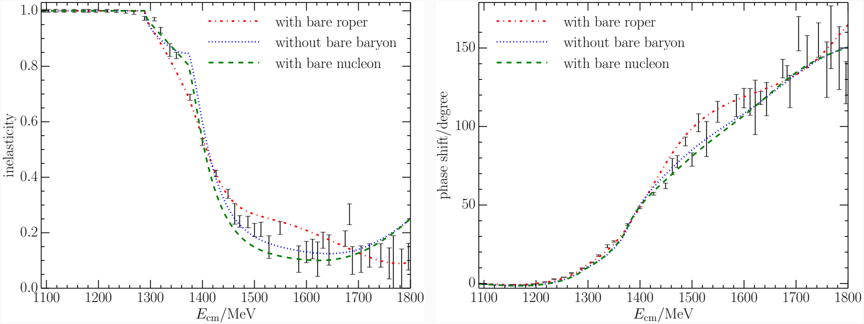
- $N^*(1440)$, usually called Roper, is the excited state $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Naive quark model predicts $m_{N^*(1440)} > m_{N^*(1535)}$
  if they are both dominated by 3-quark core. But contrary to experiment.

To check whether a 3-quark core largely exists in Roper, we consider models

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, Phys. Rev. D 95 (2017) no.3, 034034.

$N^*(1440)$ Resonance



πN scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

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- without any bare baryons
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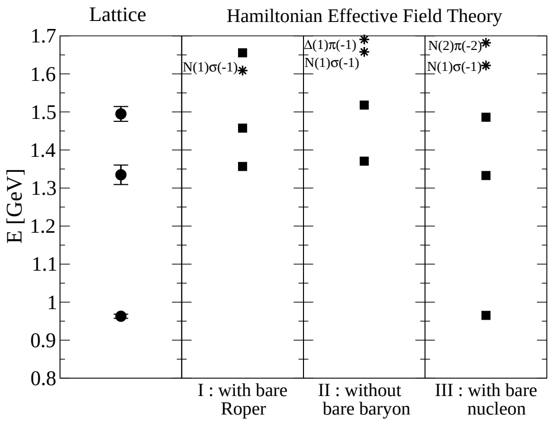
Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, Phys. Rev. D 95 (2017) no.3, 034034.

An original figure from later lattice QCD work



interpolating operators: $N(0)$, $N(0)\sigma(0)$, $N(p)\pi(-p)$, $\Delta(p)\pi(-p)$.

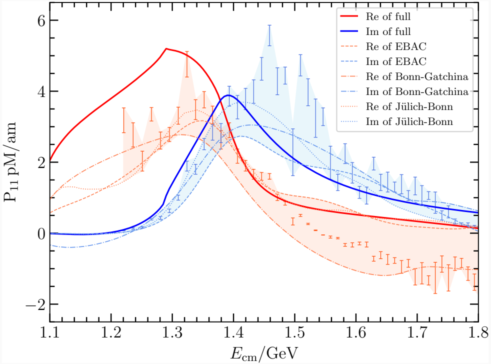
from Lang, Leskovec, Padmanath, Prelovsek, PRD95 (2017) no.1, 014510.

Extension of the Roper work

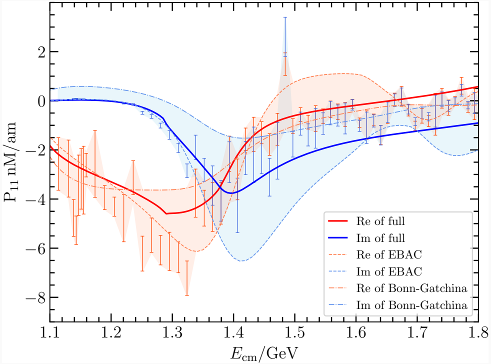
In Phys. Rev. D 97 (2018) no.9, 094509, we extended our Roper work.

- We considered effects of a resonance with bare mass/pole around 2 GeV;
- Constituent quark model with harmonic oscillator potential predicts mass of first radially excited nucleon is approximately 2 GeV.

The M_{1-} multipole amplitudes



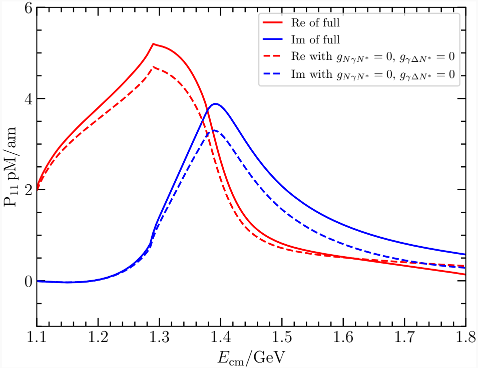
$\gamma p \to \pi N$



$\gamma n \to \pi N$

Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, arXiv: 2407.05334.

The bare core is not important for the M_{1+} in the low-energy region



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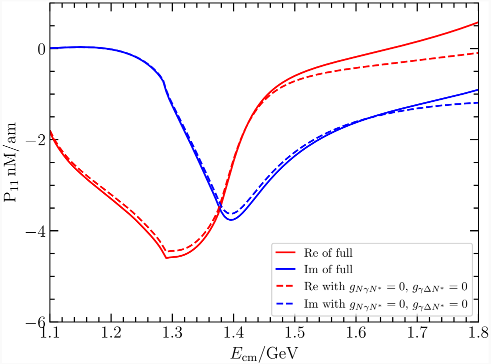

$\gamma n \to \pi N$

Yu Zhuge, Zhan-Wei Liu, Derek B. Leinweber, Anthony W. Thomas, arXiv: 2407.05334.

Summary

Summary

- Combined with pion photoproduction data, lattice QCD simulations, and so on,
  - $N^*(1535)$ contains a 3-quark core;
  - $N^*(1440)$ and $\Lambda(1405)$ contain little of 3-quark constituents;
    but the 3-quark cores should also exist and contribute to heavier resonances.

- Of course there are other interpretations for these low-lying resonances,
  and we have not considered all available data,
  such as electroproduction measurements and associated helicity amplitudes.

Thank you for your attention!