NANJING UNIVERSITY

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# Gauge sector dynamics in QCD

# Dynamical mass generation in QCD

$$\boxed{\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}^i_f\left(i\gamma^\mu D_\mu - m_f\right)_{ij}\psi^j_f + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \bar{c}^a\partial^\mu D^{ab}_\mu c^b}$$

- At the level of the Lagrangian:
  - **Gluons** are massless;
  - **Quarks** have **current masses**, but **far smaller than the hadrons they constitute**
- Perturbation theory cannot generate mass at any finite order
- Vast majority of the observable mass is **generated by the nonperturbative QCD dynamics**.
- To study **dynamical mass generation**, we look at the behavior of the nonperturbative QCD Schwinger functions (propagators and vertices):

M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).
M. Ding, C. D. Roberts and S. M. Schmidt, Particles 6, 57-120 (2023).
J. Papavassiliou, Chin. Phys. C 46, no.11, 112001 (2022).
C. D. Roberts, Symmetry 12, no.9, 1468 (2020).

Mass generation leaves **distinctive signals in the infrared** momentum region of several Schwinger functions.

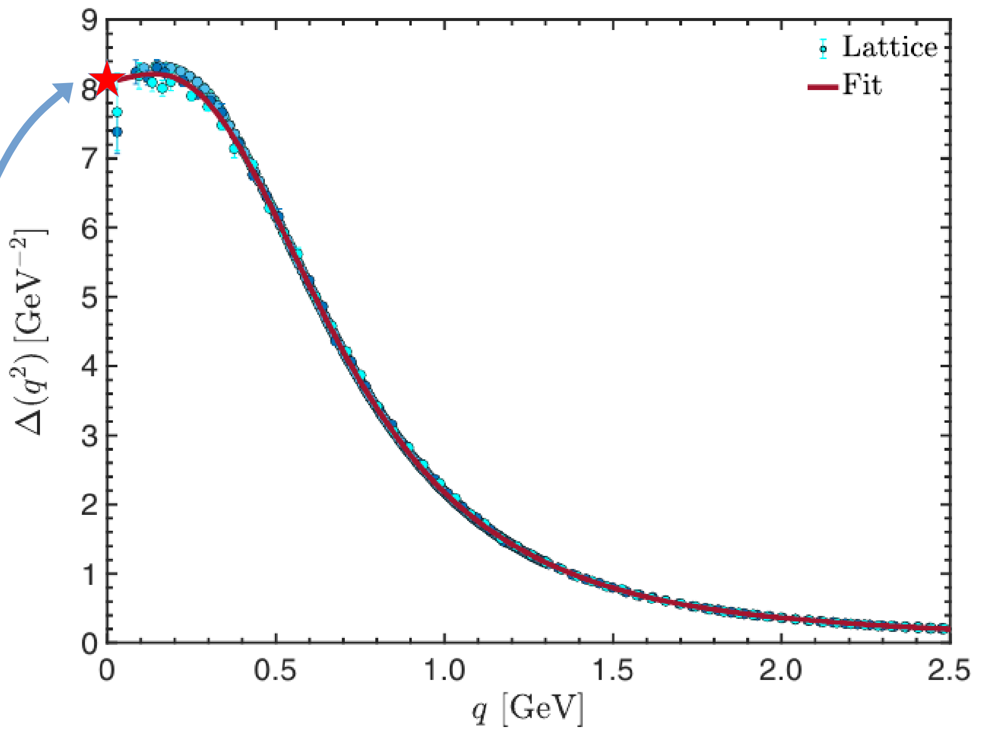
# Gluon mass generation

Gluon self-interactions can generate a dynamical mass
J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

Lattice QCD: The gluon propagator saturates at the origin:

I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou and J. Rodríguez-Quintero, Phys. Rev. D **104**, no.5, 054028 (2021).
A. Ayala, et al, Phys. Rev. D **86**, 074512 (2012).
D. Binosi, C. D. Roberts and J. Rodriguez-Quintero, Phys. Rev. D **95**, no.11, 114009 (2017).
A. C. Aguilar, et al, Eur. Phys. J. C **80**, no.2, 154 (2020).

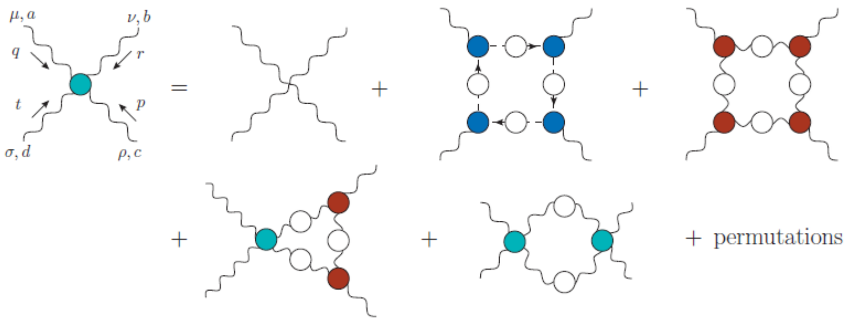
- Unequivocal signal of gluon mass generation.
- Saturation of propagator not a mass *per se*, but a Renormalization Group Invariant gluon mass of about half the proton mass scale can be constructed from it.

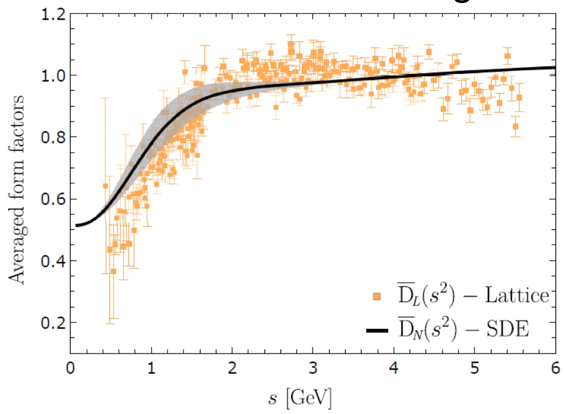



Profound implications for other Schwinger functions and eventually for observables

# Gluon mass generation: some implications

- Tames infrared divergences in many other Schwinger functions, *e.g.*, in the classical tensor of the 4-gluon vertex:



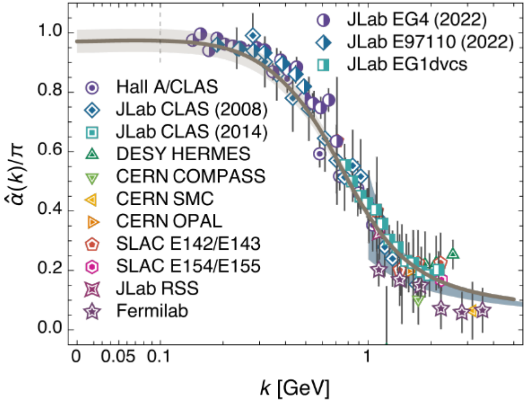




A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **84**, no.7, 676 (2024).
A. C. Aguilar, F. De Soto, M. N. F., et al, in preparation.

- Leads to an IR finite and process-independent effective charge, from which several observables can be computed.

Z. F. Cui, J. L. Zhang, D. Binosi, F. de Soto, C. Mezrag, J. Papavassiliou, C. D. Roberts, J. Rodríguez-Quintero, J. Segovia and S. Zafeiropoulos, Chin. Phys. C **44**, no.8, 083102 (2020).

D. Binosi, C. Mezrag, J. Papavassiliou, C. D. Roberts and J. Rodríguez-Quintero, Phys. Rev. D **96**, no.5, 054026 (2017).

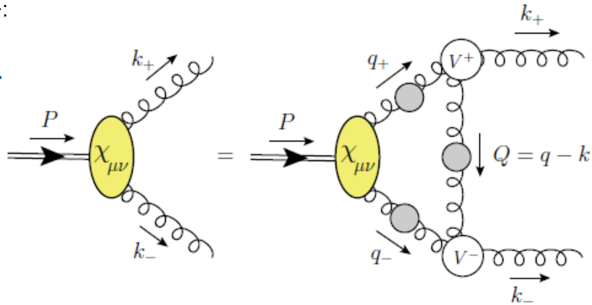



- Key ingredient in glueball Bethe-Salpeter equations, *e.g.*, pseudoscalar, $0^{-+}$:

J. Meyers and E. S. Swanson, Phys. Rev. D **87**, no.3, 036009 (2013).

E. V. Souza, M. N. F., A. C. Aguilar, et al, Eur. Phys. J. A **56**, no.1, 25 (2020).

M. Q. Huber, C. S. Fischer and H. Sanchis-Alepuz, Eur. Phys. J. C **80**, no.11, 1077 (2020).




**For more implications, see recent reviews:**

M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).
M. Ding, C. D. Roberts and S. M. Schmidt, Particles **6**, 57-120 (2023).
C. D. Roberts, Symmetry **12**, no.9, 1468 (2020).

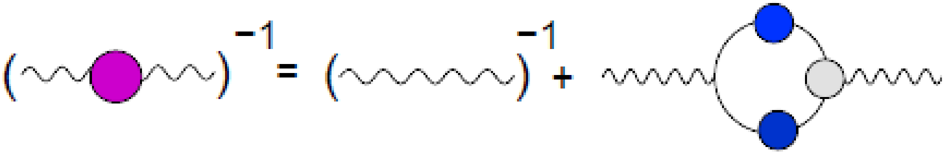
# Schwinger mechanism

How can the gluon acquire a mass gap?

- All symmetries must be explicitly preserved.
- No associated mass term, $m^2A^2$, in Lagrangian.

"A gauge boson may acquire mass, dynamically and without violating gauge symmetry if its vacuum polarization function develops a pole at zero momentum transfer." J. S. Schwinger, Phys. Rev. 125, 397 (1962); Phys. Rev. 128, 2425 (1962).

Dyson-Schwinger equation for gauge boson propagator




$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

If, for some reason

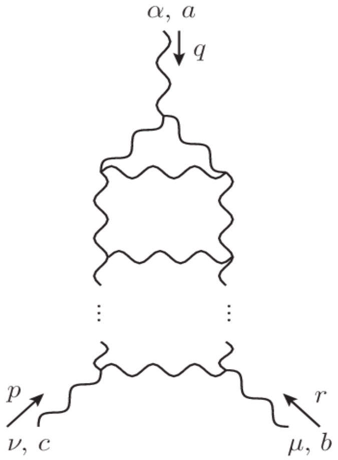
$$\lim_{q\to 0} \Pi(q^2) = \frac{c}{q^2}\,, \qquad c > 0$$

$$\Delta^{-1}(0) = c > 0$$

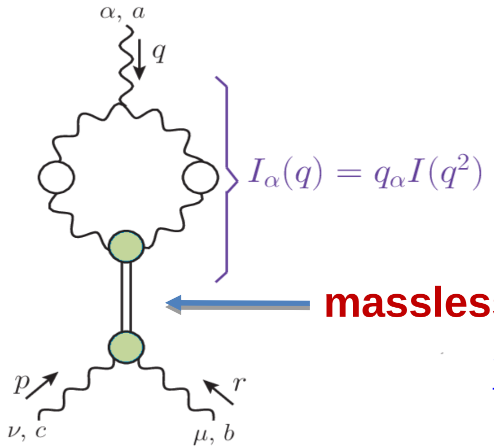
**But how can the vacuum polarization acquire such a pole?**

# Massless bound state formalism

If the interaction is sufficiently strong → formation of **massless bound states**




$q \to 0$




**Longitudinally coupled**

**massless colored excitation**

$$\frac{i\delta^{ab}}{q^2}$$

Vertices of the theory acquire **longitudinally coupled poles at zero gluon momentum, e.g.:**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \underbrace{\Gamma_{\alpha\mu\nu}(q,r,k)}_{\text{pole-free}} + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\boxed{\mathbb{C}(r^2)} + \ldots$$

**Residue functions**


E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
M. N. Ferreira and J. Papavassiliou, arXiv:2407.04392.

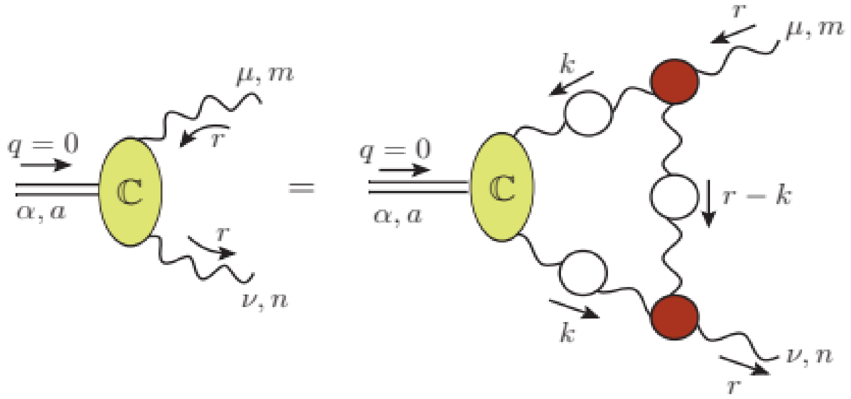
# Bethe-Salpeter equation

The formation of massless bound state is **dynamical and governed by a Bethe-Salpeter equation.**

Recalling:

$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2) + \ldots$$

The function $\mathbb{C}(r^2)$ satisfies the equation




BS amplitude

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

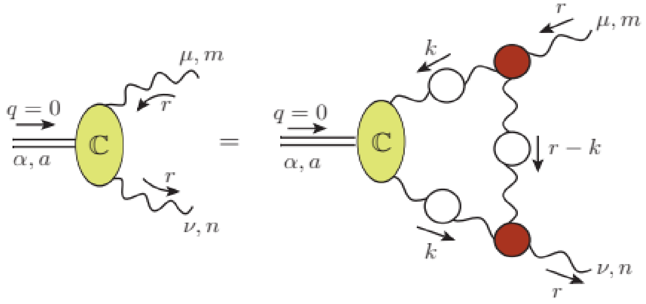
# Bethe-Salpeter equation

The Bethe-Salpeter equation admits **nontrivial solutions compatible with lattice ingredients** for the:

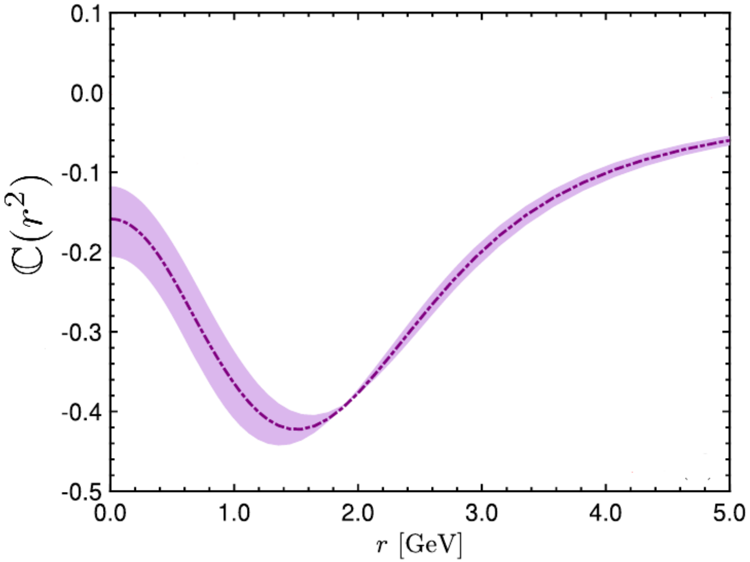
- Propagator;
- Vertex;
- and, value of the coupling

$$\alpha_s \approx 0.3 \ \ @ \ \ \mu = 4.3 \text{ GeV}$$

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi and J. Papavassiliou, Phys. Rev. D 97, no.5, 054029 (2018).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C 78, no.3, 181 (2018).
M. N. Ferreira and J. Papavassiliou, arXiv:2407.04392.




**BS amplitude**

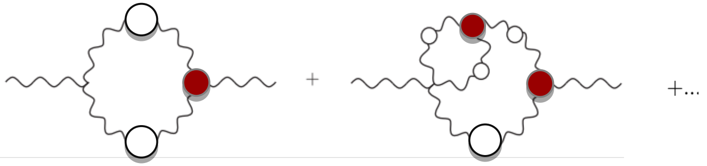



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

# Massless bound state formalism

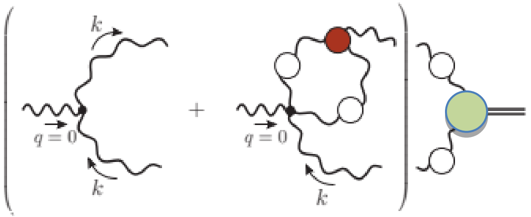
Massless poles in the three-gluon vertex lead to pole in the gluon vacuum polarization:

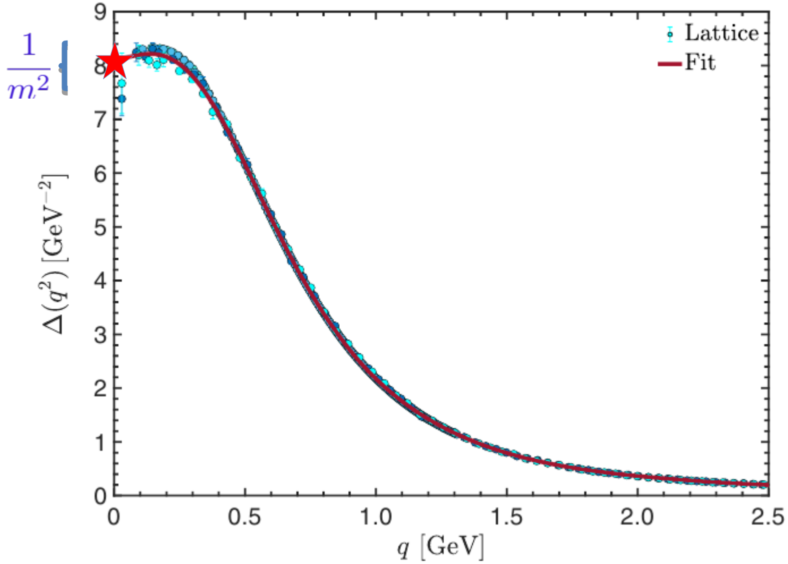
$$\Delta^{-1}(q^2) = q^2 \; +$$




$q \to 0$

$$m^2 =$$




After careful renormalization, agreement with lattice saturation value.

M. N. Ferreira and J. Papavassiliou, arXiv:2407.04392.

# Schwinger mechanism poles in lattice results?

Now, the **lattice can also compute the three-gluon vertex. Can we see longitudinal poles in it?**

**Unfortunately, no!**

The Schwinger mechanism **poles are longitudinally coupled**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \underbrace{\Gamma_{\alpha\mu\nu}(q,r,k)}_{\text{pole-free}} + \underbrace{\frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2)}_{\text{massless pole}} + \ldots$$

But **lattice simulations only access transverse tensor structures.**

**Lattice extracts the pole-free part of the vertex.**


A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).

# A smoking gun signal?

**Question:**

Is there a smoking-gun signal of the massless bound state poles, which can be tested with lattice inputs?

**Answer:**

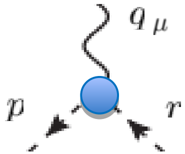
**Yes**, the **displacement of the Ward identities** satisfied by the vertices.

- The key observation is that the **Schwinger mechanism preserves the gauge symmetry**.
- If there is a massless bound state pole, the **propagators and pole-free parts of the vertices must change in shape to accommodate the pole contribution to the Ward identities**.

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

# A toy example: scalar QED

**Schwinger mechanism off**




Ward-Takahashi identity

$$\underbrace{q^\mu \Gamma_\mu(q,r,p)}_{\text{pole-free}} = D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$, $p \to -r$ ⇓ Taylor expansion

Textbook Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

**Schwinger mechanism on**

$$\mathrm{I\Gamma}_\mu(q,r,p) = \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} + \frac{q_\mu}{q^2} C(q,r,p)$$

The Ward-Takahashi identity does not change

$$q^\mu \mathrm{I\Gamma}_\mu(q,r,p) = q^\mu \Gamma_\mu(q,r,p) + C(q,r,p)$$
$$= D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$ ⇓ Taylor expansion

Displaced Ward identity

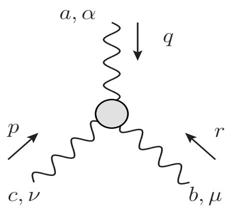
$$\underbrace{\Gamma_\mu(0,r,-r)}_{\text{pole-free}} = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

**Displacement = BS amplitude**

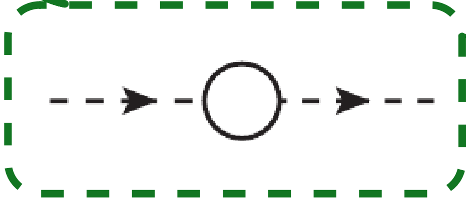
# Ward identity displacement in QCD

The **same idea applies to QCD**, just more complicated due to **non-Abelian Slavnov-Taylor identities**:

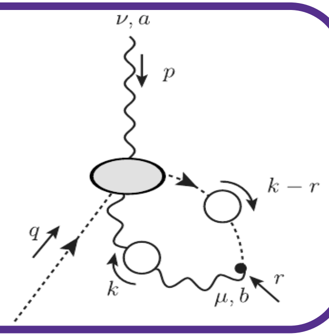
$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P^\sigma_\nu(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P^\sigma_\mu(r)H_{\sigma\nu}(r,q,p)]$$




$:= \mathrm{I\Gamma}^{abc}_{\alpha\mu\nu}(q,r,p)$



$H_{\mu\nu}(r,q,p) :=$




Then, assume the three-gluon vertex has a massless bound state pole:

$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\mathbb{C}(r^2) + \ldots$$

And expand around $q = 0$

# Ward identity displacement in QCD

$$q^\alpha \Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$

Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Displacement = BS amplitude

- **Ingredients can be computed with lattice simulations.**
- **Combine ingredients and determine if there is a nontrivial displacement.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

# Ward identity displacement in QCD

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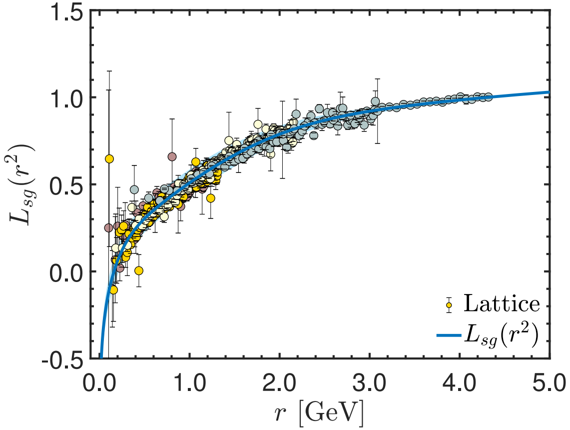


## Soft-gluon form factor of the three-gluon vertex

$$P_\mu^{\mu'}(r)P_\nu^{\nu'}(r)\Gamma_{\alpha\mu'\nu'}(0,r,-r) = 2L_{\rm sg}(r^2)r_\alpha P_{\mu\nu}(r)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$$

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

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$q \to 0$

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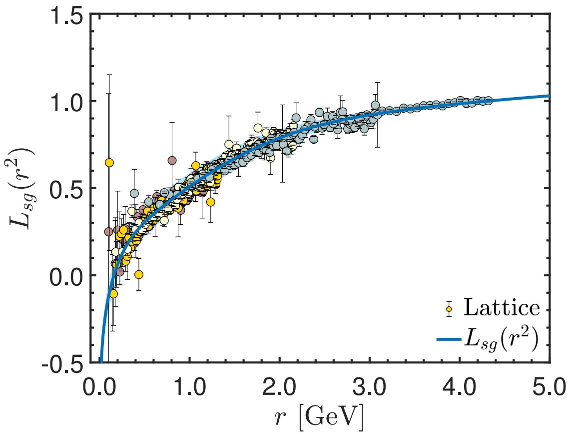


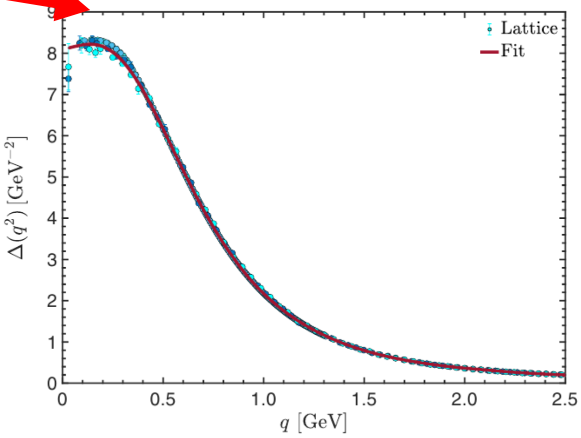

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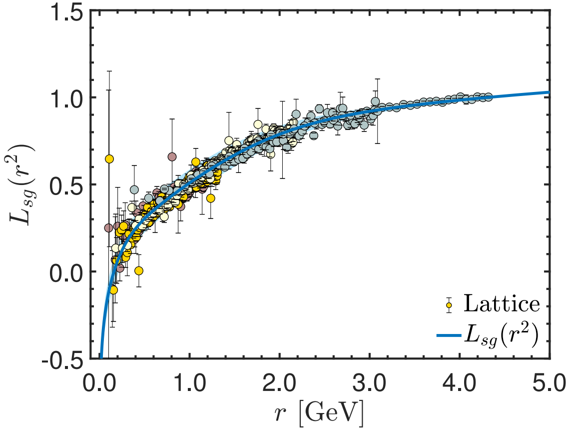


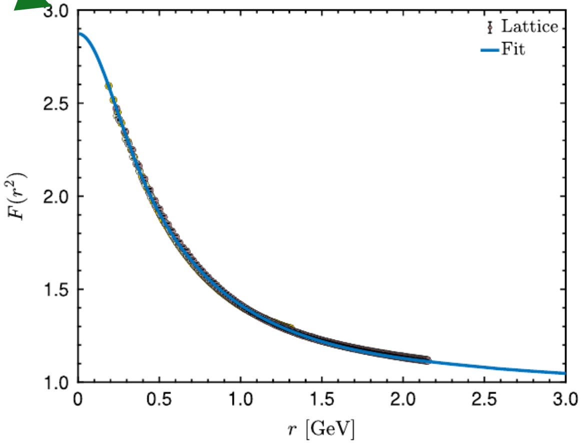


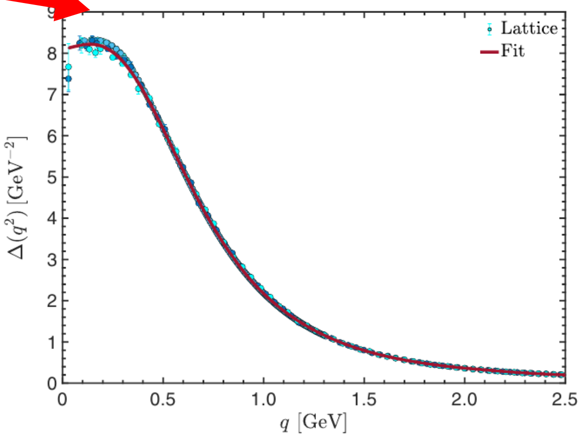

# Ward identity displacement in QCD

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

★ **Only one ingredient not yet determined directly by lattice simulations.**

# Ward identity displacement in QCD

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_{\nu}^{\sigma}(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\sigma}(r)H_{\sigma\nu}(r,q,p)]$$

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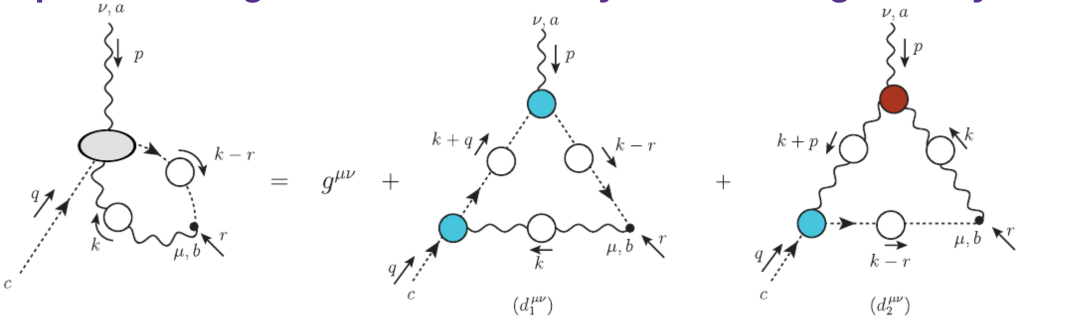
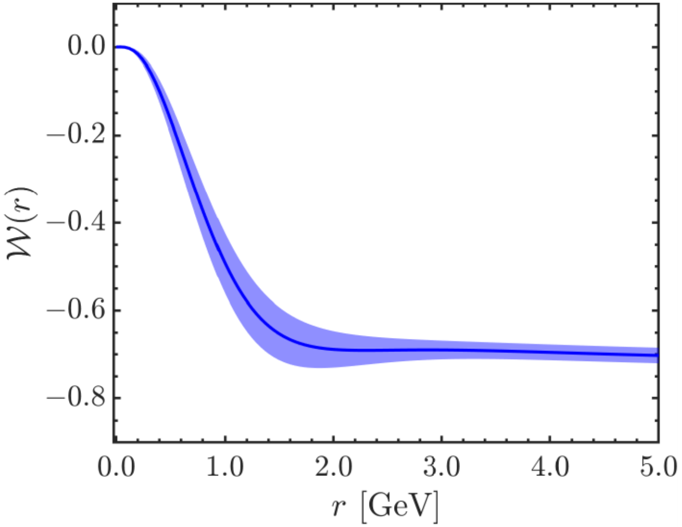
**Partial derivative of the ghost-gluon kernel**
**Computed through a lattice driven Dyson-Schwinger analysis**




A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

# Ward identity displacement in QCD

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$q \to 0$ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

Displacement = BS amplitude

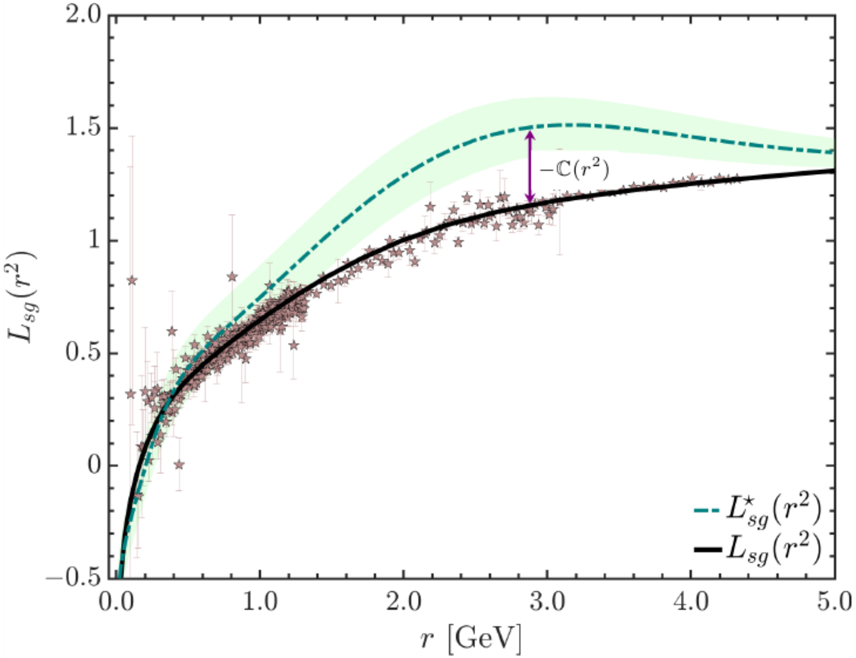
★ **Combine ingredients and determine if there is a nontrivial displacement.**

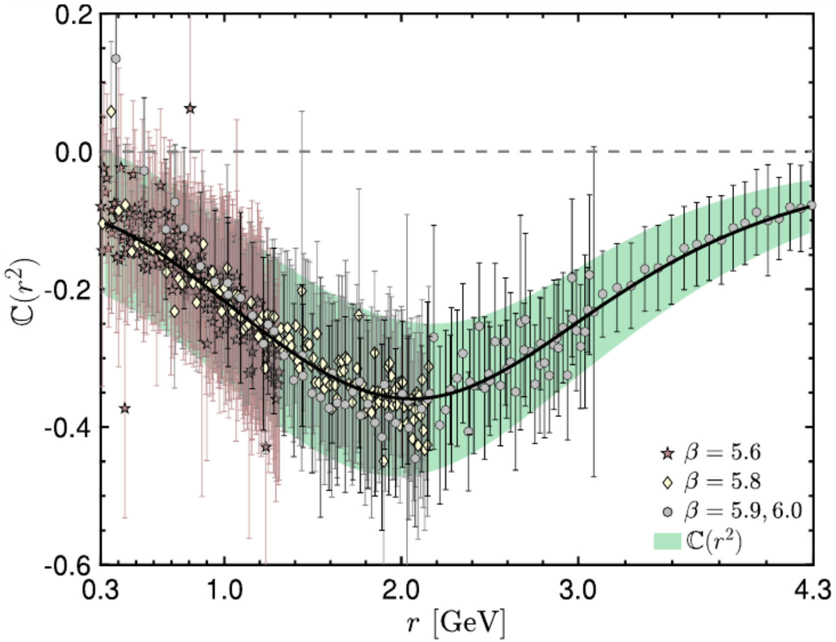
# Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $L^{\star}_{sg}(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

$$L^{\star}_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$




A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

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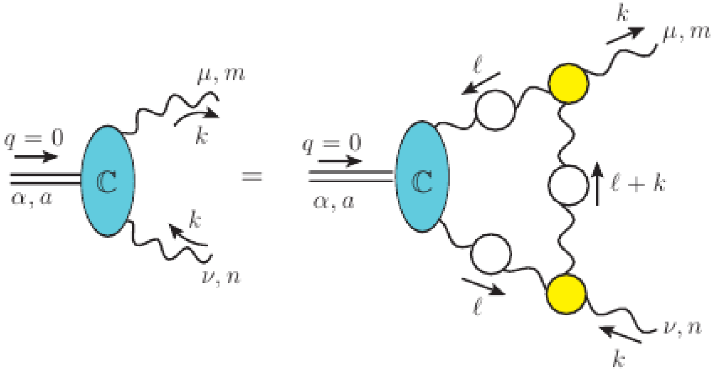
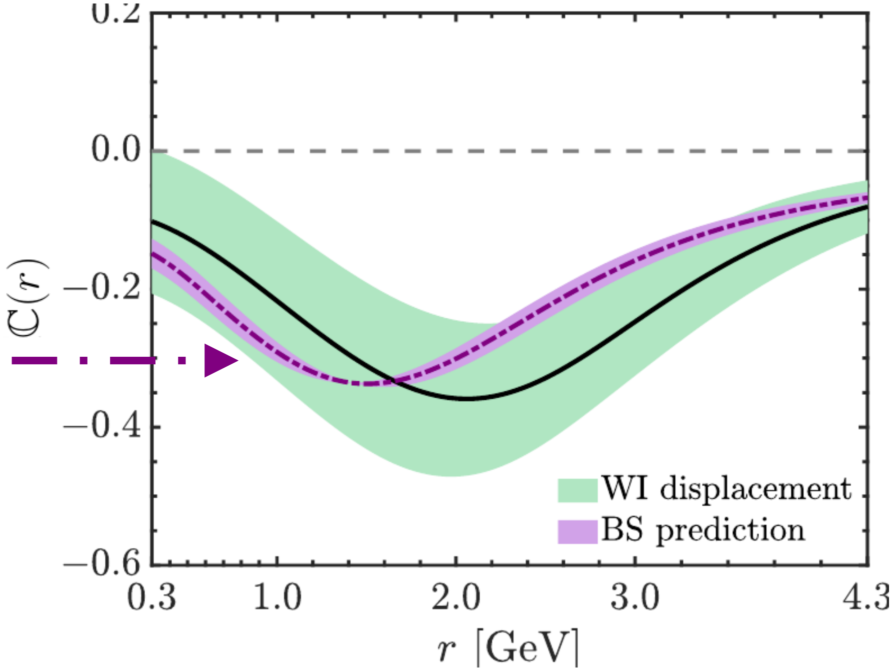
- Moreover, we find good agreement with the BSE prediction.




A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

# Conclusions

- **Dynamical mass generation in QCD is key** for hadron phenomenology.

- **Gluon self-interactions** generate a **gluon mass through the Schwinger mechanism,** via the formation of **massless bound state poles in the three-gluon vertex.**

- **Explains saturation of the gluon propagator at the origin,** seen in lattice simulations.

- Leads to **displacement of the Ward identity, whose amplitude coincides with BS amplitude of the massless bound state.**

- The **occurrence of this displacement can be tested in QCD**, by combining **lattice and Dyson-Schwinger results** for the propagators and vertices.

- **We obtain a clear displacement which agrees with the Bethe-Salpeter prediction.**

# Backup slides

# Massless bound state formalism

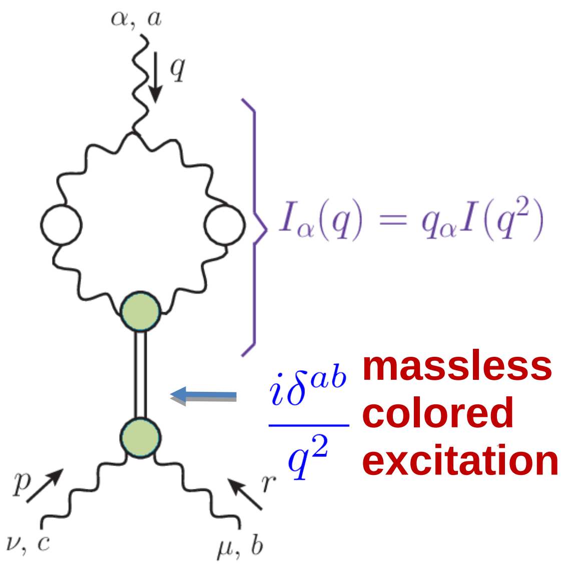
**Important: These bound states are not *glueballs*!**

**Glueballs:**

- Color singlets.
- Massive.
- Appear in the spectrum.

**Schwinger mechanism poles:**

- Colored states.
- Massless.
- Do not appear in the spectrum (would-be Goldstone boson, eaten to generate the gluon mass)




V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E 18, 1-49 (2009).

J. Smit, Phys. Rev. D **10**, 2473 (1974).
E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254-3279 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).

# Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $L^{\star}_{sg}(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

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$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

- $\mathbb{C}(r^2)$ obtained is clearly nonzero.

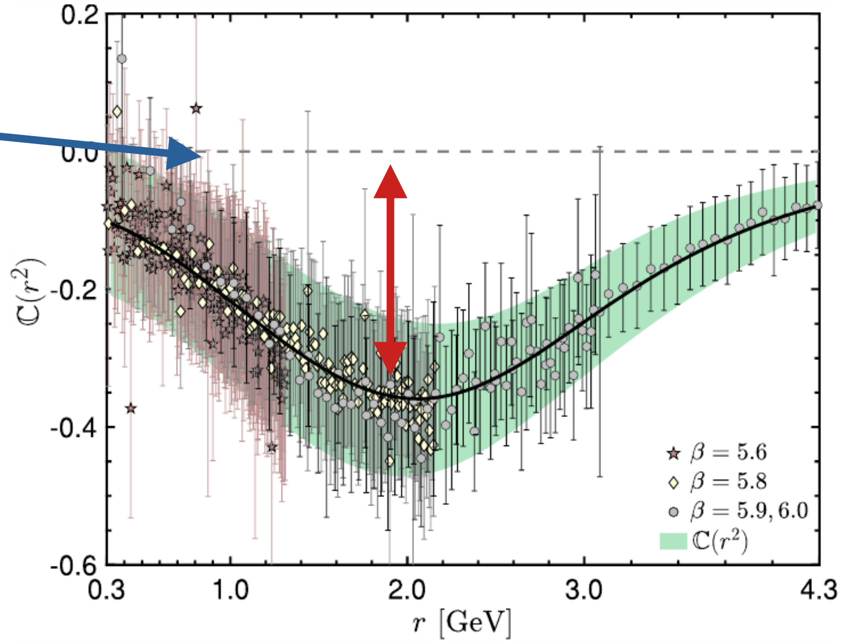
- Define the **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

***p*-value of null hypothesis is tiny:**

$$P_{\mathbb{C}_0} = \int_{\chi^2=2\,630}^{\infty} \chi^2_{\rm PDF}(515, x)dx = 7.3 \times 10^{-280}$$

- Even if the errors were doubled, the null hypothesis would still be discarded at the $5\sigma$ level.




A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

# Schwinger mechanism poles do not show in lattice results

A typical vertex form factor on the lattice is given by:

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\mathbb{\Gamma}^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

with $P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$

$$\mathbb{\Gamma}^{\alpha\mu\nu}(q,r,p) = \underbrace{\Gamma^{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}} + \underbrace{V^{\alpha\mu\nu}(q,r,p)}_{\text{poles}}$$

Given that the poles are longitudinally coupled:

$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)V^{\alpha\mu\nu}(q,r,p) = 0$$

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

**Lattice extracts the pole-free part of the vertex.**

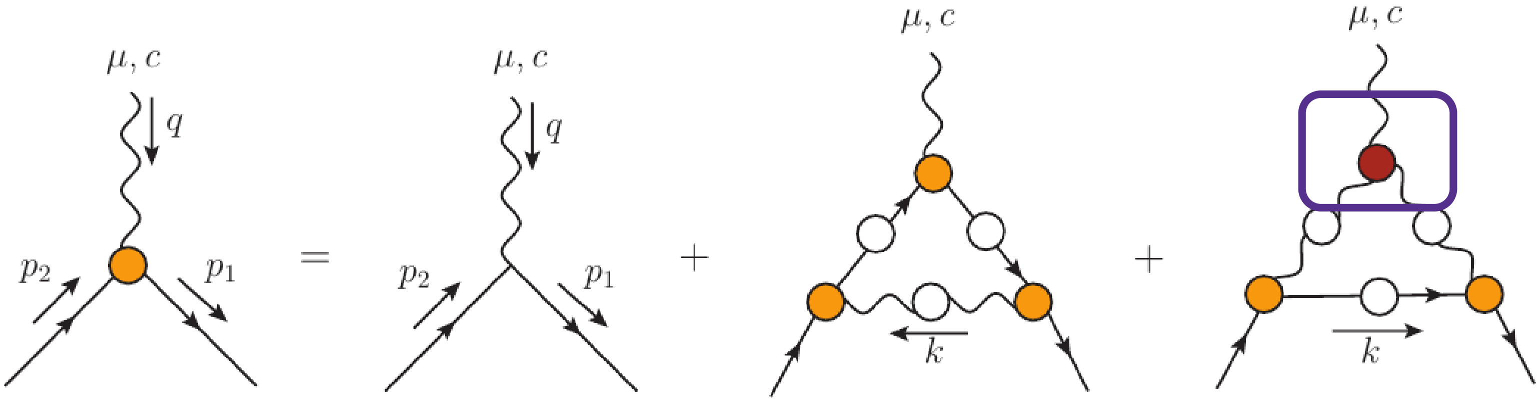


A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).

# Poles in other vertices: including dynamical quarks

The Dyson-Schwinger equations couple vertices of different species and number of external legs.

- If a **longitudinally coupled pole** is generated **in the three-gluon vertex**, it tends to **spread out to other vertices as well**.
- In particular, the **quark-gluon vertex picks up a longitudinally coupled pole**:



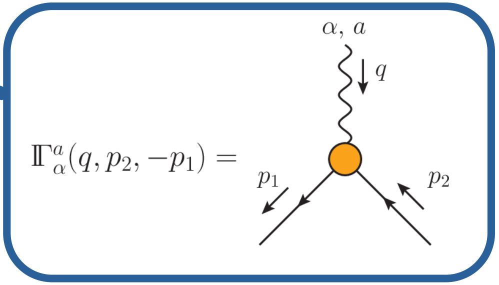

**This allows additional tests of the Schwinger mechanism, and studying the role of dynamical quarks.**

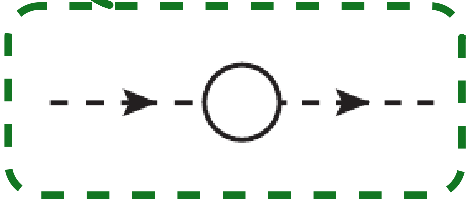
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

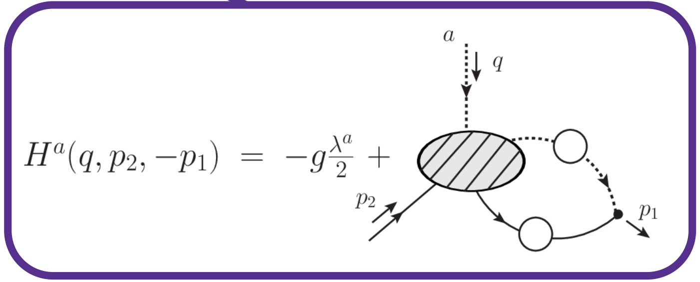
# Ward identity displacement of the quark-gluon vertex

The same idea of Ward identity displacement applies to the quark-gluon vertex. We start with the STI

$$q^\alpha \mathrm{I\!\Gamma}_\alpha(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$






Again, assume that the vertex has a massless bound state pole:

$$\mathrm{I\!\Gamma}_\alpha(q,p_2,-p_1) = \Gamma_\alpha(q,p_2,-p_1) + \frac{q_\alpha}{q^2}\not{q}\,\boxed{Q_3(p_2^2) +}\ldots$$

**BS amplitude**

And expand around *q* = *0*

# Ward identity displacement of the quark-gluon vertex

$$q^{\alpha}\Gamma_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

$q \to 0$ Isolate classical tensor structure

Ward identity

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).

$$\lambda_1(p^2) = \underbrace{F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}}_{\lambda_1^{\star}(p^2)} - \boxed{Q_3(p^2)}$$

Displacement = BS amplitude

- **Ingredients can be computed using lattice results.**

O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **99**, no.9, 094506 (2019).
A. Kizilersü, O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **103**, no.11, 114515 (2021).

- **Combine ingredients and determine if there is a nontrivial displacement.**

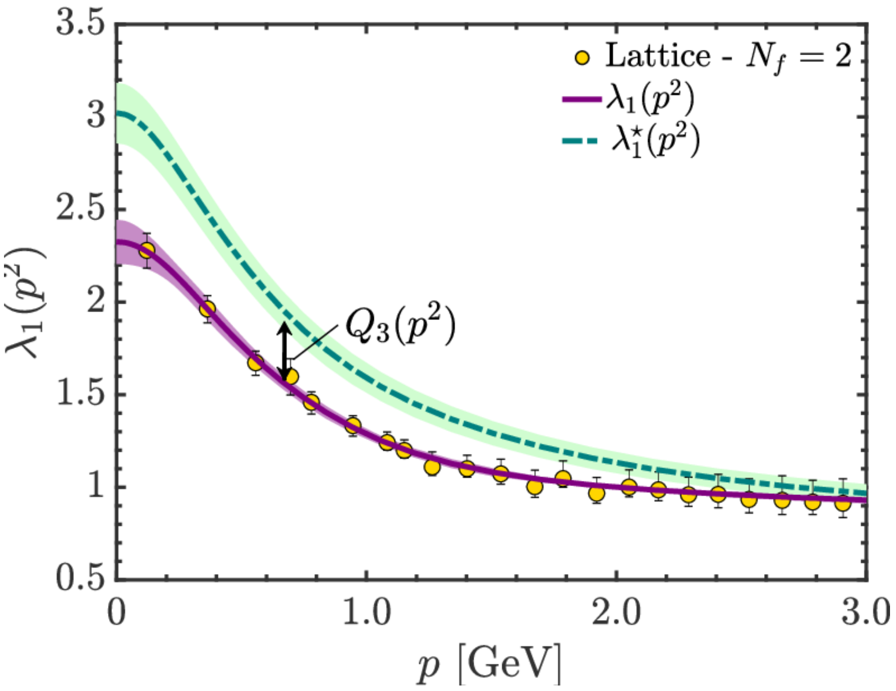
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

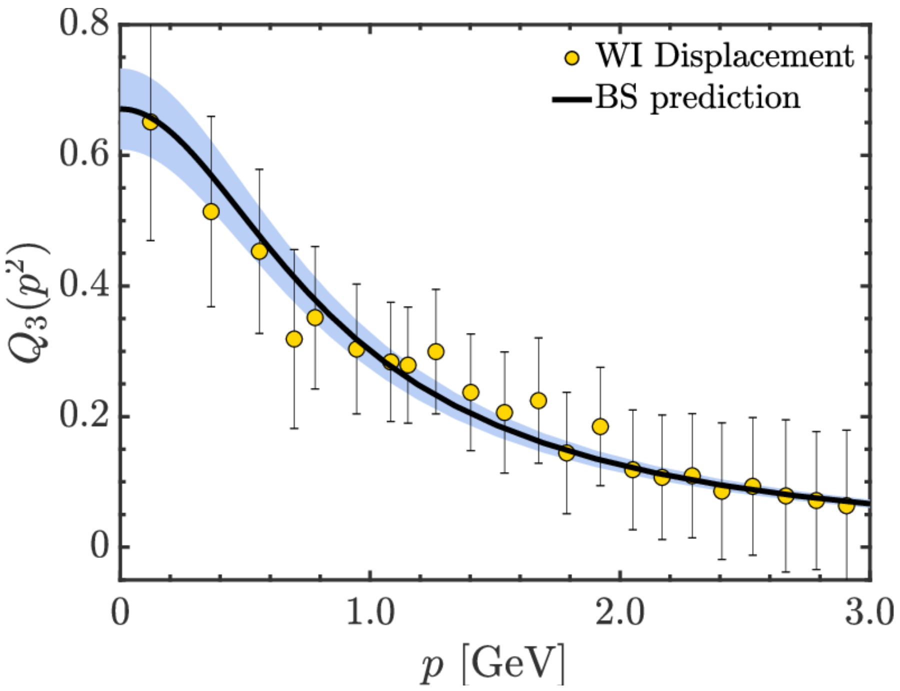
# Results for $Q_3(p^2)$

We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$

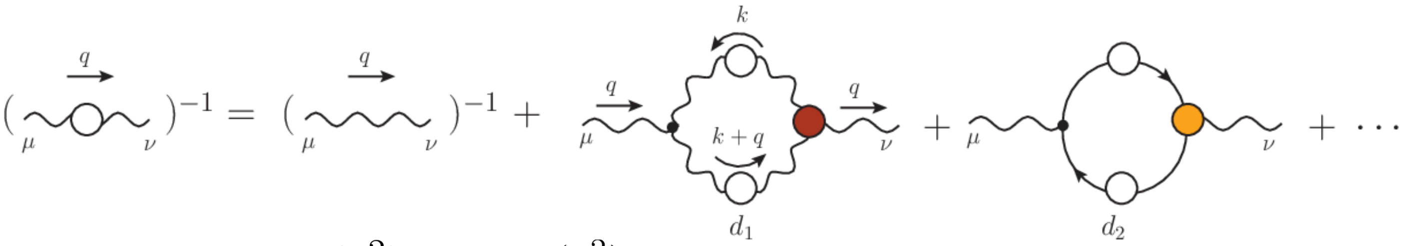



A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

# Gluon self-interaction is dominant in gluon mass generation

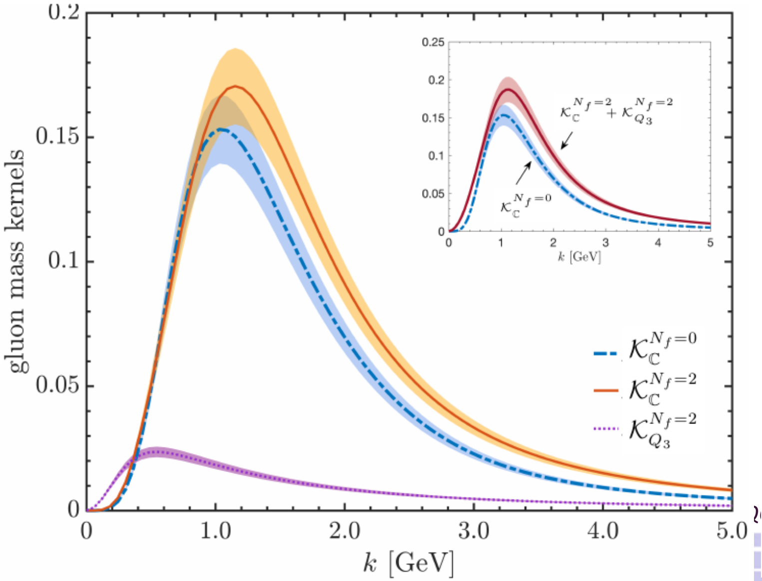
From the gluon SDE:

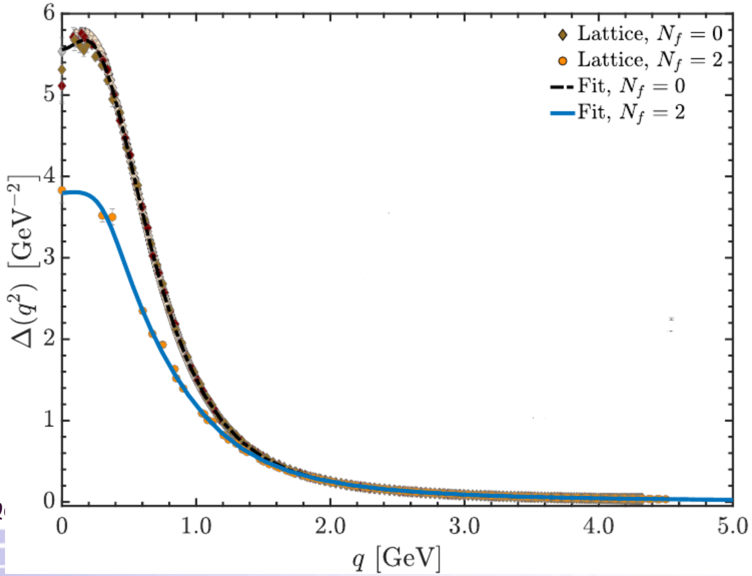



one finds an expression for the mass in terms of $\mathbb{C}(p^2)$ and $Q_3(p^2)$:

$$m^2 = \int_0^\infty dy \boxed{\mathcal{K}_{\mathbb{C}}^{N_f}(y)} + \int_0^\infty dy \boxed{\mathcal{K}_{Q_3}^{N_f}(y)}$$

**three-gluon** **quark-gluon**




- ✓ Unquenched gluon mass is larger, in agreement with lattice.
- ✓ Three-gluon is the biggest contribution.
- ✓ Gluon self-interaction drives the Schwinger mechanism in QCD.

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We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$

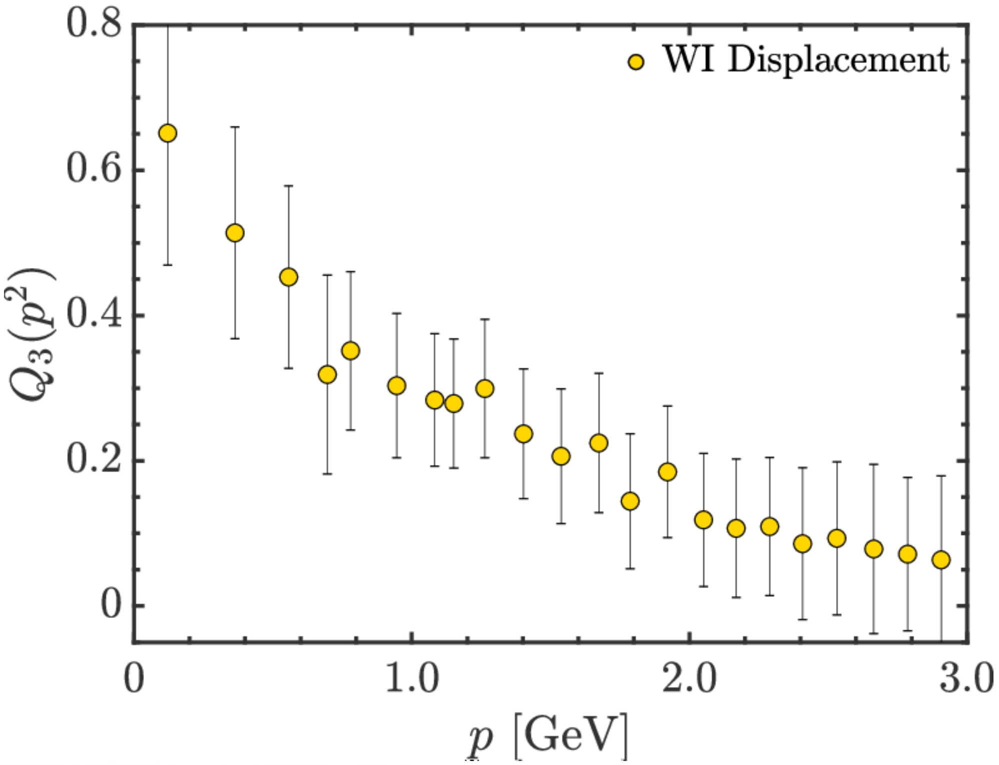
- $Q_3(p^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$Q_3(p^2) = Q_3^0(p^2) := 0$$

***p*-value of null hypothesis is very small:**

$$P_{Q_3^0} = \int_{\chi^2=119}^{\infty} \chi^2_{\text{PDF}}(18, x)dx = 6.5 \times 10^{-17}$$

- Excludes the null hypothesis at the 8σ level.



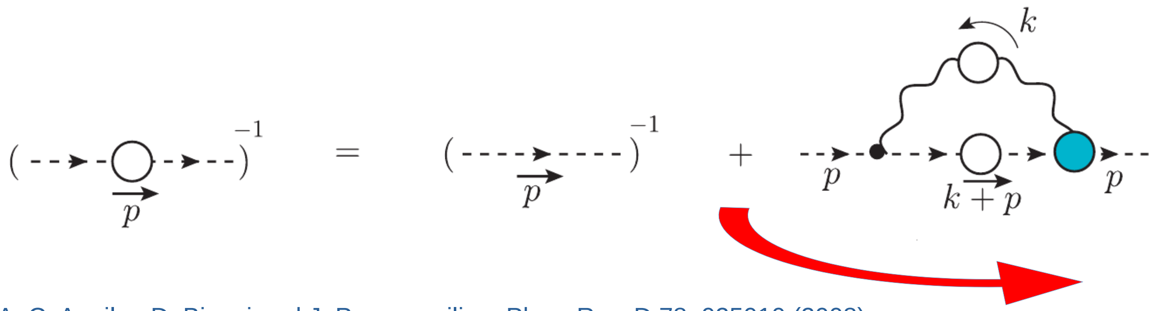
# Indirect signals: Finite ghost dressing function

The generation of a gluon mass gap leaves distinctive imprints in other Green's functions. For example:

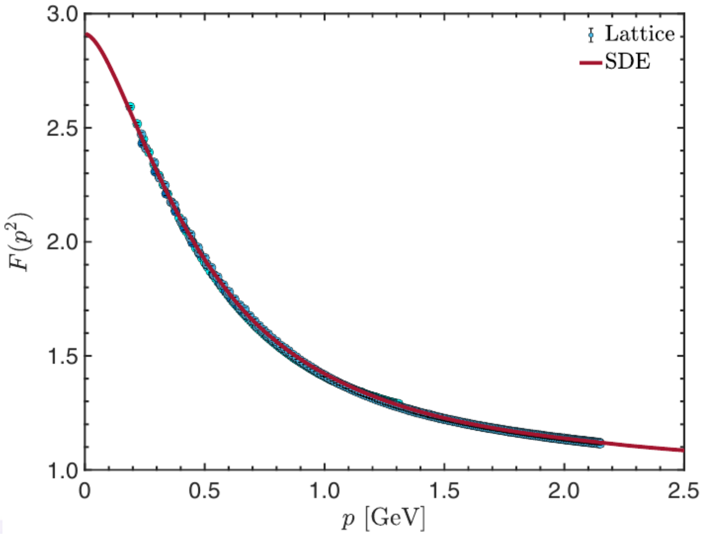
- The Schwinger mechanism leaves the **ghost propagator,** $D(q^2)$**, massless.**
- But its **dressing function**, $F(q^2)$, given by

$$\boxed{D(q^2) = \frac{iF(q^2)}{q^2}}$$

**becomes IR finite**.




A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D 78, 025010 (2008).
M. R. Pennington and D. J. Wilson, Phys. Rev. D **84**, 119901 (2011).
A. C. Aguilar, C. O. Ambrosio, F. De Soto, M. N. F., et al, Phys. Rev. D 104 no.5, 054028, (2021).




**More examples in:**

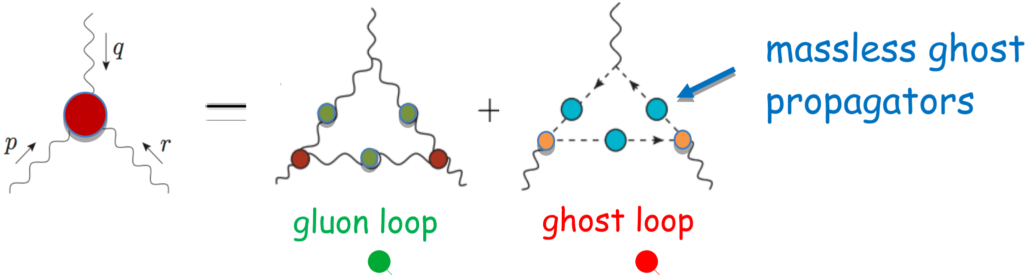
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

# Indirect signals: IR divergence of three-gluon vertex

Three-gluon vertex in the IR exhibits suppression and zero crossing

A. C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys. Rev. D 89, no. 8, 085008 (2014).
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **83**, no.6, 549 (2023).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).

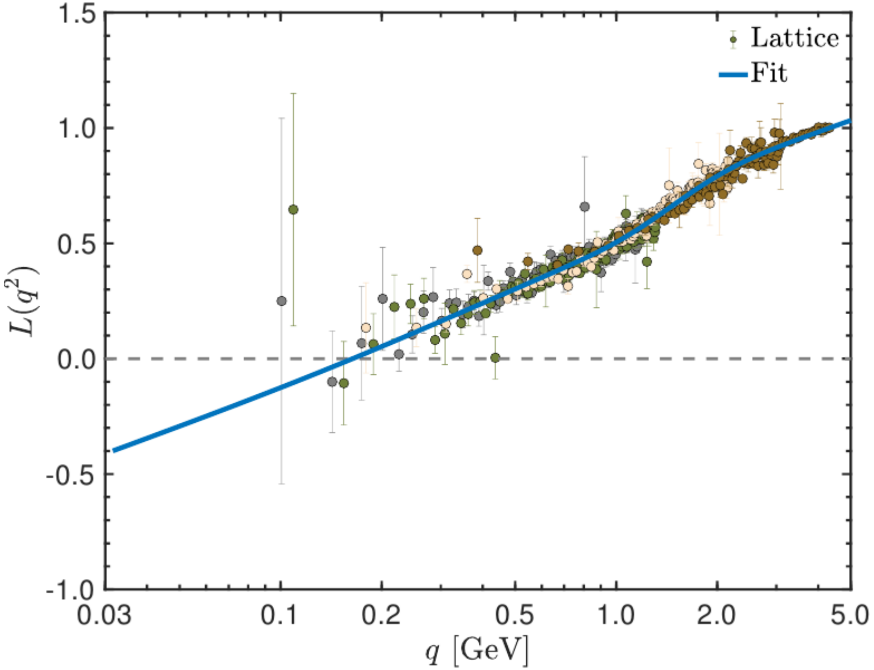
Within the Schwinger mechanism, the infrared behavior of the classical form factor of the three-gluon vertex is characterized by the interplay between two types of logarithms:




$$L_{\rm sg}(r^2) = a + b\ln\left(\frac{r^2+m^2}{\mu^2}\right) + c\ln\left(\frac{r^2}{\mu^2}\right)$$

"protected" "unprotected"

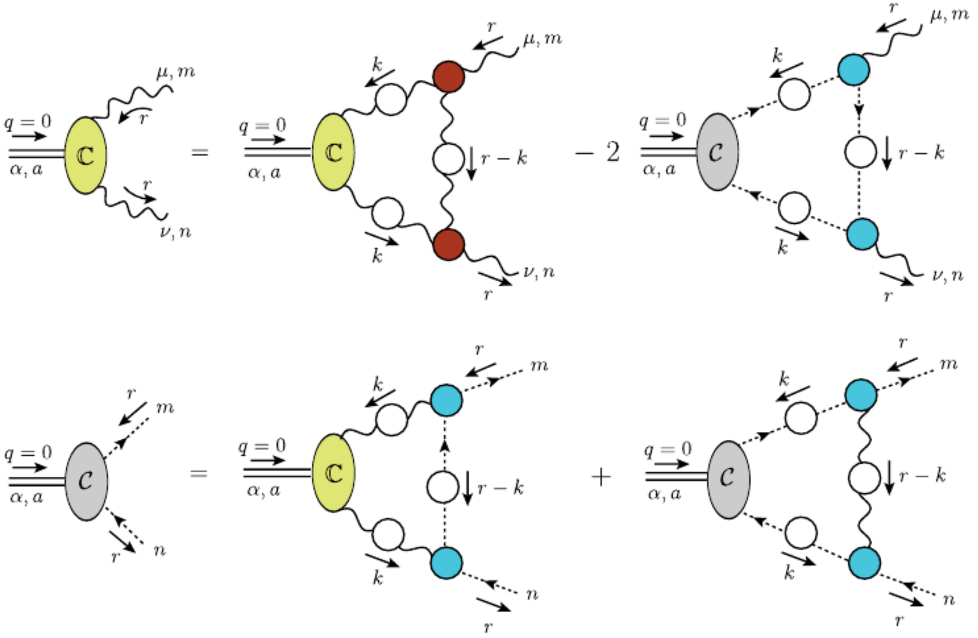
- In the IR, $L_{\rm sg}(r^2) \to -\infty$, logarithmically.
- Explains IR suppression and zero-crossing

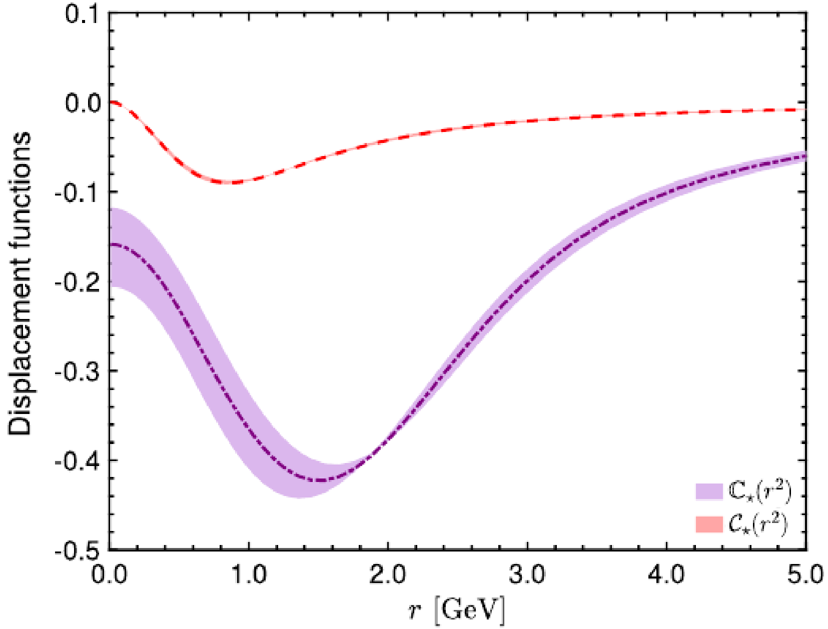



A. C. Aguilar, F. De Soto, M.N.F., J. Papavassiliou, J. Rodriguez-Quintero, Phys. Lett. B818 (2021) 136352

# Pole of the ghost-gluon vertex

The Schwinger-Dyson equation for the displacement amplitude $\mathbb{C}(r^2)$ can be coupled to a pole also in **ghost-gluon vertex**



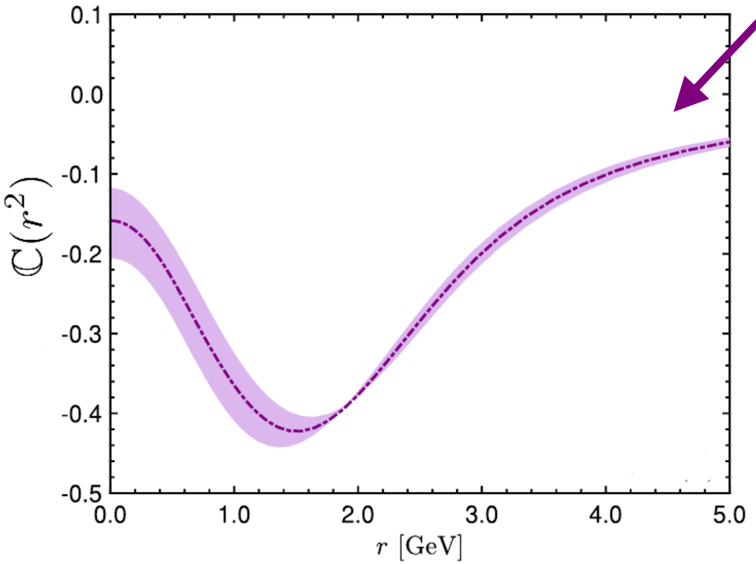



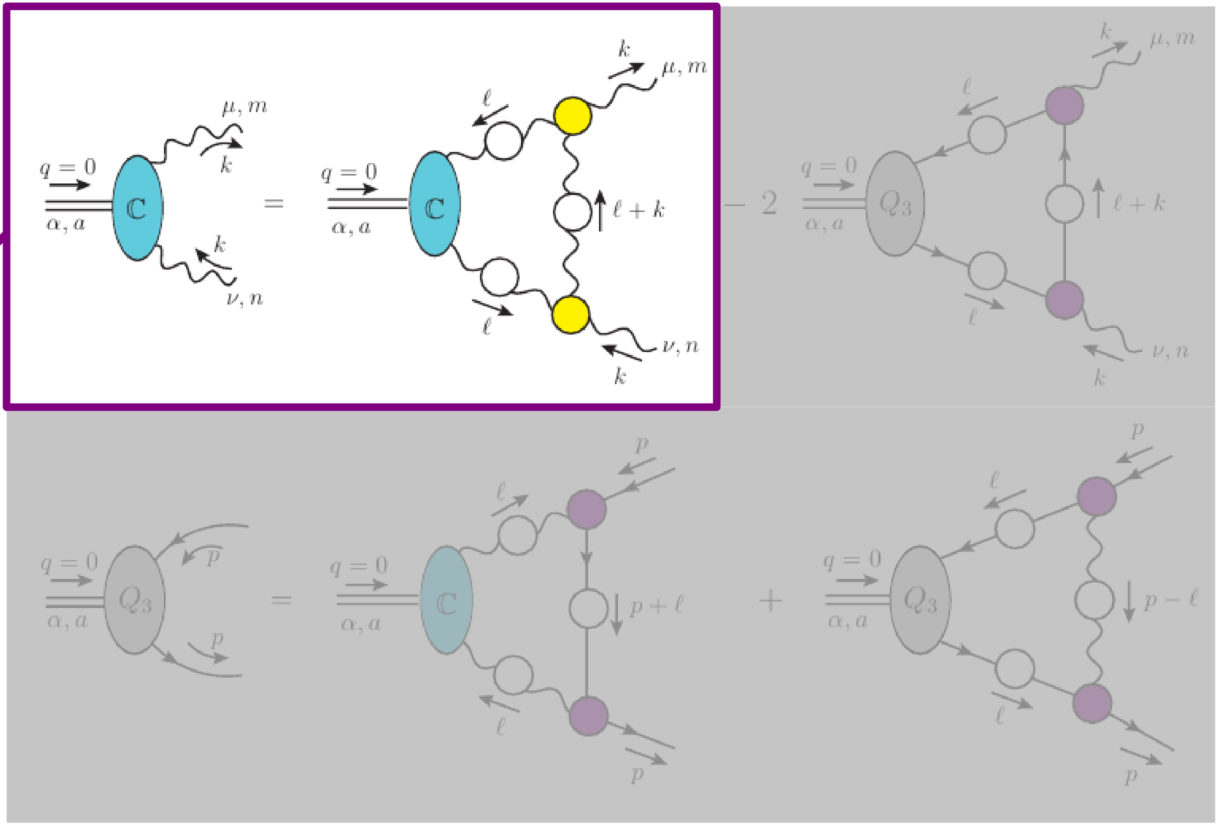

Effect on $\mathbb{C}(r^2)$ is negligible because ghost-gluon pole amplitude, $\mathcal{C}(r^2)$, is subleading.


A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

# Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.




A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

# Schwinger mechanism with dynamical quarks

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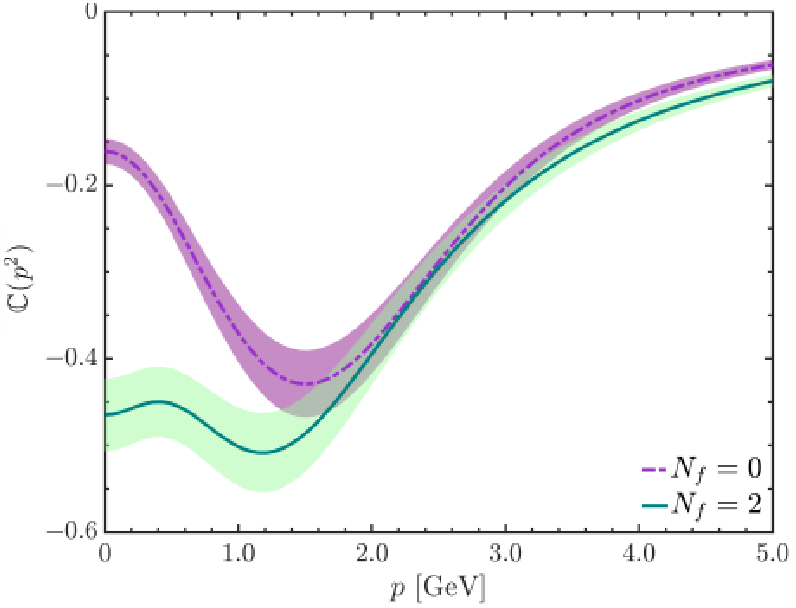
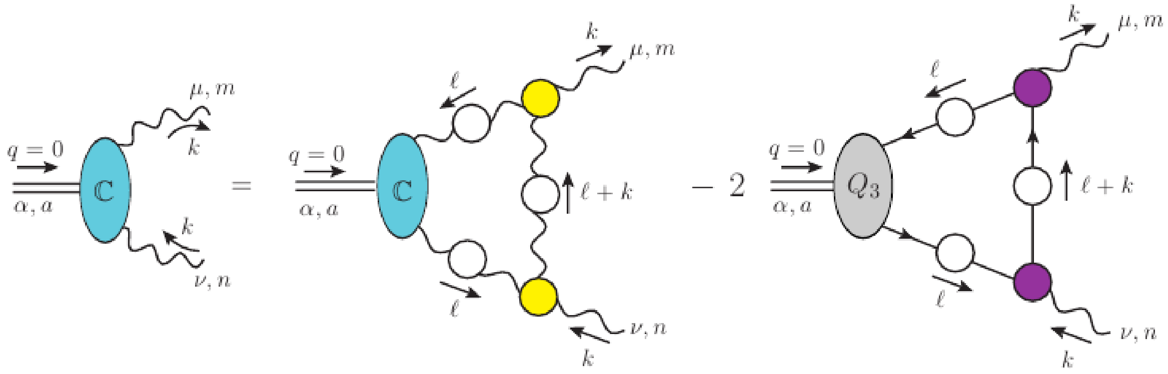
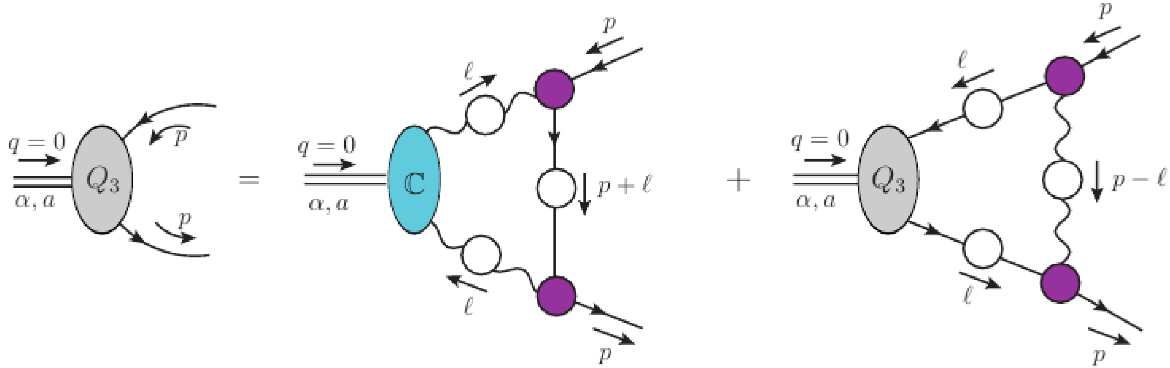
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.

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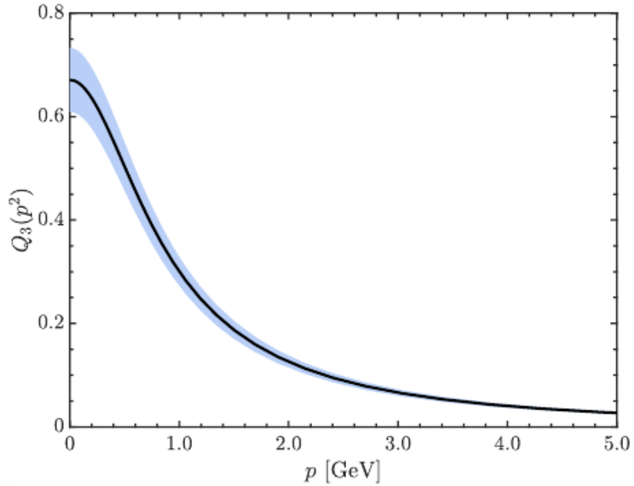
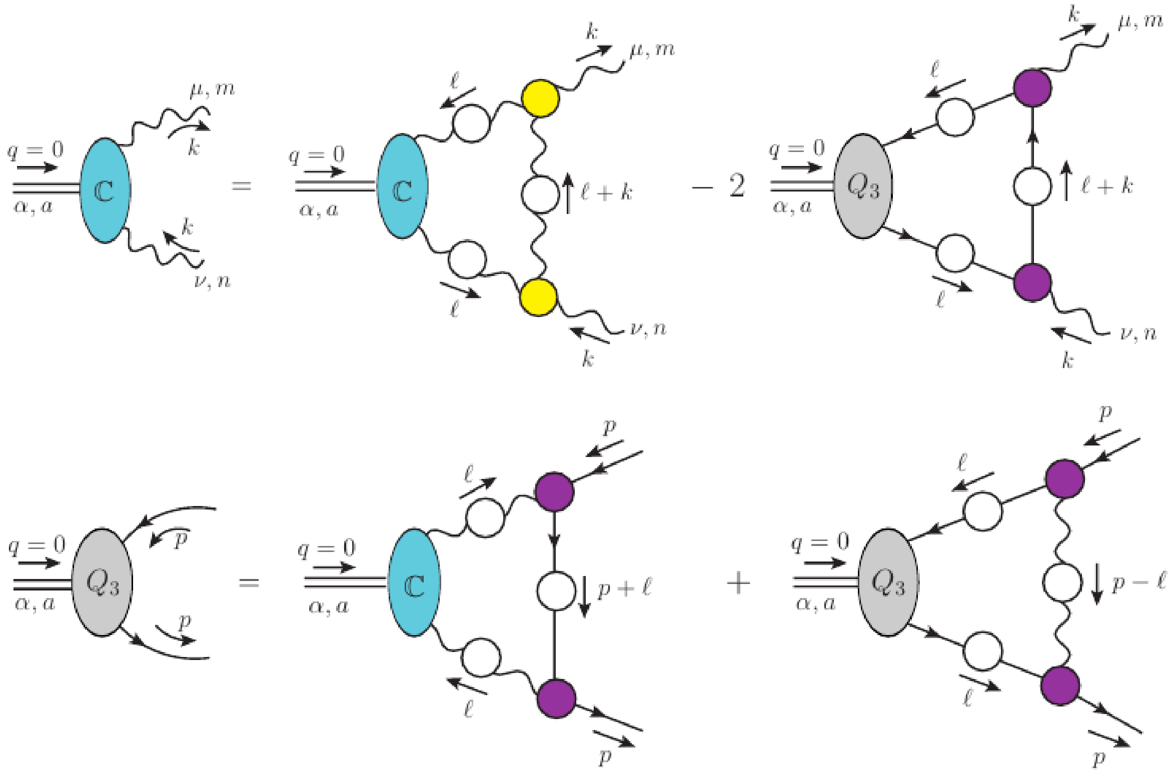
# Schwinger mechanism with dynamical quarks

- The three-gluon pole BSE has solutions even if quarks are neglected.
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.
- But also generates a **pole in the quar-gluon vertex**, with amplitude $Q_3(p^2)$.




A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

# Gluon self-interaction is dominant in gluon mass generation

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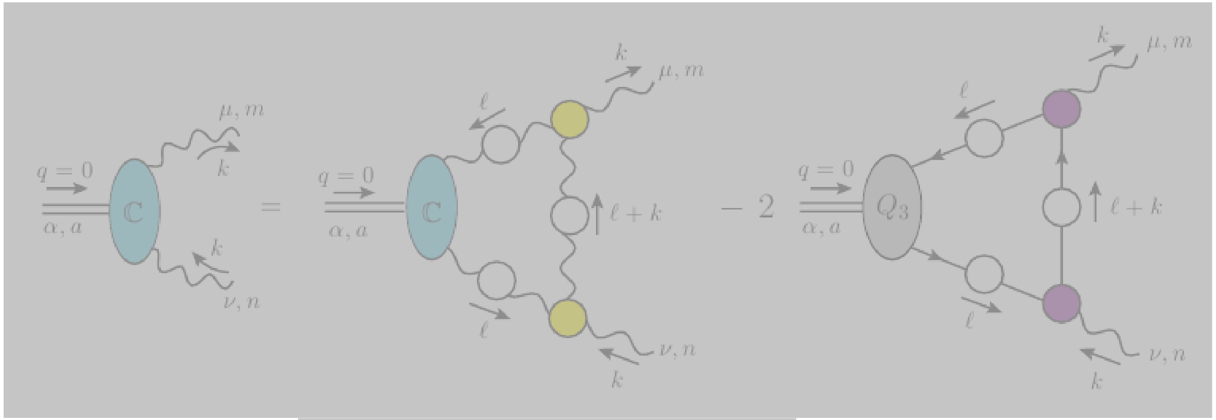
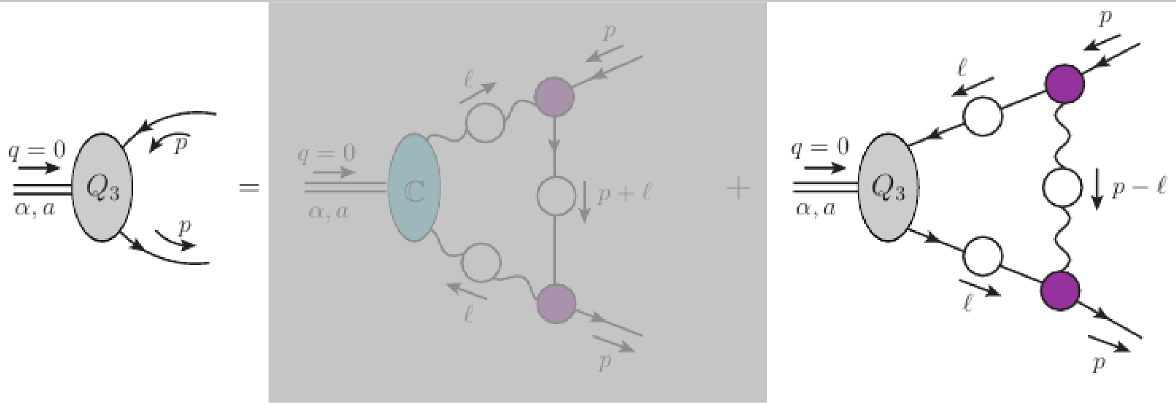
- Turning on quarks, changes the three-gluon BSE amplitude only in the deep IR.
- 
- But also generates a **pole in the quar-gluon vertex**, with amplitude $Q_3(p^2)$.
- **But turning off the three-gluon pole, no solution is found!**

**Gluon self-interaction drives gluon mass generation**




A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).

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$q \to 0$ Isolate classical tensor structure

Ward identity

$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2\boxed{K_4(p^2)}\right] - 2\boxed{K_1(p^2)}\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

**Partial derivative of the quark-ghost kernel**

$$\left.\frac{\partial H(q,p,-q-p)}{\partial q^{\mu}}\right|_{q=0} = \gamma_{\mu}K_1(p^2) + 4p_{\mu}\not{p}K_2(p^2) + 2p_{\mu}K_3(p^2) + 2\widetilde{\sigma}_{\mu\nu}p^{\nu}K_4(p^2)$$

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, Eur. Phys. J. C 83, no.10, 967 (2023).

# Ward identity displacement of the quark-gluon vertex

$$q^\alpha \Gamma_\alpha(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

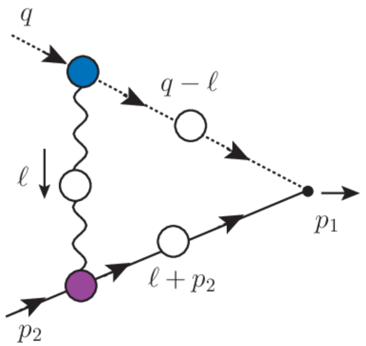
$q \to 0$ Isolate classical tensor structure

Ward identity

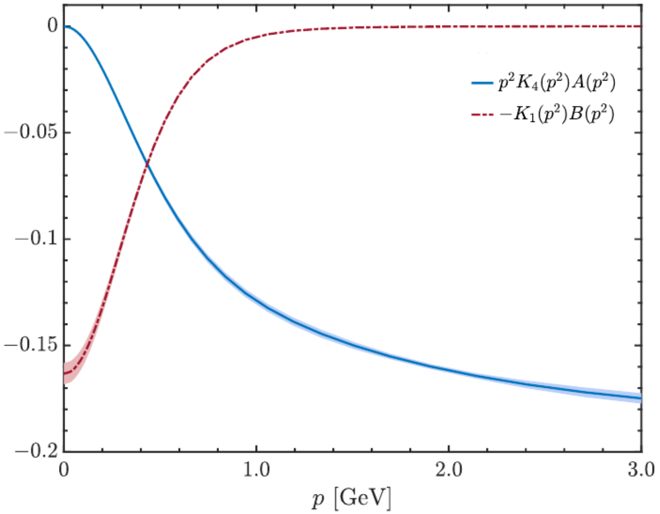
$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1 + 4p^2 \boxed{K_4(p^2)}\right] - 2\boxed{K_1(p^2)}\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

**Computed through a lattice driven Schwinger-Dyson analysis**

$$H^a(q,p_2,-p_1) = -g\frac{\lambda^a}{2} +$$

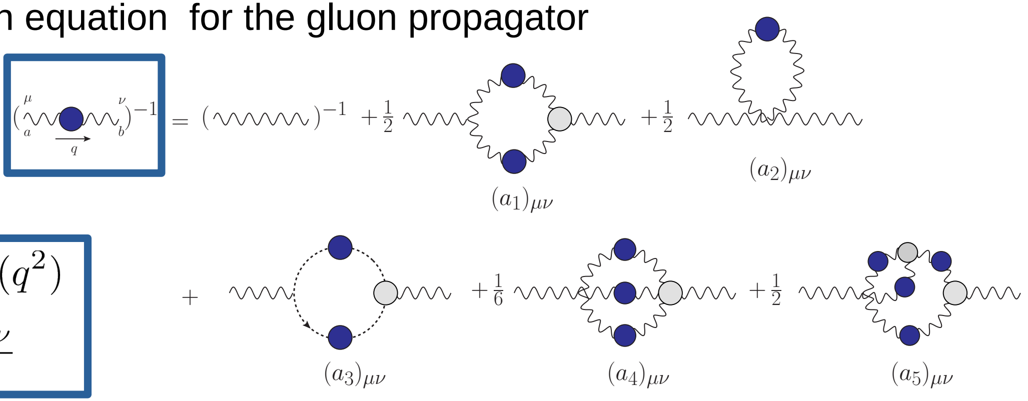



$$+ \ldots$$



# Seagull cancellation

- The **gluon mass generation must occur without violating gauge symmetry**.
- Recalling the Schwinger-Dyson equation  for the gluon propagator




$$\boxed{\begin{aligned}\Delta_{\mu\nu}(q) &= -iP_{\mu\nu}(q)\Delta(q^2)\\ P_{\mu\nu}(q) &:= g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\end{aligned}}$$

It can be shown that

**Gauge symmetry + Regular vertices at *$q^2$ = 0*** $\longrightarrow$ $\Delta^{-1}(0) = 0$

★ **The key to generate gluon mass is to have massless poles, longitudinaly coupled to the gluon momenta, in the vertices of QCD.**

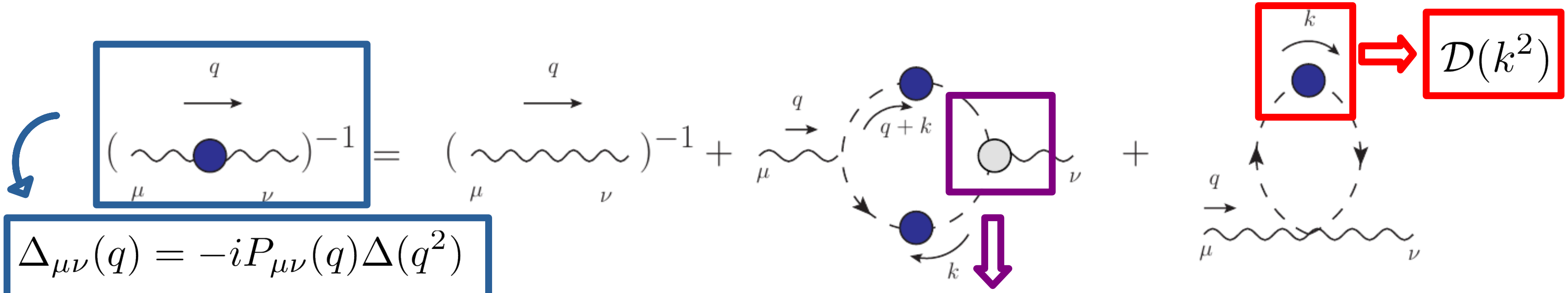

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

# Seagull cancellation

To understand **how gauge fields can become massive by the Schwinger mechanism**, let us first recall how gauge symmetry ***usually*** implies their masslessness.

A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

To this end, consider the **Schwinger-Dyson equation** for the scalar QED **photon propagator**




$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\Gamma_\nu(q, k, -q-k)$$

**At *q* = 0, we obtain:**

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

# Seagull cancellation

Now, **gauge symmetry** implies the **Ward identity**:

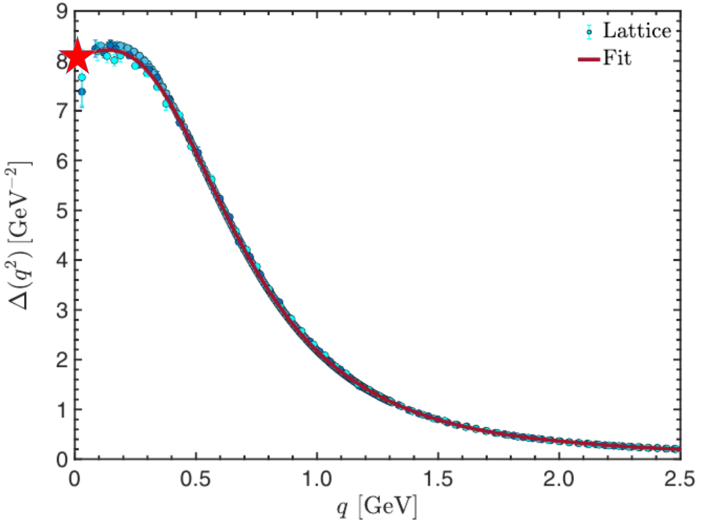
$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2) \xrightarrow{q\,=\,0} \Gamma_\mu(0,r,-r) = \frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^\mu}$$

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2) k^\mu \Gamma_\mu(0,k,-k) - 2ie^2 \int_k \mathcal{D}(k^2)$$

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\boxed{\left[\int_k k^2 \frac{\partial \mathcal{D}^{-1}(k^2)}{\partial k^2} + \frac{d}{2}\int_k \mathcal{D}(k^2)\right] = 0}$$

**Seagull identity (integration by parts in *d* dimensions).**

**Then, how can we have saturation?**





A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

# Evading the seagull cancellation

Suppose the vertex has a **pole at $q$=0, coupled longitudinally to $q$**, i.e.

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).

$$\Gamma_\mu(q,r,p) \to \mathbb{\Gamma}_\mu(q,r,p) = \boxed{\frac{q_\mu}{q^2}C(q,r,p)} + \Gamma_\mu(q,r,p)$$

Does not contribute explicitly to $\Delta(q^2)$ because it is longitudinal.

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

**However, now the regular part satisfies a "displaced" Ward identity:**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

$$\Gamma_\mu(0,r,-r) = \frac{\partial\mathcal{D}^{-1}(k^2)}{\partial k^\mu} - 2r_\mu\mathcal{C}(r^2)$$

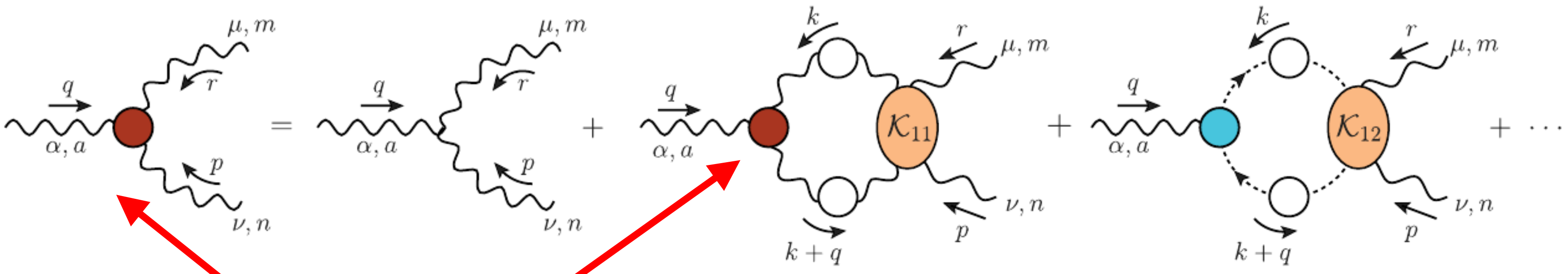
$$\mathcal{C}(r^2) := \left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}$$

**Displacement amplitude**

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\int_k k^2\mathcal{D}^2(k^2)\mathcal{C}(k^2)$$

# Derivation of the Schwinger pole Bethe-Salpeter equation

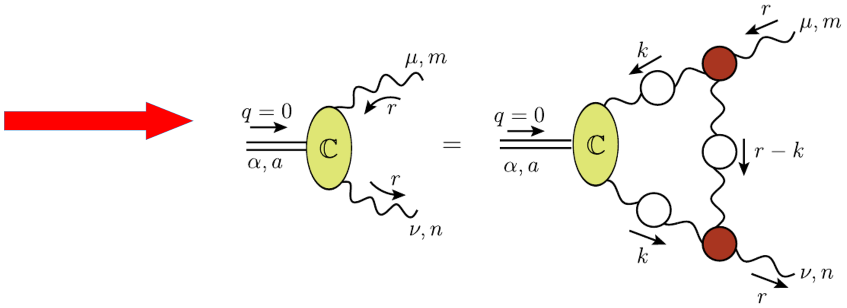
**We start with the Schwinger-Dyson (or more generaly nPI) equation for the vertex and assume the presence of a massless pole:**




$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \ldots$$

**Now multiply by $q^2$ and take $q = 0$. Only terms containing poles remain:**

- **Inhomogeneous Schwinger-Dyson** equation becomes a **Homogeneous Bethe-Salpeter** equation.



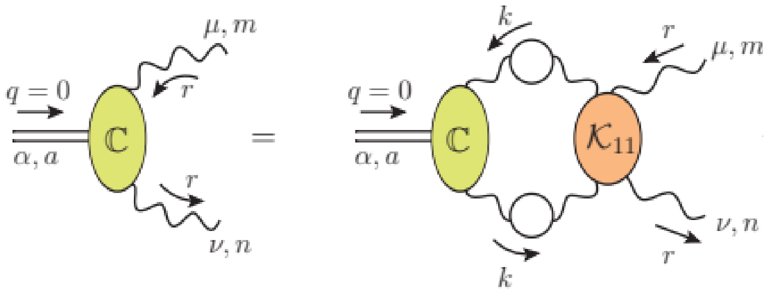


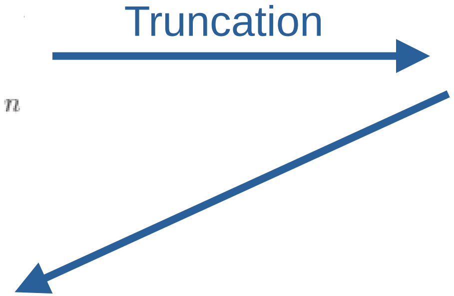


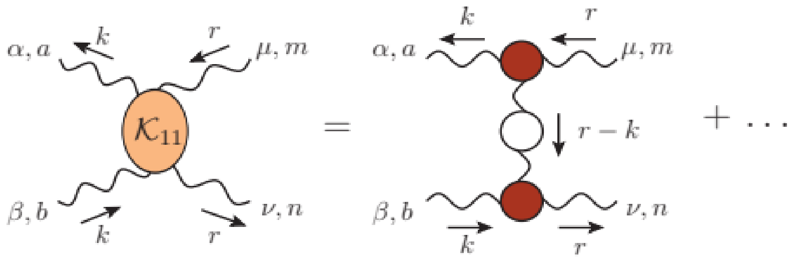
A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

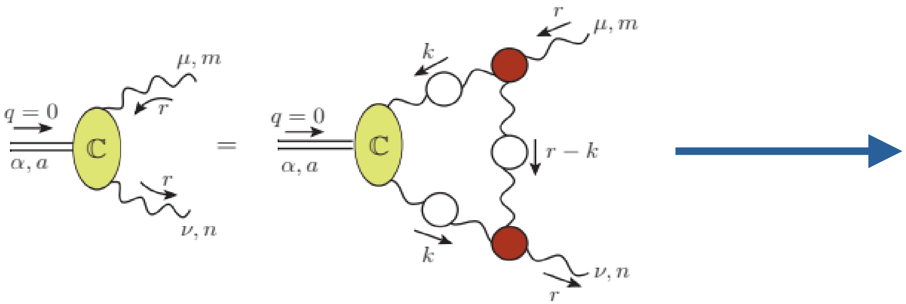
# One-gluon exchange approximation

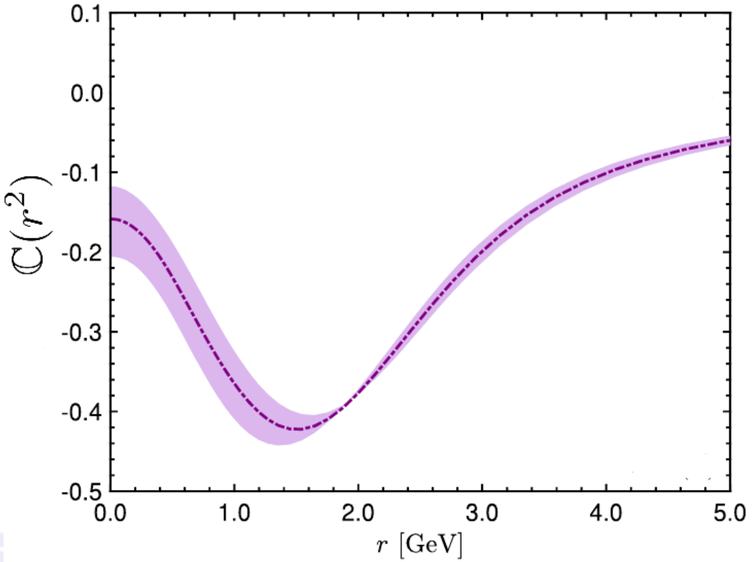
From the Bethe-Salpeter equation, we can






A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

# Method 2): Planar degeneracy

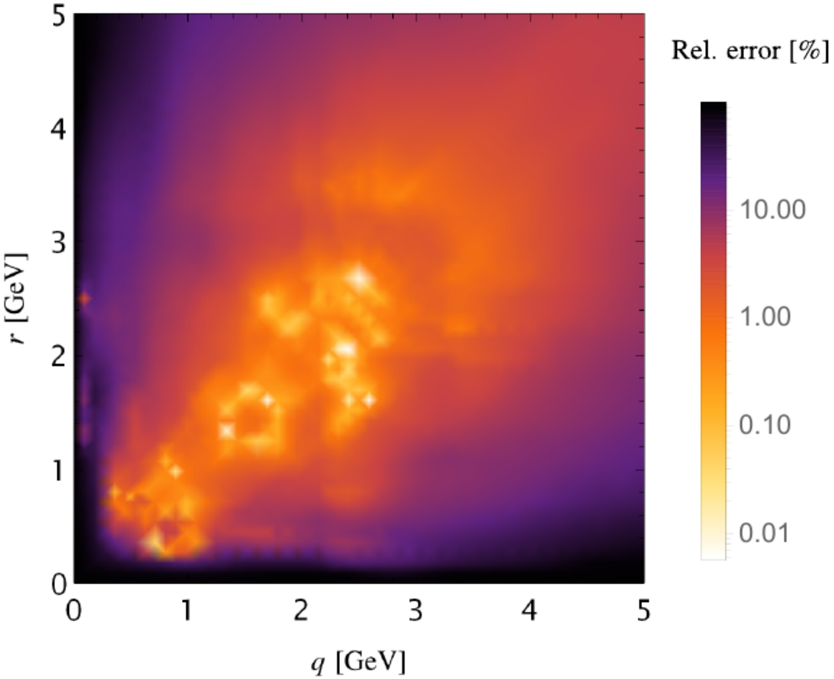
To quantify the accuracy of the approximation it is convenient to define

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)}{\overline{\mathcal{I}}^0_{\mathcal{W}}(q^2,r^2,p^2)}$$

**Planar degeneracy** $\longrightarrow$

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) \approx L_{\rm sg}(s^2)$$

Then we can measure the relative difference between $L_{\rm sg}(s^2)$ and $\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)$




- Approximation is accurate to within 1% near the diagonal.
- And within 10% for most of the kinematics.
- The measured error can then be propagated to the $\mathcal{W}(r^2)$



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

# Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

**Errors are propagated from known error of the planar degeneracy approximation.**

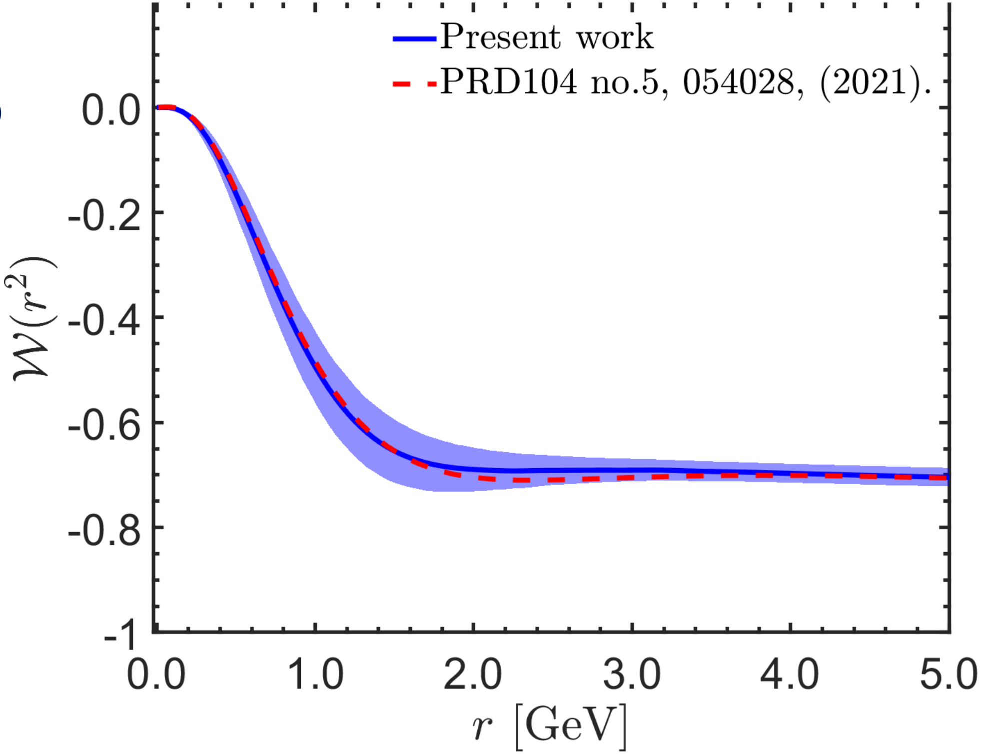
- **Result agrees well with previous calculation**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022)

- Previous result employed a particular *Ansatz* for three-gluon vertex.

- **New result stringently constrained by lattice simulation of the three-gluon vertex.**

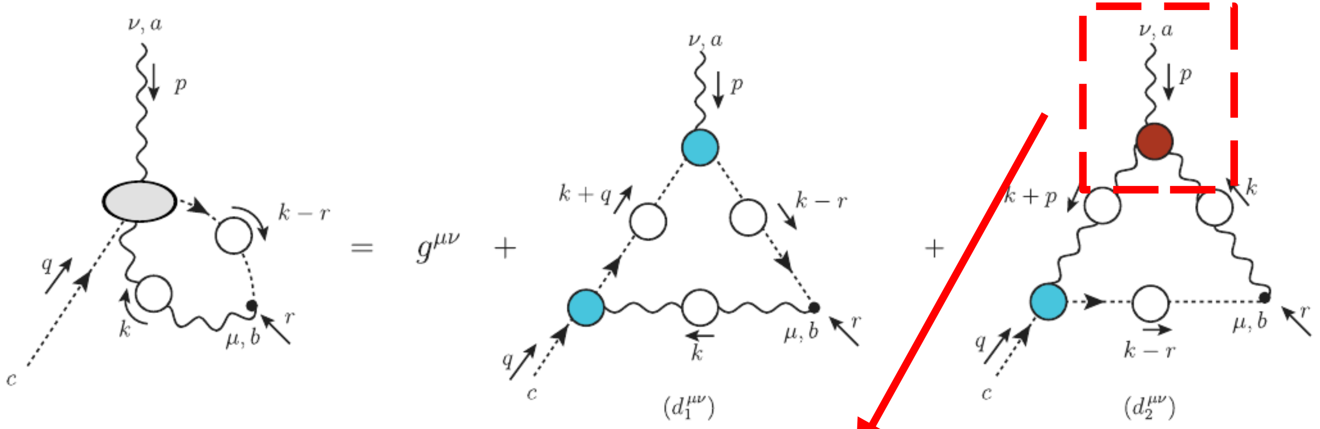
**Impact of three-gluon vertex under control**

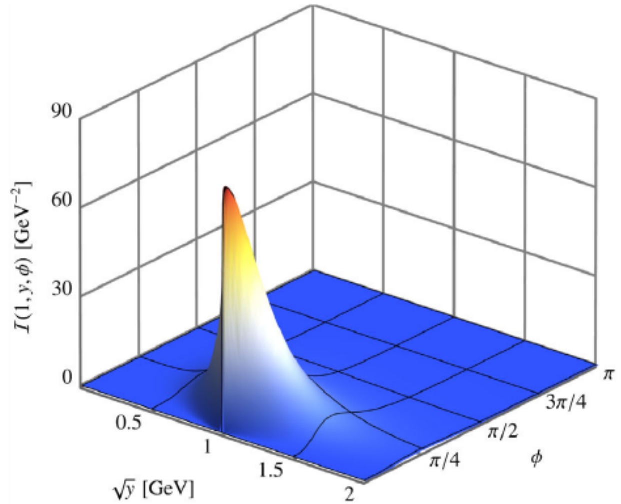



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

# Truncation error

The full Schwinger-Dyson equation for $\mathcal{W}(r^2)$ is



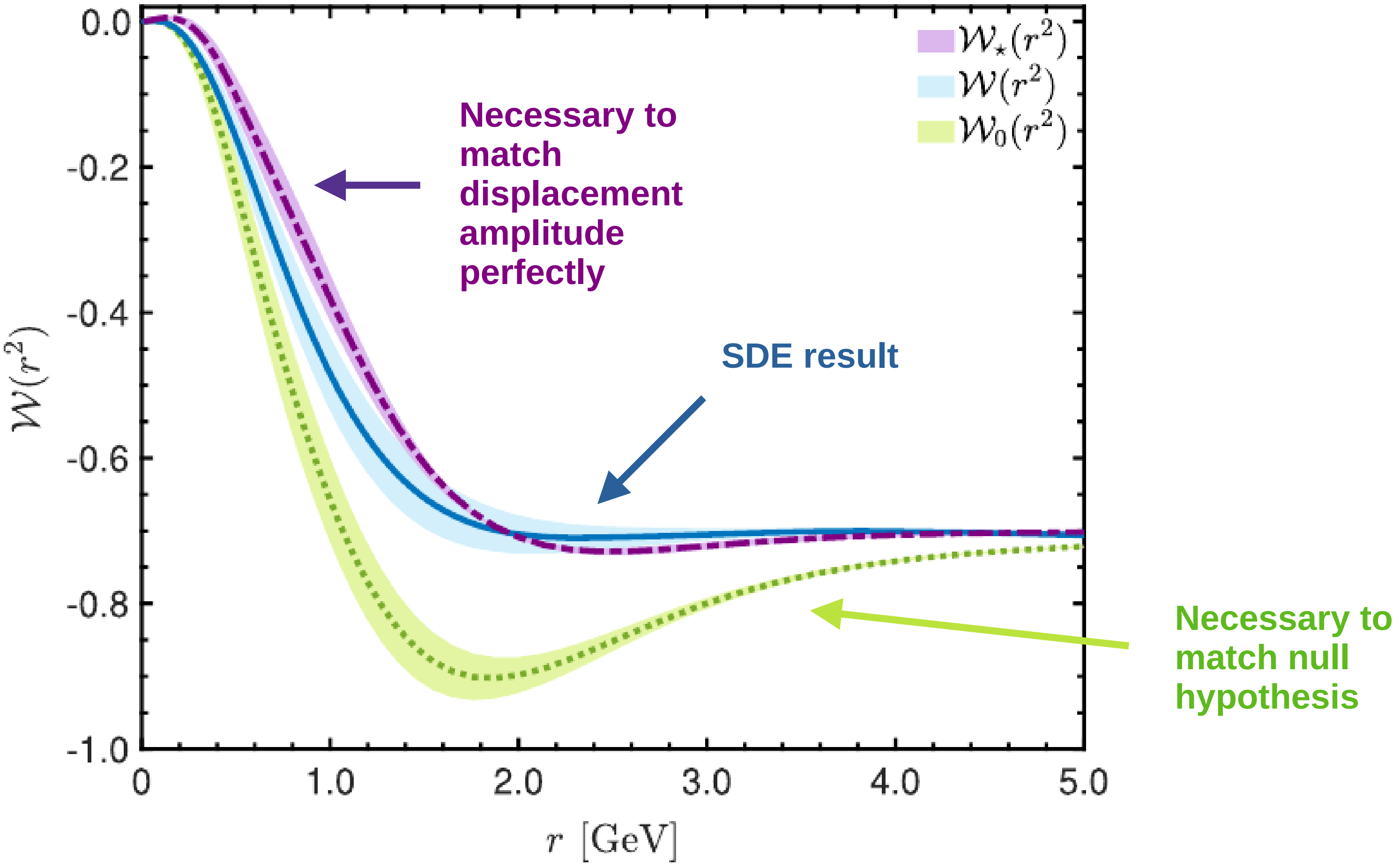

- Three-gluon vertex is a complicated object, with 14 tensor structures.

A. C. Aguilar, M. N. F., C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **99**, no.9, 094010 (2019).
J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980). [erratum: Phys. Rev. D **23**, 3085 (1981)].

- But $\mathcal{W}(r^2)$ integrand is sharply peaked, and is sensitive only to the particular projection $L_{\rm sg}(r^2)$ which is well determined by **lattice simulations.**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

# Truncation error



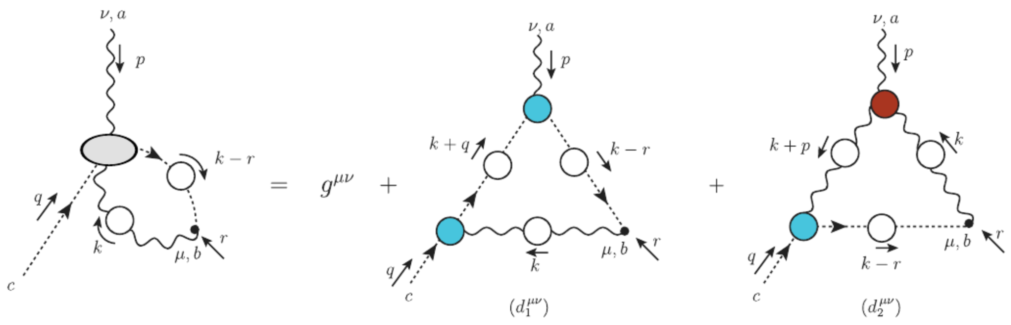


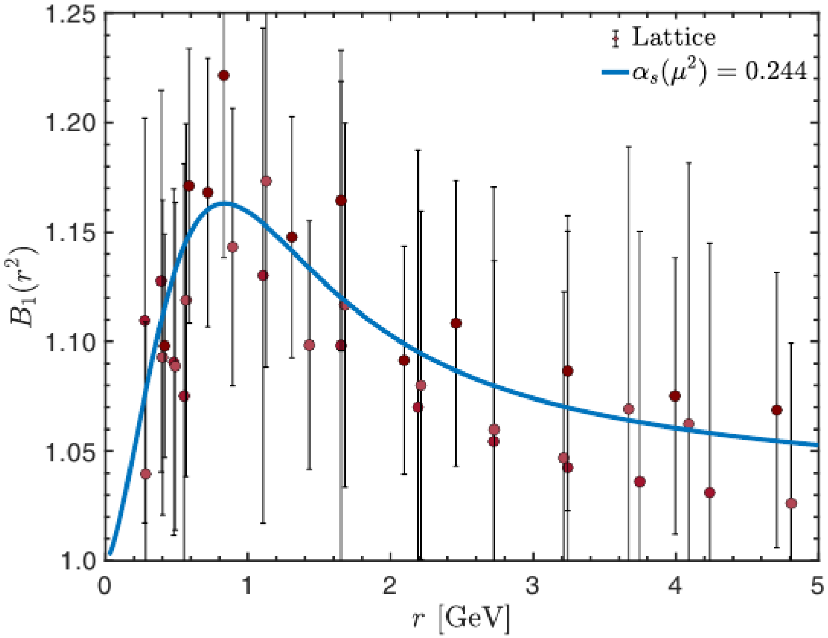


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

# Truncation error

The same truncation used to determine $\mathcal{W}(r^2)$, reproduces the available lattice data for the ghost-gluon vertex:




A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Lattice data from: A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).

# Inputs

The parametrizations to lattice data used were of the form:

$$\Delta^{-1}(r^2) = r^2\left[\frac{d}{1+(r^2/\kappa^2)}\ln\left(\frac{r^2}{\mu^2}\right) + A^{\delta}(r^2)\right] + \nu^2 R(r^2),$$

$$F^{-1}(r^2) = A^{\gamma}(r^2) + R(r^2),$$

where

$$A(r^2) := 1 + \omega \ln\left(\frac{r^2+\eta^2(r^2)}{\mu^2+\eta^2(r^2)}\right),$$

$$\eta^2(r^2) = \frac{\eta_1^2}{1+r^2/\eta_2^2},$$

$$R(r^2) = \frac{b_0 + b_1^2 r^2}{1+(r^2/b_2^2)+(r^2/b_3^2)^2} - \frac{b_0+b_1^2\mu^2}{1+(\mu^2/b_2^2)+(r^2/b_3^2)^2}.$$



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).