12th Workshop on Hadron Physics and Opportunities Worldwide

4-9 Aug, 2024 @ Dalian, China

Photon–photon fusion to three mesons

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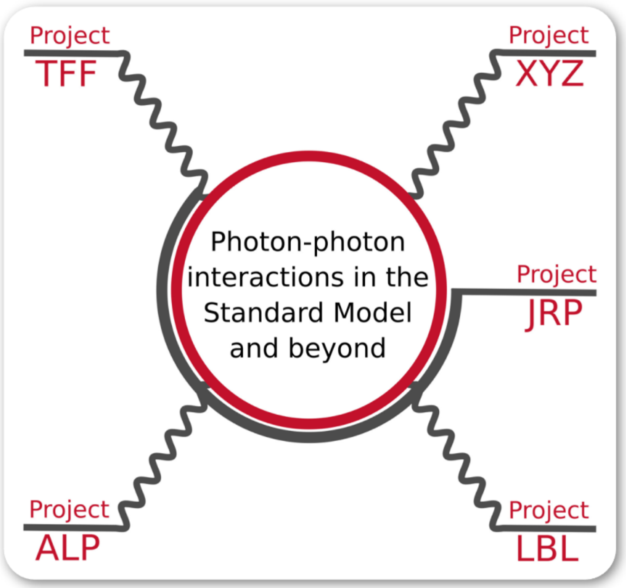


Research Unit (FOR 5327)

Deutsche Forschungsgemeinschaft

OUTLINE

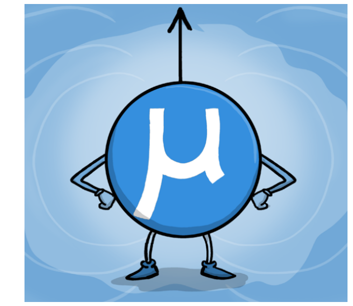
- Introduction
  - Muon g-2 in the standard model

- Photon-photon fusion to three mesons
  - $\gamma\gamma \to \pi^+\pi^-\pi^0$
  - $\gamma\gamma^* \to K\bar{K}^* \to K\bar{K}\pi$
  - $\gamma\gamma^* \to \eta\pi^+\pi^-$

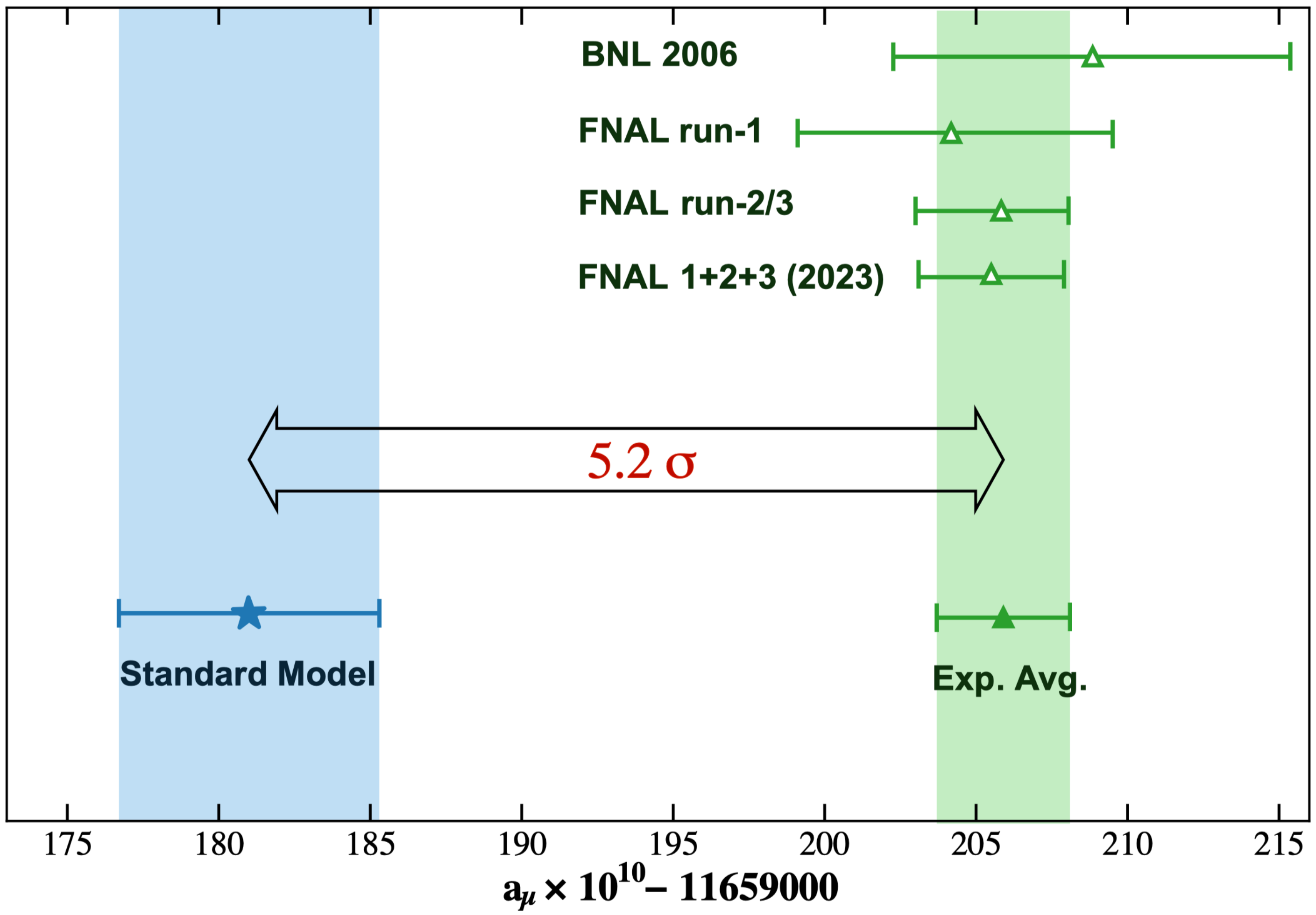
- Summary & outlook

Experiment vs. SM prediction

- Muon anomaly $a_\mu = (g_\mu - 2)/2$
  - measured with unprecedented precision (0.19 ppm)
  - calculated precisely in Standard Model (0.37 ppm)



White Paper:
Phys. Rept. 887, 1 (2020)



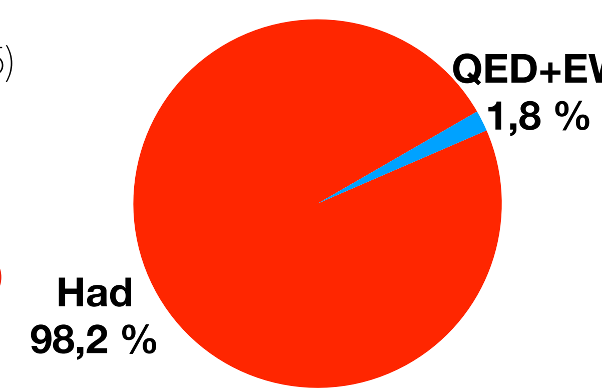
→ *Call for New Physics?*

- FNAL precision goal: $\Delta a_\mu^{\mathrm{exp}} \approx \pm 1.6 \cdot 10^{-10}$ (expected to be published in 2025)
- Decrease the theoretical error of the SM value

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{had}}$$

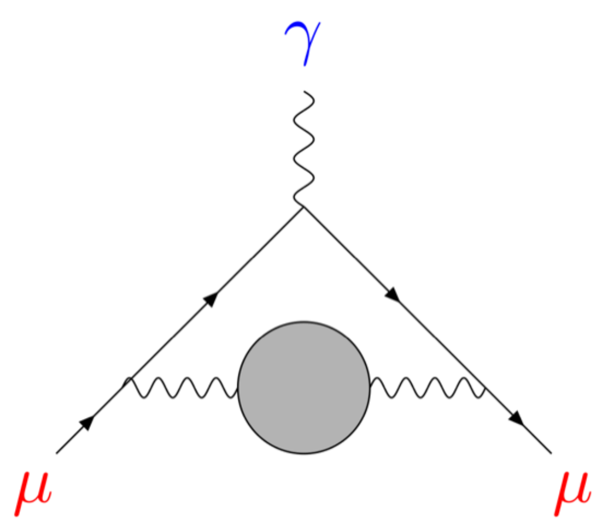
99.994%

- Non-perturbative regime of QCD
- Largest source of uncertainty

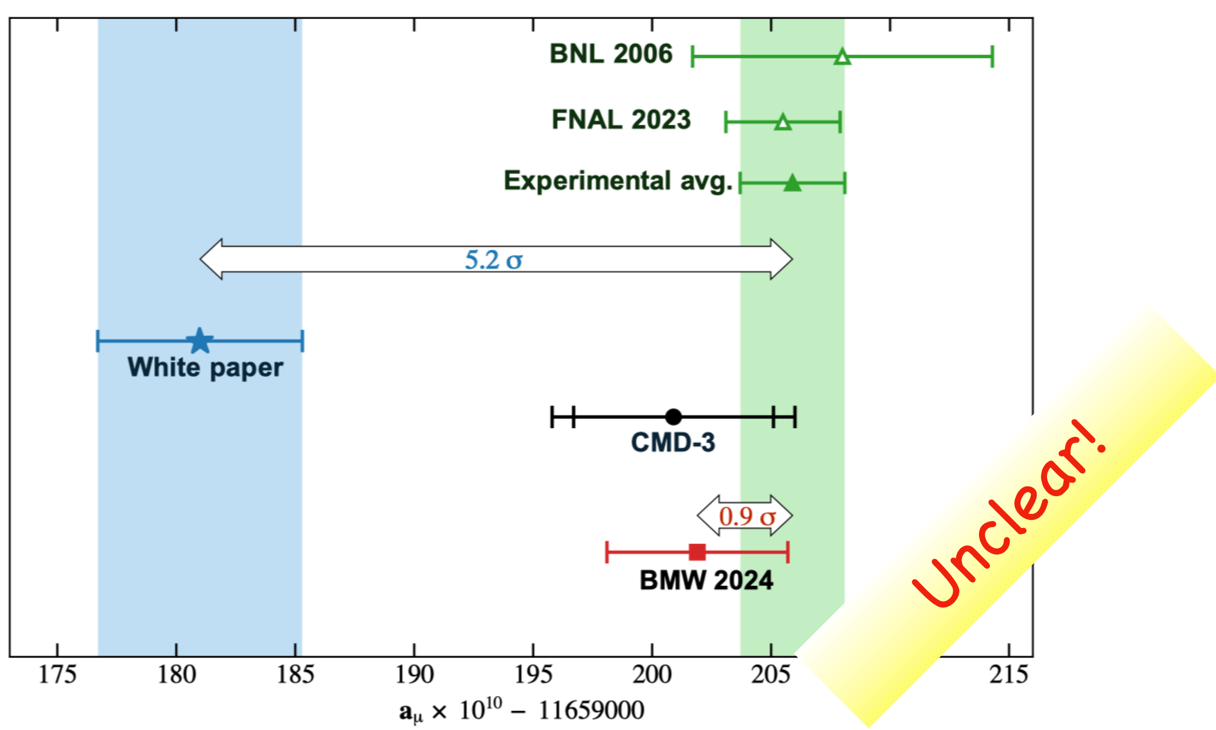


Hadronic contribution to a_μ

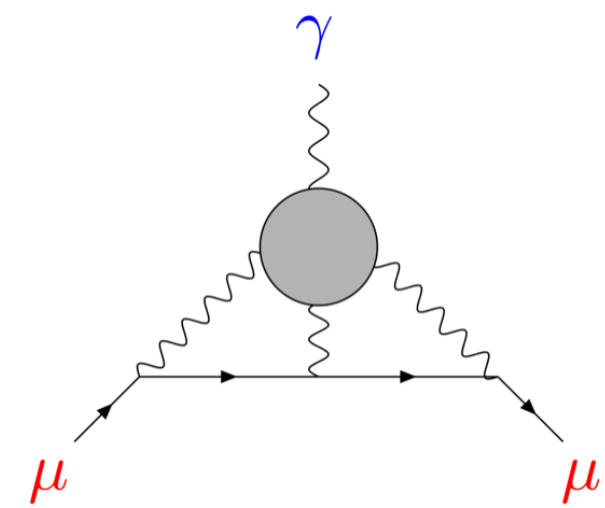
Hadronic vacuum polarization



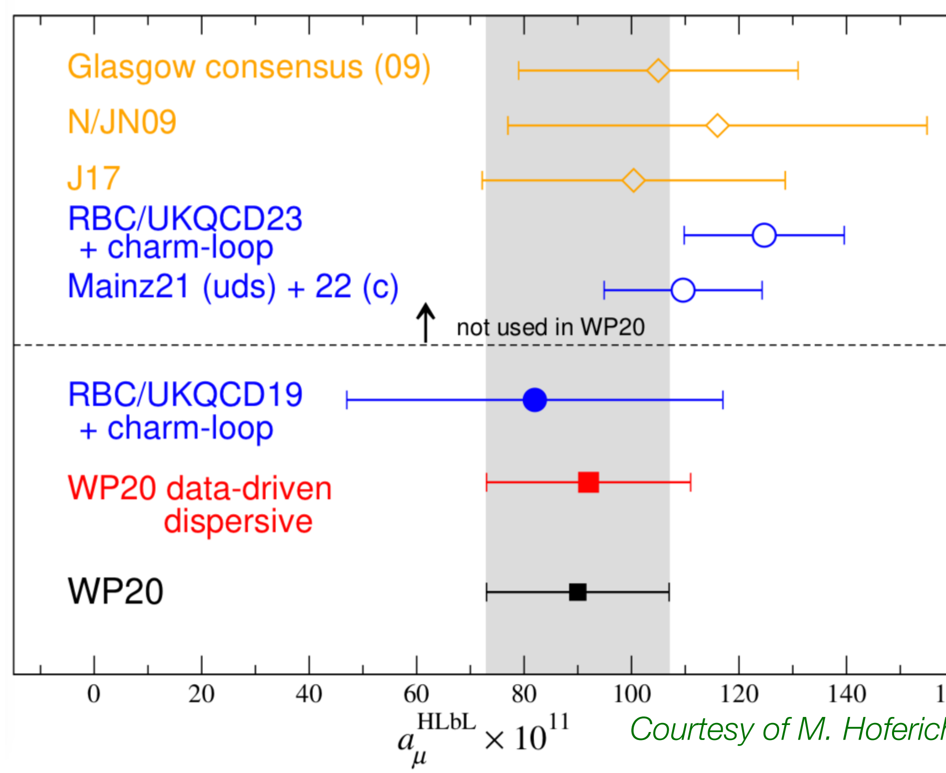
- Current **tensions** in HVP
  - ✓ WP data-driven (before CMD-3)
  - ✓ Lattice QCD: e.g. BMWc
  - ✓ Experiments: CMD-3 vs. CMD-2/others



Hadronic light-by-light scattering



- HLbL: more complicated than HVP
  - ✓ Consistent: WP / LQCD/phenomenology



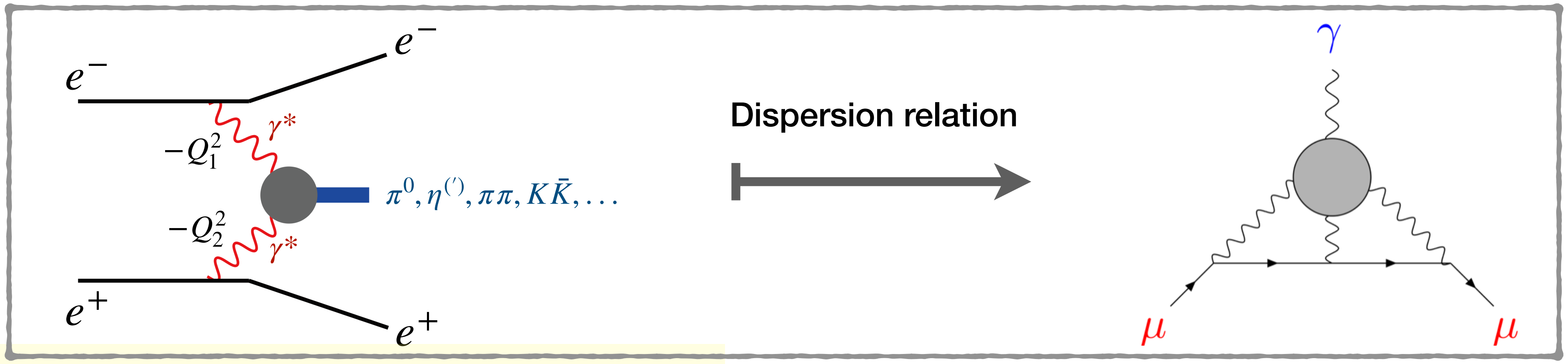
Courtesy of M. Hoferichter (2024)

  - ✓ Reduce uncertainty up to 10 % to match FNAL precision

HLbL scattering from white paper

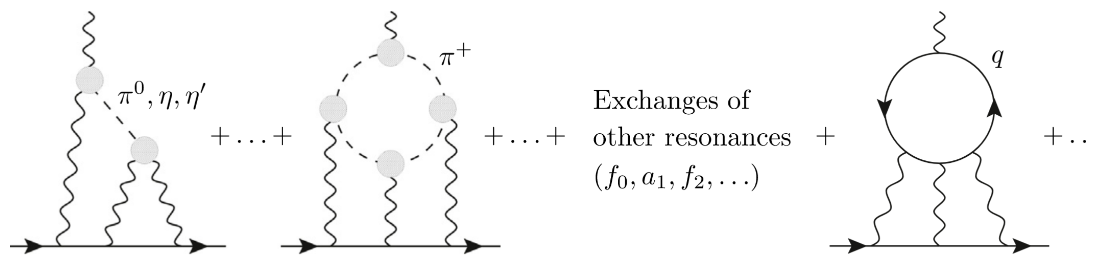
- $a_\mu^{\text{HLbL}} = 92(19) \cdot 10^{-11}$ in data-driven approach

G. Colangelo, et al. (2014–2017)



Photon-photon fusion at low Q^2

- Transition form factors (TFFs)
- (p.w.) amplitudes



- Detailed contributions

hadronic state	$a_\mu^{\text{HLbL}}\ [10^{-11}]$
pseudoscalar poles	$93.8^{+4.0}_{-3.6}$
pion box	$-15.9(2)$
S-wave $\pi\pi$ rescatt.	$-8(1)$
kaon box	$-0.5(1)$

Well determined!

hadronic state	$a_\mu^{\text{HLbL}}\ [10^{-11}]$
scalars+tensors $\gtrsim 1\,\text{GeV}$	$\sim -1(3)$
axial vectors	$\sim 6(6)$
short distance	$\sim 15(10)$
heavy quarks	$\sim 3(1)$

Major source of uncertainty!

Photon–photon fusion to three mesons

- Most relevant axial vectors and tensors

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
decay modes	$\eta\pi\pi$ (52%) $\pi\pi\pi\pi$ (33%) $K\bar{K}\pi$ (9%) $\gamma\rho^0$ (6%)	$K\bar{K}\pi$ (96%) $\eta\pi\pi$ (4%)	$\pi\pi\pi$ $KK\pi$	$\pi\pi\pi$ (70%) $\eta\pi$ (14%) $\omega\pi\pi$ (11%) $K\bar{K}$ (5%)

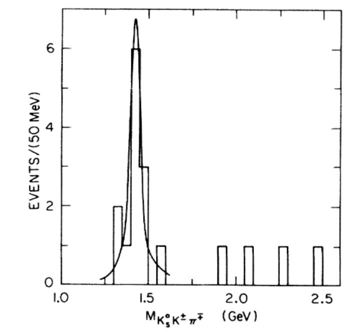
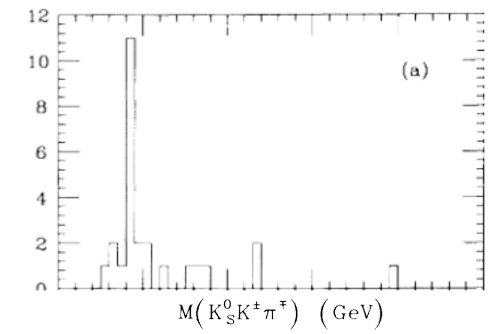
- To better control the uncertainties from those contributions
  - **Photon-photon fusion to three mesons** ⇒ TFFs
  - First priority listed in White Paper *Phys. Rept. 887, 1 (2020)*

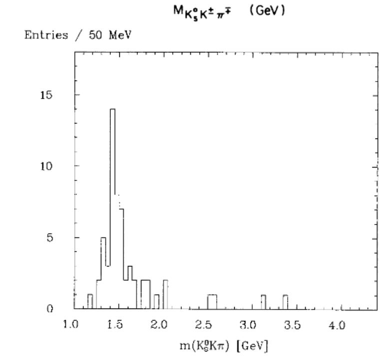
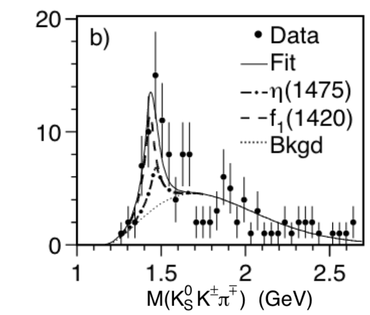
Table 14
Priorities for new experimental input and cross-checks.

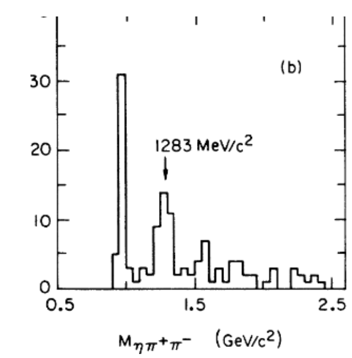
issue	experimental input [I] or cross-checks [C]
axials, tensors, higher pseudoscalars	$\gamma^{(*)}\gamma^* \to 3\pi,\ 4\pi,\ K\bar{K}\pi,\ \eta\pi\pi,\ \eta'\pi\pi$ [I]
missing states	inclusive $\gamma^{(*)}\gamma^* \to$ hadrons at 1–3 GeV [I]
dispersive analysis of $\eta^{(\prime)}$ TFFs	$e^+e^- \to \eta\pi^+\pi^-$ [I]

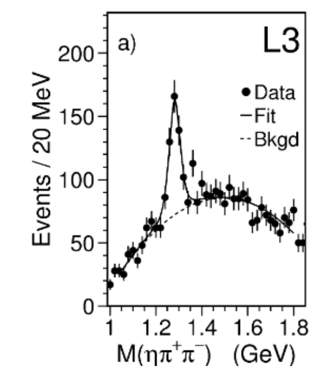
$\gamma^{(*)}\gamma^{(*)} \to 3$ mesons: experimental status

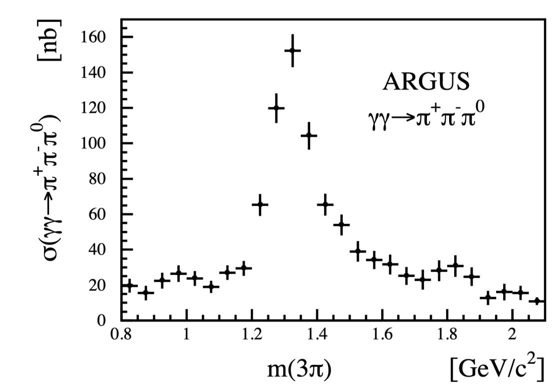
- $e^+e^- \to e^+e^-\gamma\gamma^* \to e^+e^- K_S^0 K^\pm \pi^\mp$
  - ✓ TPC/Two-Gamma Collaboration *PRL57, 2500(1986)*
  - ✓ Mark II Collaboration *PRL 59, 2016(1987)*
  - ✓ CELLO Collaboration *Z. Phys. C 42, 367-376 (1989)*
  - ✓ L3 Collaboration *JHEP 03 (2007) 018*
- $e^+e^- \to e^+e^-\gamma\gamma^* \to e^+e^- \eta\pi^+\pi^-$
  - ✓ Mark II Collaboration *PRL59,2012(1987)*
  - ✓ TPC/Two-Gamma Collaboration *PRD38,1(1988)*
  - ✓ L3 Collaboration *PLB 526 (2002) 269–277*
- $e^+e^- \to e^+e^-\gamma\gamma \to e^+e^- \pi^+\pi^-\pi^0$
  - ✓ L3 Collaboration *PLB 413, 147(1997)*
  - ✓ ARGUS Collaboration *Z. Phys. C 74, 469 (1997)*
- $e^+e^- \to e^+e^-\gamma\gamma^* \to e^+e^- \pi^+\pi^-\pi^0\pi^0$
  - ✓ TPC/Two-Gamma Collaboration *PRD 48 (1993) 3976-3987*

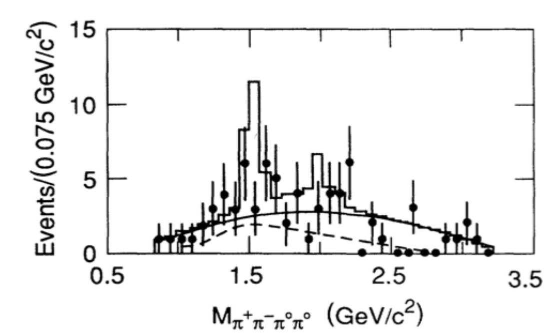







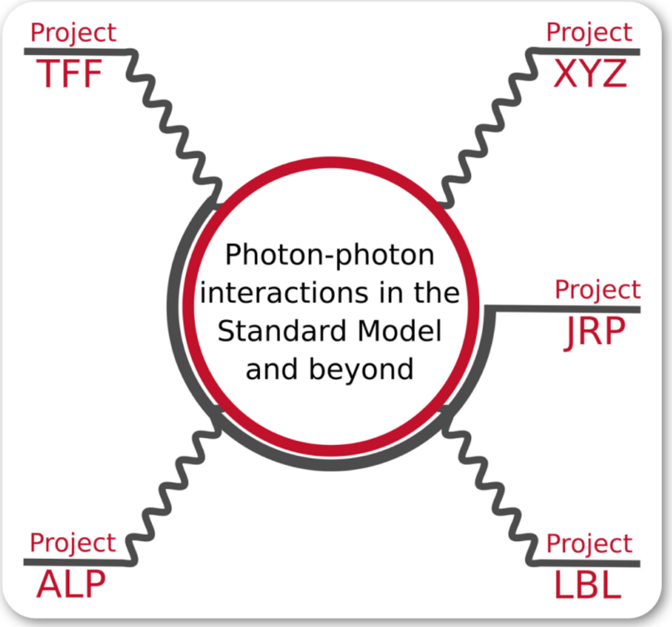




$\gamma^{(*)}\gamma^{(*)} \to 3$ mesons: experimental status

- Existing experimental data are rather old and low-statistic

- BESIII Collaboration is working on those processes
  - $\gamma\gamma^* \to \eta\pi^+\pi^-$ *On-going analysis, PhD project @ Mainz*
  - $\gamma\gamma^* \to K^\pm K^\mp \pi^0$ *Feasibility study by M. Sc. @ Mainz*
  - $\gamma\gamma^* \to \pi^+\pi^-\pi^0$ *Planned*



In this work

- Coordinate with the ongoing/planned BESIII measurements
- We focus on the photon-photon fusion to three mesons
  - $\gamma\gamma \to \pi^+\pi^-\pi^0$ to disentangle the $a_2(1320)$ contribution
  - $\gamma\gamma^* \to K\bar{K}^* \to K\bar{K}\pi$ to disentangle the $f_1(1420)$ contribution
  - $\gamma\gamma^* \to \eta\pi^+\pi^-$ to disentangle the $f_1(1285)$ contribution
- We propose the phenomenological models
  - Employ the effective Lagrangian approach
  - Include the necessary interference among the different amplitudes
  - Describe the available experimental data: L3, ARUGUS …
  - Investigate the possibilities to extract the TFFs model-independently
  - Could serve as the Monte Carlo generator for the data analysis of BESIII

XLR, I. Danilkin, M. Vanderhaeghen, PRD107,054037(2023)

XLR, I. Danilkin, M. Vanderhaeghen, arXiv: 2403.05091, PRD in press

XLR, I. Danilkin, M. Vanderhaeghen, in preparation

$\gamma^{(*)}\gamma^{(*)} \to \pi^+\pi^-\pi^0$ process

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
decay modes	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)
	$\pi\pi\pi\pi$ (33%)	$\eta\pi\pi$ (4%)	$KK\pi$	$\eta\pi$ (14%)
	$K\bar{K}\pi$ (9%)			$\omega\pi\pi$ (11%)
	$\gamma\rho^0$ (6%)			$K\bar{K}$ (5%)

XLR, I. Danilkin, M. Vanderhaeghen, PRD107,054037(2023)

Resonances in $\gamma^{(*)}\gamma^{(*)} \to \pi^+\pi^-\pi^0$

- $a_1(1260)$ production is only from $\gamma^{(*)}\gamma^* \to \pi^+\pi^-\pi^0$
  - $a_1(1260)$ is not a well-established resonance

$a_1(1260)$ [i] $I^G(J^{PC}) = 1^-(1^{++})$

T-Matrix Pole $\sqrt{s} = (1209^{+13}_{-10}) - i(288^{+45}_{-12})$ MeV
Mass (Breit-Wigner) $= 1230 \pm 40$ MeV [i]
Full width (Breit-Wigner) $= 250$ to 600 MeV [i]

- $a_2(1320)$ production is dominant in $\gamma^{(*)}\gamma^{(*)} \to \pi^+\pi^-\pi^0$
  - $a_2(1320)$ is a well-defined resonance

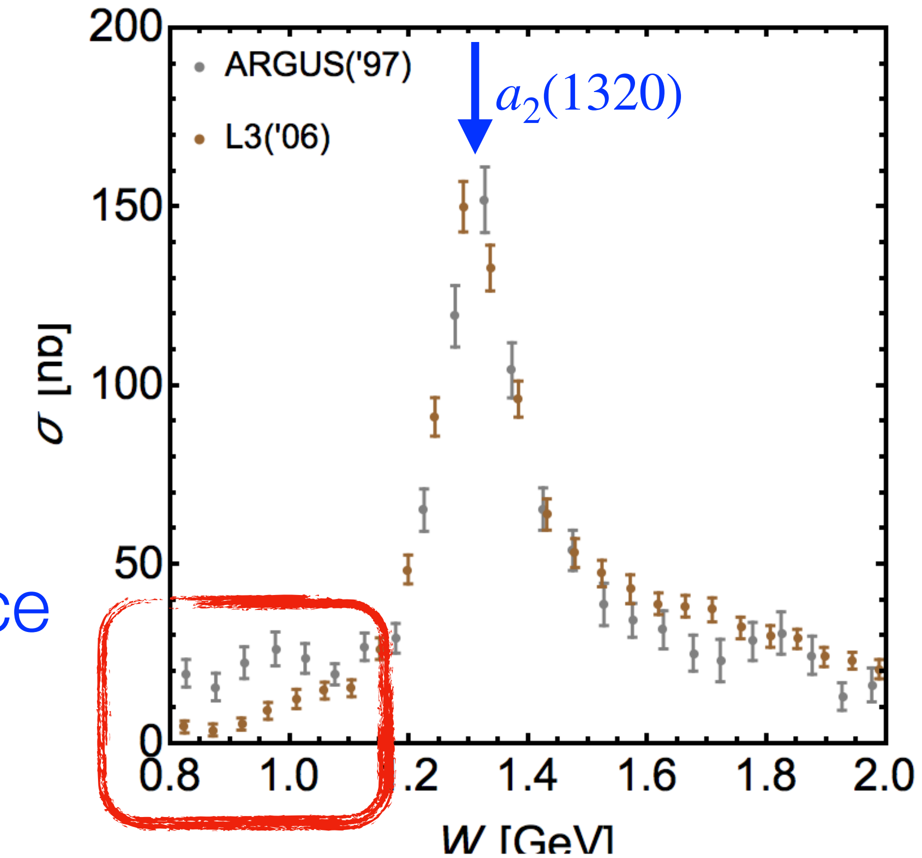
$a_2(1320)$ $I^G(J^{PC}) = 1^-(2^{++})$

T-Matrix Pole $\sqrt{s} = (1305\text{–}1321) - i(52\text{–}58)$ MeV
Mass (Breit-Wigner) $= 1318.2 \pm 0.6$ MeV $(S = 1.2)$
Full width $\Gamma = 107 \pm 5$ MeV

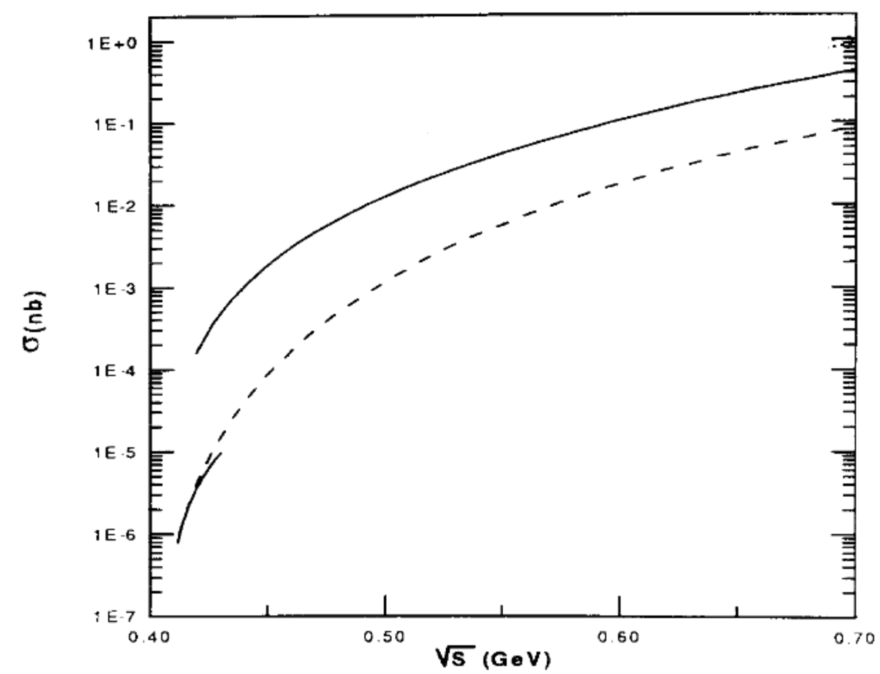
- Complicated mechanism involved in the $\pi^+\pi^-\pi^0$ final states
  - $\rho(770)$, $\sigma/f_0(500)$, $f_2(1270)$ resonances
- As a first step, we focus on the real photon-photon fusion case
  - $\gamma\gamma \to \pi^+\pi^-\pi^0$

Current status of $\gamma\gamma \to \pi^+\pi^-\pi^0$

- Experimental data of $\gamma\gamma \to \pi^+\pi^-\pi^0$
  - Data are rather old and have low statistics
    - ✓ ARGUS collab. *Z. Phys. C 74, 469 (1997)*
    - ✓ L3 collab. *PLB 413, 147(1997)*
      updated analysis EPJA27,199(2006).
  - Cross section data shows a significant difference in the low-energy region

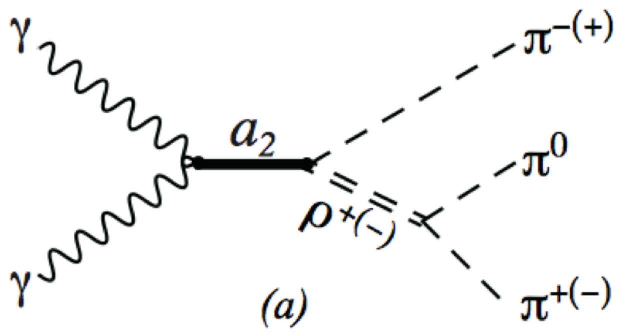


- Theoretical studies (very limited)
  - Current Algebra & Linear sigma model
    S. L. Adler, et al.,PRD(1971); T.F.Wong,PRL(1971); R. Aviv,PRD(1972)
  - Chiral perturbation theory up to NLO
    J.W. Bos, PLB337, 152(1994); P. Talavera, et al., PLB376, 186(1996)
  - Those studies focused on the very low energies
    - ✓ nearby the 3π threshold of the two-photon fusion
    - ✓ $0.41 < W < 0.7$ GeV



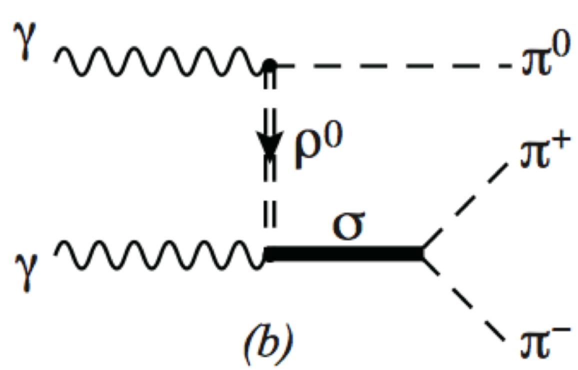
Phenomenological model for $\gamma\gamma \to \pi^+\pi^-\pi^0$

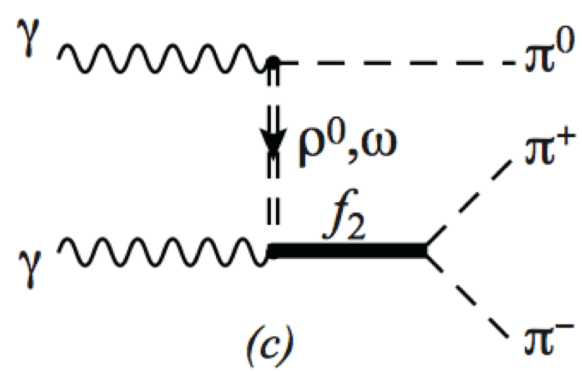
- Cover the low and the intermediate energy region
- Include all relevant channels within **effective Lagrangian method**
  - $a_2(1320)$ in s-channel (dominant)



- Vertices: described by effective Lagrangians
- Couplings: fixed by the decay widths
- Resonances: Breit-Wigner form
  - Employ the energy dependent width
  - Consider the Blatt-Weisskopf barrier factors

  - $\sigma/f_0(500)$ and $f_2(1270)$ in t/u-channel



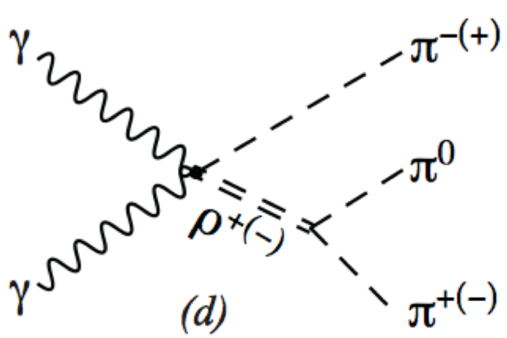


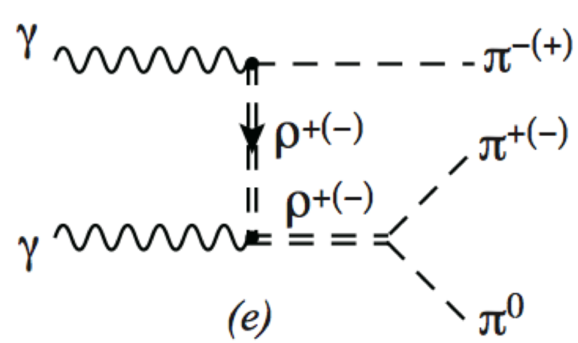
- $\sigma/f_0(500)$: S-wave **Omnes function** (I=0)

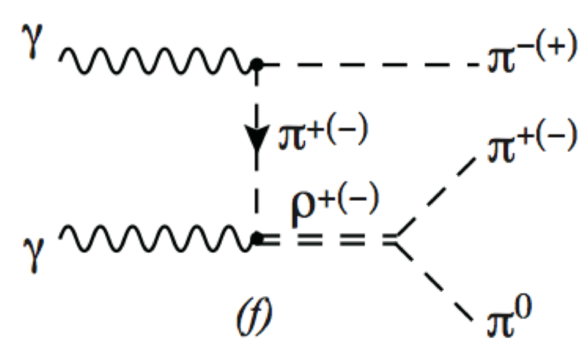
$$\Omega(x) = \exp\left\{\frac{x}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dx'}{x'}\frac{\delta(x')}{x'-x}\right\}$$

- Exchanged-ρ/ω: **Regge propagator**
  - correct behavior of the amplitude at high energy region

  - Charged $\rho(770)$, π intermediate channels







- **Essential at low-energy region**

Phenomenological model for $\gamma\gamma \to \pi^+\pi^-\pi^0$

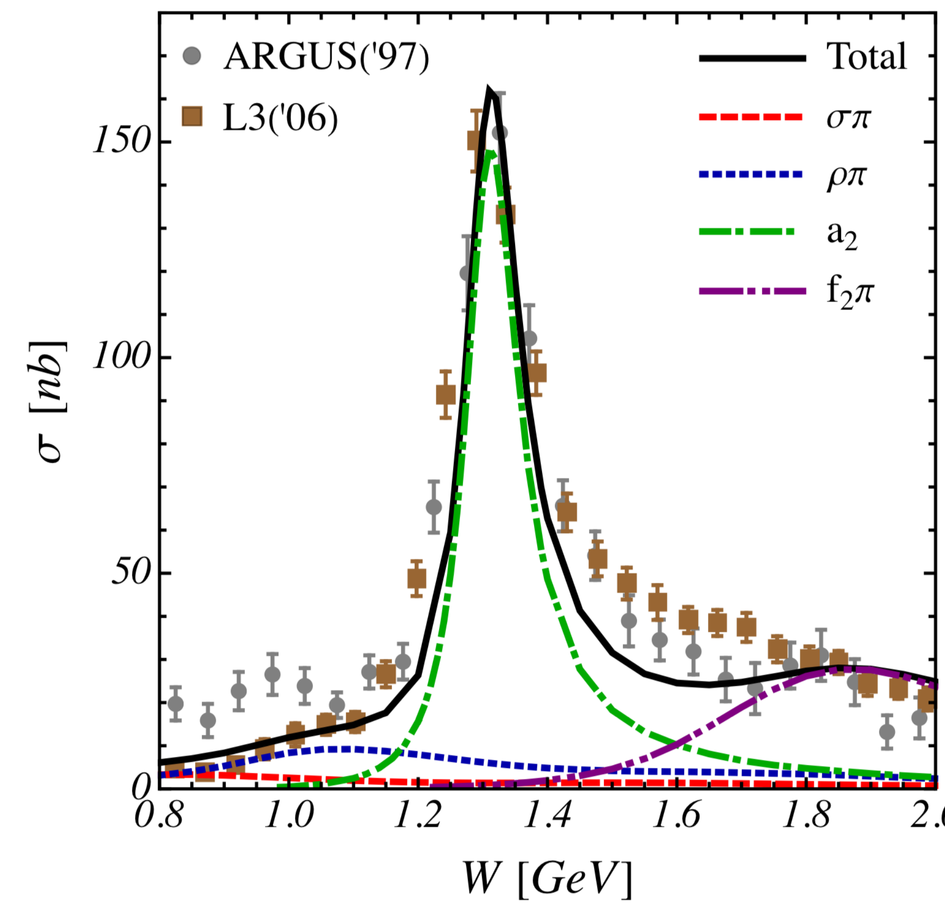
- Cover the low and the intermediate energy region
- Include all possible channels within effective Lagrangian method
  - **ONLY one free parameter** in our model
    - ✓ $g_{f_2\rho\gamma}$ is fitted by the total cross section at large energy, e.g. W=1.85 GeV
    - ✓ Other couplings are fixed via the relevant decay widths

$g_{a_2\gamma\gamma}$	$g_{a_2\rho\pi}$	$g_{\rho\pi\gamma}$	$g_{\rho\pi\pi}$	$g_{f_2\pi\pi}$	$g_{f_2\rho\gamma}$
0.151	4.9	0.102	5.97	23.67	-27.5

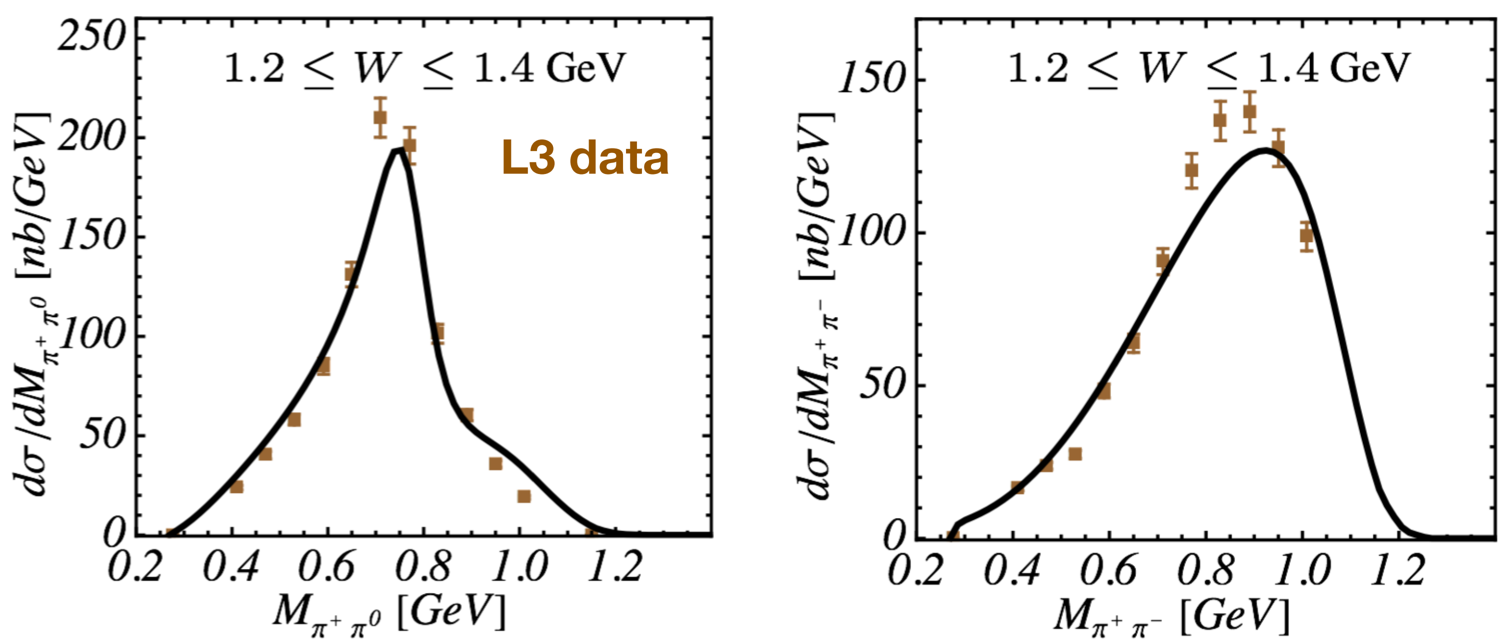
- Consider the interference among the different amplitudes
- Describe the experimental data of ARGUS and L3

Description of ARGUS and L3 data

- Total cross section
  - ARGUS and L3 data: significant differences in the low-energy region
  - **Our result: consistent with L3 data**
  - $a_2(1320)$ production: dominant contribution
  - $f_2(1270)\pi^0$ mechanism: needed to achieve the reasonable description of data (W>1.5 GeV)



- Invariant mass distribution



XLR, I. Danilkin, M. Vanderhaeghen, PRD107,054037(2023)

$\gamma\gamma^* \to K\bar{K}^* \to K\bar{K}\pi$ process

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XLR, I. Danilkin, M. Vanderhaeghen, arXiv: 2403.05091, PRD in press

$f_1(1420)$ production in $\gamma\gamma^*$ fusion

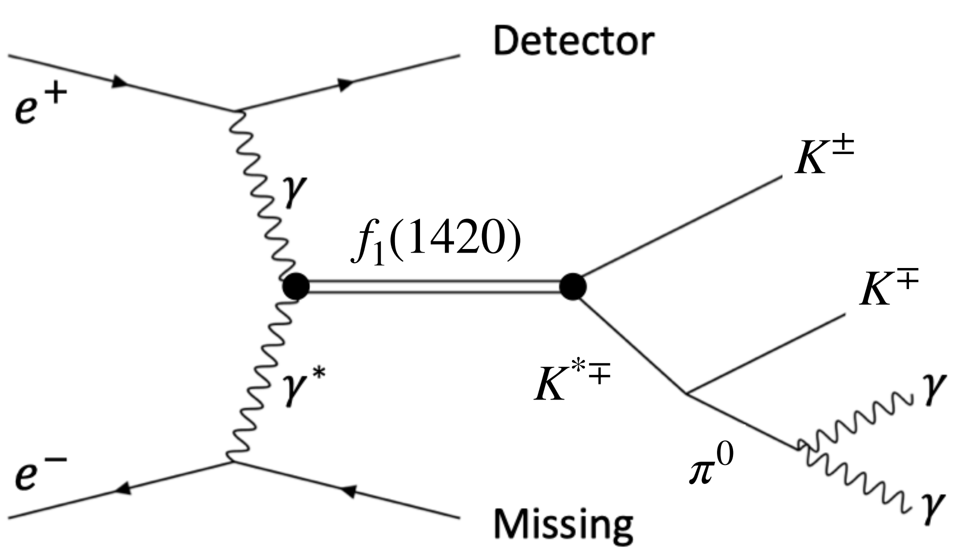
- Ideal channel to study the $f_1(1420)$ TFFs

$$\gamma^{(*)}\gamma^* \to f_1(1420) \xrightarrow{96\,\%} K\bar{K}^*(892) \to K\bar{K}\pi$$

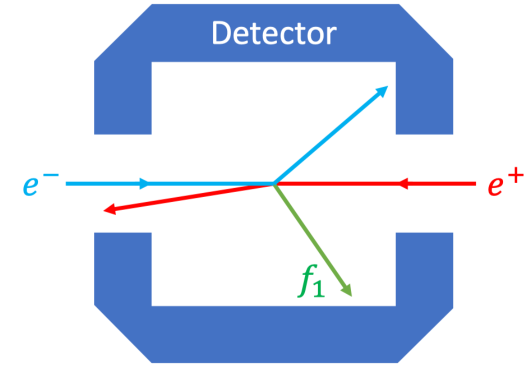
  - L3 experimental data *JHEP 03(2007) 018*

    ✓ $e^+e^- \to e^+e^-\gamma\gamma^* \to e^+e^-K_S^0K^\pm\pi^\mp$ with $Q^2 \in [0.01 - 7.0]$ GeV^2

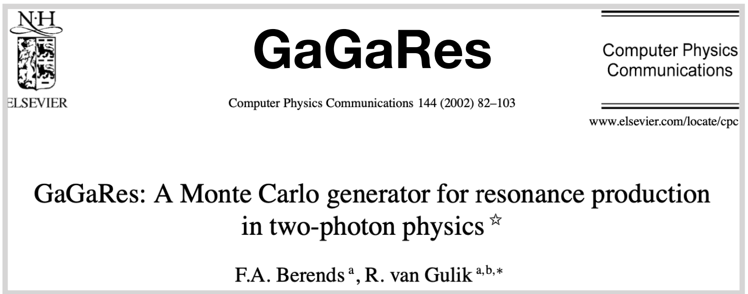
  - Ongoing analysis of BESIII measurement *Master Thesis by N. Effenberger @ Mainz*



✓ Single-tag technique



✓ Monte Carlo generator


GaGaRes

Computer Physics Communications

ELSEVIER

Computer Physics Communications 144 (2002) 82–103

www.elsevier.com/locate/cpc

GaGaRes: A Monte Carlo generator for resonance production in two-photon physics ☆

F.A. Berends a, R. van Gulik a,b,*

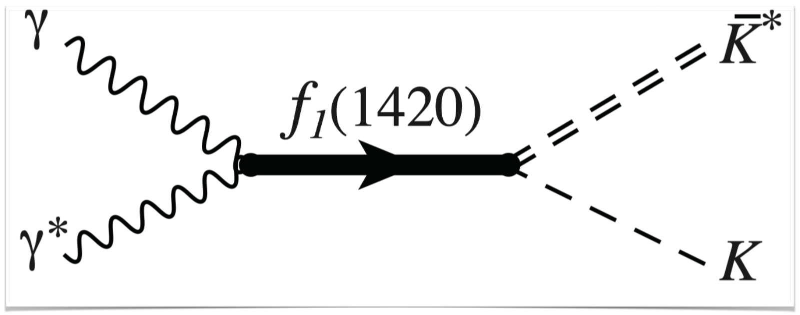
No interference among different channels

- Provide a more realistic MC generator for data analysis
  - We focus on the $\gamma\gamma^* \to K\bar{K}^*(892)$ process
  - Build up a phenomenological model in the $f_1(1420)$ region

    ✓ within the effective Lagrangian approach

Pheno. model for $\gamma\gamma^* \to K^{\pm}K^{*\mp}$

- S-channel: the $f_1(1420)$ production mechanism



- Transition amplitude of $\gamma(q_1,\lambda_1) + \gamma^*(q_2,\lambda_2) \to f_1(1420)$

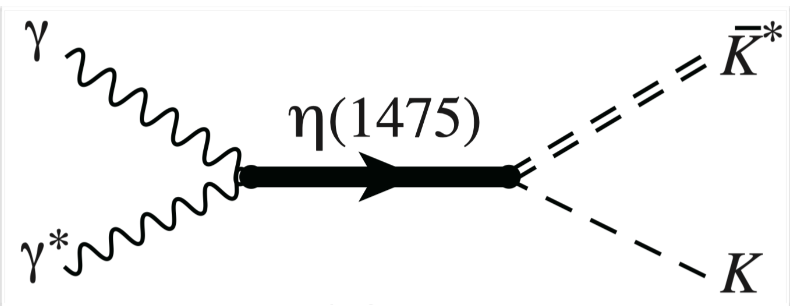
$$\mathcal{M}_{\gamma\gamma^*\to f_1} = ie^2\,\varepsilon_\mu(q_1,\lambda_1)\,\varepsilon_\nu(q_2,\lambda_2)\varepsilon^{\omega*}(q_1+q_2,\Lambda\quad) \times \left\{h_1(q_1,q_2)F^{TT}_{f_1\gamma^*\gamma^*}(0,Q_2^2) + h_2(q_1,q_2)F^{TL}_{f_1\gamma^*\gamma^*}(0,Q_2^2)\right\}$$

- Two $f_1(1420)$ TFFs estimated by the quark model

$$F^{TL}_{f_1\gamma^*\gamma^*}(0,Q_2^2) = -F^{TT}_{f_1\gamma^*\gamma^*}(0,Q_2^2) = \frac{F^{TL}_{f_1\gamma^*\gamma^*}(0,0)}{\left(1+Q_2^2/\Lambda_{f_1}^2\right)^2}$$

- Dipole form
- ~$1/Q^4$ asymptotic behavior

- S-channel: the $\eta(1475)$ production mechanism



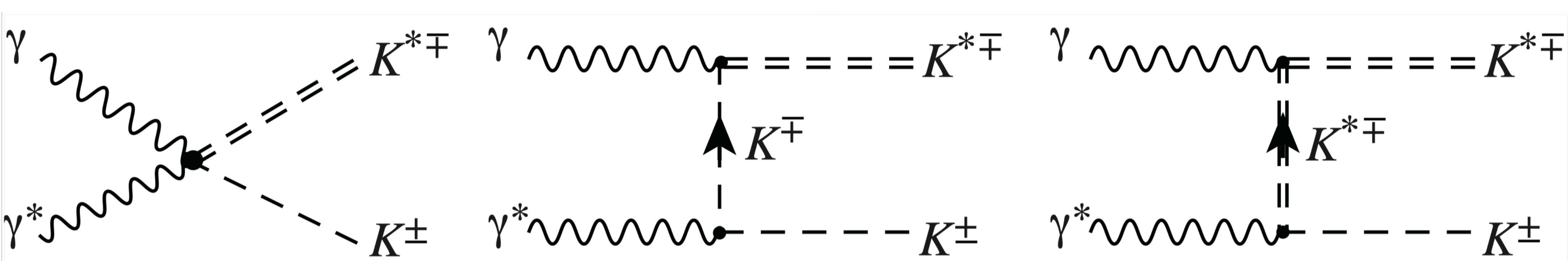
- Transition amplitude of $\gamma(q_1,\lambda_1) + \gamma^*(q_2,\lambda_2) \to \eta(1475)$

$$\mathcal{M}_{\gamma^*\gamma^*\to\bar\eta} = ie^2\varepsilon^\mu(q_1,\lambda_1)\varepsilon^\nu(q_2,\lambda_2)h(q_1,q_2)F_{\bar\eta\gamma^*\gamma^*}(0,Q_2^2)$$

- $\eta(1475)$ TFF: $F_{\bar\eta\gamma^*\gamma^*}(0,Q_2^2) = \dfrac{F_{\bar\eta\gamma^*\gamma^*}(0,0)}{1+Q_2^2/\Lambda_{\bar\eta}^2}$

- Monopole form
- Consistent with pion/eta TFF

- Non-resonant contributions via the K- and K*-exchange channels



- Amplitude is constructed using the Reggeized propagators to obtain the correct high-energy behavior

Pheno. model for $\gamma\gamma^* \to K^{\pm}K^{*\mp}$

- Use effective Lagrangian approach to evaluate the amplitudes

Table I. Values of resonance (R) parameters used in our model.

	m_R [MeV]	Γ_R [MeV]	coupling	TFF
			$g_{\bar{\eta}KK^*}$	$F_{\bar{\eta}\gamma^*\gamma^*}(0,0)$
$\eta(1475)$	1475	90	2.04	$0.0414\ \mathrm{GeV}^{-1}$
			$g_{f_1KK^*}$	$F^{TL}_{f_1\gamma^*\gamma^*}(0,0)$
$f_1(1420)$	1426.3	54.5	1.027	0.401
			$g_{\gamma KK^*}$	
$K^*(892)$	893.5	51.4	0.203	

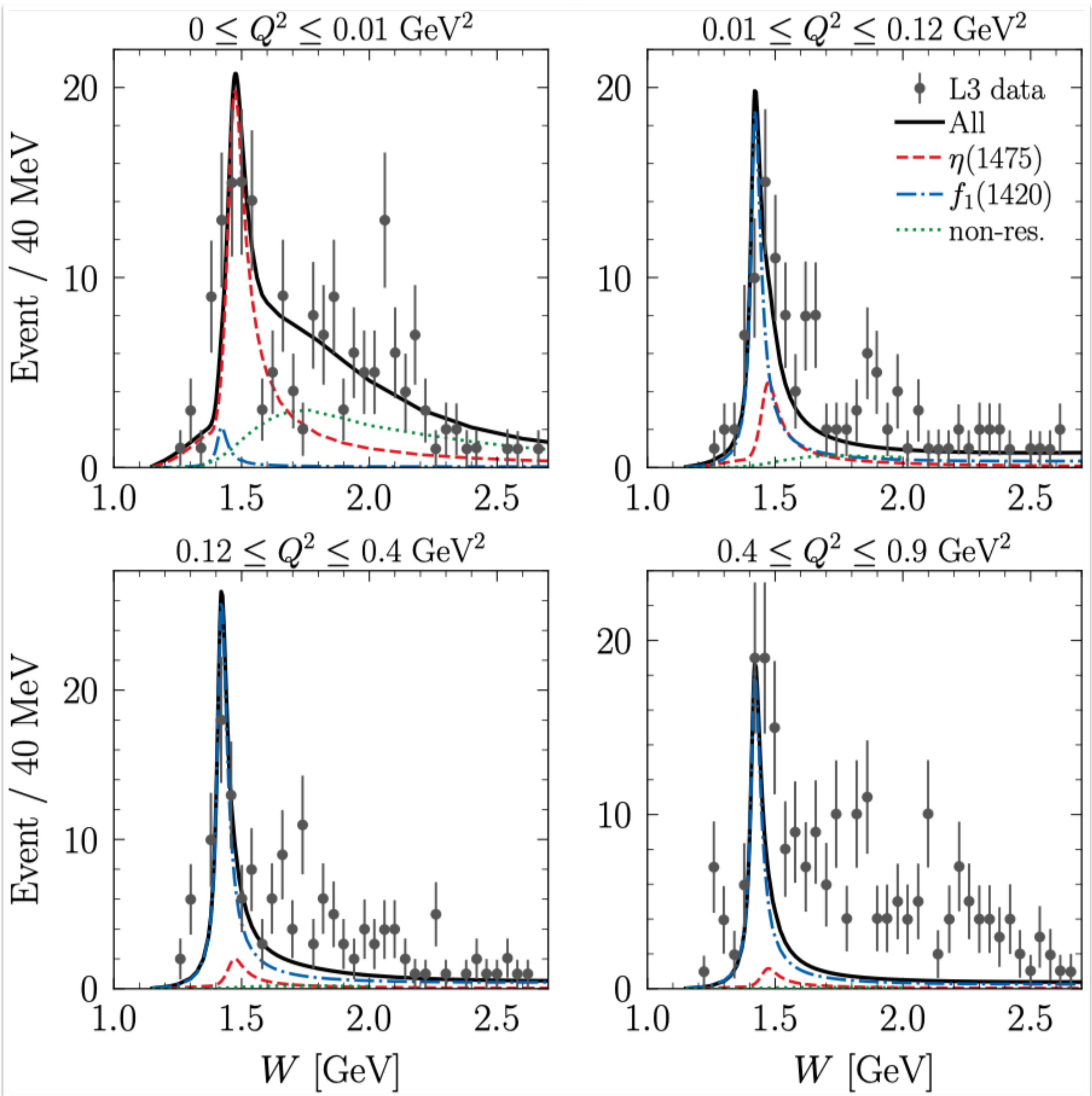
- ONLY one free parameter: dipole mass of $f_1(1420)$ TFFs

$$F^{TL}_{f_1\gamma^*\gamma^*}(0,Q_2^2) = -F^{TT}_{f_1\gamma^*\gamma^*}(0,Q_2^2) = \frac{F^{TL}_{f_1\gamma^*\gamma^*}(0,0)}{\left(1+Q_2^2/\Lambda_{f_1}^2\right)^2}$$

  - Determined by fitting L3 data of $e^+e^- \to e^+e^-\gamma\gamma^* \to e^+e^-K_S^0K^{\pm}\pi^{\mp}$
  - Consider the $K^{*-} \to (K\pi)^-$ decay effectively

Description of L3 data

- Reasonable description of L3 events: $e^+e^- \to e^+e^- K_S^0 K^\pm \pi^\mp$
  - **Constructive interference** among $\eta(1475), f_1(1420),$ and non-res. channels
  - Dipole mass scale in $f_1(1420)$ TFFs: $\Lambda_{f_1} = 920$ MeV

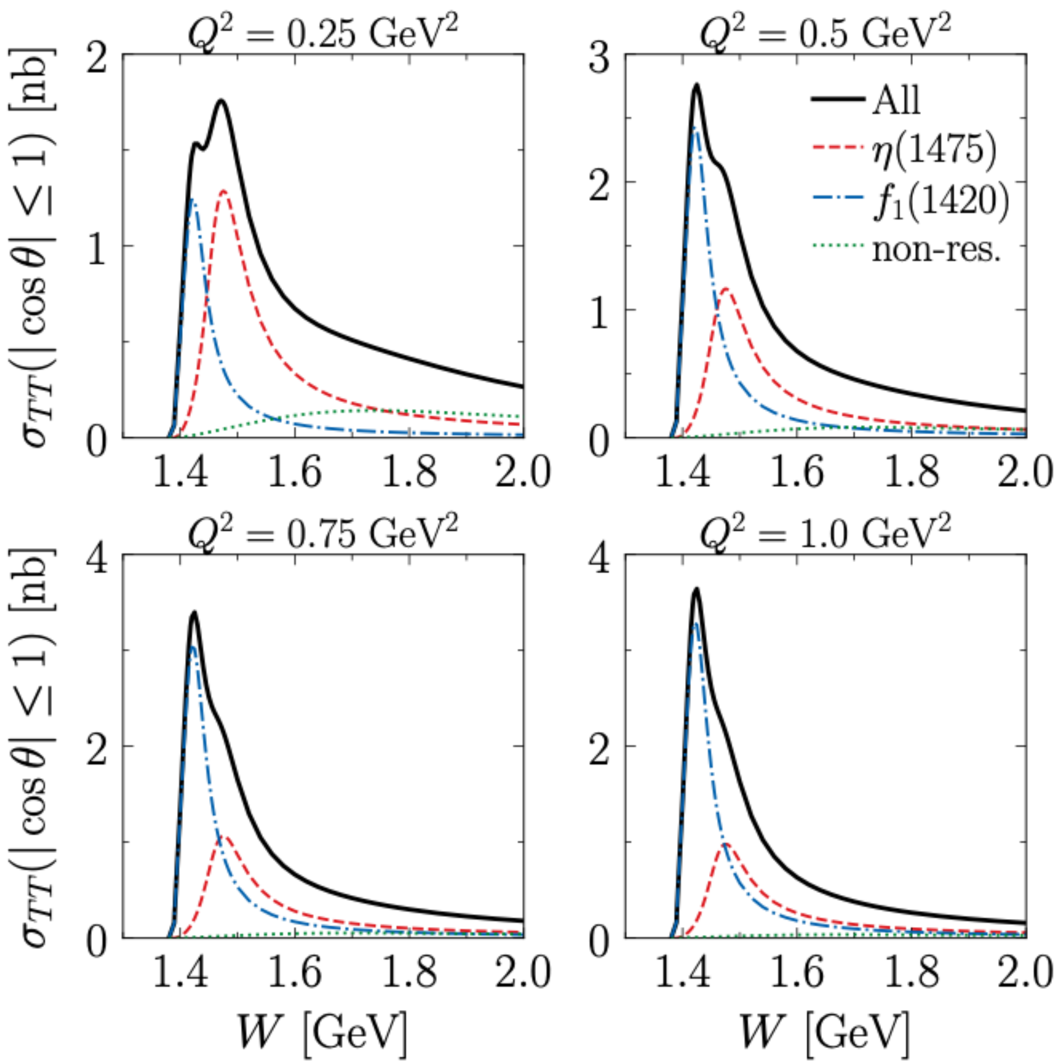


- **η(1475) contribution**
  - dominant at small Q^2 (quasi-real)
  - decreases with Q^2 increases
- **f1(1420) contribution**
  - suppressed at small Q^2
  - dominant when Q^2>0.12 GeV^2

Theoretical calculation of the $e^+e^- \to e^+e^- K_S^0 K^\pm \pi^\mp$ process is sensitive to the f1(1420) TFF

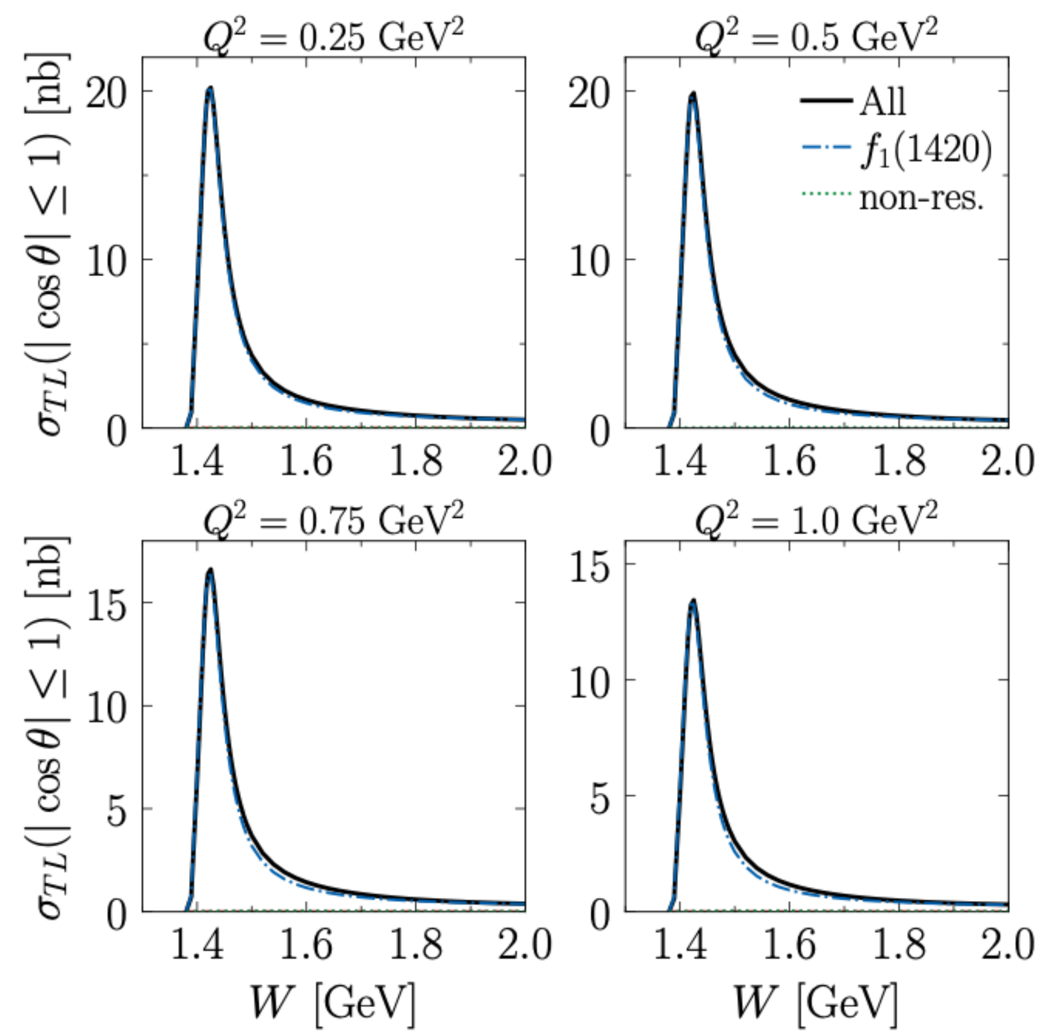
Prediction of cross section $\gamma\gamma^* \to K^\pm K^{*\mp}$

- Transverse cross section



- **Broad peak ~1.45 GeV with Q^2=0.25**
  - Interference of η(1475) and f1(1420)
  - Consistent with a preliminary analysis of BESIII *N. Effenberger Master Thesis*
- **Large Q^2, f_1(1420) channel dominant**

- Transverse-longitude cross section



- No η(1475) contribution to σ_{TL}
- f_1(1420) chanel dominant
- **Model-independent to extract f_1(1420) TFFs**

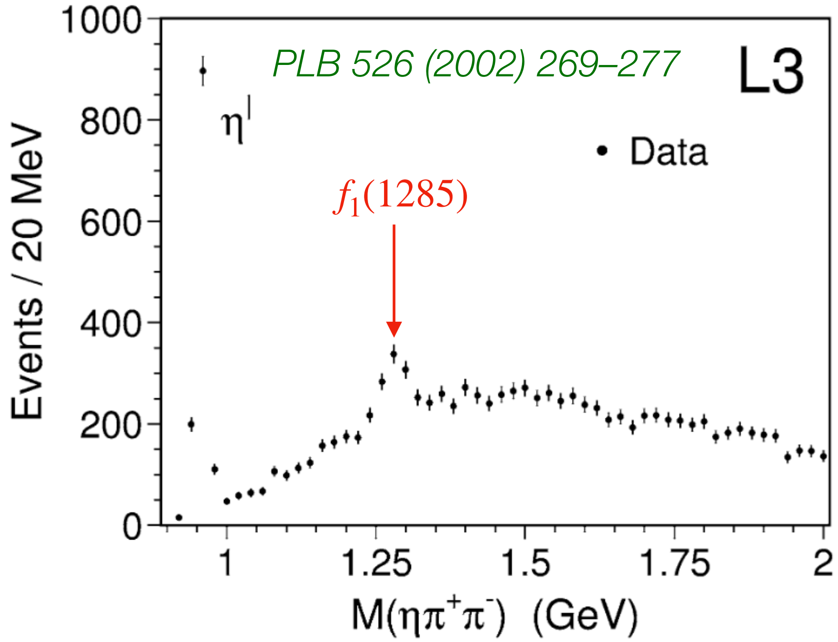
$\gamma\gamma^* \rightarrow \eta\pi\pi$ process

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
decay modes	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)
	$\pi\pi\pi\pi$ (33%)	$\eta\pi\pi$ (4%)	$KK\pi$	$\eta\pi$ (14%)
	$K\bar{K}\pi$ (9%)			$\omega\pi\pi$ (11%)
	$\gamma\rho^0$ (6%)			$K\bar{K}$ (5%)

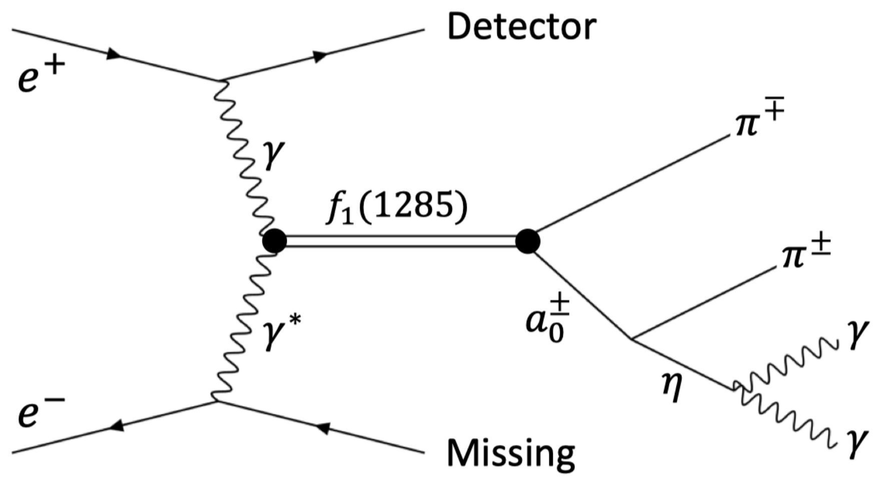
XLR, I. Danilkin, M. Vanderhaeghen, in preparation

$f_1(1285)$ production in $\gamma\gamma^* \to \eta\pi\pi$

- **Clean channel** to study the $f_1(1285)$ TFFs
  - $\eta'(958)$ and $f_1(1285)$ **well separated**
  - $f_1(1285)$ peak vanish (Q^2=0), exclude $\eta(1295)$
- Existing data: MarkII, TPC/Two-Gamma, L3
  - Low-statistic / large background
  - No analysis of different helicity states of $f_1(1285)$



- BESIII on-going analysis @ Mainz group

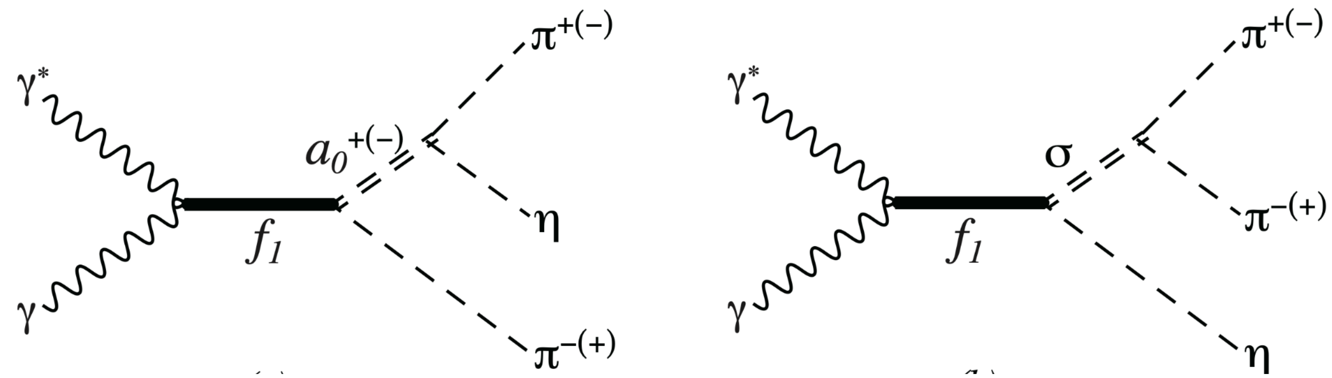


- Feasibility study M.Sc. thesis of D. Becker
- Ph.D. project (2023-) J. Muskalla

- Data analysis based on the **GaGaRes** Monte Carlo generator
- However, there is no interference among different channels

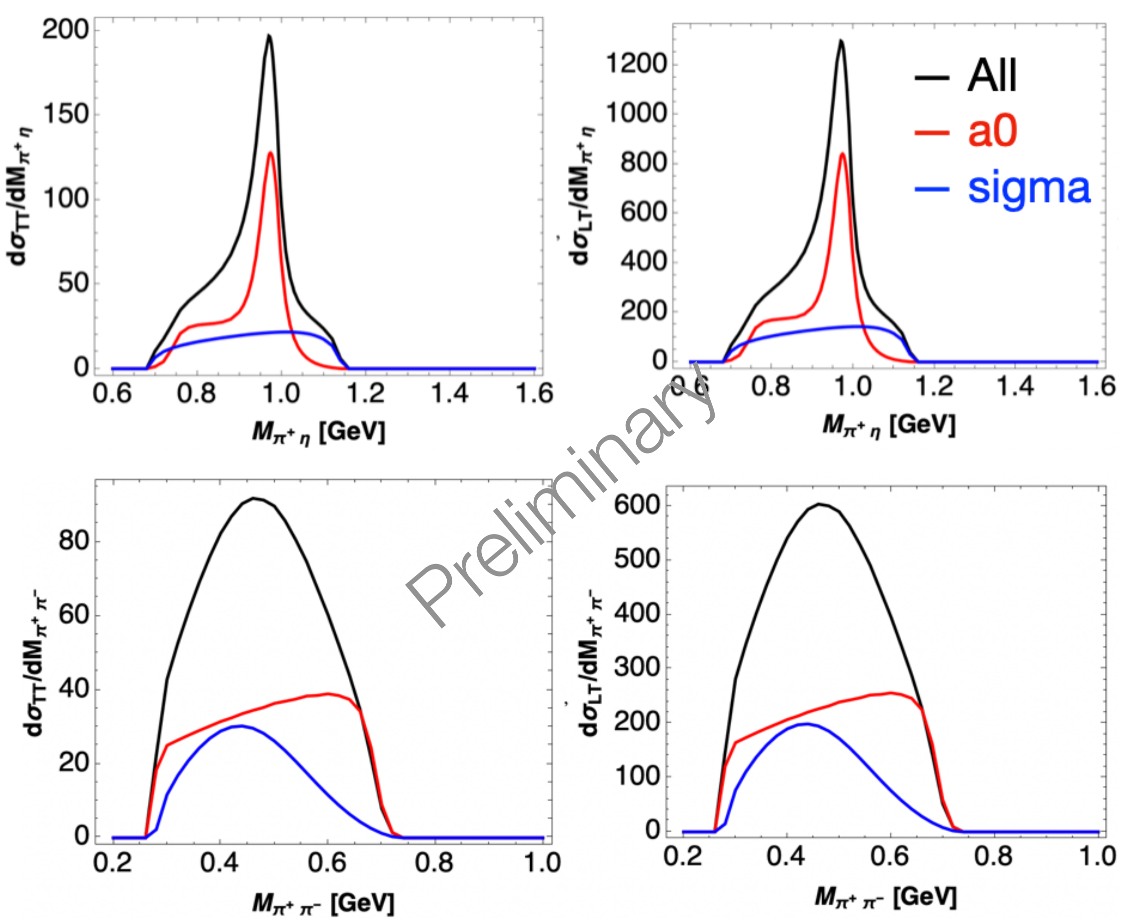
Pheno. model for $\gamma\gamma^* \to \eta\pi^+\pi^-$

- $f_1(1285)$ channel in our phenomenological model

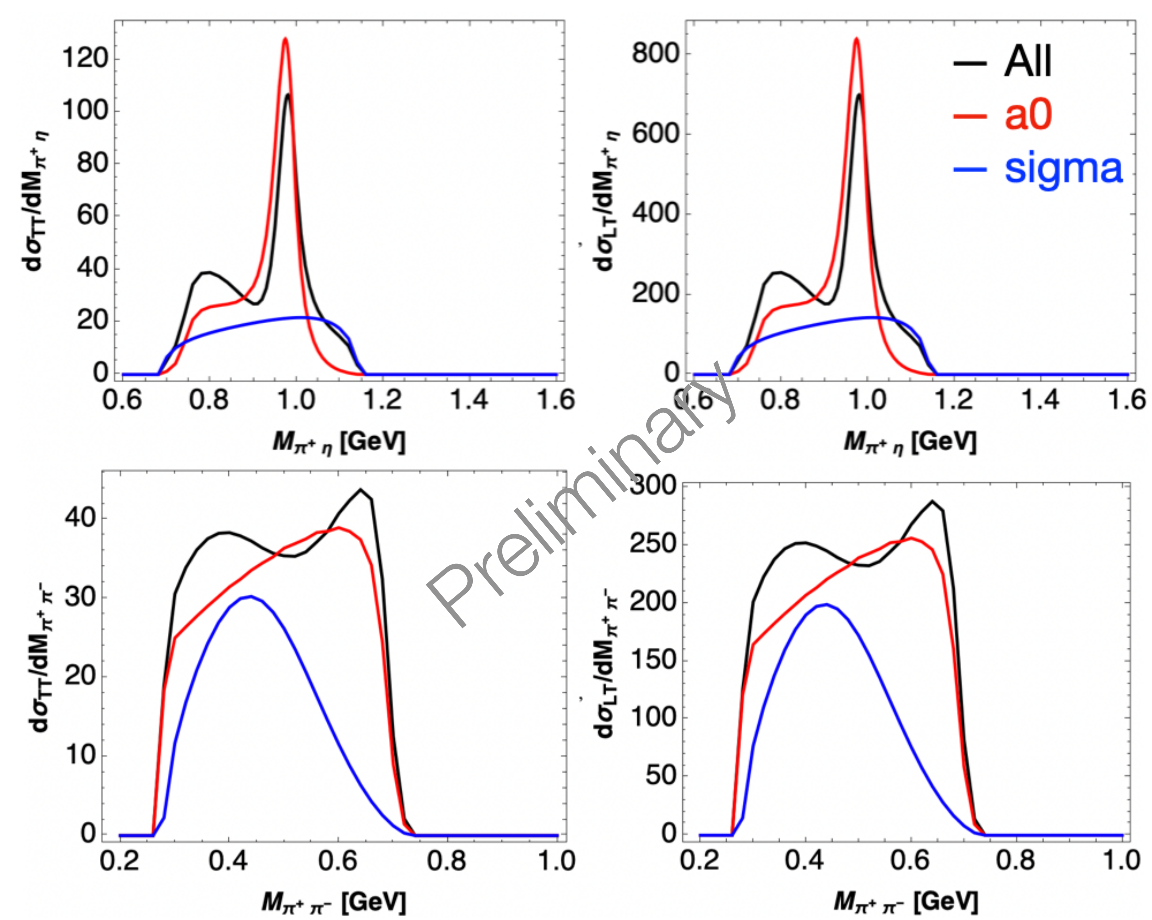


- Prediction for the invariant mass distributions

Constructive interference



Destructive interference



Interference: important to avoid the misinterpretation of the experimental data!

Summary and outlook

- Phenomenological study of photon-photon fusion to three mesons
  - Focus on the lowest-lying axial vectors and tensors

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)

  - Proposed models could serve as the Monte Carlo generators for the on-going/forthcoming analysis of BESIII Collaboration
- Next step:
  - Improve above studies by using the **Dispersion Relation** if data existing
  - Model independently determine the **meson TFFs**
  - Better control the axial vectors and tensors contributions to HLbL scattering

Thank you for your attention!

Backup slides