

# Precision Predictions for Top-quark Width

李海涛



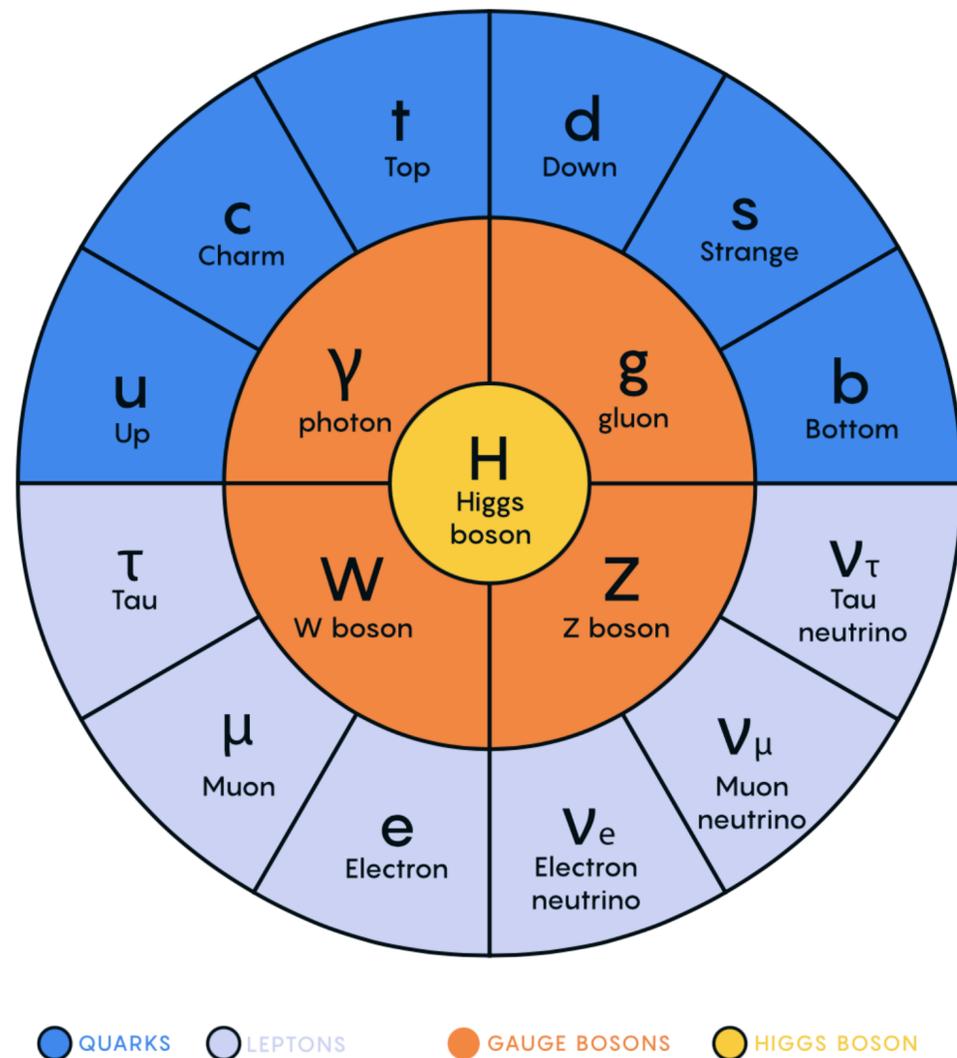
In collaboration with 陈龙斌, 王健, 王焯凡

arXiv:2212.06341

华中师范大学, 粒子物理小型工作会议

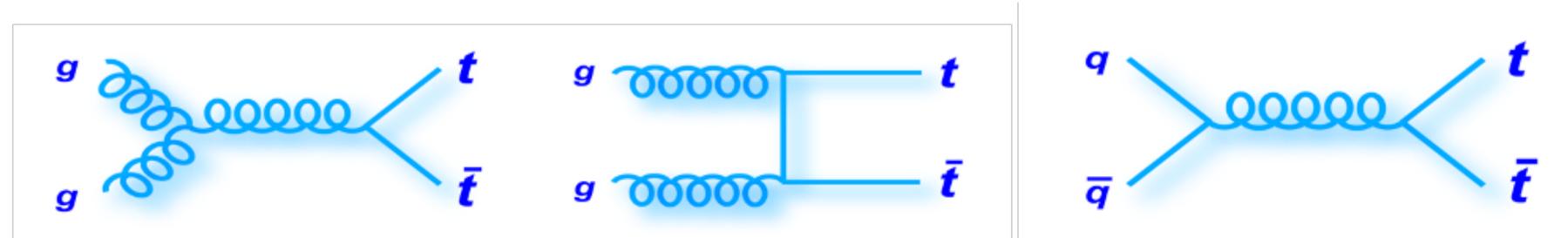
2023-04-24

# Introduction

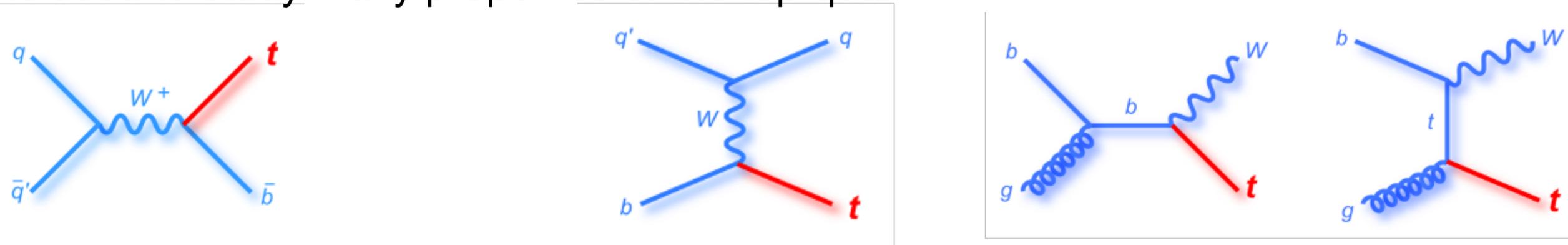


- ☑ Top quark is the heaviest elementary particle in the Standard Model.
- ☑ Top quark might play a role of electroweak symmetry breaking
- ☑ The measurements at the LHC offer the ultimate precision in top quark physics
- ☑ Top quark is a good probe of new physics

# Introduction



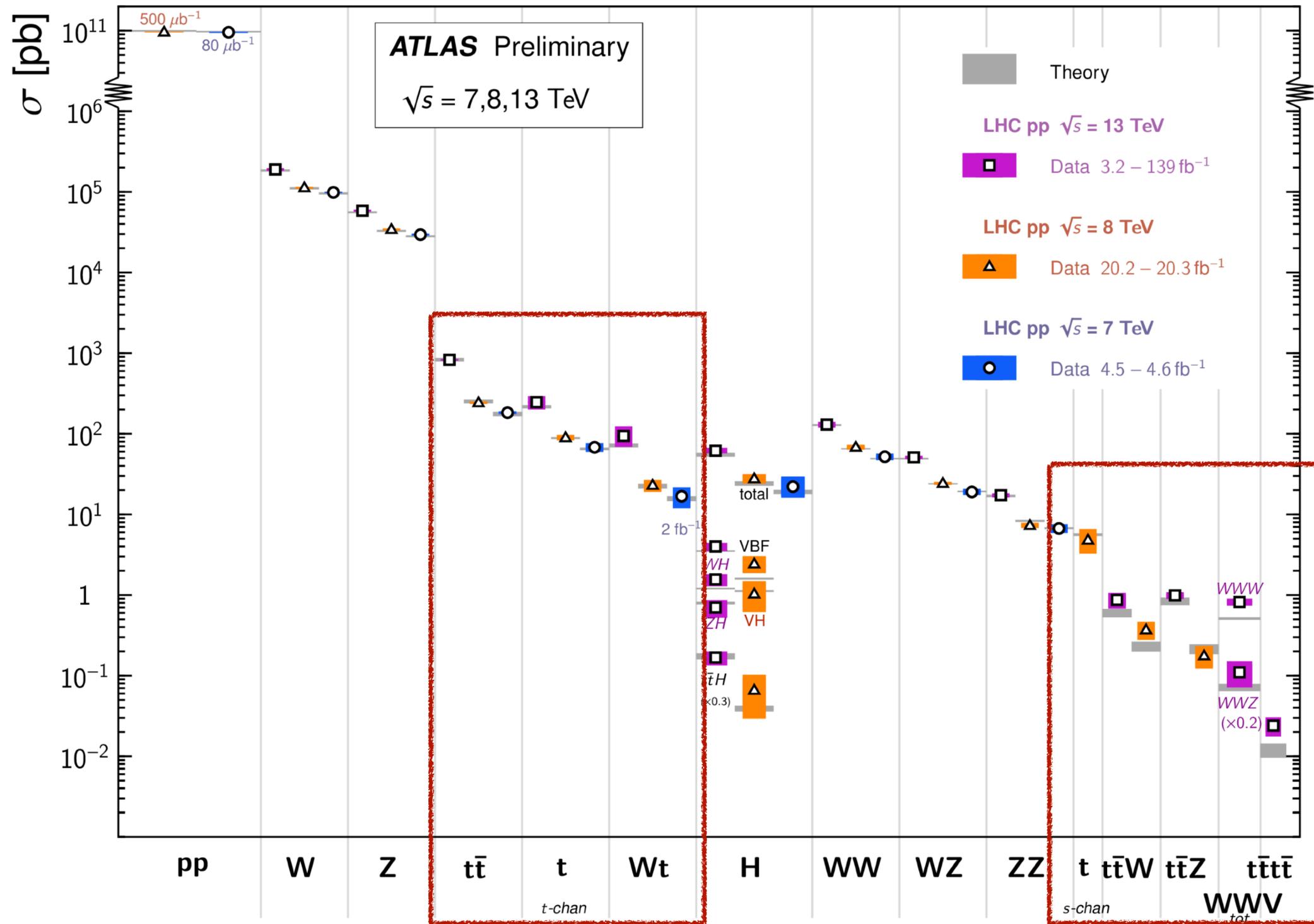
- It has high precision of the experimental measurements of total and differential cross sections
- It is used to study many properties of the top quark

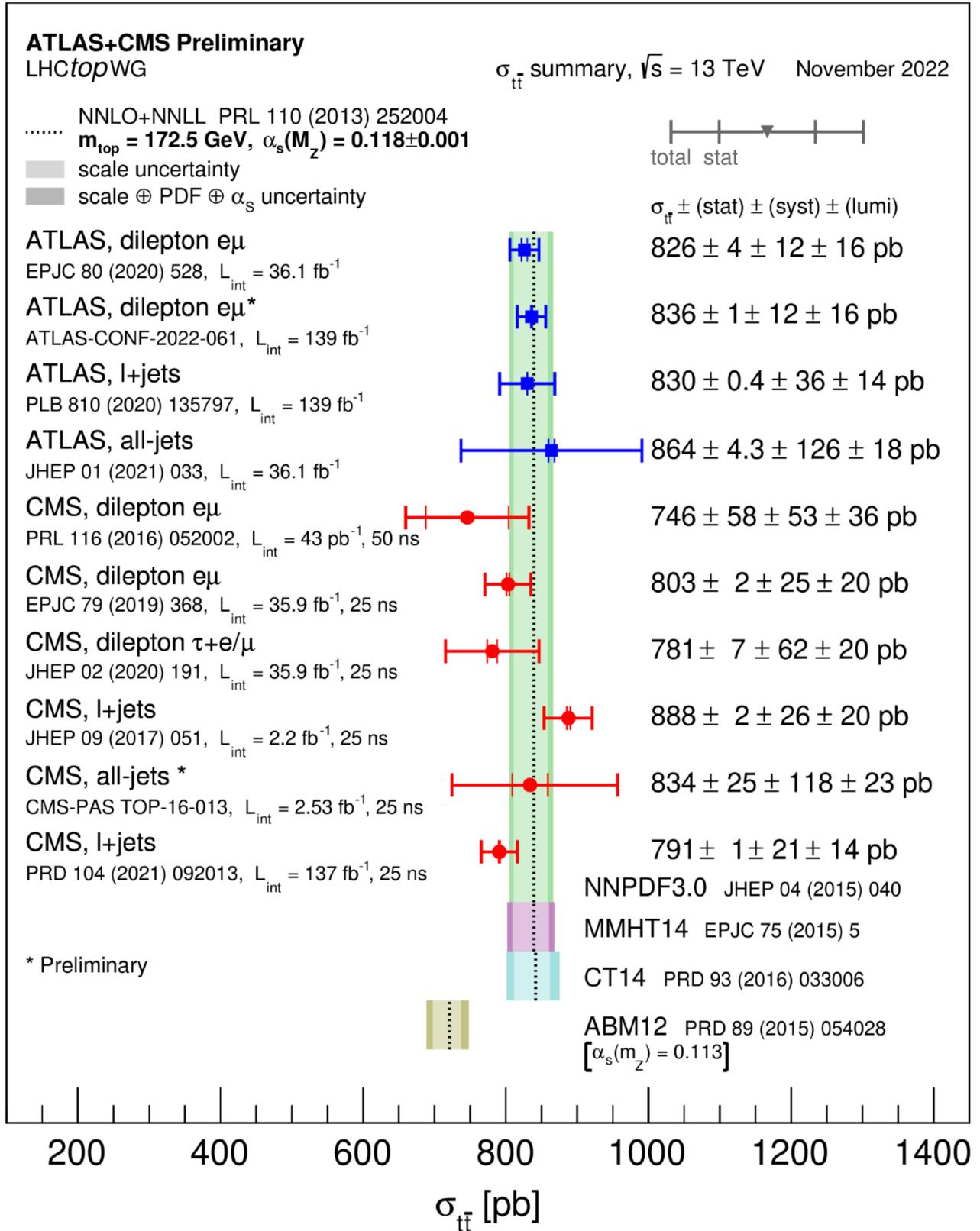


- it allows measurement of  $V_{tb}$  per channel
- It is easier to check the chiral structure of  $Wtb$  vertex than  $t\bar{t}$  production
- it-channel can be used to measurement the b quark density

# Standard Model Total Production Cross Section Measurements

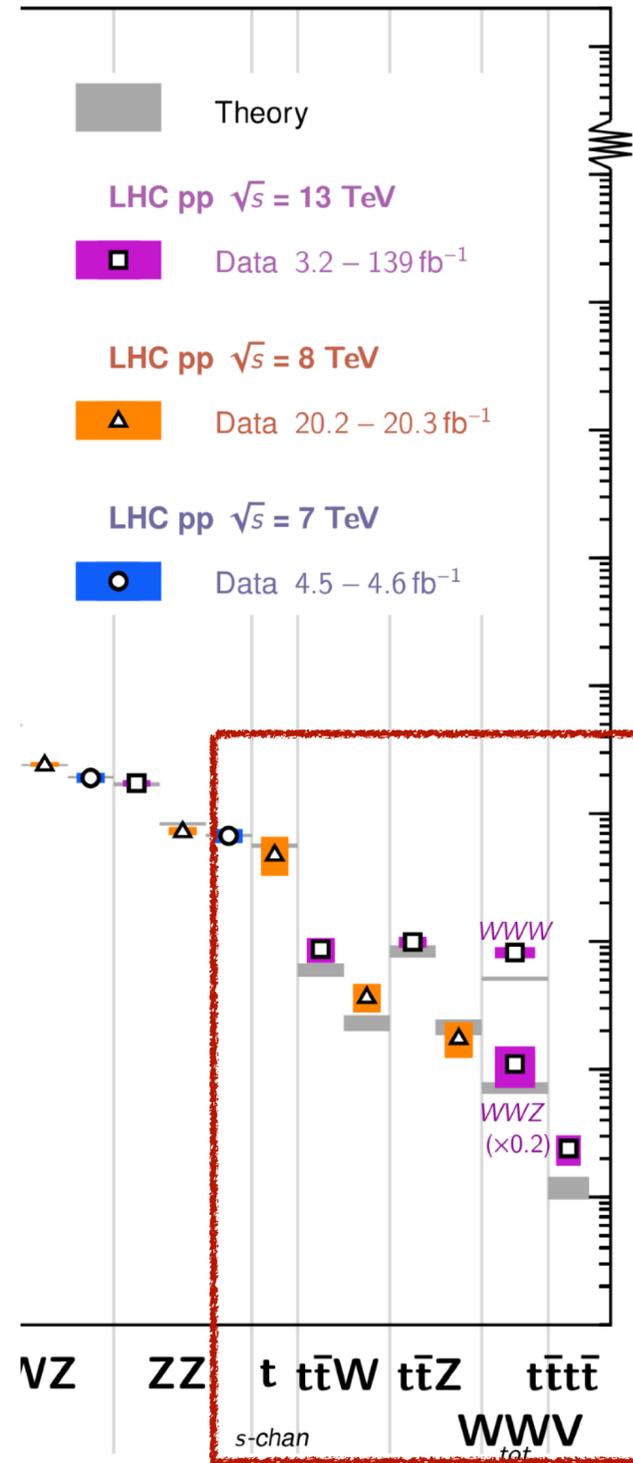
Status: February 2022





**ents**

Status: February 2022



$\sigma$  [pb]

**ATLAS+CMS Preliminary**  
LHCtopWG

$\sigma_{t\bar{t}}$  summary,  $\sqrt{s} = 13$  TeV November 2022

..... NNLO+NNLL PRL 110 (2013) 252004  
.....  $m_{top} = 172.5$  GeV,  $\alpha_s(M_Z) = 0.118 \pm 0.001$   
■ scale uncertainty  
■ scale  $\oplus$  PDF  $\oplus$   $\alpha_s$  uncertainty

**ATLAS, dilepton  $e\mu$**

EPJC 80 (2020) 528,  $L_{int} = 36.1$  fb $^{-1}$

**ATLAS, dilepton  $e\mu^*$**

ATLAS-CONF-2022-061,  $L_{int} = 139$  fb $^{-1}$

**ATLAS, l+jets**

PLB 810 (2020) 135797,  $L_{int} = 139$  fb $^{-1}$

**ATLAS, all-jets**

JHEP 01 (2021) 033,  $L_{int} = 36.1$  fb $^{-1}$

**CMS, dilepton  $e\mu$**

PRL 116 (2016) 052002,  $L_{int} = 43$  pb $^{-1}$ , 50 ns

**CMS, dilepton  $e\mu$**

EPJC 79 (2019) 368,  $L_{int} = 35.9$  fb $^{-1}$ , 25 ns

**CMS, dilepton  $\tau+e/\mu$**

JHEP 02 (2020) 191,  $L_{int} = 35.9$  fb $^{-1}$ , 25 ns

**CMS, l+jets**

JHEP 09 (2017) 051,  $L_{int} = 2.2$  fb $^{-1}$ , 25 ns

**CMS, all-jets \***

CMS-PAS TOP-16-013,  $L_{int} = 2.53$  fb $^{-1}$ , 25 ns

**CMS, l+jets**

PRD 104 (2021) 092013,  $L_{int} = 137$  fb $^{-1}$ , 25 ns

\* Preliminary

NNP  
MMF  
CT14  
ABM  
[ $\alpha_s(m_Z)$ ]

200 400 600 800 1000

$\sigma_{t\bar{t}}$  [pb]

**ents**

Status: February 2022

Inclusive cross-section [pb]

**ATLAS+CMS Preliminary**  
LHCtopWG

Single top-quark production  
November 2022

10<sup>2</sup>  
10

t-channel

tW

s-channel

t-channel

- ATLAS PRD90(2014)112006, EPJC77(2017)531, JHEP04(2017)086
- CMS JHEP12(2012)035, JHEP06(2014)090, PLB800(2019)135042
- ◆ LHC comb. JHEP05(2019)088

tW

- ATLAS PLB716(2012)142, JHEP01(2016)064, JHEP01(2018)063
- CMS PRL110(2013)022003, PRL112(2014)231802, arXiv:2208.00924
- ◆ LHC comb. JHEP05(2019)088

s-channel

- ATLAS PLB756(2016)228, arXiv:2209.08990
- CMS JHEP09(2016)027
- ◆ LHC comb. JHEP05(2019)088

--- NNLO PLB736(2014)58  
■ scale uncertainty

- - - NLO + NNLL PRD83(2011)091503,  
PRD82(2010)054018, PRD81(2010)054028

tW:  $t\bar{t}$  contribution removed  
■ scale  $\oplus$  PDF  $\oplus$   $\alpha_s$  uncertainty

— NLO NPPS205(2010)10, CPC191(2015)74  
 $\mu_R = \mu_F = m_{top}$

CT10nlo, MSTW2008nlo, NNPDF2.3nlo

tW:  $p_T^b$  veto for  $t\bar{t}$  removal = 60 GeV and  $\mu_F = 65$  GeV

■ scale uncertainty

■ scale  $\oplus$  PDF  $\oplus$   $\alpha_s$  uncertainty

stat. total

$\sqrt{s}$  [TeV]

https://arxiv.org/abs/2208.00924

# Introduction

$$\sigma = \int_{t_{\text{cut}}}^{t_{\text{max}}} \frac{d\sigma}{dt_N} + \int_0^{t_{\text{cut}}} \frac{d\sigma}{dt_N} \longrightarrow \frac{d\sigma}{dt_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left( \prod_{n=1}^N J_n \right)$$

Standard NLO

HTL, Wang, 1611.02749,1804.06358

Amplitude reads  $\mathcal{M} = \mathcal{M}_0 + \frac{\alpha_s}{4\pi} \mathcal{M}_1 + \left( \frac{\alpha_s}{4\pi} \right)^2 \mathcal{M}_2$

Tree      1-loop      2-loop

One-loop amplitude square

[arXiv:2204.13500](https://arxiv.org/abs/2204.13500)

Two-loop analytical amplitude in LC approximation

[arXiv:2208.08786](https://arxiv.org/abs/2208.08786)

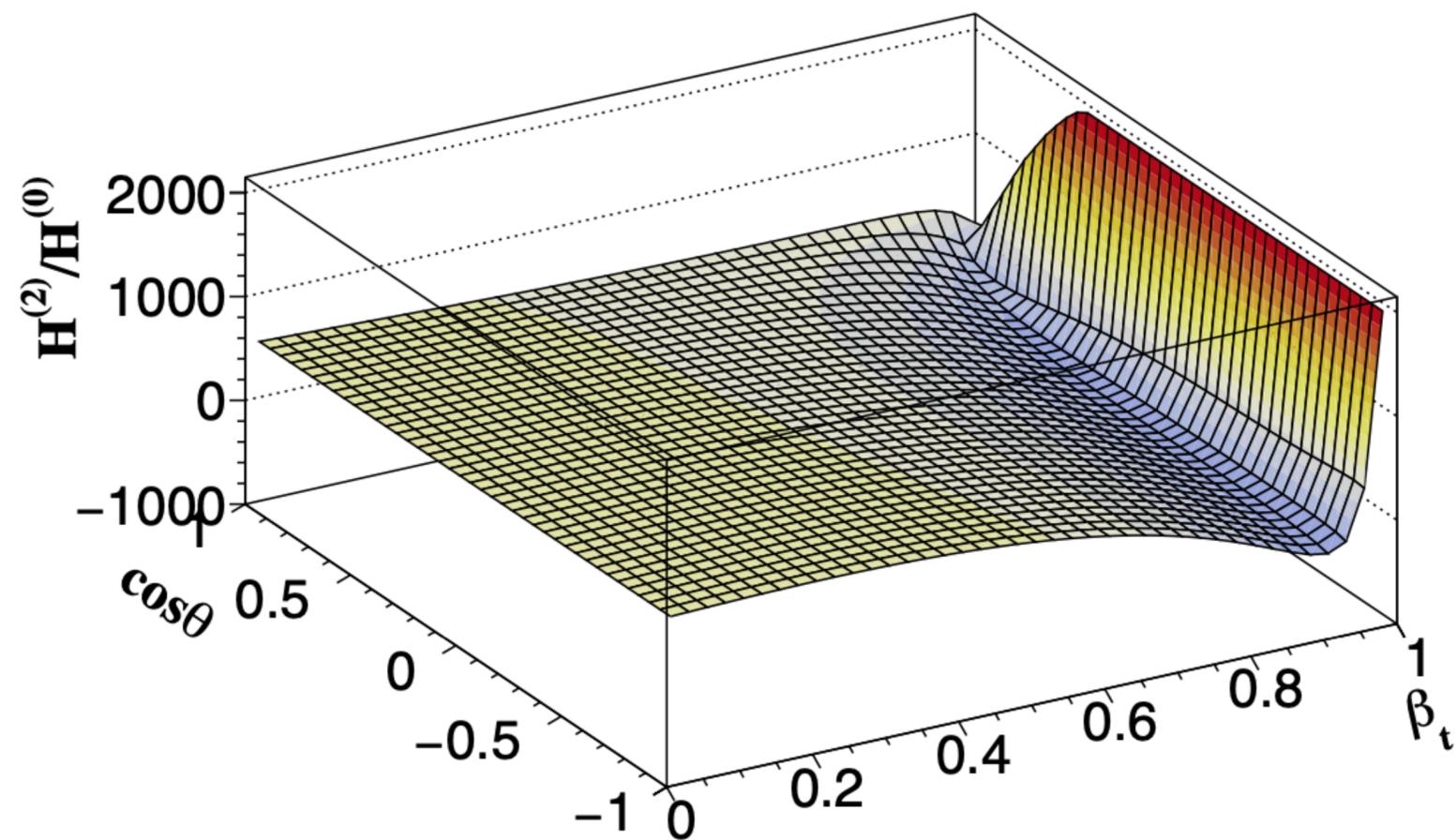
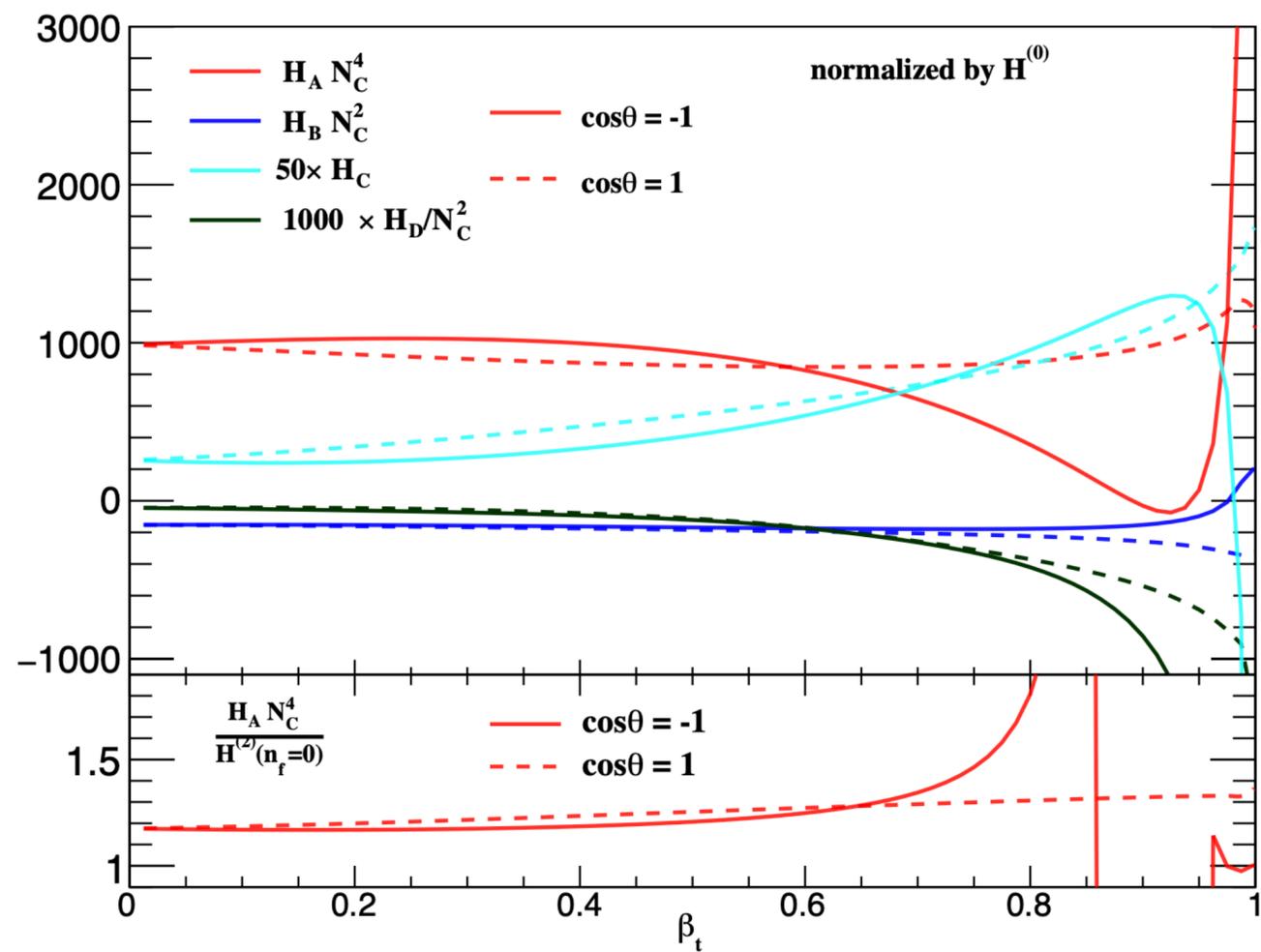
Two-loop numerical amplitude in full color

[arXiv:2212.07190](https://arxiv.org/abs/2212.07190)

by Chen, Dong, HTL, Li, Wang, Wang

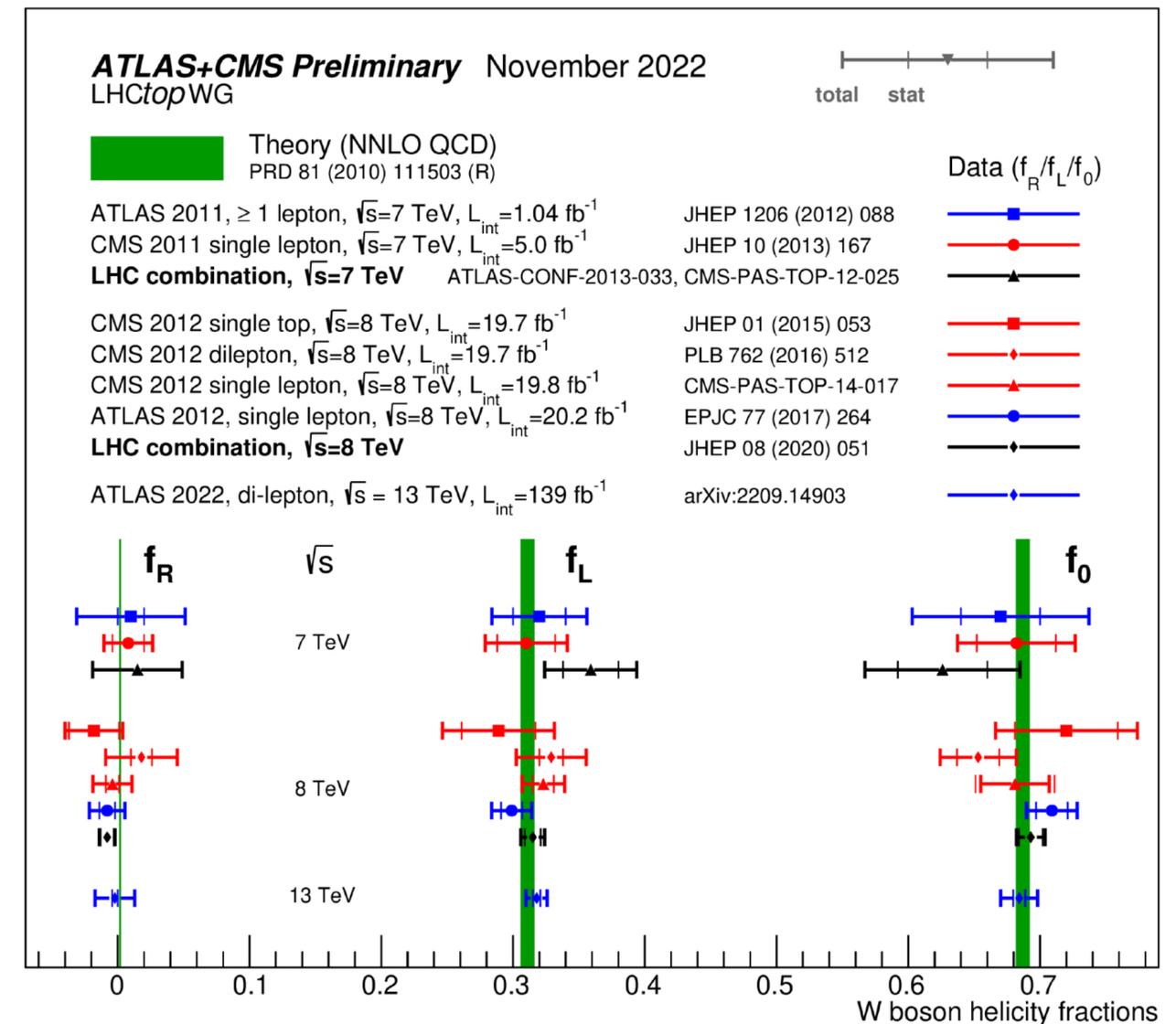
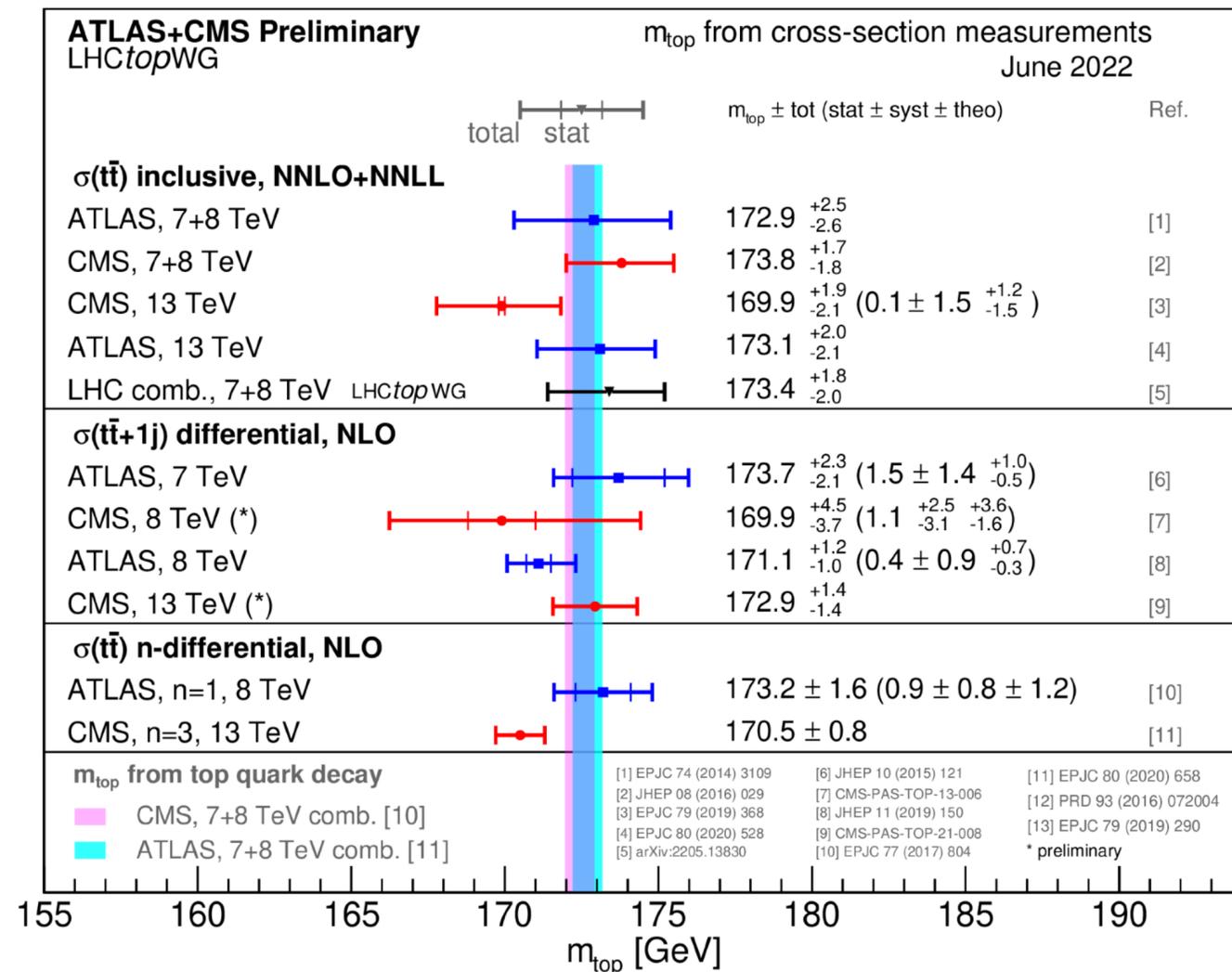
# Introduction

$$H^{(2)} = N_C^4 H_A + N_C^2 H_B + H_C + \frac{1}{N_C^2} H_D + n_l H_{nl} + n_h H_{nh}$$



# Introduction

## Top quark intrinsic properties, Mass, Width, Spin 1/2, e-charge 2/3



# Introduction

The top-quark decays almost exclusively to  $Wb$ , ie  $\Gamma_t = \Gamma(t \rightarrow Wb)$ .

At LHC, indirect techniques are precise but model dependent.

The most precise measurement is  $\Gamma = 1.36 \pm 0.2_{-11}^{+0.14}$  GeV

$$\mathcal{R} = \frac{\mathcal{B}(t \rightarrow Wb)}{\mathcal{B}(t \rightarrow Wq)} = \mathcal{B}(t \rightarrow Wb) \quad \text{Measuring } \mathcal{R} \text{ with top pair event}$$

$$\Gamma_t = \frac{\sigma_{t\text{-ch.}}}{\mathcal{B}(t \rightarrow Wb)} \cdot \frac{\Gamma(t \rightarrow Wb)}{\sigma_{t\text{-ch.}}^{\text{theor.}}}$$

**CMS, 2014**

Direct techniques are less precise but model independent.

Direct result by ATLAS is  $\Gamma_t = 1.9 \pm 0.5$  GeV **ATLAS, 2019**

Using events away from the resonance peak  $\Gamma_t = 1.28 \pm 0.3$  GeV

**Baskakov, Boos, Dudko, 2018**

**Herwig, Jazo, Nachman, 2019**

In the future  $e^+e^-$  collider,  $\Gamma_t$  can be measured with an uncertainty of 30 MeV **Martinez, Miquel 2003**

# Introduction

- NLO QCD corrections **Bigi, Dokshitzer, Khoze, Kuhn, Zerwas, 1986**  
**Jezabek, Kuhn 1989, Czarnecki 1990, Li, Oakes, Yuan 1991**
- NLO EW corrections **Denner, Sack 1991, Eilam, Mendel, Migneron, Soni 1991**
- Asymptotic expansions of NNLO QCD corrections near  $m_W \rightarrow 0$  and  $m_W \rightarrow m_t$   
**Czarnecki, Melnikov 1999, Chetyrkin, Harlander, Seidensticker, Steinhauser 1999,**  
**Blokland, Czarnecki, Slusarczyk, Tkachov 2004, 2005**
- Numerical results of full NNLO QCD corrections **Gao, Li, Zhu 2013, Brucherseifer, Caola, Melnikov 2013**
- Polarized top decay up to NNLO QCD **Czarnecki, Groote, Körner and Piclum, 2018**
- PMC scale settings up to NNLO QCD **Meng, Wang, Sun, Luo, Shen, Wu, 2022**

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**Analytical results of NNLO QCD corrections** **Chen, HTL, Wang, Wang, 2022**

# Optical Theory

Unitarity implies the  $S$ -matrix

$$S^\dagger S = 1, \text{ with } S = 1 + iT \quad T \text{ is the transfer matrix}$$

$$-i(T - T^\dagger) = TT^\dagger$$

Take the matrix element of this equation

$$\langle f | T | i \rangle = (2\pi)^4 \delta^{(4)}(p_i - p_f) \mathcal{M}(i \rightarrow f) \quad \langle \mathbf{p}_1 \mathbf{p}_2 | T^\dagger T | \mathbf{k}_1 \mathbf{k}_2 \rangle = \sum_n \int d\Phi_n \langle \mathbf{p}_1 \mathbf{p}_2 | T^\dagger | \{ \mathbf{q}_n \} \rangle \langle \{ \mathbf{q}_n \} | T | \mathbf{k}_1 \mathbf{k}_2 \rangle$$

The generalized optical theorem is

$$-i (\mathcal{M}(i \rightarrow f) - \mathcal{M}^*(i \rightarrow f)) = \sum_X \int d\Phi_X \mathcal{M}(i \rightarrow X) \mathcal{M}^*(f \rightarrow X)$$

For top quark decay width  $|i\rangle = |f\rangle = |A\rangle$

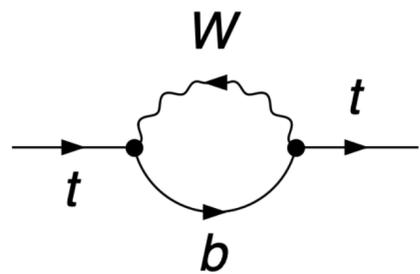
$$\text{Im } \mathcal{M}(A \rightarrow A) = m_A \sum_X \Gamma(A \rightarrow X) = m_A \Gamma_{\text{tot}}$$

# Optical Theory

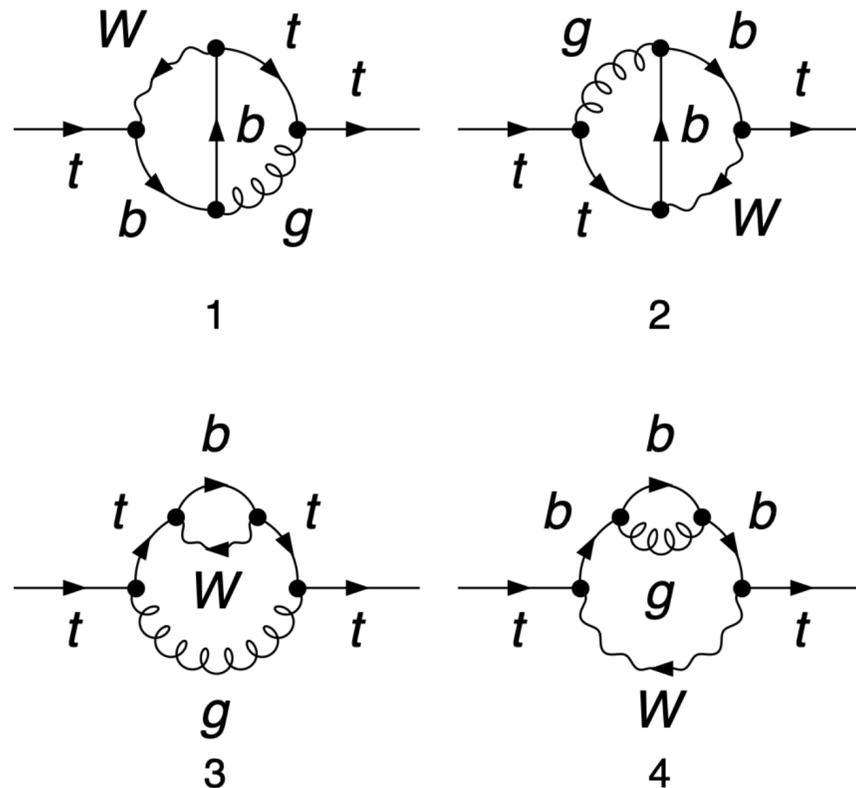
Top decaying into massless b quark and on-shell W boson

$$\Gamma_t = \frac{\text{Im}\Sigma}{m_t}$$

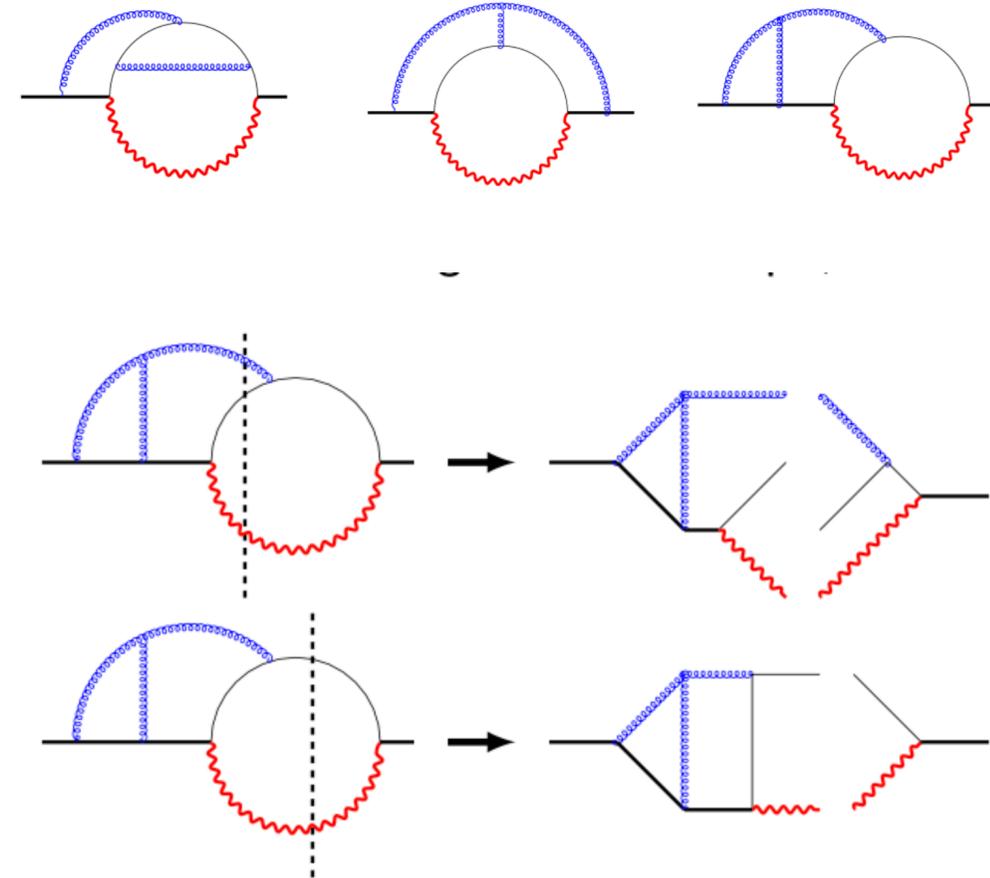
LO



NLO



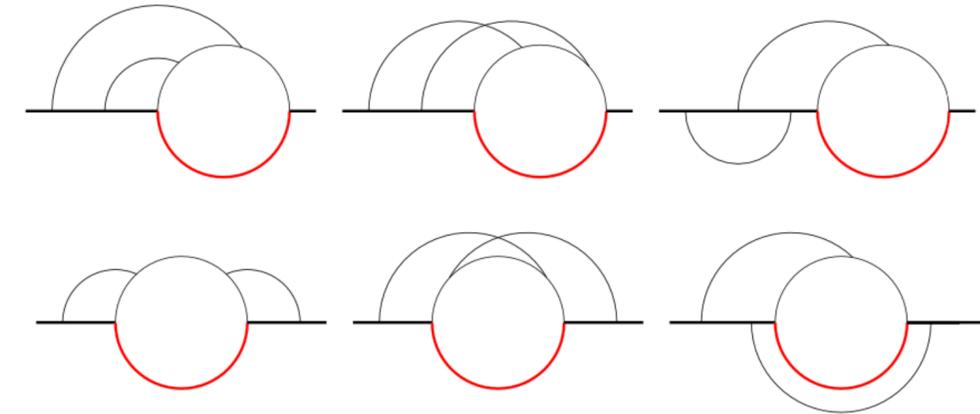
NNLO



# Amplitude and Master integrals

The amplitudes generated by FeynArts and FeynCalc

Scalar integrals reduced to master integrals using FIRE 6



Canonical differential equations constructed for the cut MIs by choosing a proper basis  $\mathbf{I}(w, \epsilon)$  [Henn, 2013](#)

$$d \mathbf{I}(w, \epsilon) = \epsilon d \left[ \sum_{i=1}^4 R_i \log(l_i) \right] \mathbf{I}(w, \epsilon)$$

Boundary conditions:  $w = \frac{m_W^2}{m_t^2} = 0$

The analytical results of some master integrals in  $w = 0$  can be found in literatures

[Blokland, Czarnecki, Slusarczyk, Tkachov 2005,](#)  
[Ritbergen, Stuart 2000.](#)

Others reconstructed by PSLQ algorithm with AMFlow [Liu, Ma 2022](#)

# Top decay width

Top decaying into massless b quark and on-shell W boson

$$\Gamma(t \rightarrow Wb) = \Gamma_0 \left[ X_0 + \frac{\alpha_s}{\pi} X_1 + \left( \frac{\alpha_s}{\pi} \right)^2 X_2 \right],$$

$$X_2 = C_F(T_R n_l X_l + T_R n_h X_h + C_F X_F + C_A X_A)$$

NNLO contributions written as HPLs

$$X_l = -\frac{X_0}{3} [H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3)] + \dots$$

$$X_F = \frac{1}{12} X_0 [-6(2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3)H_1(w)) - \pi^2 H_{1,0}(w)] + (X_0 + 4w) \left( -\frac{1}{6} \pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w) \right) + \dots]$$

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NNLO contributions written as HPLs

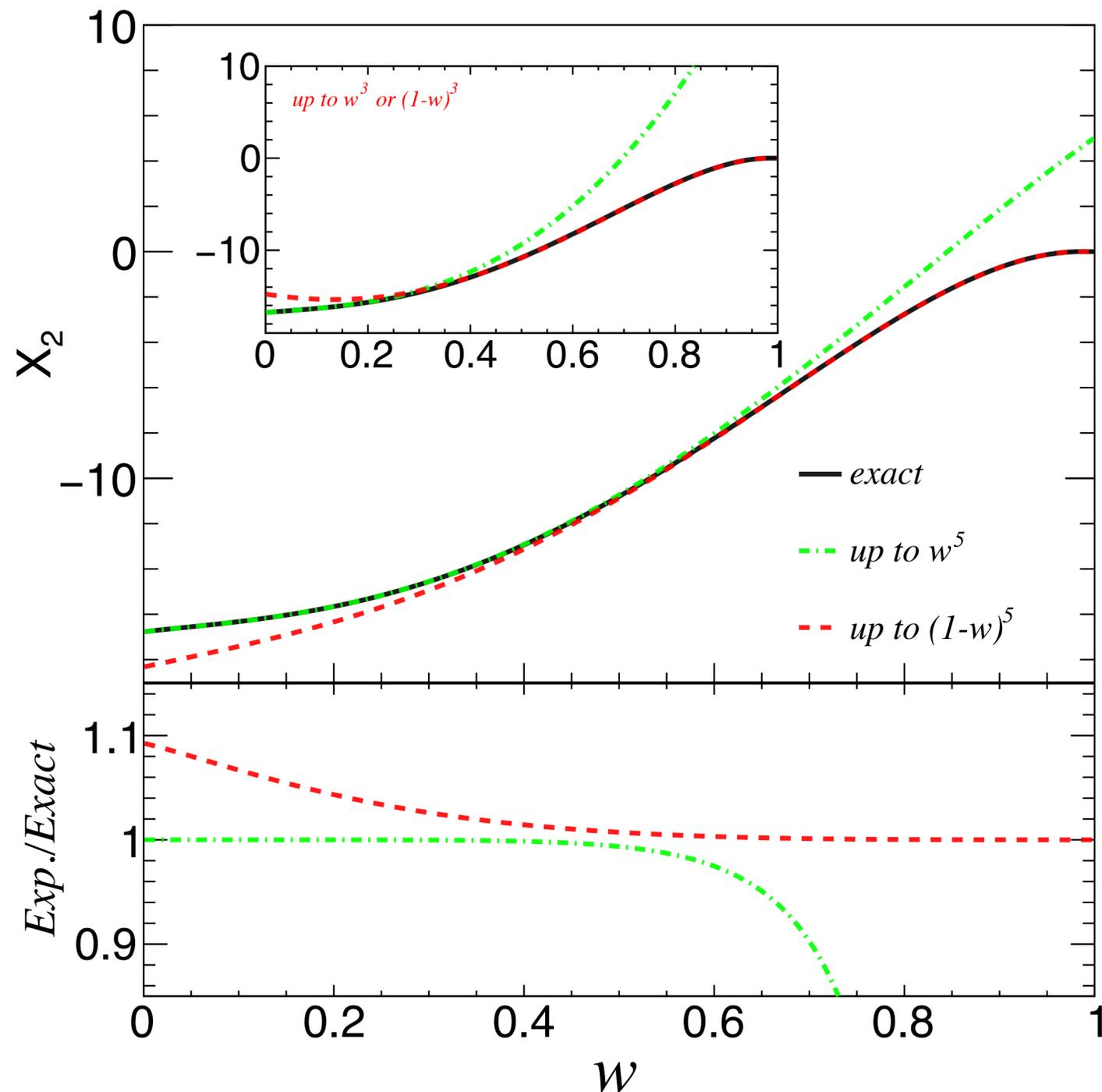
$$X_l = -\frac{X_0}{3} \left[ H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3) \right] + \dots$$

Consistent with Gao, Li, Zhu 2013

$$X_F = \frac{1}{12} X_0 \left[ -6 \left( 2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3)H_1(w) \right) - \pi^2 H_{1,0}(w) \right] + (X_0 + 4w) \left( -\frac{1}{6} \pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w) \right) + \dots$$

	width (GeV)	delta/LO width
<b>LO</b>	1.48642	—
<b>NLO</b>	1.35897	-8.58%
<b>NNLO</b>	1.32825	-2.07%

# Top decay width



Though the decay width at  $w=0$  and  $w=1$  is finite, it exhibits logarithmic structures at these two boundaries

$$X_l = \ln(w) \left( -\frac{1}{3}(2w+1)(w-1)^2(H_{0,1}(w) + 2H_{1,1}(w)) - \frac{1}{18}(38w^3 - 93w^2 + 18w + 37)H_1(w) + \frac{1}{36}w(-106w^2 + 25w + 86) \right) + \dots,$$

$$X_h = \frac{1}{54} \ln(1-w) \left( -18(2w^3 - 3w^2 - 12w + 1)H_{0,0}(w) + \frac{6(19w^4 + 32w^3 - 18w^2 - 8w + 23)H_0(w)}{w-1} - 265w^3 - 168w^2 + 498w - \frac{9}{w} - 344 \right) + \dots,$$

All the coefficients can be expanded in series of  $w$  or  $1-w$

The result expanded in  $w=0$  and  $w=1$  coincides with  
**Blokland, Czarnecki, Slusarczyk, Tkachov 2004 2005**

# Top decay width

Corrections by keeping b quark mass

$$\Gamma_0 = \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \lambda^{1/2} \left[ 1, \frac{m_b}{m_t}, \frac{m_W}{m_t} \right] \\ \times \left[ \left[ 1 - \frac{m_b^2}{m_t^2} \right]^2 + \left[ 1 + \frac{m_b^2}{m_t^2} \right] \frac{m_W^2}{m_t^2} - 2 \frac{m_W^4}{m_t^4} \right].$$

0.27% decrease compare to LO width

same order with  $\frac{m_b^2}{m_t^2}$

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0.27% decrease compare to LO width

same order with  $\frac{m_b^2}{m_t^2}$

$$\Gamma = \Gamma_0 \left\{ 1 + \frac{C_F \alpha_s}{2\pi} \left[ 2 \left[ \frac{(1 - \beta_W^2)(2\beta_W^2 - 1)(\beta_W^2 - 2)}{\beta_W^4(3 - 2\beta_W^2)} \right] \ln(1 - \beta_W^2) - \frac{9 - 4\beta_W^2}{3 - 2\beta_W^2} \ln \beta_W^2 \right. \right. \\ \left. \left. + 2\text{Li}_2 \beta_W^2 - 2\text{Li}_2(1 - \beta_W^2) - \frac{6\beta_W^4 - 3\beta_W^2 - 8}{2\beta_W^2(3 - 2\beta_W^2)} - \pi^2 \right] \right\}.$$

0.126% increase compared to LO width

Li, Oakes, Yuan, 1991

# Top decay width

For off-shell W effects, we could integrate over the

$$\Gamma(t \rightarrow W^*b) = \frac{1}{\pi} \int_0^{m^2} dq^2 \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \left( \Gamma_t \Big|_{m_W^2 \rightarrow q^2} \right) \quad \text{Taking the limits } \Gamma_W \rightarrow 0, \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \rightarrow \pi \delta(q^2 - m_W^2)$$

$$\Gamma(t \rightarrow W^*b) = \Gamma_0 \left( \tilde{X}_0 + \frac{\alpha_s}{\pi} \tilde{X}_1 + \left( \frac{\alpha_s}{\pi} \right)^2 \tilde{X}_2 \right), \quad X_i \text{ is expressed in terms of GPLs}$$

**For example**  $\tilde{X}_0 = 2rw(2w - 1) - \frac{1}{2} \left[ (2(r - i)w - i)((r - i)w + i)^2 G_{w+irw}(1) + (2(r + i)w + i)((r + i)w - i)^2 G_{w-irw}(1) \right]$

this effect can be included independently with QCD corrections

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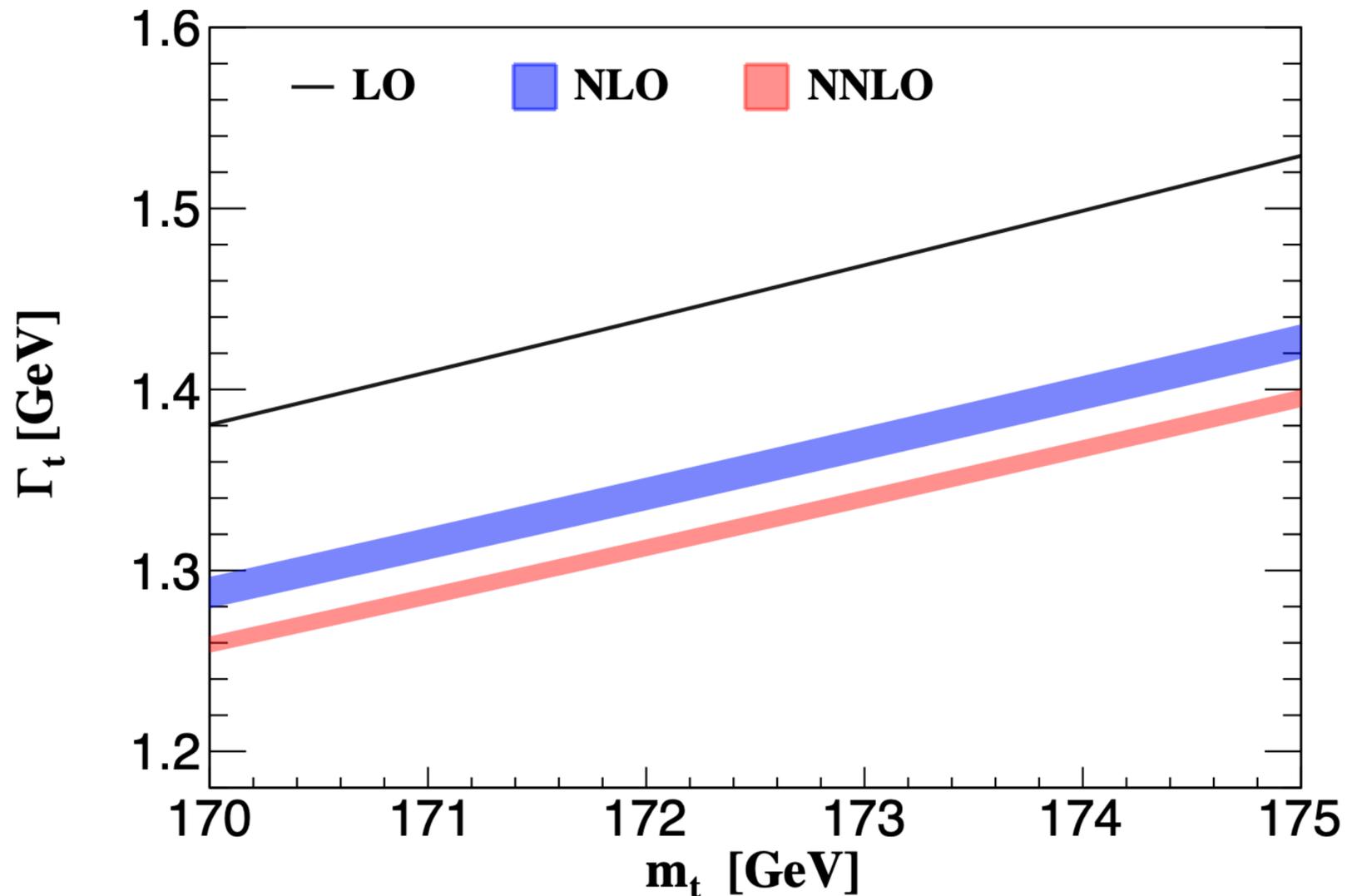
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	<b>LO</b>	<b>NLO</b>	<b>NNLO</b>
<b>delta/LO width</b>	-1.54%	0.13%	0.03%

# Top decay width

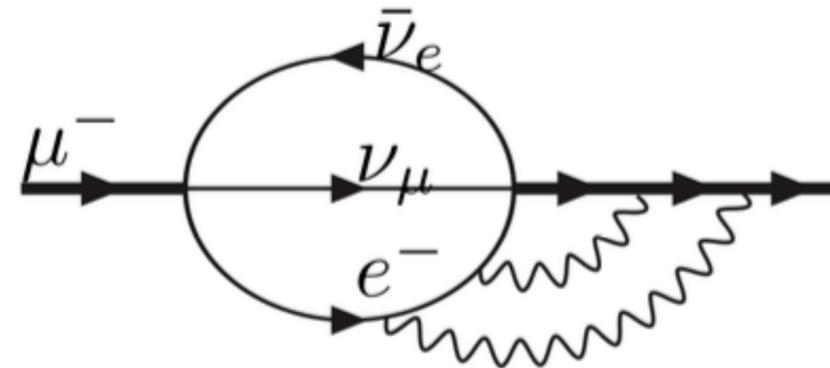
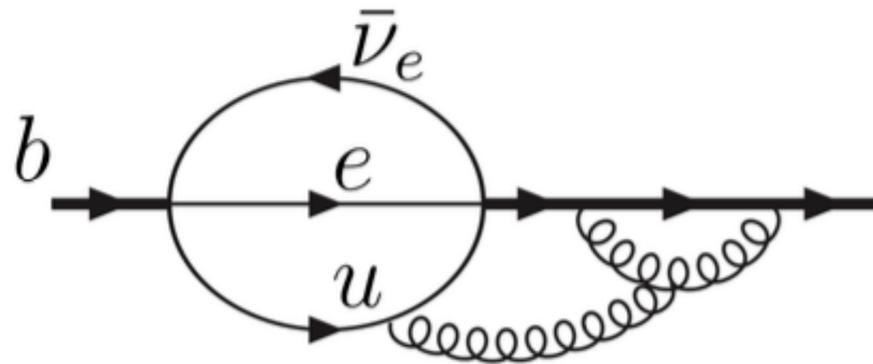
QCD renormalization scale  $\mu \in [m_t/2, 2m_t]$ , the variation is about  $\pm 0.8\%$  and  $\pm 0.4\%$  at NLO and NNLO.



- On-shell renormalization scheme adopted
- This scale uncertainty has been reduced dramatically after including NNLO QCD corrections.
- $\overline{\text{MS}}$  scheme,  $\Gamma_t^{\text{NLO}} = 1.309$  GeV,  $\Gamma_t^{\text{NNLO}} = 1.332$  GeV. QCD corrections are  $-3.79\%$  and  $0.09\%$  at NLO and NNLO.
- Assuming power like growth for QCD corrections, NNNLO corrections would be of around  $0.4\%$

# Top decay width

Cross-check and other applications



$w$  can be understood as  $q^2/m^2$

with  $q^2$  the invariant mass of lepton sector,  $m^2$  the mass of the parent particle

Integrating over  $w$  from 0 to 1, we reproduce NNLO QCD corrections in semileptonic decay  $\Gamma(b \rightarrow Xe\bar{\nu})$

**Ritbergen 1999**

Integrating  $X_F$  over  $w$ , we obtain the analytic two-loop QED correction to the muon lifetime  $\Gamma(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e)$ .

**Ritbergen, Stuart 1999**

# Mathematica Package

<https://github.com/haitaoli1/TopWidth>

main 1 branch 0 tags

Go to file Add file Code

haitaoli1 typo corrected ce94fe8 on Feb 10 30 commits

LICENSE.md	license added	4 months ago
README.md	typo corrected	2 months ago
TopWidth.m	arXiv information added	4 months ago
example.nb	typo corrected	2 months ago

README.md

## TopWidth

Mathematica Package to calculate the top decay width with NNLO corrections in QCD and NLO corrections in EW.

## Requirement

The HPL package is required to generate the numerics of the harmonic polylogarithm, which can be downloaded from <https://krone.physik.uzh.ch/data/HPL/>.

HPL is supposed to be initialized through "<<HPL`". If not please set the path "\$HPLPath="the:\path\of\the\installation".

## Download

Download the package through

git clone <https://github.com/haitaoli1/TopWidth.git>

go to the directory "TopWidth", run the example notebook "example.nb".

### About

Mathematica Package to calculate the top decay width with NNLO corrections in QCD and NLO corrections in EW.

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### Languages

Mathematica 100.0%

```
<< TopWidth`
(***** TopWidth-1.0 *****)
Authors: Long-Bin Chen, Hai Tao Li, Jian Wang, YeFan Wang
TopWidth[QCDorder, mbCorr, WwidthCorr,
EWcorr, mu] is provided for top width calculations
Please cite the paper for reference: arXiv:2212.06341

*-*-*-*-* HPL 2.0 *-*-*-*-*

Author: Daniel Maitre, University of Zurich
Rules for minimal set loaded for weights: 2, 3, 4, 5, 6.
Rules for minimal set for + - weights loaded for weights: 2, 3, 4, 5, 6.
Table of MZVs loaded up to weight 6
Table of values at I loaded up to weight 6
$HPLFunctions gives a list of the functions of the package.
$HPLOptions gives a list of the options of the package.
More info in hep-ph/0507152, hep-ph/0703052 and at
http://krone.physik.unizh.ch/~maitreda/HPL/

In[5]:= SetParameters[ $\frac{17269}{100}$ ,  $\frac{478}{100}$ , 80377 / 1000, 2085 / 1000, 911876 / 10000,
11663788  $\times 10^{-12}$ ]

In[6]:= TopWidth[2, 1 (* with mb effects *), 1 (* with rw effects*),
1 (* with NLO EW effects *),  $\frac{17269}{100}$  (*scale*)]

Out[6]= 1.33051
```

# Summary

- ☑ We provide the full analytical result of top-quark width at NNLO.
- ☑ The b mass effect is included up to NLO
- ☑ The off-shell W boson contribution is calculated analytically up to NNLO.
- ☑ The analytical result can be used to perform both fast and exact evaluations.
- ☑ The missing contributions is supposed to be less than 1%.

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{EW}^{(i)}$	$\delta_{QCD}^{(i)}$	$\Gamma_t$ [GeV]
LO	-0.273	-1.544	—	—	1.459
NLO	0.126	0.132	1.683	-8.575	1.361
NNLO	*	0.030	*	-2.070	1.331

谢谢!