Production and decay of the fully heavy tetraquark based on factorization

桑文龙

粒子物理小型会议@华中师范大学 2023/4/24

arXiv: **2009.08450, 2011.03039，2104.03887, 2304.11142** 冯锋，黄应生，贾宇，桑文龙，杨德山，张佳玥

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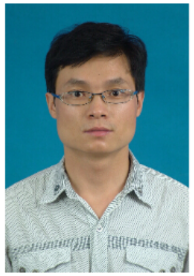

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Outline：

1. Introduction
2. T_{4c} production at colliders
3. T_{4c} electromagnetic and hadronic decay
4. Summary

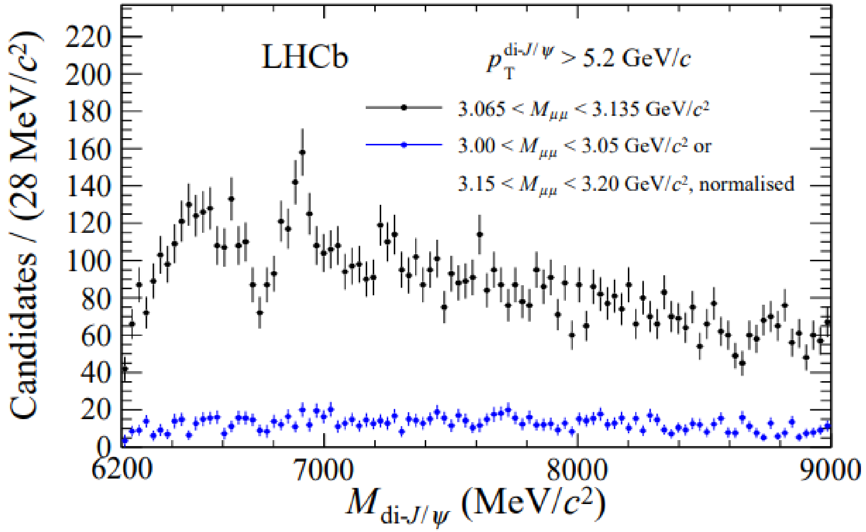
1. Introduction

Observation of structure in the J/ψ -pair mass spectrum #1

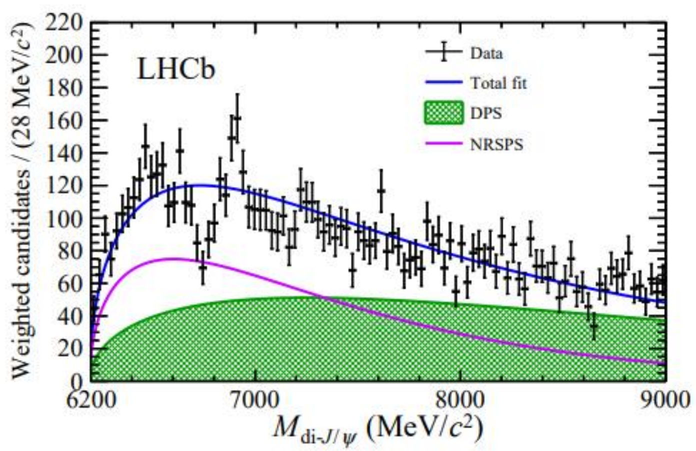
LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 30, 2020)

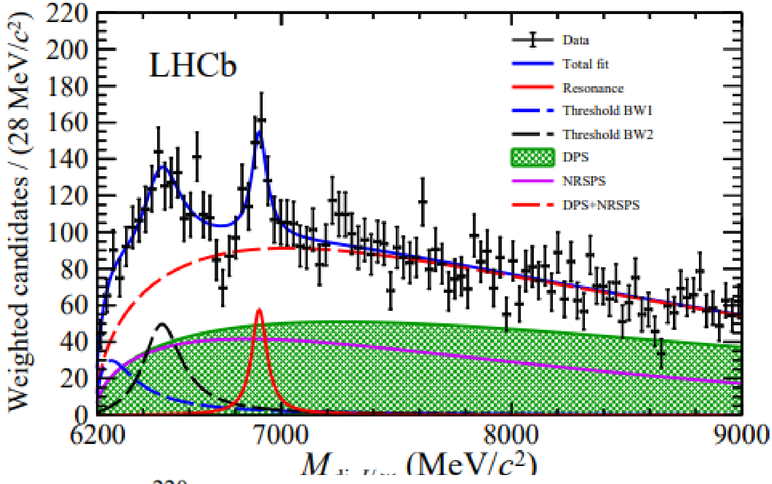
Published in: *Sci.Bull.* 65 (2020) 23, 1983-1993 • e-Print: 2006.16957 [hep-ex]

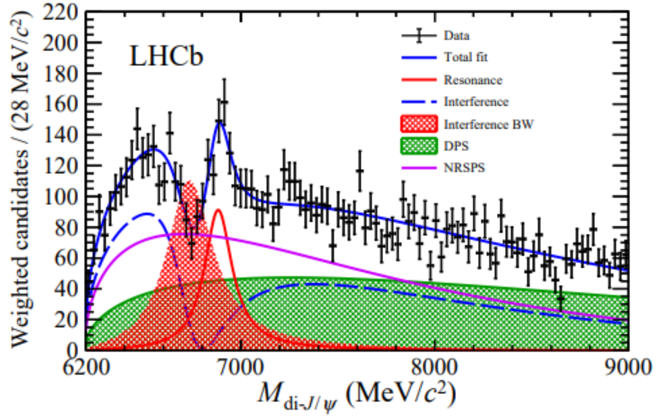
pdf links DOI cite claim reference search 261 citations



(a)







a **narrow resonance** around **6.9 GeV** with 5σ

a **broad structure** with the mass ranging from **6.2 GeV to 6.8 GeV**

a hint for another structure around 7.2 GeV.

1. Introduction

the X(6900) was confirmed in the di-J/ψ channel by the **CMS collaboration** and di-J/ψ as well as J/ψ+ψ(2S) channels by the **ATLAS collaboration**.

In addition, **some new resonances are observed**: X(6600) and X(7200) in CMS, and X(6200), X(6600) and X(7200) in ATLAS.

the mass and decay width of X(6900)

LHCb Without considering NRSPS

$$\begin{aligned} M &= 6950 \pm 11 \pm 7\,\mathrm{MeV} \\ \Gamma &= 80 \pm 19 \pm 33\,\mathrm{MeV} \end{aligned}$$

LHCb by considering NRSPS

$$\begin{aligned} M &= 6886 \pm 11 \pm 11\,\mathrm{MeV} \\ \Gamma &= 168 \pm 33 \pm 69\,\mathrm{MeV} \end{aligned}$$

CMS

$$\begin{aligned} M &= 6.87 \pm 0.03^{+0.06}_{-0.01}\,\mathrm{GeV} \\ \Gamma &= 0.12 \pm 0.04^{+0.03}_{-0.01}\,\mathrm{GeV} \end{aligned}$$

ATLAS

$$\begin{aligned} M &= 6927 \pm 9 \pm 5\,\mathrm{MeV} \\ \Gamma &= 122 \pm 22 \pm 19\,\mathrm{MeV} \end{aligned}$$

1. Introduction

Inspired extensive theoretical investigations

Based on various phenomenological models

constituent quark models

QCD sum rules

diffusion Monte Carlo

…

Citation Summary

☐ Exclude self-citations ?

	Citeable ?	Published ?
Papers	**229**	**155**
Citations	**3,914**	**3,604**
h-index ?	**38**	**37**
Citations/paper (avg)	**17.1**	**23.3**

Theoretical interpretations for them are still **controversial**.

Even the **quantum number** assignments for these states are not yet fully determined.

Due to lack of light quarks, the X(6900) is more likely to be a compact four charm quark state.

1. Introduction

It is both challenging and necessary to understand the nature of these fully charmed tetraquark systems **in the framework of QCD**.

It is worth remarking that, **lattice nonrelativistic QCD** (NRQCD) has also been used to investigate the lowest-lying spectrum of the $bb\bar{b}\bar{b}$ sector, but **found no indication** of any states below $2\eta_b$ threshold in the 0^{++}, 1^{+-} and 2^{++} channels.

By assuming $\boldsymbol{T_{4c}}$ to be **S-wave fully charmed tetraquarks** (the J^{PC} quantum number can be 0^{++}, 1^{+-} and 2^{++}), we study $\boldsymbol{T_{4c}}$ production and decay based on NRQCD factorization.

2. T_{4c} production at LHC

2009.08450 Feng, Huang, Jia, SWL, Xiong, Zhang
2304.11142 Feng, Huang, Jia, SWL, Yang, Zhang

Differential cross section of T_{4c} through fragmentation in pp collision:

$f_{a,b/p}$ denotes the parton distribution functions (PDFs) of a proton

$$\mathrm{d}\sigma\left(pp \to T_{4c}\left(p_{\mathrm{T}}\right) + X\right) = \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z \; f_{a/p}(x_a, \mu) f_{b/p}(x_b, \mu) \, d\hat{\sigma}(ab \to i(p_T/z) + X, \mu) \boxed{D_{i\to T_{4c}}\left(z, \mu\right)} + \mathcal{O}(1/p_T)$$

The partonic level cross sections are known for a long time.

Fragmentation function

2. T_{4c} production at LHC

The gluon fragmentation is defined by

Collins, Soper, 1981

$$D_{g\to T_{4c}}(z,\mu) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ \left(N_c^2-1\right)(d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+x^-}$$
$$\times \sum_X \langle 0 \left| G_c^{+\mu}(0) \mathcal{E}^\dagger \left(0,0,\mathbf{0}_\perp\right)_{cb} \right| T_{4c}(P) + X \rangle \langle T_{4c}(P) + X | \mathcal{E}\left(0,x^-,\mathbf{0}_\perp\right)_{ba} G_a^{+\nu}\left(0,x^-,\mathbf{0}_\perp\right) \left| 0 \right\rangle ,$$

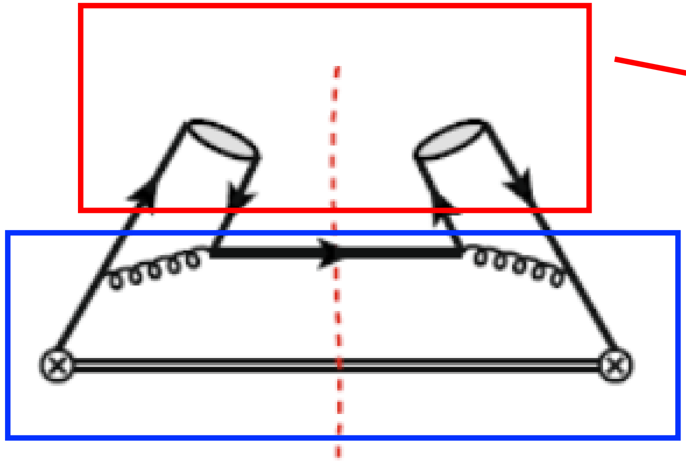
The fragmentation function obeys the renowned DGLAP evolution equation

$$\mu \frac{\partial}{\partial \mu} D_{g\to T_{4c}}(z,\mu) = \sum_{i\in\{g,c\}} \int_z^1 \frac{\mathrm{d}y}{y} P_{g\to i}\left(\frac{z}{y},\mu\right) D_{i\to T_{4c}}(y,\mu)$$

2. T_{4c} production at LHC

The fragmentation function may be computed based on NRQCD factorization

Analogous to the case of quarkonium production

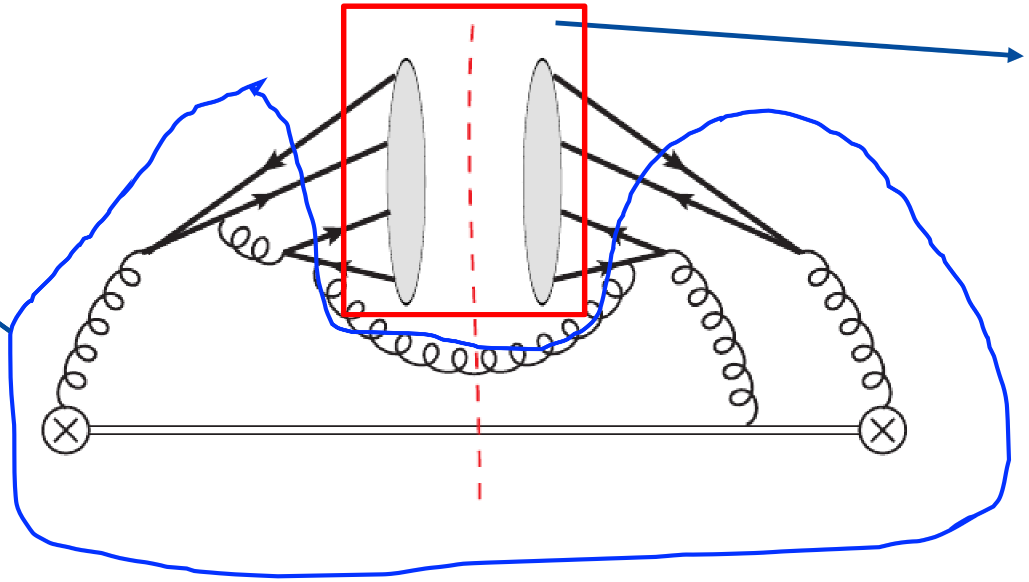


$$\sigma = \sum_i c_i \times \langle O \rangle_i$$

2. T_{4c} production at LHC

A key observation is that, prior to hadronization, two charm and two anticharm have to be created at a rather short distance ~ $1/m_c$, thus one can invoke asymptotic freedom to factorize the production rate as the product of the perturbative calculable short-distance part and the nonperturbative long-distance part

Scale as > m_c , perturbative calculable



Scale as < m_c , nonperturbative

2. T_{4c} production at LHC

The factorization formula for the fragmentation function at lowest order in velocity

$$D_{g\to T_{4c}}(z,\mu_\Lambda) = \frac{d_{3,3}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle 0\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|T^{(J)}_{4c}\right\rangle\right|^2 + \frac{d_{6,6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle 0\left|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|T^{(J)}_{4c}\right\rangle\right|^2$$
$$+\frac{d_{3,6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}2\mathrm{Re}\left[\left\langle 0\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|T^{(J)}_{4c}\right\rangle\left\langle T^{(J)}_{4c}\left|\mathcal{O}^{(J)\dagger}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|0\right\rangle\right]+\cdots$$

Short-distance coefficients

Long-distance matrix elements

2. T_{4c} production at LHC

The complement operators (S-wave) are built in the diquark-antidiquark basis

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{\alpha\beta;(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\,\Gamma^{\alpha\beta;mn}\,\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}$$

the spin configuration and color configuration of the diquark are correlated due to Fermi statistics

$$\left[\psi_a^T(i\sigma^2)\psi_b\right] = \left[\psi_b^T(i\sigma^2)\psi_a\right]$$

$$\left[\psi_a^T(i\sigma^2)\sigma^i\psi_b\right] = -\left[\psi_b^T(i\sigma^2)\sigma^i\psi_a\right]$$

$\Rightarrow$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \equiv \frac{1}{(\sqrt{2})^2}\epsilon^{abm}\epsilon^{cdn}\frac{\delta^{mn}}{\sqrt{3}} = \frac{1}{2\sqrt{N_c}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc})$$

$$\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} \equiv \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

2. T_{4c} production at LHC

The complement operators (S-wave) are built in the diquark-antidiquark basis

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}, \quad \Rightarrow \quad J^{PC} = 0^{++}$$

$$\mathcal{O}^{\alpha\beta;(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\,\Gamma^{\alpha\beta;mn}\,\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}, \quad \Rightarrow \quad J^{PC} = 2^{++}$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} \quad \Rightarrow \quad J^{PC} = 0^{++}$$

$$\psi_a(t,x) \overset{P}{\leftrightarrow} \psi_a(t,-x)$$

$$\psi_a(t,x) \overset{C}{\leftrightarrow} -i\sigma^2\chi_a^*(t,x)$$

$$\chi_a(t,x) \overset{P}{\leftrightarrow} -\chi_a(t,-x)$$

$$\chi_a(t,x) \overset{C}{\leftrightarrow} i\sigma^2\psi_a^*(t,x)$$

2. T_{4c} production at LHC

Determine the short-distance coefficients

$$D_{g\to\widetilde{T}_{4c}}(z,\mu_\Lambda) = \frac{d_{3,3}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle 0\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|\widetilde{T}^{(J)}_{4c}\right\rangle\right|^2 + \frac{d_{6,6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle 0\left|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|\widetilde{T}^{(J)}_{4c}\right\rangle\right|^2$$
$$+\frac{d_{3,6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}2\mathrm{Re}\left[\left\langle 0\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|\widetilde{T}^{(J)}_{4c}\right\rangle\left\langle\widetilde{T}^{(J)}_{4c}\left|\mathcal{O}^{(J)\dagger}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|0\right\rangle\right]+\cdots$$

A fictitious tetraquark state with the same quantum number as $\boldsymbol{T_{4c}}$

2. T_{4c} production at LHC

Relate the LDMEs to the phenomenological wave functions at the origin

$$\langle O_{3,3}^{(J)}\rangle = \langle 0|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}|T_{4c}\rangle\langle T_{4c}|\mathcal{O}^{(J)+}_{\bar{\mathbf{3}}\otimes\mathbf{3}}|0\rangle \approx 16(2J+1)\psi_3(0)\psi_3^*(0)$$
$$\langle O_{6,6}^{(0)}\rangle = \langle 0|\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}|T_{4c}\rangle\langle T_{4c}|\mathcal{O}^{(0)+}_{\mathbf{6}\otimes\bar{\mathbf{6}}}|0\rangle \approx 16\psi_6(0)\psi_6^*(0),$$
$$\langle O_{3,6}^{(0)}\rangle = \mathrm{Re}[\langle 0|\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}|T_{4c}\rangle\langle T_{4c}|\mathcal{O}^{(0)+}_{\mathbf{6}\otimes\bar{\mathbf{6}}}|0\rangle] \approx 16\psi_3(0)\psi_6^*(0),$$

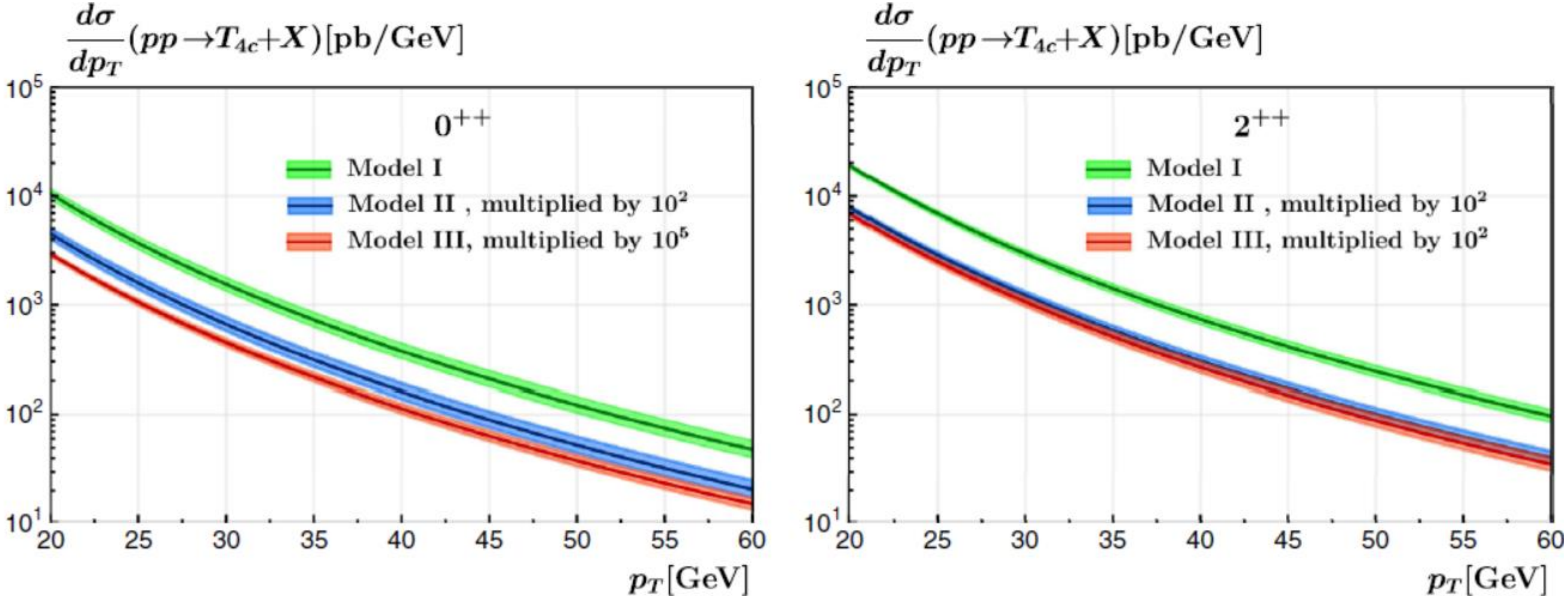
TABLE I. Numerical values of the LDMEs in models I, II, and III.

	0^{++}			2^{++}
	$\langle O_{3,3}^{(0)}\rangle$[GeV^9]	$\langle O_{3,6}^{(0)}\rangle$[GeV^9]	$\langle O_{6,6}^{(0)}\rangle$[GeV^9]	$\langle O_{3,3}^{(2)}\rangle$[GeV^9]
Model I [24]	5.77	5.01	4.35	17.4
Model II [25]	0.0347	0.0211	0.0128	0.072
Model III [26]	0.0187	–0.0161	0.0139	0.0628

PRD102,114001(2020); EPJC80,871 (2020); arXiv: 2006.11952

2. T_{4c} production at LHC

Phenomenological prediction



Whether is the fragmentation mechanism enough at low $\boldsymbol{p_T}$ range?

By roughly estimation, the fragmentation is suppressed by an extra α_s however enhanced by $\boldsymbol{p_T}^2/m_H^2$

2. T_{4c} production at LHC

We expect the **fragmentation mechanism** will denominate the **large p_T distribution**.

The fixed order **T_{4c} production** will dominate the **lower p_T distribution.**

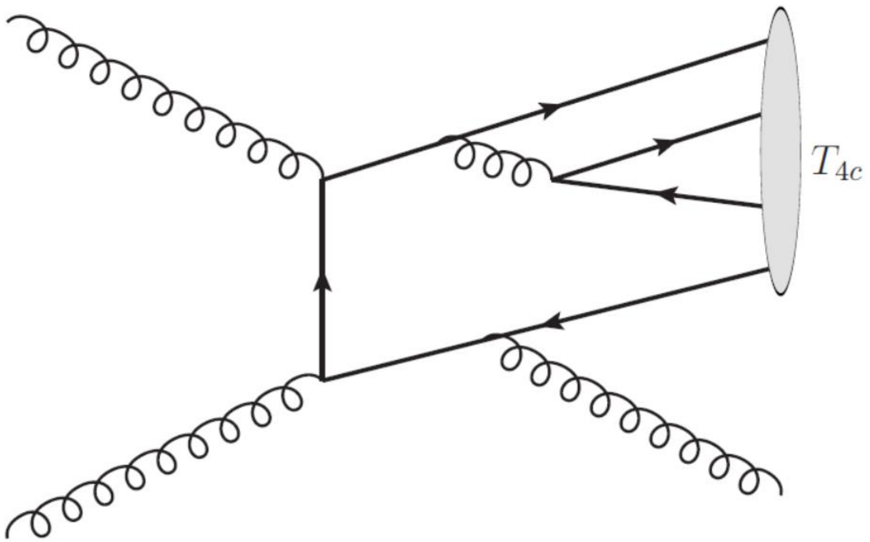


FIG. 1: One typical Feynman diagram for $gg \to T_{4c} + g$.

At small p_T , the gluon-gluon scattering will denominate the cross section.

2. T_{4c} production at LHC

At fixed order, the **$J^{PC}=1^{+-}$** S-wave T_{4c} can also be produced.

$$\frac{d\hat{\sigma}(T_{4c}^{(J)}+X)}{d\hat{t}} = \frac{2m_H}{m_c^{14}}\left[F_{3,3}^{(J)}\left\langle O_{3,3}^{(J)}\right\rangle + 2F_{3,6}^{(J)}\left\langle O_{3,6}^{(J)}\right\rangle + F_{6,6}^{(J)}\left\langle O_{6,6}^{(J)}\right\rangle\right]$$

Asymptotic expressions:

$$F_{3,3}^{0^{++}} = \frac{2209\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^4}{15552\hat{s}^5(-\hat{t})^3\left(\hat{s}+\hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right), \tag{1a}$$

$$F_{3,6}^{0^{++}} = \sqrt{6}F_{3,3}^{0^{++}} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right), \tag{1b}$$

$$F_{6,6}^{0^{++}} = \frac{2}{3}F_{3,3}^{0^{++}} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right), \tag{1c}$$

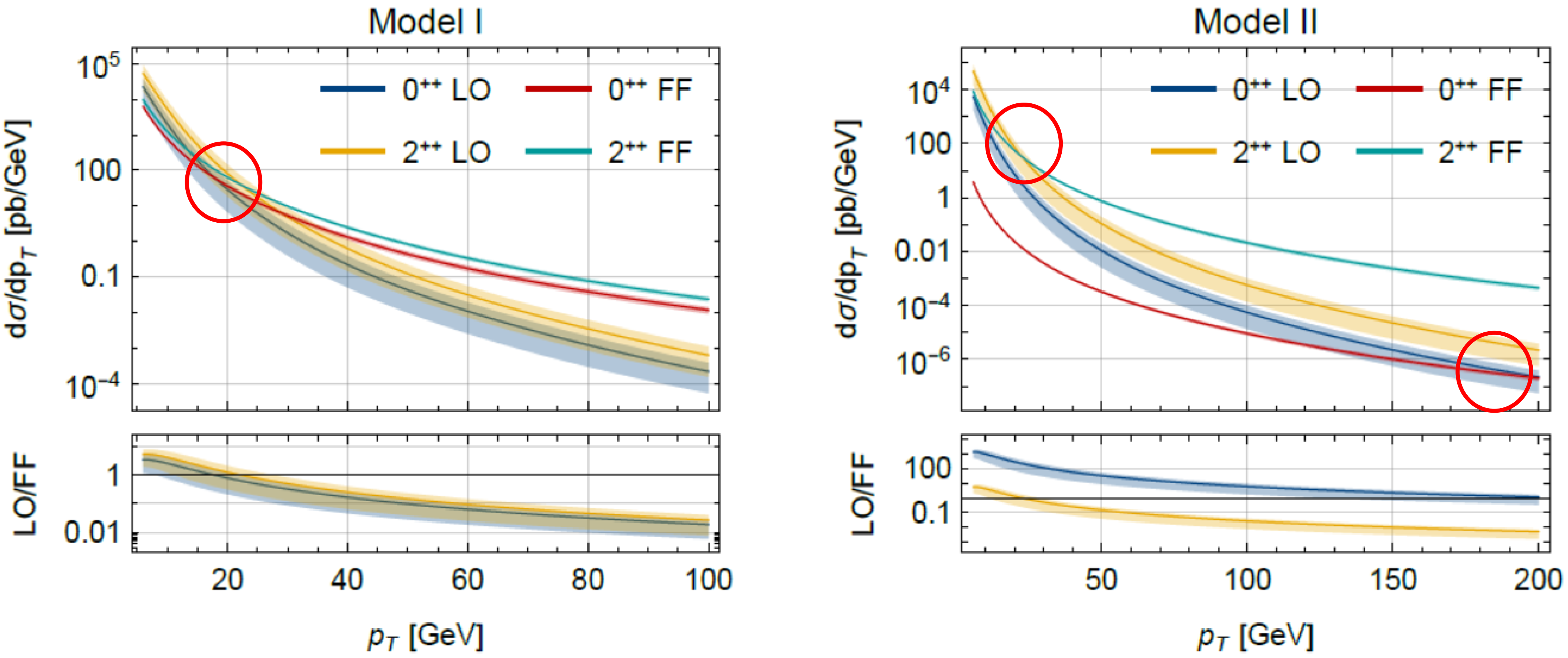
$$\boxed{F_{3,3}^{1^{+-}}} = \frac{60025\pi^4 m_c^8 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^2}{34992\hat{s}^4\hat{t}^2\left(\hat{s}+\hat{t}\right)^2} + \mathcal{O}\boxed{\left(\frac{m_c^9}{p_T^9}\right)}, \tag{1d}$$

$$F_{3,3}^{2^{++}} = \frac{17617\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^4}{38880\hat{s}^5(-\hat{t})^3\left(\hat{s}+\hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right), \tag{1e}$$

the production rate of **C-odd** tetraquark is severely suppressed with respect to the **C-even** ones due to the extra power of $1/p_T^2$

2. T_{4c} production at LHC

Comparison of the p_T distributions of the T_{4c} between **fixed order** and **fragmentation mechanism**



2. T_{4c} production at LHC

By assuming the integrated luminosity of **3000 fb^{-1}** .

	Model I		Model II	
	σ [nb]	$N_{\text{events}}/10^9$	σ [nb]	$N_{\text{events}}/10^9$
0^{++}	37 ± 26	110 ± 80	9 ± 6	27 ± 19
1^{+-}	0.28 ± 0.16	0.8 ± 0.5	0.17 ± 0.10	0.52 ± 0.29
2^{++}	93 ± 65	280 ± 200	81 ± 57	240 ± 170

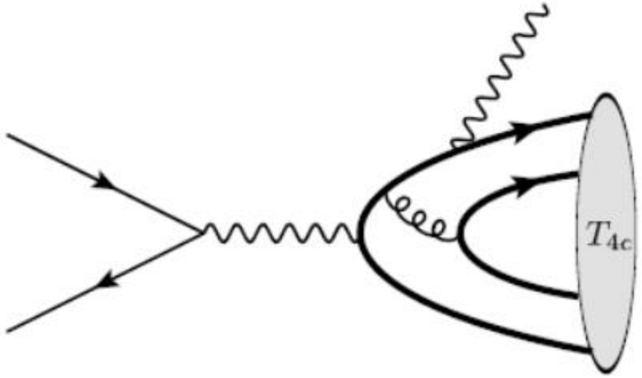
Table 1: The integrated production rates for various S-wave T_{4c} states ($6\,\text{GeV} \leq p_T \leq 100\,\text{GeV}$) and the estimated event yields.

Remarks:

1. **The cross sections and event numbers for 1^{+-} and 2^{++} are not insensitive to the Model. The condition for 0^{++} is different.**
2. **The cross section for 1^{+-} is two order-of-magnitude smaller than the other two channels.**

2. T_{4c} production at B factory

2011.03039 Feng, Huang, Jia, SWL, Zhang
2104.03887 Huang, Feng, Jia, SWL, Yang, Zhang



Only for $\mathbf{0^{++}}$ $\mathbf{2^{++}}$

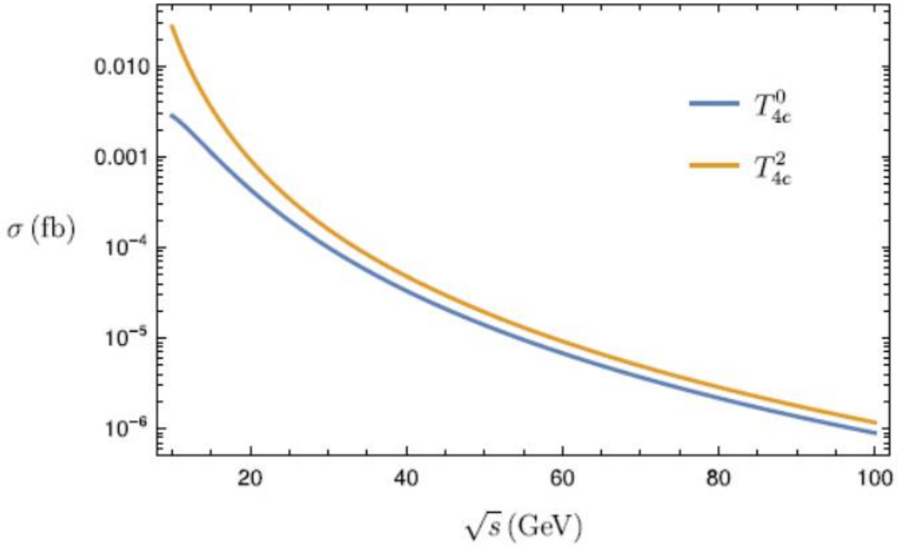
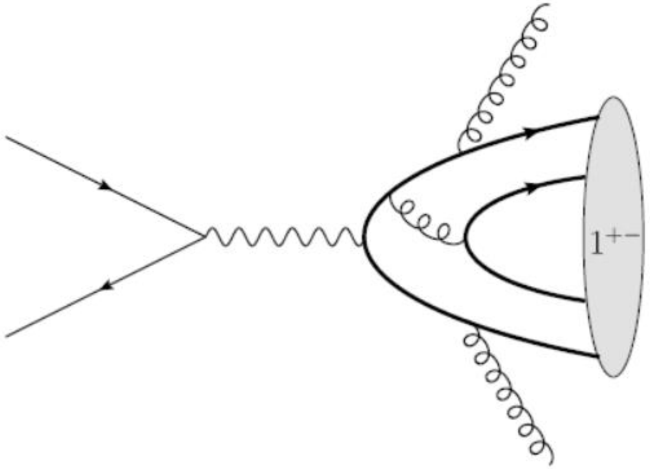
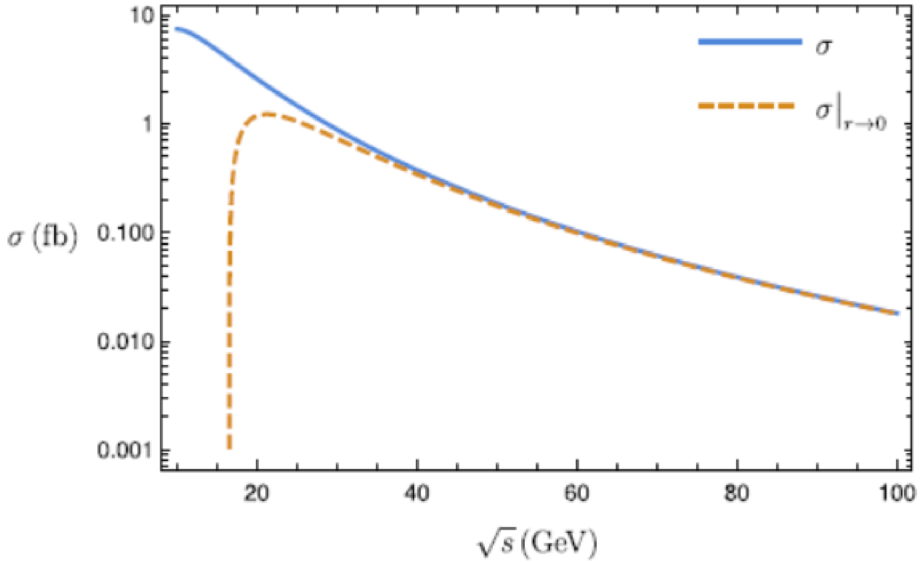


Fig. 2. The total cross sections of $e^+e^- \to \gamma + T^J_{4c}$ ($J = 0, 2$) as a function of the center-of-mass energy.



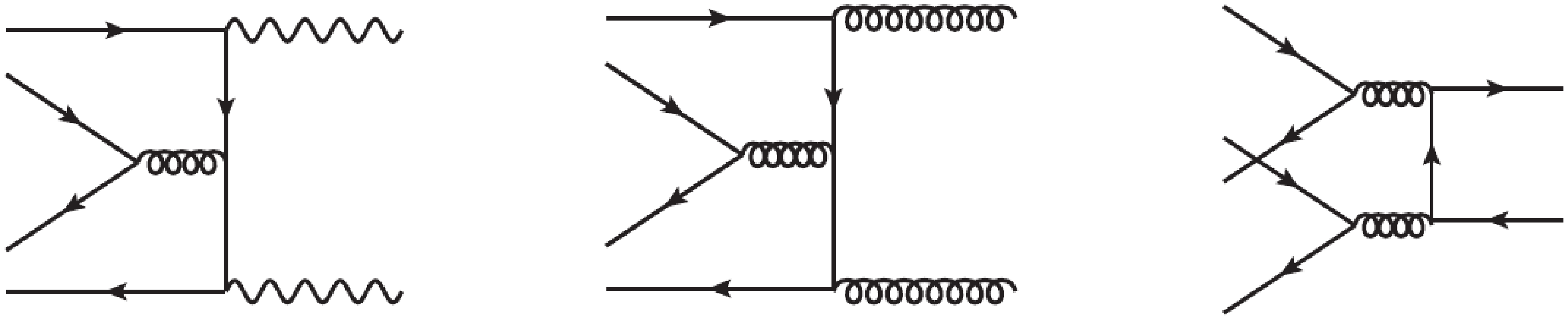
Only for $\mathbf{1^{+-}}$



3. T_{4c} electromagnetic and hadronic decay

In preparation, Feng, SWL, Wang, Zhang

To **verify the factorization formalism** at next-to-leading order in α_s



The viable decay channels

0^{++}	$T_{4c} \to \gamma\gamma, \quad T_{4c} \to \ell\ell$	$T_{4c} \to gg, \quad T_{4c} \to ggg, \quad T_{4c} \to gq\bar{q}$
1^{+-}	$T_{4c} \to \ell\ell$	$T_{4c} \to ggg, \quad T_{4c} \to gq\bar{q}$
2^{++}	$T_{4c} \to \gamma\gamma, \quad T_{4c} \to \ell\ell$	$T_{4c} \to gg, T_{4c} \to q\bar{q}, \quad T_{4c} \to ggg, \quad T_{4c} \to gq\bar{q}$

3. T_{4c} electromagnetic and hadronic decay

It is straightforward to **convert the factorization formula into the counterpart for T_{4Q} electromagnetic and hadronic decays**

$$\begin{aligned}
\Gamma[T_{4Q}^{0^{++}} \to \mathrm{LH}] &= c_{\mathrm{LH},1}^{(0)} \times \frac{2m_H \langle \mathcal{O}^{(0)}_{\bar{\mathbf{6}}\otimes\mathbf{6}} \rangle}{4^2 (2m_Q)^4} + c_{\mathrm{LH},2}^{(0)} \times \frac{2m_H \langle \mathcal{O}^{(0)}_{\mathbf{3}\otimes\bar{\mathbf{3}}} \rangle}{4^2 (2m_Q)^4} + c_{\mathrm{LH},3}^{(0)} \times \frac{2m_H \langle \mathcal{O}^{(0)}_{\mathrm{mixing}} \rangle}{4^2 (2m_Q)^4}, \\
\Gamma[T_{4Q}^{1^{+-}} \to \mathrm{LH}] &= c_{\mathrm{LH}}^{(1)} \times \frac{2m_H \langle \mathcal{O}^{(1)}_{\mathbf{3}\otimes\bar{\mathbf{3}}} \rangle}{4^2 (2m_Q)^4}, \\
\Gamma[T_{4Q}^{2^{++}} \to \mathrm{LH}] &= c_{\mathrm{LH}}^{(2)} \times \frac{2m_H \langle \mathcal{O}^{(2)}_{\mathbf{3}\otimes\bar{\mathbf{3}}} \rangle}{4^2 (2m_Q)^4},
\end{aligned}$$

Our main task is to compute the SDCs up to NLO in α_s .

3. T_{4c} electromagnetic and hadronic decay

For $\boldsymbol{T_{4c}}$ electromagnetic decay:

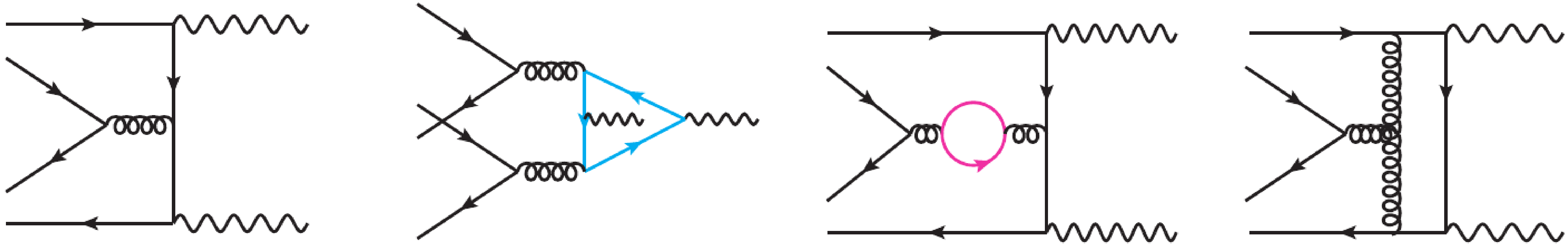
The SDC is perturbative calculable, we can expand it in powers of α_s :

$$c_{\gamma\gamma,i}^{(0)} = \frac{1}{2m_H}\frac{1}{8\pi}c_{\gamma\gamma,i}^{(0),0}\left[1+\frac{\alpha_s}{\pi}\left(\boxed{\frac{1}{2}\beta_0 \ln\frac{\mu^2}{m_Q^2}}+c_{\gamma\gamma,i}^{(0),1}\right)+\mathcal{O}(\alpha_s^2)\right],$$

$$c_{\gamma\gamma}^{(2)} = \frac{1}{2m_H}\frac{1}{8\pi}c_{\gamma\gamma}^{(2),0}\left[1+\frac{\alpha_s}{\pi}\left(\boxed{\frac{1}{2}\beta_0 \ln\frac{\mu^2}{m_Q^2}}+c_{\gamma\gamma}^{(2),1}\right)+\mathcal{O}(\alpha_s^2)\right],$$

The explicit $\ln \mu^2$ terms are deduced from the renormalization-group invariance.

3. T_{4c} electromagnetic and hadronic decay



At LO: **40** Feynman diagrams

At NLO: **856** Feynman diagrams

$$c^{(0),0}_{\gamma\gamma,1} = \frac{2048\pi^4 e_Q^4 \alpha^2 \alpha_s^2}{3m_Q^4},$$

$$c^{(0),0}_{\gamma\gamma,2} = \frac{36864\pi^4 e_Q^4 \alpha^2 \alpha_s^2}{m_Q^4},$$

$$c^{(0),0}_{\gamma\gamma,3} = \frac{2048\sqrt{6}\pi^4 e_Q^4 \alpha^2 \alpha_s^2}{m_Q^4},$$

3. T_{4c} electromagnetic and hadronic decay

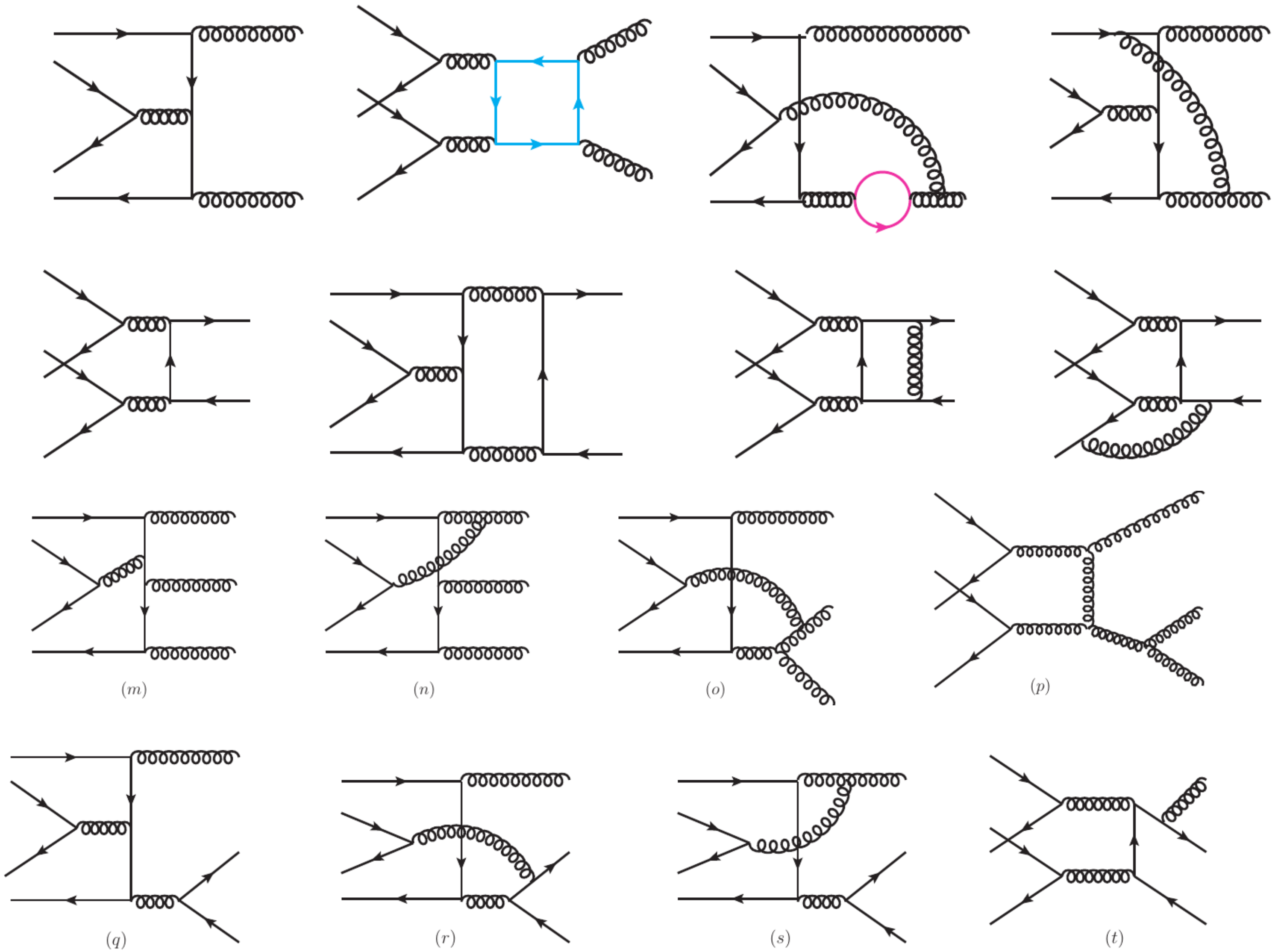
At NLO

$$c^{(0),1}_{\gamma\gamma,1} = \left\{ -\frac{1}{32} n_H \left[11\pi^2 - 32 + 56\ln^2\left(1+\sqrt{2}\right) + 12\ln^2\left(2-\sqrt{3}\right) - 16\sqrt{2}\ln\left(1+\sqrt{2}\right)\right] \right.$$
$$\left. - \frac{1}{4}(\pi^2-4) f_1 \right\}_{\mathrm{LBL}} - \frac{2}{9} n_H \left[10 + 3\sqrt{2}\ln(1+\sqrt{2})\right] + \frac{1}{9} n_L (6\ln 2 - 5) + \frac{2165}{144}\mathrm{Li}_2\left(\frac{1}{3}\right)$$
$$- \frac{91}{48} f_2 + \frac{2945}{1728}\pi^2 + \frac{1}{18} + \ln^2\left(2-\sqrt{3}\right) - \frac{65}{6}\ln^2\left(\sqrt{2}+1\right) - \frac{91}{24}\ln^2 5 + \frac{2165}{288}\ln^2 3$$
$$- \frac{91}{6}\ln 2\ln 3 + \frac{91}{12}\ln 2\ln 5 + \frac{91}{16}\ln 3\ln 5 + \frac{281}{9}\sqrt{2}\ln\left(\sqrt{2}+1\right) - \frac{1595}{54}\ln 2,$$

Numerical results

$$c^{(0),1}_{\gamma\gamma,1} = \left(-3.779 n_H - 1.467 \sum_i \frac{e_i^2}{e_Q^2} \right)_{\mathrm{LBL}} - 3.053 n_H - 0.093 n_L + 38.440,$$
$$c^{(0),1}_{\gamma\gamma,2} = \left(-0.420 n_H - 0.163 \sum_i \frac{e_i^2}{e_Q^2} \right)_{\mathrm{LBL}} - 0.167 n_H - 0.093 n_L - 2.422,$$
$$c^{(0),1}_{\gamma\gamma,3} = \left(-2.100 n_H - 0.815 \sum_i \frac{e_i^2}{e_Q^2} \right)_{\mathrm{LBL}} - 1.610 n_H - 0.093 n_L + 18.009,$$

3. T_{4c} electromagnetic and hadronic decay



$T_{4c} \to gg$

LO: **72** Feynman diagrams
NLO: **2394** Feynman diagrams

$T_{4c} \to q\bar{q}$

LO: **14** Feynman diagrams
NLO: **444** Feynman diagrams

$T_{4c} \to ggg$

682 Feynman diagrams

$T_{4c} \to gq\bar{q}$

128 Feynman diagrams

3. T_{4c} electromagnetic and hadronic decay

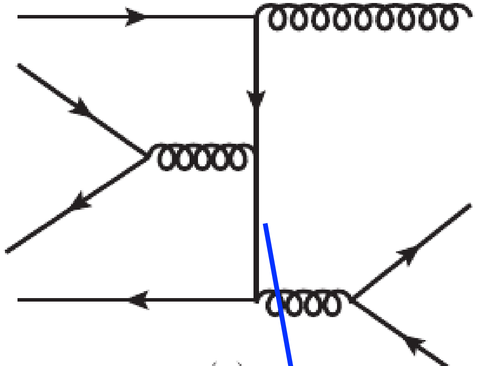
$$c^{(0),1}_{\mathrm{LH},i} = f_\epsilon \left[d^{(0)}_{\mathrm{LH},i} + f^{(0)}_{\mathrm{LH},i} \right], \qquad f_\epsilon = (4\pi e^{-\gamma_E})^\epsilon$$

Numerical results for the **virtual corrections**

$$\begin{aligned}
d^{(0),1}_{\mathrm{LH},1} &= -\frac{3}{\epsilon^2} + \frac{1}{3}\frac{n_L}{\epsilon} - \frac{7.273}{\epsilon} - 4.338 n_H + 1.434 n_L + 10.256, \\
d^{(0),1}_{\mathrm{LH},2} &= -\frac{3}{\epsilon^2} + \frac{1}{3}\frac{n_L}{\epsilon} + \frac{15.818}{\epsilon} - 1.653 n_H - 1.372 n_L - 34.102, \\
d^{(0),1}_{\mathrm{LH},3} &= -\frac{3}{\epsilon^2} + \frac{1}{3}\frac{n_L}{\epsilon} + \frac{4.272}{\epsilon} - 2.995 n_H + 0.031 n_L + 10.293
\end{aligned}$$

3. T_{4c} electromagnetic and hadronic decay

Special treatment for the **real corrections**



(a)

The following approaches **are invalid**:

A. Reverse unitarities

$$\int \frac{d^d k_i}{(2\pi)^d} 2\pi\, i\, \delta(k_i^2)\theta(k_i^0) = \int \frac{d^d k_i}{(2\pi)^d} \left(\frac{1}{k_i^2 + i\varepsilon} - \frac{1}{k_i^2 - i\varepsilon} \right)$$

B. Subtraction

The propagator may be on-shell **at certain points inside the kinematic invariants of the phase space**.

3. T_{4c} electromagnetic and hadronic decay

Special treatment for the **real corrections**

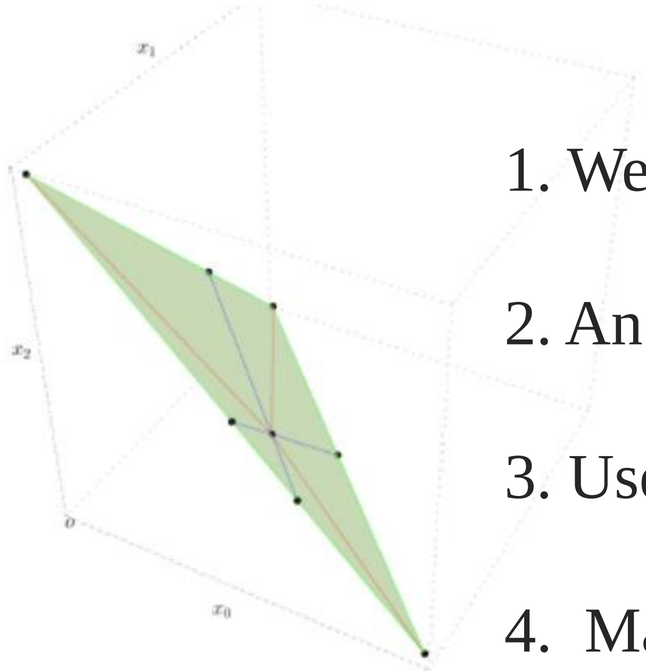
$$\mathcal{I} = \int_0^\infty d^3\boldsymbol{x}\, \frac{(x_0 x_1 x_2)^\epsilon \delta(1 - x_0 - x_1 - x_2)}{(x_0 + x_1 - 2x_2 + i\varepsilon)(x_0 - x_1 - x_2 - i\varepsilon)}$$

the key point is to divide the integration domain with $x_i \geq 0$ into sub-domains, and **the both denominators ($x_0 + x_1 - 2\, x_2$) and ($x_0 - x_1 - x_2$) have definite sign**, i.e., always positive or negative in each sub-domain.

3. T_{4c} electromagnetic and hadronic decay

Special treatment for the **real corrections**

$$\mathcal{I} = \int_0^{\infty} d^3\boldsymbol{x}\, \frac{(x_0 x_1 x_2)^{\epsilon}\delta(1 - x_0 - x_1 - x_2)}{(x_0 + x_1 - 2x_2 + i\varepsilon)(x_0 - x_1 - x_2 - i\varepsilon)}$$



1. We decomposed the integration domain into several parts.

2. An additional regulator is required to handle the singularities.

3. Use sector decomposition to separate the divergence.

4. Make Taylor expansion and perform the numerical integration.

3. T_{4c} electromagnetic and hadronic decay

Numerical values for the **real corrections**

$$
\begin{aligned}
f^{(0),1}_{\mathrm{LH},1} \quad = \quad & \boxed{\frac{3}{\epsilon^2} - \frac{1}{3}\frac{n_L}{\epsilon} + \frac{7.2731429241897471 96}{\epsilon}} \\
& -13.89907809078159655335027518512524601420753012728681(5)n_L \\
& +60.5162124478415296920854439137218160774326081358961 4(7),
\end{aligned}
$$

$$
d^{(0),1}_{\mathrm{LH},1} \quad = \quad \boxed{-\frac{3}{\epsilon^2} + \frac{1}{3}\frac{n_L}{\epsilon} - \frac{7.273}{\epsilon}} - 4.338 n_H + 1.434 n_L + 10.256,
$$

The SDCs are finally finite. **We verify the factorization formula for the T_{4c} hadronic decay**.

4. Summary

1. We **propose a factorization formula** for T_{4c} production and decay in the NRQCD framework.

2. We study T_{4c} production at LHC and B factory. **The cross sections at LHC are considerable, and very small at B factory**.

3. To verify the factorization, we carry out the NLO corrections for T_{4c} electromagnetic and hadronic decay.

Thank you for your attention！