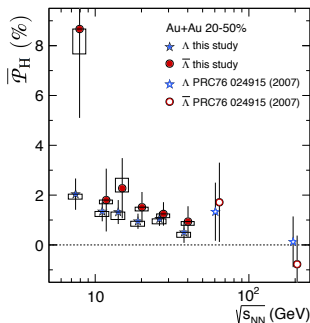


The Chiral and Deconfinement Transitions induced by Rotation in the PNJL model

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and Mei Huang_(UCAS)

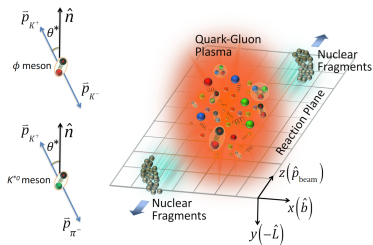
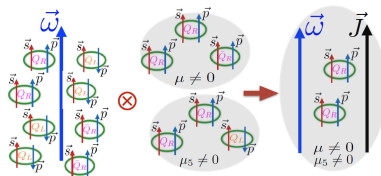
November 28, 2023

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- The diagram illustrates the experimental setup for studying the quark-gluon plasma. Two Beam Beam Chambers (BBCs) are used to detect particles. A forward-going beam fragment is shown, with a quark (q) and a gluon (g) being produced. The quark is labeled with a momentum vector $\vec{p}_q$ and the gluon with a momentum vector $\vec{p}_g$. The angle θ is shown between the beam direction and the quark's path. A vector $\vec{P}_\Lambda$ is also indicated.



Angular momentum at the order of $10^4 \sim 10^5 \hbar$! ($\omega \sim 10^{22} \text{s}^{-1}$)

- Rotation gives rise to lots of interesting phenomena...



The influence of rotation effect on the chiral condensate...

A (very incomplete) list of references on chiral condensate

Y. Jiang, J. Liao, Phys. Rev. Lett. 117(19) (2016).

H.-L. Chen, K. Fukushima, X.-G. Huang, K. Mameda, Phys. Rev. D 93(10) (2016).

S. Ebihara, K. Fukushima, K. Mameda, Phys. Lett. B 764 (2017) 94-99.

M. Chernodub, S. Gongyo, J. High Energy Phys. 01 (2017) 136, Phys. Rev. D 95(9) (2017) 096006.

X. Wang, M. Wei, Z. Li, M. Huang, Phys. Rev. D 99(1) (2019) 016018.

L. Wang, Y. Jiang, L. He, and P. Zhuang, Phys. Rev. D 100, 114009 (2019).

H. Zhang, D. Hou, J. Liao, Chin. Phys. C 44(11) (2020) 111001.

Yin Jiang, Eur. Phys. J. C 82 (2022) 10, 949.

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How the rotation affects the deconfinement transition?

X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, JHEP 07, 132 (2021).

Y. Fujimoto, K. Fukushima, and Y. Hidaka, Phys. Lett. B 816, 136184 (2021).

M. Chernodub, Phys. Rev. D 103, 054027 (2021).

V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, and A. A. Roenko, Phys. Rev. D 103, 094515 (2021).

Ji-Chong Yang, Xu-Guang Huang, arXiv:2307.05755 [hep-lat].

Gaoqing Cao, arXiv: 2310. 03310.

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PNJL model incorporates confinement effects...

What are the influences of rotation on the order parameters?

How does the rotation affect the chiral and deconfinement phase transitions in the PNJL model?

Based on following work:

Fei Sun, kun Xu, Mei Huang, Phys. Rev. D 108 (2023) 9, 096007.

Fei Sun, Shuang Li, Rui Wen, Anping Huang, Wei Xie, arXiv:2310.18942v1.

Fei Sun, Jingdong Shao, Mei Huang, (in progress).

Fei Sun, Anping Huang, Phys. Rev. D 106 (2022) 7, 076007.

For the rotating frame, the space-time metric reads

$$g_{\mu\nu} = \begin{pmatrix} 1 - \vec{v}^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix}.$$

The Lagrangian in the two-flavor NJL model under rotation:

$$\mathcal{L}_{NJL} = \sum_f \bar{\psi}_f \left\{ i\bar{\gamma}^\mu (\partial_\mu + \Gamma_\mu) - m + \gamma^0 \mu \right\} \psi_f + G(\bar{\psi}\psi)^2,$$

The Lagrangian of Polyakov-loop extended NJL model under rotation

$$\mathcal{L}_{PNJL} = \mathcal{L}_{NJL} + \bar{\psi} \gamma^\mu A_\mu \psi - \mathcal{U}(\Phi, \bar{\Phi}, T),$$

$$\Phi = \frac{1}{N_c} \langle \text{tr} L \rangle, \bar{\Phi} = \frac{1}{N_c} \langle \text{tr} L^\dagger \rangle, L(\bar{x}) = \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\bar{x}, \tau) \right].$$

Expanding the Lagrangian up to the first order of angular velocity, the PNJL model under rotation has the form:

$$\begin{aligned} \mathcal{L}_{PNJL} = & \bar{\psi} \left[i\gamma^\mu D_\mu - m + \gamma^0 \mu + (\gamma^0)^{-1} \left((\vec{\omega} \times \vec{x}) \cdot (-i\vec{\partial}) + \vec{\omega} \cdot \vec{S}_{4 \times 4} \right) \right] \psi \\ & + G(\bar{\psi}\psi)^2 - \mathcal{U}(\Phi[A], \bar{\Phi}[A], T), \end{aligned}$$

Carrying out the mean field approximation and the path integral formulation

$$\log \mathcal{Z} = -\frac{1}{T} \int d^3x \left(\frac{(M-m)^2}{4G} \right) + 2 \log \det \frac{D^{-1}}{T},$$

$$\log \det \frac{\hat{D}^{-1}}{T} = \text{tr} \log \frac{\hat{D}^{-1}}{T} = \int d^3x \int \frac{d^3p}{(2\pi)^3} \left\langle \psi_p(x) \left| \log \frac{\hat{D}^{-1}}{T} \right| \psi_p(x) \right\rangle.$$

$$D^{-1} = \begin{pmatrix} \left(-i\omega_l + \left(n + \frac{1}{2} \right) \omega + \mu - iA_4 \right) - M & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & - \left(-i\omega_l + \left(n + \frac{1}{2} \right) \omega + \mu - iA_4 \right) - M \end{pmatrix}$$

Considering the eigenstate of the complete set of commuting operators $\{\hat{H}, \hat{p}_z, \hat{p}_t^2, \hat{J}_z, \hat{h}_t\}$

$$u = \frac{1}{2} \sqrt{\frac{E+m}{E}} \begin{pmatrix} e^{ip_z z} e^{in\theta} J_n(p_t r) \\ se^{ip_z z} e^{i(n+1)\theta} J_{n+1}(p_t r) \\ \frac{p_z - isp_t}{E+m} e^{ip_z z} e^{in\theta} J_n(p_t r) \\ \frac{-sp_z + ip_t}{E+m} e^{ip_z z} e^{i(n+1)\theta} J_{n+1}(p_t r) \end{pmatrix},$$

$$v = \frac{1}{2} \sqrt{\frac{E+m}{E}} \begin{pmatrix} \frac{p_z - isp_t}{E+m} e^{-ip_z z} e^{in\theta} J_n(p_t r) \\ \frac{-sp_z + ip_t}{E+m} e^{-ip_z z} e^{i(n+1)\theta} J_{n+1}(p_t r) \\ e^{-ip_z z} e^{in\theta} J_n(p_t r) \\ -se^{-ip_z z} e^{i(n+1)\theta} J_{n+1}(p_t r) \end{pmatrix}.$$

Here, $\mathcal{H} = \bar{\psi} (i\gamma^0 \partial_0) \psi - \mathcal{L}$, the transverse helicity
 $\hat{h}_t = \gamma^5 \gamma^3 \vec{p}_t \cdot \vec{S}$, and $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$.

Taking trace over color space and using grand potential
 $\Omega = -\frac{T}{V} \log \mathcal{Z}$ we obtain

$$\begin{aligned} \Omega_{PNJL} = & G \langle \bar{q}q \rangle^2 - \frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\Lambda} p_t dp_t \int_{-\sqrt{\Lambda^2 - p_t^2}}^{\sqrt{\Lambda^2 - p_t^2}} dp_z \left((J_{n+1}(p_t r)^2 + J_n(p_t r)^2) T \times \right. \\ & \left\{ \log \left[1 + 3\Phi e^{-\frac{\varepsilon_{n-}\mu}{T}} + 3\bar{\Phi} e^{-2\frac{\varepsilon_{n-}\mu}{T}} + e^{-3\frac{\varepsilon_{n-}\mu}{T}} \right] + \log \left[1 + 3\bar{\Phi} e^{\frac{\varepsilon_{n-}\mu}{T}} + 3\Phi e^{2\frac{\varepsilon_{n-}\mu}{T}} + e^{3\frac{\varepsilon_{n-}\mu}{T}} \right] \right. \\ & \left. + \log \left[1 + 3\bar{\Phi} e^{-\frac{\varepsilon_{n+}\mu}{T}} + 3\Phi e^{-2\frac{\varepsilon_{n+}\mu}{T}} + e^{-3\frac{\varepsilon_{n+}\mu}{T}} \right] + \log \left[1 + 3\Phi e^{\frac{\varepsilon_{n+}\mu}{T}} + 3\bar{\Phi} e^{2\frac{\varepsilon_{n+}\mu}{T}} + e^{3\frac{\varepsilon_{n+}\mu}{T}} \right] \right\} \\ & + \mathcal{U}(\Phi, \bar{\Phi}, T). \end{aligned}$$

Here, $\varepsilon_n = \sqrt{(m - 2G \langle \bar{q}q \rangle)^2 + p_t^2 + p_z^2} - (\frac{1}{2} + n) \omega$ with the dynamic quark mass $M = m - 2G \langle \bar{q}q \rangle$.

Stationary condition

$$\frac{\partial \Omega}{\partial \langle \bar{q}q \rangle} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0.$$

Causality condition

$$\omega r < 1.$$

Case 1:

$$\mathcal{U}(\Phi, \bar{\Phi}, T)$$

Polyakov loop potential:

$$\frac{\mathcal{U}}{T^4} = -\frac{1}{2}b_2(T)\Phi\bar{\Phi} - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2.$$

$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3.$$

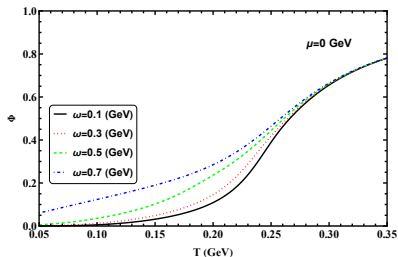
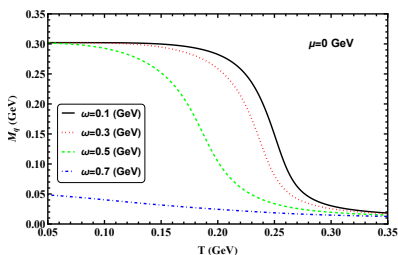
Input parameters

$$m = 0.005 \text{ GeV}, \Lambda = 0.65 \text{ GeV}, G = 4.93 \text{ GeV}^{-2}, n = 0, \pm 1, \pm 2, \dots, r = 0.1 \text{ GeV}^{-1}.$$

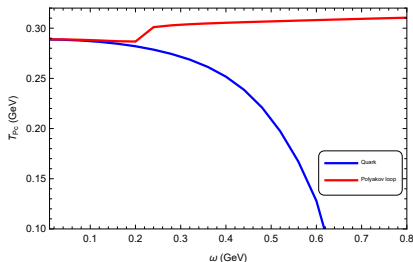
a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	

Table: Parameters of the Polyakov loop sector of the model.

- The light quark effective mass and Polyakov loop as functions of temperature T at $\mu = 0$ GeV for different angular velocities.



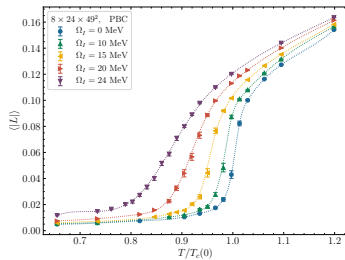
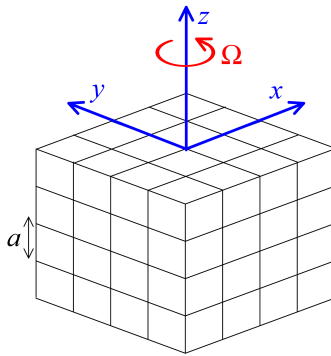
- The pseudocritical temperatures of the quark and Polyakov loop according to the angular velocity at zero chemical potential with $T_0 = 0.32$ GeV.



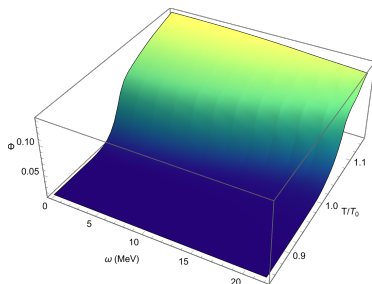
Case 2:

$$\mathcal{U}(\Phi, \bar{\Phi}, T, \omega)$$

- (Left) Rotating lattice. (Right) The Polyakov loop as a function of temperature for different values of imaginary angular velocity Ω_I (the figure is taken from Ref: PRD 103, 094515 (2021)).



- The Polyakov loop as a function of real angular velocity and temperature after the analytical continuation.



Polyakov loop potential under rotation:

$$\mathcal{U}(\Phi, \bar{\Phi}, T, \omega) = -\left(\frac{T}{T_0}\right)^2 \Phi \bar{\Phi} - \frac{2f(T, \omega)^2 \left(\frac{T}{T_0}\right)^4 - 1}{3f(T, \omega) \left(\frac{T}{T_0}\right)^2} (\Phi^3 + \bar{\Phi}^3) + \frac{1}{\left(\frac{T}{T_0}\right)^2} \Phi^2 \bar{\Phi}^2,$$

with

$$f(T, \omega) = a(\omega) \tanh \left(b(\omega) \left(\frac{T}{T_0} - c(\omega) \right) \right) + d(\omega),$$

here

$$a(\omega) = a_0 + a_2 \omega^2 + a_4 \omega^4,$$

$$b(\omega) = b_0 + b_2 \omega^2 + b_4 \omega^4,$$

...

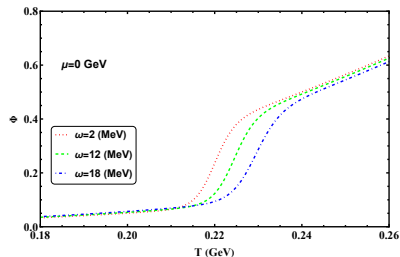
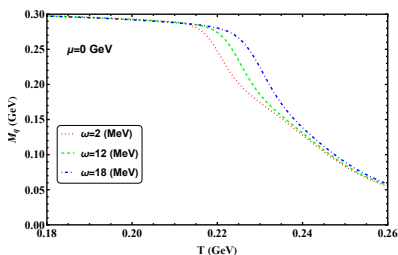
Input parameters

$$m = 0.005 \text{ GeV}, \Lambda = 0.65 \text{ GeV}, G = 4.93 \text{ GeV}^{-2}, n = 0, \pm 1, \pm 2 \dots, r = 0.1 \text{ GeV}^{-1}, C=0.2.$$

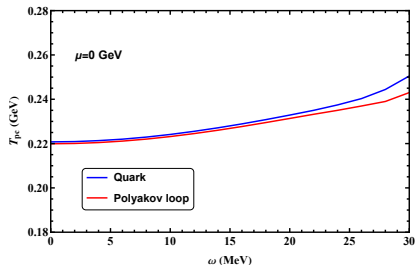
Parameters of the Polyakov loop potential fitted:

a_0	0.0454431
a_2	-1.27942×10^{-5}
a_4	-5.43339×10^{-9}
b_0	46.8263
b_2	-0.0210165
b_4	-2.15394×10^{-5}
c_0	1.00298
c_2	0.000155157
c_4	-5.99032×10^{-8}
d_0	0.0600157
d_2	-5.74388×10^{-6}
d_4	-8.24192

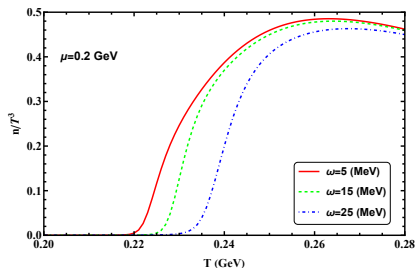
- The light quark effective mass and Polyakov loop as functions of temperature T at $\mu = 0$ GeV for different angular velocities.



- The pseudocritical temperatures of the quark and Polyakov loop according to the angular velocity at zero chemical potential.

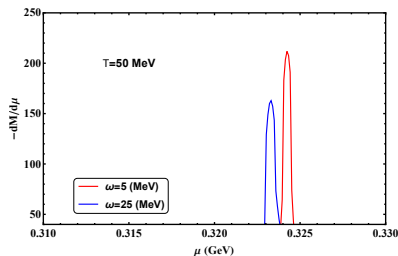
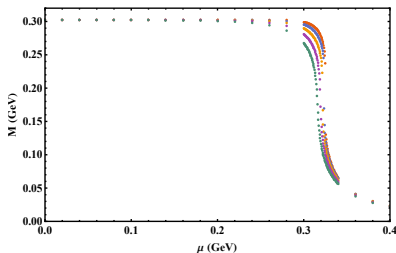


- The scaled quark number density with T^3 as a function of T in the PNJL model under rotation with $\mu = 0.2$ GeV for different angular velocities.

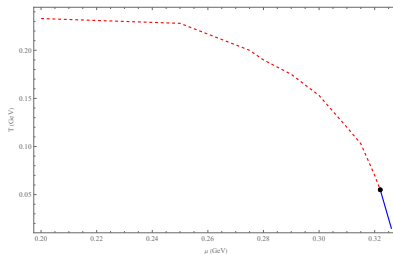


- The influence of the total rotation on the phase diagram of $T - \mu$ plain (in progress).

...



- The influence of the total rotation on the phase diagram of $T - \mu$ plain (in progress).
- ...



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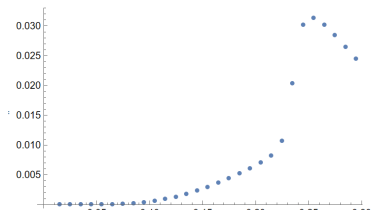
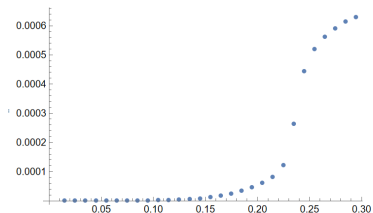
[illegible]

- The influence of the total rotation on the angular momentum (in progress).

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ListPlot[{netspinlistML $\omega$ 0025[[2, All, {1, 3}]]}]
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```
ListPlot[{netspinlistscaledML $\omega$ 0025[[2, All, {1, 3}]]}]
```



- The influence of the total rotation on the angular momentum (in progress).

...

1. The contribution of the quark sector is small.
2. The contribution of gluon sector is large, and may have a nonmonotonic behavior.

When considering $\mathcal{U} = \mathcal{U}(\Phi, \bar{\Phi}, T)$:

Conclusion 1

The deconfinement transition is not so sensitive to the presence of the rotating effect. The insensitivity of the deconfinement transition under rotation is similar to that of finite chemical potential.

Conclusion 2

The chiral dynamics and gluon dynamics can be split by rotation, which means a quarkyonic phase can be induced by rotation.

When considering $\mathcal{U} = \mathcal{U}(\Phi, \bar{\Phi}, T, \omega)$:

Conclusion 1

The chiral condensate is enhanced and the Polykov loop is suppressed by the rotation in the transition regime.

Conclusion 2

The rotation in the fermionic sector tends to decrease the critical temperatures, and the contribution from gluons increases the critical temperatures. Taken together, the critical temperatures of deconfinement and chiral transitions increase with angular velocity.

Conclusion 3

The contribution from the rotating gluon may lead to some interesting thermodynamic phenomena.

Thank you for your attention ! !