量子化与规范对称性

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背景：下一步物理（Next Physics）研究

- 半个世纪新物理的理论和实验研究：“0”
- “0”结果告诉了我们什么？
- 考察物理学基础：…1974…1948…1873…1687…

1. 为什么量子化?

- 为了解释实验：经典力学到新力学（不得不量子化）
- 有没有其他物理原因？（量子化是自然的）

Feynman's proof of the Maxwell equations

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I. THE PROOF

As I mentioned in my talk at the Feynman Memorial Session of the AAAS meeting in San Francisco,[1] Feynman showed me in October 1948 a proof of the Maxwell equations, assuming only Newton's law of motion and the commutation relation between position and velocity for a single nonrelativistic particle. In response to many enquiries, I here publish the proof in a form as close as I can come to Feynman's 1948 exposition. Unfortunately, I preserved neither Feynman's manuscript nor my original notes. What follows is a version reconstructed at some unknown time from notes which I discarded.

Assume a particle exists with position x_j ($j = 1,2,3$) and velocity $\dot{x}_j$ satisfying Newton's equation

$$m\ddot{x}_j = F_j(x,\dot{x},t), \tag{1}$$

with commutation relations

$$[x_j,x_k] = 0, \tag{2}$$

$$m[x_j,\dot{x}_k] = i\hbar\,\delta_{jk}. \tag{3}$$

Then there exist fields $E(x,t)$ and $H(x,t)$ satisfying the Lorentz force equation

$$F_j = E_j + \epsilon_{jkl}\dot{x}_k H_l, \tag{4}$$

and the Maxwell equations

$$\operatorname{div} H = 0, \tag{5}$$

$$\frac{\partial H}{\partial t} + \operatorname{curl} E = 0. \tag{6}$$

$$[x_j, F_k] + m[\dot{x}_j, \dot{x}_k] = 0. \tag{9}$$

$$[x_j, F_k] = -(i\hbar/m)\epsilon_{jkl}H_l. \tag{13}$$

quation (13) is the definition of the field H, which would

The definition (13) of H may be written

$$H_l = -(im^2/2\hbar)\epsilon_{jkl}[\dot{x}_j, \dot{x}_k] \tag{16}$$

by virtue of (9). Another application of the Jacobi identity gives

$$\epsilon_{jkl}[\dot{x}_l, [\dot{x}_j, \dot{x}_k]] = 0. \tag{17}$$

Equations (16) and (17) imply

$$[\dot{x}_l, H_l] = 0, \tag{18}$$

which is equivalent to (5). It remains to prove the second

Next we satisfy (4) by assuming it to be the definition of the field E. Again, E will in general depend on x, $\dot{x}$, and t,

Take the total derivative of Eq. (16) with respect to time. This gives

$$\frac{\partial H_l}{\partial t} + \dot{x}_m \frac{\partial H_l}{\partial x_m} = -\frac{im^2}{\hbar}\epsilon_{jkl}[\ddot{x}_j, \dot{x}_k]. \tag{19}$$

Now by (1) and (4), the right side of (19) becomes

$$\begin{aligned} &-(im/\hbar)\epsilon_{jkl}[E_j + \epsilon_{jmn}\dot{x}_m H_n, \dot{x}_k] \\ &\quad = -(im/\hbar)(\epsilon_{jkl}[E_j, \dot{x}_k] + [\dot{x}_k H_l, \dot{x}_k] - [\dot{x}_l H_k, \dot{x}_k]) \\ &\quad = \epsilon_{jkl}\frac{\partial E_j}{\partial x_k} + \dot{x}_k \frac{\partial H_l}{\partial x_k} - \dot{x}_l \frac{\partial H_k}{\partial x_k} \\ &\qquad + (im/\hbar)H_k[\dot{x}_l, \dot{x}_k]. \end{aligned} \tag{20}$$

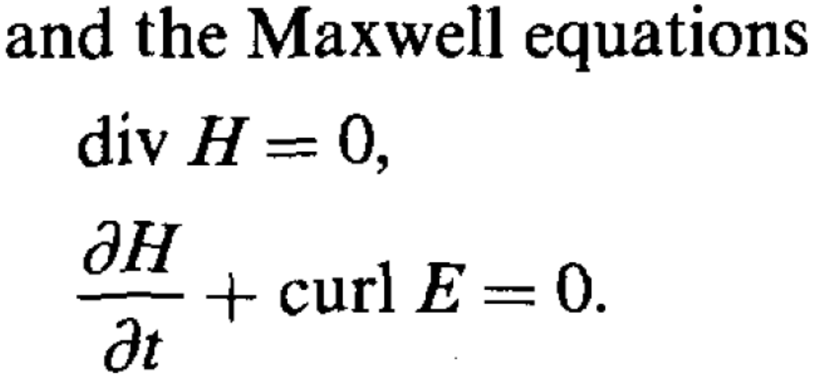
On the right side of Eq. (20), the last term is zero by symmetry because of (16), the third term is zero because of (5), and the second term is equal to the second term on the left of (19). The remaining terms in Eqs. (19) and (20) give

$$\frac{\partial H_l}{\partial t} = \epsilon_{jkl}\frac{\partial E_j}{\partial x_k}, \tag{21}$$

which is equivalent to (6). End of proof.

对费曼工作的讨论

- 只是推导了麦克斯韦的两个方程，即引入规范对称性的方程
- 不是完整麦克斯韦方程组的证明！（戴森文章受到抨击的原因）
- 费曼和戴森可能没有意识到：量子化与规范对称性的共生关系！
- 所有场都要量子化吗？



and the Maxwell equations

$$\operatorname{div} H = 0,$$

$$\frac{\partial H}{\partial t} + \operatorname{curl} E = 0.$$

2. 引力需要量子化?

- 最核心的理论问题

  广相在宏观很成功（引力波发现）

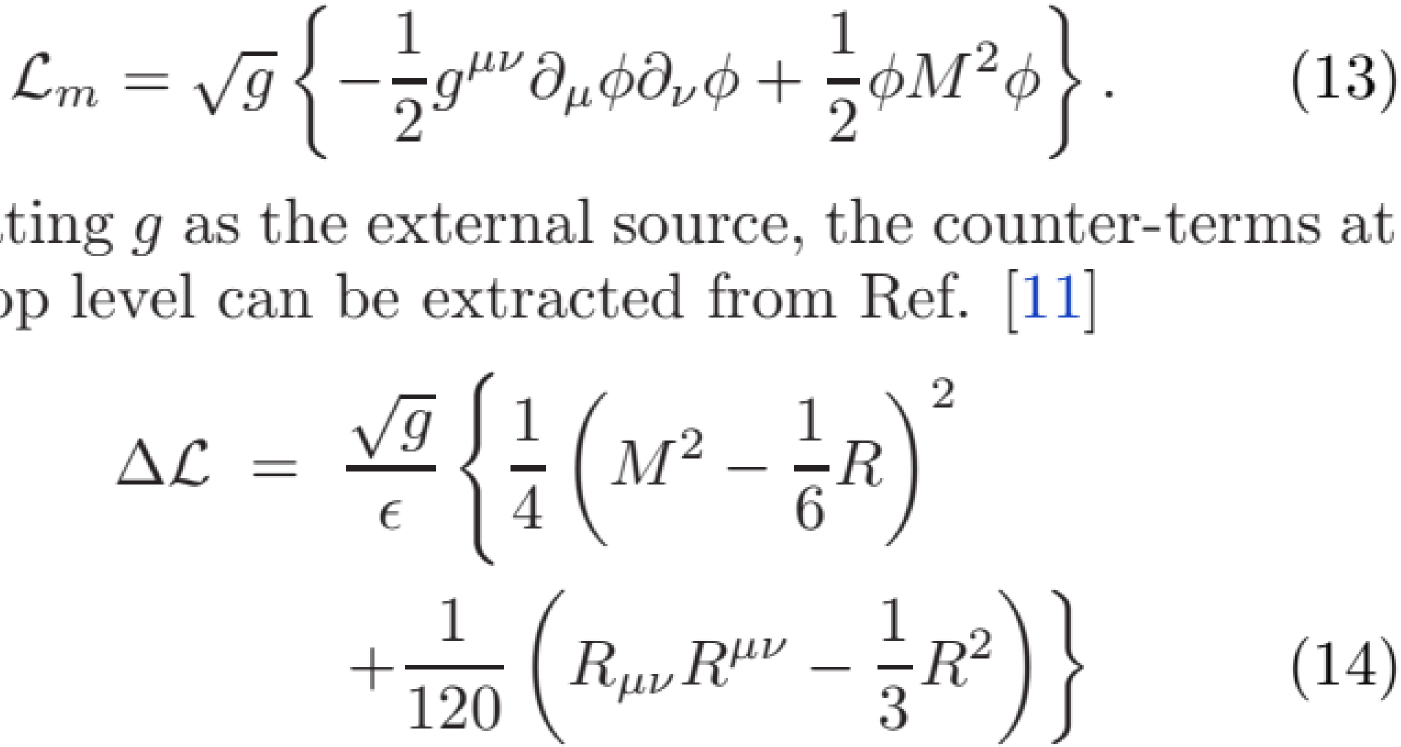
  可重整量子场论在微观很成功

  广相在微观和宇观是否成立?

$F = G\frac{m_1 m_2}{r^2}$， 对比静电库伦定律： $F = \frac{q_1 q_2}{r^2}$

量纲分析： $[G] = 质量^{-2}$， 不可重整!

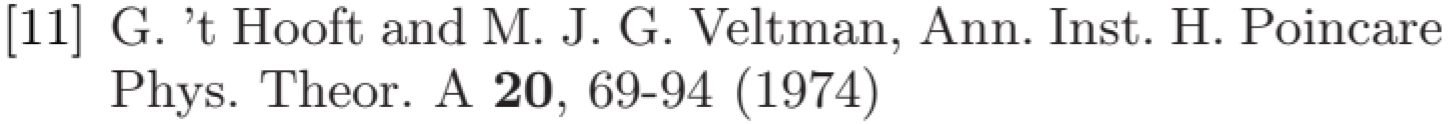
一个具体例子



$$\mathcal{L}_m = \sqrt{g}\left\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\phi M^2\phi\right\}. \tag{13}$$

Treating g as the external source, the counter-terms at one-loop level can be extracted from Ref. [11]

$$\begin{aligned}\Delta\mathcal{L} \;=\; & \frac{\sqrt{g}}{\epsilon}\left\{\frac{1}{4}\left(M^2 - \frac{1}{6}R\right)^2\right. \\ & \left. +\frac{1}{120}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\right\}\end{aligned} \tag{14}$$



[11] G. ’t Hooft and M. J. G. Veltman, Ann. Inst. H. Poincare Phys. Theor. A **20**, 69-94 (1974)

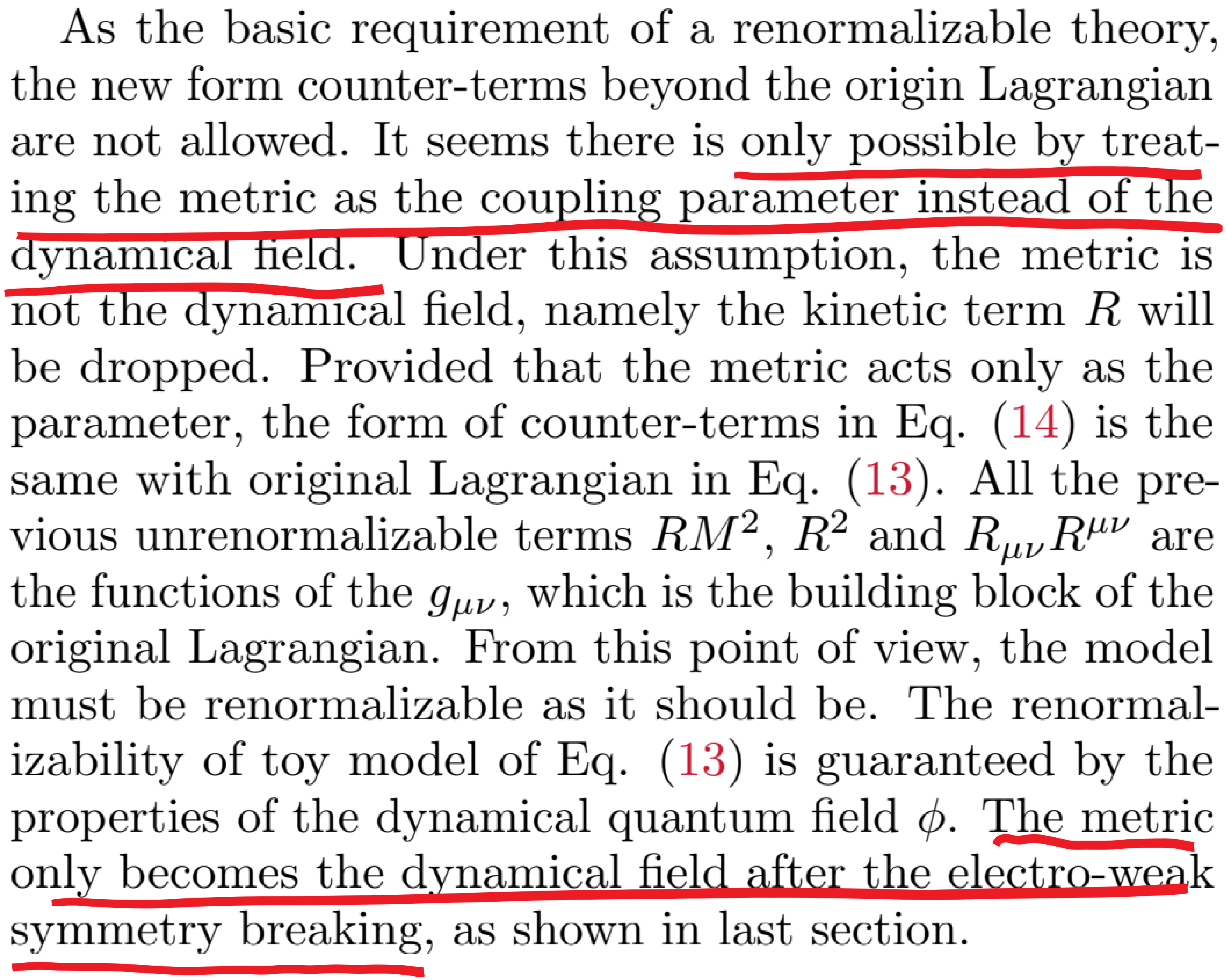
Ref. [11] has argued that the unrenormalizable term in Eq. (14), namely the RM^2 term, can be eliminated by adding the specific term $\frac{1}{12}R\phi^2$ to the original Lagrangian of Eq. (13). However the unrenormalizable terms R^2 and $R_{\mu\nu}R^{\mu\nu}$ remain. The situation becomes even worse after including contributions from the gravitons in the loops. It seems impossible to generally eliminate all unrenormalizable terms by modifying the original Lagrangian. This is the key argument that gravity is an unrenormalizable theory. As shown in this simple excise, there exists fundamental difficulty to renormalize the gravity in this way. Some fundamental aspect of the gravity has to be changed.

Renormalizability and associated infinity are usually thought as annoying, however it can be treated as the tool, even a principle to construct a meaningful theory. In

怎么最简单的构造可重整理论？

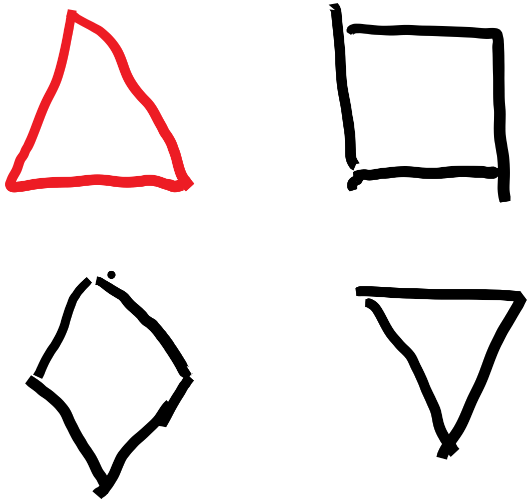
① QFT (90s) with SSB

② Φ (2012)



As the basic requirement of a renormalizable theory, the new form counter-terms beyond the origin Lagrangian are not allowed. It seems there is only possible by treating the metric as the coupling parameter instead of the dynamical field. Under this assumption, the metric is not the dynamical field, namely the kinetic term R will be dropped. Provided that the metric acts only as the parameter, the form of counter-terms in Eq. (14) is the same with original Lagrangian in Eq. (13). All the previous unrenormalizable terms RM^2, R^2 and $R_{\mu\nu}R^{\mu\nu}$ are the functions of the $g_{\mu\nu}$, which is the building block of the original Lagrangian. From this point of view, the model must be renormalizable as it should be. The renormalizability of toy model of Eq. (13) is guaranteed by the properties of the dynamical quantum field ϕ. The metric only becomes the dynamical field after the electro-weak symmetry breaking, as shown in last section.

量子行为的根源来自相位空间
引力是时空性质，无须量子化！



常数c联系时空

$$t = \frac{x}{c}$$

常数h联系时空与相位空间

$$\theta = \frac{S}{h}$$

在电弱对称性破坏前“热冻”引力，即扔掉动能项！

The Lagrangian of proposed model can be written as

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m .$$

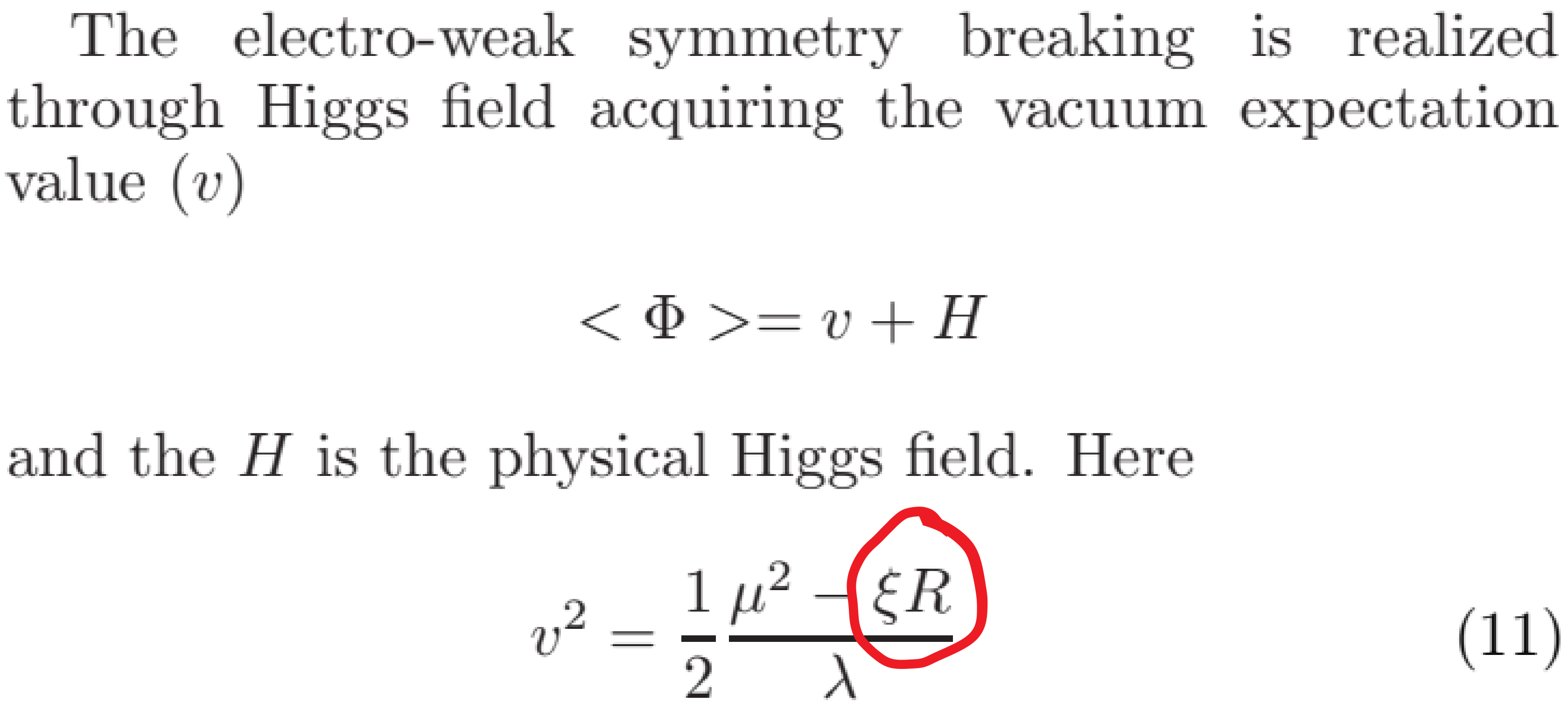
Here the general coordinate invariant pure gravity and matter Lagrangians with the metric field g and the weak doublet Higgs field Φ are

$$\mathcal{L}_g = \sqrt{g}\,\{-\kappa R\} \tag{9}$$

$$\kappa = \begin{cases} 1 & \text{Hilbert-Einstei} \\ 0 & \text{“热冻”} \end{cases}$$

$$\begin{aligned} \mathcal{L}_m \;=\; & \sqrt{g}\Big\{-g^{\mu\nu}\partial_\mu\Phi^\dagger\partial_\nu\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2 \\ & +\Phi^\dagger\Phi\left(\boxed{\xi R} - \mu^2\right) + \Lambda_0\Big\} . \end{aligned} \tag{10}$$

电弱对称性破坏后产生动能项



The electro-weak symmetry breaking is realized through Higgs field acquiring the vacuum expectation value (v)

$$< \Phi >= v + H$$

and the H is the physical Higgs field. Here

$$v^2 = \frac{1}{2}\frac{\mu^2 - \boxed{\xi R}}{\lambda} \tag{11}$$

"Universal"

should be $O(10^{-3}) \sim O(10^{-4})$. Can the contribution from ξR be so large? Basically ξ is an arbitrary dimensionless parameter, and which can be determined empirically. We will argue theoretically that the contribution can be large. Due to the renormalizable criteria, as discussed in next section, we will drop R-term in Eq. (9). In order to reproduce the usual Hilert-Einstein gravity, ξ is fixed to be $O(m_P^2/v^2)$. Such value is much larger than the usual assumption, for example for the case of Higgs inflation models. Usually the range with the sizable gravity effect is estimated as

$$r \sim G_N E = \frac{E}{m_P^2},$$

where E is the effective collider energy. Due to the ξ enhancement, the range becomes

$$r \sim \boxed{\xi} \frac{E}{m_P^2} \sim \frac{m_P^2}{v^2} \frac{E}{m_P^2} = \frac{E}{v^2}.$$

After the electro-weak symmetry breaking with $v^2 = \mu^2/(2\lambda)$,

$$-\kappa + \xi v^2 = -1$$

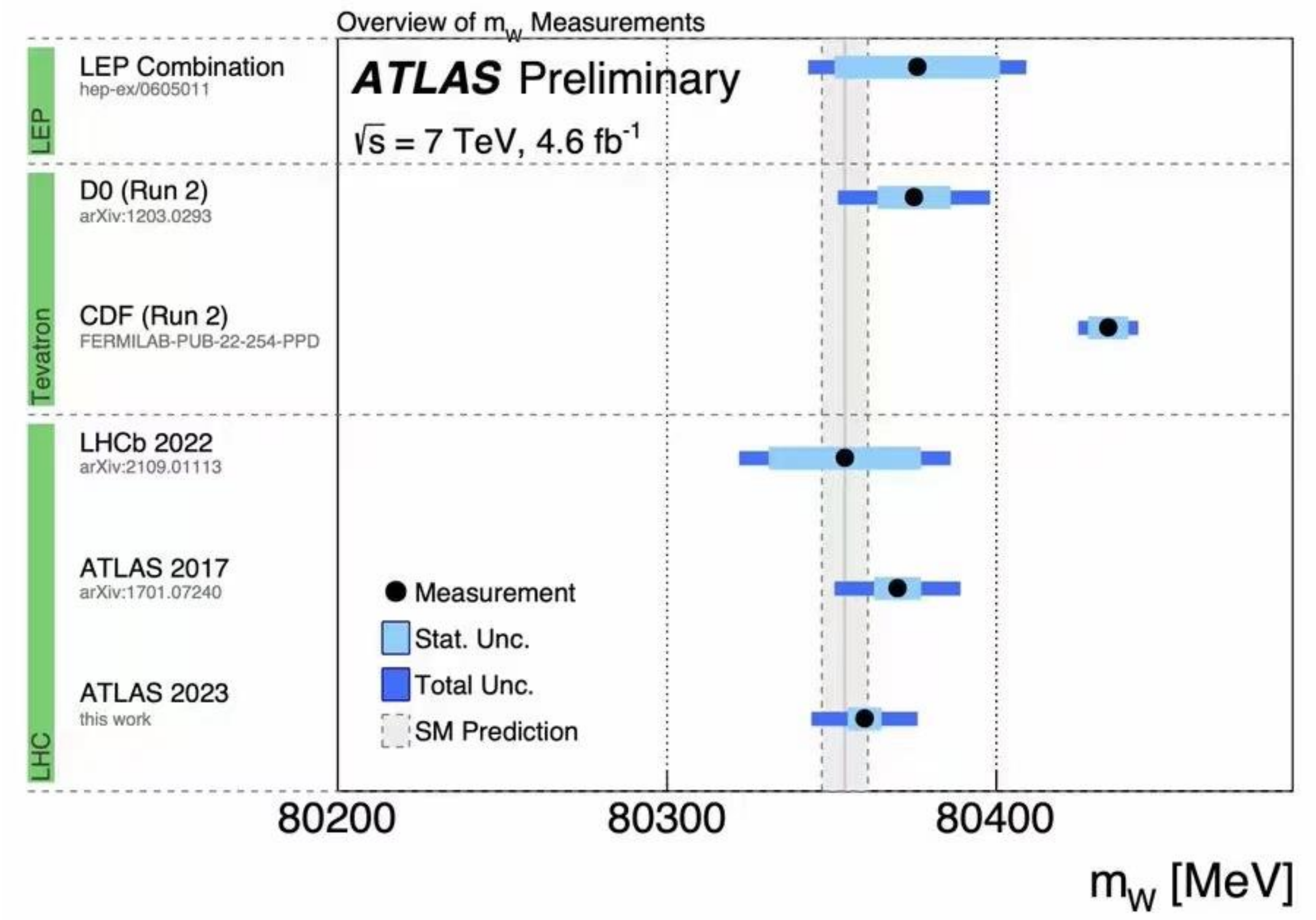
which is empirically required by the Newtonian gravity. And

$$\lambda v^4 - \mu^2 v^2 + \Lambda_0 = \Lambda$$

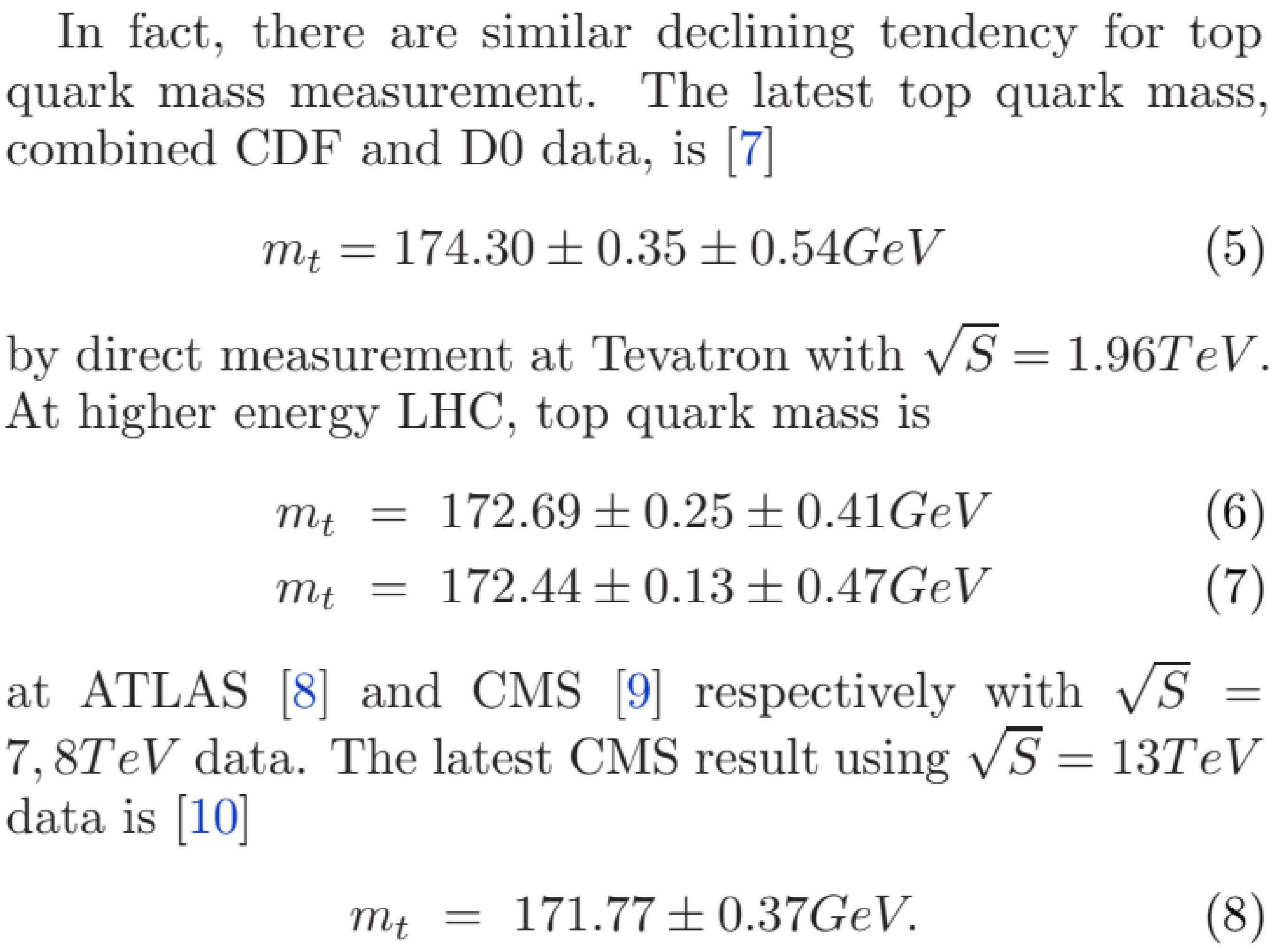
which is the cosmological constant. After symmetry breaking, the induced Lagrangian becomes

$$\mathcal{L} = \sqrt{g}\,\{-R + \Lambda + \cdots\}\,. \tag{12}$$

3. 唯象学：W质量随加速器能量变化？



顶夸克



In fact, there are similar declining tendency for top quark mass measurement. The latest top quark mass, combined CDF and D0 data, is [7]

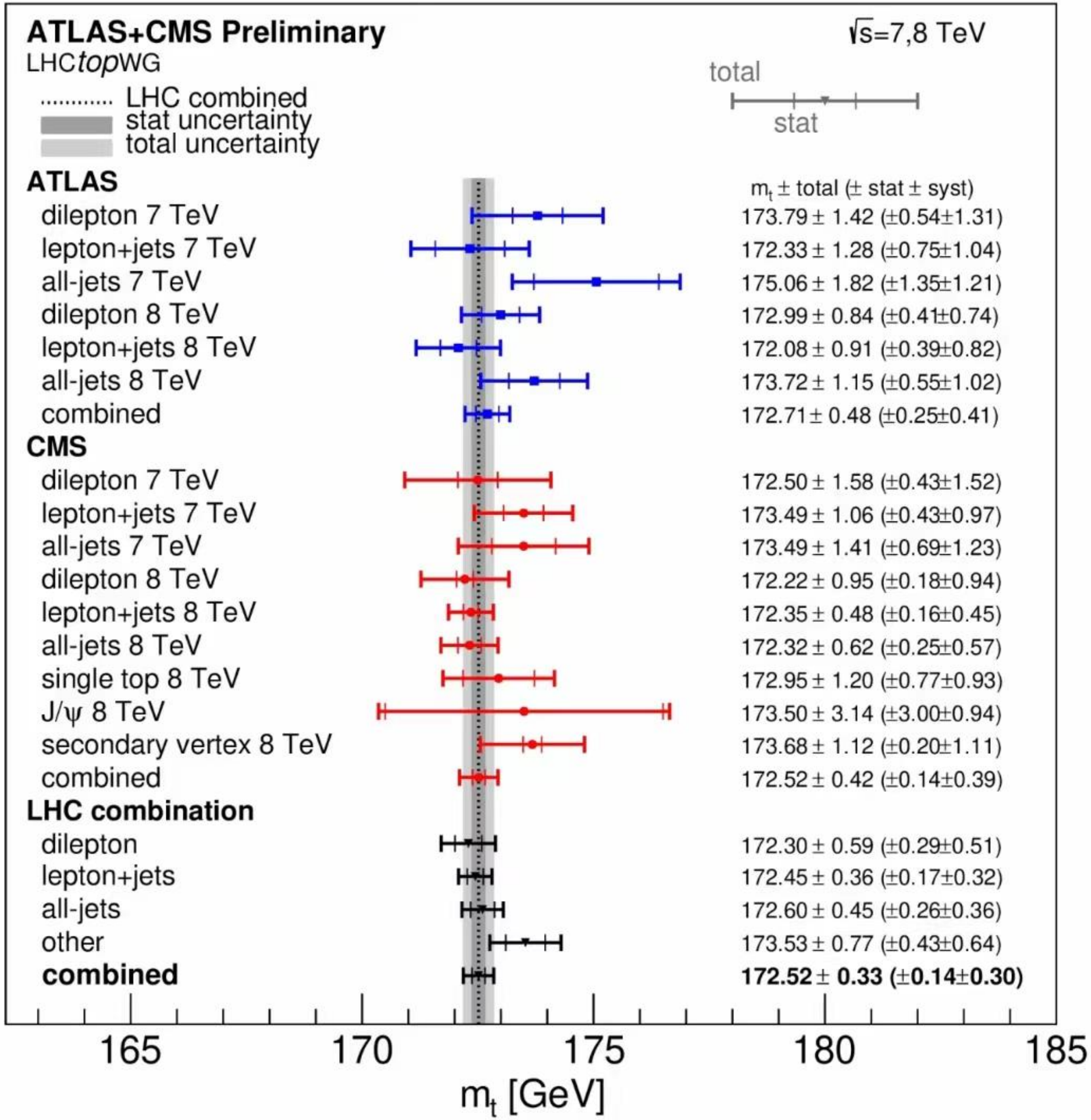
$$m_t = 174.30 \pm 0.35 \pm 0.54 GeV \tag{5}$$

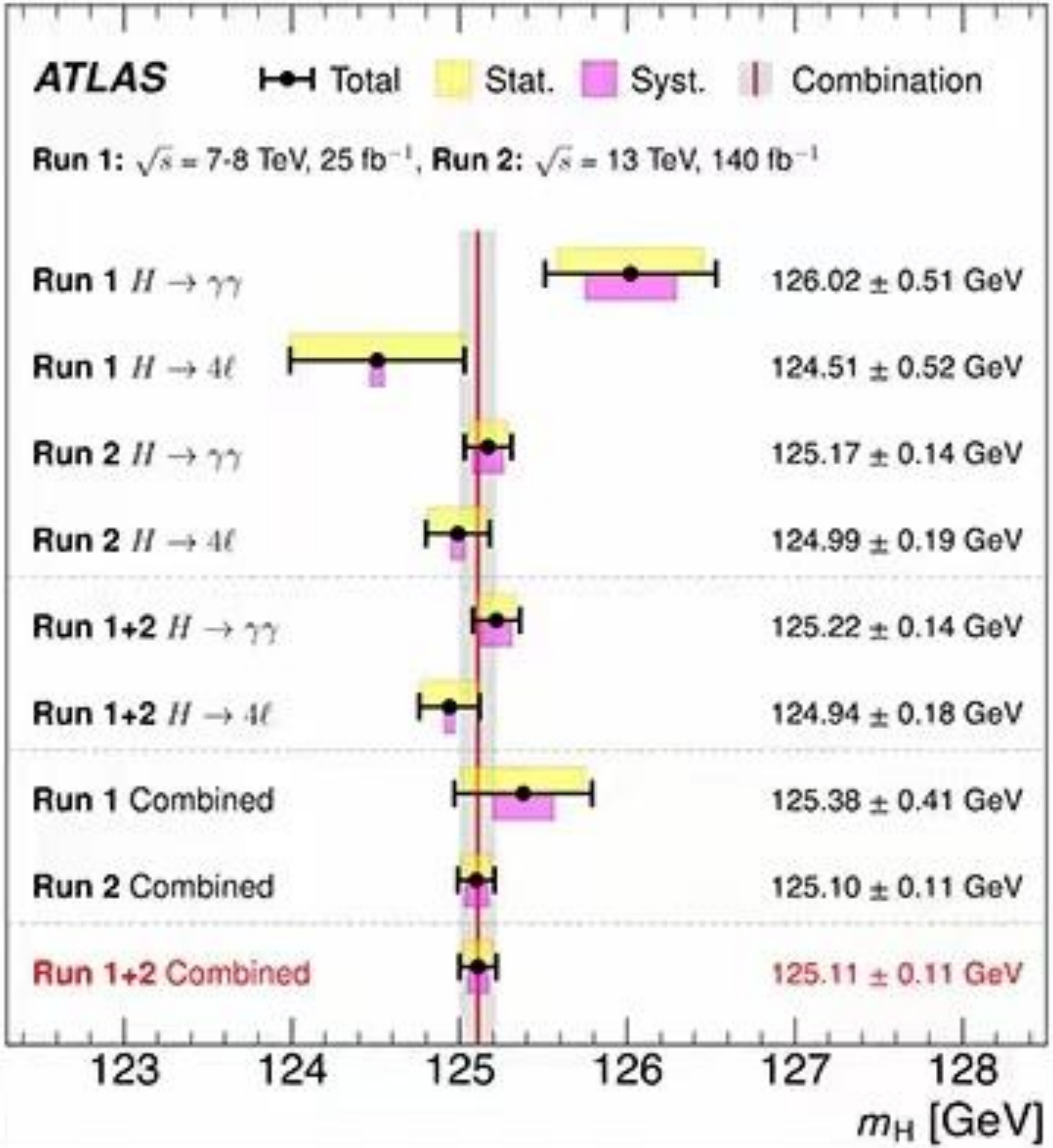
by direct measurement at Tevatron with $\sqrt{S} = 1.96 TeV$. At higher energy LHC, top quark mass is

$$\begin{aligned} m_t &= 172.69 \pm 0.25 \pm 0.41 GeV \qquad (6) \\ m_t &= 172.44 \pm 0.13 \pm 0.47 GeV \qquad (7) \end{aligned}$$

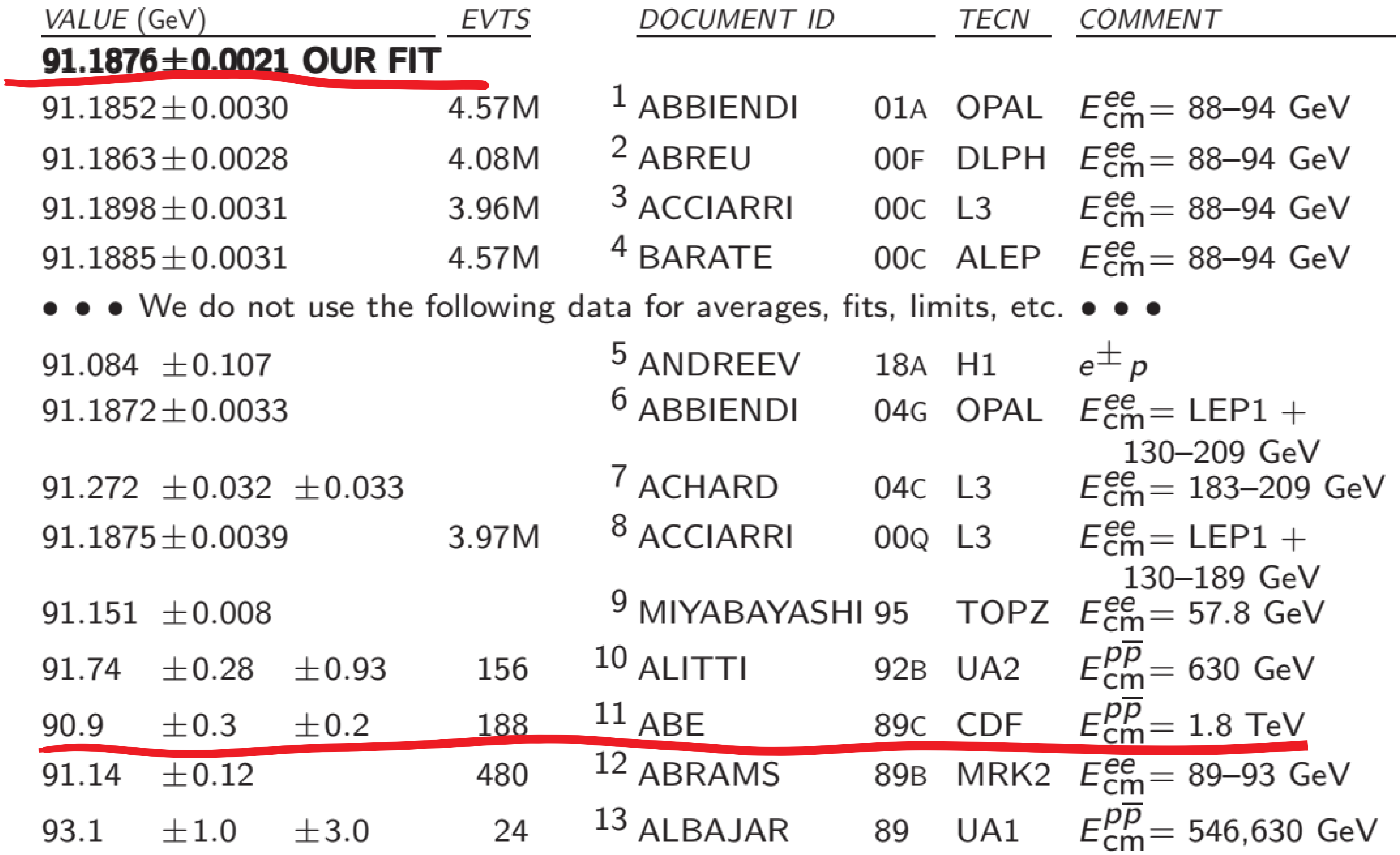
at ATLAS [8] and CMS [9] respectively with $\sqrt{S} = 7, 8 TeV$ data. The latest CMS result using $\sqrt{S} = 13 TeV$ data is [10]

$$m_t = 171.77 \pm 0.37 GeV. \tag{8}$$


ATLAS+CMS Preliminary
LHC*top*WG
$\sqrt{s}$=7,8 TeV
LHC combined
stat uncertainty
total uncertainty
total
stat
ATLAS
m_t ± total (± stat ± syst)
dilepton 7 TeV 173.79 ± 1.42 (±0.54±1.31)
lepton+jets 7 TeV 172.33 ± 1.28 (±0.75±1.04)
all-jets 7 TeV 175.06 ± 1.82 (±1.35±1.21)
dilepton 8 TeV 172.99 ± 0.84 (±0.41±0.74)
lepton+jets 8 TeV 172.08 ± 0.91 (±0.39±0.82)
all-jets 8 TeV 173.72 ± 1.15 (±0.55±1.02)
combined 172.71 ± 0.48 (±0.25±0.41)
CMS
dilepton 7 TeV 172.50 ± 1.58 (±0.43±1.52)
lepton+jets 7 TeV 173.49 ± 1.06 (±0.43±0.97)
all-jets 7 TeV 173.49 ± 1.41 (±0.69±1.23)
dilepton 8 TeV 172.22 ± 0.95 (±0.18±0.94)
lepton+jets 8 TeV 172.35 ± 0.48 (±0.16±0.45)
all-jets 8 TeV 172.32 ± 0.62 (±0.25±0.57)
single top 8 TeV 172.95 ± 1.20 (±0.77±0.93)
J/ψ 8 TeV 173.50 ± 3.14 (±3.00±0.94)
secondary vertex 8 TeV 173.68 ± 1.12 (±0.20±1.11)
combined 172.52 ± 0.42 (±0.14±0.39)
LHC combination
dilepton 172.30 ± 0.59 (±0.29±0.51)
lepton+jets 172.45 ± 0.36 (±0.17±0.32)
all-jets 172.60 ± 0.45 (±0.26±0.36)
other 173.53 ± 0.77 (±0.43±0.64)
combined 172.52 ± 0.33 (±0.14±0.30)
165
170
175
180
185
m_t [GeV]

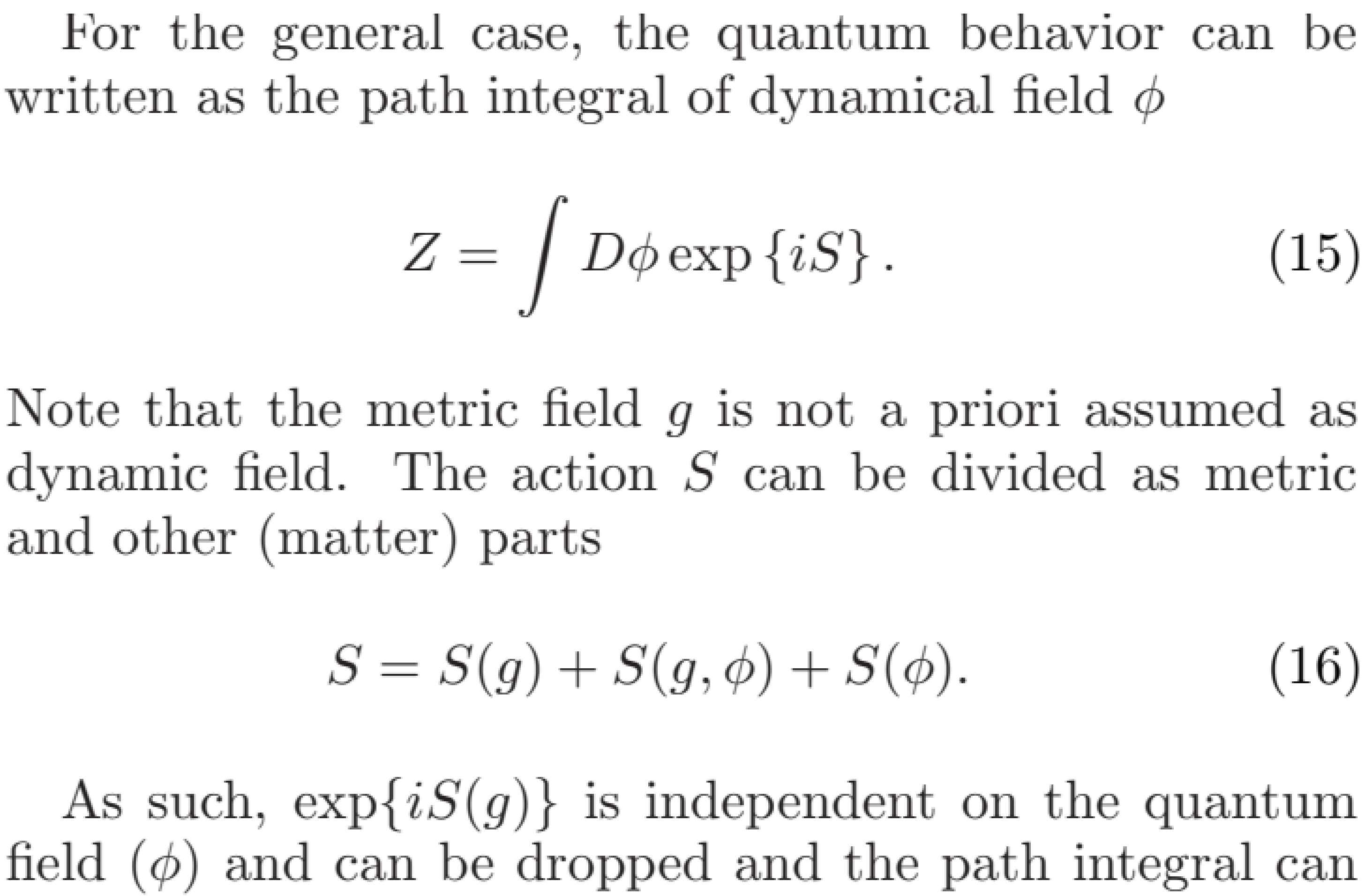

ATLAS
Total
Stat.
Syst.
Combination
Run 1: $\sqrt{s}$ = 7-8 TeV, 25 fb^{-1}, Run 2: $\sqrt{s}$ = 13 TeV, 140 fb^{-1}
Run 1 $H \to \gamma\gamma$ 126.02 ± 0.51 GeV
Run 1 $H \to 4\ell$ 124.51 ± 0.52 GeV
Run 2 $H \to \gamma\gamma$ 125.17 ± 0.14 GeV
Run 2 $H \to 4\ell$ 124.99 ± 0.19 GeV
Run 1+2 $H \to \gamma\gamma$ 125.22 ± 0.14 GeV
Run 1+2 $H \to 4\ell$ 124.94 ± 0.18 GeV
Run 1 Combined 125.38 ± 0.41 GeV
Run 2 Combined 125.10 ± 0.11 GeV
Run 1+2 Combined 125.11 ± 0.11 GeV
123
124
125
126
127
128
m_H [GeV]

请在LHC上测量Z质量！



VALUE (GeV)	EVTS	DOCUMENT ID		TECN	COMMENT
91.1876±0.0021 OUR FIT					
91.1852±0.0030	4.57M	[1] ABBIENDI	01A	OPAL	E^{ee}_{cm}= 88–94 GeV
91.1863±0.0028	4.08M	[2] ABREU	00F	DLPH	E^{ee}_{cm}= 88–94 GeV
91.1898±0.0031	3.96M	[3] ACCIARRI	00C	L3	E^{ee}_{cm}= 88–94 GeV
91.1885±0.0031	4.57M	[4] BARATE	00C	ALEP	E^{ee}_{cm}= 88–94 GeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
91.084 ±0.107		[5] ANDREEV	18A	H1	$e^{\pm} p$
91.1872±0.0033		[6] ABBIENDI	04G	OPAL	E^{ee}_{cm}= LEP1 + 130–209 GeV
91.272 ±0.032 ±0.033		[7] ACHARD	04C	L3	E^{ee}_{cm}= 183–209 GeV
91.1875±0.0039	3.97M	[8] ACCIARRI	00Q	L3	E^{ee}_{cm}= LEP1 + 130–189 GeV
91.151 ±0.008		[9] MIYABAYASHI	95	TOPZ	E^{ee}_{cm}= 57.8 GeV
91.74 ±0.28 ±0.93	156	[10] ALITTI	92B	UA2	$E^{p\overline{p}}_{cm}$= 630 GeV
90.9 ±0.3 ±0.2	188	[11] ABE	89C	CDF	$E^{p\overline{p}}_{cm}$= 1.8 TeV
91.14 ±0.12	480	[12] ABRAMS	89B	MRK2	E^{ee}_{cm}= 89–93 GeV
93.1 ±1.0 ±3.0	24	[13] ALBAJAR	89	UA1	$E^{p\overline{p}}_{cm}$= 546,630 GeV

一般情形



For the general case, the quantum behavior can be written as the path integral of dynamical field ϕ

$$Z = \int D\phi \exp\{iS\}\,. \tag{15}$$

Note that the metric field g is not a priori assumed as dynamic field. The action S can be divided as metric and other (matter) parts

$$S = S(g) + S(g,\phi) + S(\phi). \tag{16}$$

As such, $\exp\{iS(g)\}$ is independent on the quantum field (ϕ) and can be dropped and the path integral can be simplified as

$$Z = \int D\phi \exp\{iS(g,\phi) + iS(\phi)\}\,. \tag{17}$$

The metric, as the dynamical field after electro-weak symmetry breaking, manifests itself only classically. Eq. (10) is only the specific realization of the general case.

4. 结论和讨论

- 为了物理理论可重整，需要丢掉爱因斯坦-希尔伯特项。实现了引力与电、弱、强相互作用的统一描述，不引入额外参数
- 电弱对称性破缺和黑格斯场处于核心地位。对称性破缺产生了爱因斯坦-希尔伯特项，没有普朗克标度!
- 引力不需量子化，真空的一部分

$$v^2 = \frac{1}{2}\frac{\mu^2 - \boxed{\xi R}}{\lambda}$$

- 不同高能加速器测量的粒子质量不同，比如CDF的W质量反常可能来自引力的影响，顶夸克质量也有类似迹象
- 目前特别需要Z质量在LHC的精确数值，从而与LEP精确测量值进行比较
- 还会影响早期宇宙演化，f(R)或者“暗物质”【”weak miracle”】，以及黑洞物理等。
- 请提出宝贵意见，谢谢!