对撞机与引力波实验中的电弱相变物理

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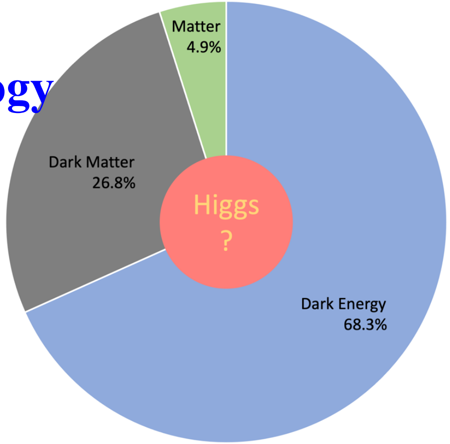
The 27th LHC Mini-Workshop @ Zhuhai, Jan. 20, 2024

Outline

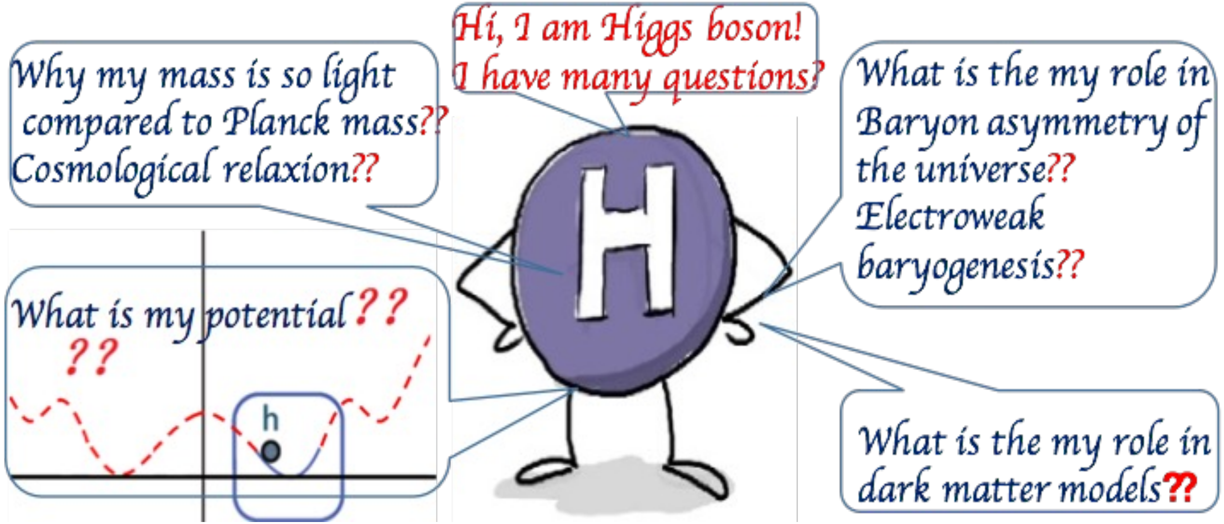
1. Motivation
2. Electroweak phase transition (SFOPT) and phase transition gravitational wave (GW) in a nutshell
3. New Higgs potential in effective field theory
4. Electroweak baryogenesis
5. Dark matter (DM)
6. Phase transition dynamics (bubble wall velocity)
7. Summary and outlook

Motivation Post Higgs Era: Higgs particle cosmology

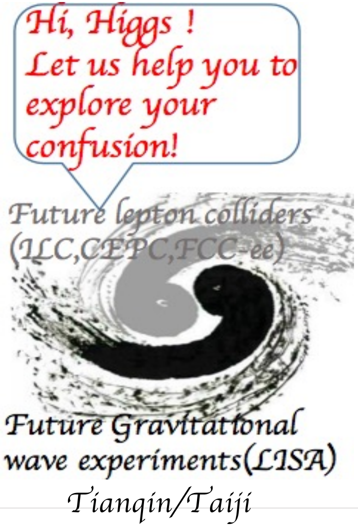


After the discovery of 125 GeV scalar, it becomes a realistic portal to study the fundamental physics and its deep connections to cosmology.

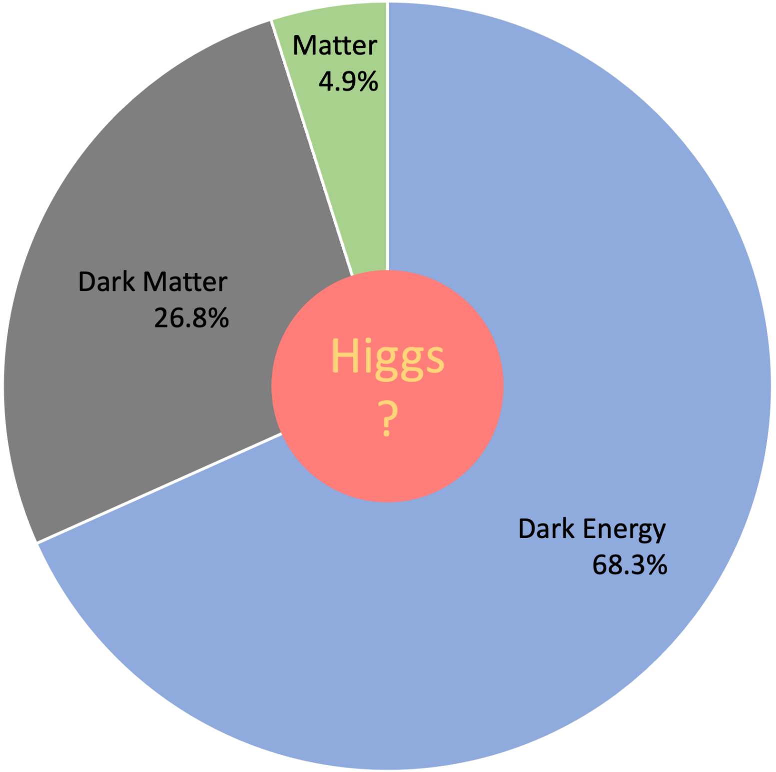
What is the role of Higgs in the early universe?



How to test?



Motivation Post Higgs Era: Higgs particle cosmology



Higgs' deep connections to cosmology, such as EW phase transition/baryogenesis, dark matter(DM) testable by colliders &GW signals

$\mathcal{O} = M^2 - g\phi$

Relaxion

$\mathcal{O} = ll$

neutrino mass and leptogenesis

$\mathcal{O} = R \longleftrightarrow +\mathcal{O}_{hidden} H^\dagger H \longleftrightarrow \mathcal{O} = \phi\phi$

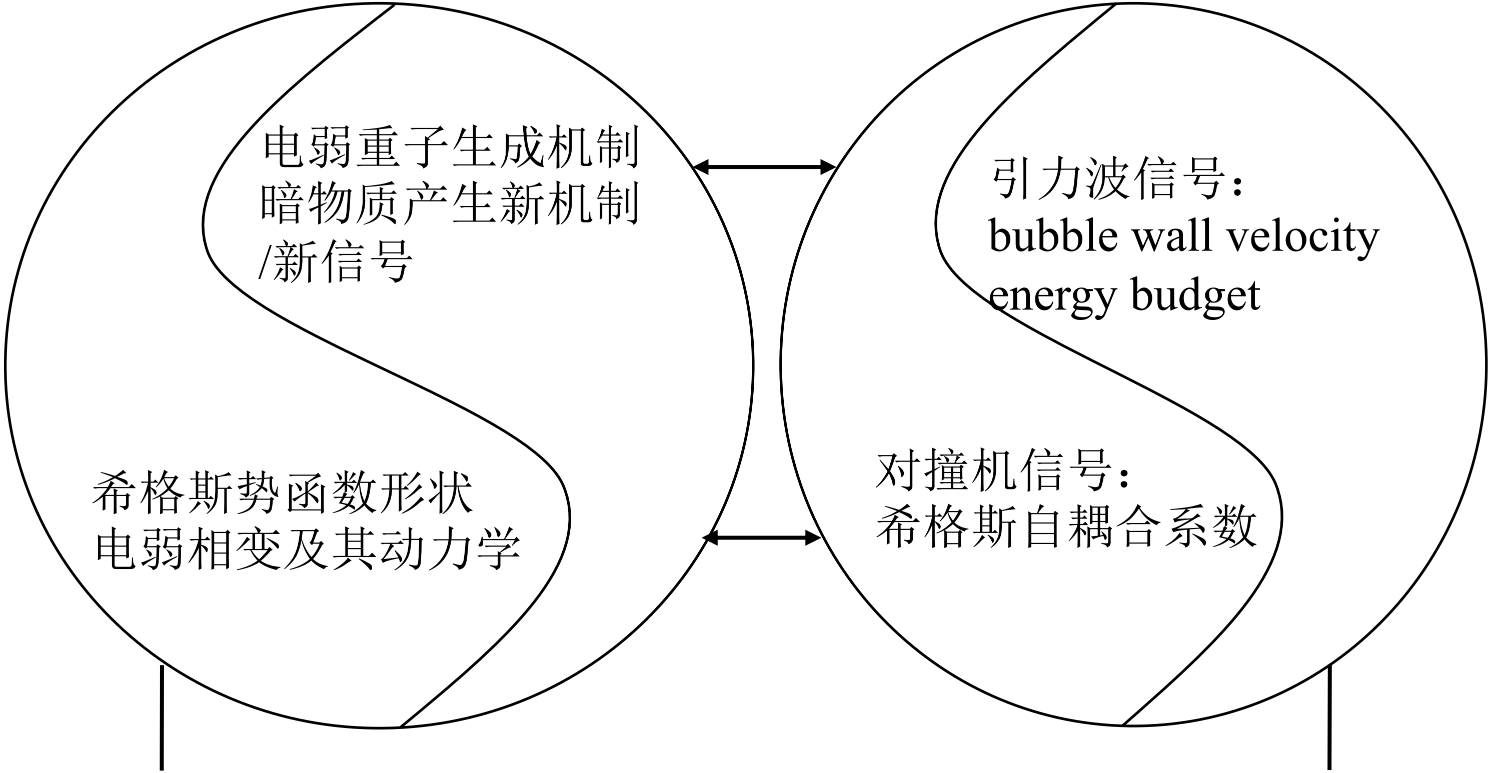
Higgs (bouncing) inflation Preferred by CMB data at Planck

Higgs portal dark matter

$\mathcal{O} = \phi S, \phi\phi$

EW phase transition/ baryogenesis

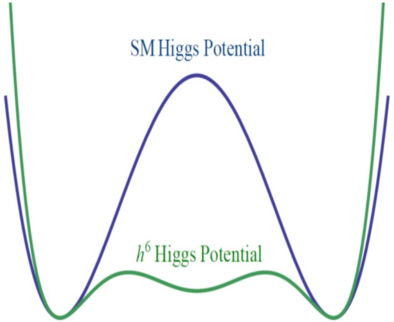
Motivation **Post Higgs Era: Higgs particle cosmology**



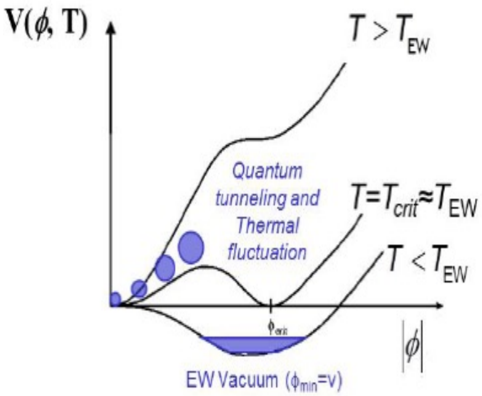
(有限温度)量子场论、相对论流体力学、广义相对论

Motivation

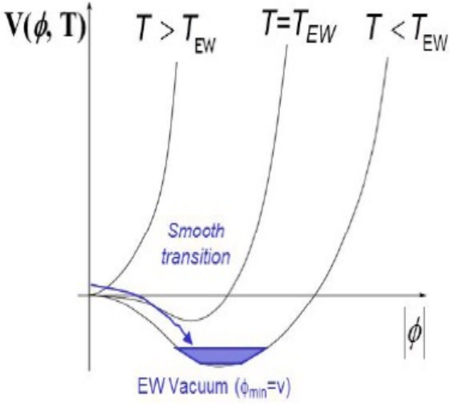
From lattice simulation



SFOPT for m_H < 75 GeV



Cross over for m_H > 75 GeV

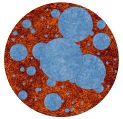


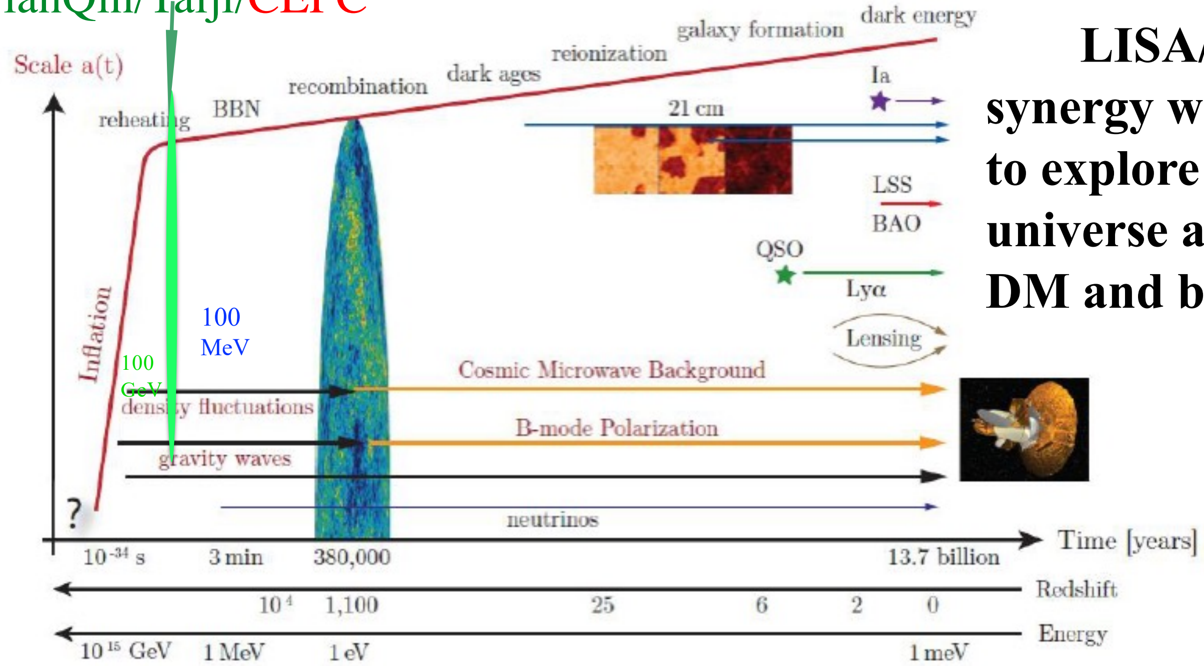
Extension of the Higgs sector is needed to SFOPT for 125 GeV Higgs boson.

We discuss well-motivated extensions (baryogenesis, DM…) of Higgs section to realize strong first-order phase transition (SFOPT) with abundant cosmological effects.
EW phase transition and its GW signals becomes realistic after the discovery of Higgs by LHC and GW by LIGO.

Motivation

EW phase transition/ baryogenesis: LISA/TianQin/Taiji/CEPC





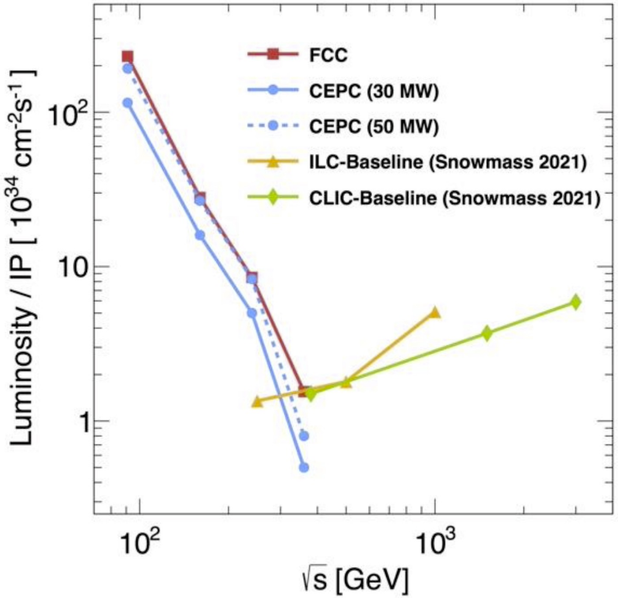
LISA/Tianqin/Taiji in synergy with CEPC helps to explore the early universe around 100 GeV, DM and baryogenesis.

credit:D.Baumann

Motivation

Complementary

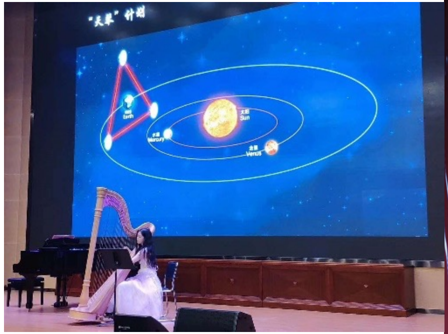
Particle approach
CEPC/SppC, FCC etc.

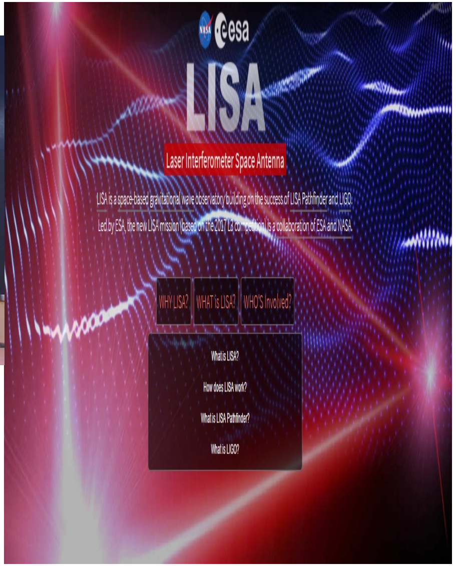


Relate by Higgs physics:EW phase transition/ baryogenesis

Double test on the Higgs potential and baryogenesis, DM

Wave approach
LISA/TianQin/Taiji ~2034





“天琴”
“Harpe in space”

GW in a nutshell

The quadruple nature of GW !

EM wave radiation

$$\ddot{\boldsymbol{d}} = e\ddot{\boldsymbol{x}}$$

GW radiation

$$\ddot{\boldsymbol{d}} = \sum_{\text{particles } A} m_A \ddot{\boldsymbol{x}}_A = \dot{\boldsymbol{p}} = 0$$

momentum conservation

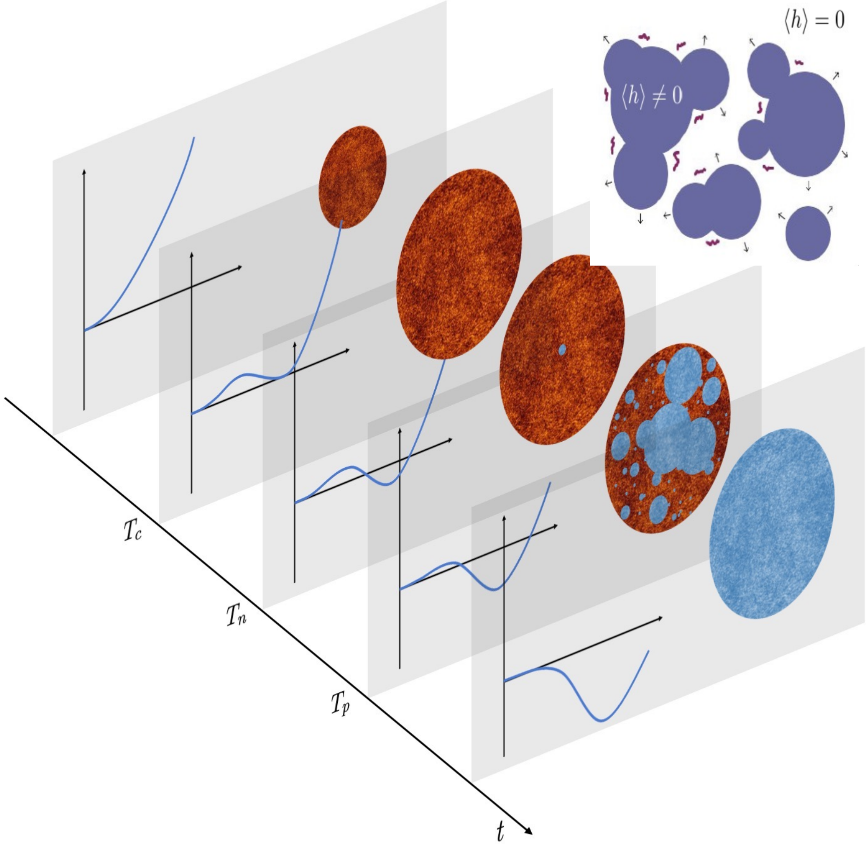
$$L_{\text{electric quadrupole}} = \frac{1}{20}\dddot{Q}^2 \equiv \frac{1}{20}\dddot{Q}_{jk}\dddot{Q}_{jk}$$

$$L_{\text{mass quadrupole}} = \frac{1}{5}\langle \dddot{I}^2 \rangle \equiv \frac{1}{5}\langle \dddot{I\!\!\!-}_{jk}\dddot{I\!\!\!-}_{jk} \rangle$$

$$Q_{jk} \equiv \sum_A e_A \left(x_{Aj}x_{Ak} - \frac{1}{3}\delta_{jk} r_A{}^2 \right)$$

$$I\!\!\!-_{jk} \equiv \sum_A m_A \left(x_{Aj}x_{Ak} - \frac{1}{3}\delta_{jk} r_A{}^2 \right)$$

Phase transition GW in a nutshell



calculate the finite-temperature effective potential using the thermal field theory: free energy density.

$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \Big[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\big(p^2 + m_i^2(\bar{\phi})\big) + J_{\mathrm{B,F}}\Big(\frac{m_i^2(\bar{\phi})}{T^2}\Big) \Big]$$

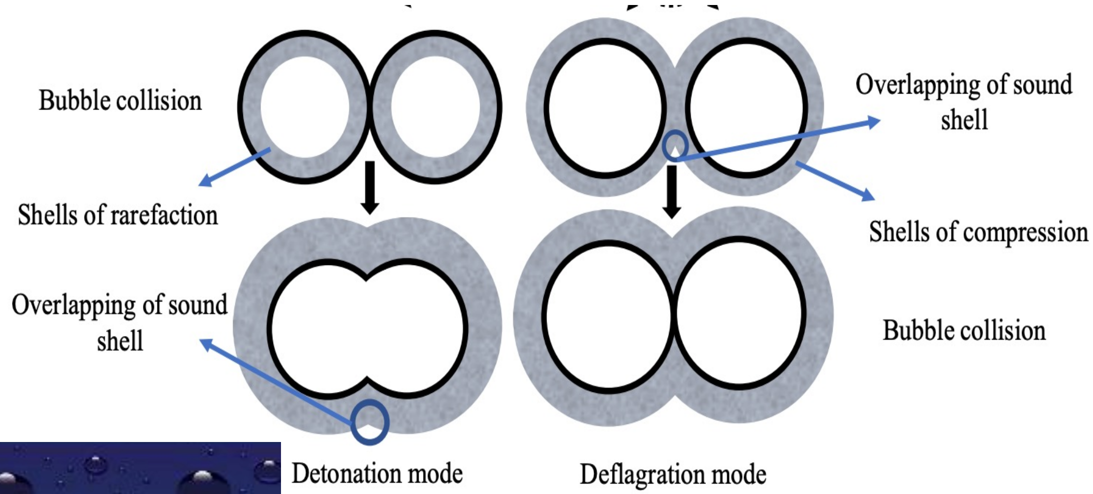
$$S(T) = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\text{eff}}(\phi, T) \right]$$

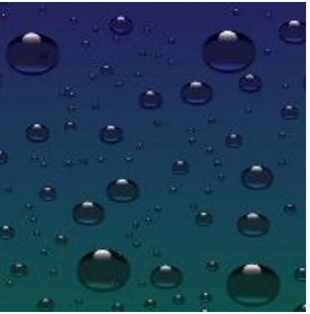
$$\Gamma = \Gamma_0 e^{-S(T)}$$

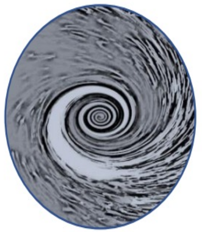
这世上的热闹，源自隧穿

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell



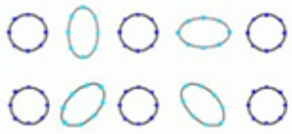




Turbulence

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983); M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

characteristic frequency of the GW signal

$$f_* = \frac{1}{\ell_*} \geq H_*$$

$$\epsilon_* = \ell_* H_*$$

Ratio of the typical length-scale of the GW sourcing process (size of the anisotropic stresses) and the Hubble scale at the generation time

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\mathrm{GeV}} \, \mathrm{Hz}$$

电弱相变对应的峰值频率在mHz附近，刚好也在空间引力波实验(LISA、天琴、太极)的探测区间

Phase transition GW in a nutshell

Bubble collisions

$$h^2\Omega_{\rm co}(f) \simeq 1.67\times10^{-5}\left(\frac{H_*R_*}{(8\pi)^{1/3}}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_\star}\right)^{1/3}\frac{0.11v_b}{0.42+v_b^2}\,\frac{3.8(f/f_{\rm co})^{2.8}}{1+2.8(f/f_{\rm co})^{3.8}}$$

Turbulence

$$h^2\Omega_{\rm turb}(f) \simeq 1.14\times10^{-4}H_*R_*\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{100}{g_*}\right)^{1/3}\frac{(f/f_{\rm turb})^3}{(1+f/f_{\rm turb})^{11/3}(1+8\pi f/H_*)}$$

Sound wave

$$h^2\Omega_{\rm sw}(f) \simeq 1.64\times10^{-6}(H_*\tau_{\rm sw})(H_*R_*)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

E. Witten, Phys. Rev. D 30, 272 (1984); C. J. Hogan, Phys. Lett. B 133, 172 (1983); M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))Mark Hindmarsh, et al., PRL 112, 041301 (2014); Lots of unlisted papers.

See Huai-ke and Ligong's works for current constraints from GW data

Phase transition GW in a nutshell

早期宇宙普遍的引力波源

各向异性
剪切应力张量

$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

GW sources	Sources of tensor anisotropic stress	General form Π_{ij}
Collisions of bubble walls	scalar field gradients	$[\partial_i\phi\partial_j\phi]^{TT}$
Sound waves and turbulence	bulk fluid motion	$[\gamma^2(\rho+p)v_iv_j]^{TT}$
Primordial magnetic fields	gauge fields	$[-E_iE_j - B_iB_j]^{TT}$
Scalar perturbations	second order scalar perturbations	$\partial_i\Psi, \partial_i\Phi$

Phase transition dynamics

Theory: 相变引力波信号、相变暗物质、早期宇宙电弱重子数产生机制最核心却最难计算的参数是泡泡膨胀速度

v_b

Experiment: 因此实验上最重要的相变参数也是泡泡膨胀速度

Finite-temperature effective potential

$V_{eff}(\phi, T)$

α T_p $R_* H_*$

(1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
(2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
(3). No perturbative calculations: lattice calculations and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity

v_b

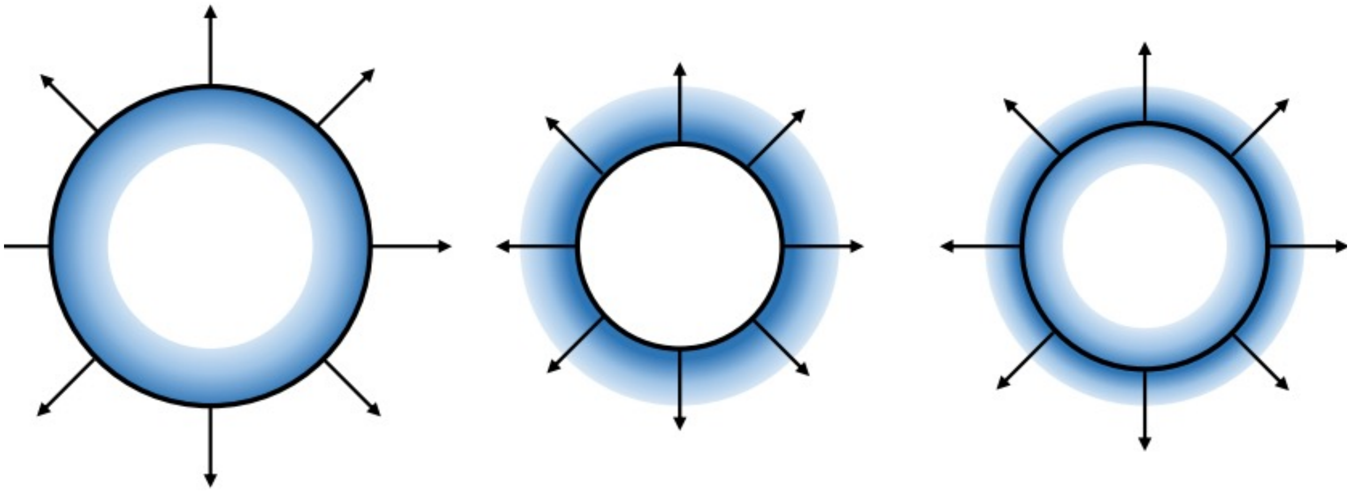
Energy budget

κ

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang ， arXiv:2007.10343 ,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith , arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, FPH, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

F. Giese, T. Konstandin, K. Schmitz and J. van de Vis ,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, JCAP 07 (2023) 006

Phase transition dynamics



Detonation
$v_b > c_s$

Deflagration
$v_b < c_s$

Hybrid
$v_b > c_s$

Stronger GW signal favored **EW baryogengesis favored**

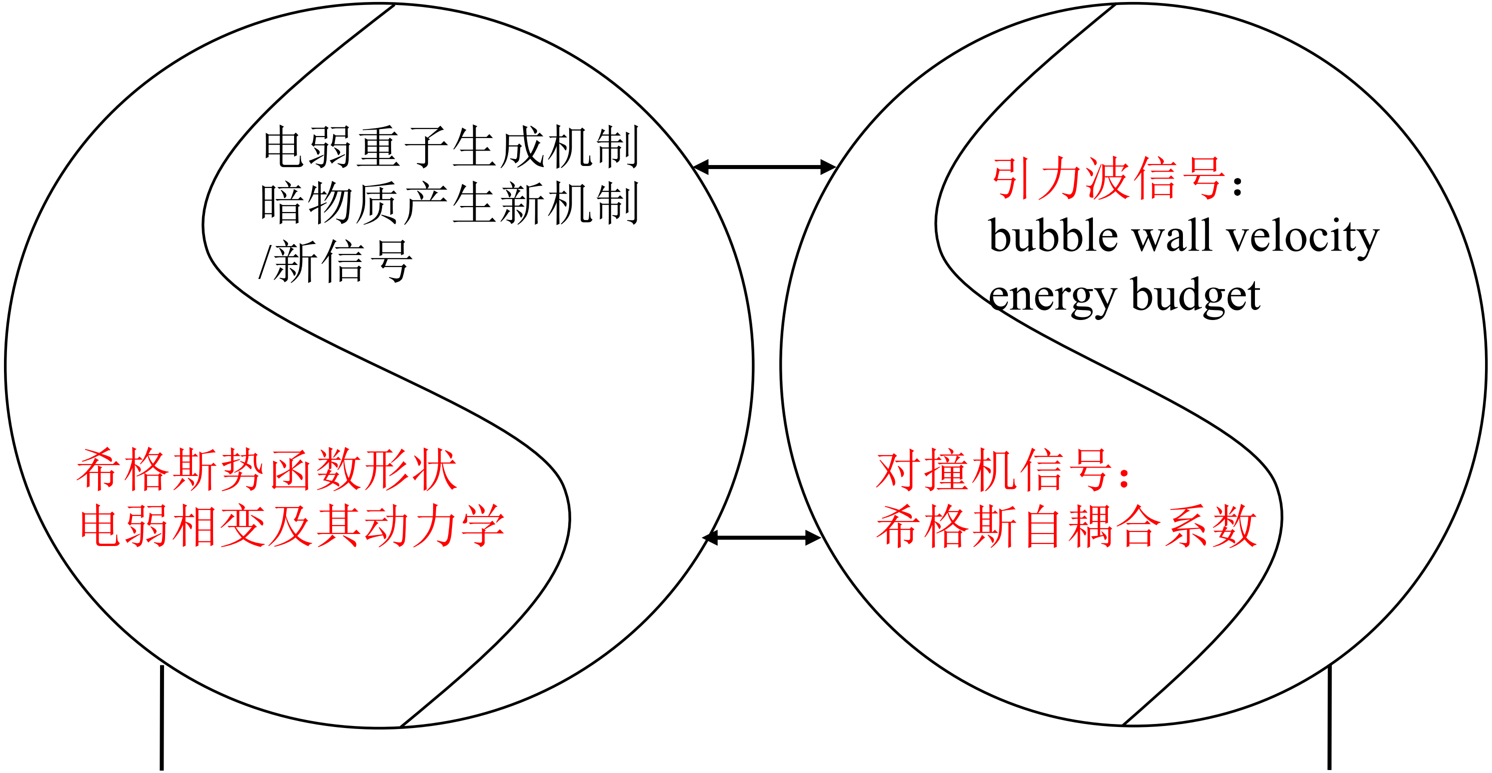
Phase transition dynamics

Classify the SFOPT into four cases:

- **Slight supercooling**: $\alpha_p < 0.1$ the GW is too weak to be detected by LISA, might be within the sensitivity of BBO and ultimate-DECIGO.
- **Mild supercooling:** $0.1 < \alpha_p < 0.5$ the GW could be detected by LISA, TianQin, Taiji, BBO, DECIGO.
- **Strong supercooling:** $0.5 < \alpha_p < 1$
- **Ultra supercooling:** $\alpha_p > 1$

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

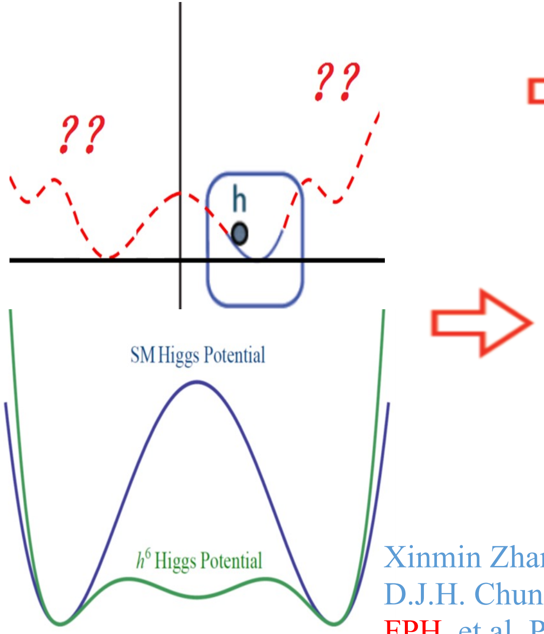
Motivation



(有限温度)量子场论、相对论流体力学、广义相对论

SFOPT and Higgs potential

What is the shape of Higgs potential?



Current data tells us nothing but the quadratic oscillation around the VEV 246 GeV with 125 GeV mass. **mass**

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

or $$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4} h^4 + \frac{1}{\Lambda^2} h^6$$

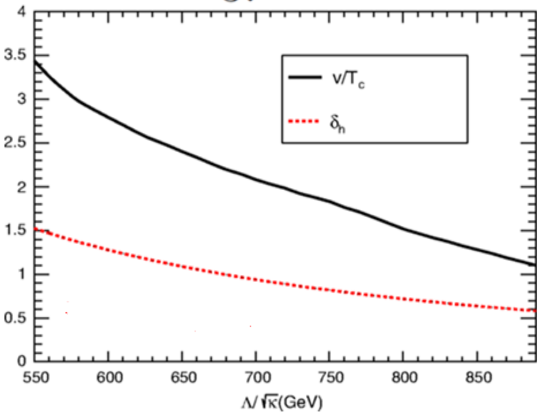
Produce a SFOPT, large deviation of Higgs trilinear coupling, and GW

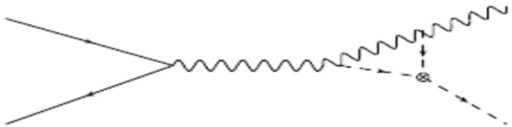
Xinmin Zhang Phys.Rev. D47 (1993) 3065-3067;C. Grojean, G. Servant, J. Well PRD71(2005)036001
D.J.H. Chung, A. J. Long, Lian-tao Wang Phys.Rev. D87(2013) 023509
FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; **FPH**, et.al, Phys.Rev.D93 (2016) no.10,103515
arXiv:1511.06495, Nima Arkani-Hamed et. al.; PreCDR of CEPC; arXiv: 1811.10545,CDR of CEPC

SFOPT and Higgs potential

SFOPT leads to obvious deviation of the tri-linear Higgs coupling

$$\mathcal{L}_{hhh} = -\frac{1}{3!}(1+\delta_h)A_h h^3$$





At one-loop level, deviation of the tri-linear Higgs coupling

$$\delta_h \in (0.6, 1.5)$$

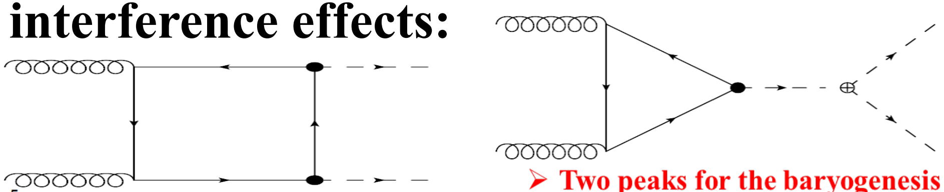
The Circular Electron Positron Collider (CEPC), ILC, FCC-ee can precisely test this scenario by precise measurements of the hZ cross section ($e^- e^+ \longrightarrow hZ$).
SM NNLO hZ cross section recently by Lilin Yang, et al 2016, Yu Jia et at 2016

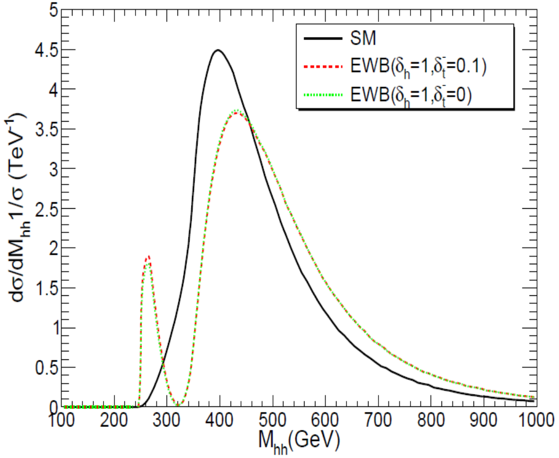
$$\delta_\sigma = \frac{\sigma_{hz,\delta_h \neq 0}}{\sigma_{hz,SM}} - 1$$

See the new work of Lilin Yang, Zhao Li Yu Jia et.al,

SFOPT and Higgs potential

Hints at hadron collider: Modify the invariant mass distribution of Higgs pair due to interference effects:



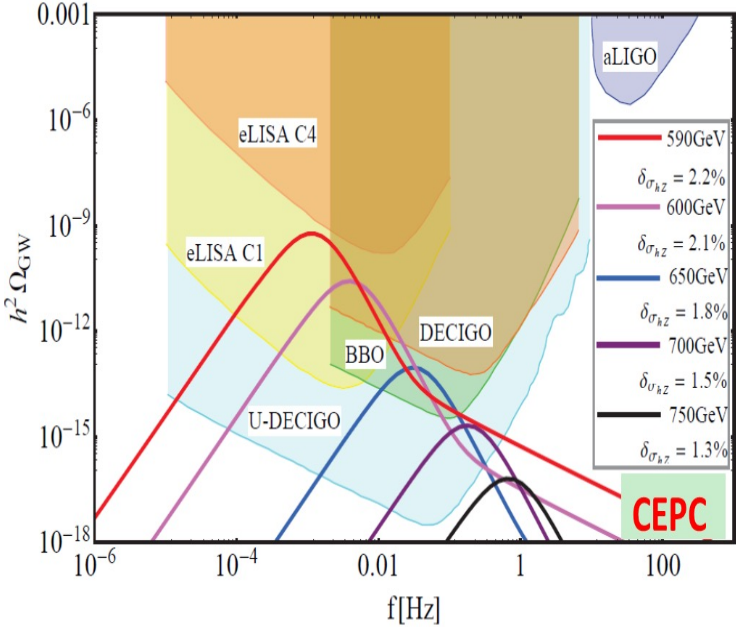


- **Two peaks for the baryogenesis scenario, one peak for the SM.**
- **Due to the difficulties to suppress backgrounds at the LHC, it will be difficult to completely pin down these anomalous coupling at 14 TeV LHC, even with 3000 ab^{-1} integrated luminosity.**
- **Exploiting boosted tricks helps to increase ability to extract the anomalous couplings.**
- **More precise information may come from future100 TeV hadron collider, such as SppC, or future lepton collider, such as CEPC .**

LHC上的详细讨论见
成瞳光、李数老师的报告

SFOPT and Higgs potential

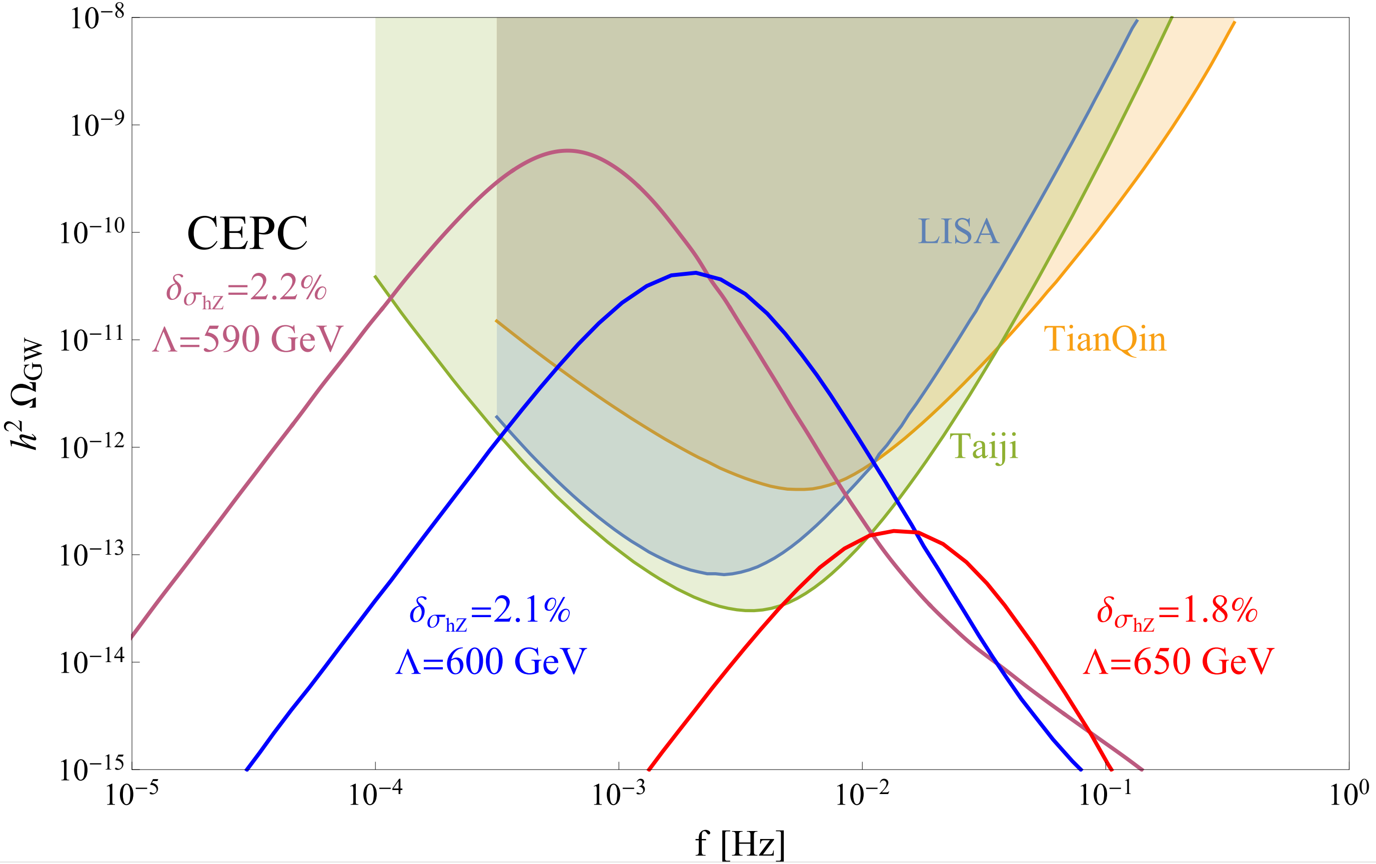
Correlate particle collider and GW signals: double test on Higgs potential from particle to wave



- **For CEPC with 10 ab^{-1} at $\sqrt{s} = 240$ GeV, precision of σ_{zh} may be about 0.4% and can test the scenario.**
- **LISA, BBO,U-DECIGO are capable of detection**
- **The study on EW phase transition naturally bridges the particle physics at collider with GW survey and baryogenesis**

FPH, et.al, Phys.Rev.D94(2016)no.4,041702； **FPH**, et.al, Phys.Rev.D93 (2016) no.10,103515

SFOPT and Higgs potential



SFOPT and Higgs potential

SM EFT

$$\mathcal{L} \supset -\mu^2|H|^2 - \lambda|H|^4 + c_6|H|^6$$
$$+ c_T\mathcal{O}_T + c_{WW}\mathcal{O}_{WW} + \text{other dimension-six operators}$$

$$\delta_{\sigma(hZ)} \approx (0.26c_{WW} + 0.01c_{BB} + 0.04c_{WB} - 0.06c_H - 0.04c_T + 0.74c_L^{(3)\ell} + 0.28c_{LL}^{(3)\ell} + 1.03c_L^{\ell} - 0.76c_R^{e}) \times 1\ \text{TeV}^2 + 0.016\delta_h,$$

SFOPT produces large modification of trilinear Higgs coupling δ_h

c_6 dominates the hZ cross section deviation

Taking a general study of the scalar extended models and the composite Higgs model as examples, we find that the Higgs sextic scenario still works well after considering all the dim-6 operators and the precise measurements.

Qing-Hong Cao, FPH, Ke-Pan Xie, Xinmin Zhang, arXiv:1708.0473,

SFOPT and Higgs potential

One examples

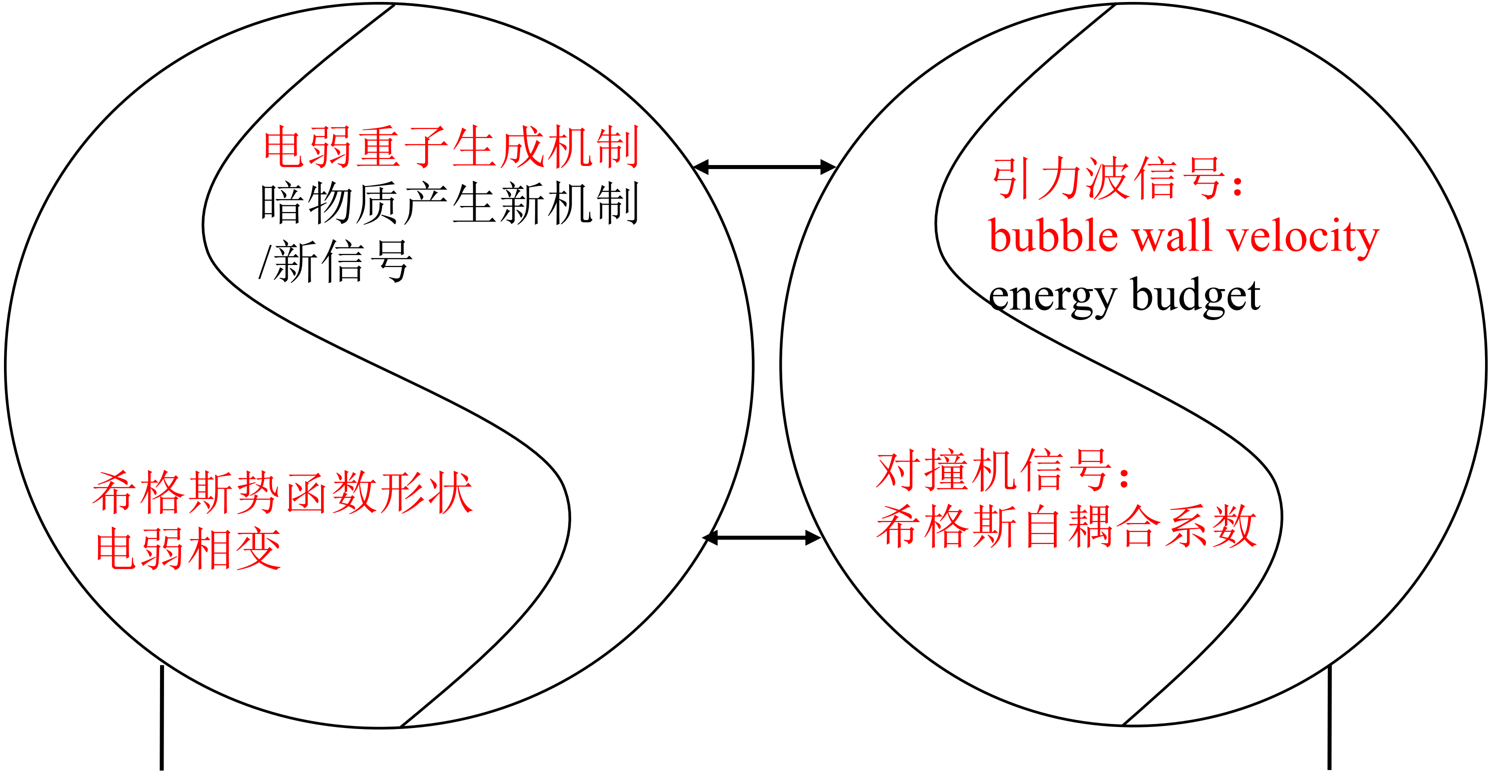
Qing-Hong Cao, **FPH**, Ke-Pan Xie, Xinmin Zhang, arXiv:1708.0473,

$$\delta\mathcal{L} = D_\mu\Phi^\dagger D^\mu\Phi - M_\Phi^2\Phi^\dagger\Phi - \frac{\lambda_\Phi}{4}(\Phi^\dagger\Phi)^2 - \lambda_1\Phi^\dagger\Phi H^\dagger H - \lambda_2|\Phi\cdot H|^2$$
$$- \lambda_3[(\Phi\cdot H)^2 + h.c.] + (\eta_H|H|^2 + \eta_\Phi|\Phi|^2)(\Phi\cdot H + h.c.)\,,$$

Using Covariant Derivative Expansion method, the matched dim-6 operators and their coefficients in the doublet scalar models are obtained:

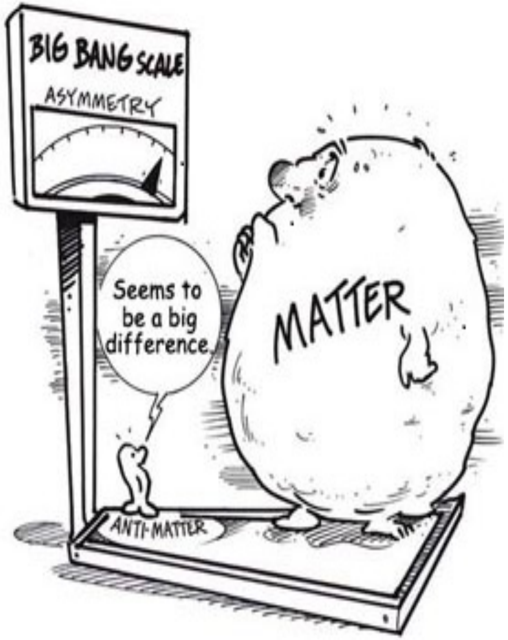
Dimension-six operator	Wilson coefficient
$\mathcal{O}_{WW} = g^2\|H\|^2 W^a_{\mu\nu}W^{a,\mu\nu}$	$c_{WW} = \frac{1}{(4\pi)^2}\frac{1}{48}(2\lambda_1+\lambda_2)\frac{1}{M_\Phi^2}$
$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$	$c_{2W} = \frac{1}{(4\pi)^2}\frac{g^2}{60}\frac{1}{M_\Phi^2}$
$\mathcal{O}_{3W} = \frac{1}{3!}g\epsilon^{abc}W^{a\mu}_{\rho}W^{b\nu}_{\mu}W^{c\rho}_{\nu}$	$c_{3W} = \frac{1}{(4\pi)^2}\frac{g^2}{60}\frac{1}{M_\Phi^2}$
$\mathcal{O}_{BB} = g'^2\|H\|^2 B_{\mu\nu}B^{\mu\nu}$	$c_{BB} = \frac{1}{(4\pi)^2}\frac{1}{48}(2\lambda_1+\lambda_2)\frac{1}{M_\Phi^2}$
$\mathcal{O}_{WB} = gg'H^\dagger\sigma^a H W^a_{\mu\nu}B^{\mu\nu}$	$c_{WB} = \frac{1}{(4\pi)^2}\frac{\lambda_2}{24}\frac{1}{M_\Phi^2}$
$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B^{\mu\nu})^2$	$c_{2B} = \frac{1}{(4\pi)^2}\frac{g'^2}{60}\frac{1}{M_\Phi^2}$
$\mathcal{O}_H = \frac{1}{2}(\partial_\mu\|H\|^2)^2$	$c_H = \frac{1}{(4\pi)^2}[6\eta_\Phi\eta_H + \frac{1}{12}(4\lambda_1^2+4\lambda_1\lambda_2+\lambda_2^2+4\lambda_3^2)]\frac{1}{M_\Phi^2}$
$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$	$c_T = \frac{1}{(4\pi)^2}\frac{1}{12}(\lambda_2^2-4\lambda_3^2)\frac{1}{M_\Phi^2}$
$\mathcal{O}_r = \|H\|^2\|D_\mu H\|^2$	$c_r = \frac{1}{(4\pi)^2}(6\eta_\Phi\eta_H + \frac{1}{6}(\lambda_2^2+4\lambda_3^2))\frac{1}{M_\Phi^2}$
$\mathcal{O}_6 = \|H\|^6$	$c_6 = \eta_H^2 + \frac{1}{(4\pi)^2}[\frac{3}{2}\lambda_\Phi\eta_H^2 + 6\eta_\Phi(\lambda_1+\lambda_2) - \frac{1}{6}(2\lambda_1^3+3\lambda_1^2\lambda_2+3\lambda_1\lambda_2^2+\lambda_2^3) - 2(\lambda_1+\lambda_2)\lambda_3^2]\frac{1}{M_\Phi^2}$

Motivation



(有限温度)量子场论、相对论流体力学、广义相对论

SFOPT and EW baryogenesis



$$\eta_B = n_B / n_\gamma = 5.8 - 6.6 \times 10^{-10}$$

(CMB, BBN)

A long standing problem in particle cosmology is the origin of baryon asymmetry of the universe.

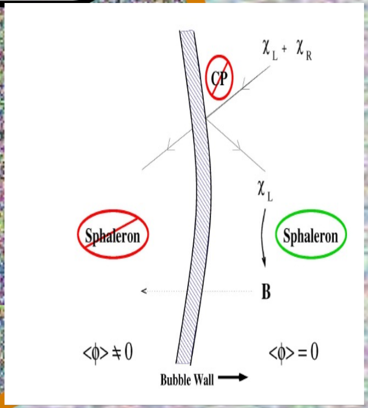
After discovery of Higgs@LHC & GW @aLIGO, EW baryogenesis becomes a testable scenario.

SM technically has all the 3 elements for baryogenesis(Sakharov conditions)

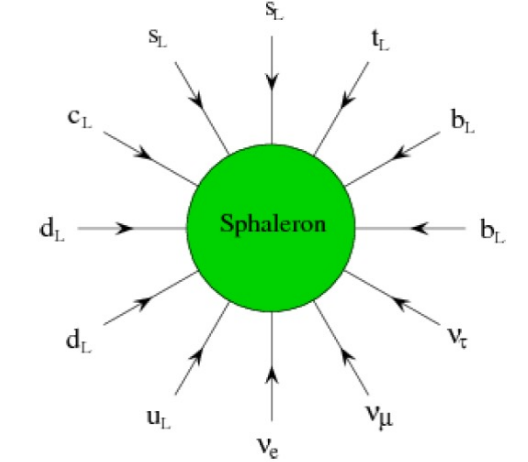
- **B violation from anomaly in B+L current;**
- **C and CP-violation: CKM matrix, but too weak, need new CP-violating sources;**
- **Departure from thermal equilibrium: SFOPT with expanding Higgs bubble wall**

SFOPT and EW baryogenesis

$$\Gamma_S \sim \mathrm{Exp}(-\phi_C/T_C)$$



$$\Gamma_S \sim T^4$$



D. E. Morrissey and M. J. Ramsey-Musolf, New J. Phys. 14, 125003 (2012).

Credit: T. Cohen

SFOPT and EW baryogenesis

$$\Delta\left(n_B - \overline{n}_B\right) = 0$$

$$\Delta\left(n_B - \overline{n}_B\right) \neq 0$$

Credit:
T. Cohen

SFOPT and EW baryogenesis

理论困境:Bubble wall velocity is essential in EW baryogenesis

low/high bubble wall velocity, thin/thick wall, local/non-local

Precise calculation of EW baryogenesis requires to solve the transport equations

vacuum expectation value insertion approximation (VIA)

$$D_i n_i'' + v_w n_i' - \mathcal{C}_i^{VIA}[n_j] = S_{VIA,i},$$

$$\int d^4y \, \mathrm{tr}(S^>(x,y)\Sigma^<(y,x) + \{x \leftrightarrow y\}) - \{\Sigma \leftrightarrow S\}.$$

A.Riotto, Phys.Rev.D53, 5834(1996), Nucl.Phys.B518,339(1998).

semiclassical (WKB) formalism

$$A_i \begin{pmatrix} \mu_i \\ u_i \end{pmatrix}' + (m_i^2)' B_i \begin{pmatrix} \mu_i \\ u_i \end{pmatrix} - \mathcal{C}_i^{WKB} = \begin{pmatrix} v_w S_{WKB,i} \\ S_{WKB,i} \end{pmatrix},$$

$$-v_w \gamma_w h s_p \frac{(m^2\theta')'}{2EE_z} f'_{v_w}$$

$$+v_w \gamma_w h s_p \frac{m^2 (m^2)'\theta'}{4E^2 E_z} \left(\frac{f'_{v_w}}{E} - \gamma_w f''_{v_w} \right)$$

M. Joyce,T.Prokopec,andN.Turok, Phys.Rev.Lett.75,1695 (1995); J.M.Cline and K.Kainulainen, Phys.Rev.Lett.85,5519 (2000).

In Phys.Rev.D 101 (2020) 6, 063525, James Cline and Kimmo Kainulainen make a comparison between the two methods in a given model, they found that the predictions typically differed by factors of 10-40.

SFOPT and EW baryogenesis

Recently, it has been pointed out that the VIA source terms exactly vanish by performing correct resummation of 1PI self energy.

EW baryogenesis with high bubble wall velocity

P.Auclair, C.Caprini, D.Cutting, M.Hindmarsh, K.Rummukainen, D.A.Steer and D.J.Weir, [arXiv:2205.02588]
J.Dahl, M.Hindmarsh, K.Rummukainen and D.J.Weir, [arXiv:2112.12013].

CP-violating transport theory for electroweak baryogenesis with thermal corrections

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Abstract. We derive CP-violating transport equations for fermions for electroweak baryogenesis from the CTP-formalism including thermal corrections at the one-loop level. We consider both the VEV-insertion approximation (VIA) and the semiclassical (SC) formalism. We show that the VIA-method is based on an *assumption* that leads to an ill-defined source term containing a pinch singularity, whose regularisation by thermal effects leads to ambiguities including spurious ultraviolet and infrared divergences. We then carefully review the derivation of the semiclassical formalism and extend it to include thermal corrections. We present the semiclassical Boltzmann equations for thermal WKB-quasiparticles with source terms up to the second order in gradients that contain both dispersive and finite width corrections. We also show that the SC-method reproduces the current divergence equations and that a correct implementation of the Fick's law captures the semiclassical source term even with conserved total current $\partial_\mu j^\mu = 0$. Our results show that the VIA-source term is not just ambiguous, but that it does not exist. Finally, we show that the collisional source terms reported earlier in the semiclassical literature are also spurious, and vanish in a consistent calculation.

Keywords: baryon asymmetry, cosmological phase transitions, particle physics - cosmology connection

ArXiv ePrint: 2108.08336

JCAP11(2021)042

Resummation and cancellation of the VIA source in electroweak baryogenesis

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[c] *Kavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*

E-mail: mpostma@nikhef.nl, jorinde.van.de.vis@desy.de, graham.white@ipmu.jp

Abstract: We re-derive the vev-insertion approximation (VIA) source in electroweak baryogenesis. In contrast to the original derivation, we rely solely on 1-particle-irreducible self-energy diagrams. We solve the Green's function equations both perturbatively and resummed over all vev-insertions. The VIA source corresponds to the leading order contribution in the gradient expansion of the Kadanoff-Baym (KB) equations. We find that it vanishes both for bosons and fermions, both in the perturbative and in the resummed approach. The non-existence of the source is a result of a cancellation between different terms in the KB equations, and persists after resumming the masses.

Keywords: Baryo-and Leptogenesis, Cosmology of Theories BSM, Early Universe Particle Physics

ArXiv ePrint: 2206.01120

JHEP 2)121

SFOPT and EW baryogenesis

实验困难：

Large enough CP-violating source for successful EW baryogenesis

Strong tension in most cases

pretty small CP-violation to avoid strong EDM constraints

$$|d_e| < 4.1 \times 10^{-30}\, e\,\mathrm{cm}$$

Science 381 (2023) 6653

How to alleviate this tension for successful baryogenesis?

Dynamical CP violation for baryogengesis ?

SFOPT and EW baryogenesis

Question: How to alleviate the tension for successful baryogenesis ?

Answer: Dynamical CP-violating source

Large enough CP-violating source for successful EW baryogenesis

Alleviate by assuming the CP-violating source is time dependent

Dynamical/cosmological evolve

Negligible CP-violating source at current time to avoid strong EDM constraints

- **Effective field theory**: **FPH**, Zhuoni Qian, Mengchao Zhang,Phys.Rev. D98 (2018) no.1, 015014;**FPH**,Chong Sheng Li, Phys. Rev. D 92, 075014 (2015); lots of works
- **Renormalizble model**: Complex 2HDM,Xiao Wang, FPH, Xinmin Zhang, arXiv: 1909.02978, work in progress with Eibur Senaha, Xiao Wang in an extended IDM model

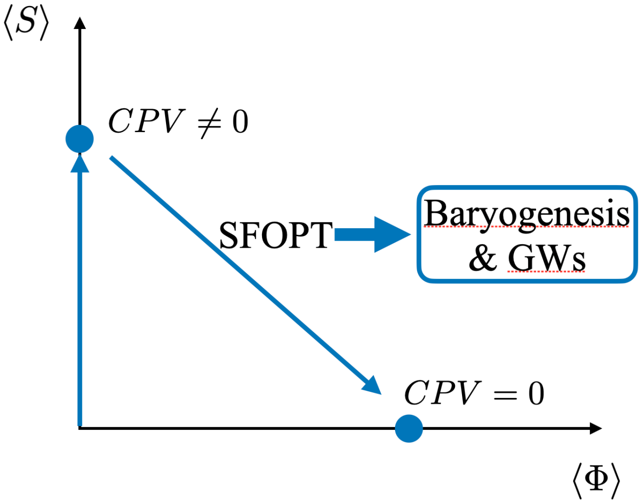
Baldes, T. Konstandin and G. Servant, arXiv:1604.04526;I. Baldes, T. Konstandin and G. Servant, JHEP 1612, 073 (2016);S. Bruggisser,T.Konstandin and G. Servant, JCAP 1711, no. 11, 034 (2017)

See Matthew Reece's recent study on the dynamical CP-violation.

SFOPT and EW baryogenesis

Taking the effective scenario as a representative example:

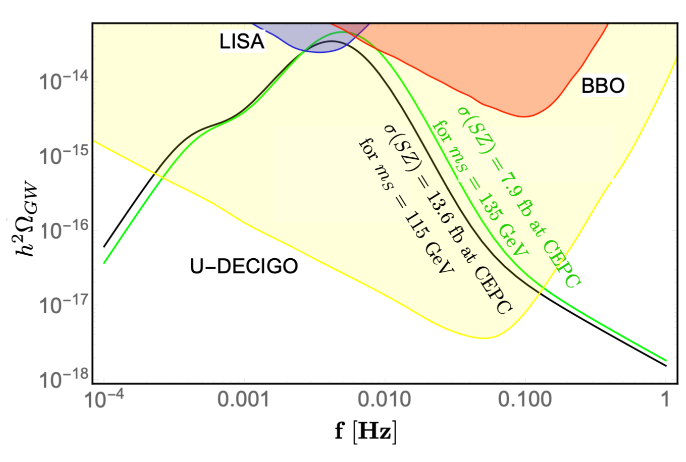
$$\mathcal{L}_{\rm SM} - y_t \frac{\eta}{\Lambda} S \bar{Q}_L \tilde{\Phi} t_R + {\rm H.c} + \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\mu^2 S^2 - \frac{1}{4}\lambda S^4 - \frac{1}{2}\kappa S^2 (\Phi^\dagger \Phi)$$



The singlet and the dim-5 operator can come From many types composite Higgs model, arXiv:0902.1483 , arXiv:1703.10624 ,arXiv:1704.08911,

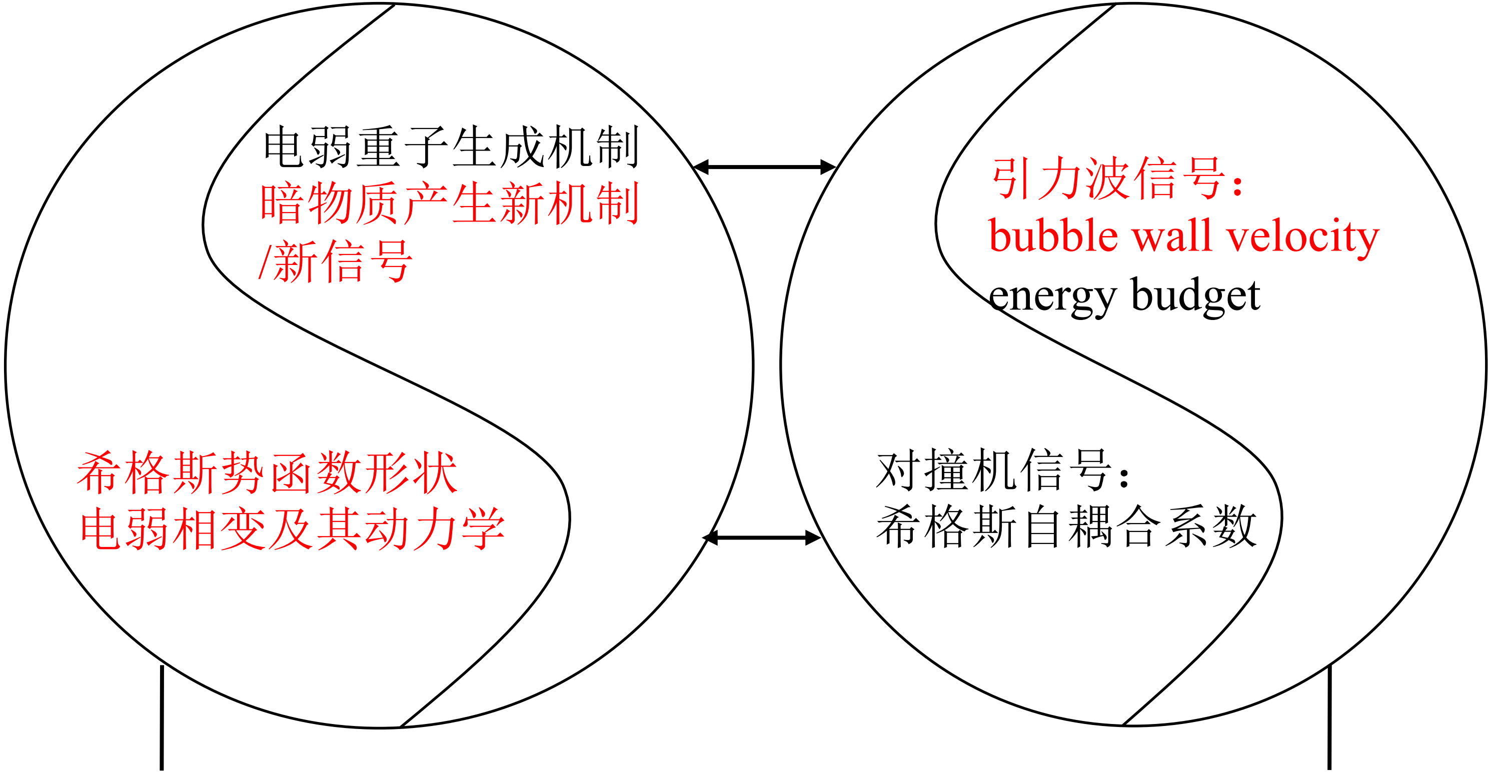
J. R. Espinosa, B. Gripaios, T. Konstandin and F. Riva, JCAP **1201**, 012 (2012)

J. M. Cline and K. Kainulainen, JCAP **1301**, 012 (2013)



Phys.Rev. D98 (2018) no.1, 015014 , **FPH**, Zhuoni Qian, Mengchao Zhang

Motivation

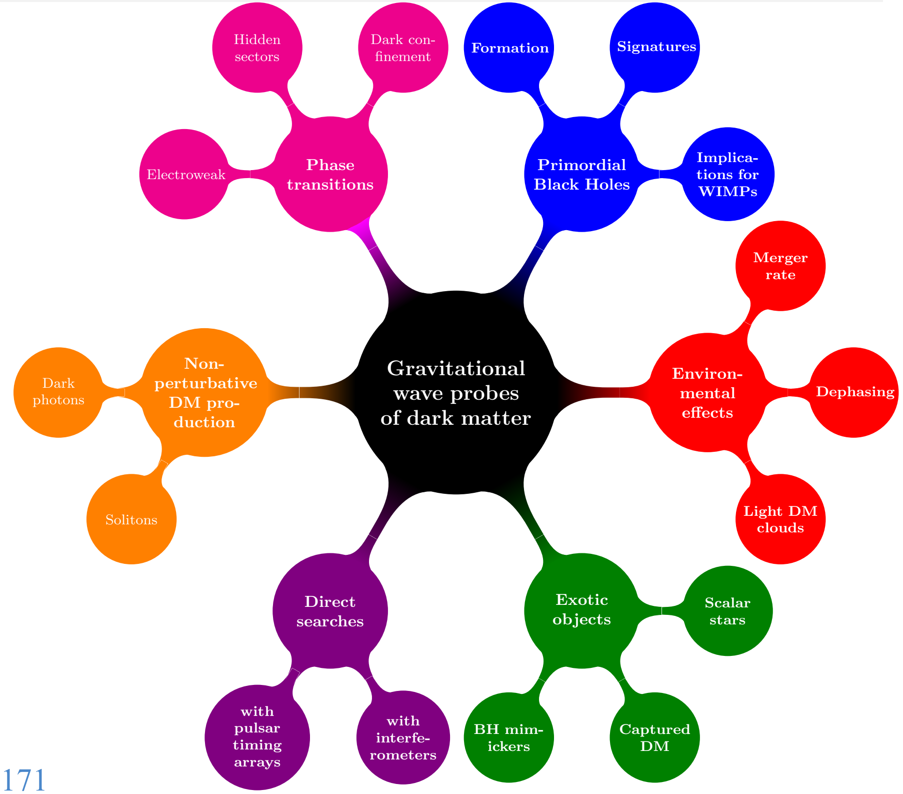


(有限温度)量子场论、相对论流体力学、广义相对论

SFOPT and new DM mechanism/signal

- ➢ The observation of GW@LIGO initiates a new era of exploring DM by GW.
- ➢ DM can trigger a SFOPT in the early universe and detectable GW signals.
- ➢ SFOPT could provide a new approach for DM production.

J.Jaeckel, V. V. Khoze, M. Spannowsky, Phys.Rev. D94 (2016) no.10, 103519
Zhaofeng Kang,et.al. arXiv:2101.03795, arXiv:2003.02465
Yan Wang, Chong Sheng Li, and **FPH**, arXiv:2012.03920
FPH, Eibun Senaha Phys.Rev. D100 (2019) no.3, 03501
FPH PoS ICHEP2018 (2019) 397
FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-
Haipeng An, et.al, arXiv: 2208.14857, arXiv:2009.12381, arXiv:2201.05171



Credit: Gianfranco Bertone et. al.

SFOPT and new DM mechanism/signal

DM Case I:

Inert Doublet Models

$$V_0 = M_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 \Phi^\dagger \Phi D^\dagger D + \lambda_4 |\Phi^\dagger D|^2 + (\lambda_5/2)[(\Phi^\dagger D)^2 + h.c.],$$

mixed singlet-doublet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_D^2 H_2^\dagger H_2 + \frac{1}{2}\lambda_S S^2 |\Phi|^2 + \lambda_3 \Phi^\dagger \Phi H_2^\dagger H_2 + \lambda_4 |\Phi^\dagger H_2|^2 + \frac{\lambda_5}{2}[(\Phi^\dagger H_2)^2 + \mathrm{H.c.}] + A[S\Phi H_2^\dagger + \mathrm{H.c.}].$$

mixed singlet-triplet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_\Sigma^2 \mathrm{Tr}(H_3^2) + \kappa_\Sigma \Phi^\dagger \Phi \mathrm{Tr}(H_3^2) + \frac{\kappa}{2}|\Phi|^2 S^2 + \xi S \Phi^\dagger H_3 \Phi.$$

provide natural DM candidate

produce SFOPT and phase transition GW

FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022

Yan Wang, Chong Sheng Li, and **FPH**, *Phys.Rev.D* 104 (2021) 5, 053004;

SFOPT and new DM mechanism/signal

DM Case II: anti-fileter case

New DM production scenario filtered by the bubbles during a SFOPT in the early universe.

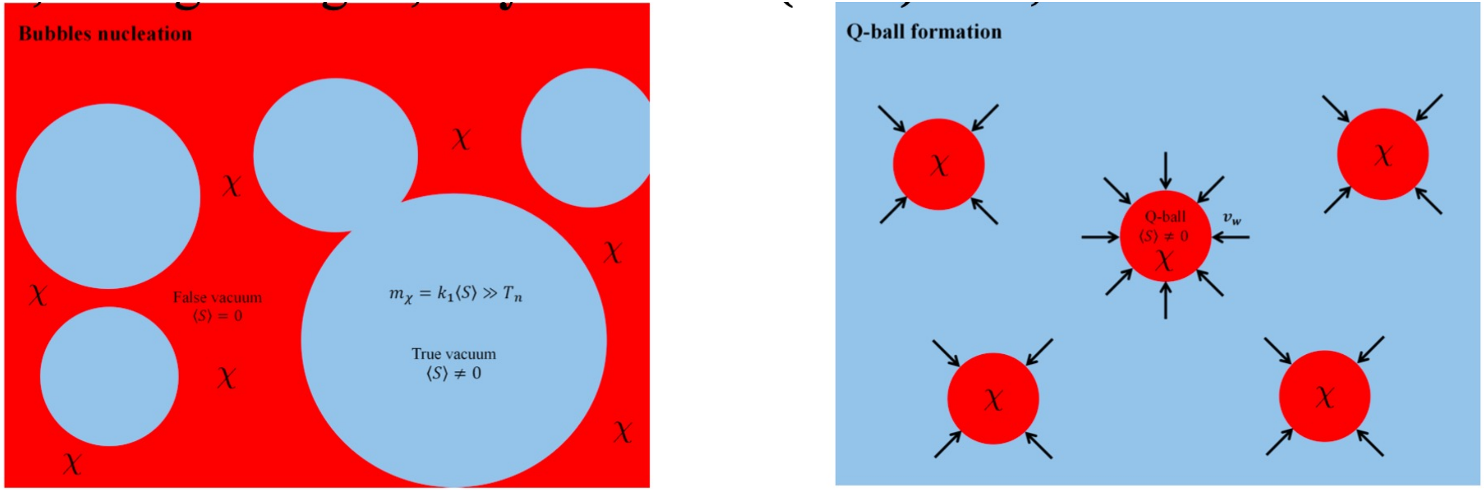
The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously, where constraints on DM mass and reverse dilution are significantly relaxed. We study how to probe this scenario by GW signals and collider signals at QCD NLO.

FPH, Chong Sheng Li, Phys. Rev. D96 (2017) no.9, 095028

$$\rho_{\rm DM}^4 v_b^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

SFOPT and new DM mechanism/signal

SFOPT naturally correlates DM, baryogenesis, particle collider and GW signals.



(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression

(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

The global Q-ball model proposed by T.D. Lee

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

SFOPT and new DM mechanism/signal

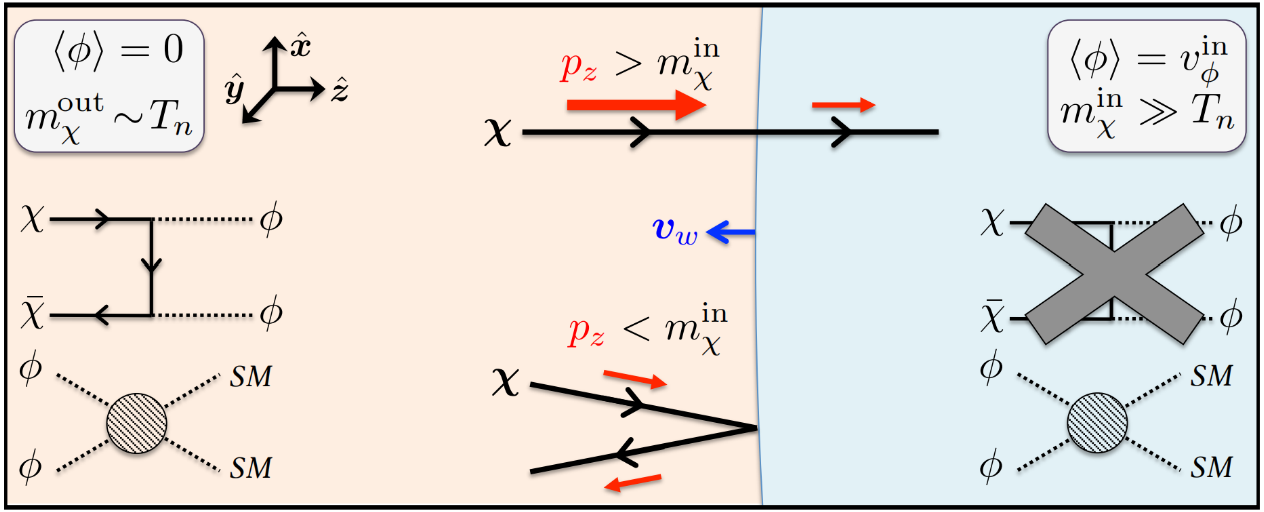
DM Case III: fileter case

In the recent two years, this dynamical DM formed by phase transition has became a new idea and attracted more and more attentions.
Namely, bubbles in SFOPT can be the “filters” to packet your needed heavy DM.

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin
arXiv:1912.02830, Michael J. Baker, Joachim Kopp,and Andrew J. Long
arXiv:2012.15113, Wei Chao, Xiu-Fei Li, Lei Wang
arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin
arXiv:2103.09827, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2103.09822, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2008.04430 Jeong-Pyong Hong， Sunghoon Jung, Ke-pan Xie
Haipeng An, et.al, arXiv: 2208.14857
Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218
more and more new works…

SFOPT and new DM mechanism/signal

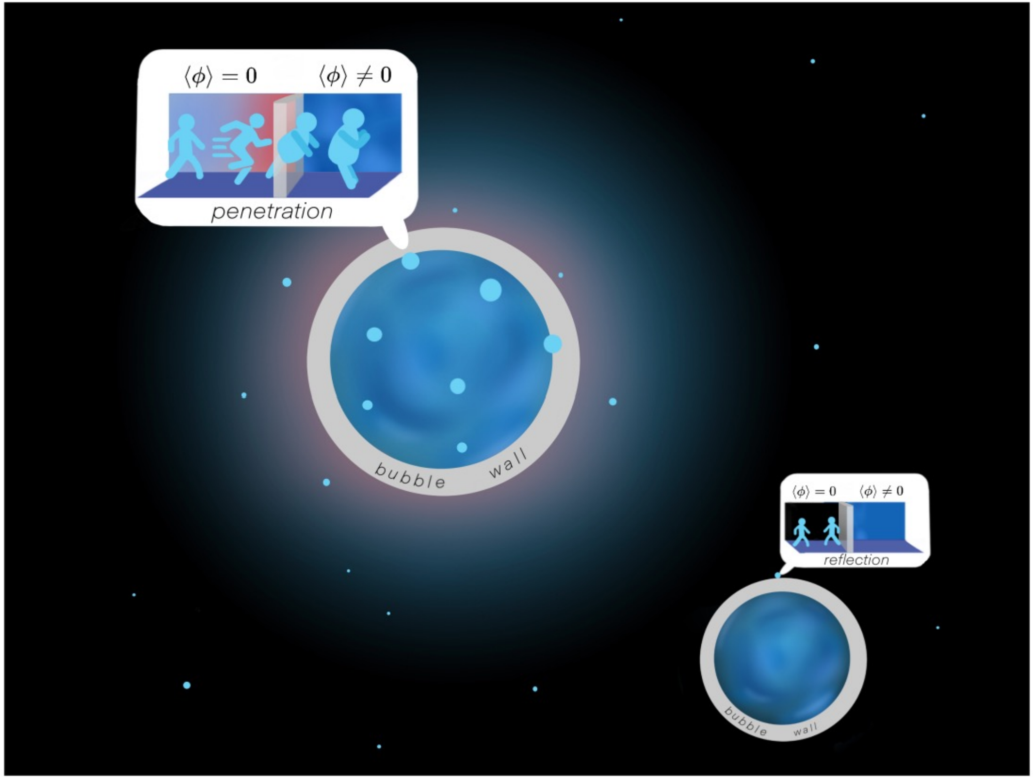
Michael J. Baker, Joachim Kopp, Andrew J. Long,Phys.Rev.Lett. 125 (2020) 15, 151102



$$\Omega_{\rm DM} h^2 \approx 0.17 \left(\frac{T_n}{\rm TeV}\right)\left(\frac{m_\chi^\infty}{30 T_n}\right)^{-\frac{5}{2}} \exp\left(-\frac{m_\chi^\infty}{30 T_n}\right)$$

SFOPT and new DM mechanism/signal

Bubble wall dynamics plays an essential role in the filtered DM mechanism.

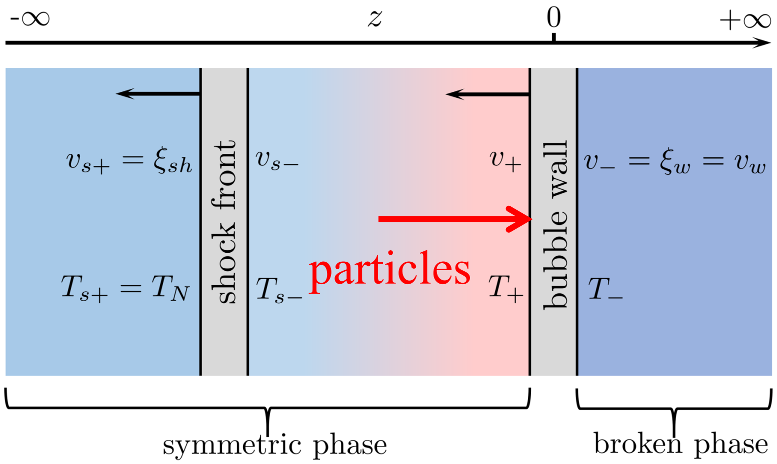


Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218, Phys.Rev.D 108 (2023) 6, 063508

SFOPT and new DM mechanism/signal

Original work:

$$\tilde{v}_{\rm pl} = v_w, \quad T = T' = T_n$$



$$\tilde{v}_{\rm pl} = \tilde{v}_+, \quad T = T_+, \quad T' = T_- \qquad \text{(this work with hydrodynamic effects)}\ .$$

$$J_w^{\rm in} = \frac{g_\chi}{(2\pi)^2}\int_0^{-1} d\cos\theta\cos\theta\int_{-\frac{m_\chi^{\rm in}}{\cos\theta}}^{\infty} dp\frac{p^2}{e^{\tilde{\gamma}_+(1+\tilde{v}_+\cos\theta)p/T_+}} = \frac{g_\chi T_+^3\left(1+\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+\right)}{4\pi^2\tilde{\gamma}_+^3(1-\tilde{v}_+)^2}e^{-\tilde{\gamma}_+ m_\chi^{\rm in}(1-\tilde{v}_+)/T_+}.$$

$$n_\chi^{\rm in} = \frac{J_w^{\rm in}}{\gamma_w v_w} \qquad \Omega_{\rm DM}^{\rm (hy)}h^2 = \frac{m_\chi^{\rm in}(n_\chi^{\rm in}+n_{\bar\chi}^{\rm in})}{\rho_c/h^2}\frac{g_{\star 0}T_0^3}{g_\star(T_-)T_-^3} \simeq 6.29\times 10^8\frac{m_\chi^{\rm in}}{\rm GeV}\frac{(n_\chi^{\rm in}+n_{\bar\chi}^{\rm in})}{g_\star(T_-)T_-^3}$$

SFOPT and new DM mechanism/signal

$$T_\phi^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V_{T=0}(\phi)\right]$$

Energy-momentum tensor of scalar field

$$T_{\rm pl}^{\mu\nu} = \sum_i \int \frac{d^3k}{(2\pi)^3 E_i} k^\mu k^\nu f_i^{\rm eq}(k)$$

Energy-momentum tensor of fluid

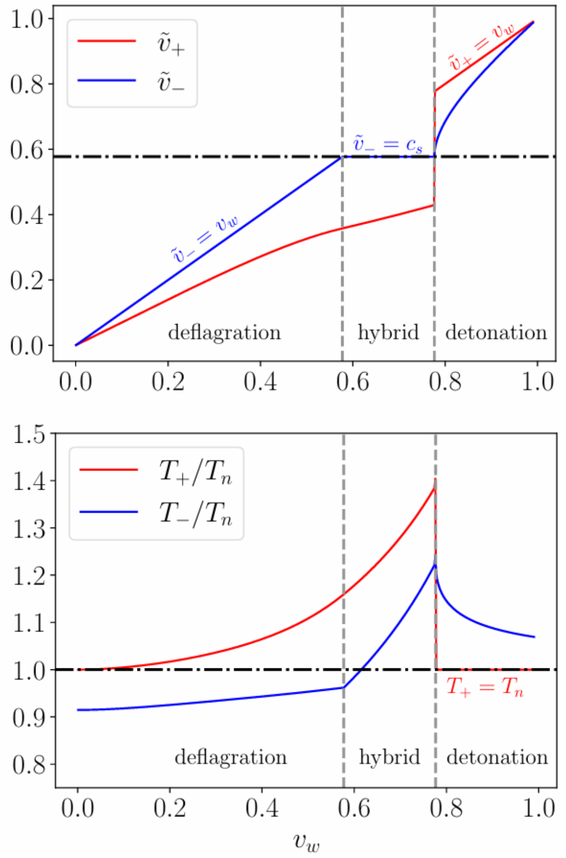
$$T_{\rm fl}^{\mu\nu} = T_\phi^{\mu\nu} + T_{\rm pl}^{\mu\nu} = \omega u^\mu u^\nu - p g^{\mu\nu}$$

Energy-momentum conservation

$$\omega_+ \tilde{v}_+^2 \tilde{\gamma}_+^2 + p_+ = \omega_- \tilde{v}_-^2 \tilde{\gamma}_-^2 + p_-, \quad \omega_+ \tilde{v}_+ \tilde{\gamma}_+^2 = \omega_- \tilde{v}_- \tilde{\gamma}_-^2$$

$$\alpha_+ \equiv \epsilon / \left(a_+ T_+^4\right) \qquad r_\omega = \omega_+/\omega_- = (a_+ T_+^4)/(a_- T_-^4)$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \longleftrightarrow \quad \begin{aligned} j\frac{v}{\xi} &= \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v \\ \frac{\partial_\xi \omega}{\omega} &= \left(1 + \frac{1}{c_s^2}\right)\gamma^2 \mu \partial_\xi v\ . \end{aligned}$$



SFOPT and new DM mechanism/signal

General phase-transition model

$$V_{\rm eff}(\phi,T)=\frac{\mu^2+DT^2}{2}\phi^2-CT\phi^3+\frac{\lambda}{4}\phi^4-\frac{g_\star\pi^2T^4}{90}$$

$$\langle\phi\rangle=0,\quad \frac{3CT}{2\lambda}\left[1+\sqrt{1-\frac{4\lambda\left(\mu^2+DT^2\right)}{9C^2T^2}}\right]$$

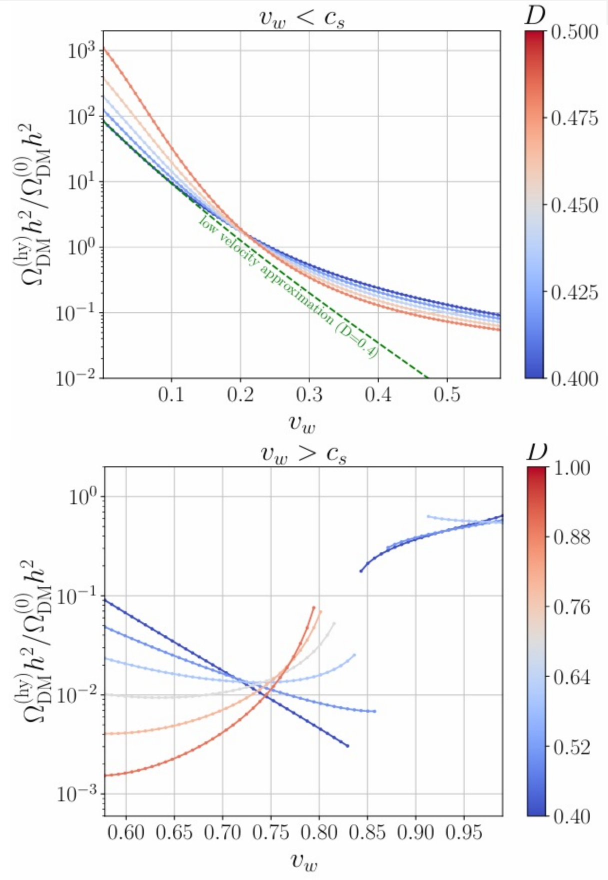
$$m_\chi^{\rm in}=\frac{y_\chi\phi_-}{\sqrt{2}}\qquad\qquad \phi_-=\phi(T_-)$$

$$n_\chi^{\rm in}\simeq\frac{g_\chi T_+^3}{\gamma_w v_w}\left(\frac{\tilde\gamma_+\left(1-\tilde v_+\right)m_\chi^{\rm in}/T_+ +1}{4\pi^2\tilde\gamma_+^3\left(1-\tilde v_+\right)^2}\right)e^{-\tilde\gamma_+(1-\tilde v_+)m_\chi^{\rm in}(T_-)/T_+}$$

We found:

(a) for $v_w\lesssim 0.2$: the DM relic density is enhanced

(b) for $v_w\gtrsim 0.2$: the DM relic density is reduced



SFOPT and new DM mechanism/signal

Boltzmann equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

$$f_\chi = \mathcal{A}(z,p_z)\, f^{\rm eq}_{\chi,+} = \mathcal{A}(z,p_z)\exp\left(-\frac{\tilde{\gamma}_+(E-\tilde{v}_+ p_z)}{T_+}\right)$$

$$\mathbf{L}[f_\chi] = \frac{p_z}{E}\frac{\partial f_\chi}{\partial z} - \frac{m_\chi}{E}\frac{\partial m_\chi}{\partial z}\frac{\partial f_\chi}{\partial p_z} \qquad m_\chi(z) \equiv \frac{m_\chi^{\rm in}(\phi_-)}{2}\left(1+\tanh\frac{2z}{L_w}\right)$$

$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{L}[f_\chi] \approx \left[\left(\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\tilde{\gamma}_+\tilde{v}_+}{T_+}\right)\mathcal{A}(z,p_z)\right]\frac{g_\chi m_\chi T_+}{2\pi\tilde{\gamma}_+}e^{\tilde{\gamma}_+\left(\tilde{v}_+ p_z - \sqrt{m_\chi^2+p_z^2}\right)/T_+}$$

including $\chi\bar{\chi} \leftrightarrow \phi\phi, \chi\phi \leftrightarrow \chi\phi, \chi\chi \leftrightarrow \chi\chi, \chi\bar{\chi} \leftrightarrow \chi\bar{\chi}, \ldots$

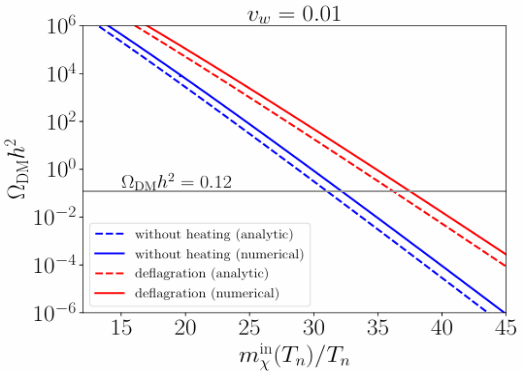
$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2}\mathbf{C}[f_\chi] = -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[f_{\chi_p} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right]$$

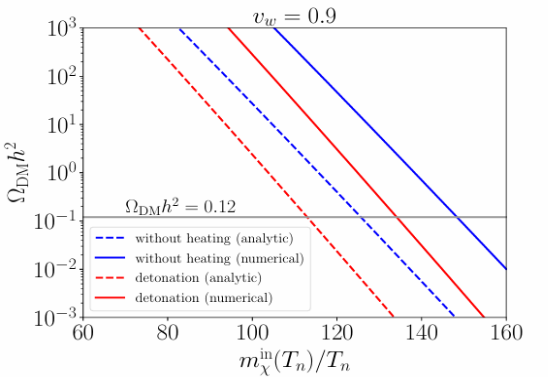
$$= -g_\chi g_{\bar{\chi}} \int \frac{dp_x dp_y}{(2\pi)^2 2E_p^{\mathcal{P}}} d\Pi_{q^{\mathcal{P}}} 4F\sigma_{\chi\bar{\chi}\to\phi\phi}\left[\mathcal{A} f^{\rm eq}_{\chi_p,+} f^{\rm eq}_{\bar{\chi}_q,+} - f^{\rm eq}_{\chi_p} f^{\rm eq}_{\bar{\chi}_q}\right]$$

$$\equiv \Gamma_{\rm P}(z,p_z)\mathcal{A}(z,p_z) - \Gamma_{\rm I}(z,p_z)\ ,$$

SFOPT and new DM mechanism/signal

$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right)/T_+\right]\left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$





$v_w = 0.01$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$
BP_1	31	66	32	71
BP_2	31.1	7.9	32.2	8.1
BP_3	30.8	778.8	31.9	858.5
BP_4	$\star$	$\star$	$\star$	$\star$

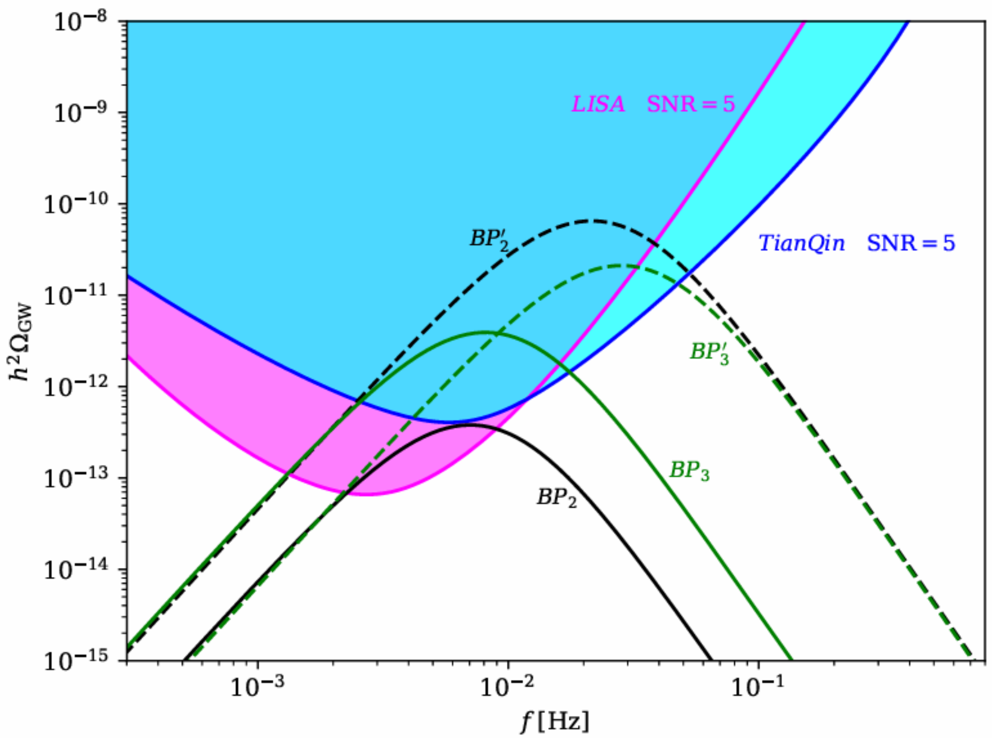
$v_w = 0.9$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)}h^2/\Omega_{\rm DM}^{(0)}h^2$
BP_1	125.3	1/19	147.8	1/27
BP_2	125.9	1/7	148.7	1/9
BP_3	124.6	1/10	147.3	1/12
BP_4	123.8	$1/(1.2 \times 10^{13})$	146.5	$1/(2.2 \times 10^{15})$

SFOPT and new DM mechanism/signal

The hydrodynamic effects play essential roles in the filtered DM mechanism. For the deflagration mode with low bubble wall velocity, the hydrodynamic effects significantly enhance the relic density. In contrast, for the detonation mode, the relic density is obviously reduced. For the hybrid mode, the hydrodynamic correction is extremely large.

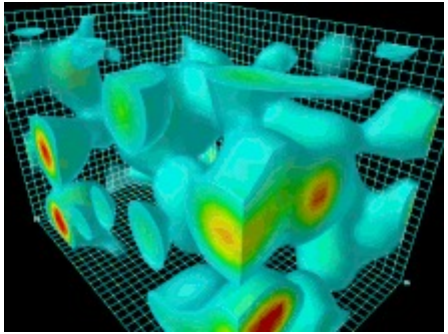
Precise calculation of filtered DM relic density can help to decide the phase-transition parameters precisely. This gives more accurate phase-transition GW spectra.

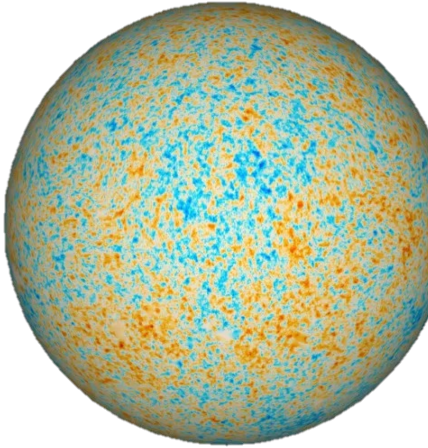


Anisotropy and primordial seeds

Primordial power spectra

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left\langle |\mathcal{R}_k|^2 \right\rangle\Big|_{aH=k} = \frac{H^2}{\pi \varepsilon_{\rm sr} m_{\rm pl}^2}\Big|_{aH=k} = A_s \left(\frac{k}{k_*}\right)^{n_s-1}$$



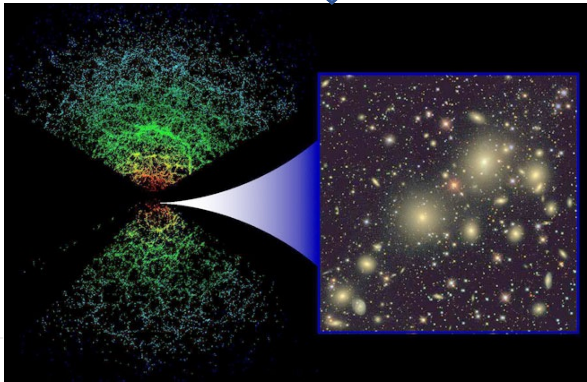


Angular power spectra

$$C_l = \frac{2}{\pi}\int_0^\infty dk k^2 P_{\mathcal{R}}(k)\,|\mathcal{T}_l(k)|^2$$

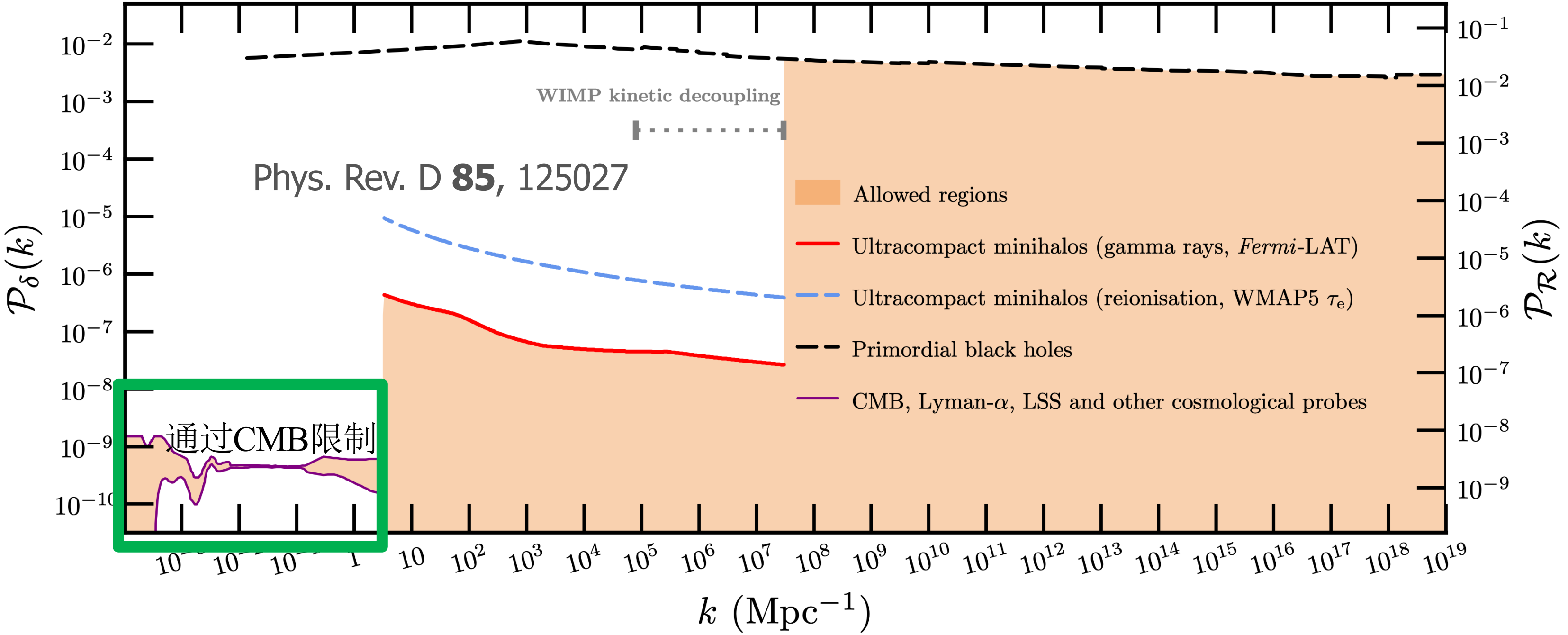
$$C_\ell^{\rm obs} = \frac{1}{2\ell+1}\sum_{\ell m} a_{\ell m} a_{\ell m}^*$$

The primordial density perturbation (quantum fluctuation) from inflation or alternative as the origin of structure



Anisotropy and primordial seeds

The quest for small-scale power spectrum——The road less traveled



✓ A complete model of inflation requires a solid understanding of the small-scale primordial power spectrum;

✓ however, it is hard!

Anisotropy and primordial seeds

Phys. Lett. B, 2017, 771: 9-12 ; Phys. Rev. D, 2019, 100(12): 121501 ; Phys. Rev. D, 2021, 103 (2): 023522 ; Phys. Rev. Lett., 2021, 127(27): 271301

distribution function $$f(\eta, \boldsymbol{x}, \boldsymbol{p}) = \bar{f}(\eta, p) - p\frac{\partial \bar{f}(\eta, p)}{\partial p}\boxed{\mathcal{G}(\eta, \boldsymbol{x}, \hat{\boldsymbol{p}})}$$

$\mathcal{G}(\eta, \boldsymbol{x}, \hat{\boldsymbol{p}})$

quantify the imhomogeneity of graviton

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1-2\Phi)\delta_{ij}dx^i dx^j$$

Boltzmann equation $$\frac{df}{dt} = C[f] \Rightarrow \boxed{\mathcal{G}' + ik\mu\mathcal{G} = \Phi' - ik\mu\Psi}$$

$$\begin{aligned}
\mathcal{G}(\eta_0, k, \mu) =& \mathcal{G}(\eta_{\rm pt}, k, \mu)e^{ik\mu(\eta_{\rm pt}-\eta_0)} + \int_{\eta_{\rm pt}}^{\eta_0} d\eta \left[\Phi'(\eta, k) - ik\mu\Psi(\eta, k)\right] e^{ik\mu(\eta-\eta_0)} \\
=& \mathcal{G}(\eta_{\rm pt}, k, \mu)e^{ik\mu(\eta_{\rm pt}-\eta_0)} + \int_{\eta_{\rm pt}}^{\eta_0} d\eta \left[\Phi'(\eta, k)e^{ik\mu(\eta-\eta_0)} - \frac{d}{d\eta}\left(\Psi(\eta, k)e^{ik\mu(\eta-\eta_0)}\right) + \Psi'(\eta, k)e^{ik\mu(\eta-\eta_0)}\right] \\
=& \underbrace{\left[\mathcal{G}(\eta_{\rm pt}, k) + \Psi(\eta_{\rm pt}, k)\right] e^{ik\mu(\eta_{\rm pt}-\eta_0)}}_{\rm SW} + \underbrace{\int_{\eta_{\rm pt}}^{\eta_0} d\eta \left[\Phi'(\eta, k) + \Psi'(\eta, k)\right] e^{ik\mu(\eta-\eta_0)}}_{\rm ISW}
\end{aligned}$$

Anisotropy and primordial seeds

$$(\mathcal{G}+\Psi)(\eta_{\rm pt},k) = -\frac{1}{3}\mathcal{R}(k)$$ radiation dominate

$$\mathcal{G}_\ell^{\rm SW}(\eta_0,k) = (\mathcal{G}+\Psi)(\eta_{\rm pt},k)\, j_\ell[k(\eta_{\rm pt}-\eta_0)]$$

$$C_\ell^{\mathcal{G},\rm SW} = \frac{4\pi}{9}\int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2\left[k\left(\eta_0-\eta_{\rm pt}\right)\right]$$

$$\int_{\eta_{\rm pt}}^{\eta_0} d\eta \left[\Phi'(\eta,k)+\Psi'(\eta,k)\right] e^{ik\mu(\eta-\eta_0)}$$

$$\mathcal{G}_\ell^{\rm ISW}(\eta_0,k) = \int_{\eta_{pt}}^{\eta_0} d\eta \left(\Phi'+\Psi'\right)(\eta,k) j_\ell\left[k\left(\eta-\eta_0\right)\right]$$

$$\left(\Phi'+\Psi'\right)(\eta,k) \approx -(\Phi+\Psi)(\eta_{\rm pt},k)\,\delta\left(\eta-\eta_k\right)$$

$$(\Phi+\Psi)(\eta_{\rm pt},k) = -\frac{4}{3}\mathcal{R}(k)$$

small scale

$$C_\ell^{\mathcal{G},\rm ISW} = \frac{64\pi}{9}\int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2\left[k\left(\eta_0-\eta_{\rm pt}\right)\right]$$

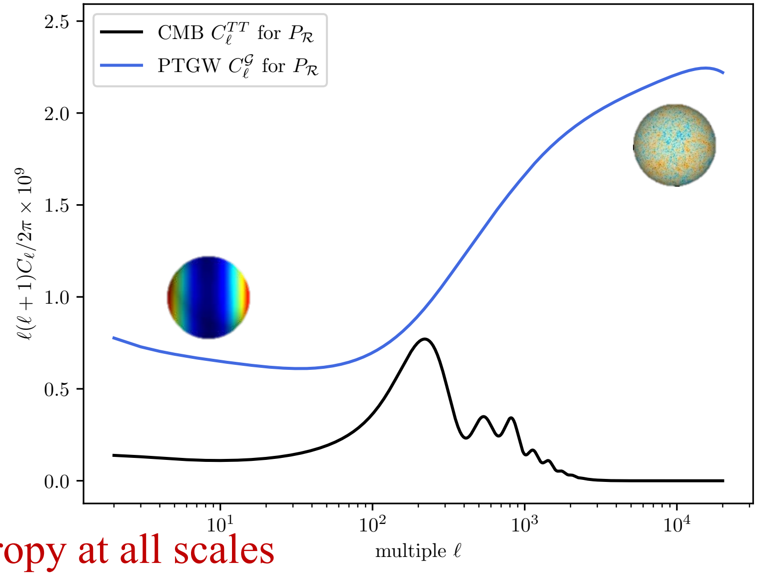
Anisotropy and primordial seeds

Planck: $\ln(10^{10}A_s) = 3.040 \pm 0.016 \qquad n_s = 0.9626 \pm 0.0057$

$$C_\ell^{\mathcal{G}} \approx C_\ell^{\mathcal{G},\mathrm{SW}} = \frac{4\pi}{9}\int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2\left[k\left(\eta_0 - \eta_{\mathrm{pt}}\right)\right]$$

$$\begin{aligned}\mathcal{G}_\ell\left(\eta_0, k\right) &= \left(\mathcal{G}_\ell^{\mathrm{SW}} + \mathcal{G}_\ell^{\mathrm{ISW}}\right)\left(\eta_0, k\right) \\ &\approx \left[(\mathcal{G} + \Psi) - (\Phi + \Psi)\right]\left(\eta_{pt}, k\right) j_\ell\left[k\left(\eta_{\mathrm{pt}} - \eta_0\right)\right]\end{aligned}$$

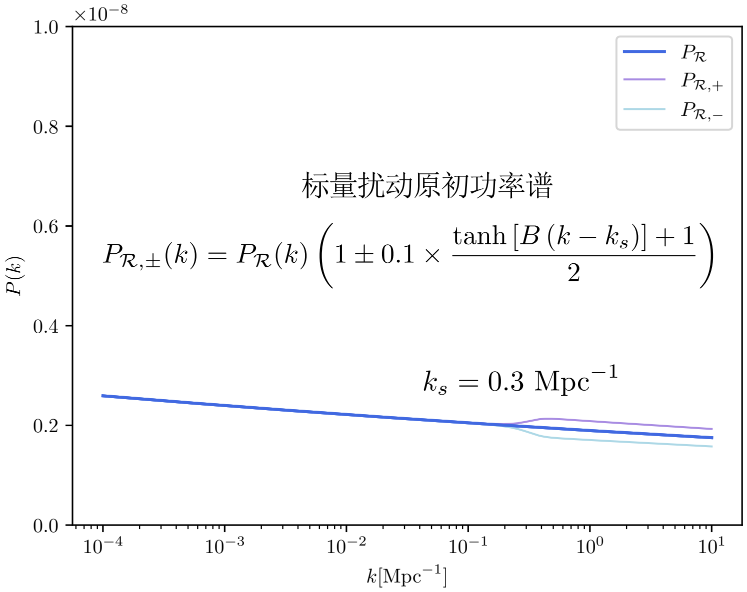
$$C_\ell^{\mathcal{G}} = 4\pi \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2\left[k\left(\eta_0 - \eta_{\mathrm{pt}}\right)\right]$$

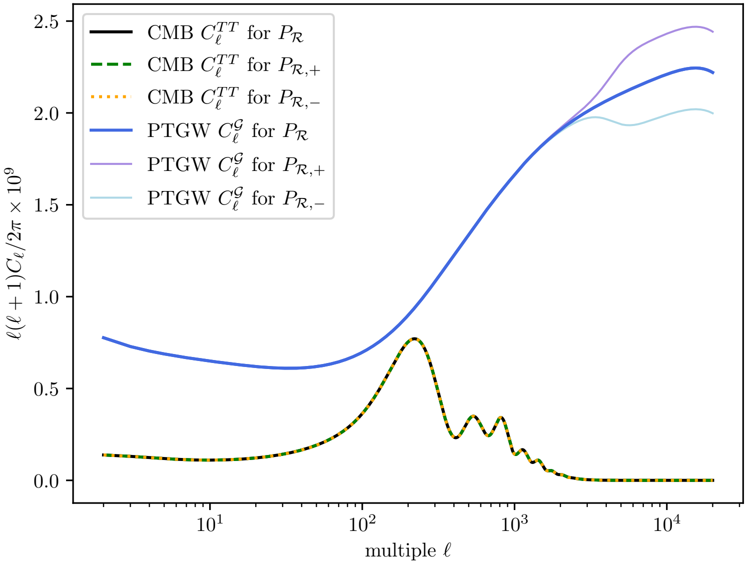


Angular power spectra

- ✓ The anisotropy of PTGW is stronger than CMB anisotropy at all scales
- ✓ The anisotropy of PTGW goes up at small scale

Anisotropy and primordial seeds

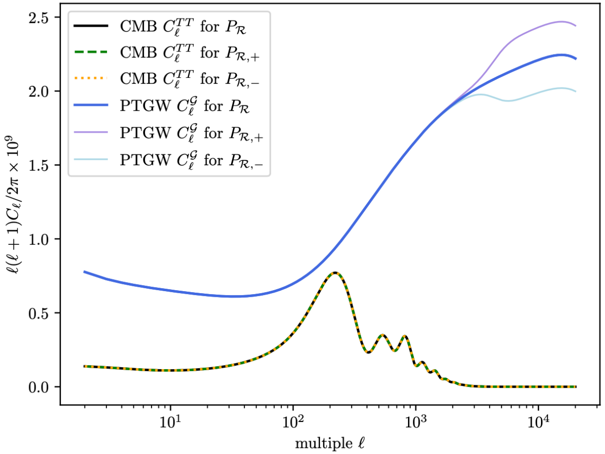


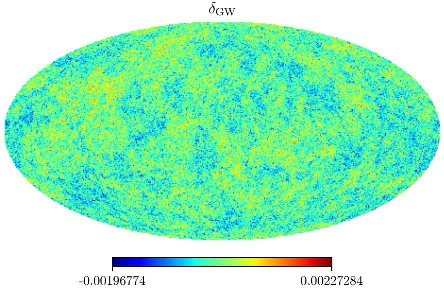


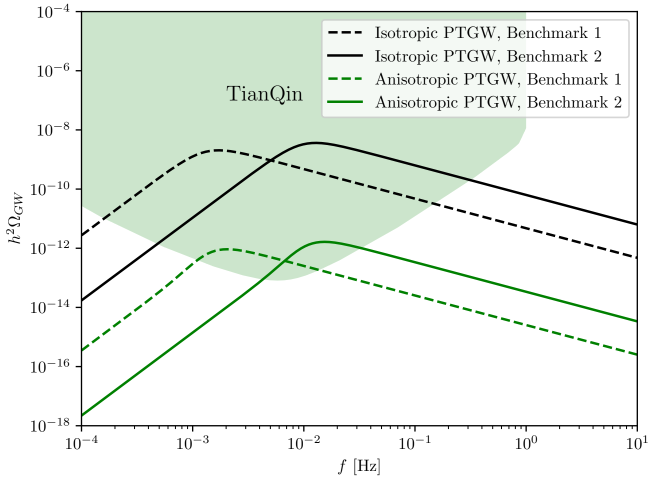
✓ The anisotropy of PTGW keeps more small-scale information.

Anisotropy and primordial seeds

Application to EWPT：相变引力波的各向同性能谱可能探测暗物质的信号，各向异性信息有助于了解暴涨产生的原初扰动信息

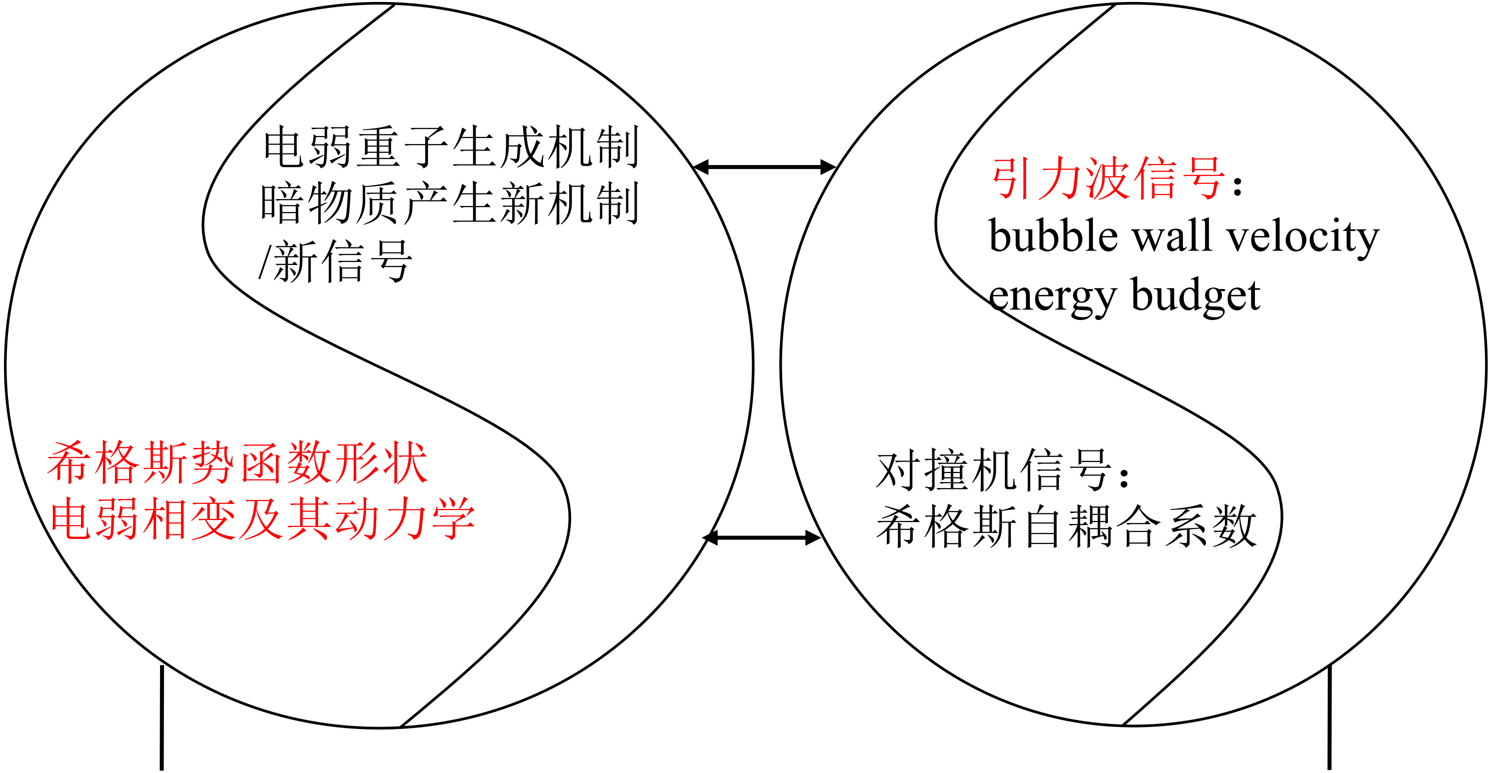






Yongping Li, **FPH**, Xiao Wang, Xinmin Zhang, Phys.Rev.D 105 (2022) 083527
and work in progress

Motivation

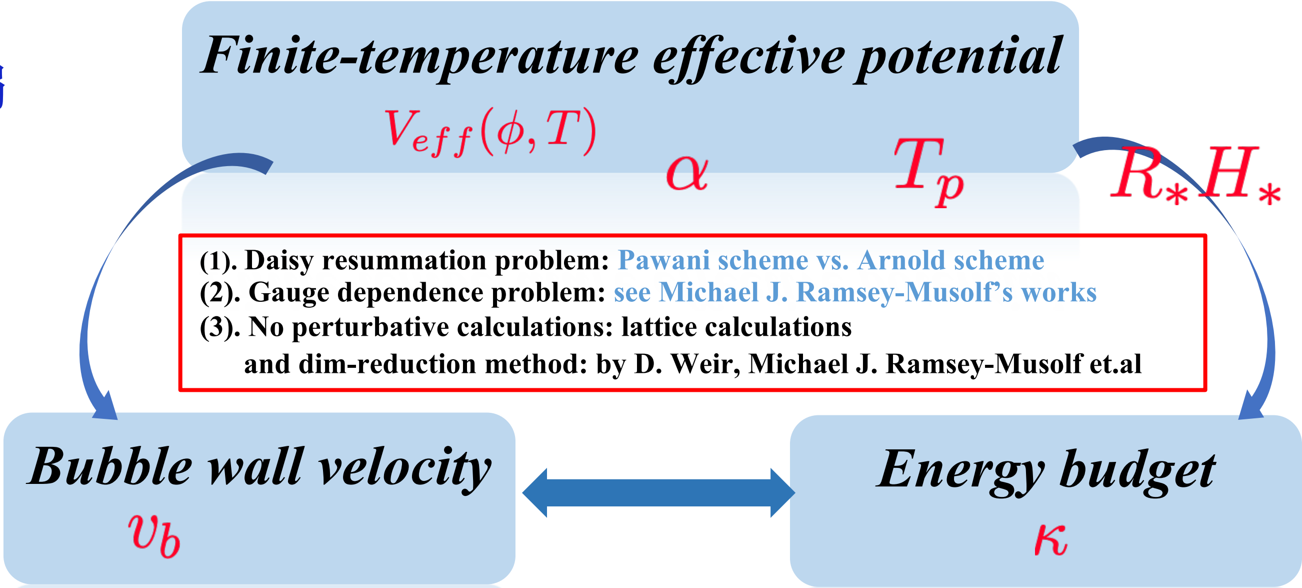


(有限温度)量子场论、相对论流体力学、广义相对论

Phase transition dynamics

Theory: 相变引力波信号、相变暗物质、早期宇宙电弱重子生成机制最核心却最难计算的是泡泡膨胀速度

v_b



Experiment: 实验上最重要的相变参数也是泡泡膨胀速度

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang ， arXiv:2007.10343 ,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith , arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, FPH, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

F. Giese, T. Konstandin, K. Schmitz and J. van de Vis ,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, JCAP 07 (2023) 006

Phase transition dynamics

Systematically calculation of bubble wall velocity in specific model:

Standard Model (small Higgs mass):

Guy D. Moore,Tomislav Prokopec, How fast can the wall move? A Study of the electroweak phase transition dynamics, Phys.Rev.D 52 (1995) 7182-7204

Minimal Supersymmetric Standard Model:

P. John, M.G. Schmidt, Do stops slow down electroweak bubble walls?, Nucl.Phys.B 598 (2001) 291-305

Higgs + scalar singlet:

Jonathan Kozaczuk, Bubble Expansion and the Viability of Singlet-Driven Electroweak Baryogenesis, JHEP 10 (2015) 135

Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, Wall speed and shape in singlet-assisted strong electroweak phase transitions, Phys.Rev.D 103 (2021) 5, 055020

Inert Doublet Model:

Siyu Jiang, FPH, Xiao Wang, Bubble wall velocity during electroweak phase transition in the inert doublet model, Phys.Rev.D 107 (2023) 9, 095005

Phase transition dynamics

The Guy Moore's method would be invalid at around sound velocity, there are some other solutions:

New ansatz:

Benoit Laurent, James M. Cline, Phys.Rev.D 102 (2020) 6, 063516

James M. Cline, Avi Friedlander, Dong-Ming He, Kimmo Kainulainen, Benoit Laurent, Phys.Rev.D 103 (2021) 12, 123529

Marek Lewicki, Marco Merchand, Mateusz Zych, JHEP 02 (2022) 017

Benoit Laurent, James M. Cline, Phys.Rev.D 106 (2022) 2, 023501

Stefania De Curtis, Luigi Delle Rose, Andrea Guiggiani, Ángel Gil Muyor, Giuliano Panico, JHEP 03 (2022) 163

Higher order corrections in Guy Moore's ansatz

Glauber C. Dorsch, Stephan J. Huber, Thomas Konstandin, JCAP 04 (2022) 04, 010

Glauber C. Dorsch, Daniel A. Pinto, arXiv:2312.02354

Phenomenological parametrization of friction (friction=ηv_w)

Ariel Megevand, et.al, Nucl.Phys.B 820 (2009) 47-74, Nucl.Phys.B 825 (2010) 151-176 …

Phase transition dynamics

Hydrodynamical backreaction:

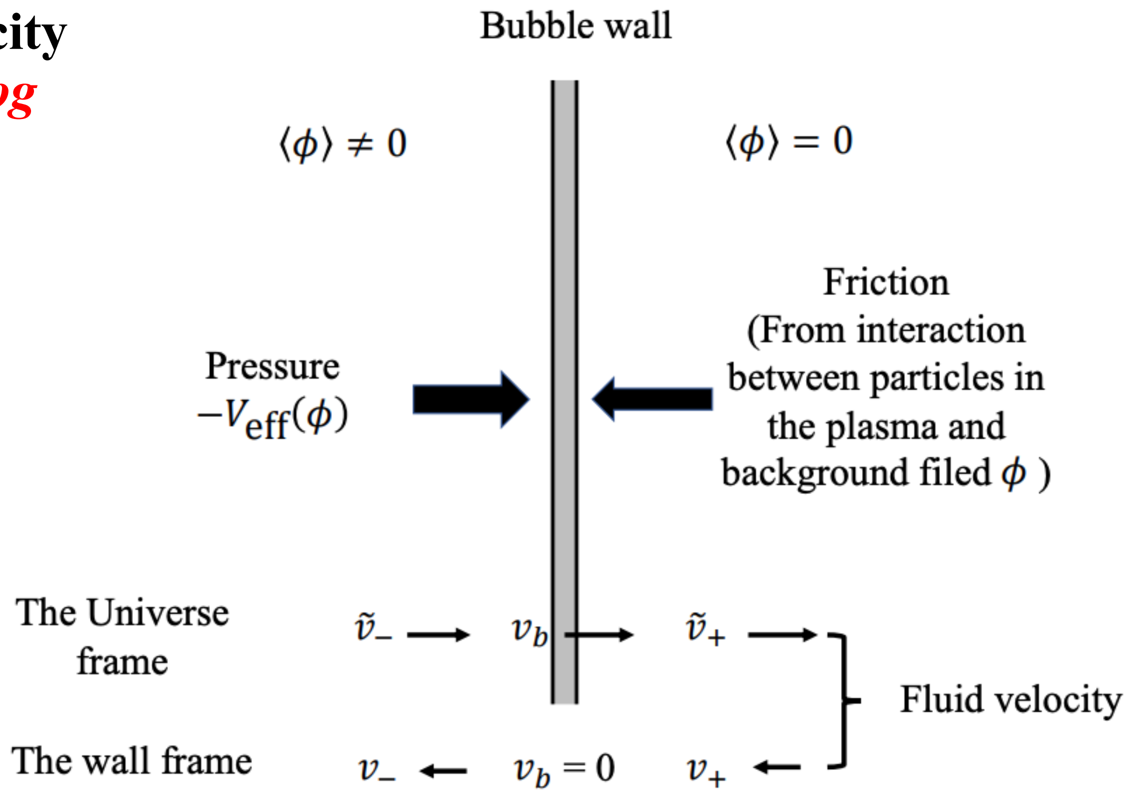
Marc Barroso Mancha, Tomislav Prokopec, Bogumila Swiezewska, JHEP 01 (2021) 070
Wen-Yuan Ai, Bjorn Garbrecht, Carlos Tamarit, JCAP 03 (2022) 03, 015
Wen-Yuan Ai, Benoit Laurent, Jorinde van de Vis, JCAP 07 (2023) 002
Shao-Jiang Wang, Zi-Yan Yuwen, Phys.Rev.D 107 (2023) 2, 023501
Jun-Chen Wang, Zi-Yan Yuwen, Yu-Shi Hao, Shao-Jiang Wang, arXiv:2310.07691
Tomasz Krajewski, Marek Lewicki, Mateusz Zych, Phys.Rev.D 108 (2023) 10, 103523

Bubble wall velocity for ultra-relativistic bubble walls (run-away criterion):

Dietrich Bodeker, Guy D. Moore, JCAP 05 (2009) 009
Dietrich Bodeker, Guy D. Moore, JCAP 05 (2017) 025
Stefan Höche, Jonathan Kozaczuk, Andrew J. Long, Jessica Turner, Yikun Wang, JCAP 03 (2021) 009
Aleksandr Azatov, Miguel Vanvlasselaer, JCAP 01 (2021) 058
Yann Gouttenoire, Ryusuke Jinno, Filippo Sala, JHEP 05 (2022) 004
Wen-Yuan Ai, JCAP 10 (2023) 052
见康召丰老师的报告

Phase transition dynamics

Bubble wall velocity
beyond *leading-log*
approximation
in electroweak
phase transition



Phase transition dynamics

Bubble wall velocity beyond *leading-log* approximation in electroweak phase transition

The EOM of Higgs filed (order parameter field)

- Energy momentum conservation of scaler-plasma system in WKB approximation

$$\Box\phi + \frac{\partial V_0(\phi)}{\partial\phi} + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E} f(p,x) = 0 \ ,$$

Zero-temperature part of effective potential

$$f \equiv f_0 + \delta f .$$

$$\Box\phi + \frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E} \delta f(p,x) = 0 \ ,$$

Full thermal effective potential

Friction term

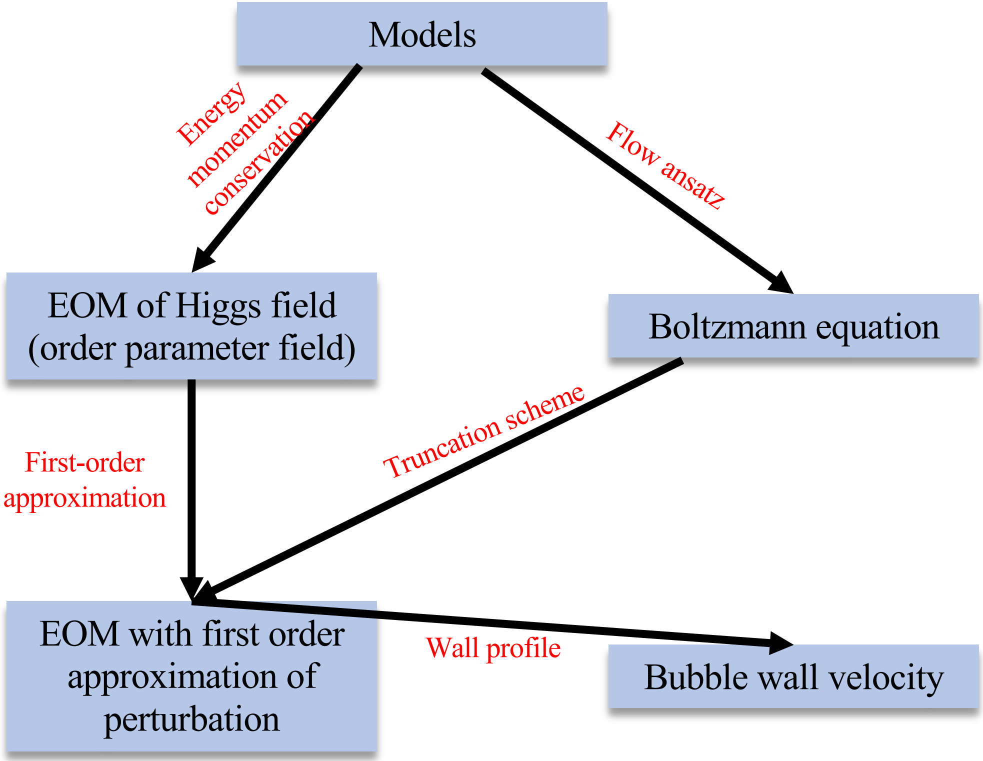
Deviation from equilibrium

Field dependent mass

Phase transition dynamics

Bubble wall velocity
beyond *leading-log*
approximation
in electroweak
phase transition

arXiv:2011.12903, Xiao wang, FPH, Xinmin Zhang



Phase transition dynamics

The deviation distribution part for each massive particle is crucial

With WKB approximation $p \gg 1/L_w$, we can describe it with Boltzmann equation

$$\frac{d}{dt}f = \left(\frac{\partial}{\partial t} + \dot{z}\frac{\partial}{\partial z} + \dot{p_z}\frac{\partial}{\partial p_z}\right) f = -C[f] \ ,$$

To solve the Boltzmann equation

- *Appropriate form for the distribution function (flow ansatz)*
- *Specific truncation scheme*
- *Proper treatment of collision term*

Phase transition dynamics

Key point: calculate the particle scattering amplitude in thermal plasma

The collision term in Boltzmann equation

$$C[f] = \sum_i \frac{1}{2E_p} \int \frac{d^3k d^3p' d^3k'}{(2\pi)^9 2E_k 2E_{p'} 2E_{k'}} \bar{\Sigma}|M|^2 (2\pi)^4 \delta^4(p+k-p'-k') \mathcal{P}[f_i] \ ,$$

$$\mathcal{P}[f_i] = f_1 f_2 (1 \pm f_3)(1 \pm f_4) - f_3 f_4 (1 \pm f_1)(1 \pm f_2) \ .$$

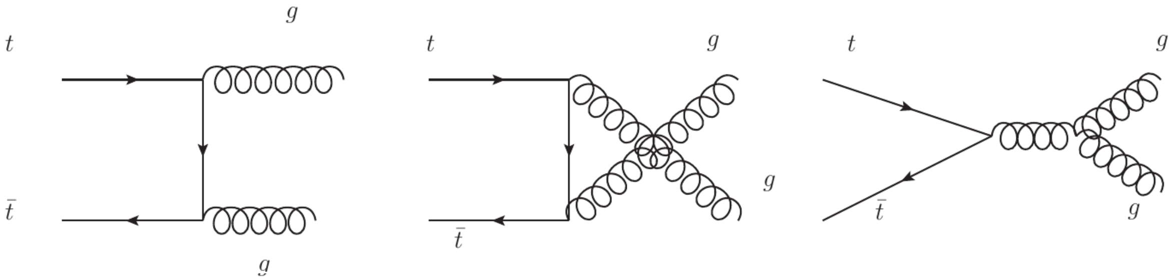
To the first-order approximation:

$$\mathcal{P}[f_i] \approx \frac{\delta_1 + \delta_2 - \delta_3 - \delta_4}{T} f_1 f_2 (1 \pm f_3)(1 \pm f_4) \ .$$

Phase transition dynamics

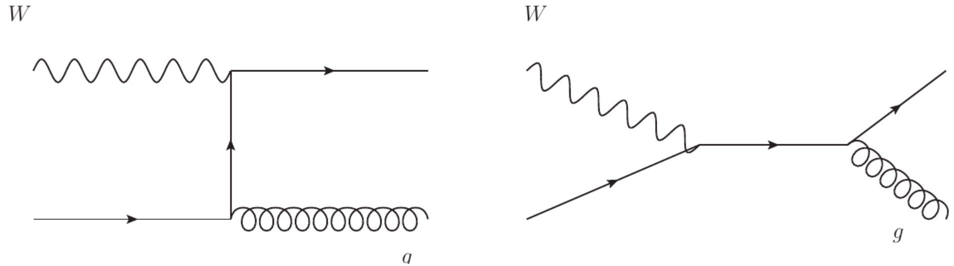
Top quark: $t\bar{t} \to gg$

- Interactions of $O(g_s^4)$



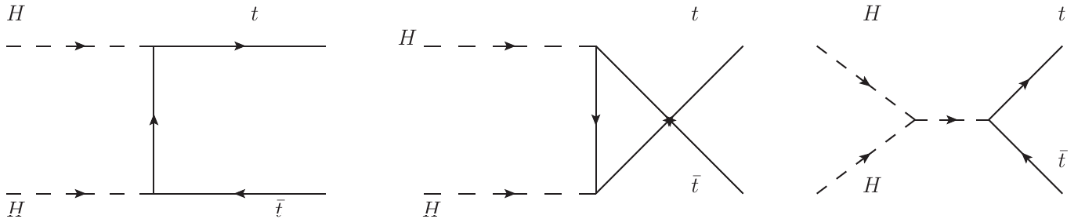
W/Z boson: $Wu \to dg$

- Interactions of $O(g_s^2 g_w^2)$



Higgs boson: $HH \to t\bar{t}$

- Interactions of $O(y_t^4)$



Phase transition dynamics

$$\delta = -\mu - \mu_{bg} - \frac{E}{T}(\delta T + \delta T_{bg}) - p_z(\delta v + \delta v_{bg})$$

To the first order of perturbations, we can derive the following EOM of the Higgs field.

$$\begin{aligned}
-(1-v_w^2)\phi'' + \frac{\partial V_{\text{eff}}(\phi,T)}{\partial \phi} + \delta T_{\text{bg}} \frac{\partial^2 V_{\text{eff}}(\phi,T)}{\partial T \partial \phi} \\
+ \frac{N_t T}{2}\frac{dm_t^2}{d\phi}\left(c_1^f \mu_t + c_2^f(\delta T_t + \delta T_{bg})\right) \\
+ \frac{N_W T}{2}\frac{dm_W^2}{d\phi}\left(c_1^b \mu_W + c_2^b(\delta T_W + \delta T_{bg})\right) \\
+ \frac{N_H T}{2}\frac{dm_H^2}{d\phi}\left(c_1^b \mu_H + c_2^b(\delta T_H + \delta T_{bg})\right) = 0
\end{aligned}$$

$$T = T_+ + \delta T_{bg}(z) \longrightarrow T_n = T_+ = T$$

Here we take an approximation

Phase transition dynamics

Bubble wall velocity beyond *leading-log* approximation in electroweak phase transition

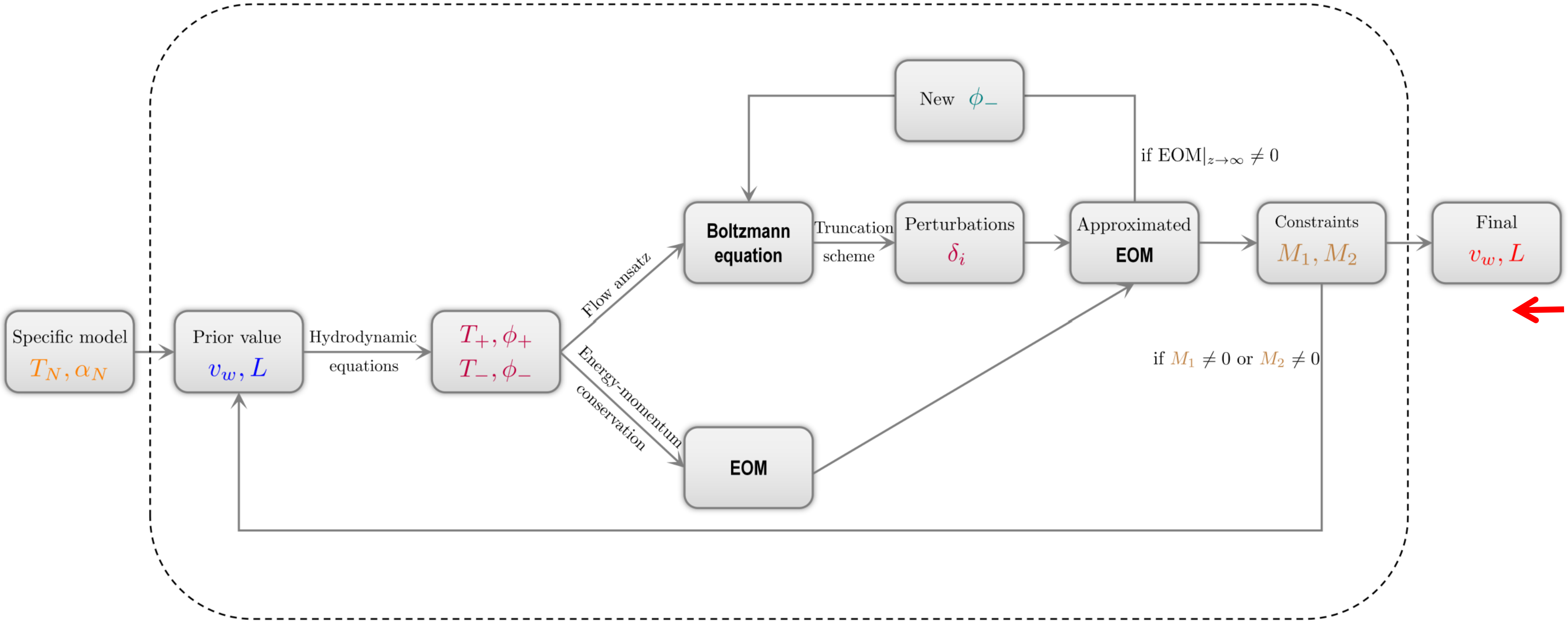
- For a specific benchmark point, the results of beyond leading-log approximation with the contribution of Higgs boson

	$\Lambda/\sqrt{\kappa}$ [GeV]	v_w	$L_w T$	v_c/T_c	T_n [GeV]
Two-particle	780	0.3382	20.1863	1.1044	100.977
Three-particle	780	0.2499	18.1759	1.1044	100.977

arXiv:2011.12903, Xiao wang, FPH, Xinmin Zhang

Phase transition dynamics

A simple DM Model: **Bubble wall velocity in inert doublet model**



← More accurate processes

Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005

Phase transition dynamics

$$V_0 = \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4 + \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \text{H.c.}\}\ ,$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h + v + iG^0) \end{pmatrix}, \quad \eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix},$$

$$V_{\text{CW}}(\phi, T=0) = \sum_i \frac{n_i}{64\pi^2}\left[m_i^4(\phi)\left(\ln\frac{m_i^2(\phi)}{\overline{m}_i^2} - \frac{3}{2}\right) + 2\overline{m}_i^2 m_i^2(\phi)\right],$$

$$V_{\text{T}}(\phi, T>0) = \sum_i n_i \frac{T^4}{2\pi^2} I_b\left(\frac{M_i^2}{T^2}\right),$$

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_{\text{CW}}(\phi) + V_{\text{T}}(\phi, T)\ .$$

Phase transition dynamics

$$f=\frac{1}{e^{(E+\delta)/T}\pm 1}$$

$$(-f_0')\left(\frac{p_z}{E}\left[\partial_z\mu+\frac{E}{T}\partial_z\left(\delta T+\delta T_{bg}\right)+p_z\partial_z\left(\delta v+\delta v_{bg}\right)\right]+\partial_t\mu\right.$$
$$\left.+\frac{E}{T}\partial_t\left(\delta T+\delta T_{bg}\right)+p_z\partial_t\left(\delta v+\delta v_{bg}\right)\right)+TC[\mu,\delta T,\delta v]=(-f_0')\frac{\partial_t\left(m^2\right)}{2E}$$

truncation scheme →

$$\hat{A}\delta'+\Gamma\delta=\Sigma,$$

source term

collision term

$$\delta=(\mu_t,\delta T_t,T\delta v_t,\mu_W,\delta T_W,T\delta v_W,\mu_A,\delta T_A,T\delta v_A),$$

$$\Sigma=\frac{v_w}{2T}\left(c_1^t(m_t^2)',c_2^t(m_t^2)',0,c_1^W(m_W^2)',c_2^W(m_W^2)',0,c_1^A(m_A^2)',c_2^A(m_A^2)',0\right),$$

$$\hat{A}=\begin{pmatrix}\hat{A}_t & 0 & 0\\ 0 & \hat{A}_W & 0\\ 0 & 0 & \hat{A}_A\end{pmatrix},\quad \text{where}\quad \hat{A}_i=\begin{pmatrix}v_wc_2^i & v_wc_3^i & \frac{1}{3}c_3^i\\ v_wc_3^i & v_wc_4^i & \frac{1}{3}c_4^i\\ \frac{1}{3}c_3^i & \frac{1}{3}c_4^i & \frac{1}{3}v_wc_4^i\end{pmatrix},$$

Phase transition dynamics

Collision terms (Monte Carlo integration)

$$\Gamma_{\mu 1,t} \simeq \left(5.0 \times 10^{-4} g_s^4 + 5.8 \times 10^{-4} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{T1,t} \simeq \Gamma_{\mu 2,t} \simeq \left(1.1 \times 10^{-3} g_s^4 + 1.3 \times 10^{-3} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{T2,t} \simeq \left(1.1 \times 10^{-2} g_s^4 + 4.0 \times 10^{-3} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{v,t} \simeq \left(2.0 \times 10^{-2} g_s^4 + 1.8 \times 10^{-3} g_s^2 y_t^2\right) T \,,$$

$$\Gamma_{\mu 1,W} \simeq \left(2.3 \times 10^{-3} g_s^2 g_w^2 + 2.0 \times 10^{-3} g_w^4\right) T \,,$$

$$\Gamma_{T1,W} \simeq \Gamma_{\mu 2,W} \simeq \left(4.7 \times 10^{-3} g_s^2 g_w^2 + 4.1 \times 10^{-3} g_w^4\right) T$$

$$\Gamma_{T2,W} \simeq \left(1.5 \times 10^{-2} g_s^2 g_w^2 + 1.5 \times 10^{-2} g_w^4\right) T \,,$$

$$\Gamma_{v,W} \simeq \left(5.7 \times 10^{-2} g_s^2 g_w^2 + 1.5 \times 10^{-2} g_w^4\right) T \,,$$

$$\Gamma_{\mu 1,A} \simeq 1.0 \times 10^{-2} \lambda_3^4 T \,,$$

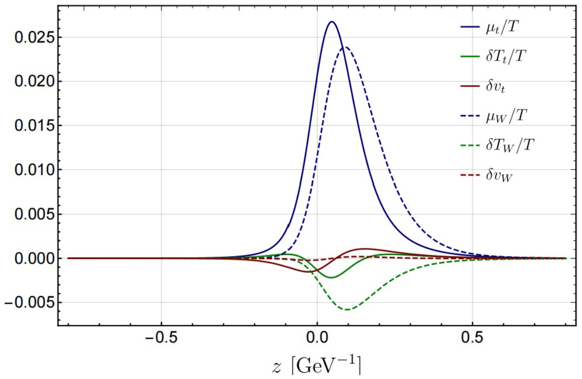
$$\Gamma_{T1,A} \simeq \Gamma_{\mu 2,A} \simeq 4.9 \times 10^{-3} \lambda_3^4 T \,,$$

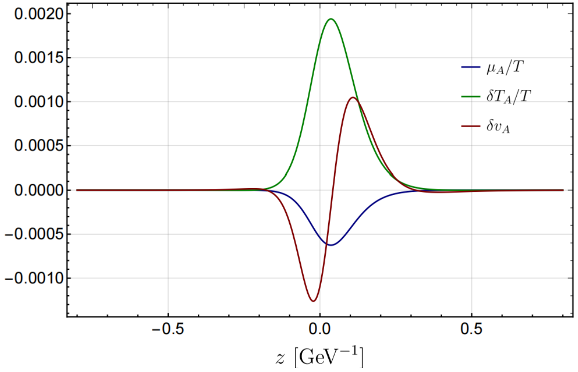
$$\Gamma_{T2,A} \simeq 5.1 \times 10^{-3} \lambda_3^4 T \,,$$

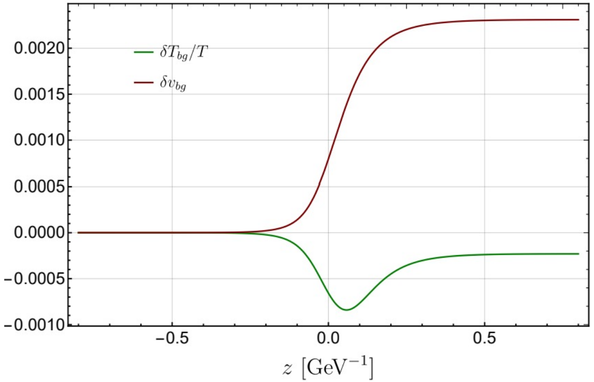
$$\Gamma_{v,A} \simeq 1.8 \times 10^{-3} \lambda_3^4 T \,.$$

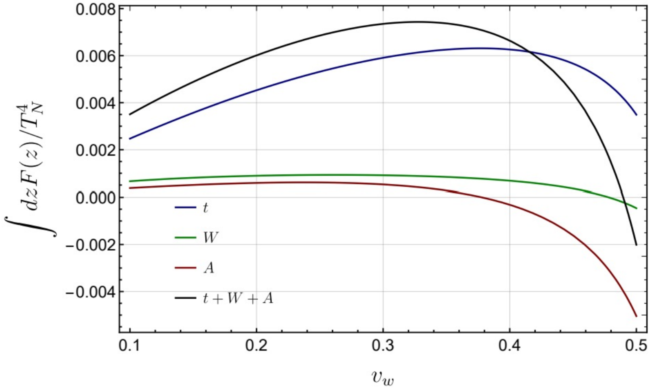
Phase transition dynamics

Solving perturbation equations:
Green Function Method







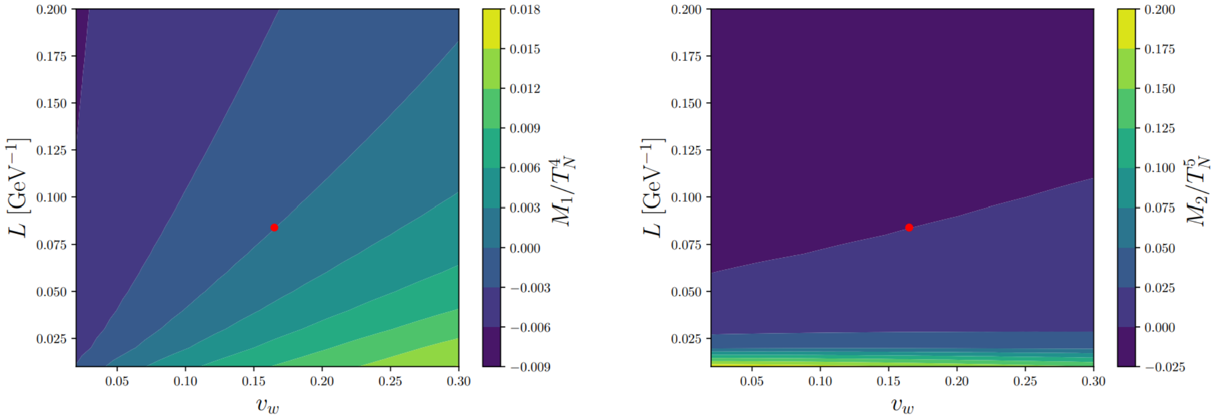


Phase transition dynamics

$$S_{\rm EOM} \equiv \left(1 - v_w^2\right)\phi'' + \frac{\partial V_{\rm eff}\left(\phi, T_+\right)}{\partial \phi} + \frac{N_t T_+}{2}\frac{dm_t^2}{d\phi} \times \left(c_1^t \mu_t + c_2^t\left(\delta T_t + \delta T_{bg}\right)\right)$$
$$+ \sum_b \frac{N_b T_+}{2}\frac{dm_b^2}{d\phi}\left(c_1^b \mu_b + c_2^b\left(\delta T_b + \delta T_{bg}\right)\right) = 0\ ,$$

Solving the EOM

$$M_1 = \int S_{\rm EOM}\phi' dz = 0, \quad M_2 = \int S_{\rm EOM}(2\phi - \phi_-)\phi' dz = 0\ .$$



Phase transition dynamics

	T_c [GeV]	T_N [GeV]	v_w	L [GeV^{-1}]
Benchmark A	118.3	117.1	0.165	0.084
Benchmark B	118.6	117.5	0.164	0.085
Benchmark C	119.4	118.4	0.164	0.088

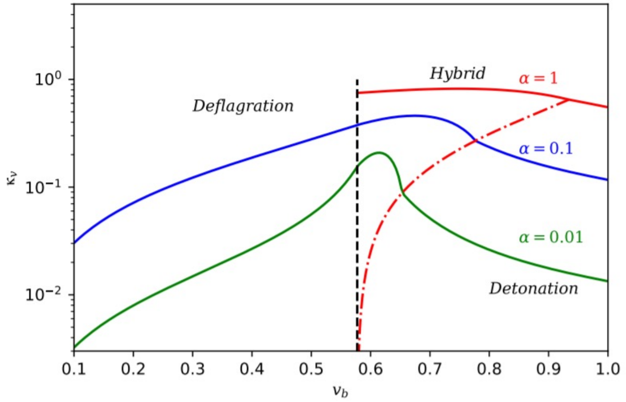
In the allowed parameter spaces, the bubble wall velocity varies slightly around 0.165.

The basic procedure in this work can also be used for any other SFOPT model.

Phase transition dynamics

Energy budget (to measure the efficiency of the energy released by a SFOPT converting to the kinetic energy of sounding plasma)

- GW signals generated during a SFOPT are directly related to efficiency parameter: $h\Omega_{GW} \propto \kappa^2$ or $\kappa^{\frac{3}{2}}$;
- Most of current studies of the efficiency parameter are based on bag EoS, which assume the sound velocity is $1/\sqrt{3}$ in both phases. But for a realistic SFOPT, particle can obtain the mass, hence, the sound velocity can deviate from pure radiation phase.



LISA Cosmology working group *JCAP* 03 (2020) 024

New trend: the energy budget/efficiency parameter beyond the bag model:

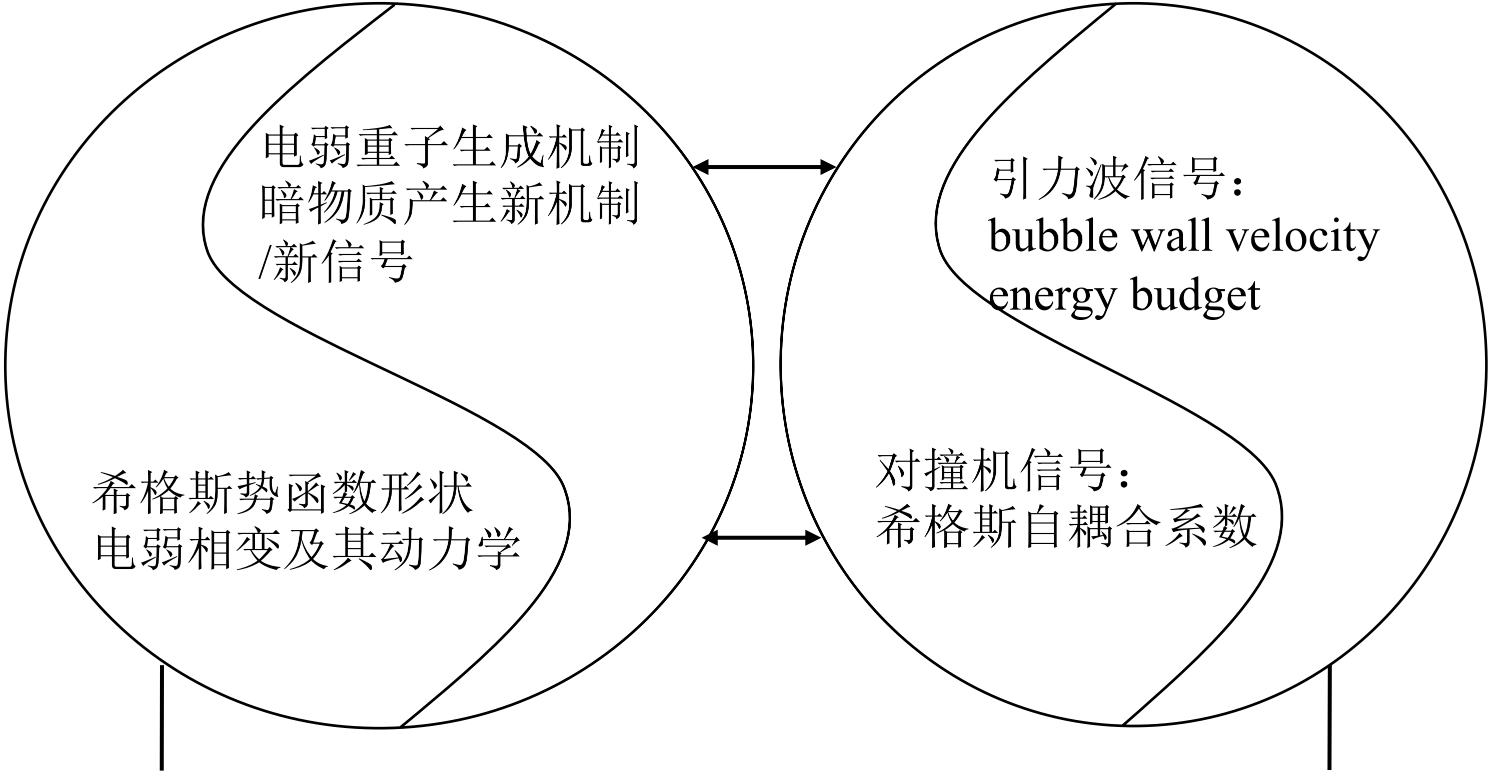
Xiao Wang, FPH and Xinmin Zhang, PRD 103 (2021) 10, 103520
Xiao Wang, Chi Tian, FPH, JCAP 07 (2023) 006

J. R. Espinosa, T. Konstandin, J. M. No, and G. Servant, JCAP 06 (2010) 028.

Xiao Wang, **FPH** and Xinmin Zhang, JCAP 05, 045 (2020)

Summary

Post Higgs Era: Higgs particle cosmology



(有限温度)量子场论、相对论流体力学、广义相对论

Summary and outlook

- **EW first-order phase transition has abundant collider and cosmological effects in baryogenesis, DM, GW…**

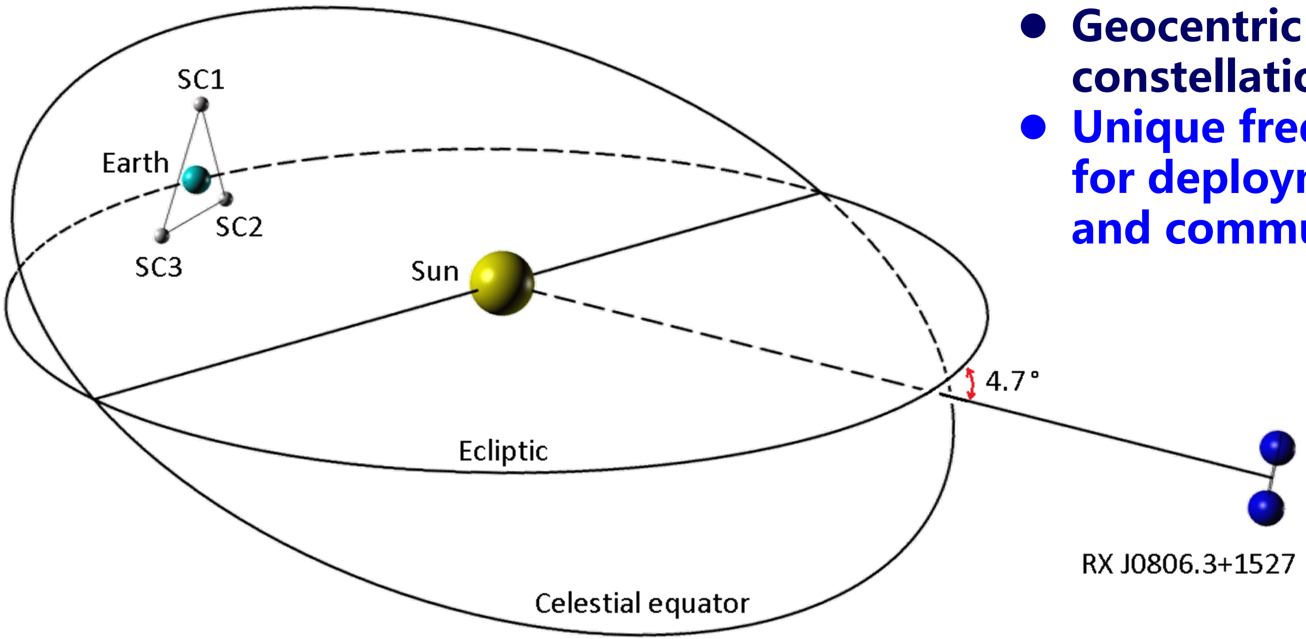
- **The correlation between GW and collider signals at CEPC can make complementary test on the Higgs nature, baryogenesis, DM and the cosmic evolution history at 100 GeV.**

- **More precise study: reliable resummation, non-perturbative calculations, bubble dynamics (wall velocity, energy budget).**

Thanks! Comments and collaborations are welcome!

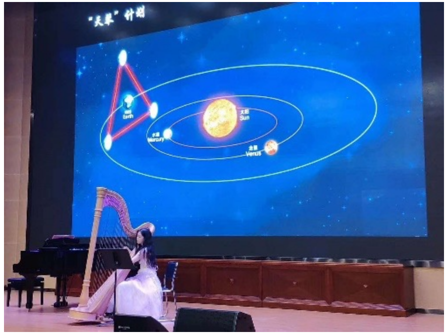
Email:huangfp8@sysu.edu.cn

Backup slides



J. Luo et al. TianQin: a space-borne gravitational wave detector, Class. Quant. Grav. 33 (2016) no.3, 035010.

- **Expected in 2035**
- **Geocentric orbit, normal triangle constellation, radius ~10^5km**
- **Unique frequency band, easier for deployment, tracking, control, and communication**



"天琴"
"Harpe in space"

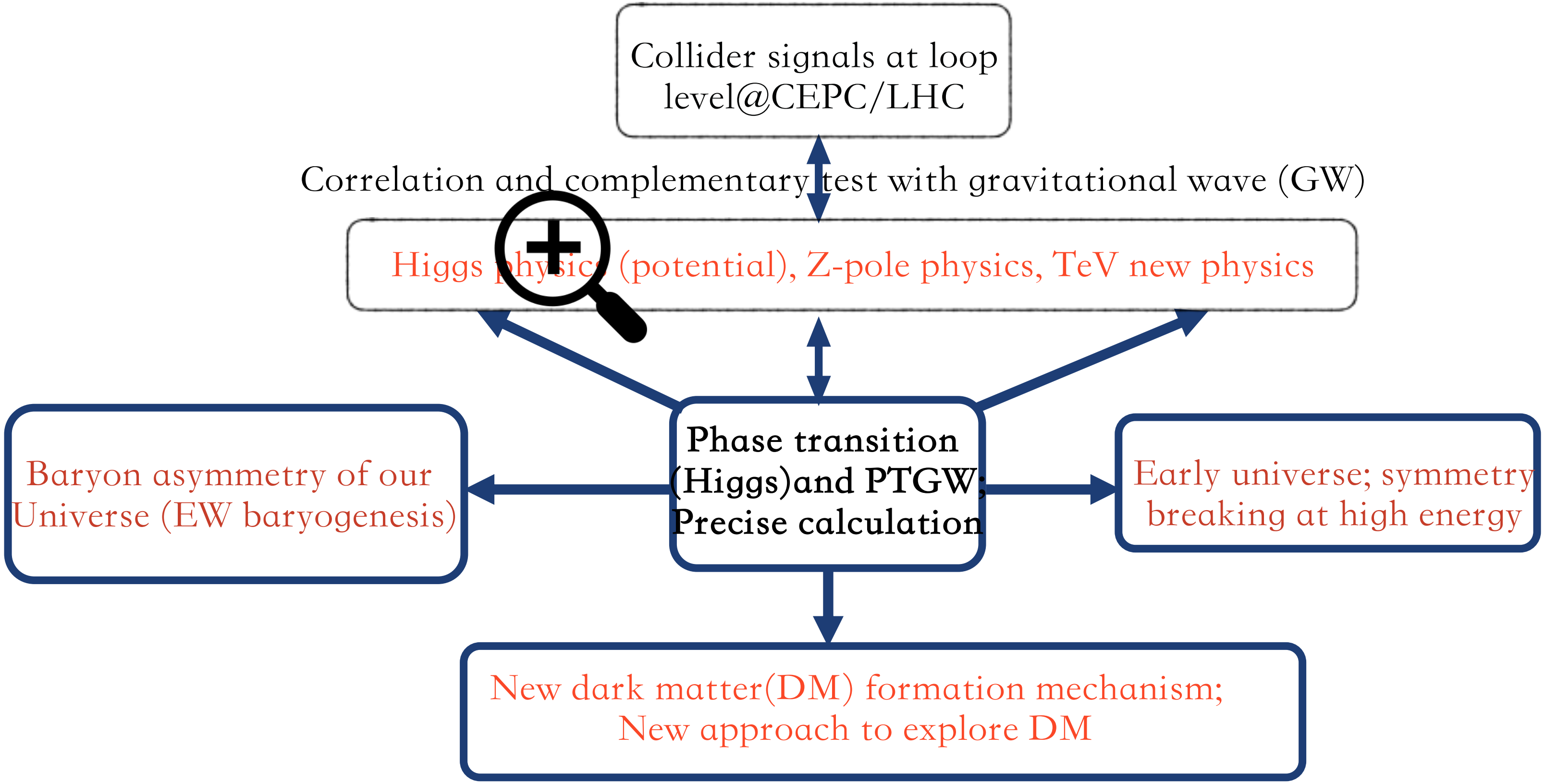
SFOPT and new DM mechanism/signal

Phase transition GW can provide a unique way to probe many important physics processes: inflation, PQ-symmetry breaking, neutrino physics, axion physics, extra dimension, primordial magnetic field, cosmic defects...

Wei-Chih Huang, et. al, arXiv: 2012.11614
Mark Hindmarsh, et. al. arXiv: 2011.12878
Bhupal Dev, et. al. arXiv: 1905.00891
Yiyang Zhang, et. al. arXiv:1902.02751
Yang Bai, et. al. arXiv:1810.04360
Andrew Long, et. al. arXiv:1703.04902
Graciela Gelmini, et. al. arXiv:2009.01903
Stephen King, et.al. arXiv:2005.13549
Bhupal Dev, et.al. arXiv:1602.04203
Astrid Eichhorn et.al. arXiv:2010.00017
Yuefeng Di, et. al., arXiv: 2012.15625
Haipeng An,et.al. arXiv:2009.12381
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29,
Jia Liu, et.al. arXiv:2104.06421
Zhao Zhang, et. al. arXiv:2102.01588
Wei Liu，et.al. arXiv:2101.10469
Cheng-wei Chiang, et.al. arXiv:2012.14071
Ke-Pan Xie, et.al. arXiv:2011.04821
Ligong Bian，et.al. arxiv:1907.13589
Zhaofeng Kang,et.al. arXiv:2101.03795
Zhaofeng Kang, et. al. arXiv:2003.02465

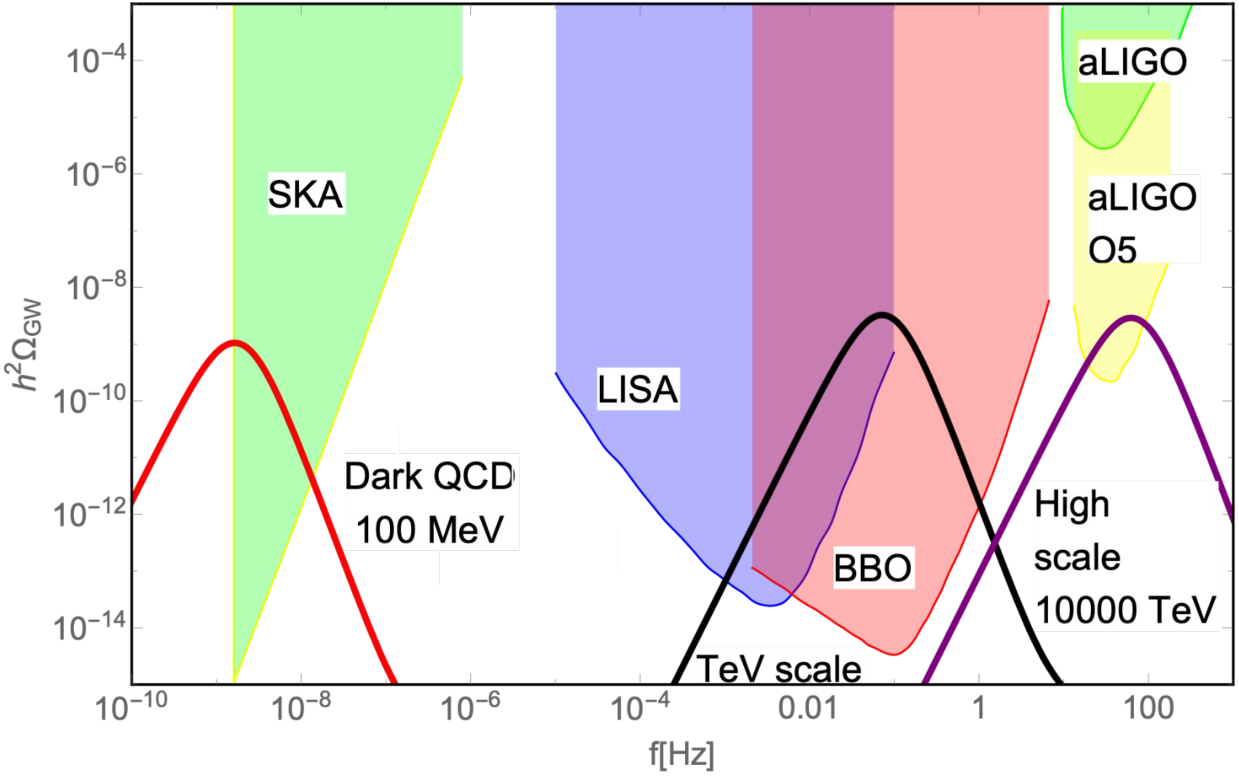
a lot of new and nice works unmentioned here

Motivation


Collider signals at loop level@CEPC/LHC
Correlation and complementary test with gravitational wave (GW)
Higgs physics (potential), Z-pole physics, TeV new physics
Baryon asymmetry of our Universe (EW baryogenesis)
Phase transition (Higgs)and PTGW; Precise calculation
Early universe; symmetry breaking at high energy
New dark matter(DM) formation mechanism; New approach to explore DM

SFOPT and new physics/early universe

Generally



FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29,

Phase transition dynamics

Energy budget for phase transition GW

Bag EoS

$$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \,,$$

$$p_- = \frac{1}{3}a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \,.$$

$$\alpha_\theta = \frac{4}{3}\frac{\Delta\epsilon}{w_+}, \quad \epsilon_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$$

Strength parameter

$$\partial p/\partial e = c_s^2 = \text{constant}$$

EoS with different sound velocity (DSVM)

$$p_+ = c_+^2 a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \,,$$

$$p_- = c_-^2 a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \,,$$

$$\alpha_{\bar\theta} = \frac{\Delta\bar\theta}{3w_+}, \quad \bar\theta = e - p/c_-^2$$

Strength parameter

F. Giese, T. Konstandin, and J. van de Vis, JCAP. 07 (2020) 057.

Phase transition dynamics

Energy budget for phase transition GW

- Matching condition

$$w_- v_-^2 \gamma_-^2 + p_- = w_+ v_+^2 \gamma_+^2 + p_+ ,$$
$$w_- v_- \gamma_-^2 = w_+ v_+ \gamma_+^2 .$$

$$\longrightarrow \quad v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-} , \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-} .$$

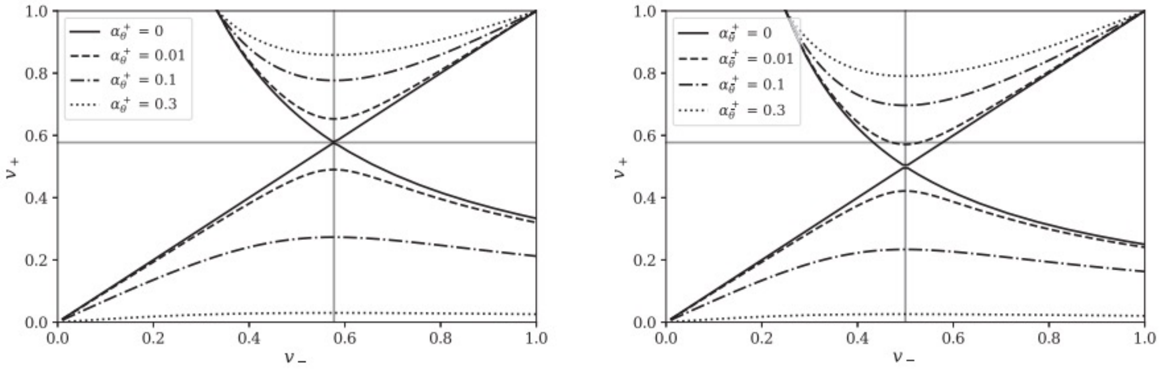


FIG. 1. The fluid velocities v_+ and v_- in the reference frame of bubble wall for different definitions and values of phase transition strength parameter. The horizontal and vertical gray lines indicate the sound velocities of symmetric and broken phase. Left panel: the bag model. Right panel: the DSVM with $c_+^2 = 1/3$ and $c_-^2 = 0.25$.

X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase transition dynamics

Energy budget for phase transition GW

- Energy momentum conservation derive fluid equation:

$$(\xi - v)\frac{\partial_\xi e}{w} = 2\frac{v}{\xi} + \gamma^2(1 - v\xi)\partial_\xi v \ ,$$

$$(1 - v\xi)\frac{\partial_\xi p}{w} = \gamma^2(\xi - v)\partial_\xi v \ .$$

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v$$

Velocity profile

$$\frac{\partial_\xi w}{w} = \left(1 + \frac{1}{c_s^2}\right)\mu\gamma^2\partial_\xi v$$

Enthalpy profile

$$\frac{\partial_\xi T}{T} = \gamma^2\mu\partial_\xi v$$

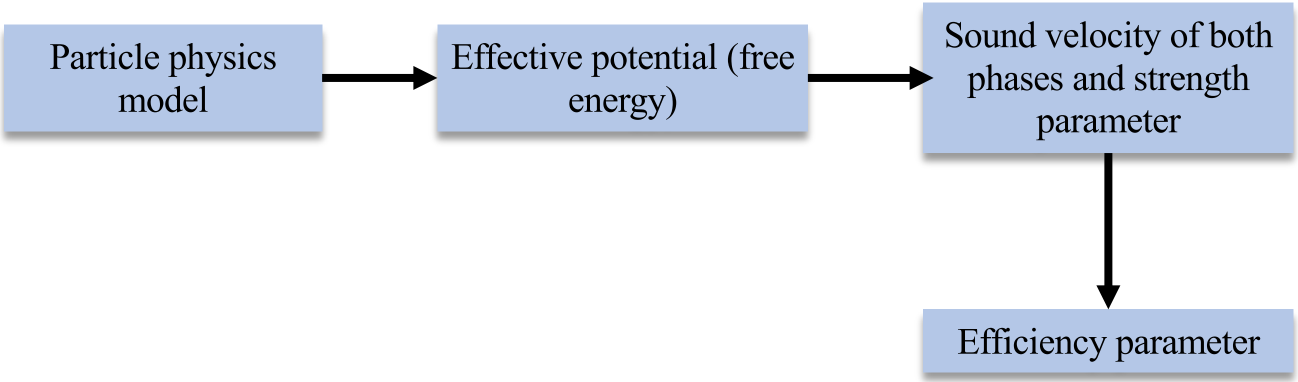
Temperature profile

Different boundary conditions give different hydrodynamical modes.

Phase transition dynamics

Energy budget for phase transition GW

- The method to map a particle physics model on the DSVM to get efficiency parameter

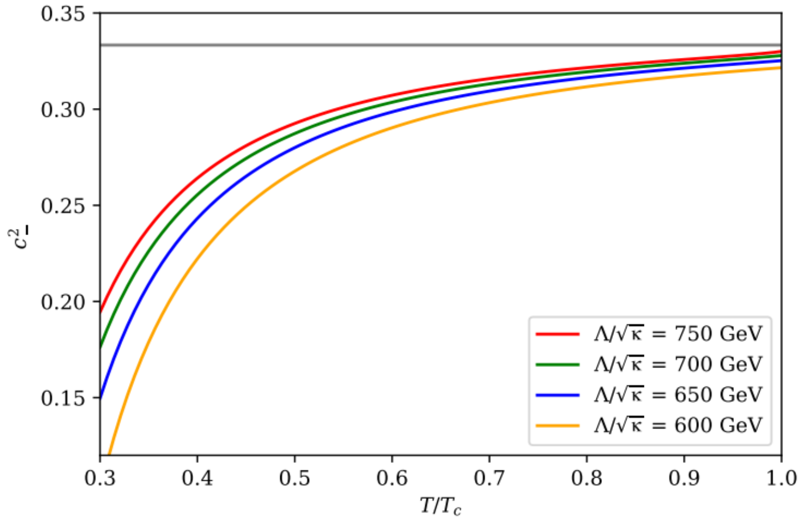


Phase transition dynamics

Energy budget for phase transition GW

The evolution of sound velocity of broken phase and symmetric phase in Dim-6 effective model:

$$\mathcal{F}(\phi, T) \approx -\frac{a_{\pm}}{3}T^4 + \frac{\mu^2 + cT^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\kappa}{8\Lambda^2}\phi^6$$

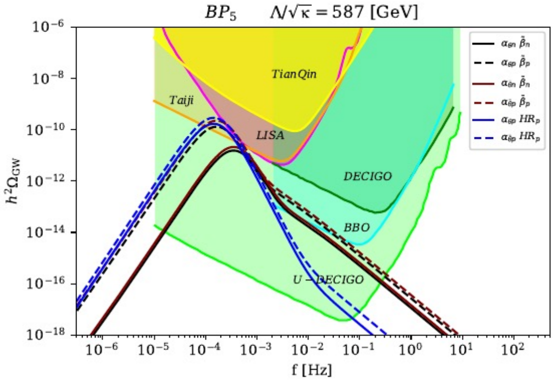


X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase transition dynamics

Energy budget for phase transition GW

GW spectrum and SNR for different EoS with different parameter combination:



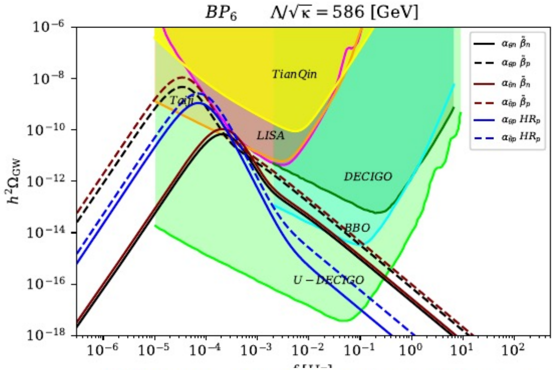


TABLE II. The SNR of BP_5 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde{\beta}_n$	$\alpha_{\theta p}\,\tilde{\beta}_p$	$\alpha_{\bar{\theta} n}\,\tilde{\beta}_n$	$\alpha_{\bar{\theta} p}\,\tilde{\beta}_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar{\theta} p}\,HR_p$
$SNR_{(LISA)}$	7.949	16.930	10.913	28.836	16.009	27.468
$SNR_{(Taiji)}$	14.760	58.607	20.271	100.343	66.216	113.609
$SNR_{(TianQin)}$	0.452	1.506	0.620	2.576	1.629	2.794

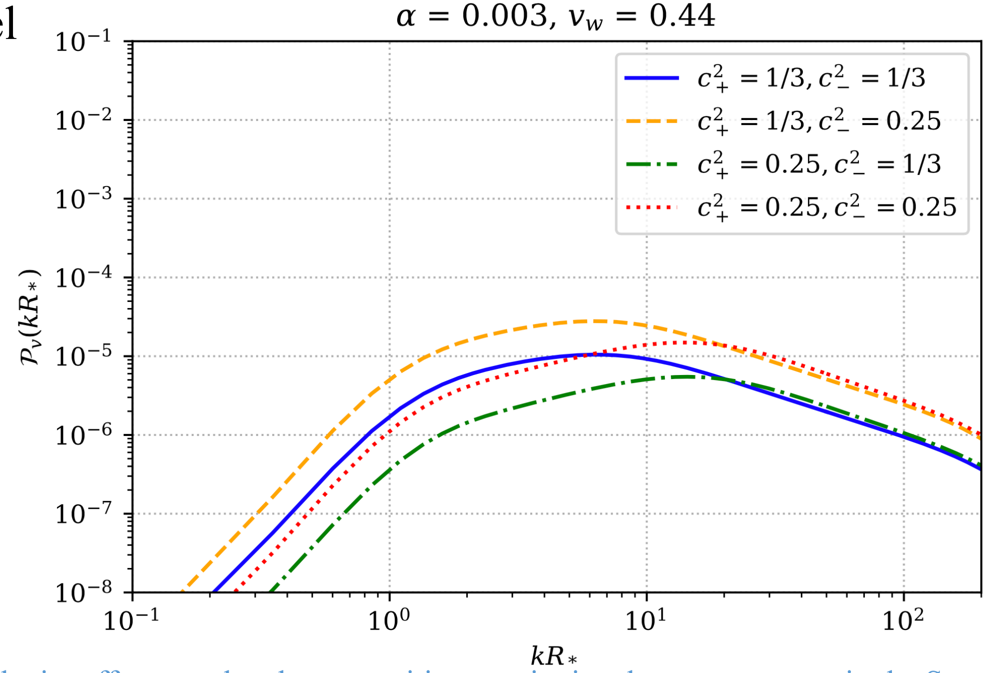
TABLE III. The SNR of BP_6 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde{\beta}_n$	$\alpha_{\theta p}\,\tilde{\beta}_p$	$\alpha_{\bar{\theta} n}\,\tilde{\beta}_n$	$\alpha_{\bar{\theta} p}\,\tilde{\beta}_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar{\theta} p}\,HR_p$
$SNR_{(LISA)}$	14.230	15.368	22.470	26.382	17.367	40.816
$SNR_{(Taiji)}$	38.666	427.813	61.208	1000.501	213.123	500.668
$SNR_{(TianQin)}$	1.060	5.569	1.678	12.934	3.973	9.333

X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

Phase Transition dynamics

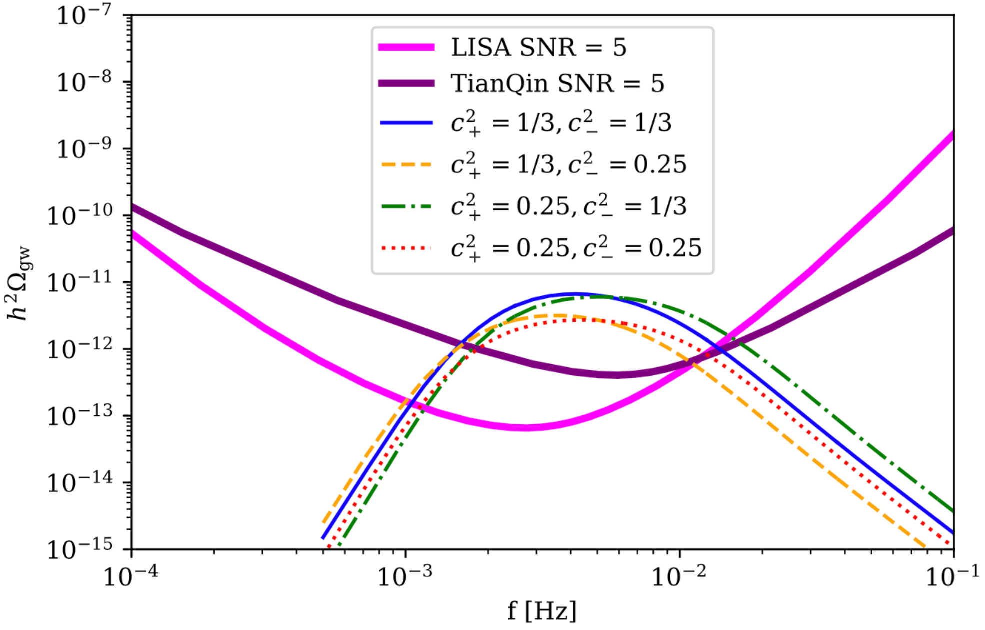
Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model



Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, arXiv:2112.14650

Xiao Wang, **FPH,** Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model ,Phys.Rev.D 103 (2021) 10, 103520

Phase transition dynamics



Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, arXiv:2112.14650

Xiao Wang, **FPH,** Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model ,Phys.Rev.D 103 (2021) 10, 103520

Anisotropy and primordial seeds

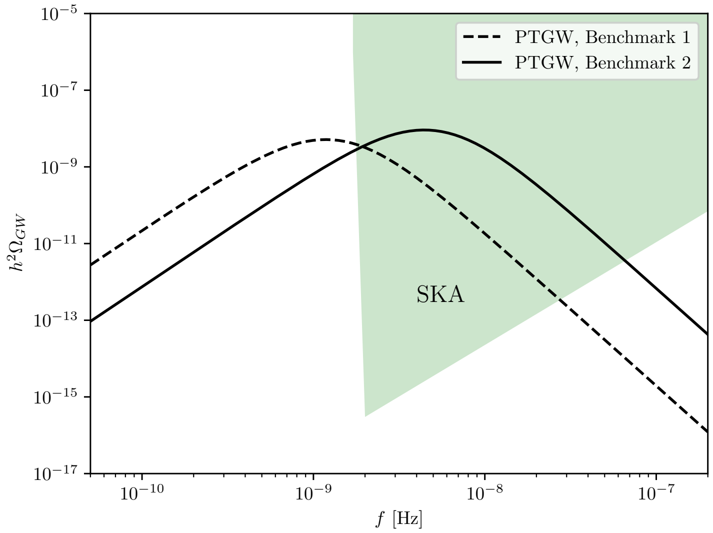
$$\boxed{H_*^2 = \rho/3M_{\rm pl}^2}$$

$$h^2\Omega_{\rm GW}(f) \simeq 1.64\times 10^{-6}\left(\frac{4}{3}\right)^{\frac{1}{2}}\boxed{(H_*R_*)^2}\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^{\frac{3}{2}}\times\left(\frac{100}{g_*}\right)^{\frac{1}{3}}(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{\frac{7}{2}}$$

$$f_{\rm sw}\simeq 2.6\times 10^{-5}\ {\rm Hz}\frac{1}{H_*R_*}\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$ Peak frequency

QCD-like DM model

	Benchmark 1	Benchmark 2
phase transition strength	$\alpha = 0.5$	
D.O.F	$g_* = 10$	
efficiency factor	$\kappa_v \approx 0.44$	
bubble wall velociy	$v_b = 0.95$	
characteristic temperature	$T_* = 1$ MeV	$T_* = 5$ MeV
mean bubble separation	$H_*R_* = 0.15$	$H_*R_* = 0.2$

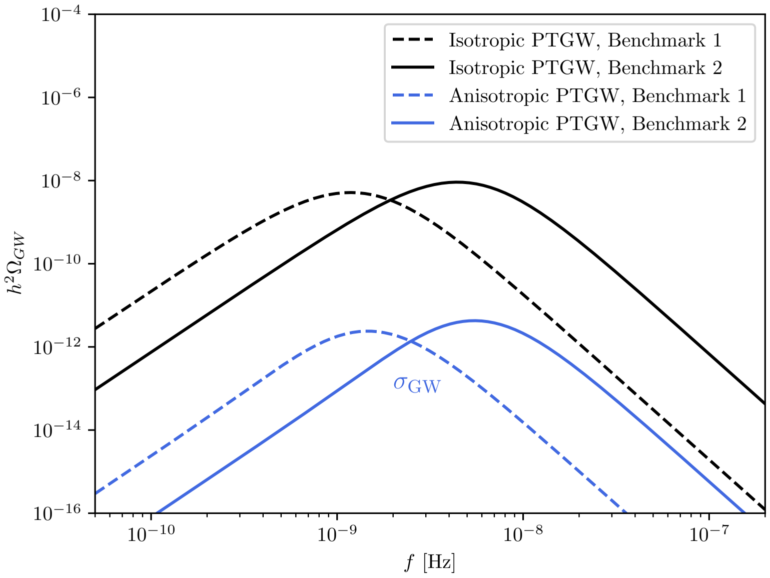


Anisotropy and primordial seeds

$$\mathrm{Var}^{\mathcal{G}} = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_{\ell}^{\mathcal{G}}$$

$$\sigma_{\mathrm{GW}}(p) \equiv h^2\Omega_{\mathrm{GW}}(p)\sqrt{\mathrm{Var}^{\delta_{\mathrm{GW}}}(p)}$$

CMB TT anisotropy	4×10^{-5}
PTGW anisotropy	1×10^{-4}
PTGW energy spectra anisotropy	8×10^{-4} $(> f_{\mathrm{sw}})$



Anisotropy and primordial seeds

$$\rho_{\rm GW}(\eta,\boldsymbol{x}) = \int d^3\boldsymbol{p}\, p f(\eta,\boldsymbol{x},\boldsymbol{p}) = \int dp d\hat{\boldsymbol{p}} p^3 f(\eta,\boldsymbol{x},p,\hat{\boldsymbol{p}})$$

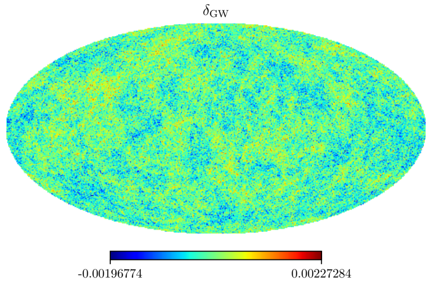
$$\Omega_{\rm GW}(\eta,\boldsymbol{x},p) = \int \frac{d\hat{\boldsymbol{p}}}{4\pi}\bar{\Omega}_{\rm GW}(\eta,p)\left[1+\delta_{\rm GW}(\eta,\boldsymbol{x},p,\hat{\boldsymbol{p}})\right]$$

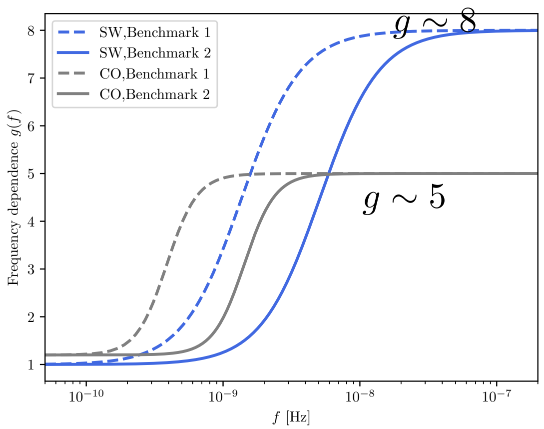
$$= \int d\hat{\boldsymbol{p}}\frac{p^4}{\rho_c}\left[\bar{f}(\eta,p) - p\frac{\partial \bar{f}(\eta,p)}{\partial p}\mathcal{G}(\eta,\boldsymbol{x},\hat{\boldsymbol{p}})\right]$$

anisotropy of GW energy spectra $\delta_{\rm GW} = \dfrac{\delta\Omega_{\rm GW}(\eta,\boldsymbol{x},p,\hat{p})}{\bar{\Omega}_{\rm GW}(\eta,p)}$

$$\delta_{\rm GW}(\eta,\boldsymbol{x},p,\hat{p}) = \left[4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}(\eta,p)}{\partial \ln p}\right]\mathcal{G}(\eta,\boldsymbol{x},\hat{p})$$

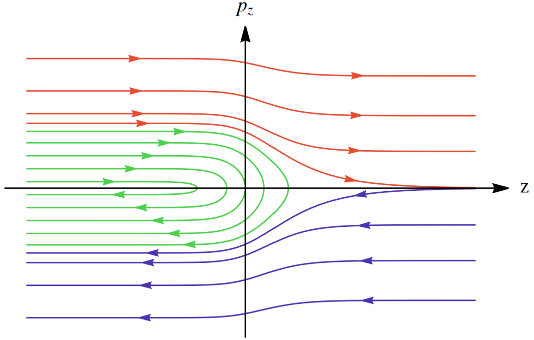
$$C_l^{\delta_{\rm GW}}(p) = g^2(p) C_l^{\mathcal{G}}$$





SFOPT and new DM mechanism/signal

$$p_z^2 + m_\chi^2 \equiv \text{const}$$



$$\frac{p_z}{m_\chi}\frac{\partial}{\partial z} - \left(\frac{\partial m_\chi}{\partial z}\right)\frac{\partial}{\partial p_z} \quad \rightarrow \quad \frac{p_z}{m_\chi}\frac{d}{dz}$$

$$\frac{d\mathcal{A}}{dz} = c(\mathcal{A}, p_z, z)$$

$$\mathcal{A}\left(z \ll -L_w, p_z > 0\right) = 1$$