中国科学院大学
University of Chinese Academy of Sciences
ICTP-AP
International Centre
for Theoretical Physics Asia-Pacific
国际理论物理中心-亚太地区

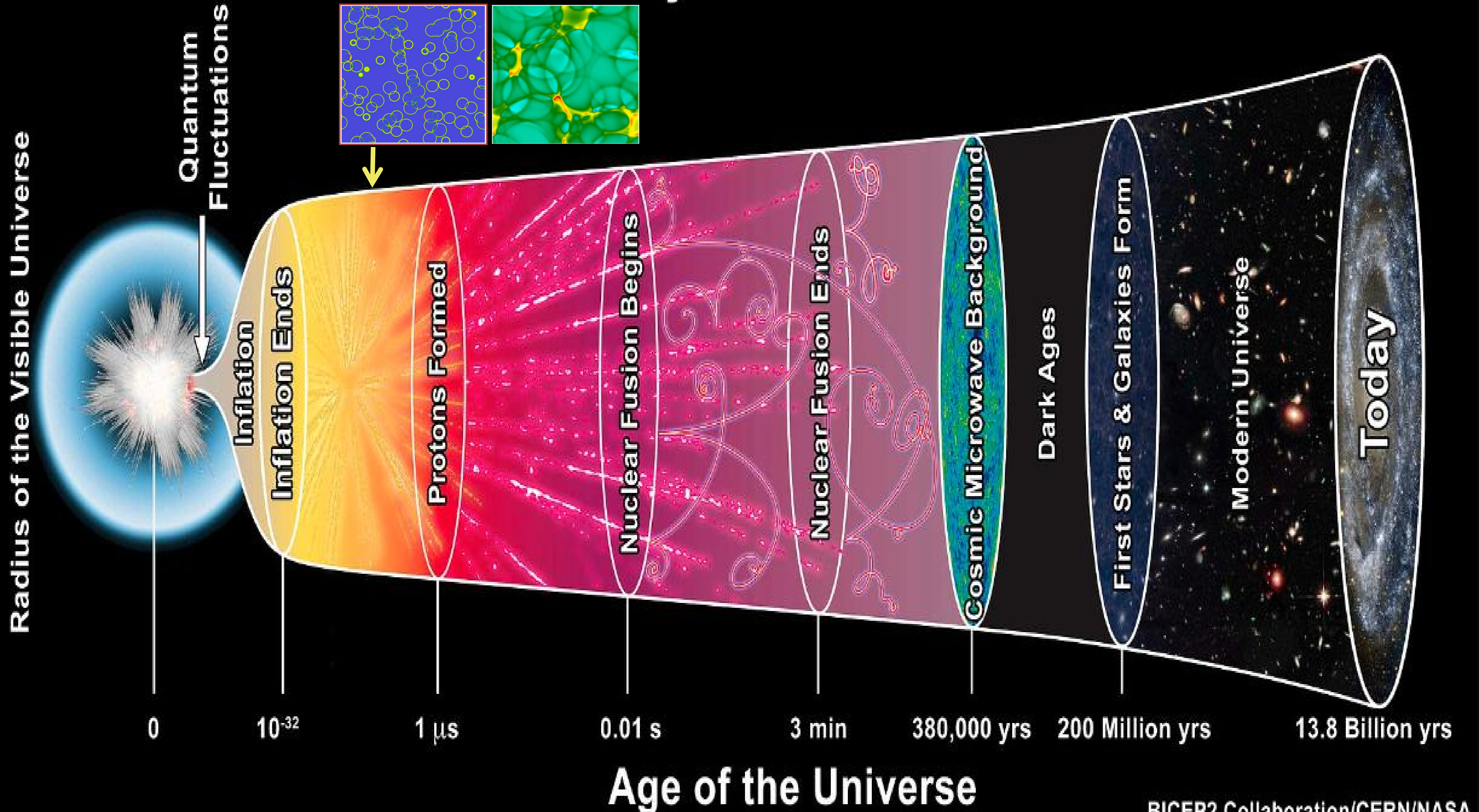
耗散效应作为宇宙相变的新观测量

郭怀珂

2024年1月20日

27th LHC Mini-workshop

Based on HG [2310.10927]



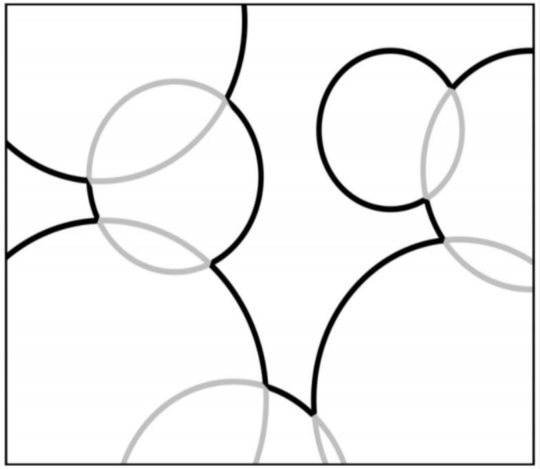
BICEP2 Collaboration/CERN/NASA

Gravitational Wave Sources

The current understanding:

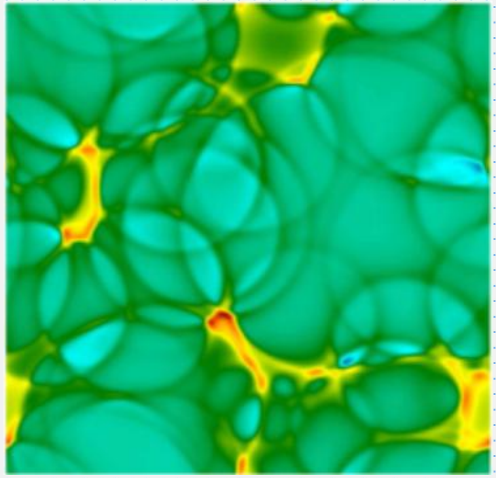
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

energy near the wall



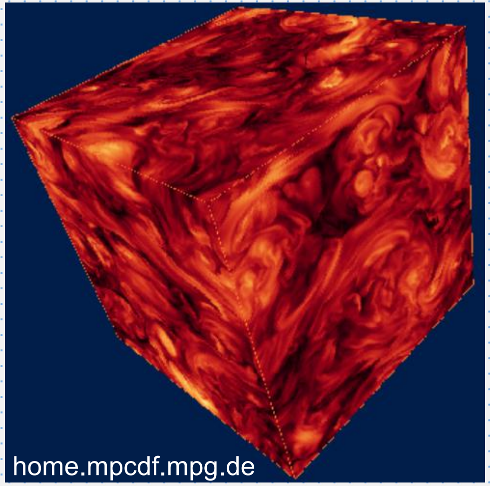
Bubble Collisions

fluid kinetic energy



Sound Waves

turbulent fluid + magnetic field



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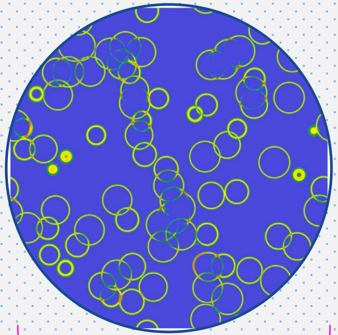
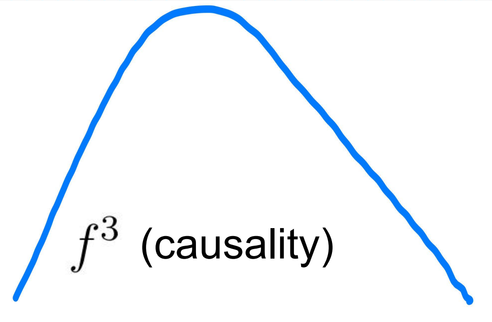
Magnetohydrodynamic Turbulence

Properties



Horizon size: 1/H*

$$f_{\text{now}} = 1.65\times10^{-5}\underbrace{\left(\frac{f_{\text{PT}}}{\beta}\right)\left(\frac{\beta}{H_*}\right)}_{\sim 100\text{-}1000}\left(\frac{T_*}{100\text{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\text{Hz}$$



Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

~mHz : (~100GeV) weak scale

~100Hz (~PeV - EeV) high scale

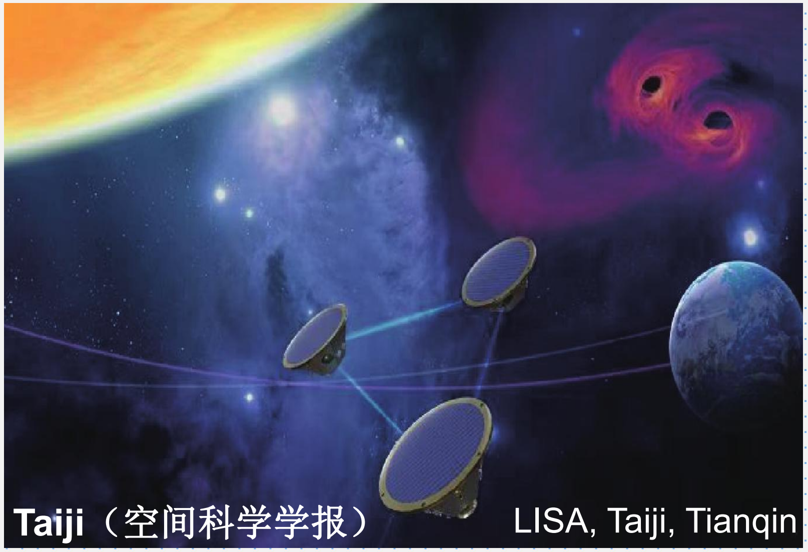
EPTA

NANOGrav
Physics Frontiers Center

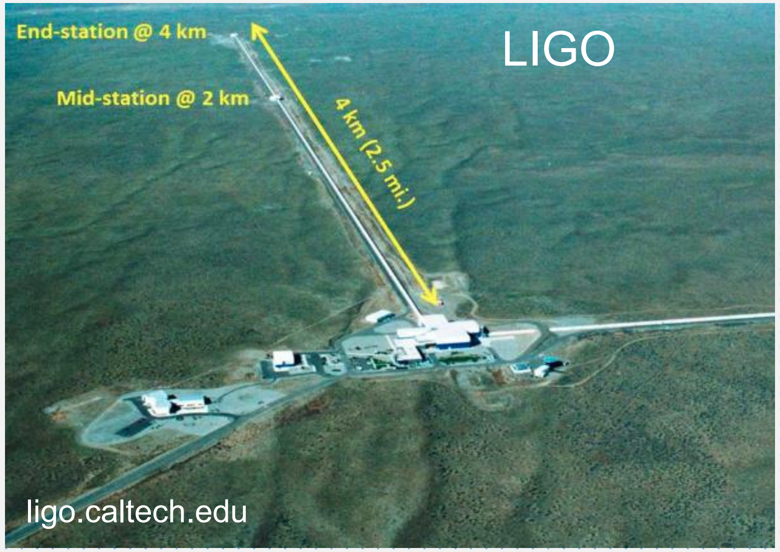
PPTA

InPTA
Indian Pulsar Timing Array

中国脉冲星测时阵列（CPTA）



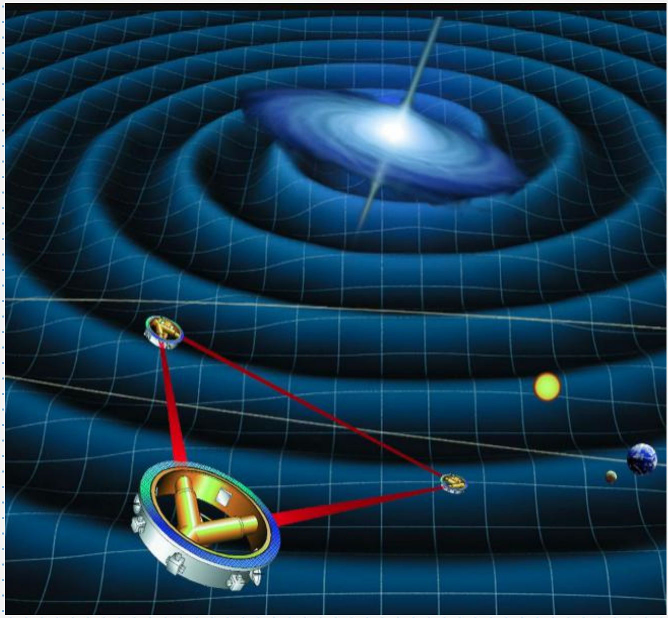
Taiji（空间科学学报） LISA, Taiji, Tianqin



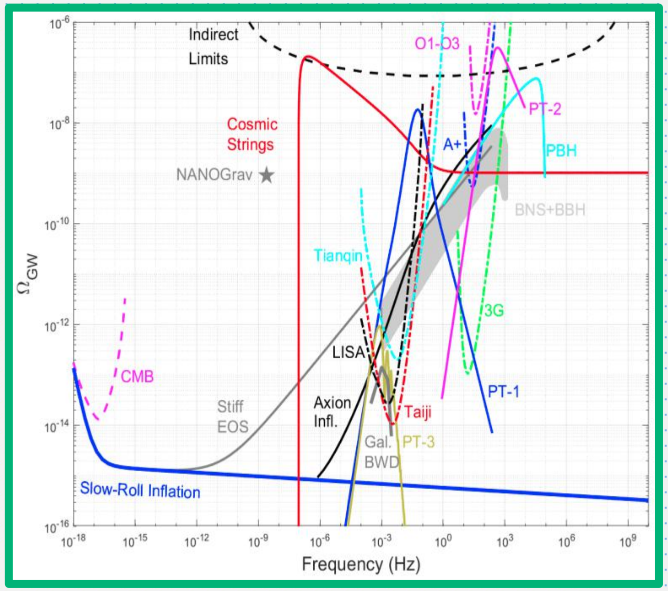
ligo.caltech.edu

From Theory to Experiment

theorist



LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum

α

β

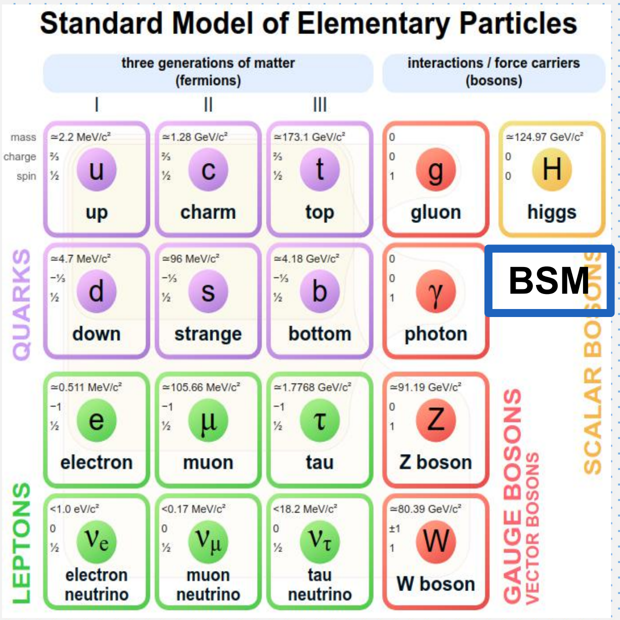
v_w

T_*

g_s

…

Phase Transition Parameters



Standard Model of Elementary Particles

three generations of matter (fermions)

interactions / force carriers (bosons)

I II III

mass / charge / spin

QUARKS: ≈2.2 MeV/c², ⅔, ½, u up; ≈1.28 GeV/c², ⅔, ½, c charm; ≈173.1 GeV/c², ⅔, ½, t top; ≈4.7 MeV/c², −⅓, ½, d down; ≈96 MeV/c², −⅓, ½, s strange; ≈4.18 GeV/c², −⅓, ½, b bottom

LEPTONS: ≈0.511 MeV/c², −1, ½, e electron; ≈105.66 MeV/c², −1, ½, μ muon; ≈1.7768 GeV/c², −1, ½, τ tau; <1.0 eV/c², 0, ½, ν_e electron neutrino; <0.17 MeV/c², 0, ½, ν_μ muon neutrino; <18.2 MeV/c², 0, ½, ν_τ tau neutrino

GAUGE BOSONS / VECTOR BOSONS: 0, 0, 1, g gluon; 0, 0, 1, γ photon; ≈91.19 GeV/c², 0, 1, Z boson; ≈80.39 GeV/c², ±1, 1, W boson

SCALAR BOSONS: ≈124.97 GeV/c², 0, 0, H higgs

BSM

Particle Physics Model

experimentalist

Problem: parameter degeneracy

Models	Strong 1st order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
SM charged				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
SM uncharged				
S_r (xSM) [37–49]	✓	✓	✗	✗
2 S_r's [50]	✓	✓	✓	✗
S_c (cxSM) [49, 51–54]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (no interaction with SM) [55]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (Higgs Portal) [56]	✓	✓	✓	
$\mathrm{U(1)_D}$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite SU(7)/SU(6) [58]	✓	✓	✓	
$\mathrm{U(1)_L}$ [59]	✓	✓	✓	✗
$\mathrm{SU(2)_D} \to$ global $SO(3)$ by a doublet [60–62]			✓	✗
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ by a triplet [63–65]			✓	✓
$\mathrm{SU(2)_D} \to Z_2$ by two triplets [66]			✓	✗
$\mathrm{SU(2)_D} \to Z_3$ by a quadruplet [67, 68]			✓	✗
$\mathrm{SU(2)_D} \times \mathrm{U(1)_{B\text{-}L}} \to Z_2 \times Z_2$ by a quintuplet and a S_c [69]			✓	✗
$\mathrm{SU(2)_D}$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$\mathrm{SU(3)_D} \to Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$\mathrm{SU(3)_D}$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times Z_2$ [74]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times G_{\mathrm{D,SM}} \cdots$ [75]	✓	✓	✓	
Current work				
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ (see the text)	✓	✓	✓	✓

Ghosh,HG,Han,Liu, JHEP [2012.09758]

Many models can lead to the same PT parameter values

Solutions: New Observables

- Anisotropy

  Geller, Hook, Sundrum, Yuhsin Tsai, PRL [1803.10780]
  Li, Huang, Wang, Zhang, PRD [2112.01409]
  Li, Yan, Huang, PRD [2211.03368]

- Primordial magnetic field

  Di,Wang,Zhou,Bian,Cai, PRL [2012.15625]
  Yang,Bian,PRD [2102.01398], ...

- Primordial black holes and solitons

  Hong, Jung, Xie, PRD [2008.04430]
  Kawana,Xie,PLB [2106.00111]
  Liu,Bian,Cai,Guo,Wang, PRD [2106.05637]
  Lu,Kawana,Xie, PRD [2202.03439]

- Curvature perturbations

  Liu,Bian,Cai,Guo,Wang,PRL[2208.14086]
  Jiang,Liu,Sun,Wang, PLB [1512.07538]

Anything directly readable from the isotropic GW spectrum?

Dissipative Effects as New Observables

GW depends on (large) bulk velocity of the system

$$h \sim 10^{-22} \frac{M/M_\odot}{r/100\mathrm{Mpc}} \left(\frac{\boxed{v}}{c}\right)^2$$

Dissipative effects dissipate away the bulk kinetic energy (leaves imprint)

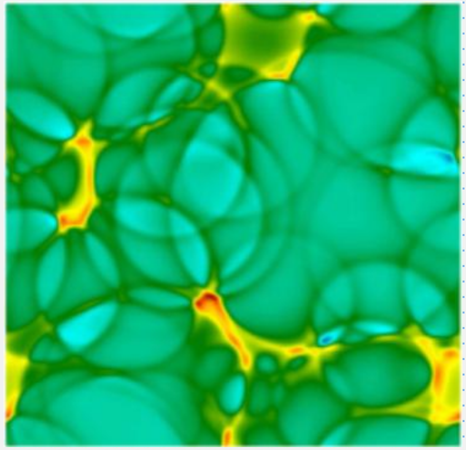
$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \boxed{\mu\nabla^2\mathbf{v} + (\zeta + \frac{1}{3}\mu)\nabla(\nabla\cdot\mathbf{v})}$$

Navier–Stokes equations (Newtonian fluid mechanics)

studiousguy.com

GW calculation requires: relativistic (magneto-)hydrodynamics

Sound Waves



Usually the dominant source (Hindmarsh,Huber,Rummukainen,Weir, PRL [1304.2433])

$$T^{ij} \propto (p+e)v^i v^j$$

$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$

$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2} \qquad f_* = \frac{2\beta}{\sqrt{3}v_w} \approx \frac{3.4}{R_*}$$

Hindmarsh,Huber,Rummukainen,Weir, PRD [1504.03291]

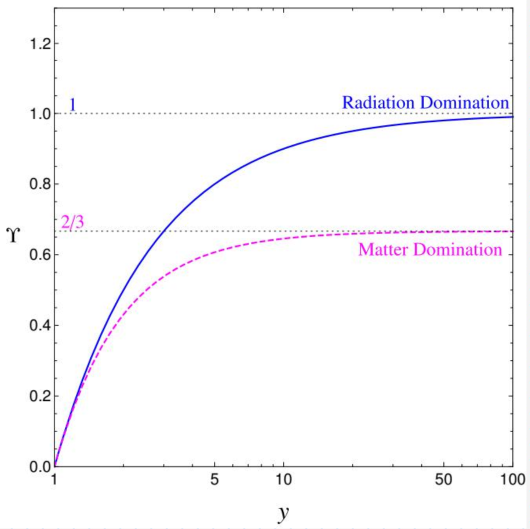
$$\Omega_{\rm SW}(f \gtrsim f_{\rm peak}) \propto f^{-4}$$

$$\Omega_{\rm SW}(f \lesssim f_{\rm peak}) \propto f^3$$

Slight different fit obtained by the same group, PRD [1704.05871]

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (radiation domination)

HG, Sinha, Vagie, White, JCAP [2007.08537]



Sound Waves: Recent Development

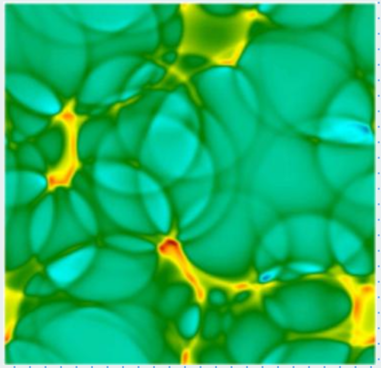
Analytical Modelling

- Refine the sound shell model
- Synergy with simulations

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
Pol, Procacci, Caprini [2308.12943]

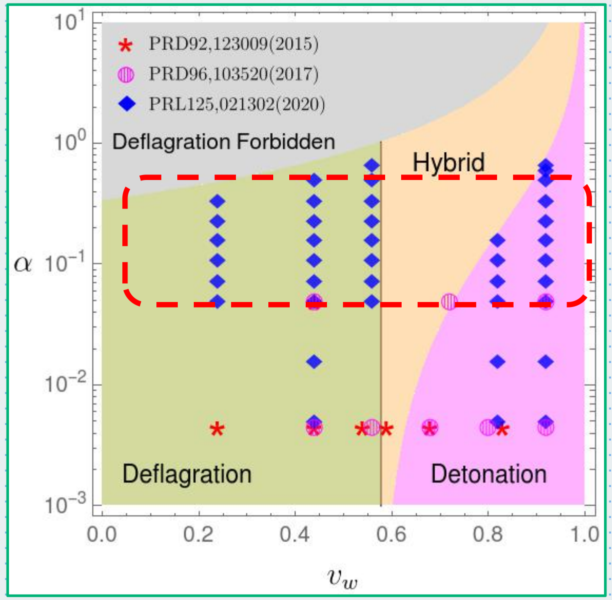
$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

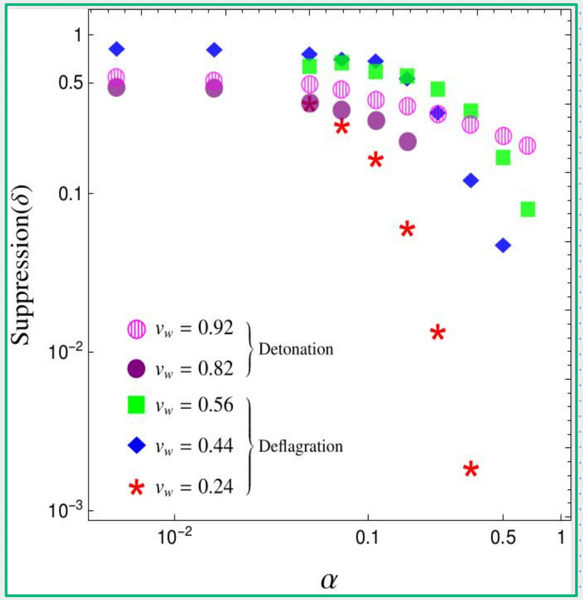


Numerical Simulation

- Suppression found for strong transitions with small vw
- Need to cover more parameter space (very strong PT)

$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$





Cutting, Hindmarsh, Weir, PRL [1906.00480]

All based on perfect fluid approximation

Sound Waves: Modelling

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
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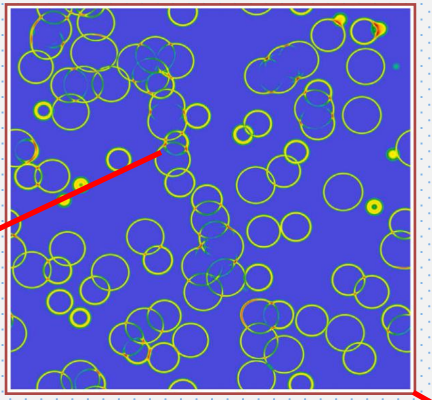
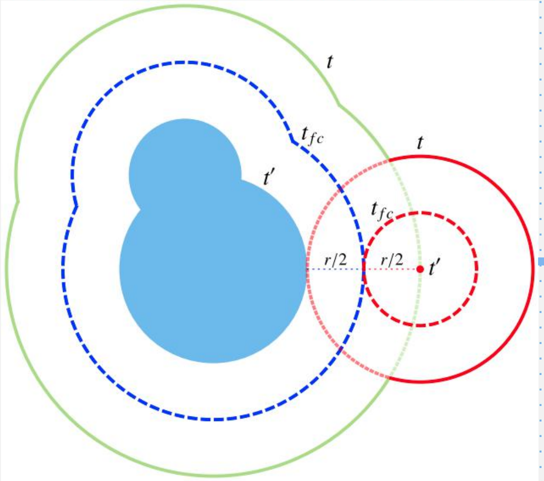


$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\left[v^i_{\mathbf{q}} e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + v^{i*}_{\mathbf{q}} e^{i\omega\eta - i\mathbf{q}\cdot\mathbf{x}}\right]$$



linear superposition
(core of SSM)

$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

forced fluid motion

freely propagating sound

$$\left.\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k}\right|_{\mathrm{SW}} = \left.\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k}\right|_{\mathrm{SW}}^{\mathrm{forced}} + \left.\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k}\right|_{\mathrm{SW}}^{\mathrm{free}}.$$

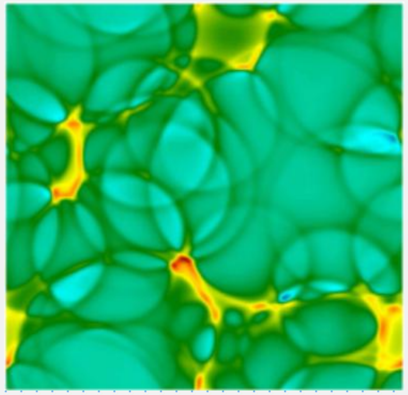
Cai, Wang, Yuwen, PRD Letter [2305.00074]

Neglect possible forced motion in the following.

Effects of Dissipation

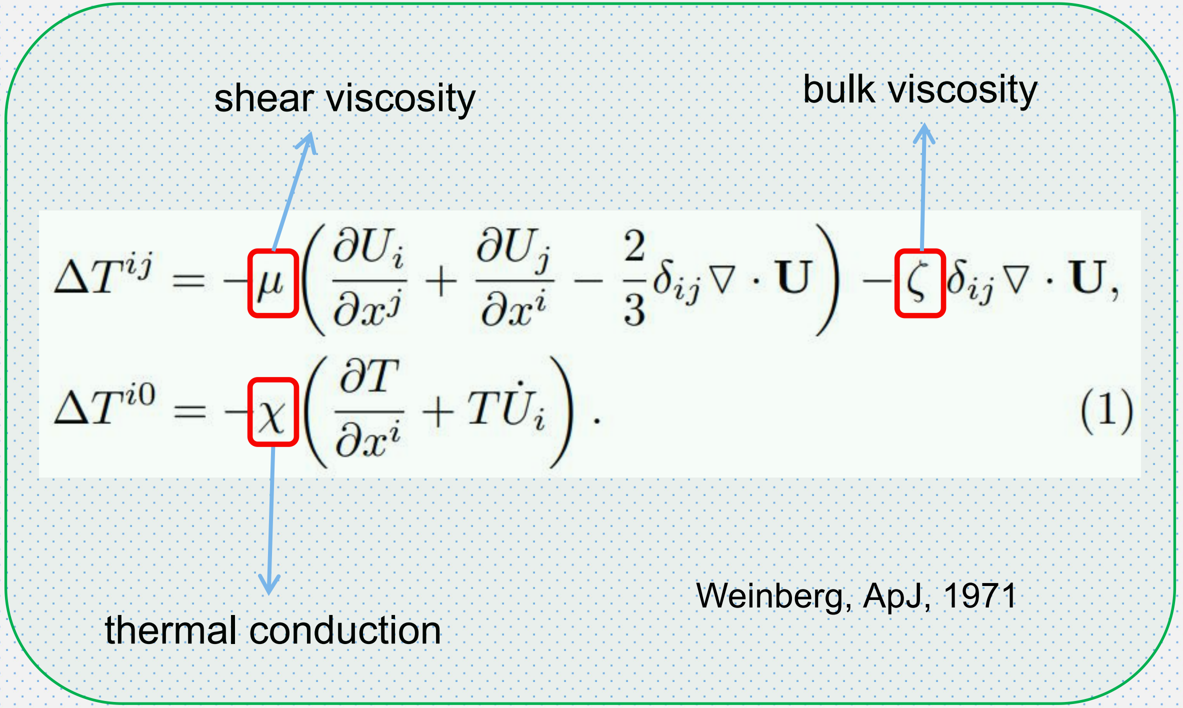
- Disturbed fluid comes into rest eventually

$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\left[v^i_{\mathbf{q}} e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + c.c.\right]$$



$$v^i_{\mathbf{q}}(\eta) \propto \exp\left[-\int \Gamma(\mu, \zeta, \xi)\, d\eta\right]$$

$$\boxed{\Gamma \propto q^2}$$



shear viscosity

bulk viscosity

$$\Delta T^{ij} = -\boxed{\mu}\left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{U}\right) - \boxed{\zeta}\,\delta_{ij}\nabla\cdot\mathbf{U},$$

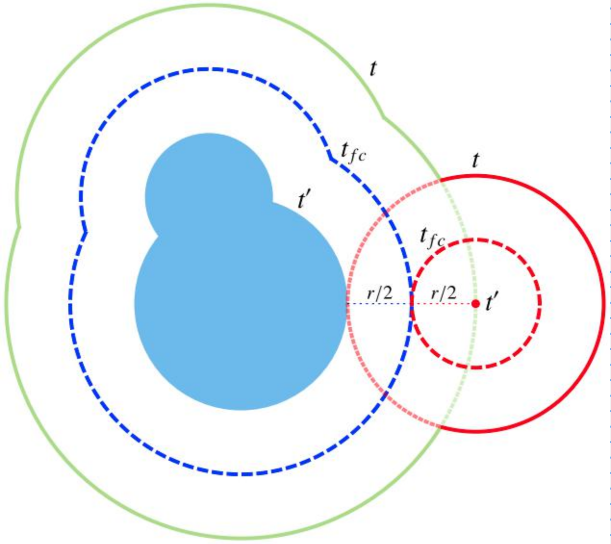
$$\Delta T^{i0} = -\boxed{\chi}\left(\frac{\partial T}{\partial x^i} + T\dot{U}_i\right). \qquad (1)$$

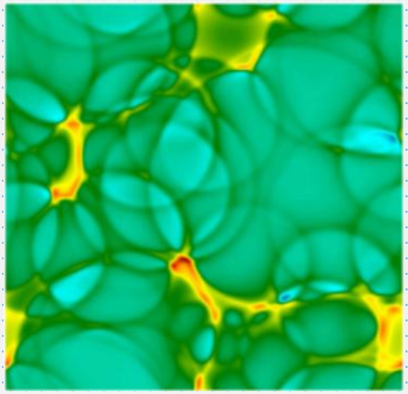
thermal conduction

Weinberg, ApJ, 1971

Euler equation -> Navier–Stokes equations
-> Relativistic hydrodynamics

Sound Shell Model with Dissipation





$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

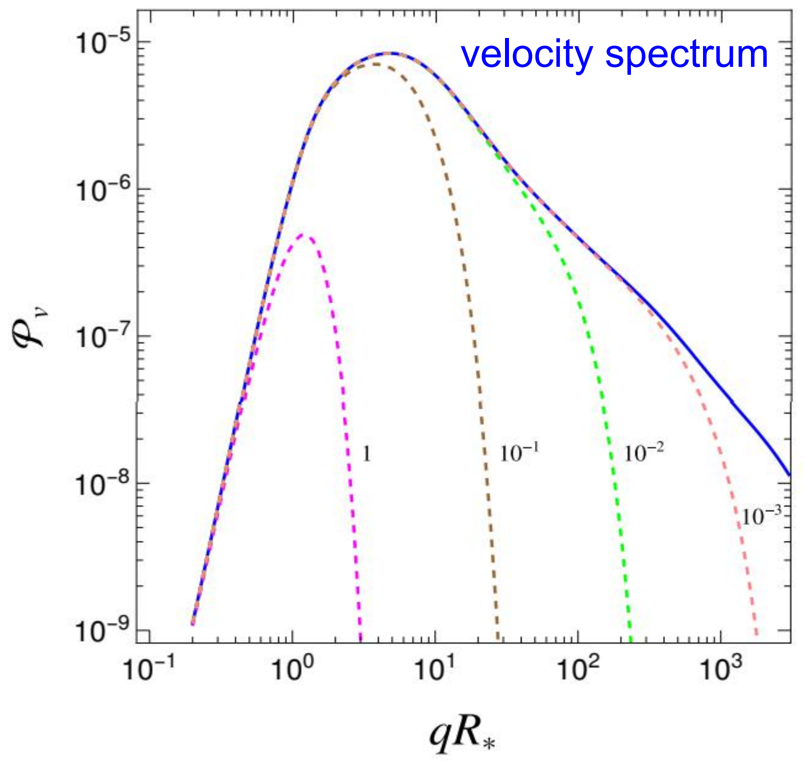
$$v^i_{\mathbf{q}}(\eta) = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}} \boxed{\exp\left[-\int_{\eta_d^{(n)}}^{\eta} \Gamma d\bar{\eta}\right]} \theta(\eta - \eta_d^{(n)})$$

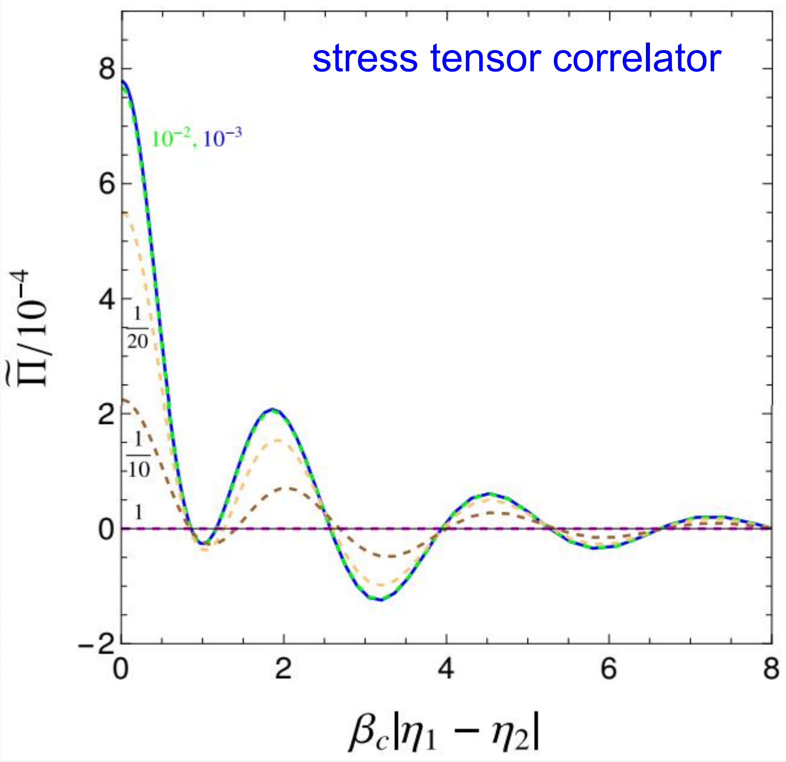
bubble destruction time
(when SW forms)

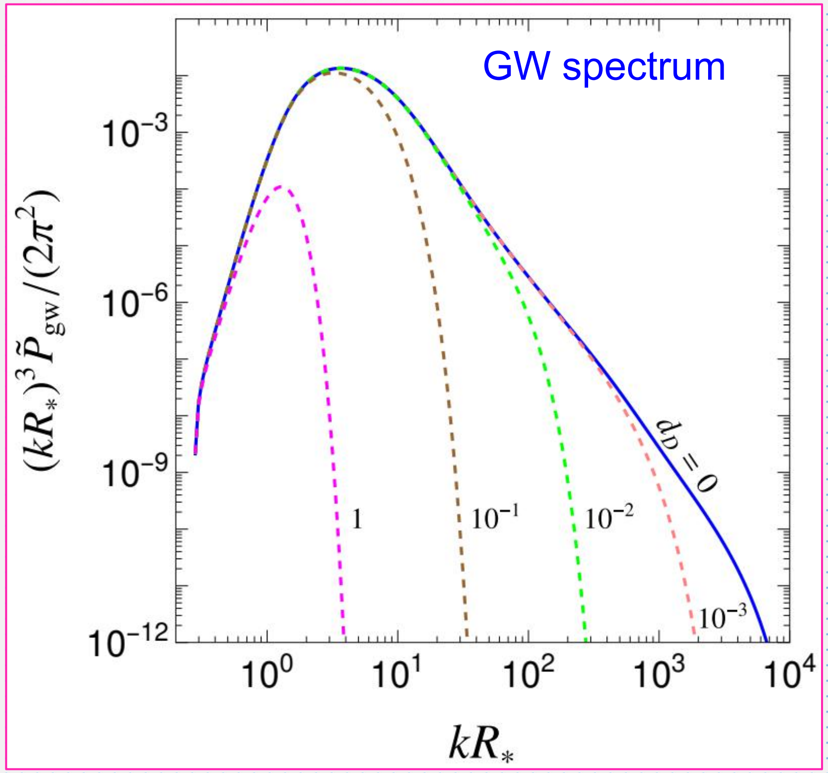
Dampings due to Dissipation

- Velocity power spectrum and stress tensor correlator are generally non-stationary

  (unequal time correlator depends not just on time difference)
- Damping at large frequencies (small scales)

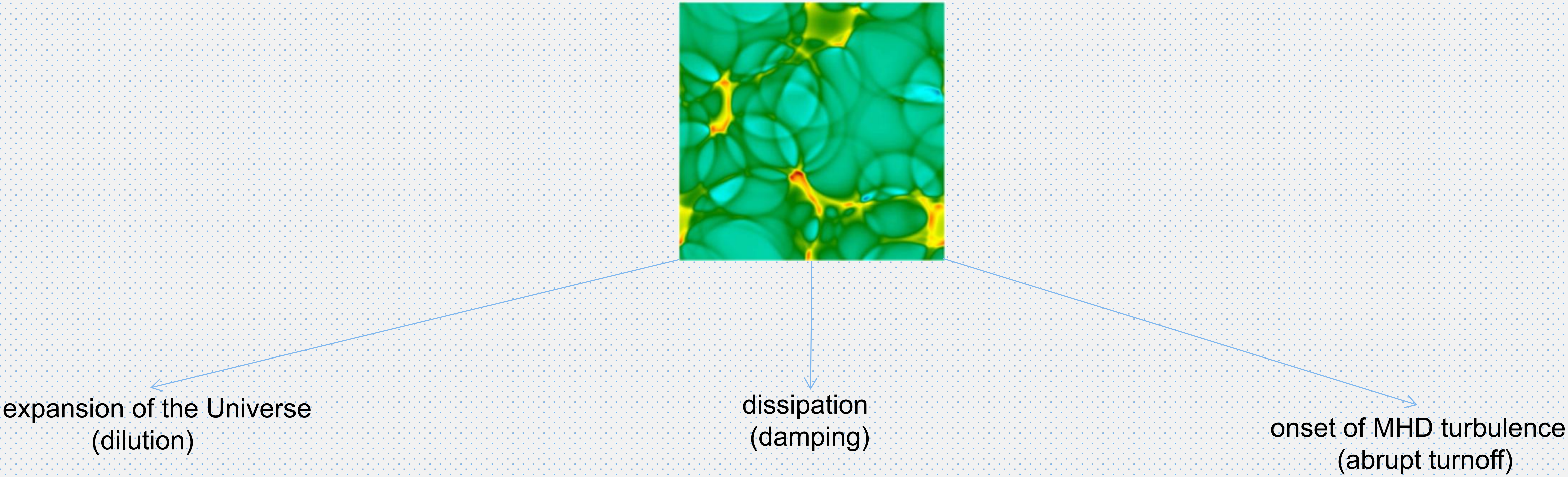






All plots assuming constant effective damping length for illustration (leads to stationary spectrum)

Lifetime of Sound Waves



expansion of the Universe
(dilution)

dissipation
(damping)

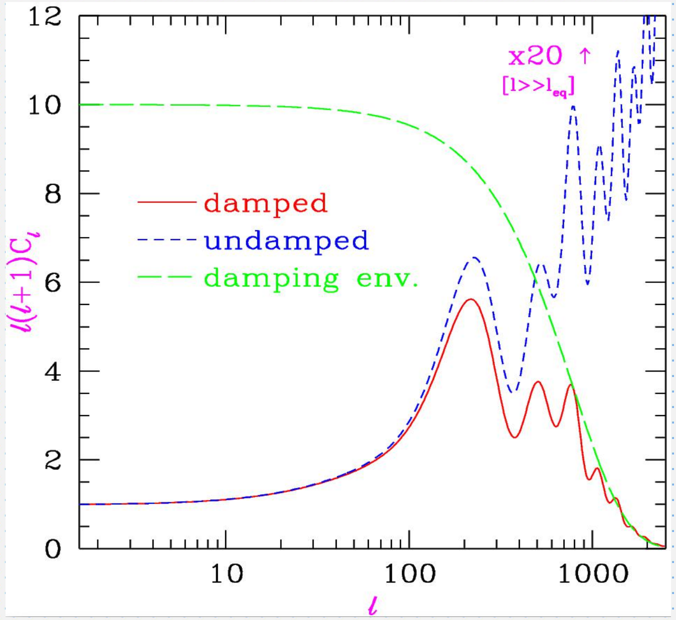
onset of MHD turbulence
(abrupt turnoff)

- Realistic cases: intertwining of these effects (makes GW spectrum model dependent)
- GW spectrum carries information about each model (break parameter degeneracy)
- Upsilon factor becomes frequency and model dependent

Microscopic Origin

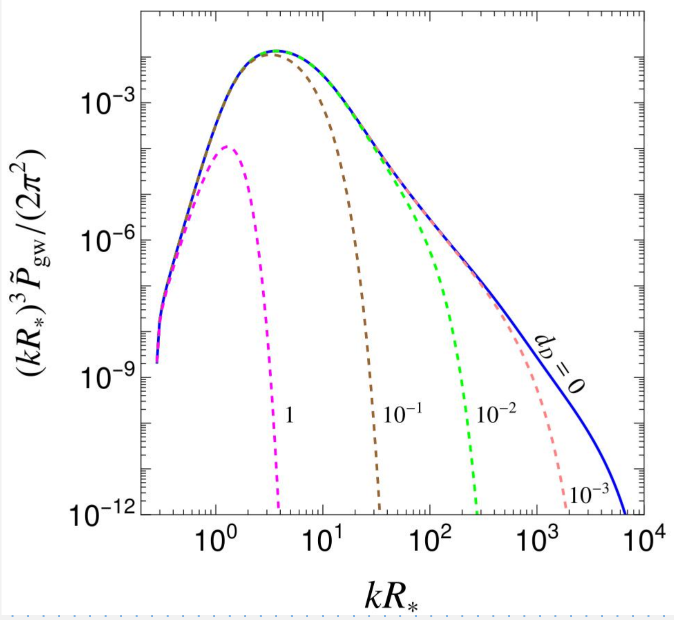
- Viscosity in the early universe is very small
- But can be significant for phase transitions in the dark sector
- Can also be stronger when BSM physics are included (from very weak interactions)
- Viscosity and transport coeffieicnts calculable from semi-classical kinetic theory or Green-Kubo relations

Analogy: Silk damping of CMB Anisotropy



Hu,White, ApJ [9609079]

damping of GW



HG [2310.10927]

Summary

- Dissipative effects can serve as new obervables for cosmic phase transitions
- New portals to probe microscopic particle (very weak) interactions
- Experimental searches of new spectrum are desired

Thanks!